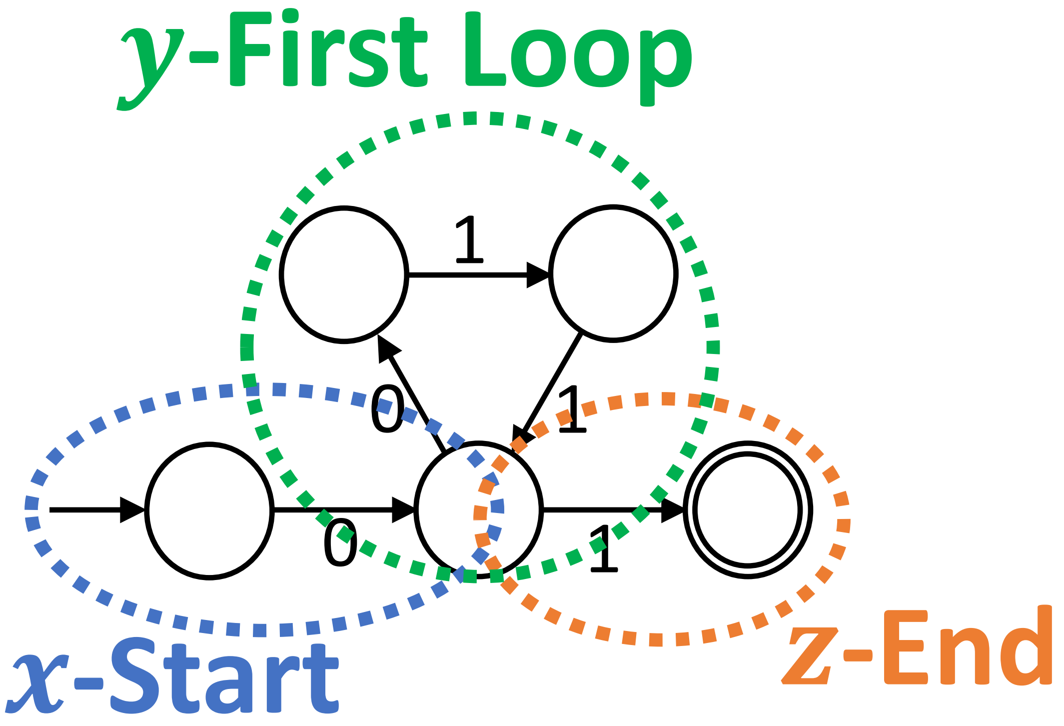
Pumping Lemma
CSCI 338

Pumping Lemma

<u>Pumping Lemma</u>: Given a regular language L, $\exists$ a number p such that <u>any</u> string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

1. $xy^iz \in L, \forall i \geq 0$.
2. $|y| > 0$.
3. $|xy| \leq p$.



Non-Regularity Proofs

Pumping Lemma: **Given a regular language L**, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

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2. Select p from pumping lemma.

Non-Regularity Proofs

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2. **$|y| > 0$.**
3. **$|xy| \leq p$.**

1. Suppose language is regular.
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3. Carefully select string $\in L$ and $|s| \geq p$.
4. Determine what y must consist of.

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5. Make new string by selecting i.

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3. Carefully select string $\in L$ and $|s| \geq p$.
4. Determine what y must consist of.
5. Make new string by selecting i.
6. Show new string is not in language.

Pumping Lemma Example 1

Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: ?

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Then, $\boldsymbol{L}$ must abide by the pumping lemma. I.e. There must be some number $\boldsymbol{p}$ such that any string $\boldsymbol{s \in L}$, with $\boldsymbol{|s| \geq p}$, …

Pumping Lemma: Given a regular language L, $\exists$ a number p **such that any string $\boldsymbol{s \in L}$, with $\boldsymbol{|s| \geq p}$,** can be divided into three pieces, $s = xyz$ satisfying:

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Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ **satisfying:**

1. $\boldsymbol{xy^i z \in L, \forall i \geq 0}$.
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Pumping Lemma Example 1

Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Then, L must abide by the pumping lemma. I.e. There must be some number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying the three conditions.

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Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Now, we need some appropriate string ($s \in L$ and $|s| \geq p$) that will break condition 1 when we allow multiple y's.

Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

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Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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To break condition 1, we need to learn more about $\boldsymbol{y}$

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Consider $s = 0^p 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$,
since ?

