Pumping Lemma
CSCI 338

Regular Language

A language is called a regular language if some DFA, NFA, or regular expression recognizes it.

How do you prove a language is regular?

Regular Language

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How do you prove a language is regular?

Make a DFA, NFA, or regular expression that recognizes it.

DFA/NFA Limitations

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What are some properties that require a language to be regular?

Quest for Regular Language Properties

Claim: All languages that are finite in size are regular.

Proof: ?

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Proof: Since there are a finite number of strings, build a DFA for each individual string in the language.

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DFAs/NFAs can process strings of arbitrary length, but not infinite length.

E.g. $L = \{\omega\colon \omega \text{ contains an even number of 0s}\}$

Quest for Regular Language Properties

Claim: All languages that are finite in size are regular.

Proof: Since there are a finite number of strings, build a DFA for each individual string in the language.

Connecting all start states to a new start state via ε-transitions gives an NFA that will recognize the (regular) language.

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Proof: Consider a language where each string has size $\leq n$.

Since the alphabet is finite, there is a finite number of strings constructible with n characters.

Thus, the language is finite and regular.

Non-Regular Languages

What do we know about non-regular languages?

Non-Regular Languages

What do we know about non-regular languages?

- They must be infinite in length
- They must have arbitrarily long strings in them.

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Suppose some regular language L contains strings of arbitrary length (i.e. $\forall n \geq 0, \exists s \in L$ such that $|s| \geq n$). Then:

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Let $p =$ number of states.
Let s be <u>any</u> string in L such that $|s| \geq p$.
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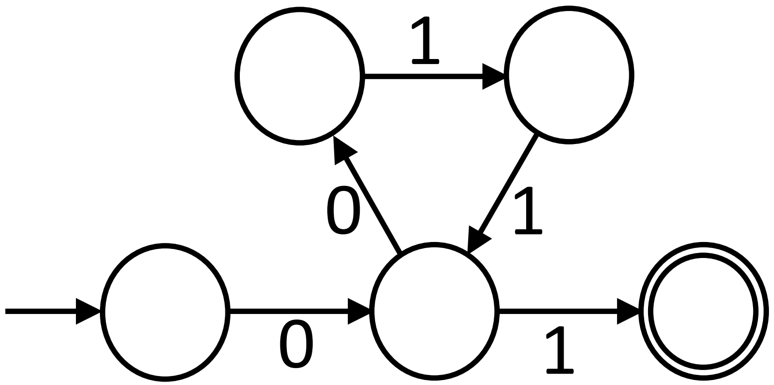

e.g. $s = 00111 \in L$

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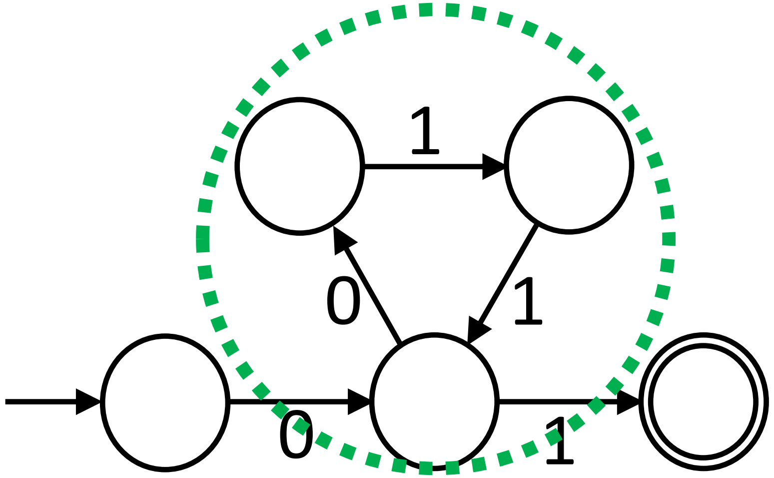

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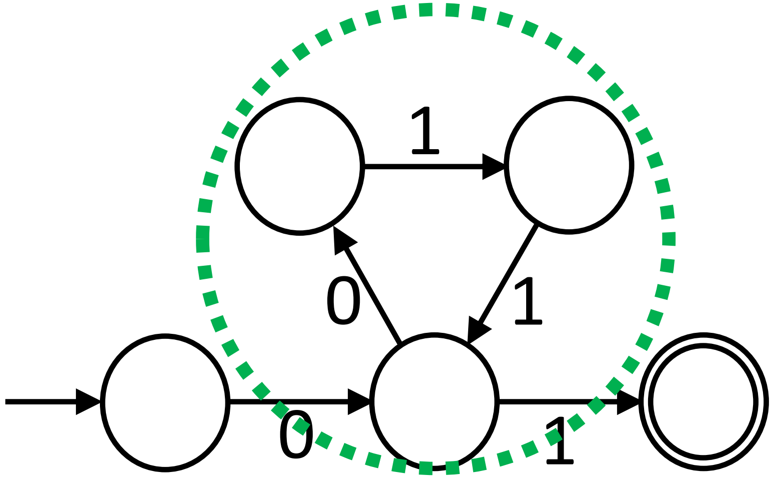

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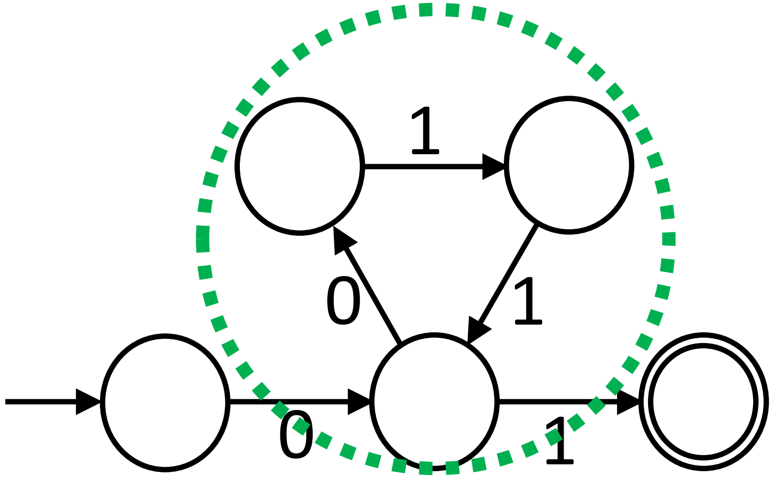

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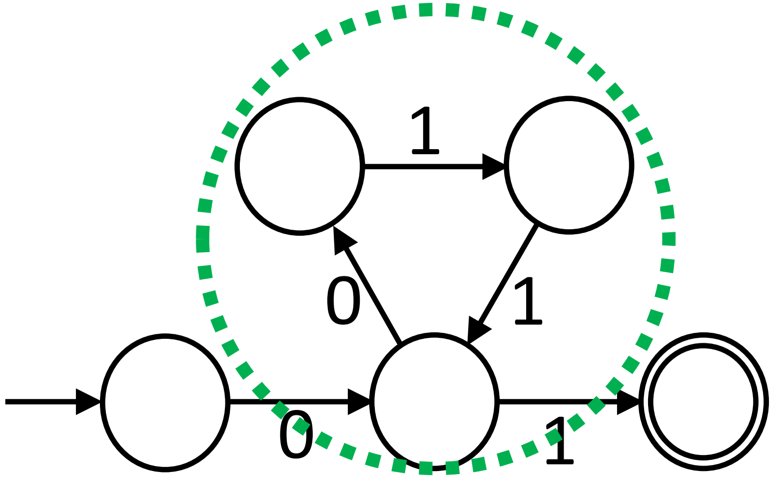

