Introduction

CSCI 338

True or False?

Finding shortest paths in a graph can be done easily (i.e. in polynomial time).

True or False?

Finding shortest [illegible] can be done [illegible] ynomial time).

TRUE – Dijkstra’s Algorithm

True or False?

That cost at most 20 “units”

Finding shortest paths in a graph can be done easily (i.e. in polynomial time).

True or False?

That cost at most 20 “units”

Finding short[illegible] ph can be done [illegible] polynomial time).

UNKNOWN!!!

True or False?

If efficiency does not matter, a computer can solve any computational problem you can give it.

True or False?

If efficiency do

solve an   can give it.

FALSE – Unsolvable problems exist, regardless of the computer!

338 Goals:

338 Goals:

1. Become better problem solvers.
2. Understand and identify fundamental limitations of computing.

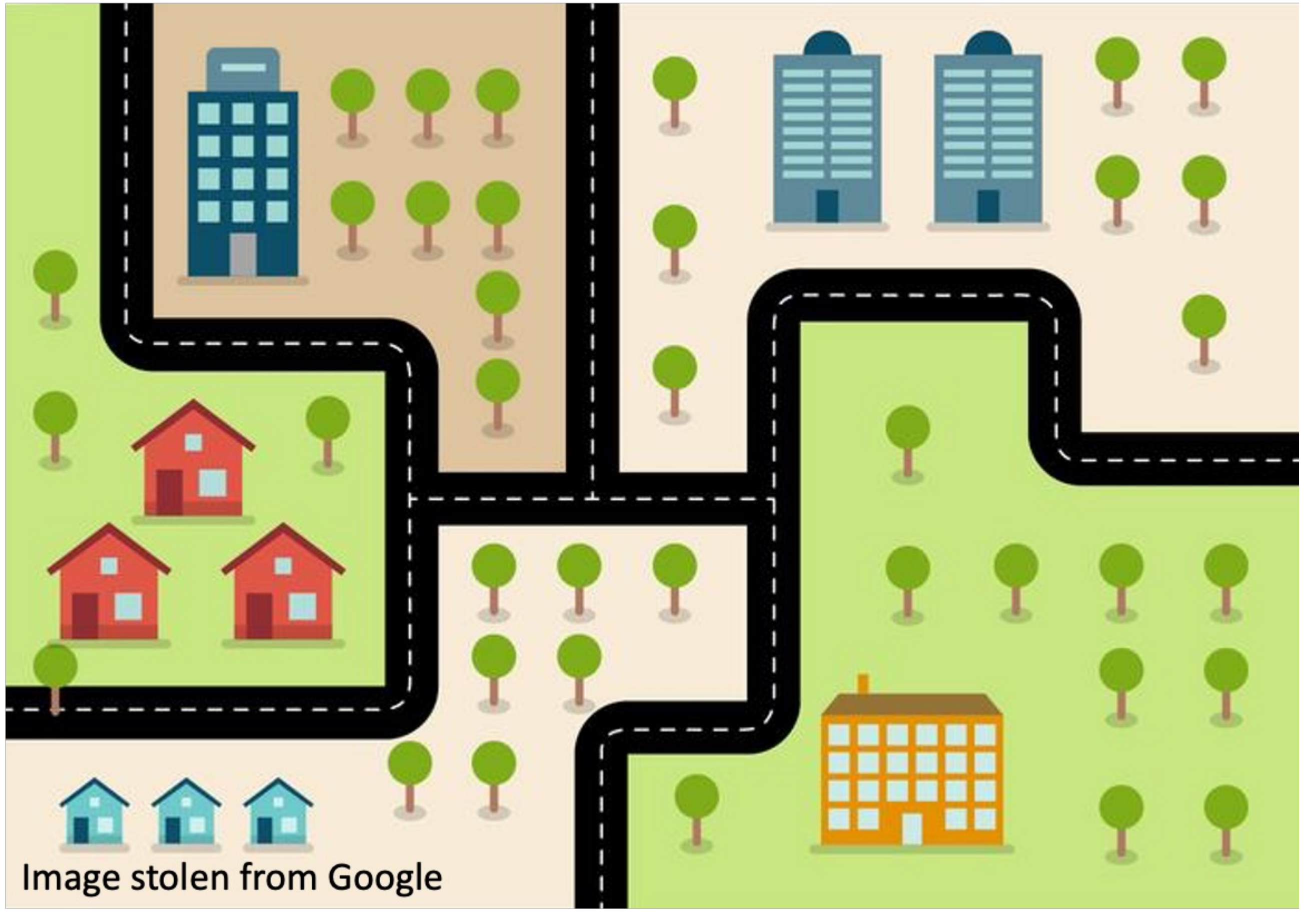
Mathematical Model

Mathematical Model

Mathematical Model:

- A rigorous mathematical formulation of reality.
- Used to make predictions.

