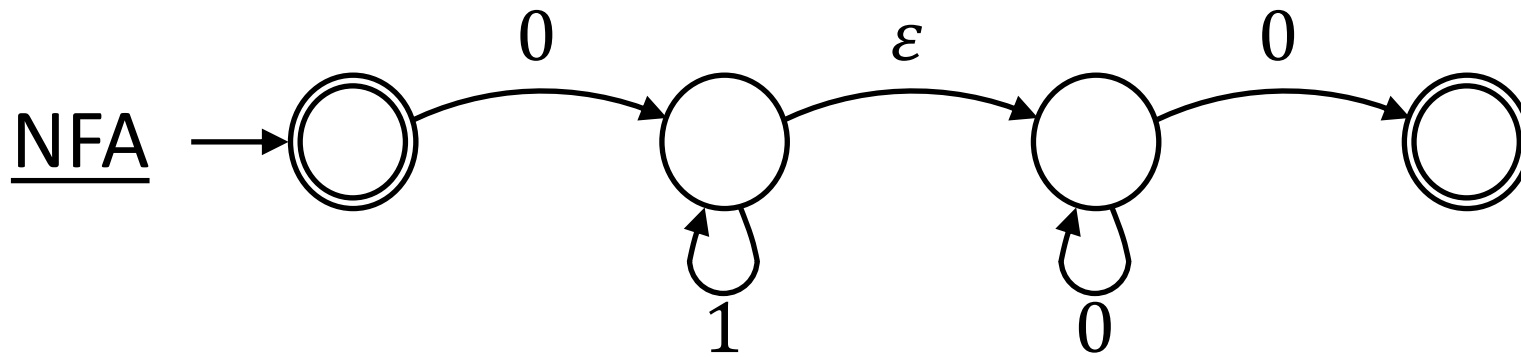


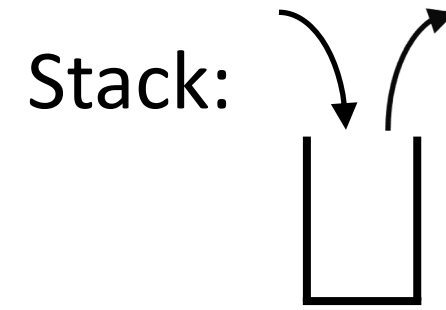
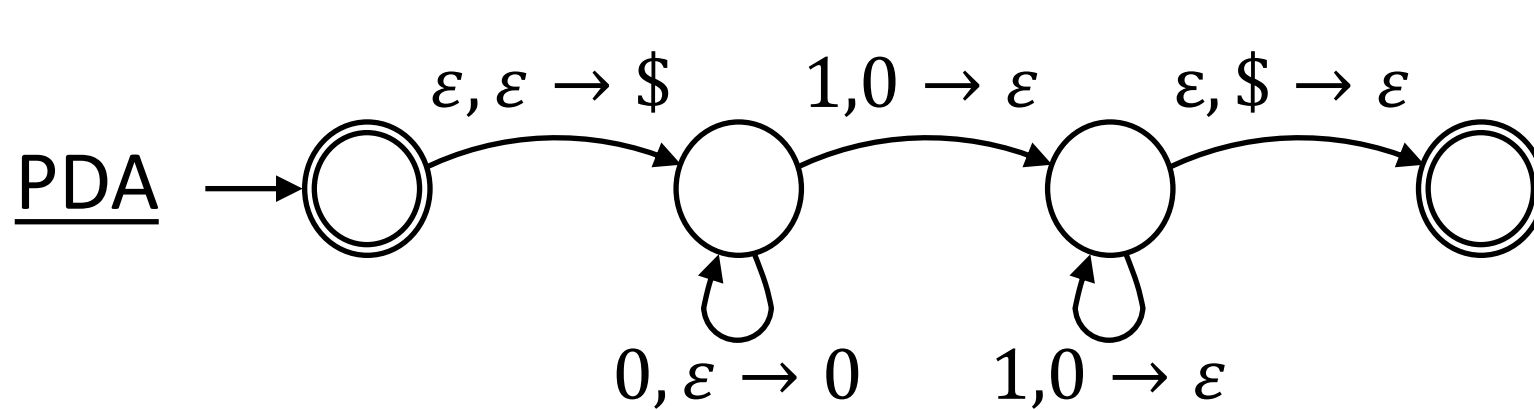
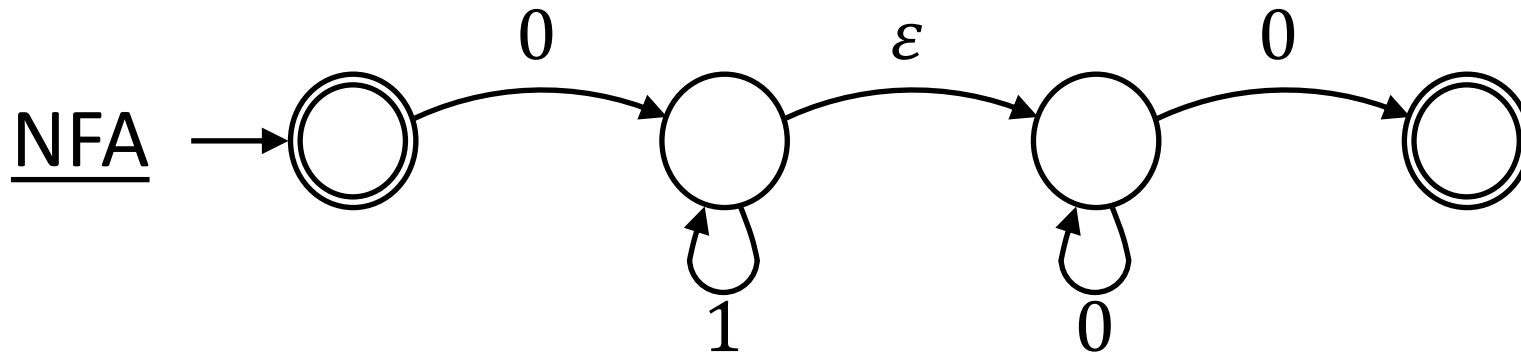
Turing Machines

CSCI 338

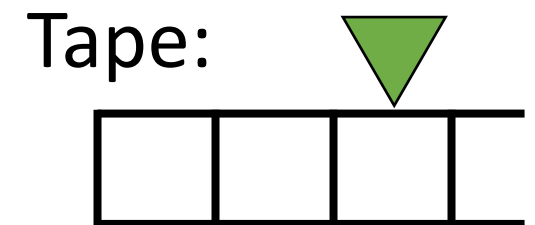
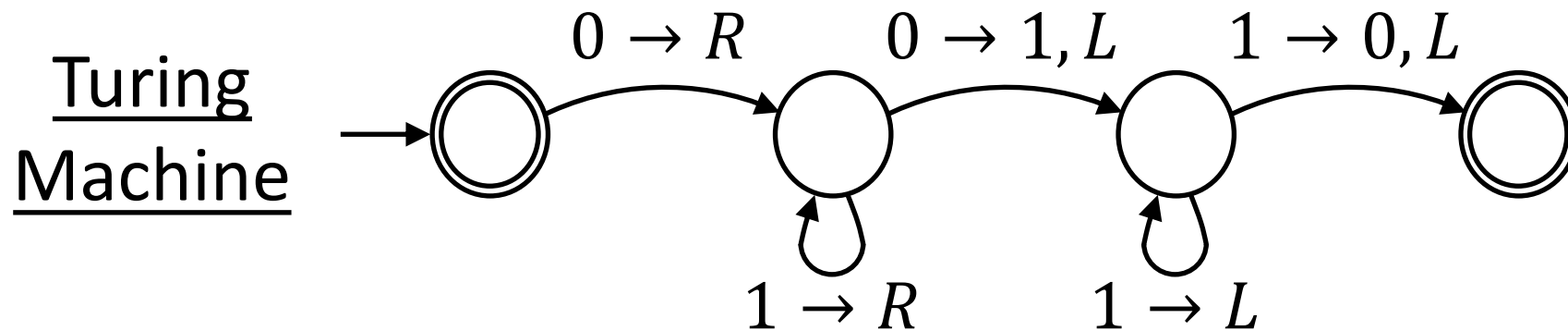
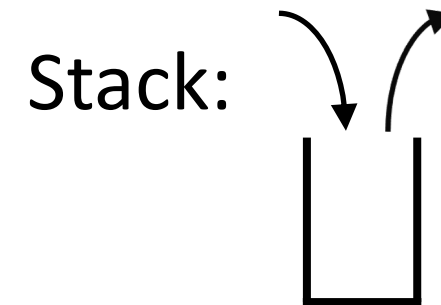
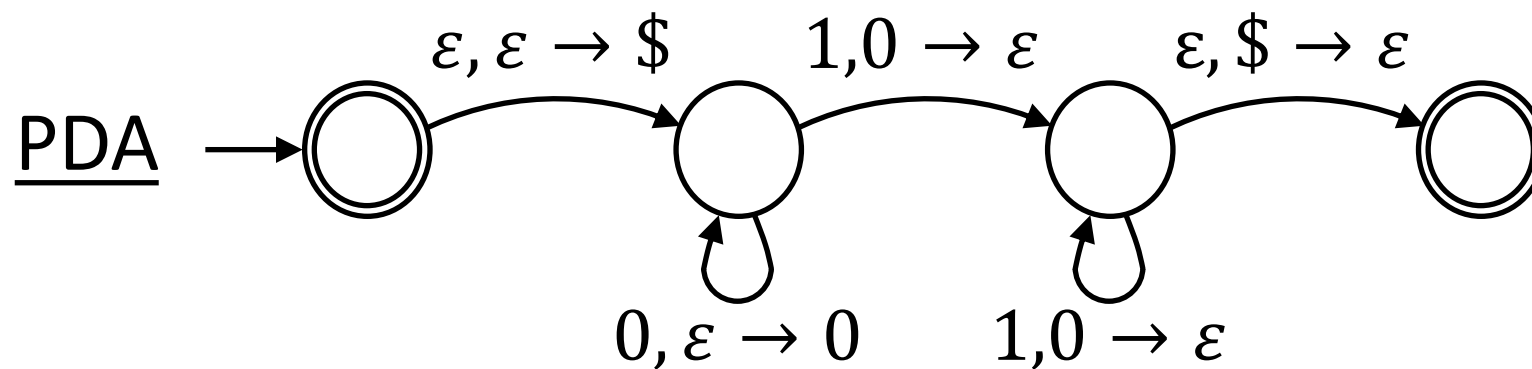
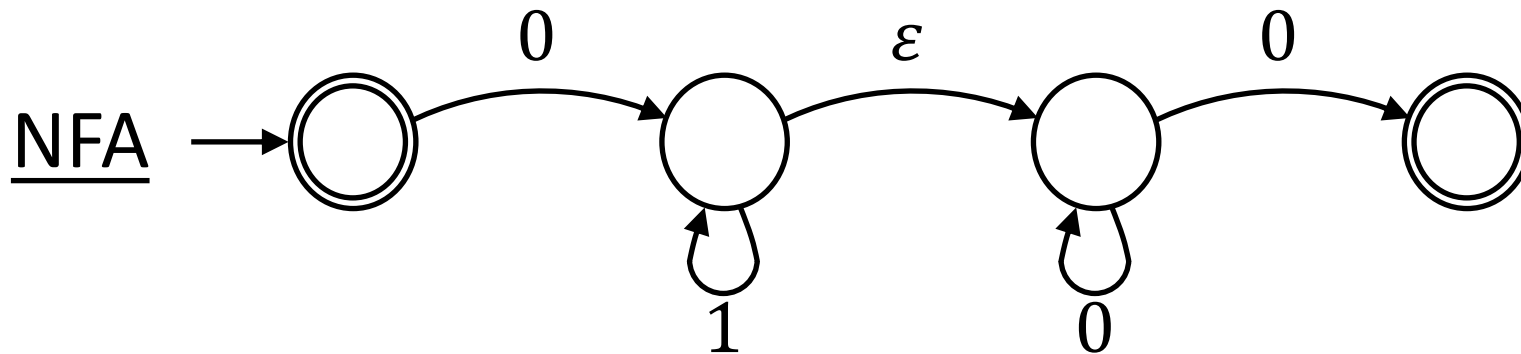
Computational Models



Computational Models

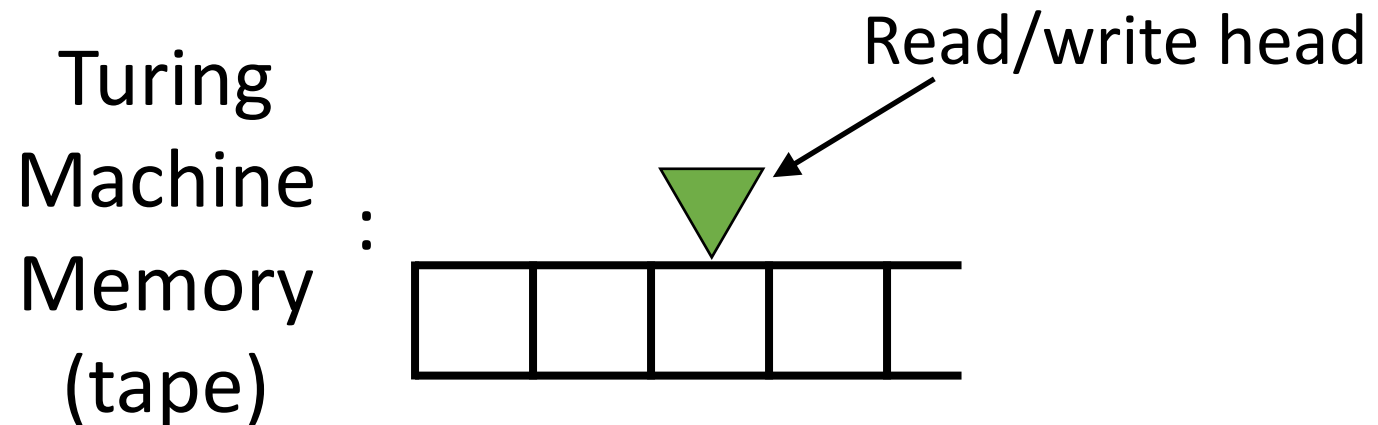
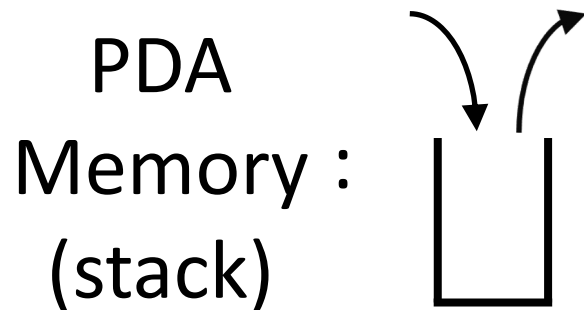


Computational Models



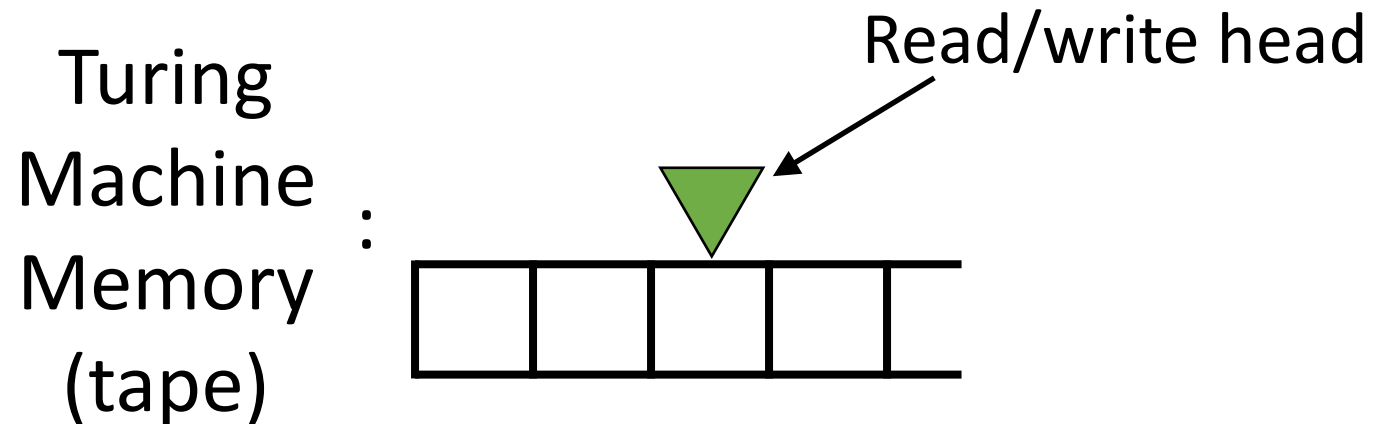
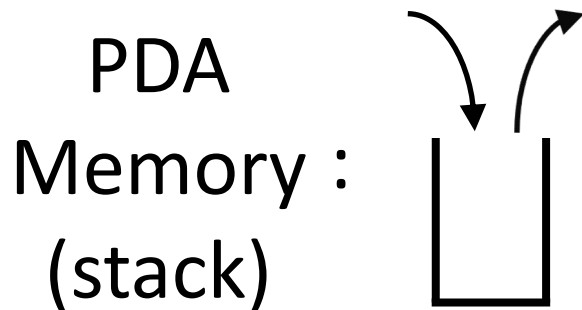
Turing Machine

- Finite automaton with unrestricted memory (tape).



