

CSCI 338

Homework 5

Assigned 10/4/2022, due by start of class (3:05 pm) on 10/11/2022. Please submit this assignment to the appropriate dropbox on D2L. You must follow the collaboration policy detailed on the course website.

Problem 1 (5 points). Provide a high-level description of a TM that decides the following language: $L = \{\text{same number of 0's and 1's}\}$

Solution. $M =$ on input $\langle w \rangle$.

1. Scan tape and mark first unmarked 0. If none found, skip to step 4.
2. Scan tape and mark first unmarked 1. If none found, reject.
3. Loop to step 1.
4. Scan tape for unmarked 1's. If one is found, reject. Otherwise, accept.

□

Problem 2 (3 points). Read the following article on the Church-Turing Thesis:

<https://cacm.acm.org/magazines/2019/1/233526-the-church-turing-thesis/fulltext>

In several sentences, summarize the article. Also, provide an explanation of the Church-Turing thesis that would be understandable to someone without any computational/mathematical background.

Problem 3 (5 points). Let $DNA_{DFA} = \{\langle B, \omega \rangle : B \text{ is a DFA that does not accept } \omega\}$. Show that DNA_{DFA} is decidable.

Solution. Construct the following machine:

$M =$ on input $\langle B, \omega \rangle$.

1. Run B on ω .
2. If B accepts, reject. If B rejects, accept.

M halts since B is guaranteed to halt processing ω .

□

Problem 4 (5 points). Let $ALL_{DFA} = \{\langle A \rangle : A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

Solution. $L(A) = \Sigma^*$ if and only if $L(\bar{A}) = \emptyset$. Construct the following machine:
 $M =$ on input $\langle A \rangle$.

1. Construct DFA $B = \bar{A}$ using the procedure discussed in class.
2. Run M_3 from the E_{DFA} problem on $\langle B \rangle$.
3. If M_3 accepts, accept. If M_3 rejects, reject.

M halts since building \bar{A} is done deterministically and M_3 is a decider. □