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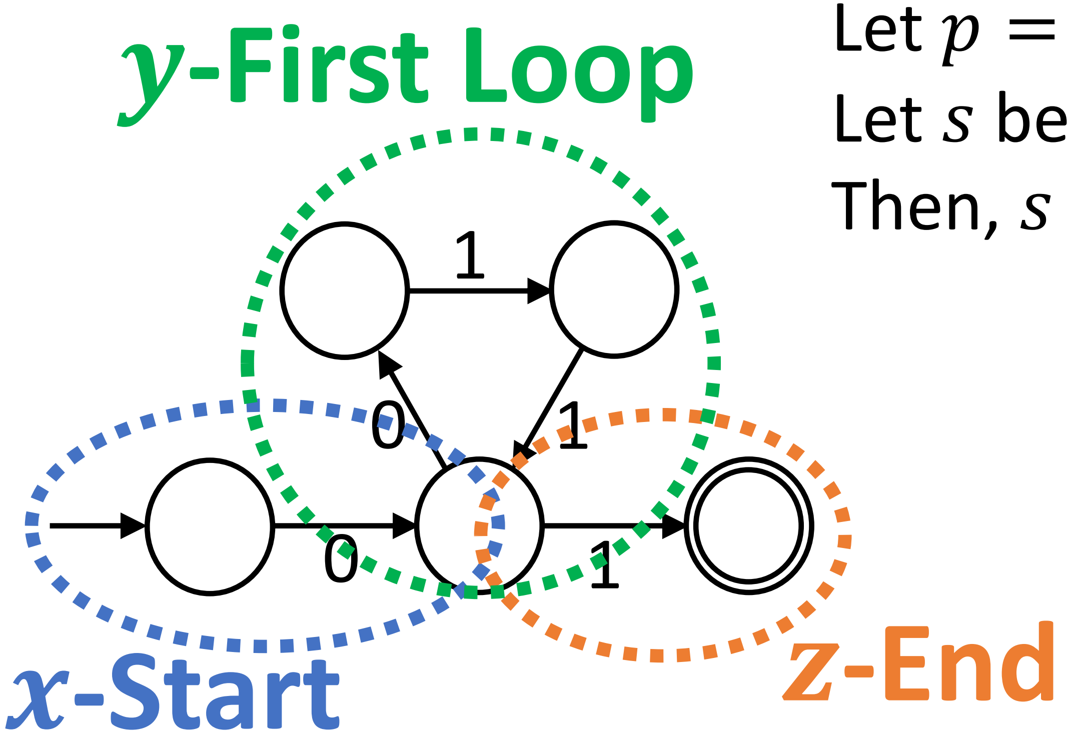

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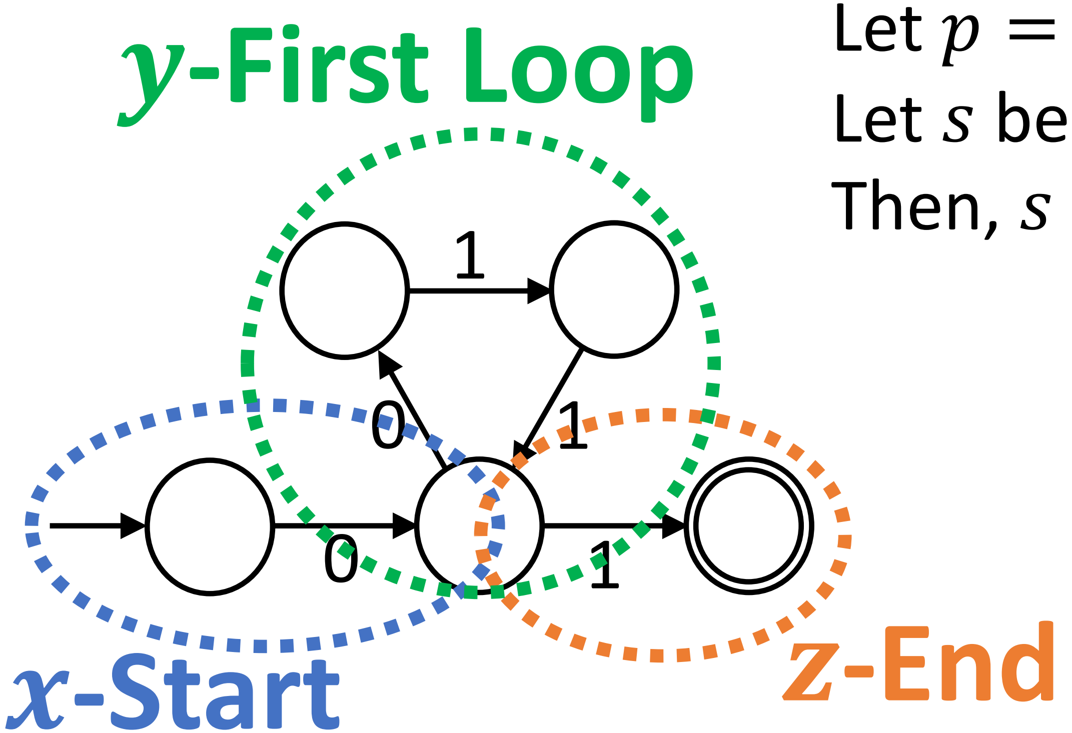

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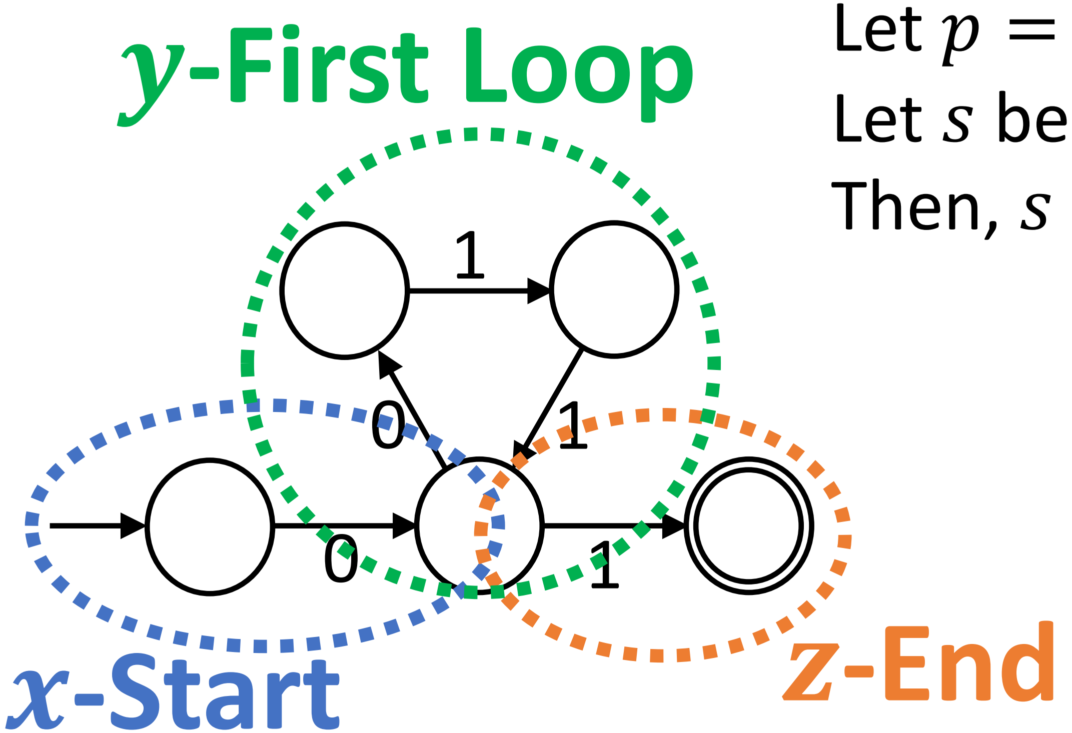