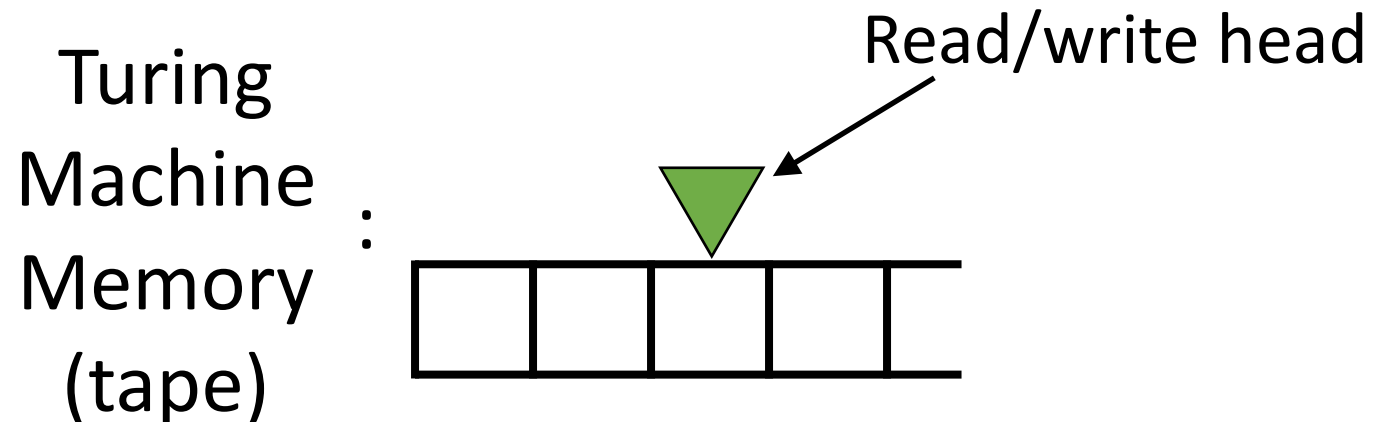
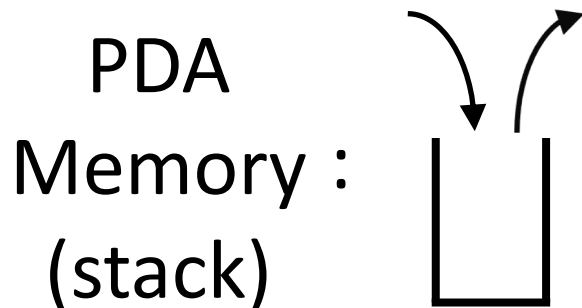
Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.



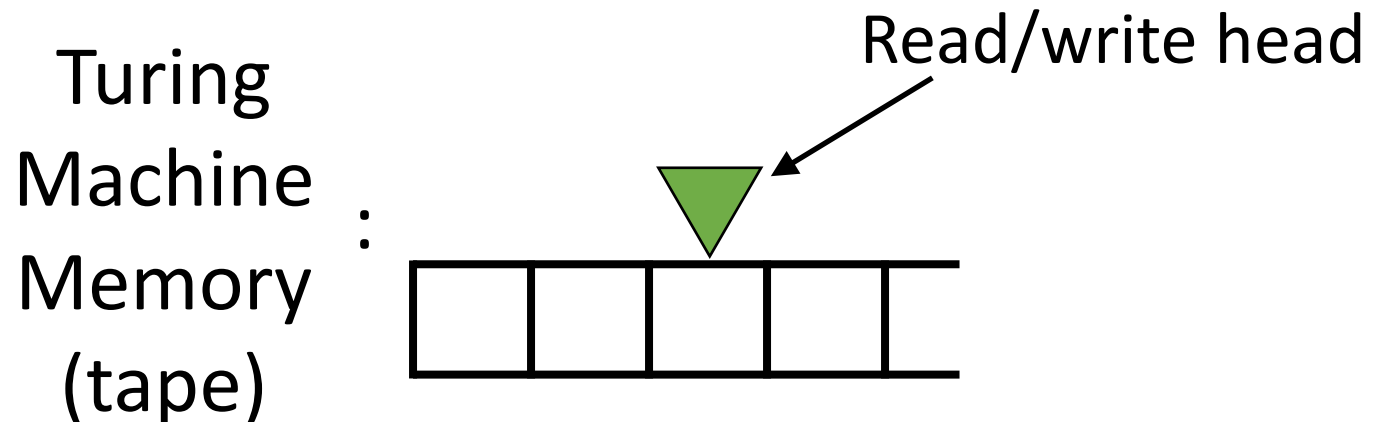
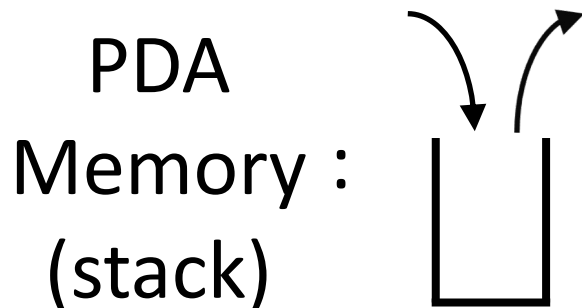
Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.
- Can access any spot in memory.



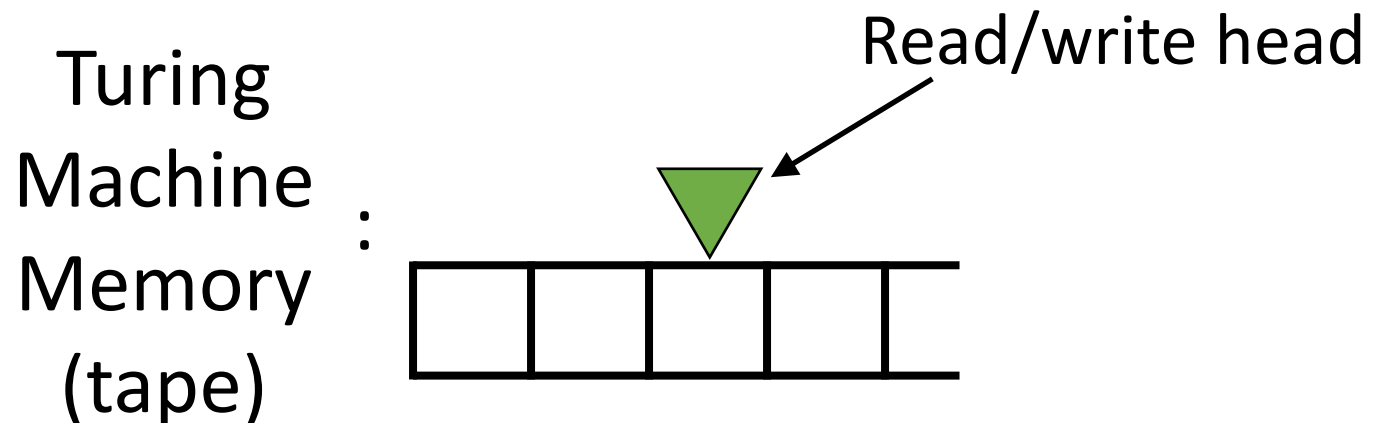
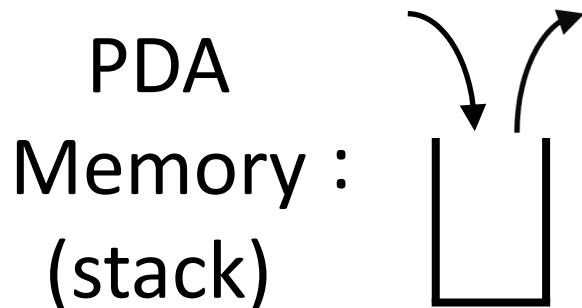
Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.
- Can access any spot in memory.
- Infinite memory.



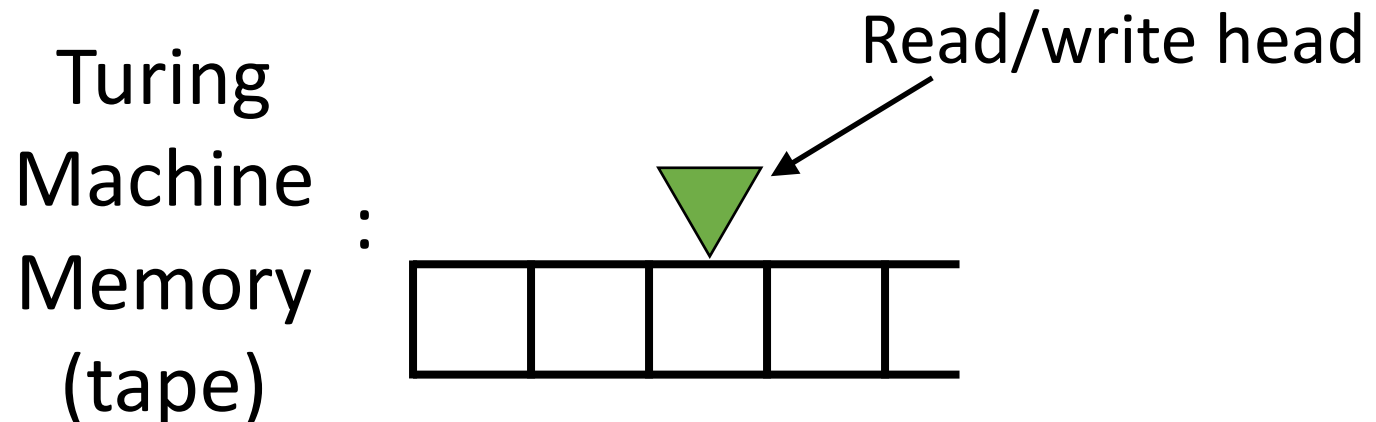
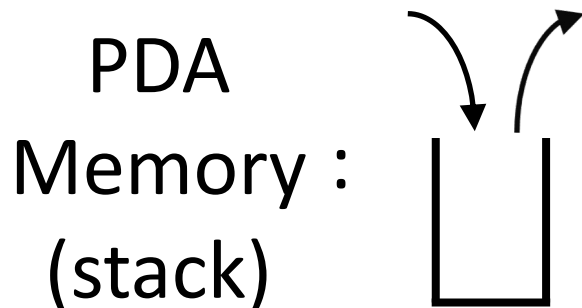
Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.
- Can access any spot in memory.
- Infinite memory.
- Start configuration: start state, input on tape, r/w head far left.



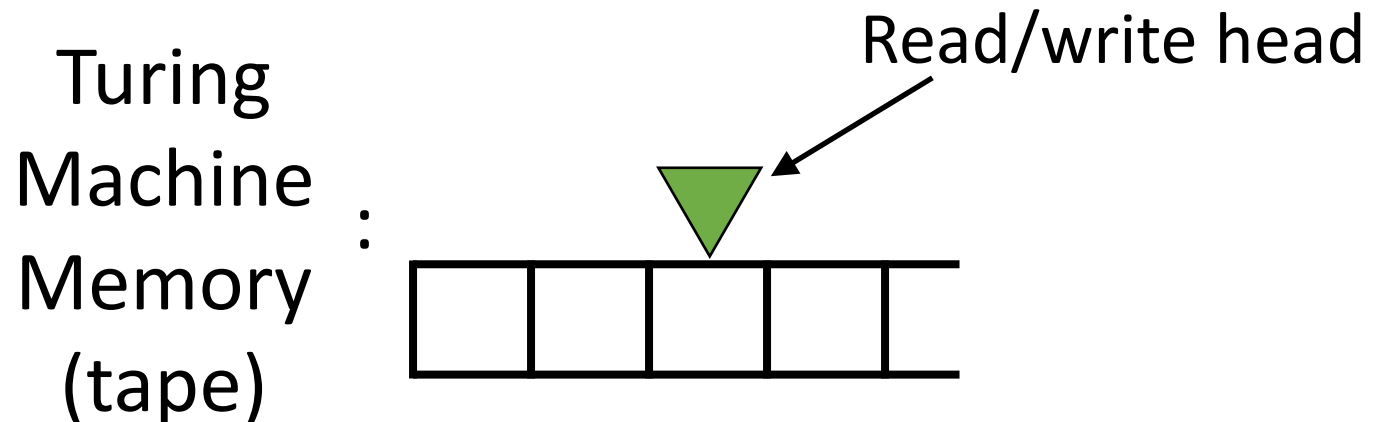
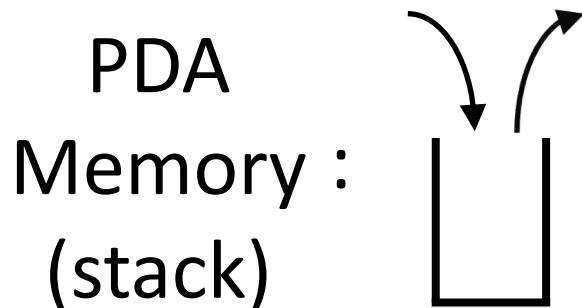
Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.
- Can access any spot in memory.
- Infinite memory.
- Start configuration: start state, input on tape, r/w head far left.
- \exists states other than accept and reject.



Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.
- Can access any spot in memory.
- Infinite memory.
- Start configuration: start state, input on tape, r/w head far left.
- \exists states other than accept and reject.
- Accept and reject take effect immediately.



Turing Machine Formal Definition

TMs consist of:

1. Finite set of states, Q .
2. Finite input alphabet, Σ (does not blank symbol \sqcup).
3. Finite tape alphabet, Γ (includes \sqcup , $\Sigma \subset \Gamma$).
4. Transition function, $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$.
5. Start state, $q_0 \in Q$.
6. Accept state, $q_{\text{accept}} \in Q$.
7. Reject state, $q_{\text{reject}} \in Q$, where $q_{\text{accept}} \neq q_{\text{reject}}$.

Turing Machine Example

How would you use a TM's tape to see if a string is in the language

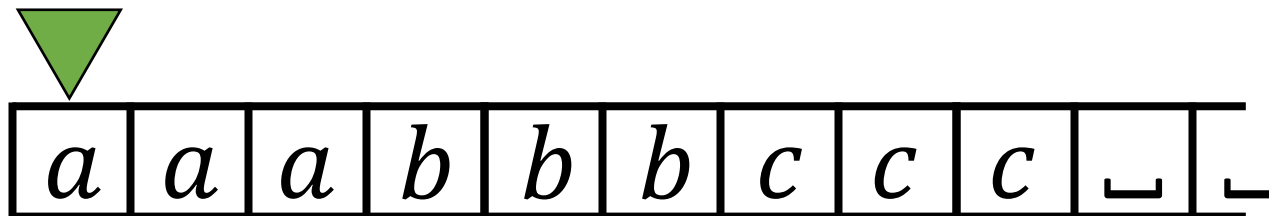
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Turing Machine Example

How would you use a TM's tape to see if a string is in the language

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Memory initial state: input on tape, read/write head at start.



