CSCI 338
Homework 6

Assigned 10/11/2022, due by start of class (3:05 pm) on 10/18/2022. Please submit this assignment to the appropriate dropbox on D2L. You must follow the collaboration policy detailed on the course website.

Problem 1 (5 points). Let $111_{DFA} = \{\langle A \rangle : A \text{ is a DFA and } A \text{ accepts at least one string that has 111 as a substring}\}$. Show that 111_{DFA} is decidable.

Solution. Construct the following machine:
$M =$ on input $\langle A \rangle$.

1. Construct DFA B where $L(B) = \Sigma^* 111 \Sigma^*$.
2. Construct DFA $C = A \cap B$.
3. Run M_3 from the E_{DFA} problem on $\langle C \rangle$.
4. If M_3 accepts, reject. If M_3 rejects, accept.

M halts since building B and C is done deterministically and M_3 is a decider. □

Problem 2 (5 points). Let $ALL_{DFA} = \{\langle A \rangle : A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable by using the decider for EQ_{DFA}.

Solution. Construct the following machine:
$M =$ on input $\langle A \rangle$.

1. Construct DFA B where $L(B) = \Sigma^*$ (i.e., a DFA that accepts everything).
2. Run decider from the EQ_{DFA} problem on $\langle A, B \rangle$.
3. If the decider accepts, accept. If it rejects, reject.

M halts since B is finite and the decider from EQ_{DFA} halts. □

Problem 3 (5 points). Let $SUBSET_{DFA} = \{\langle A, B \rangle : A \text{ and } B \text{ are DFAs and } L(A) \subseteq L(B)\}$. Show that $SUBSET_{DFA}$ is decidable.

Solution. $L(A) \subseteq L(B)$ if and only if $L(A \cap \bar{B}) = \emptyset$. Construct the following machine:
$M =$ on input $\langle A, B \rangle$.

1. Construct DFA $C = A \cap \bar{B}$ using the procedures discussed in class.
2. Run M_3 from the E_{DFA} problem on $\langle C \rangle$.
3. If M_3 accepts, accept. If M_3 rejects, reject.

M halts since building C is done deterministically and M_3 is a decider. □

Problem 4 (5 points). A prunable state in a DFA is some state that is never entered while processing any input string. Show that the following language is decidable:
$SIMP_{DFA} = \{\langle A \rangle : A \text{ is a DFA with no prunable states}\}$

Solution. Construct the following machine:
$M =$ on input $\langle A \rangle$.

1. For each state q in the DFA,
2. Make q the only accept state to get DFA B.
3. Run M_3 from the E_{DFA} problem on $\langle B \rangle$.
4. If M_3 accepts, reject (since we found a prunable state). Otherwise, continue.
5. If completed for all states without rejecting, accept (as each state has been shown to be reachable).

M halts since M_3 is a decider and there are a finite number of states in each DFA. □