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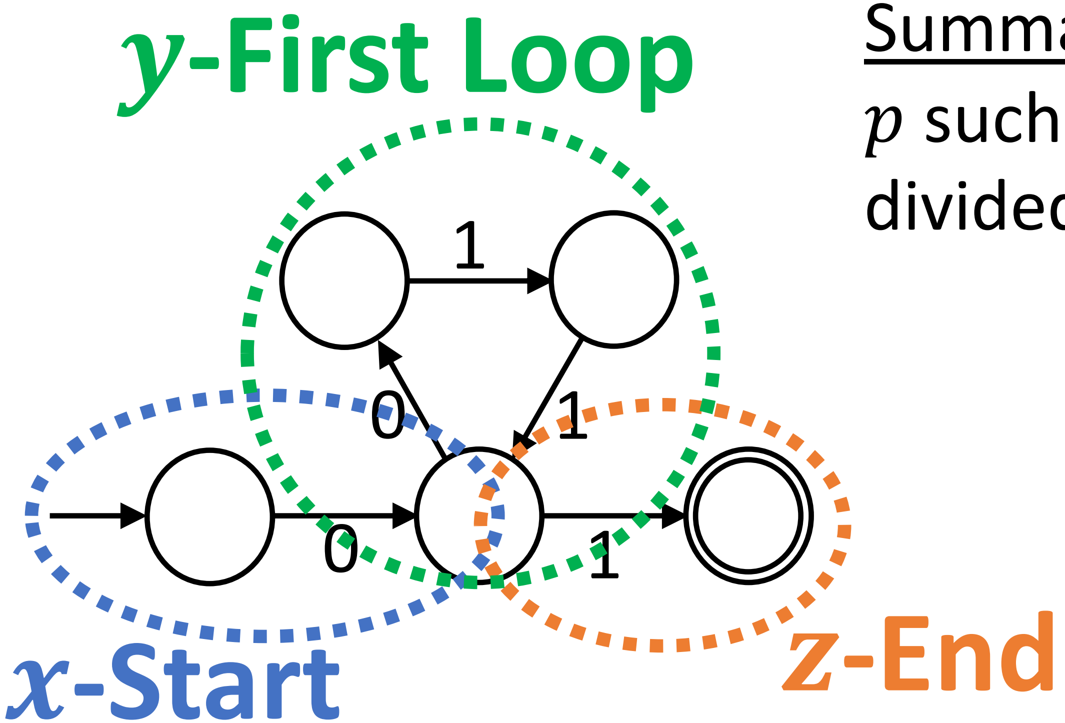

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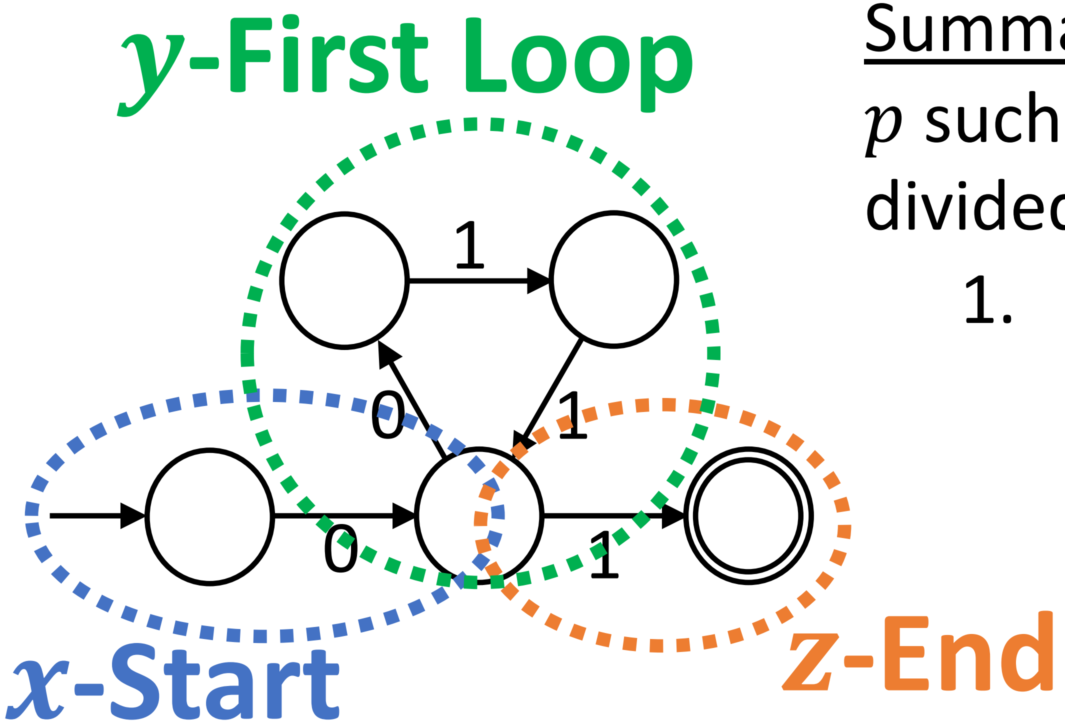

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From our previous argument.

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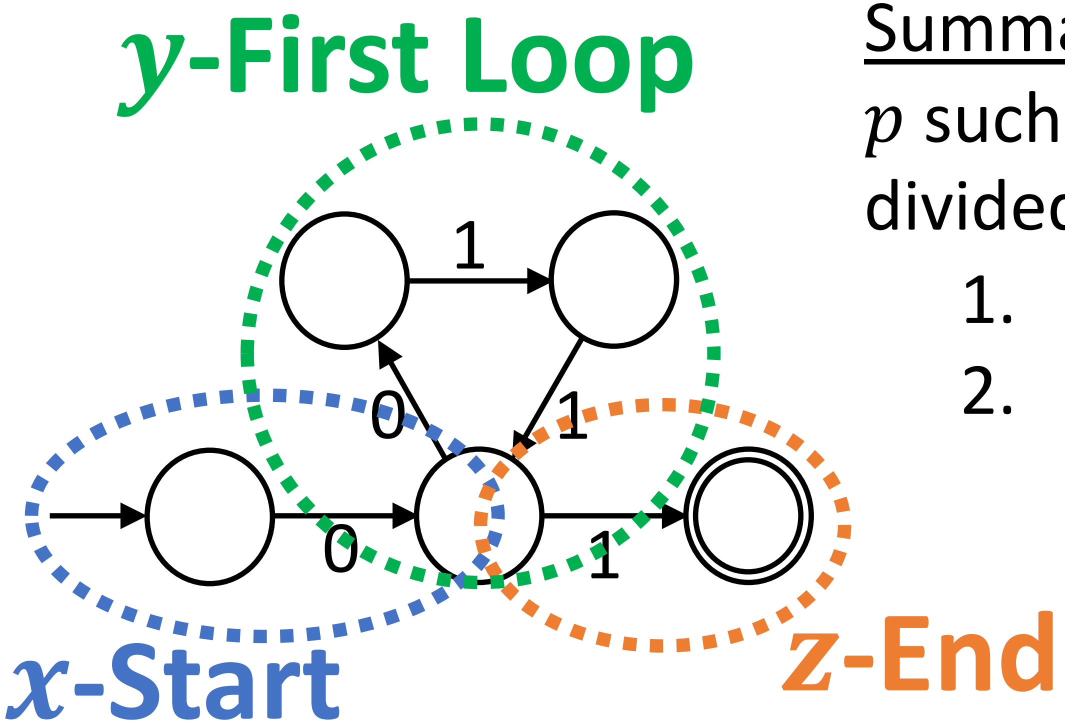

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2. $|y| > 0.$

Since $|s| \geq p$, we must have repeated states.

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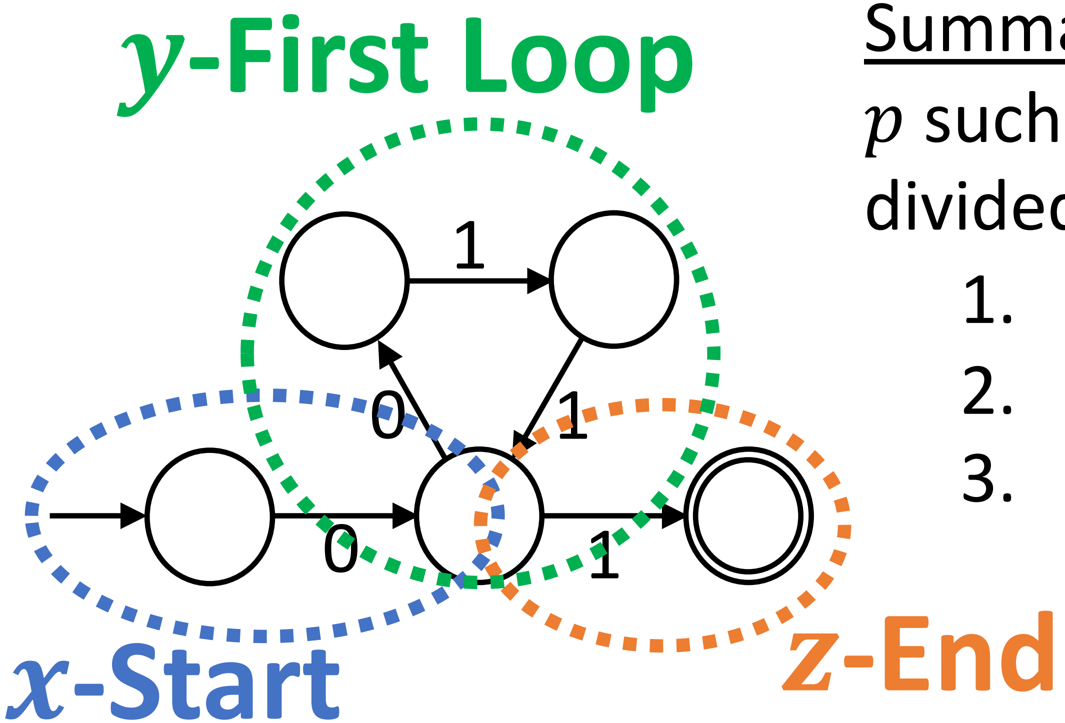

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Since there have to be repeated states within the first p transitions.