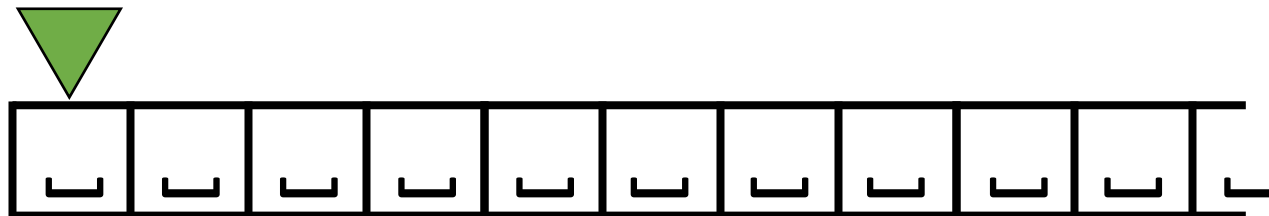
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TM M: on input ω

1. If $\omega = \varepsilon$, ?



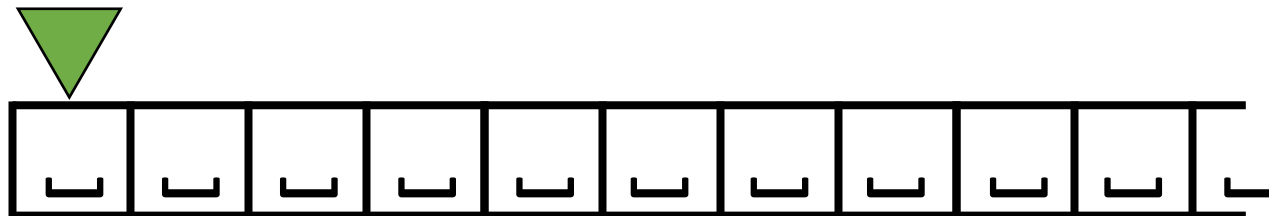
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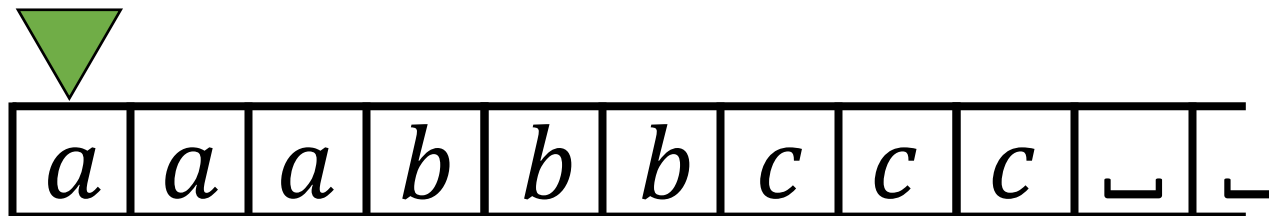
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Suppose $\omega = a^+ b^+ c^+$