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$y = 0^k$ for some $k > 0$,
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$$s = \overbrace{000 \ldots 00}^{p}\overbrace{11 \ldots 111}^{p}$$

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There is no possible way to partition s into xyz where both:

1. $|xy| \leq p$
2. y **has 1s in it.**

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$y = 0^k$ for some $k > 0$,
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Since $|xy| \leq p$, y must be in the first p characters of every string. Since the first p characters of this string are all 0, y must contain all 0s.

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$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Rightarrow s = 0^{p-k} 0^k 1^p$

Consider the string $s' = xy^2 z = 0^{p-k} 0^{2k} 1^p$

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$\Rightarrow s' \notin L$.

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Consider the string $s' = xy^2z = 0^{p-k} 0^{2k} 1^p$

But, the number of zeros $= p - k + 2k = p + k >$ number of ones $= p$.

$\Rightarrow s' \notin L$.

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But, the number of zeros $= p - k + 2k = p + k >$ number of ones $= p$.

$\Rightarrow s' \notin L$. **But the pumping lemma said this should work for every string in L!**

Pumping Lemma Example 1

Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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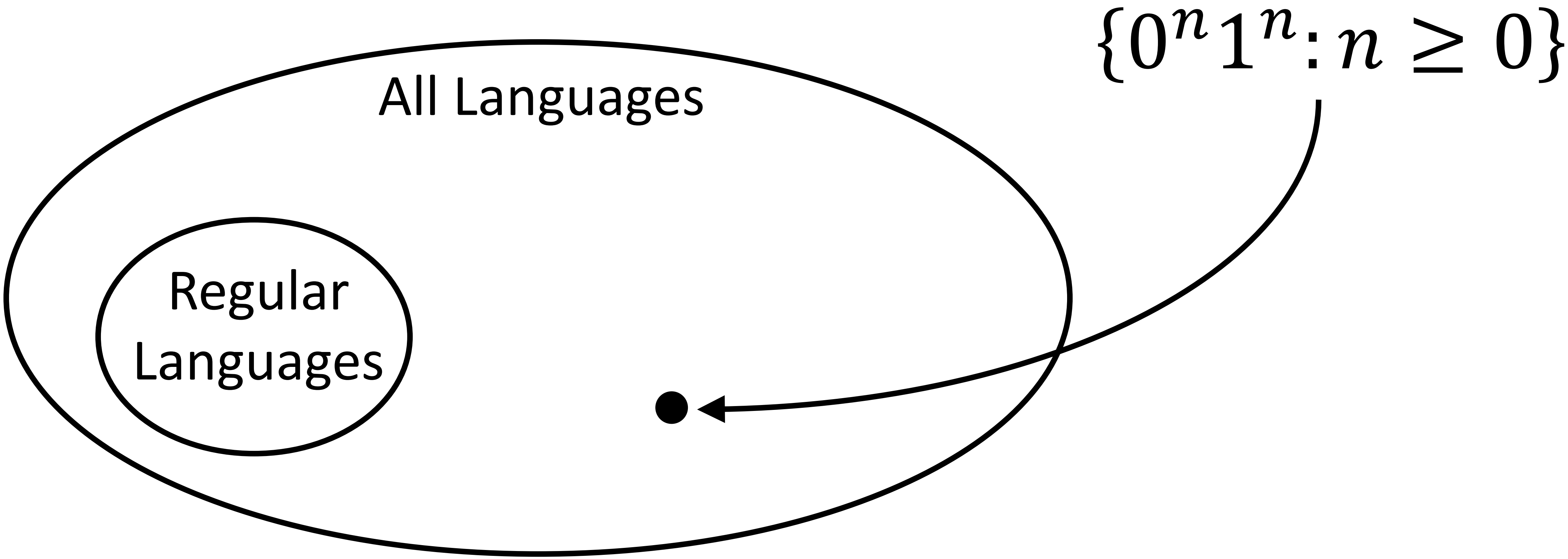
Therefore, the language is not regular.

DFA/NFA Limitations



Pumping Lemma Proof Blueprint

<u>Claim</u>: Some language L is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = \mathbf{?}$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

?

Consider the string $s' = xy^{?}z = \mathbf{?}$

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Proof Blueprint

Claim: Some language L is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$ **?**. **1 – Select s that will work with $s \in L$ and $|s| \geq p$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z =$ **?**

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<u>Claim</u>: Some language L is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$?. **1 – Select s that will work with $\boldsymbol{s \in L}$ and $\boldsymbol{|s| \geq p}$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that $\boldsymbol{y}$ must meet**

Consider the string $s' = xy^{?}z =$?