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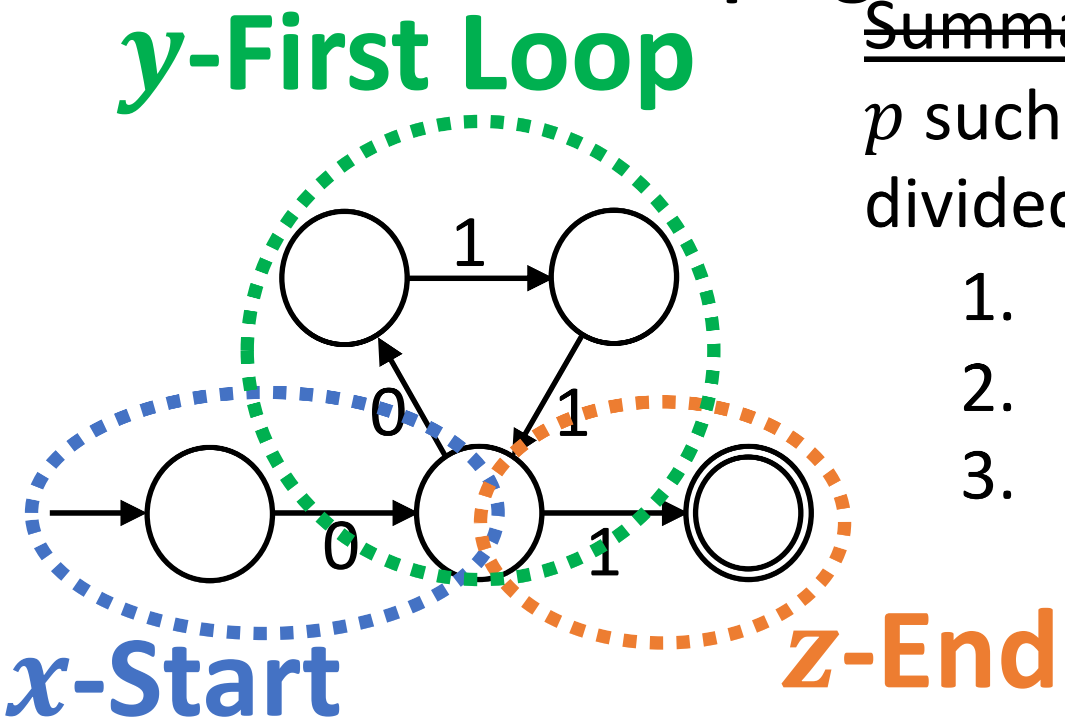
Pumping Lemma



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Quest for Regular Language Properties

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(i.e. $\forall n$ …

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The Pumping Lemma is our property that all regular languages must have.

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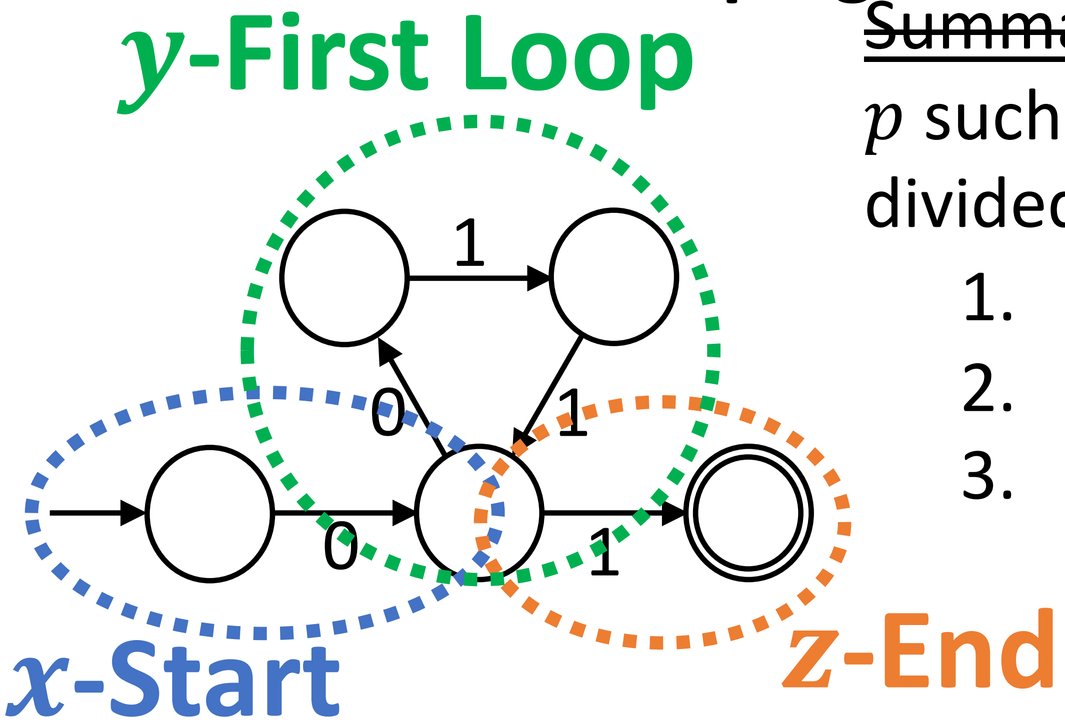

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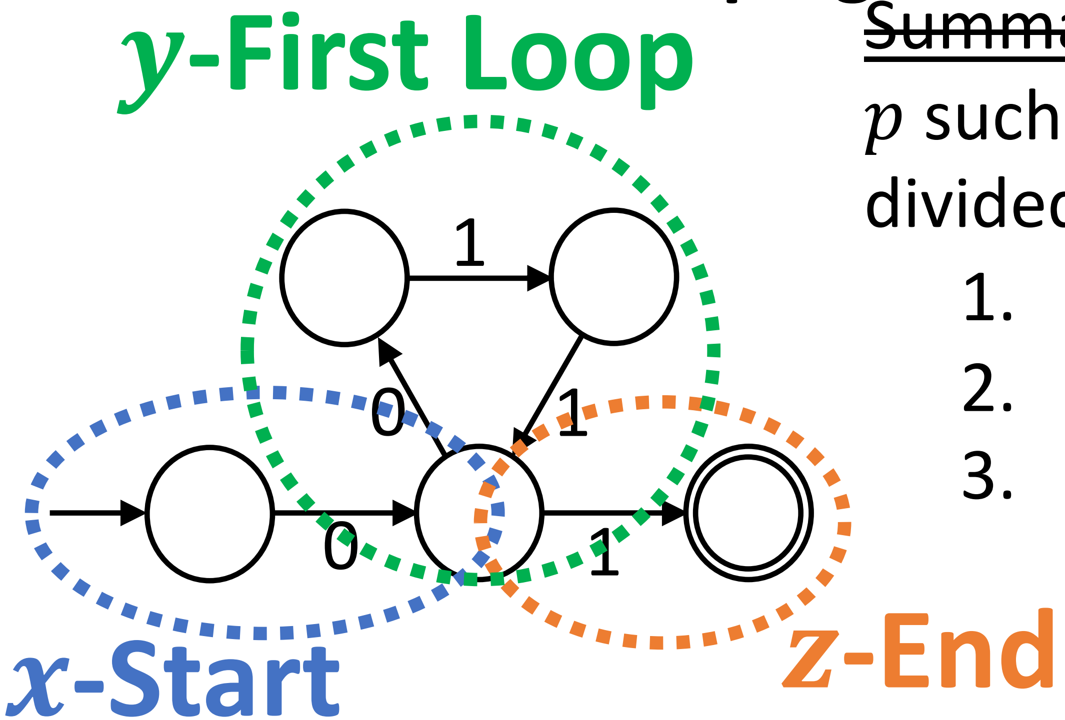

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Pumping Lemma User Manual:

1. The pumping lemma says all regular languages have property $\boldsymbol{P}$.
2. If we can show a language does not have property $\boldsymbol{P}$, then it cannot be regular.