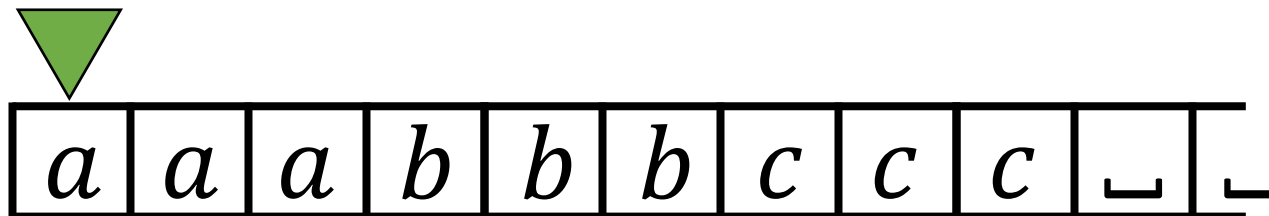
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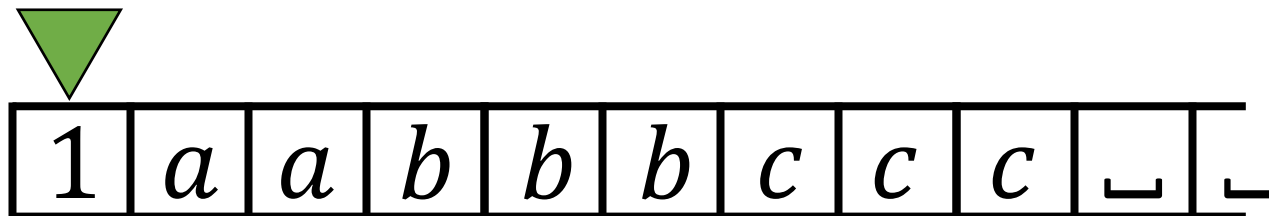
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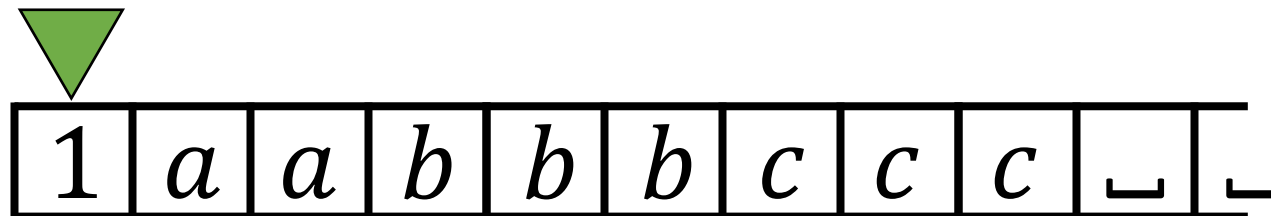
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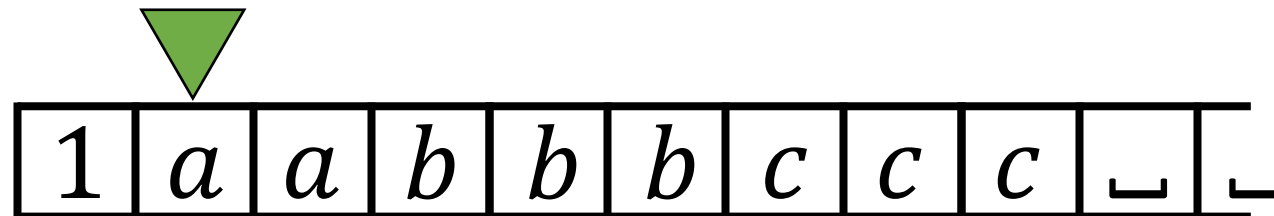
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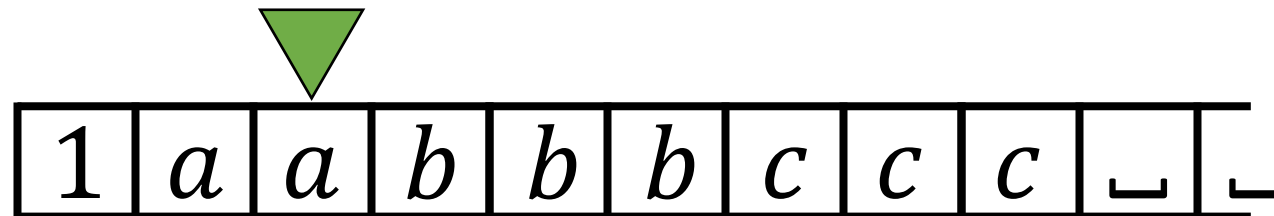
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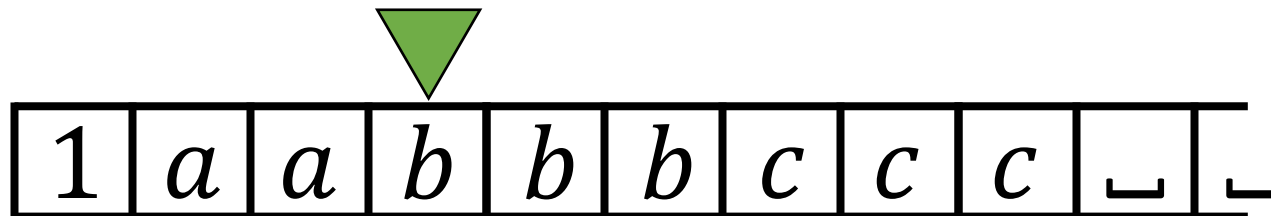
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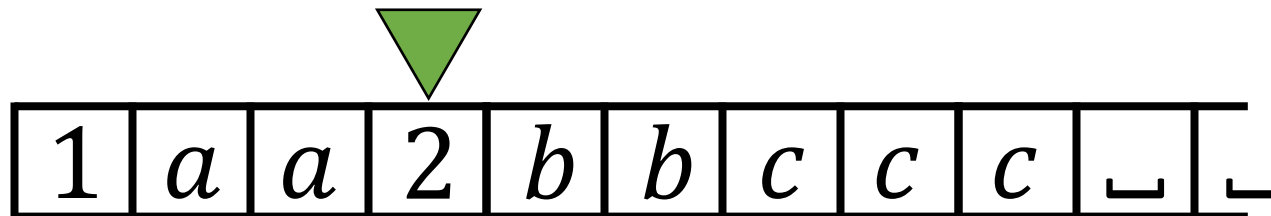
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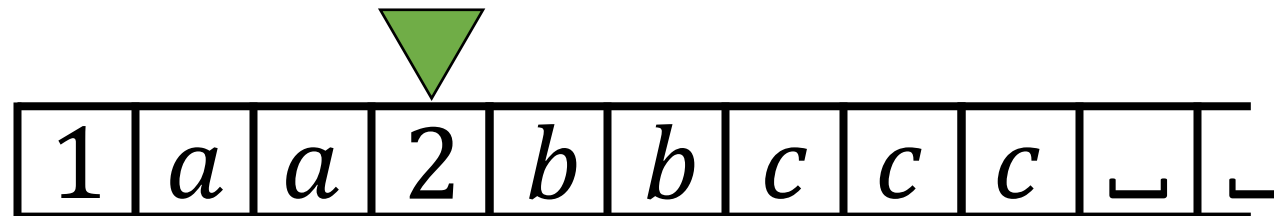
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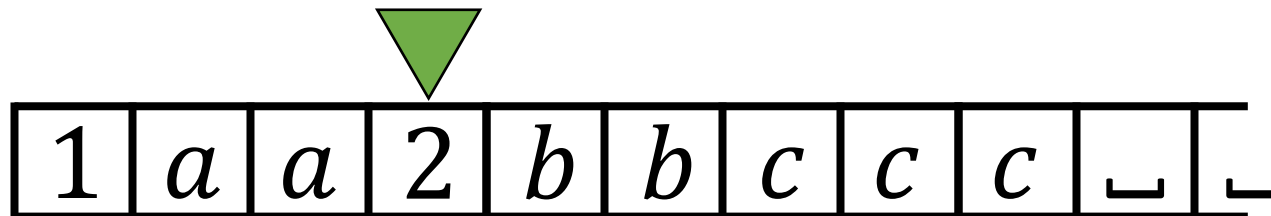
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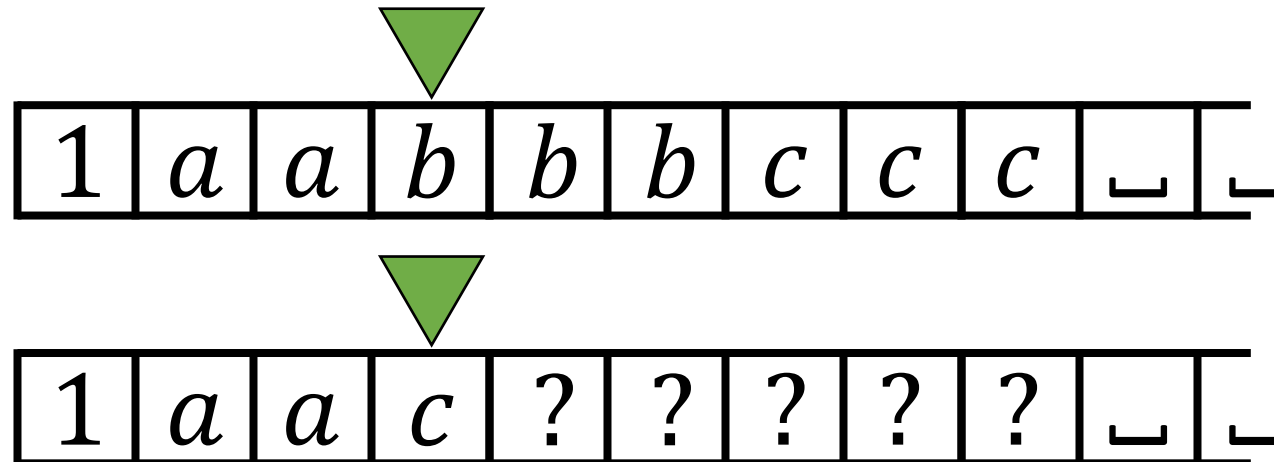
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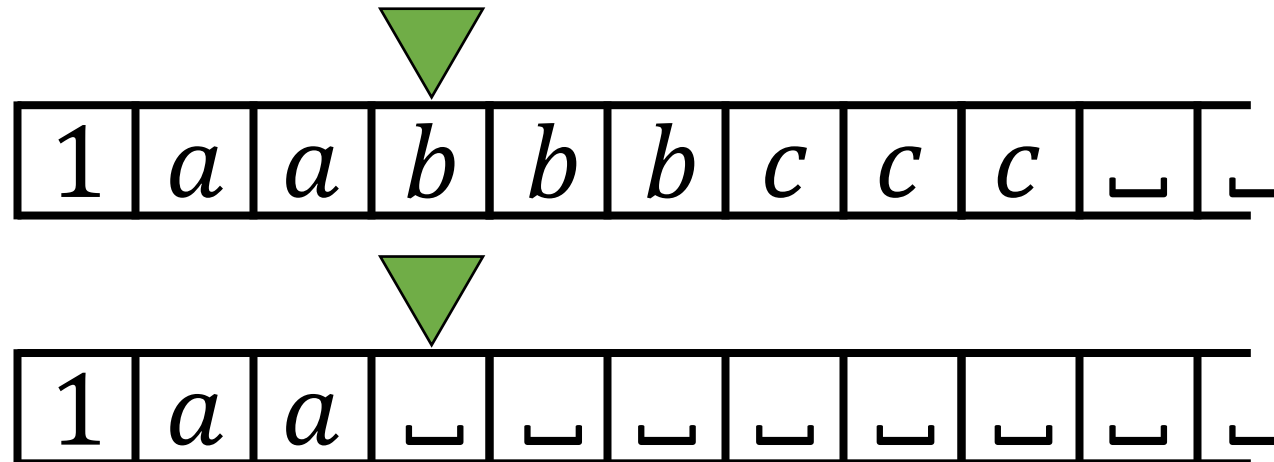
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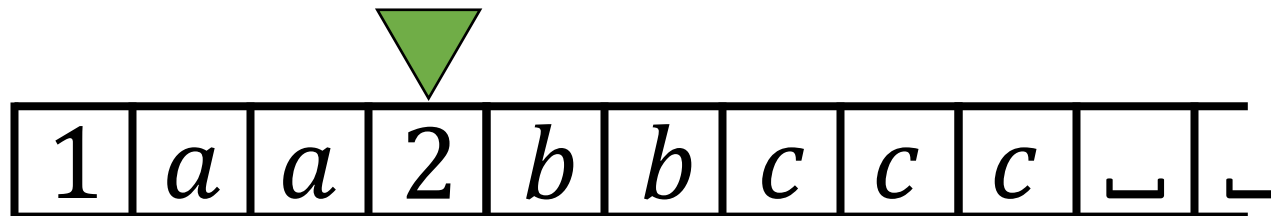
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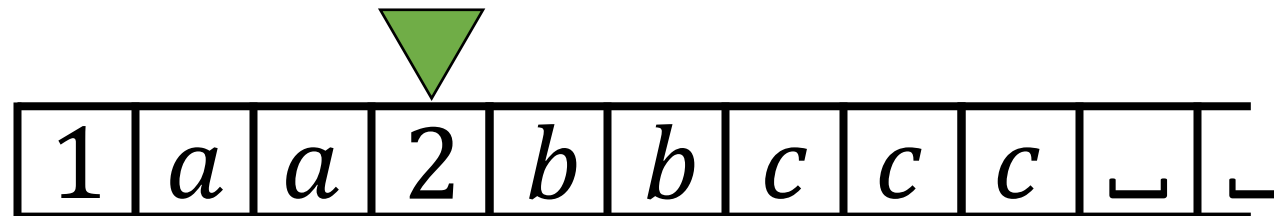
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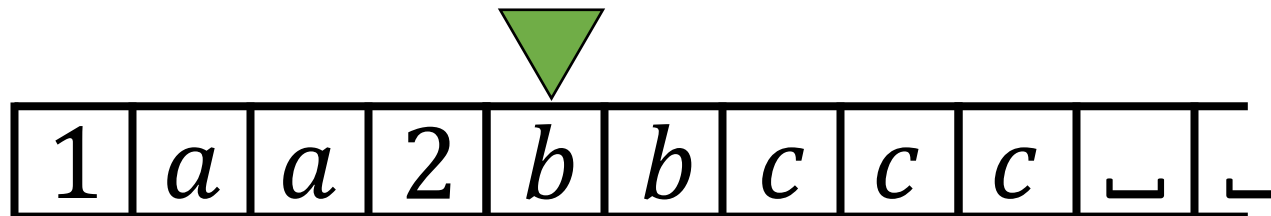
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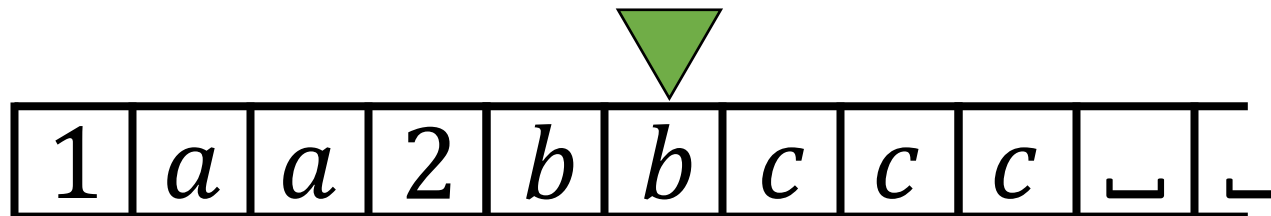
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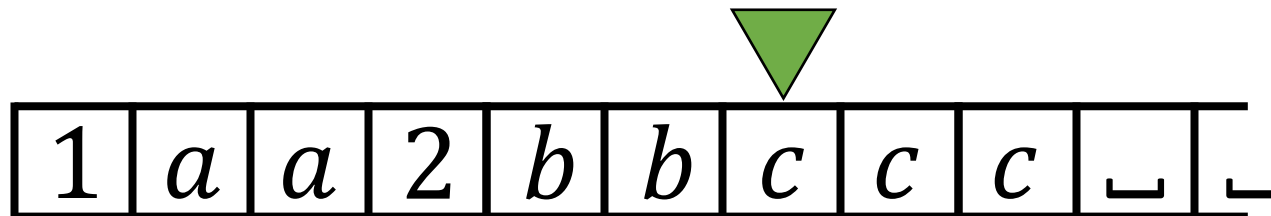
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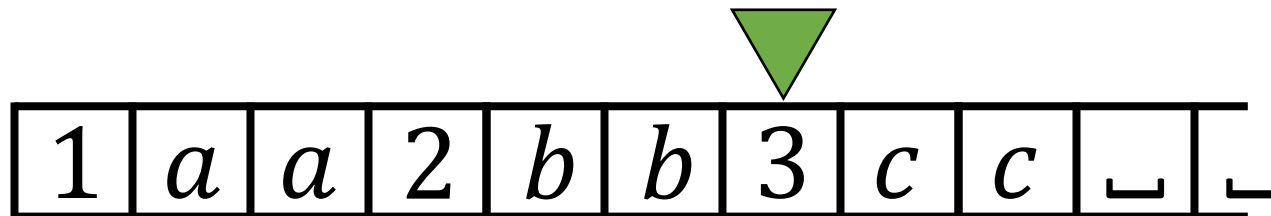
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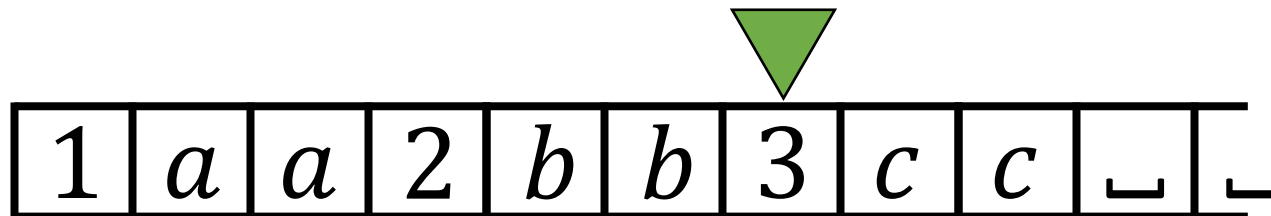
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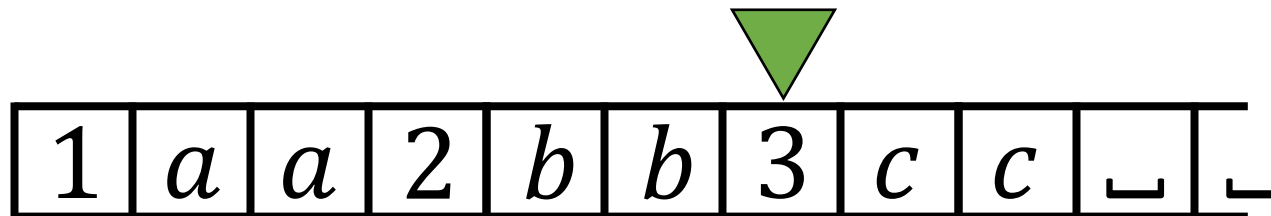
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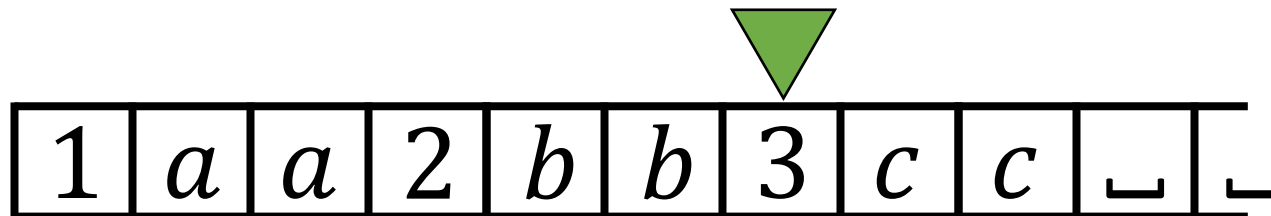
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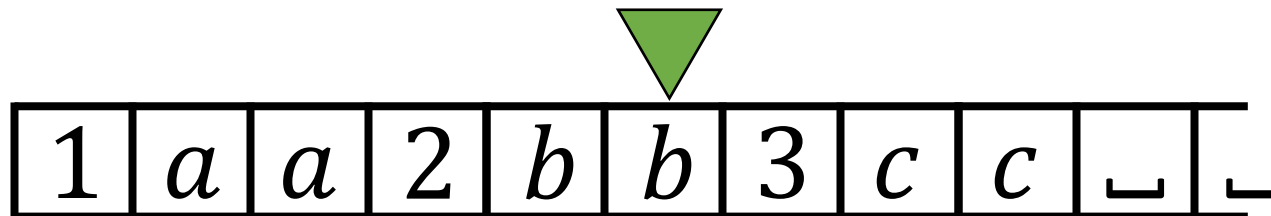
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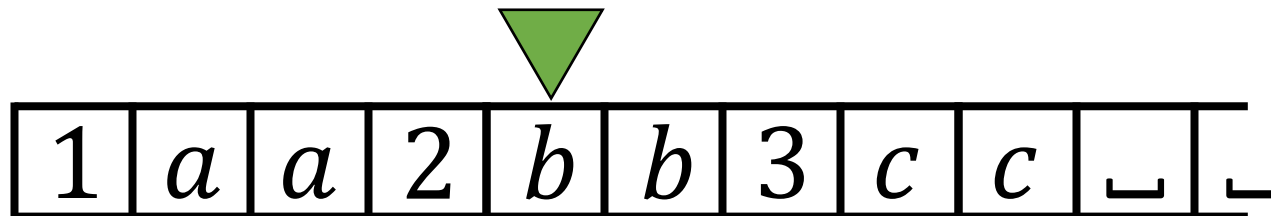
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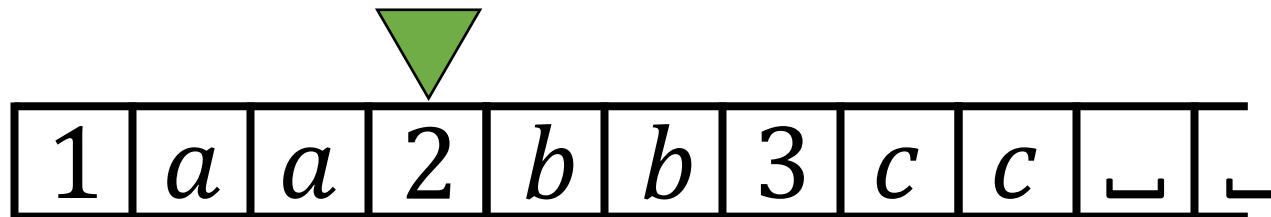
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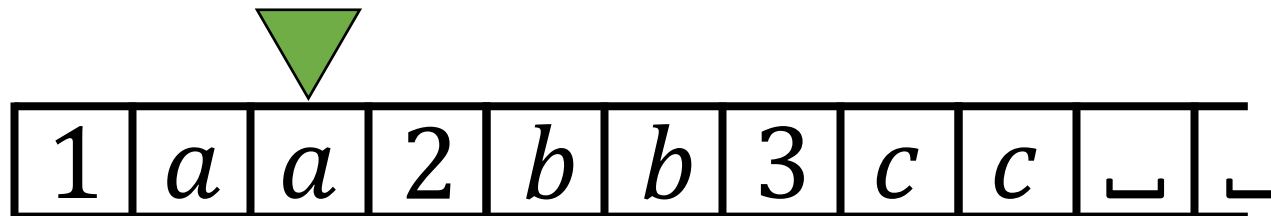
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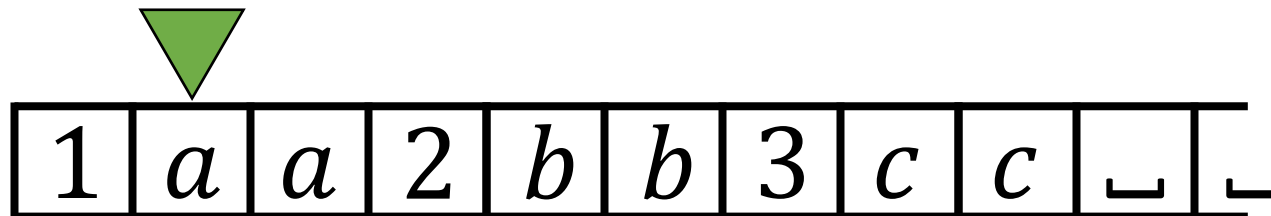
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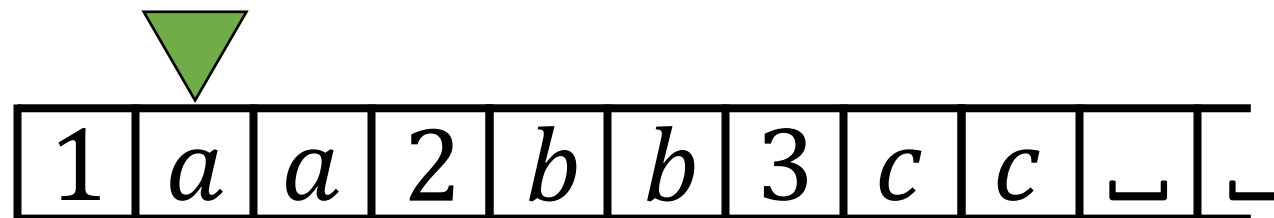
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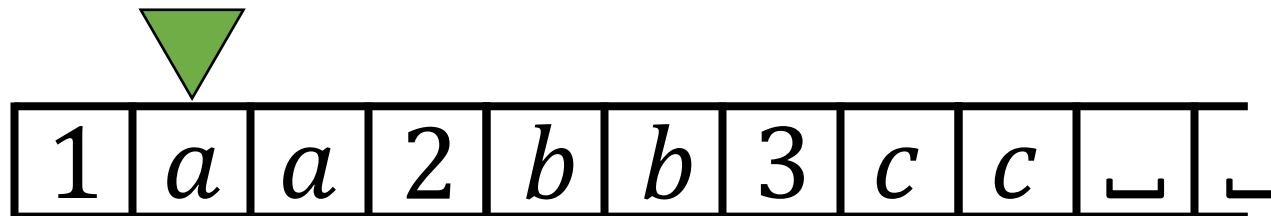
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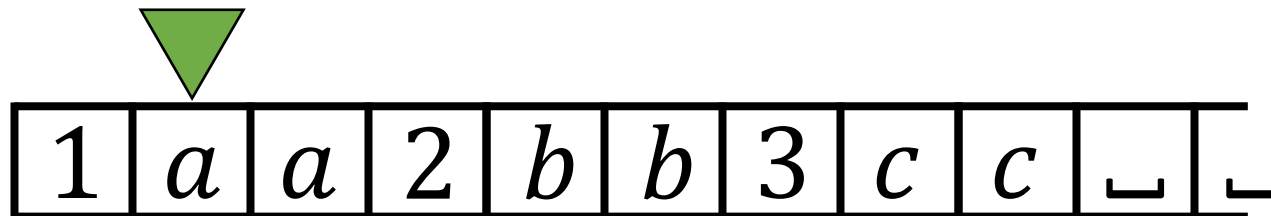
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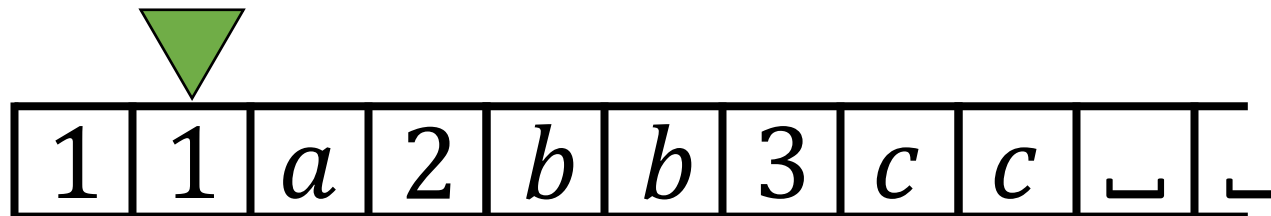
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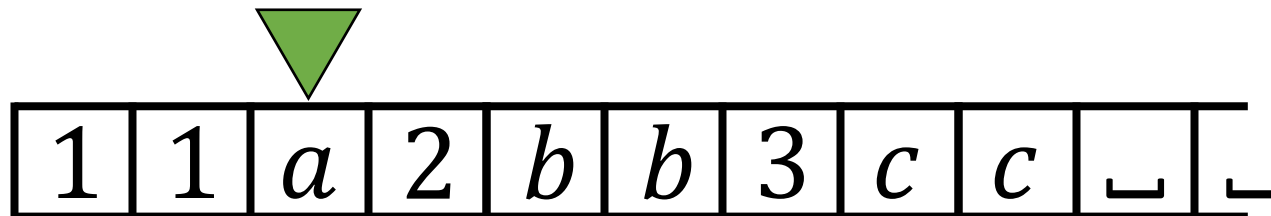
