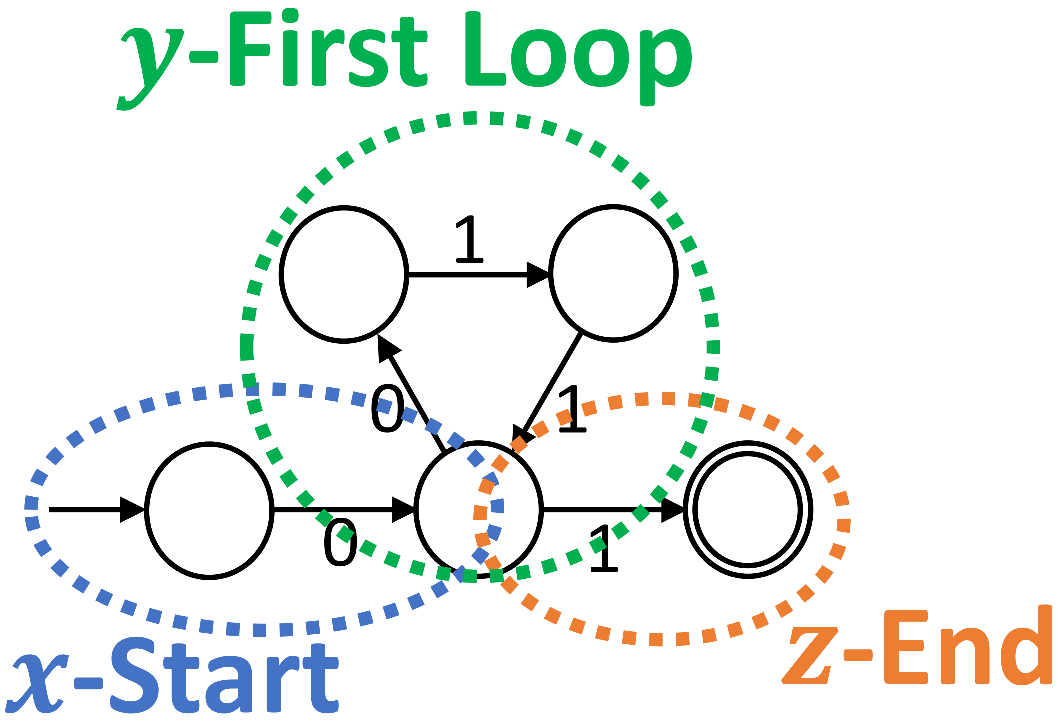
Pumping Lemma
CSCI 338

Pumping Lemma

Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

1. $xy^iz \in L, \forall i \geq 0.$
2. $|y| > 0.$
3. $|xy| \leq p.$



Pumping Lemma Proof Blueprint

Claim: Some language L is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$?. **1 – Select s that will work with $s \in L$ and $|s| \geq p$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that y must meet**

3 – Select an i (number of times to repeat y)

Consider the string $s' = xy^{?}z =$? **4 – Show what s' equals**

? **5 – Show s' is not in L**

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 4

Claim: The language $L = \{0^m 1^n : m \neq n\}$ is not regular.

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Consider $s = 0^p 1^{p+1}$.

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Clearly, y is all 0's.

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For us to violate the pumping lemma, we must violate a condition for ***every*** xyz partition.

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Let $y = 00$

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Goal: Pick an s such that repeating y (no matter what y is) is guaranteed (at some point) to make #0's equal #1's

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Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^{p+\alpha}$. $\boldsymbol{\alpha = ?}$

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Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$

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Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$$y = 0^k \text{ for some } k > 0 \Longrightarrow s = 0^{p-k} 0^k 1^{p+\alpha}$$

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Consider the string $s' = xy^i z = 0^{p-k} 0^{ik} 1^{p+\alpha}$ $\boldsymbol{i = ?}$

If #0's = #1's, then…

If we can find an $\boldsymbol{i}$ such that #0's = #1's, we have contradicted the pumping lemma.

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If #0's = #1's, then $p + (i-1)k = p + \alpha$

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If #0's = #1's, then $p + (i-1)k = p + \alpha \Rightarrow i = \frac{\alpha}{k} + 1$, for $0 < k \leq p$.

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So, α needs to be evenly divisible by k for all possible $0 < k \leq p$. Let $\alpha = ?$

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$$y = 0^k \text{ for some } k > 0 \Longrightarrow s = 0^{p-k} 0^k 1^{p+p!}$$

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 4

Claim: The language $L = \{0^m 1^n : m \neq n\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^{p+p!}$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0 \Longrightarrow s = 0^{p-k} 0^k 1^{p+p!}$

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$$A \cap B = \overline{\bar{A} \cup \bar{B}}$$

(De Morgan's Laws)

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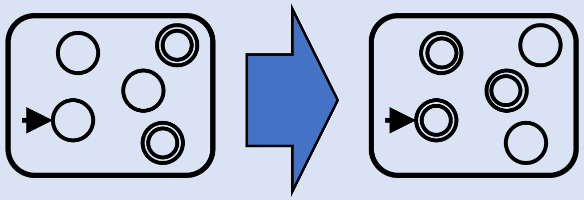