Example with Regular Language

<u>Pumping Lemma</u>: Given a regular language L, $\exists$ a number p such that <u>any</u> string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

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$\{\omega : \omega$ contains an even number of 0's$\}$

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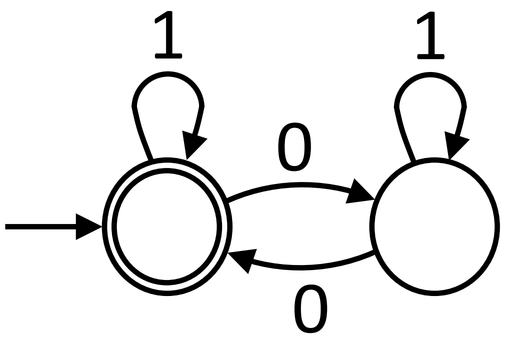

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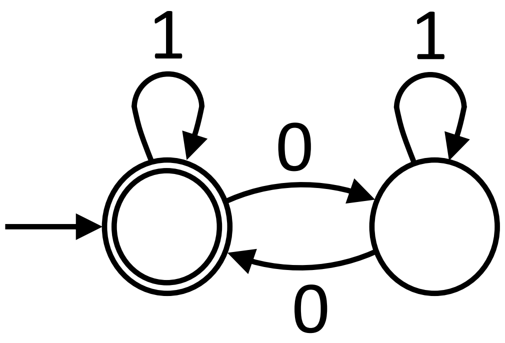

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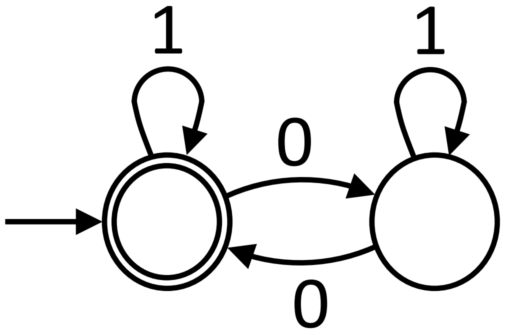

$$s = 01110$$

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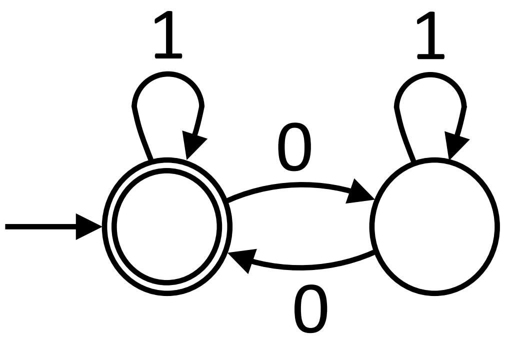

$$s = \underbrace{0}_{x}|\underbrace{1}_{y}|\underbrace{110}_{z}$$

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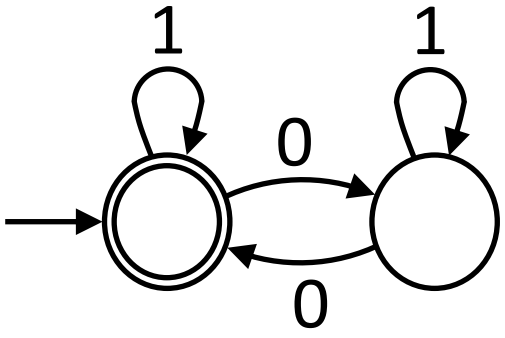

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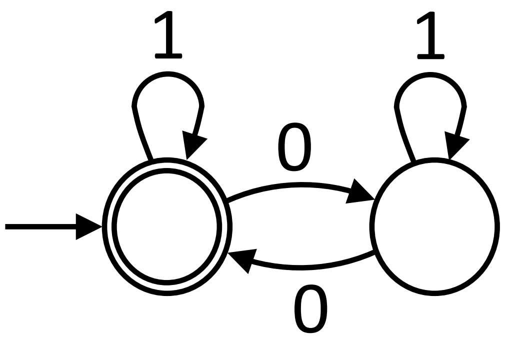

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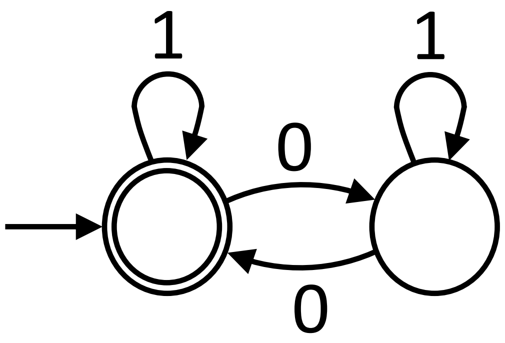

$$s = \overset{x}{0}|\overset{y}{1}|\overset{z}{110}$$

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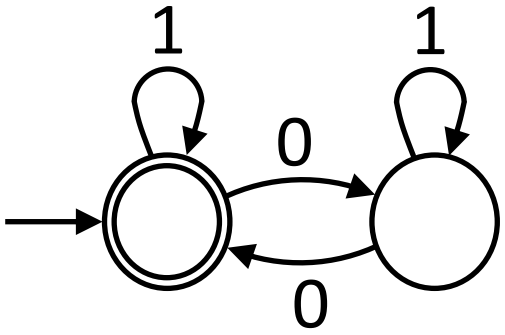

$$s = \overset{x}{0}|\overset{y}{1}|\overset{z}{110}$$

$$s' = \overset{x}{0}|\overset{y}{1}|\overset{y}{1}|\overset{z}{110}$$

$$s'' = \overset{x}{0}|\overset{y}{1}|\overset{y}{1}|\overset{y}{1}|\overset{y}{1}|\overset{z}{110}$$

Example with Regular Language

<u>Pumping Lemma</u>: Given a regular language L, $\exists$ a number p such that <u>any</u> string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ **satisfying:**

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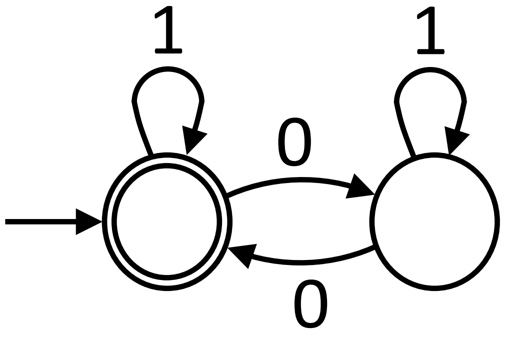

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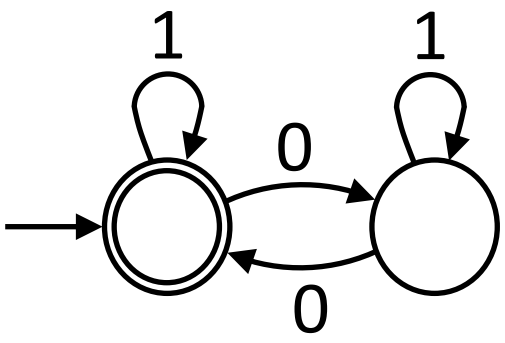

$$s = 0000$$

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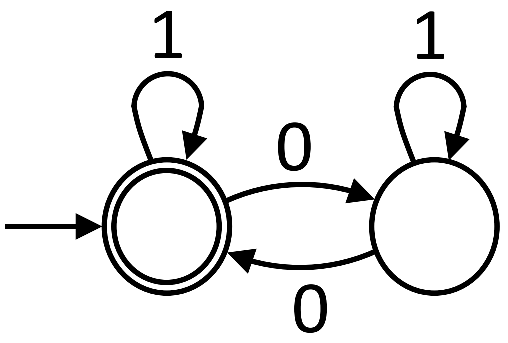

$$s = \overset{x}{0}\,|\,\overset{y}{0}\,|\,\overset{z}{00}$$

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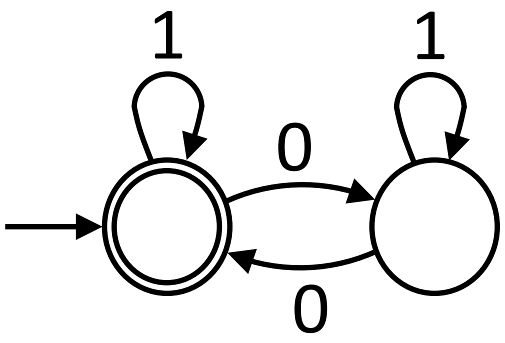