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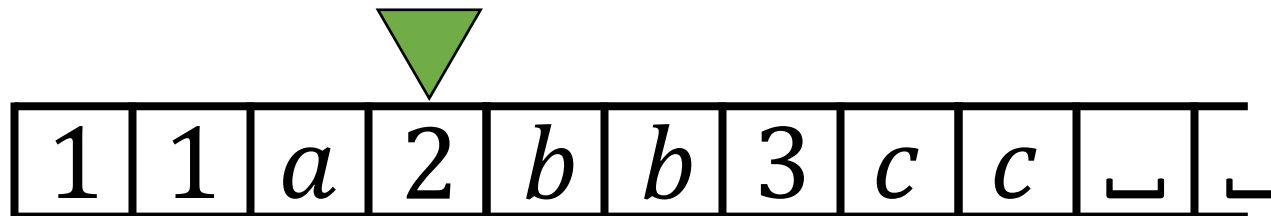
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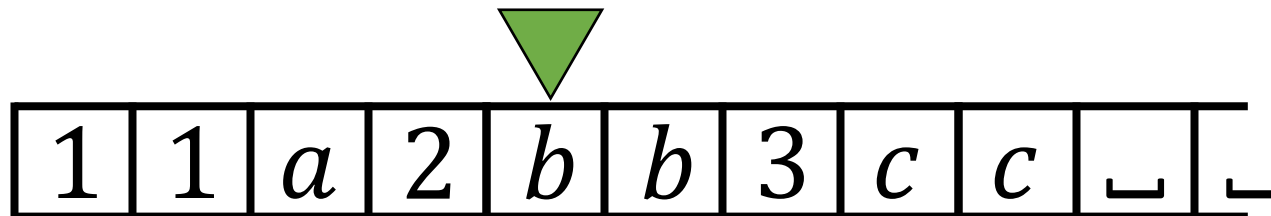
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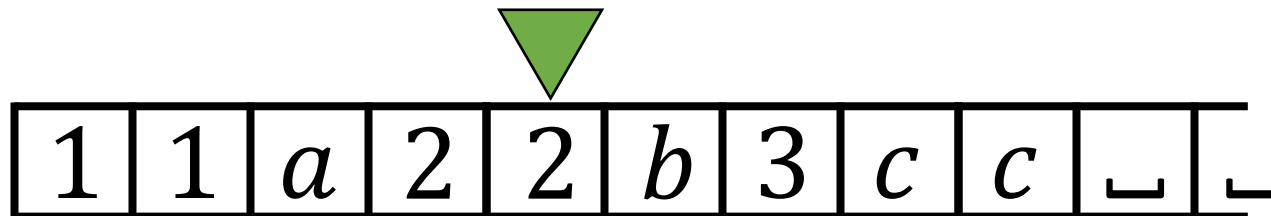
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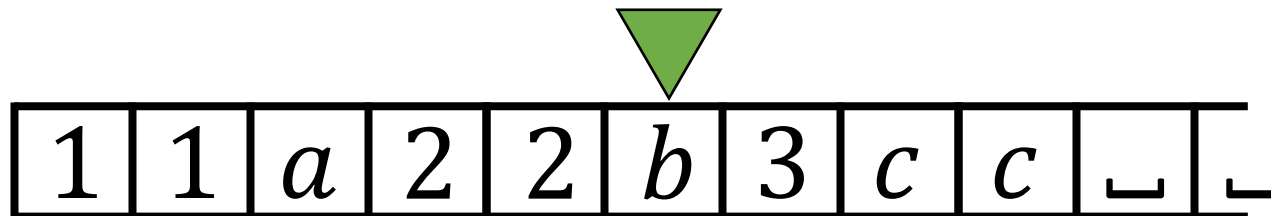
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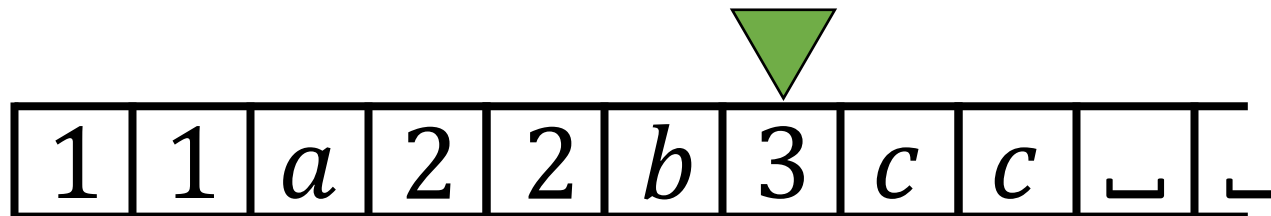
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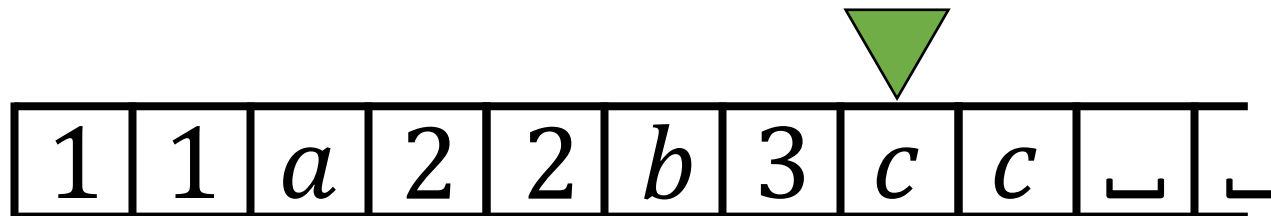
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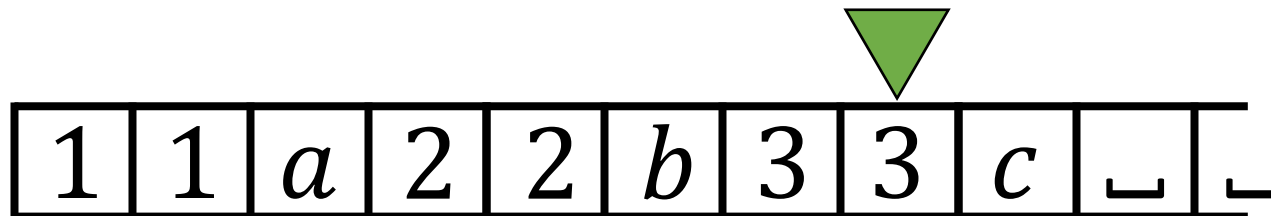
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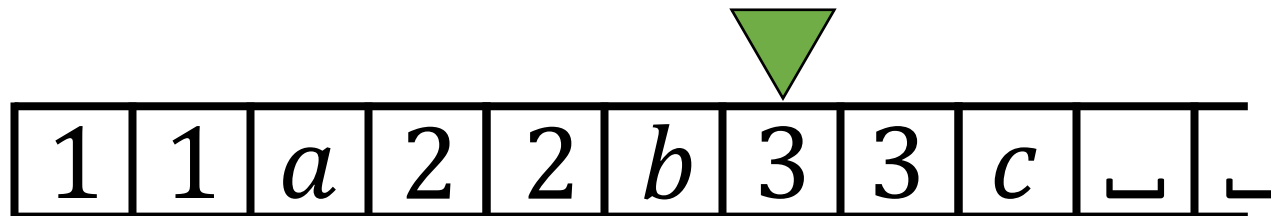
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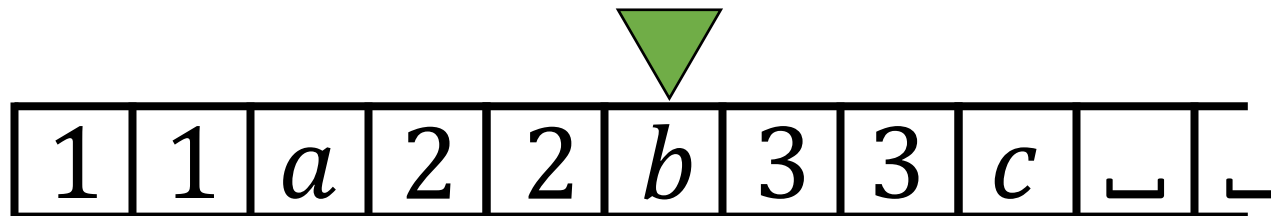
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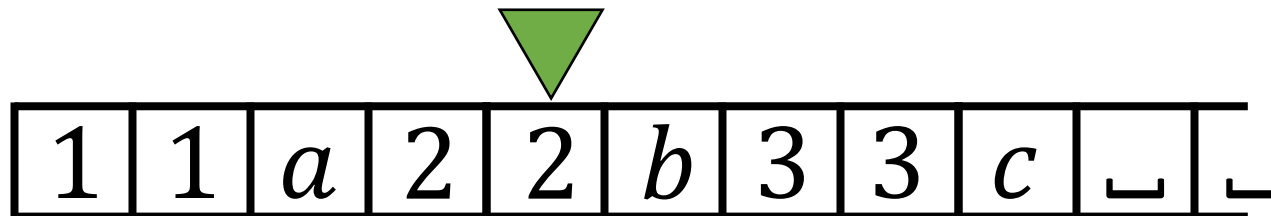
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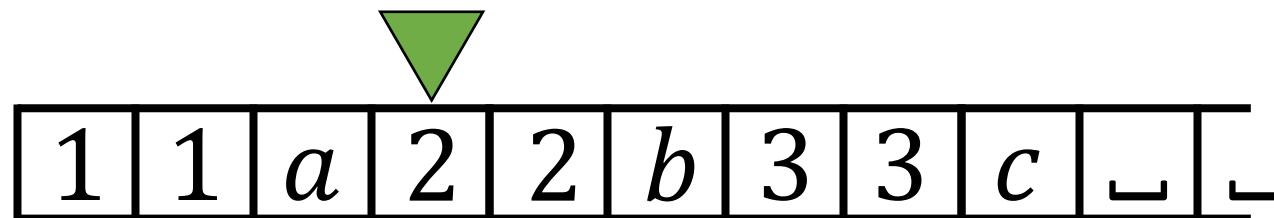
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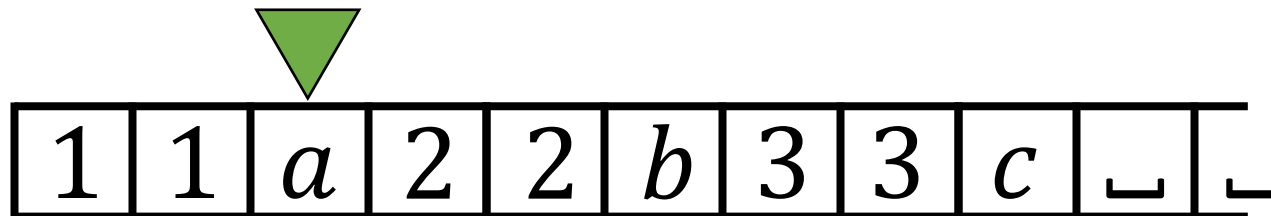
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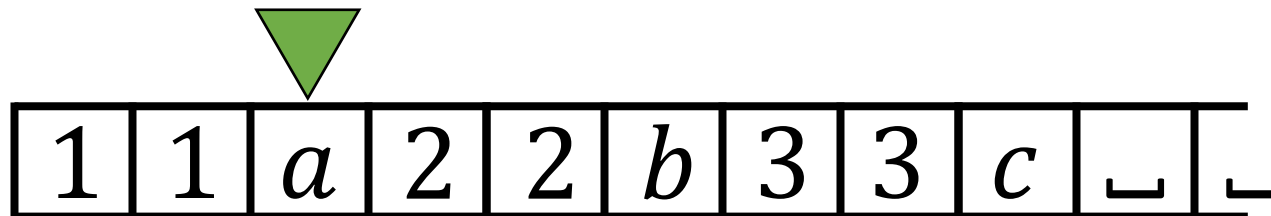
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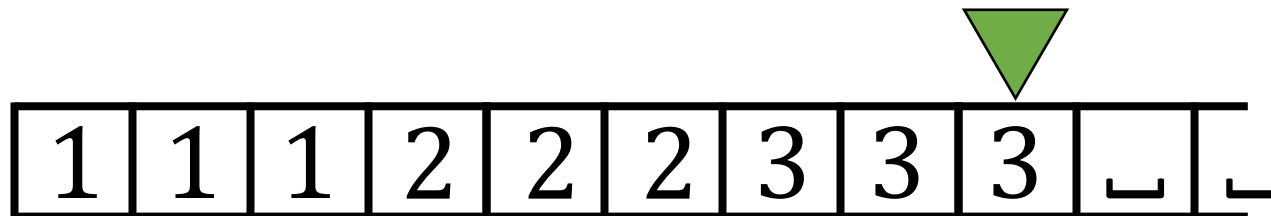
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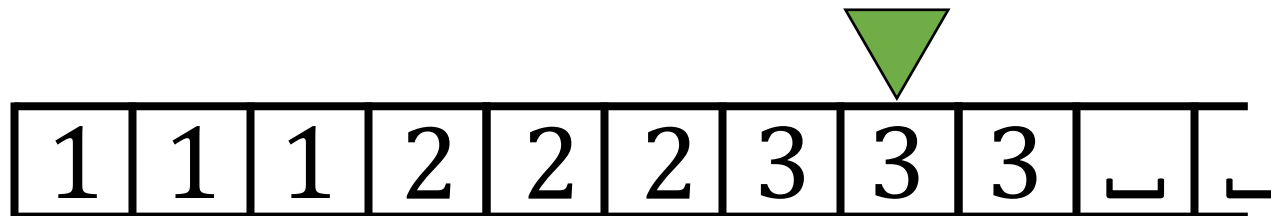
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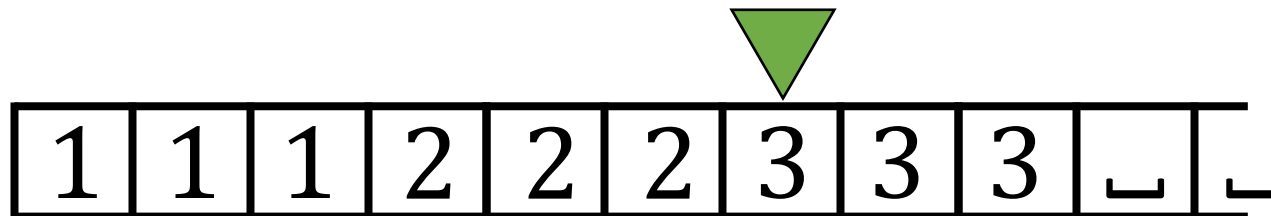
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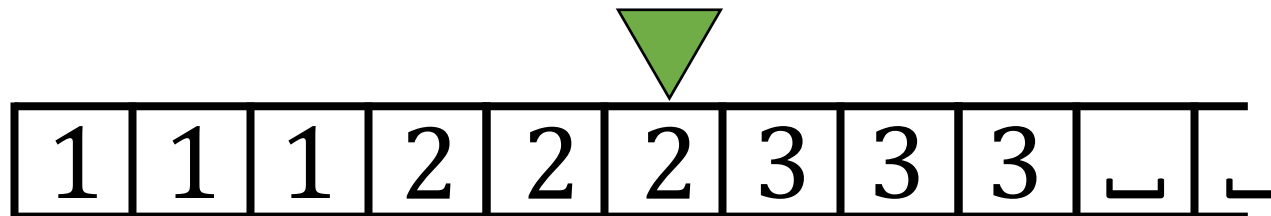
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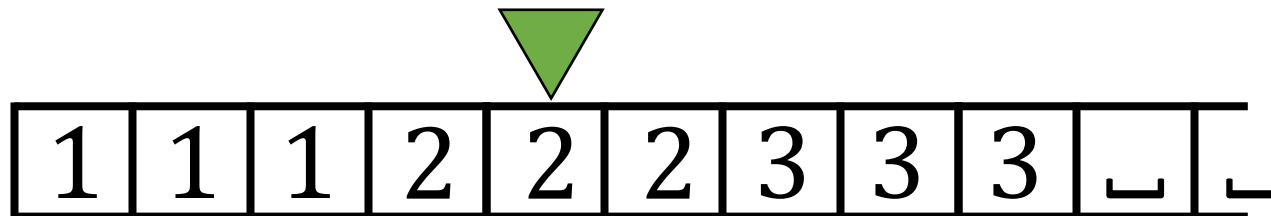
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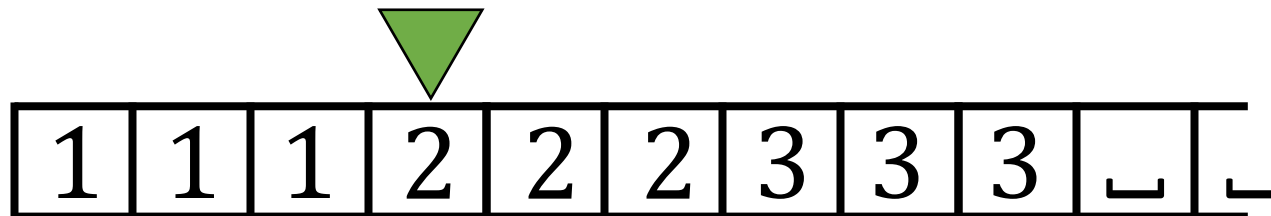
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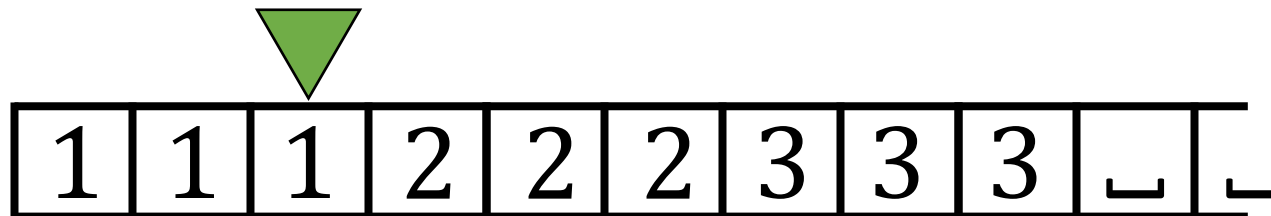
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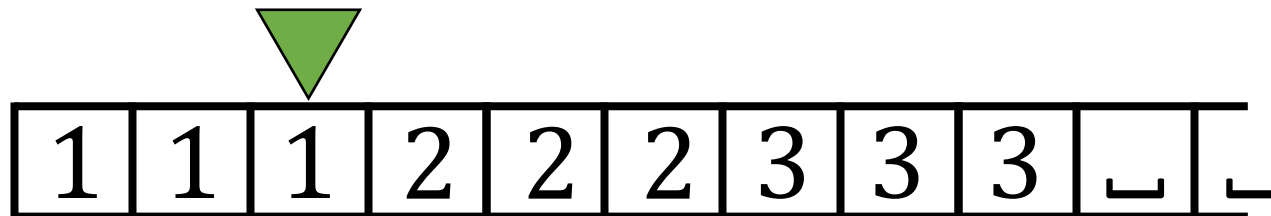
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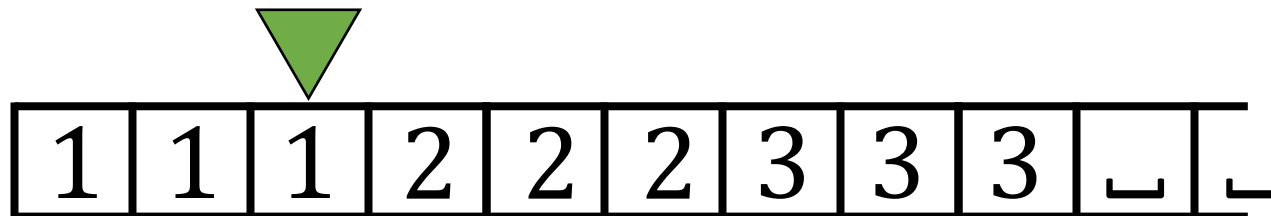
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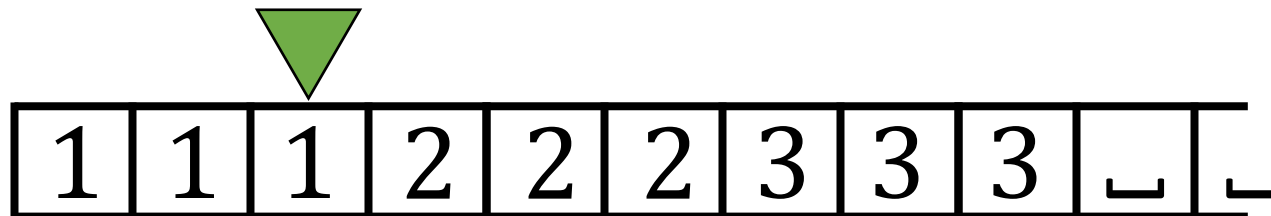
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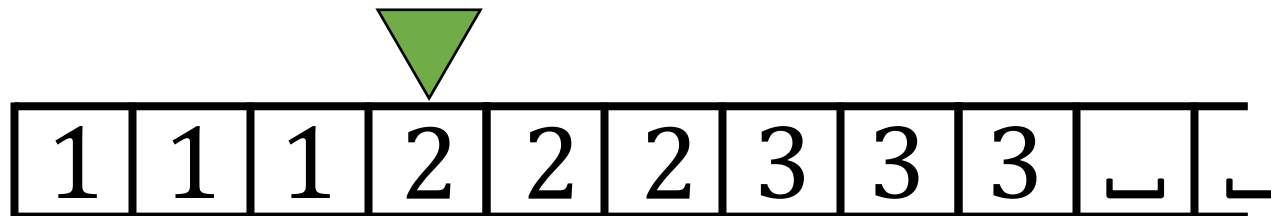
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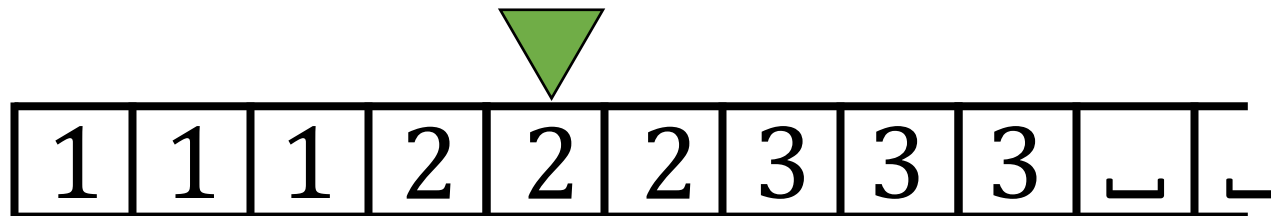
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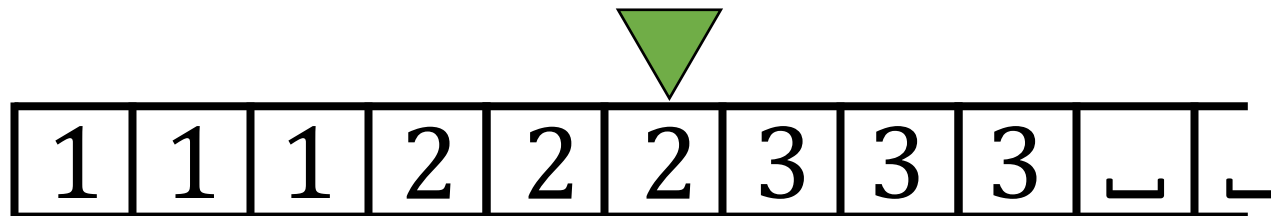
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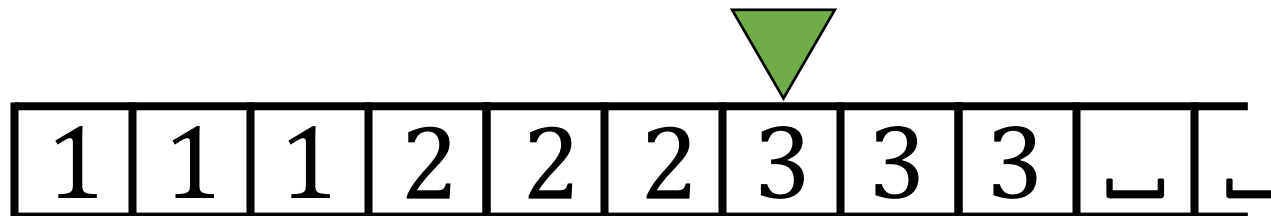
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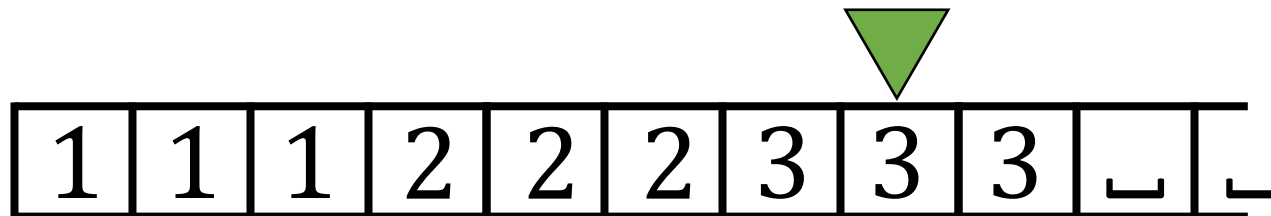
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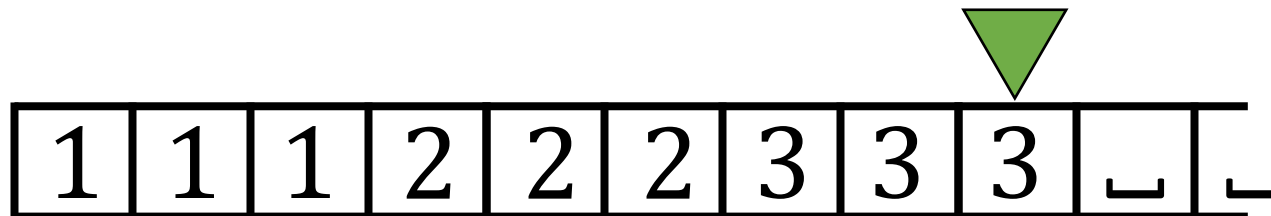
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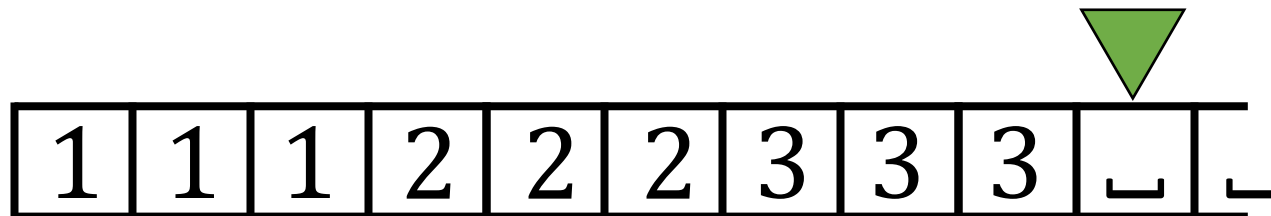
Turing Machine Example