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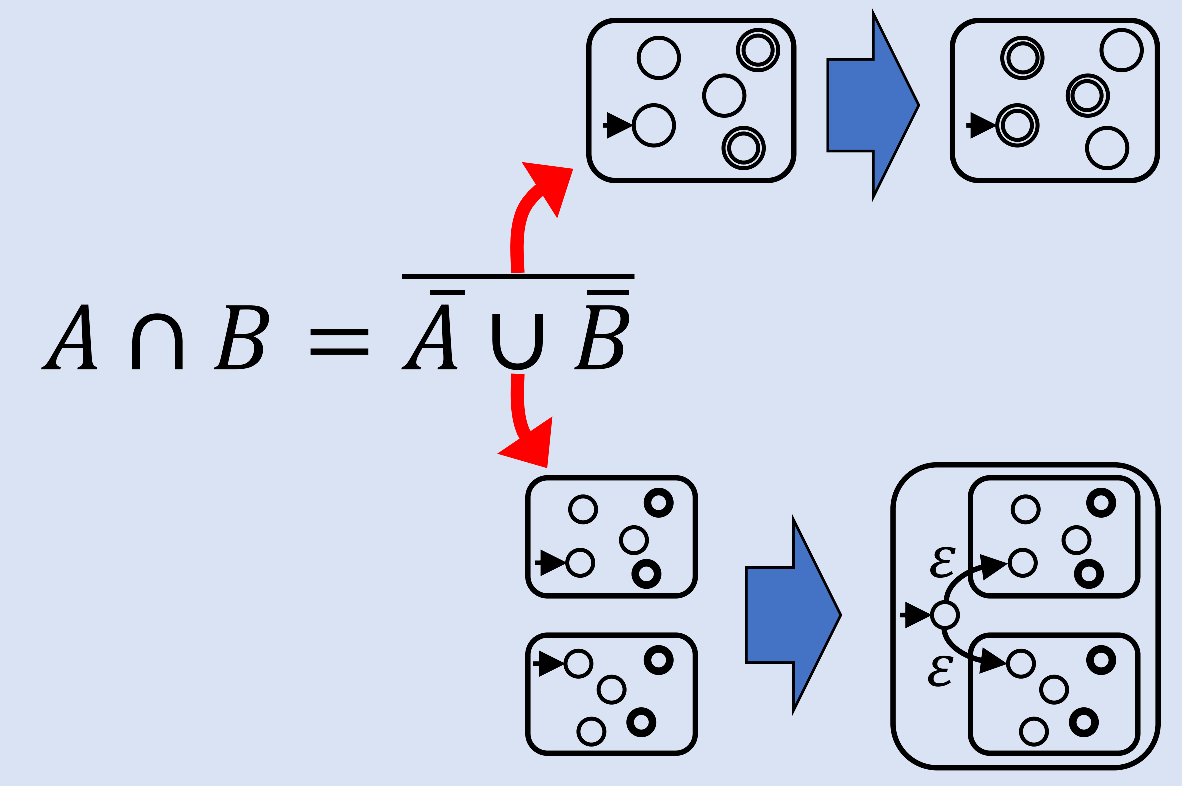

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If $\bar{L}$ was regular, so would $0^n 1^n$
(regular ∩ regular = regular)

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Consider the language $L = \{a0^n1^n : n \geq 1\} \cup \{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ over the alphabet $\Sigma = \{0,1,a\}$.

<u>Claim:</u> L satisfies the pumping lemma.

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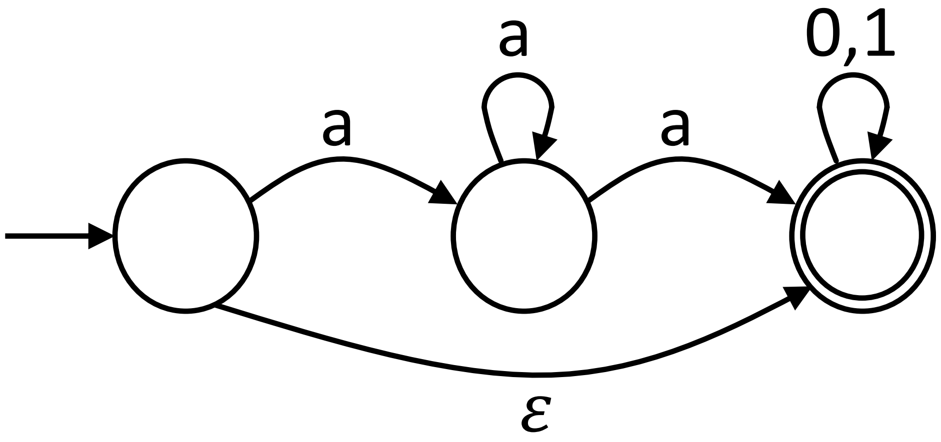

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Let $p > 0$ be any number and let s be any string from $\{a0^n1^n : n \geq 1\}$ where $|s| \geq p$.

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Let $s = xyz$ where $x = \varepsilon$, $y = a$, and z is everything else.

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If y is pumped up or down, $s' \in \{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ (0 or > 1 a).

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So L satisfies the pumping lemma.

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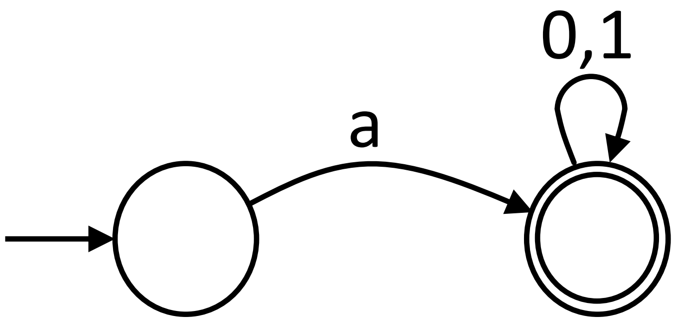

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Consider $s' = xy^0z$. Then, s' either:

- has no a (and $s' \notin L$)
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This contradicts the pumping lemma, so $L \cap M$ is not regular.

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So, M is regular and $L \cap M$ is not regular. What about L?

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Thus, L cannot be regular.

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