Mathematical Model



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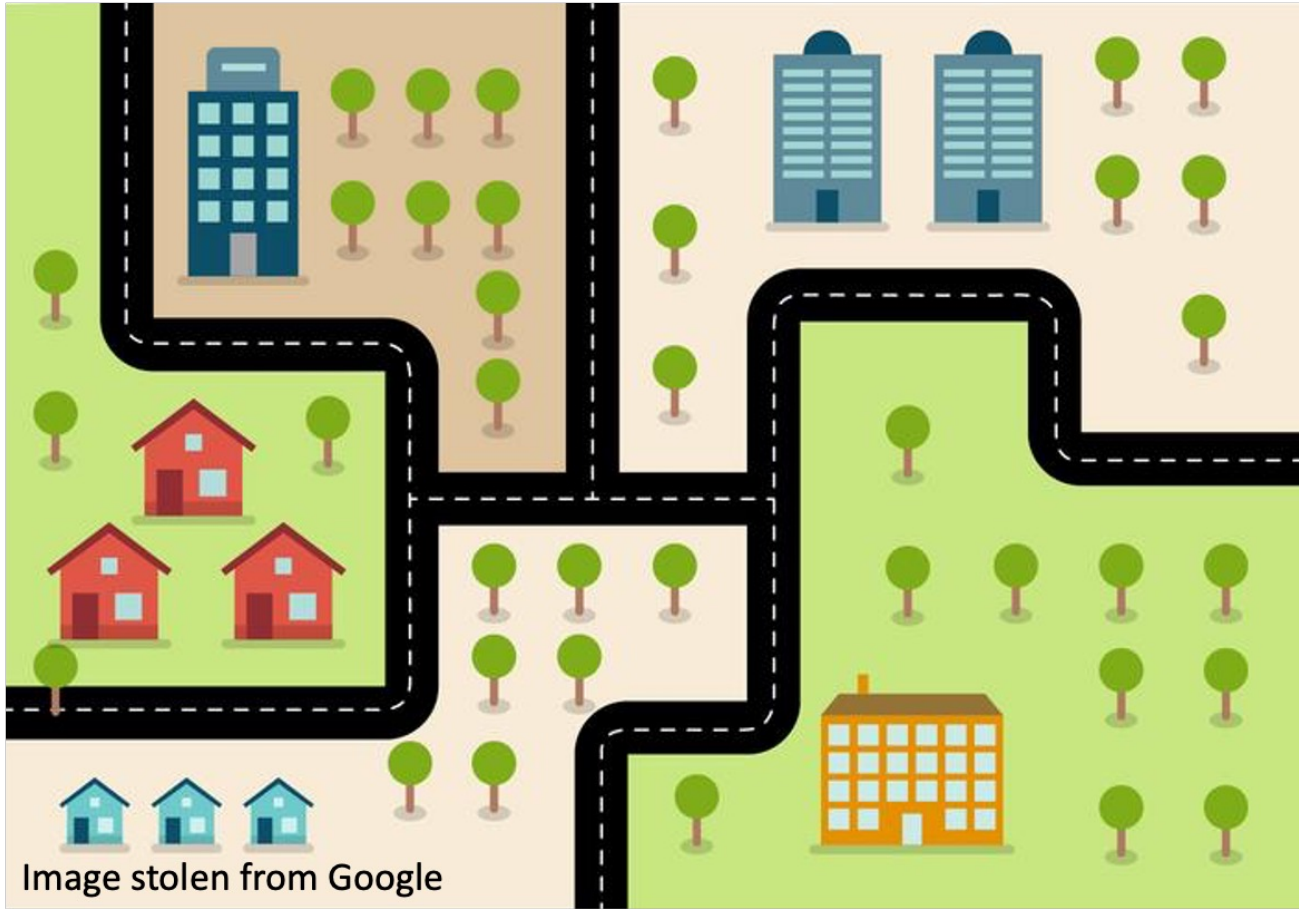
Can we represent a road network as a mathematical model?