How would you use a TM's tape to see if a string is in the language

$$L = \{a^n b^n c^n : n \geq 0\}?$$

TM M: on input ω

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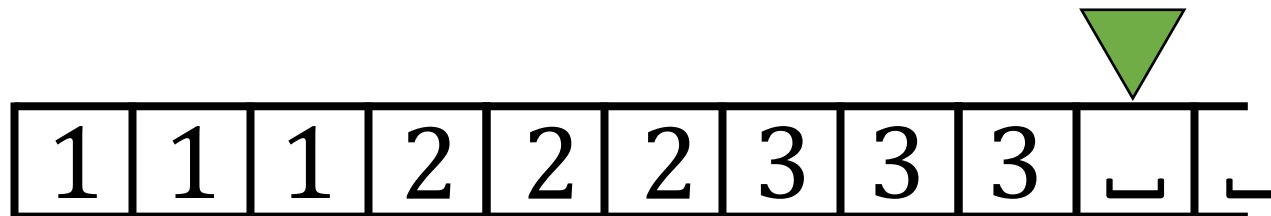
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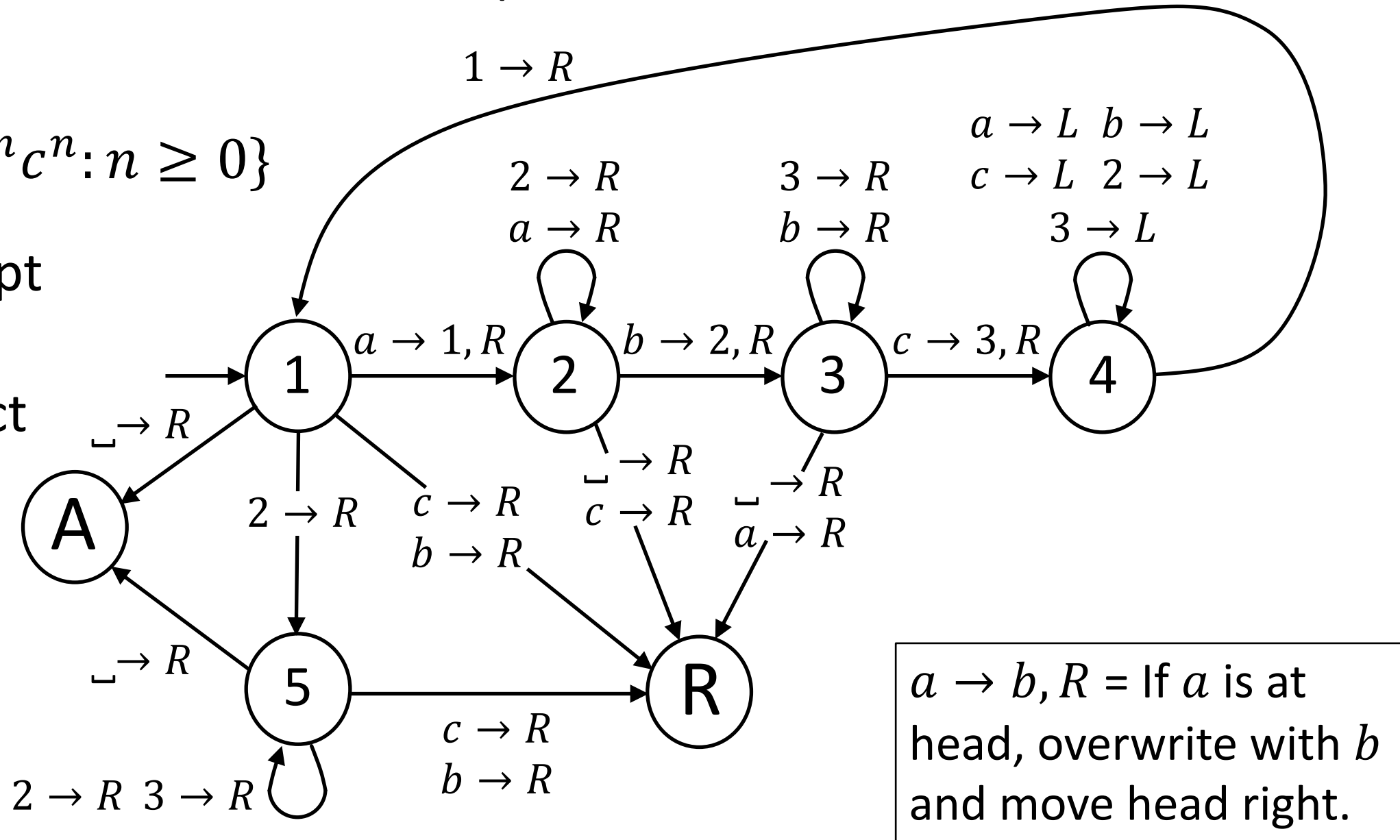


Turing Machine Example

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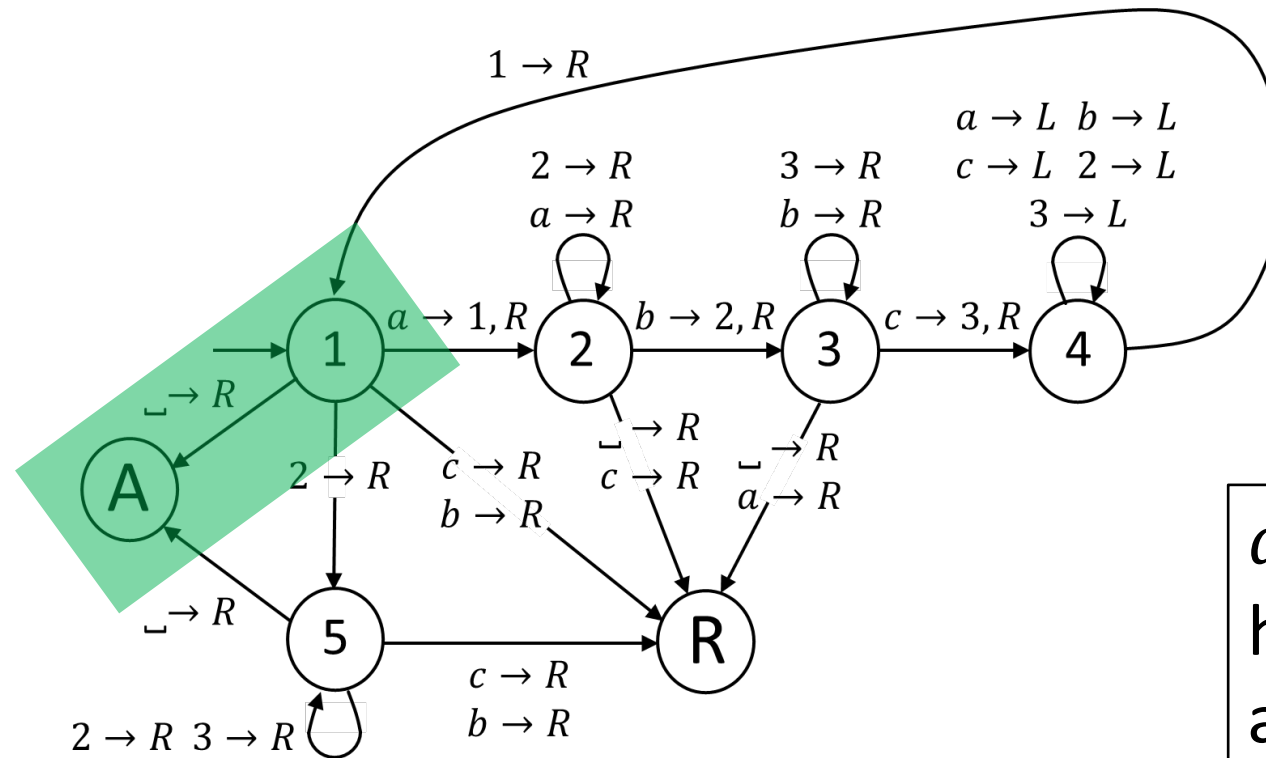
A = Accept