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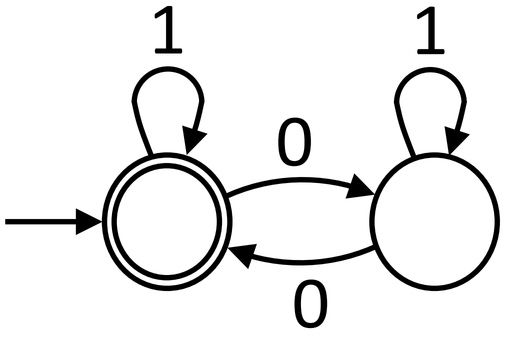

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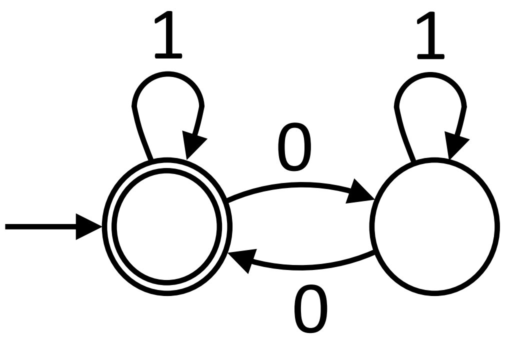

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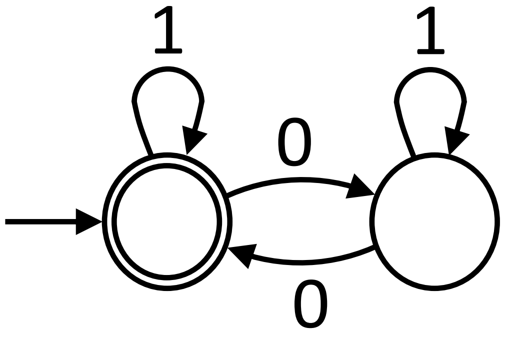

$$s = \overset{x}{0}|\overset{y}{0}|\overset{z}{0\,0}$$

$$s' = \overset{x}{0}|\overset{y}{0}|\overset{y}{0}|\overset{z}{0\,0} \notin L$$

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s <u>**can**</u> be divided into three pieces such that…

$\{\omega: \omega$ contains an even

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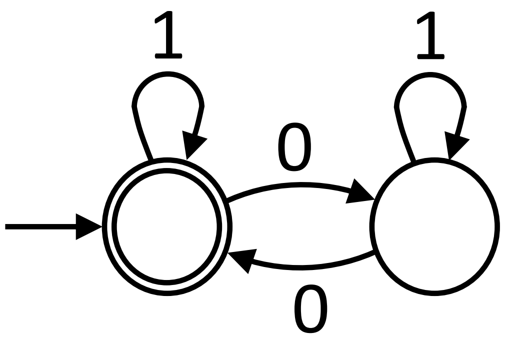

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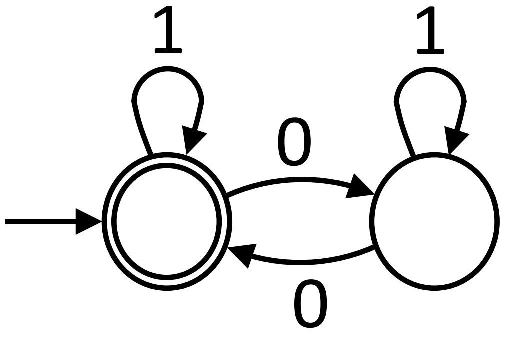

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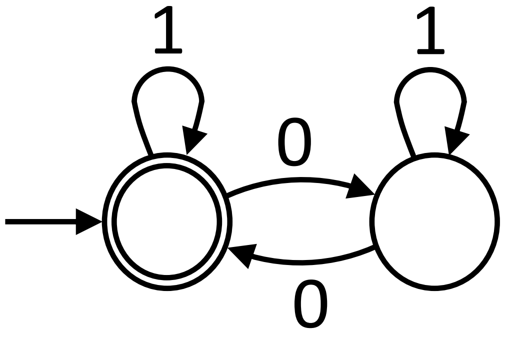

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$s = \overset{y}{00}\,|\,\overset{z}{00}$

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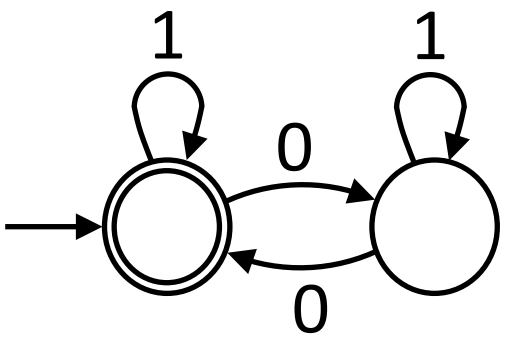

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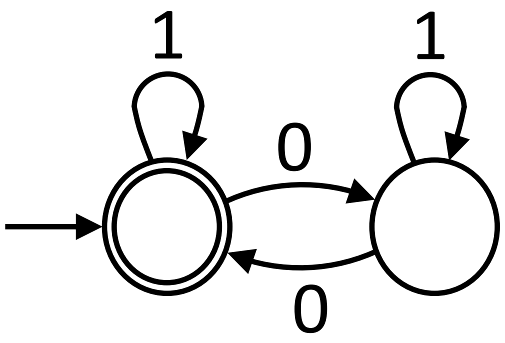

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$s = \overset{y}{00}|\overset{z}{00}$

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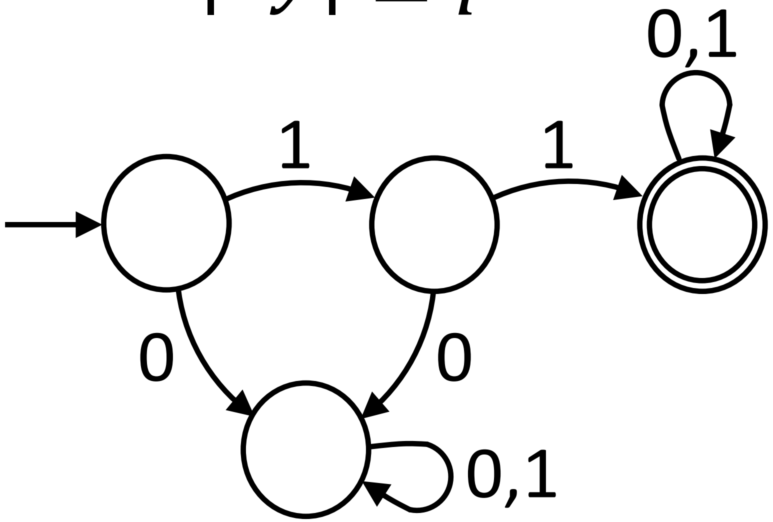
$\{\omega : \omega \text{ starts with } 11\}$



Example with Regular Language

Pumping Lemma: Given a regular language L, **∃ a number $\boldsymbol{p}$** such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

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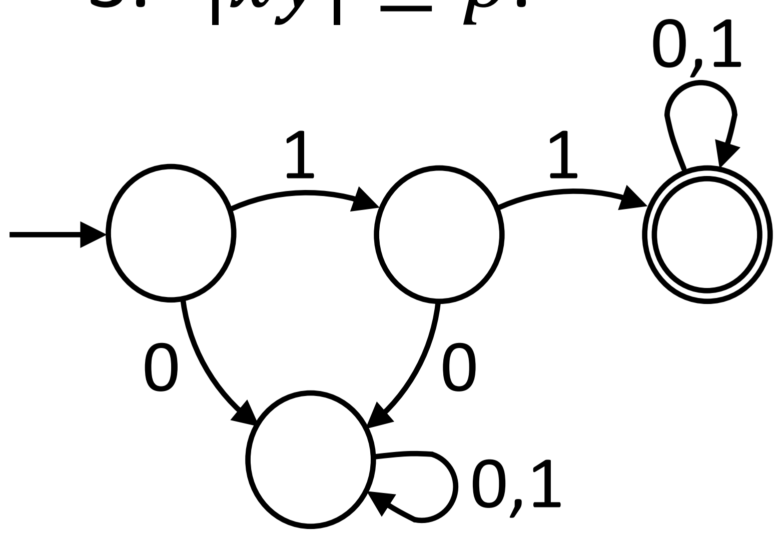
$p = 4$