Mathematical Model



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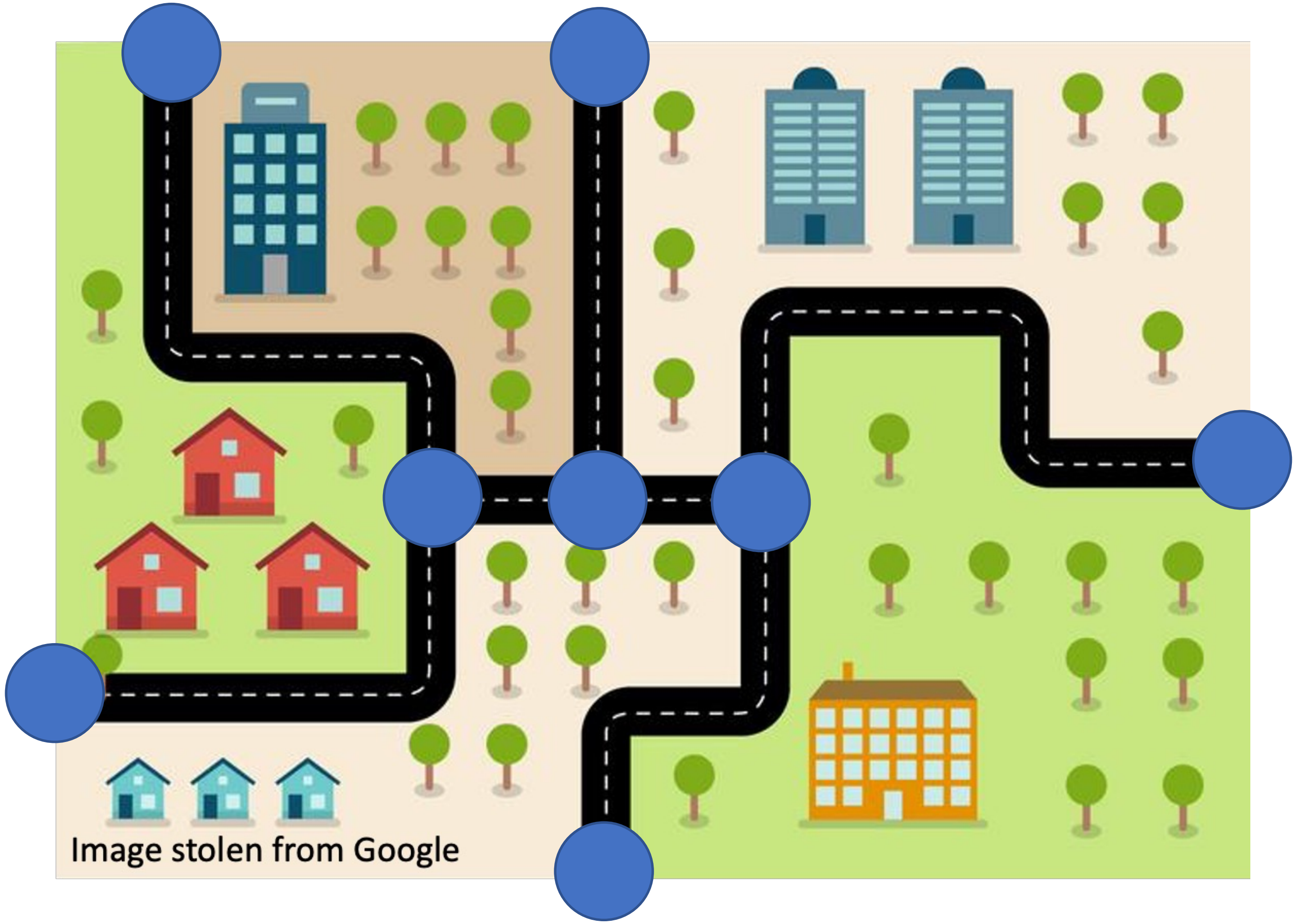