R = Reject



TM M: on input ω

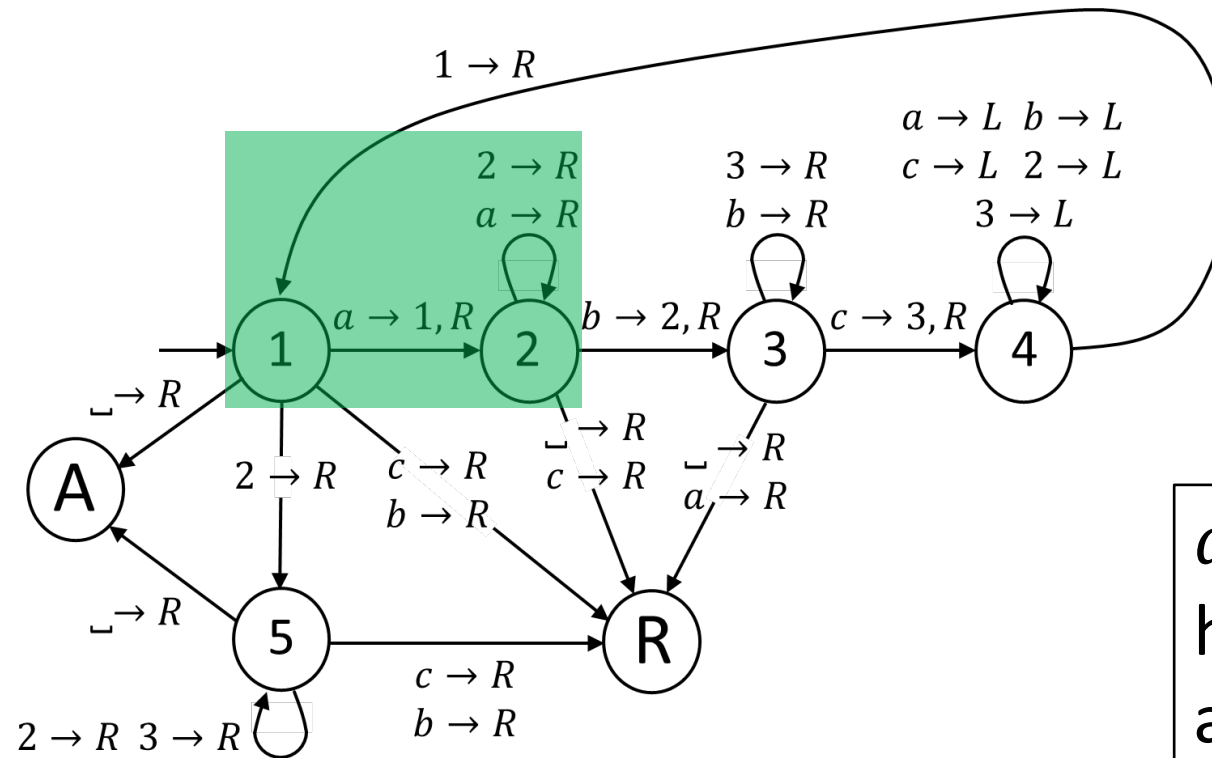
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$a \rightarrow b, R$ = If a is at head, overwrite with b and move head right.

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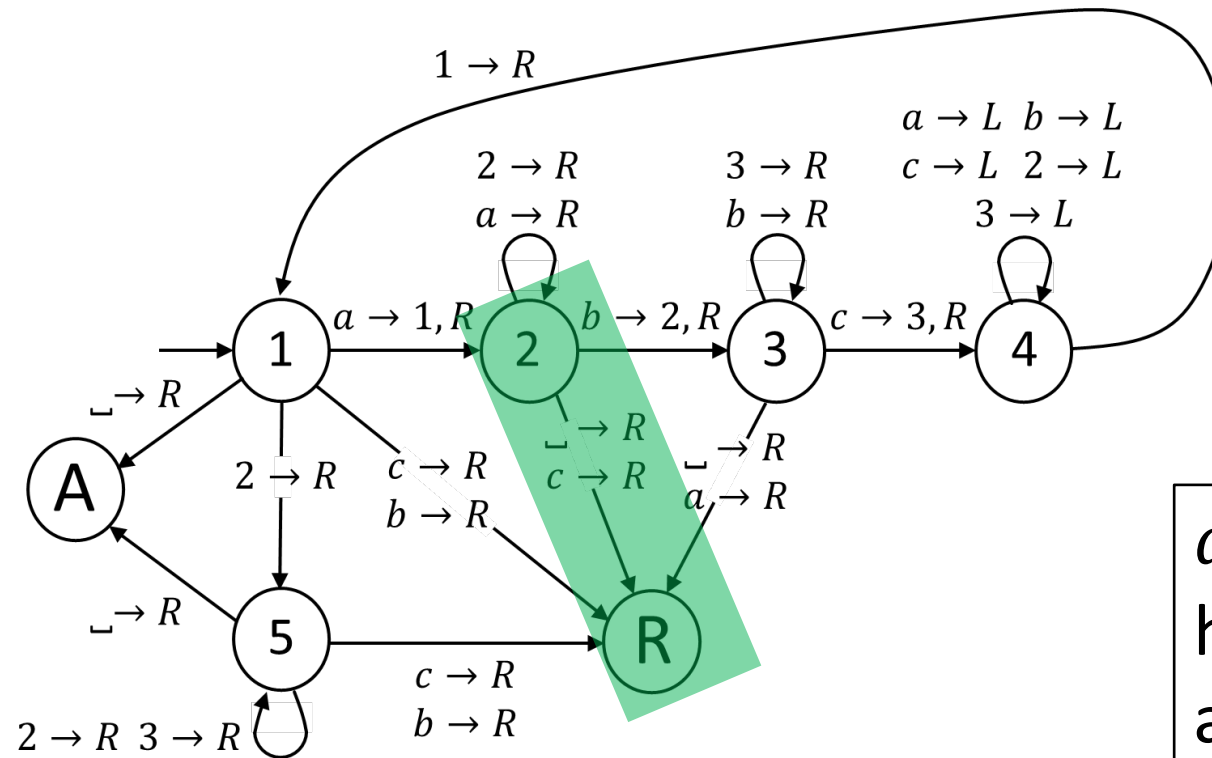
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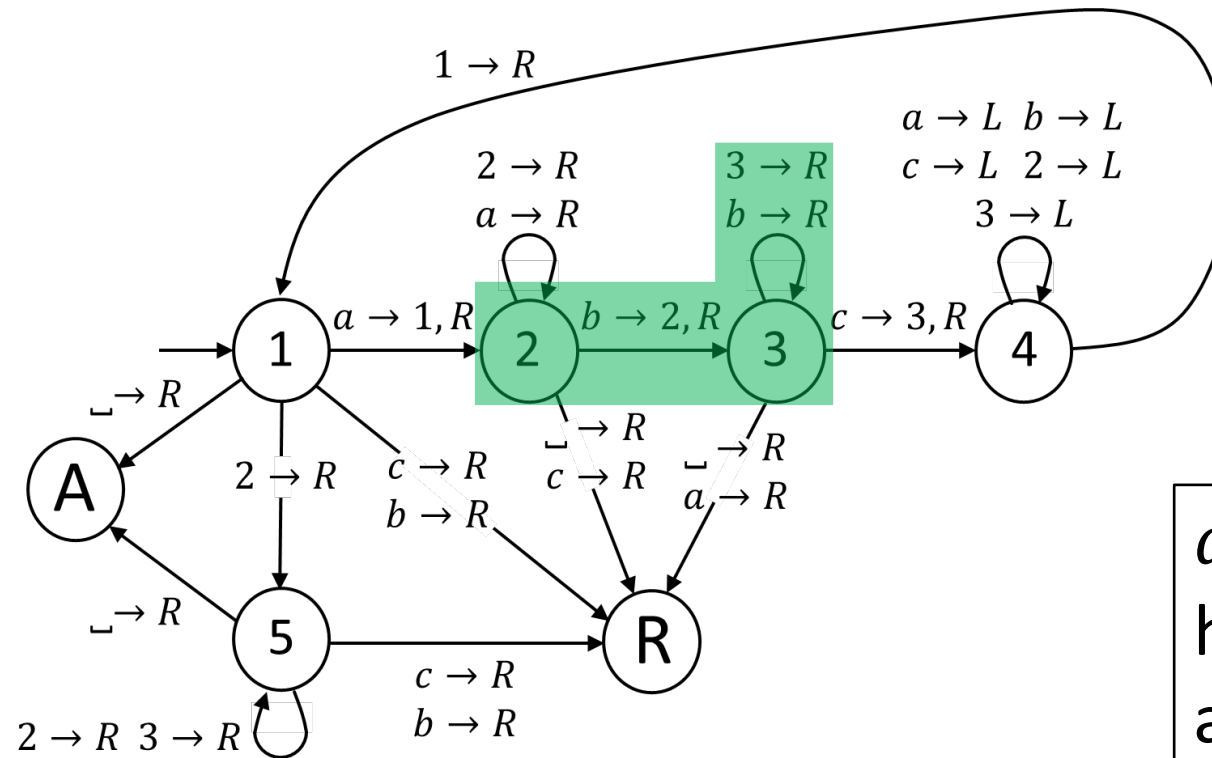
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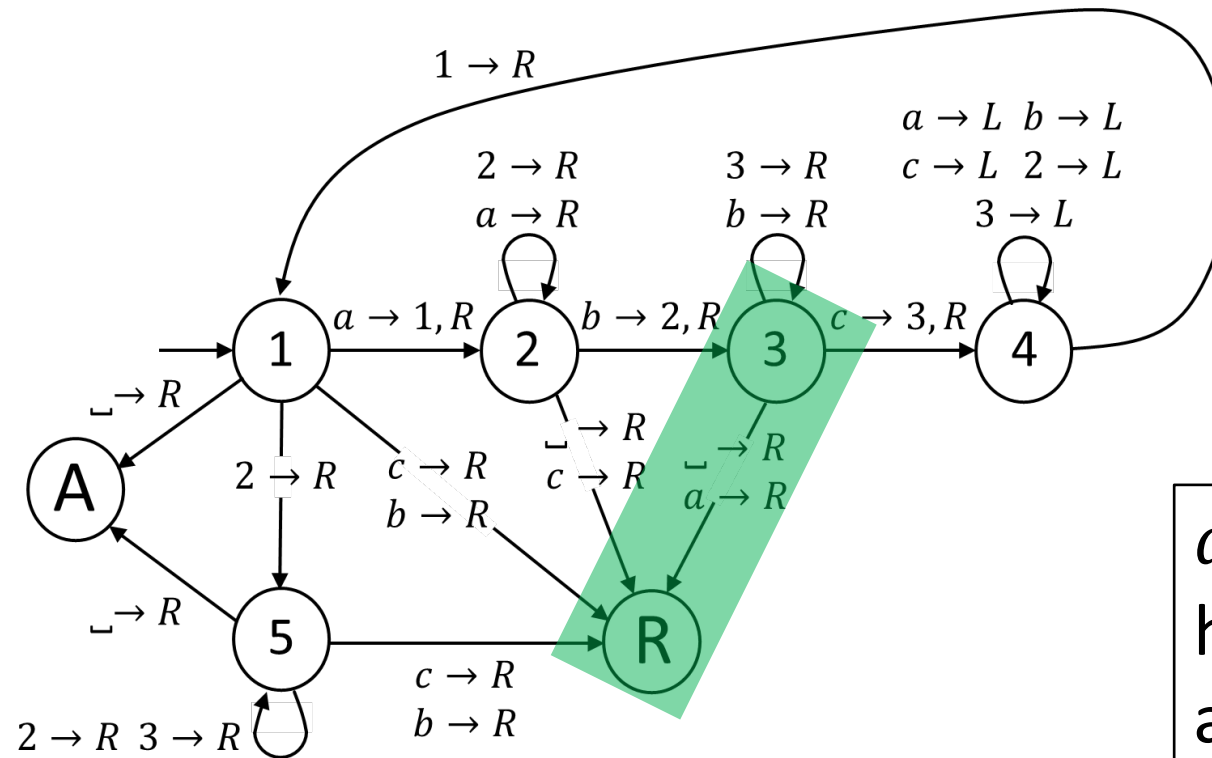
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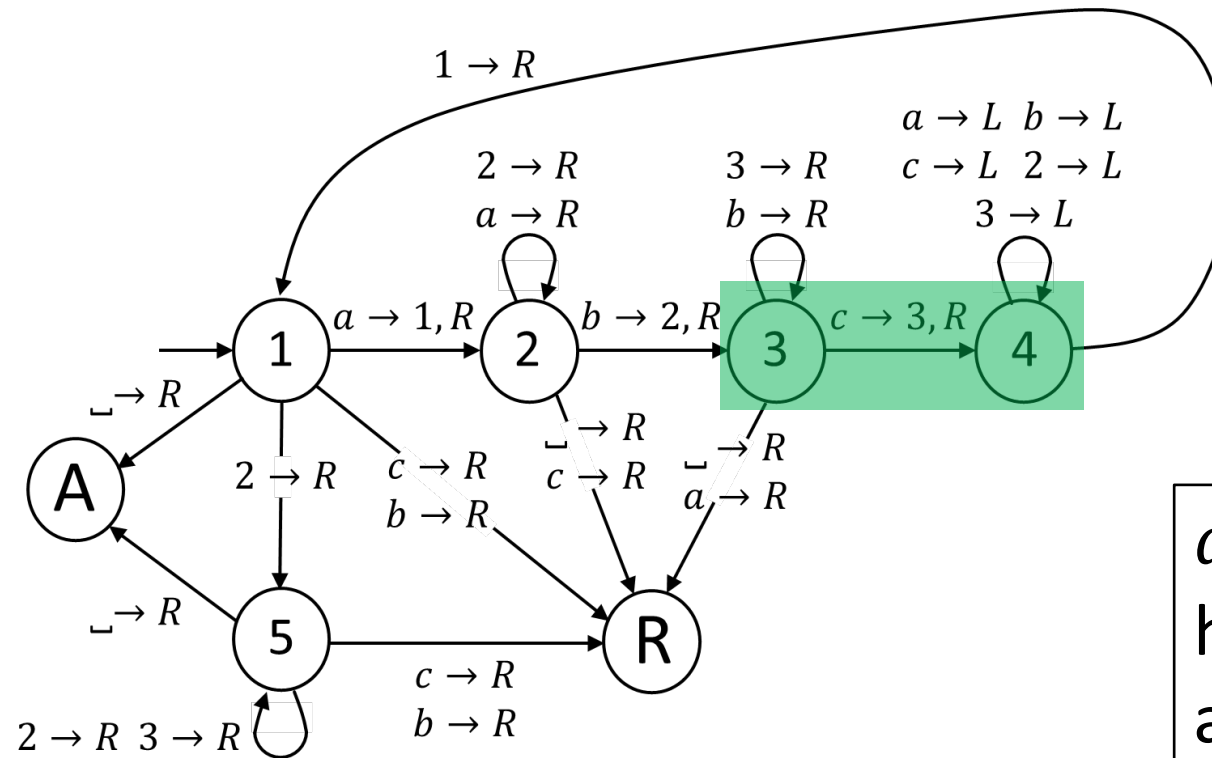
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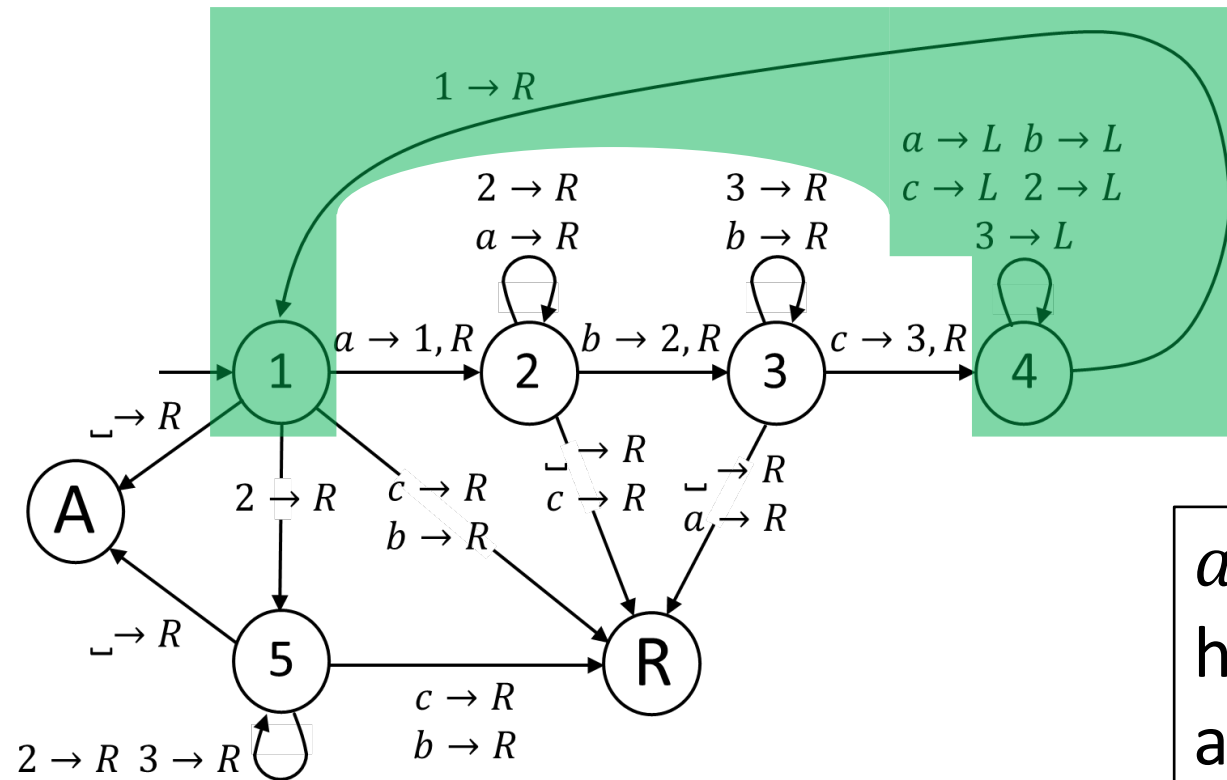
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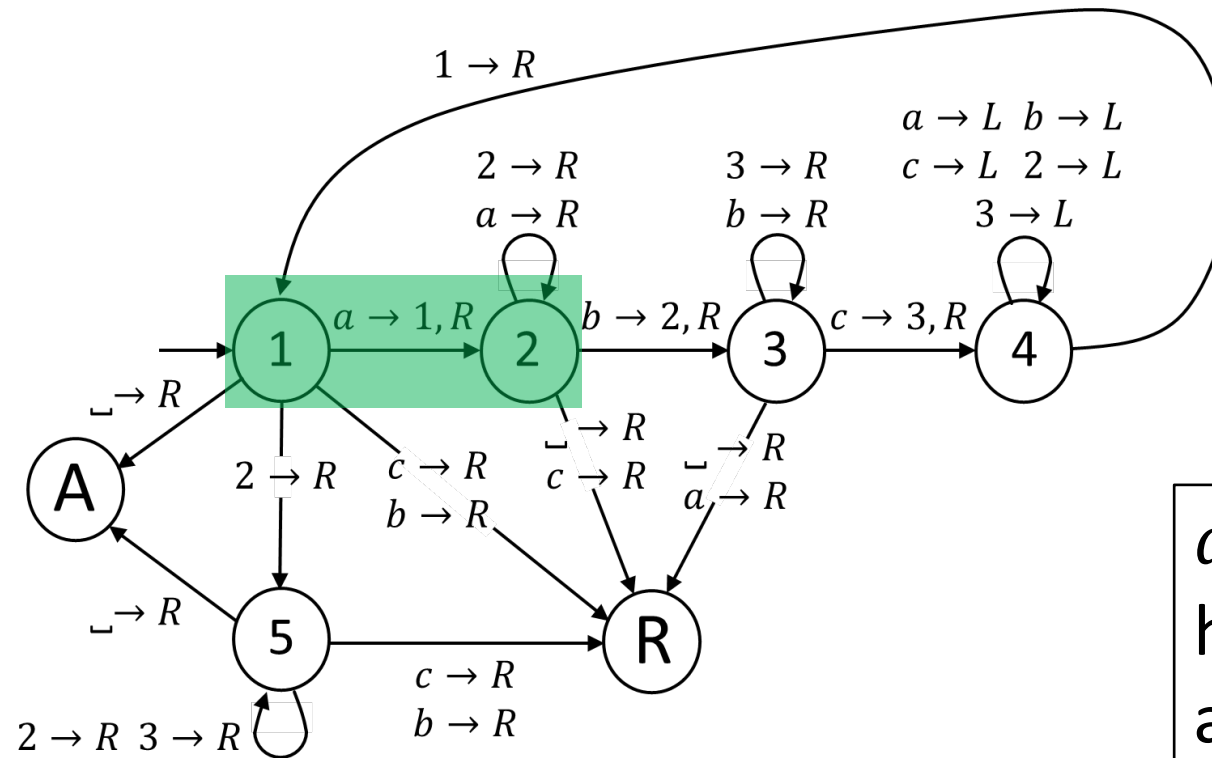
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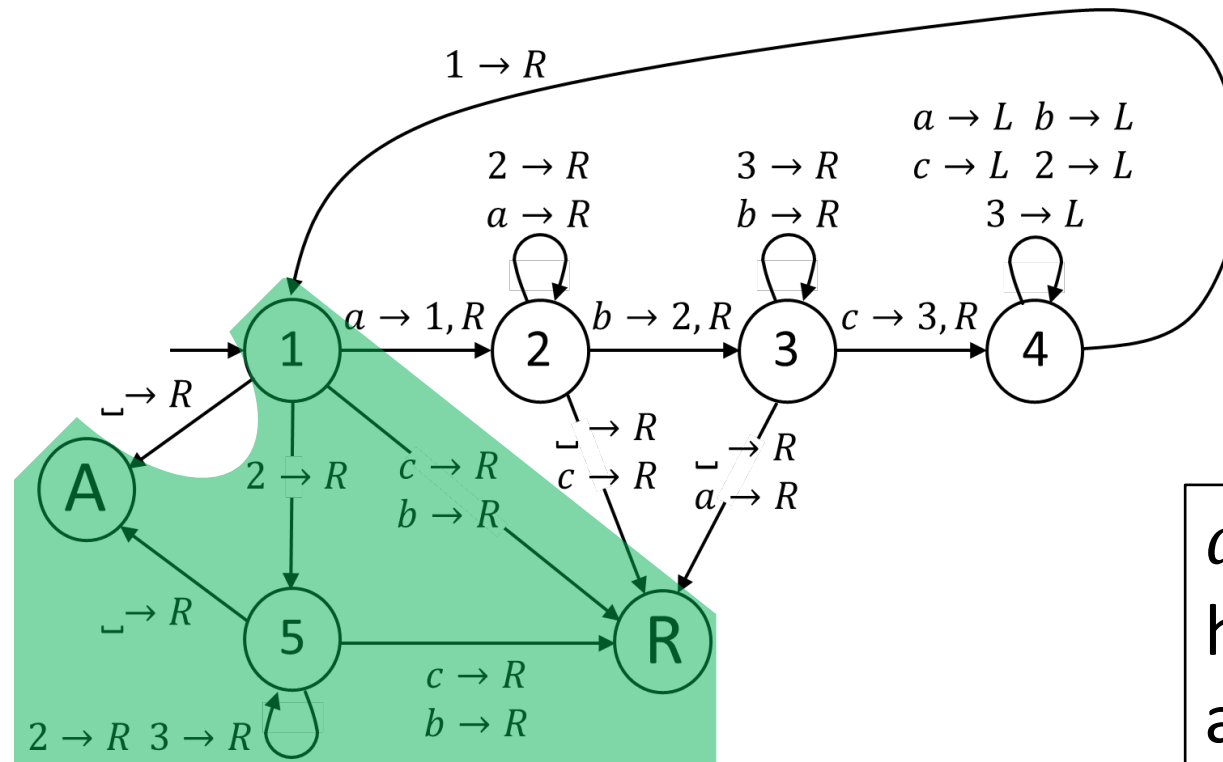
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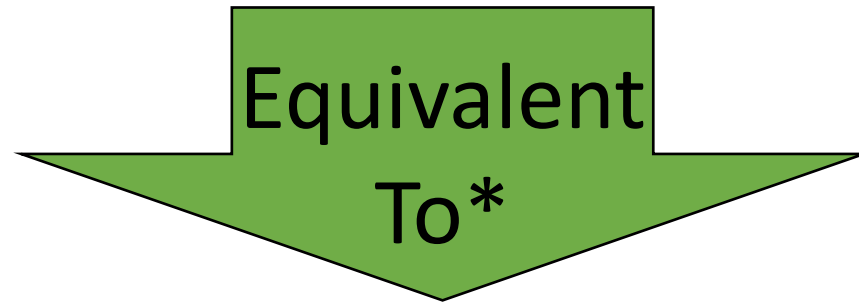
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Turing Machines – Equivalent Models