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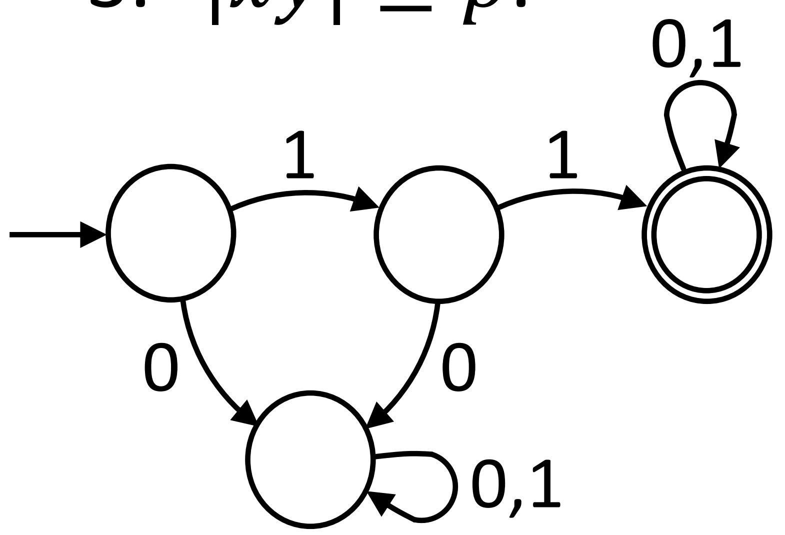

$s = 1110$

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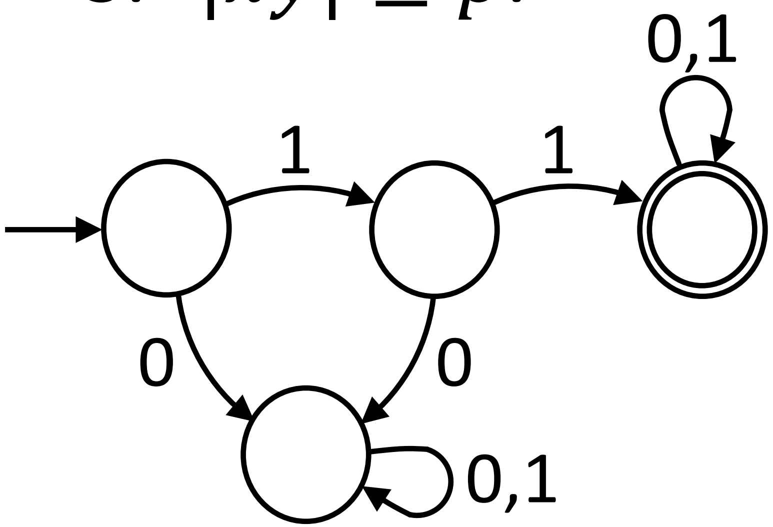

$\{\omega: \omega \text{ starts with } 11\}$

$p = 4$

$$s = 1\overset{y}{1}10$$

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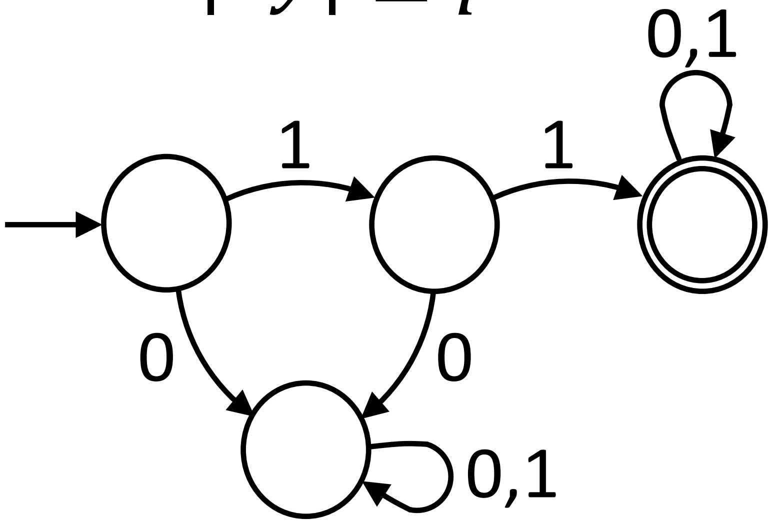

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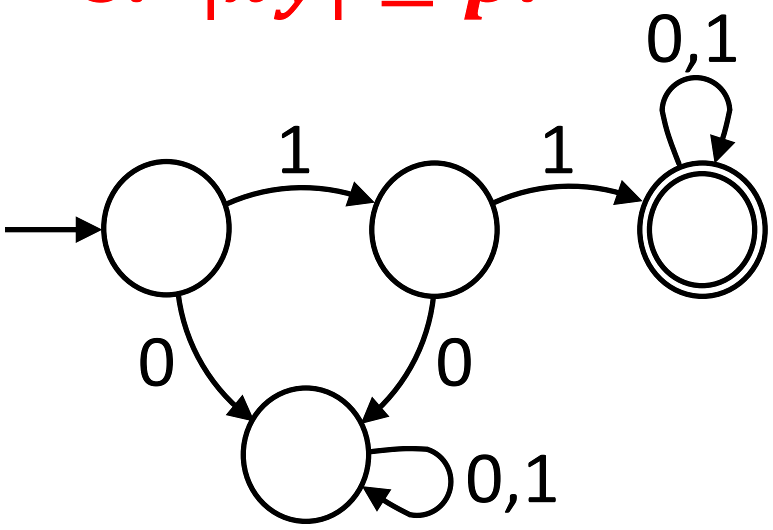

$\boldsymbol{y}$

$s = 1110$

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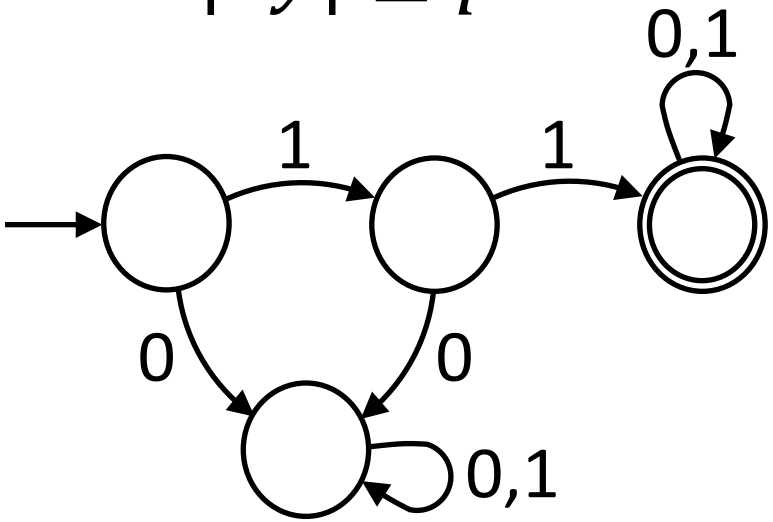

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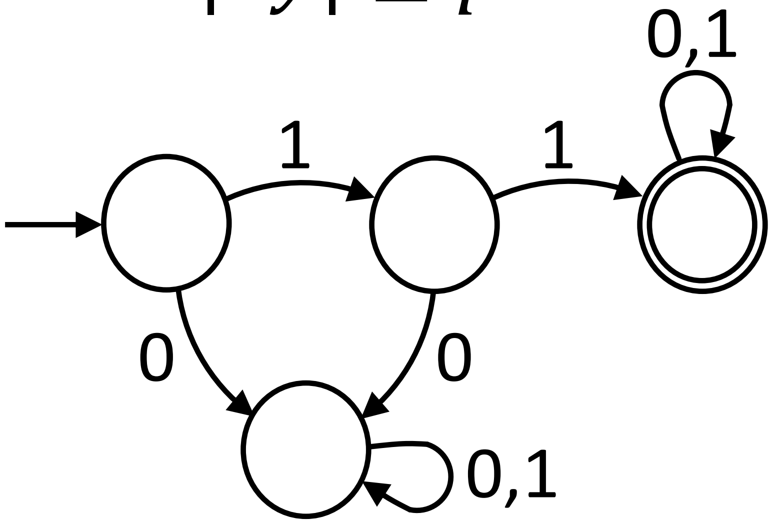