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

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Claim: Some language L is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$?. **1 – Select s that will work with $s \in L$ and $|s| \geq p$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that y must meet**

3 – Select an i (number of times to repeat y)

Consider the string $s' = xy^{?}z =$?

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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? **2 – Find some conditions that y must meet**

3 – Select an i (number of times to repeat y)

Consider the string $s' = xy^{?}z = ?$ **4 – Show what s' equals**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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Consider the string $s' = xy^{?}z =$? **4 – Show what s' equals**

? **5 – Show s' is not in L**

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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Pumping Lemma Example 2

Claim: The language $L = \{0^i 1^j : i > j\}$ is not regular.

Proof: ?

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<u>Claim</u>: The language $L = \{0^i 1^j : i > j\}$ is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$ **?**.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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Consider $s = \boxed{?}$.

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

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Claim: The language $L = \{0^i 1^j : i > j\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^{p+1}1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k+1}0^k1^p$

Consider the string $s' = xy^{?}z =$ **?**

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 2

Claim: The language $L = \{0^i 1^j : i > j\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^{p+1} 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
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Consider the string $s' = xy^2z = 0^{p-k+1}0^{2k}1^p = 0^{p+k+1}1^p$

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Consider the string $s' = xy^2z = 0^{p-k+1} 0^{2k} 1^p = 0^{p+k+1} 1^p \in L$

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Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

1. $xy^i z \in L, \forall i \geq 0$.
2. $|y| > 0$.
3. $|xy| \leq p$.

Pumping Lemma Example 2

Claim: The language $L = \{0^i 1^j : i > j\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^{p+1} 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k+1} 0^k 1^p$

Consider the string $s' = xy^0 z =$ **?**

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Consider the string $s' = xy^0 z = 0^{p-k+1} 1^p$

But, $k > 0 \Longrightarrow p - k + 1 \leq p$. I.e. the number of 0s is not larger than the number of 1s.

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 3

Claim: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

Proof:

E.g. 1101011 is a palindrome
1001 is a palindrome
10100 is not

Pumping Lemma Example 3

Claim: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$ **?**.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

?

Consider the string $s' = xy^{?}z =$ **?**

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Consider $s = 0^p 0^p$.

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Consider the string $s' = xy^2z = 0^{p-k} 0^{2k} 0^p$

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Consider the string $s' = xy^2z = 0^{p-k}0^{2k}0^p = 0^{2p+k}$

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Pumping Lemma Example 3

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Consider $s = 0^p 0^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
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Consider the string $s' = xy^2 z = 0^{p-k} 0^{2k} 0^p = 0^{2p+k}$ = $0^p 0^k 0^p$

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 3

<u>Claim</u>: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 0^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k} 0^k 0^p$

Consider the string $s' = xy^3z =$

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 3

Claim: The language $L = \{w: w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 0^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k} 0^k 0^p$

Consider the string $s' = xy^4z =$

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 3

<u>Claim</u>: The language $L = \{w\colon w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 0^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k}0^k0^p$

Consider the string $s' = xy^0z =$

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 3

Claim: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

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Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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$\Longrightarrow s = 0^{p-k} 0^k 0^p$

Consider the string $s' = xy^0z =$

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

No matter what we set i to,
s' will always be a palindrome.

Pumping Lemma Example 3

Claim: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

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Consider $s = 0^p 0^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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~~$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.~~

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No matter what we set i to,
s' will always be a palindrome.

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$y = 0^k$ for some $k > 0$, since $|xy| \leq p$

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Does this mean L is regular?

Consider the string $s' = xy^0z =$

?

~~$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.~~

~~Therefore, the language is not regular.~~

Pumping Lemma Example 3

<u>Claim</u>: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

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Consider $s = 0^p 0^p$.

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Consider the string $s' = xy^0z =$

?

~~$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.~~

~~Therefore, the language is not regular.~~

Does this mean L is regular?

NO! It could be, but perhaps we just chose a poor string s.

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Consider $s =$ **?**.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

?

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Claim: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1 0^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z =$ **?**

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Consider the string $s' = xy^2z = 0^{p-k} 0^{2k} 1 0^p$

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Pumping Lemma Example 3

Claim: The language $L = \{w : w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p10^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k}0^k10^p$

Consider the string $s' = xy^2z = 0^{p-k}0^{2k}10^p$

But, $k > 0 \Longrightarrow p + k \neq p$. I.e. More 0s before the 1 than after $\Rightarrow s'$ is not a palindrome

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.