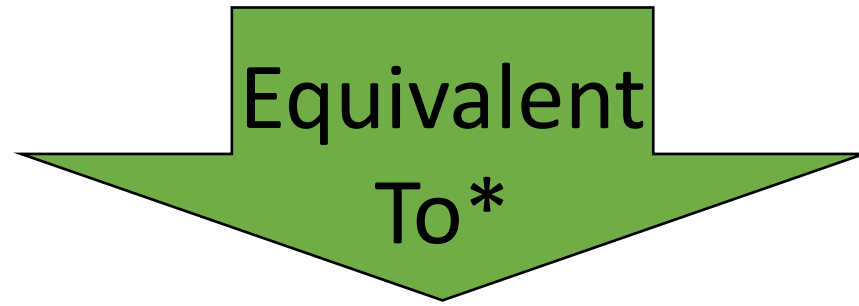


Turing Machines

* I.e. Can solve the same problems.

Turing Machines – Equivalent Models

Multi-tape TM



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Turing Machines – Equivalent Models

Multi-tape TM

Non-deterministic TM



Turing Machines

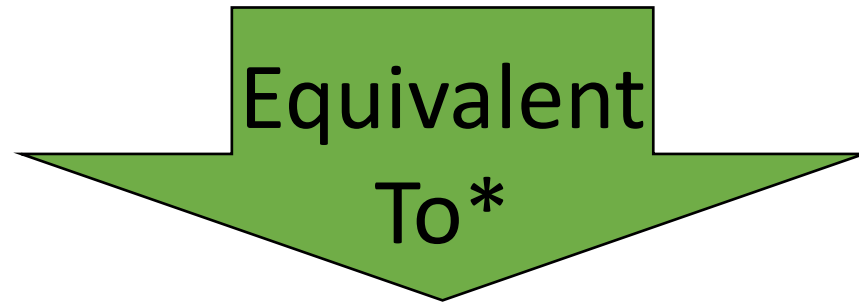
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Turing Machines – Equivalent Models

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Non-deterministic TM

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Turing Machines – Equivalent Models

Multi-tape TM

λ -Calculus

Non-deterministic TM

General Recursive Functions

Quantum TM



Turing Machines

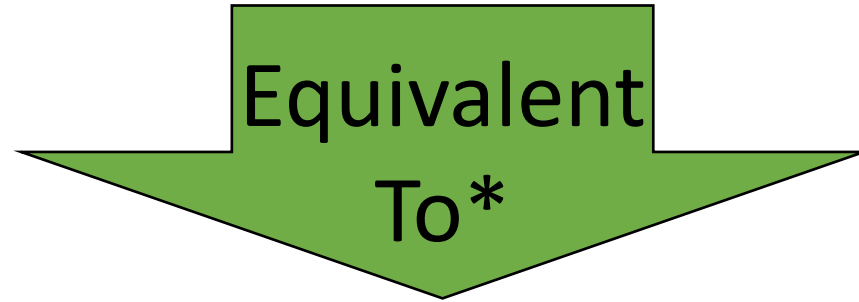
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Turing Machines – Equivalent Models

Multi-tape TM
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Turing Machines

Not Equivalent:

- Oracle

Turing Machines – Equivalent Models

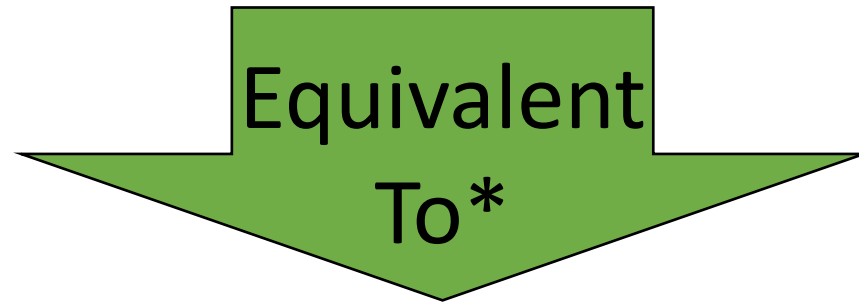
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Not Equivalent:

- Oracle
- Computer orbiting a rotating, electrically charged, black hole

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Multi-tape TM
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Actually, ALL computational models that allow unrestricted access to unlimited memory are equivalent to TMs (with basic assumptions) !

Church-Turing Thesis

Intuitive notion
of algorithms. = Turing Machine
algorithms.

Church-Turing Thesis

Intuitive notion
of algorithms.

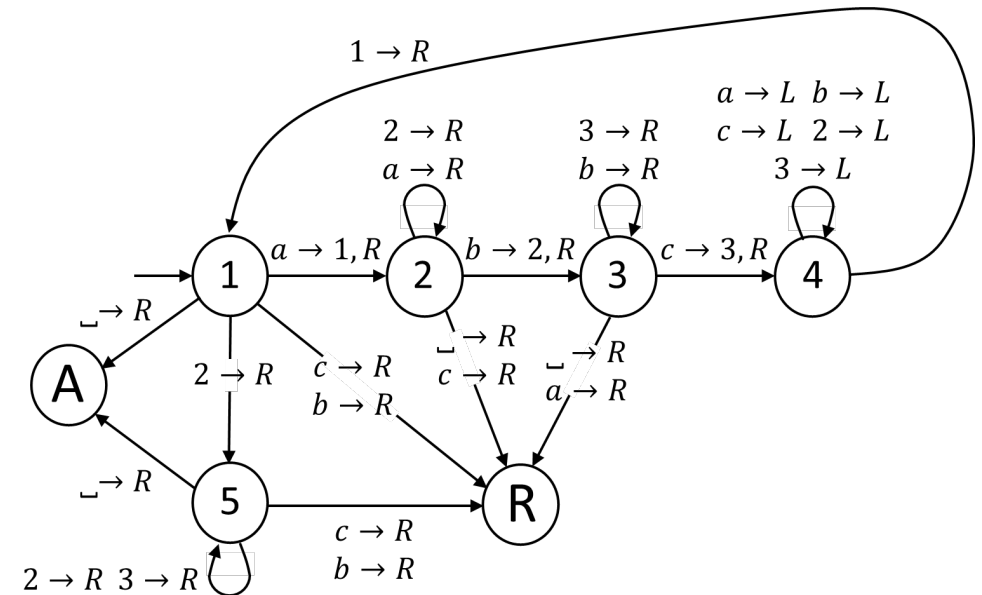
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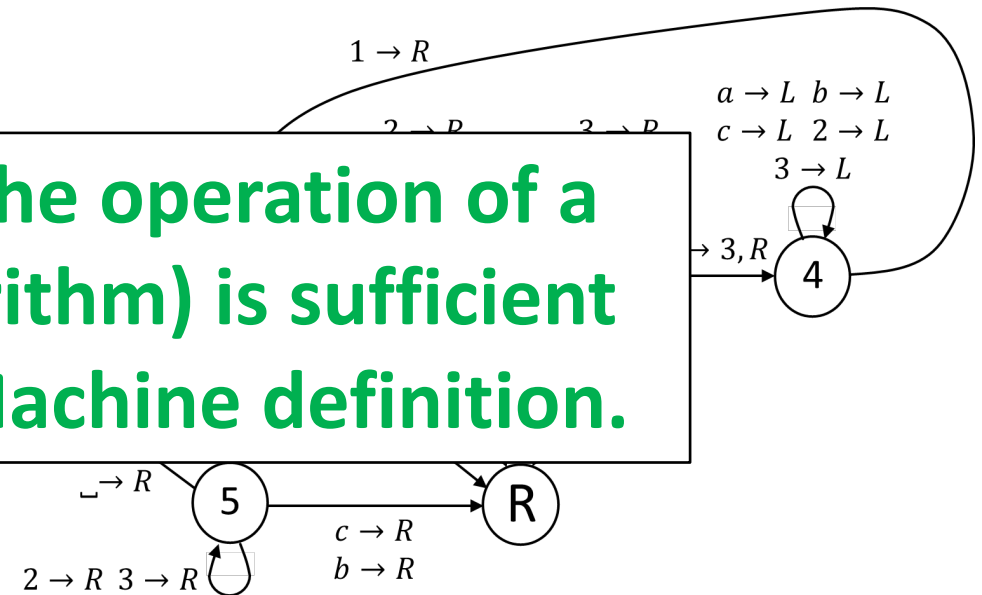
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Intuitive notion
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TM M: on input ω

1. If $\omega =$
2. Move r
3. Move r
4. Move b
5. Move r

Takeaway: A description of the operation of a Turing Machine (i.e. an algorithm) is sufficient in place of a formal Turing Machine definition.



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Computational Complexity Church-Turing Thesis*: Every physically realizable computation device can be simulated by a TM with a polynomial number of extra steps.

*Controversial