$\{\omega : \omega \text{ starts with } 11\}$

$p = 4$

$$s = 11\overset{y}{1}0$$

What if $i = 0$? ✘

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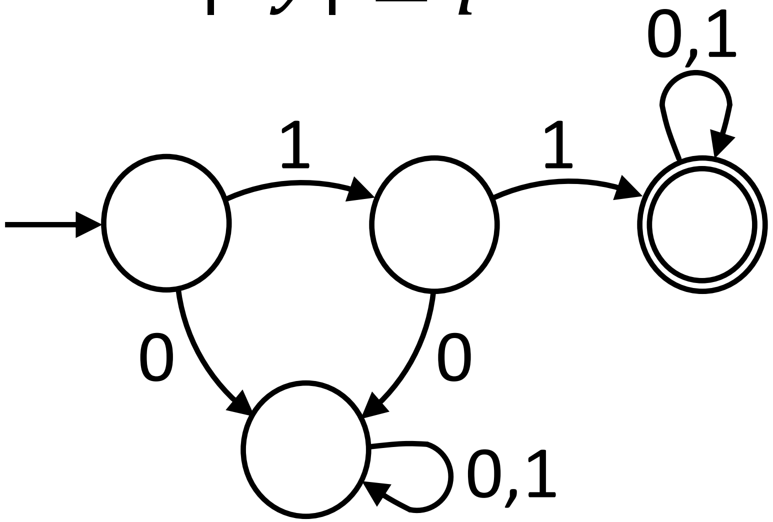

$$s = \overset{y}{1}110 \longleftarrow$$ What if $i = 0$? ✗

$$s = \overset{x}{11}|\overset{y}{10}$$

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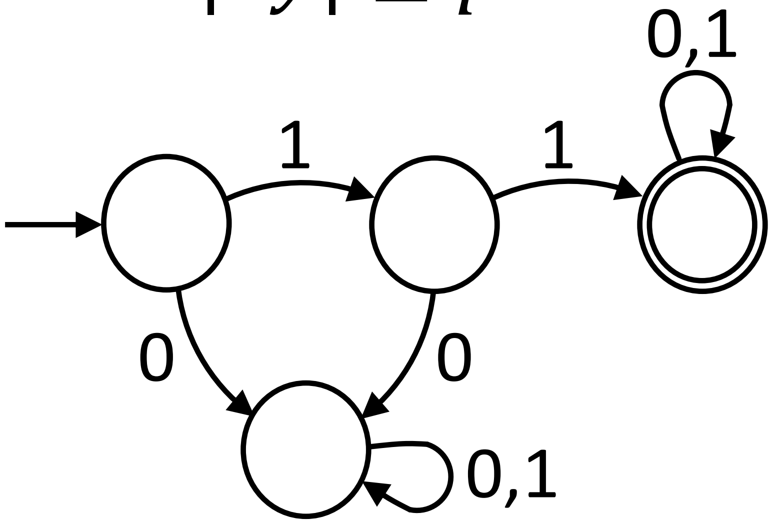

$$s = 11\overset{y}{1}0 \longleftarrow \text{What if } i = 0?$$

$$s = \overset{x}{11}|\overset{y}{10}$$

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$\{\omega : \omega \text{ starts with } 11\}$

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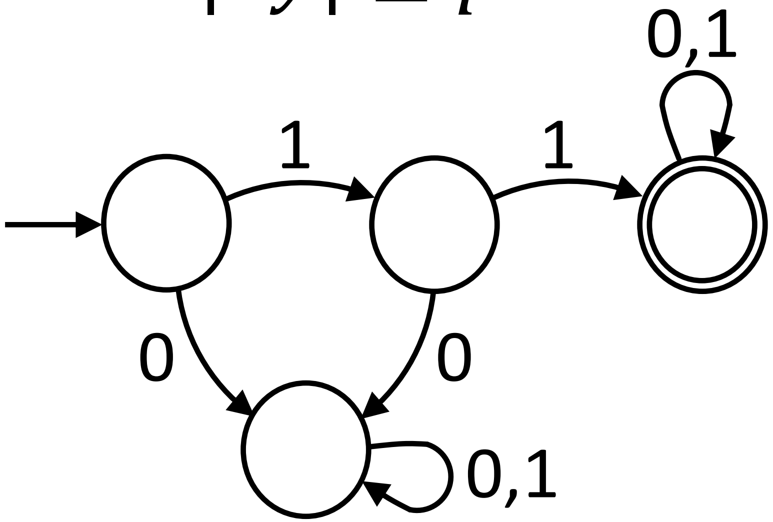

$s = 1\overset{y}{1}10$ ← What if $i = 0$? ✗

$s = \overset{x}{11}|\overset{y}{10}$ ✓

$s = \overset{x}{1}|\overset{y}{1}|\overset{z}{10}$

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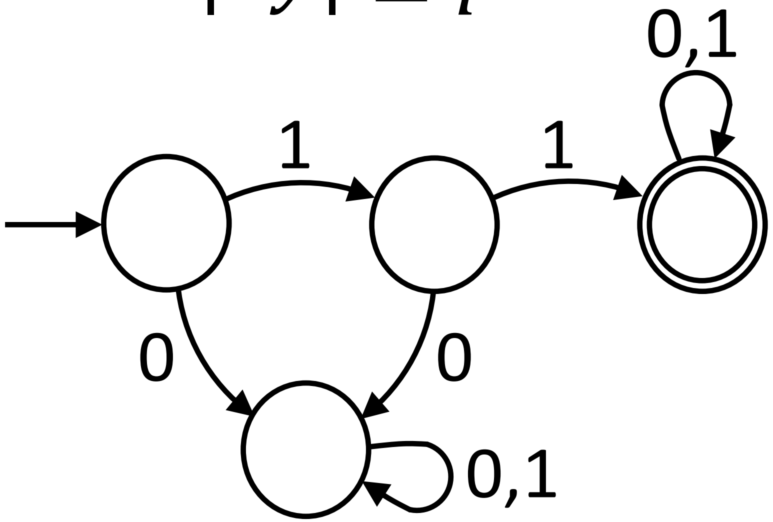

$$s = \overset{y}{1110}$$ ⟵ What if $i = 0$? ✗

$$s = \overset{x}{11}|\overset{y}{10}$$ ✓

$$s = \overset{x}{1}|\overset{y}{1}|\overset{z}{10}$$ ✓

Non-Regularity Proofs

Pumping Lemma: **Given a regular language L**, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

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1. Suppose language is regular.

Non-Regularity Proofs

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1. Suppose language is regular.
2. Select p from pumping lemma.

Non-Regularity Proofs

<u>Pumping Lemma</u>: Given a regular language L, $\exists$ a number p **such that <u>any</u> string $\boldsymbol{s} \in \boldsymbol{L}$, with $|\boldsymbol{s}| \geq \boldsymbol{p}$**, can be divided into three pieces, $s = xyz$ satisfying:

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2. $|y| > 0.$
3. $|xy| \leq p.$

1. Suppose language is regular.
2. Select p from pumping lemma.
3. Carefully select $s \in L$ and $|s| \geq p$.

Non-Regularity Proofs

Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, **can be divided into three pieces, $\boldsymbol{s = xyz}$ satisfying:**

1. $xy^iz \in L, \forall i \geq 0.$
2. **$|\boldsymbol{y}| > \mathbf{0}.$**
3. **$|\boldsymbol{xy}| \leq \boldsymbol{p}.$**

1. Suppose language is regular.
2. Select p from pumping lemma.
3. Carefully select $s \in L$ and $|s| \geq p$.
4. Determine what y must consist of.

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1. Suppose language is regular.
2. Select p from pumping lemma.
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4. Determine what y must consist of.
5. Make new string by selecting i.
6. Show new string is not in language.