CSCI 338
Homework 4

Assigned 9/20/2022, due by start of class (3:05 pm) on 9/27/2022. Please submit this assignment to the appropriate dropbox on D2L. You must follow the collaboration policy detailed on the course website.

Problem 1 (5 points). Show that the language $L = \{0^{2^n} : n \in \mathbb{N}\}$ is not regular.

Solution. Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^{2^p}$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p \implies s = 0^{p-k}0^k0^{2^p-p}$

Consider the string $s' = xy^2z = 0^{p-k}0^{2k}0^{2^p-p} = 0^{2^p+k}$

But since $k \leq p$, $2^p + k \leq 2^p + p < 2^p + 2^p = 2^{p+1}$. Thus, $2^p + k$ is not a power of two. I.e., There is no $n \in \mathbb{N}$ such that $2^p + k = 2^n$.

$\implies s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular. $\square$

Problem 2 (5 points). Show that the language $L = \{www : w \in \{0, 1\}^*\}$ is not regular.

Solution. Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = www = 0^p10^p10^p1$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$, since $|xy| \leq p \implies s = 0^{p-k}0^k10^p10^p1$

Consider the string $s' = xy^2z = 0^{p-k}0^{2k}10^p10^p1$

But, $k > 0 \implies p - k + 2k > p$, so there are more 0s in the first w than the subsequent ones. In other words, since there are exactly three 1s, there must be a 1 in each w. Since there are no trailing 0s, w must end with the 1. Thus, each w must consist of all the 0s since the last w, followed by its 1. But since the first set of 0s contains more than the subsequent sets, each w is not identical.

$\implies s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular. $\square$

Problem 3 (5 points). Show that the language $L = \{0^n1^m0^n : m, n \geq 0\}$ is not regular.

Solution. Suppose L is regular. Let p be the number from the pumping lemma.
Consider $s = 0^p10^p$.
Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.
$y = 0^k$ for some $k > 0$, since $|xy| \leq p \implies s = 0^{p-k}0^k10^p$
Consider the string $s' = xy^2z = 0^{p-k}0^{2k}10^p$
But, $k > 0 \implies p - k + 2k > p$, so there are more 0s before the 1 than after.
$\implies s' \notin L$, which is a contradiction of the pumping lemma.
Therefore, the language is not regular. $\square$

Problem 4 (5 points). Why can the string $s = 0^p1^00^p$ not be used to prove that $L = \{0^n1^m0^n : m, n \geq 0\}$ is not regular?

Solution. Suppose we were to use $s = 0^p1^00^p = 0^{2p}$. Clearly y consists of all 0s. So, we need to pump y up or down and make is so that the number of 0s cannot be broken into two equal parts (0^j0^j). The only way for that to happen is if the total number of 0s (after pumping) is odd. But, the pumping lemma says that any string *can* be divided into an xyz that suffices, so by having the length of y be even, pumping it up or down any amount results in an even number of 0's (since $2p$ is even and even $\pm$ even is still even). $\square$