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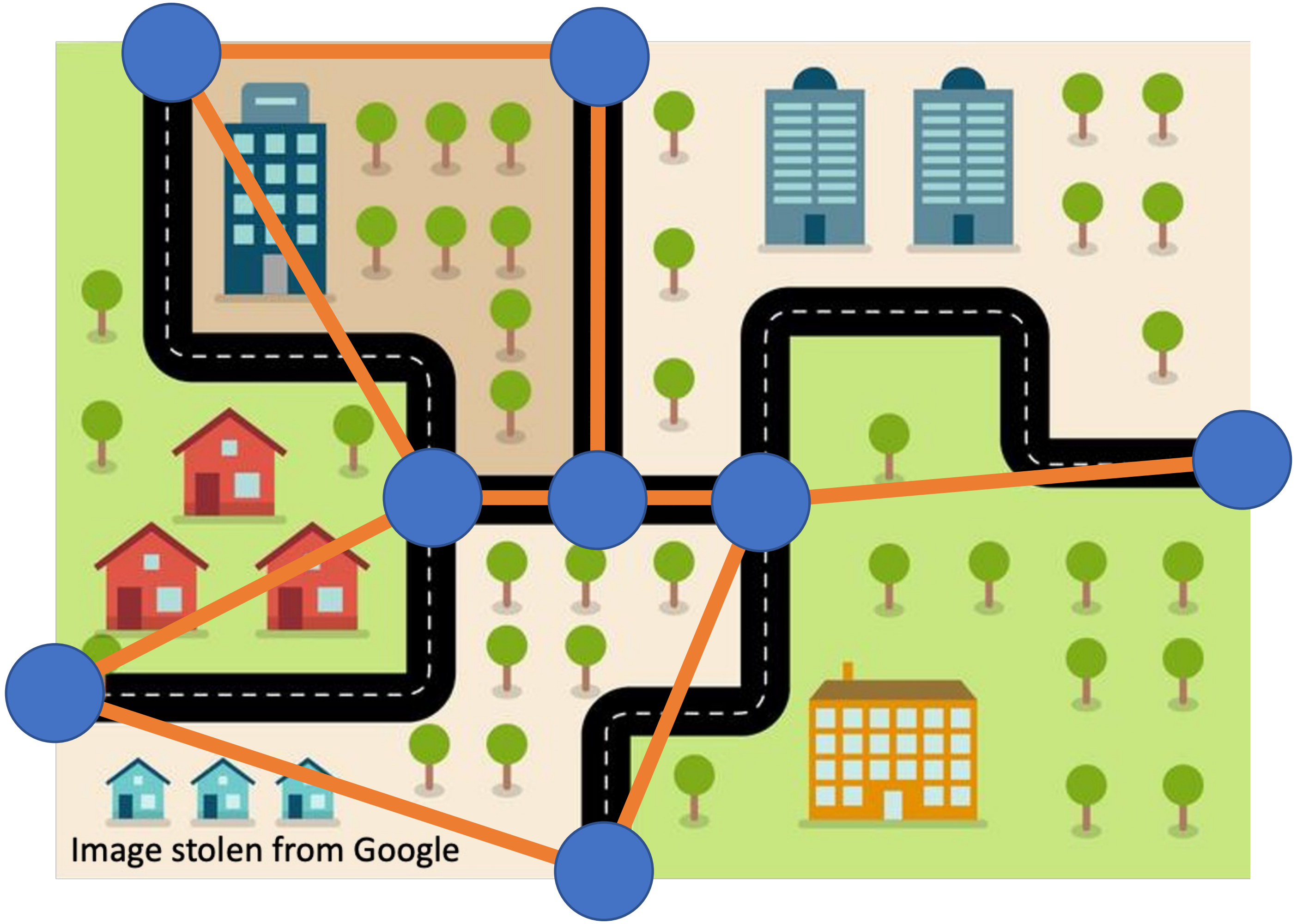

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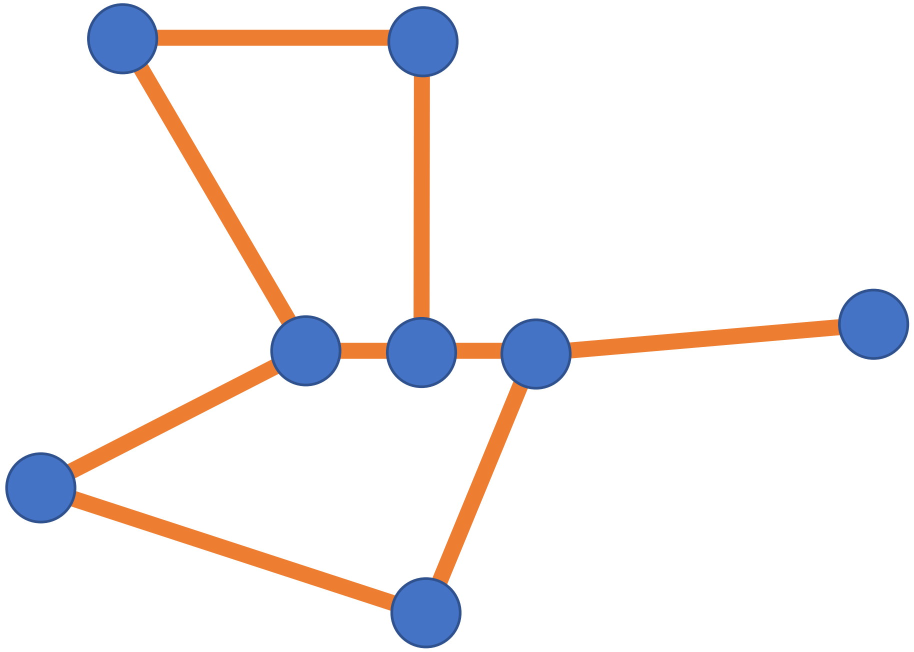
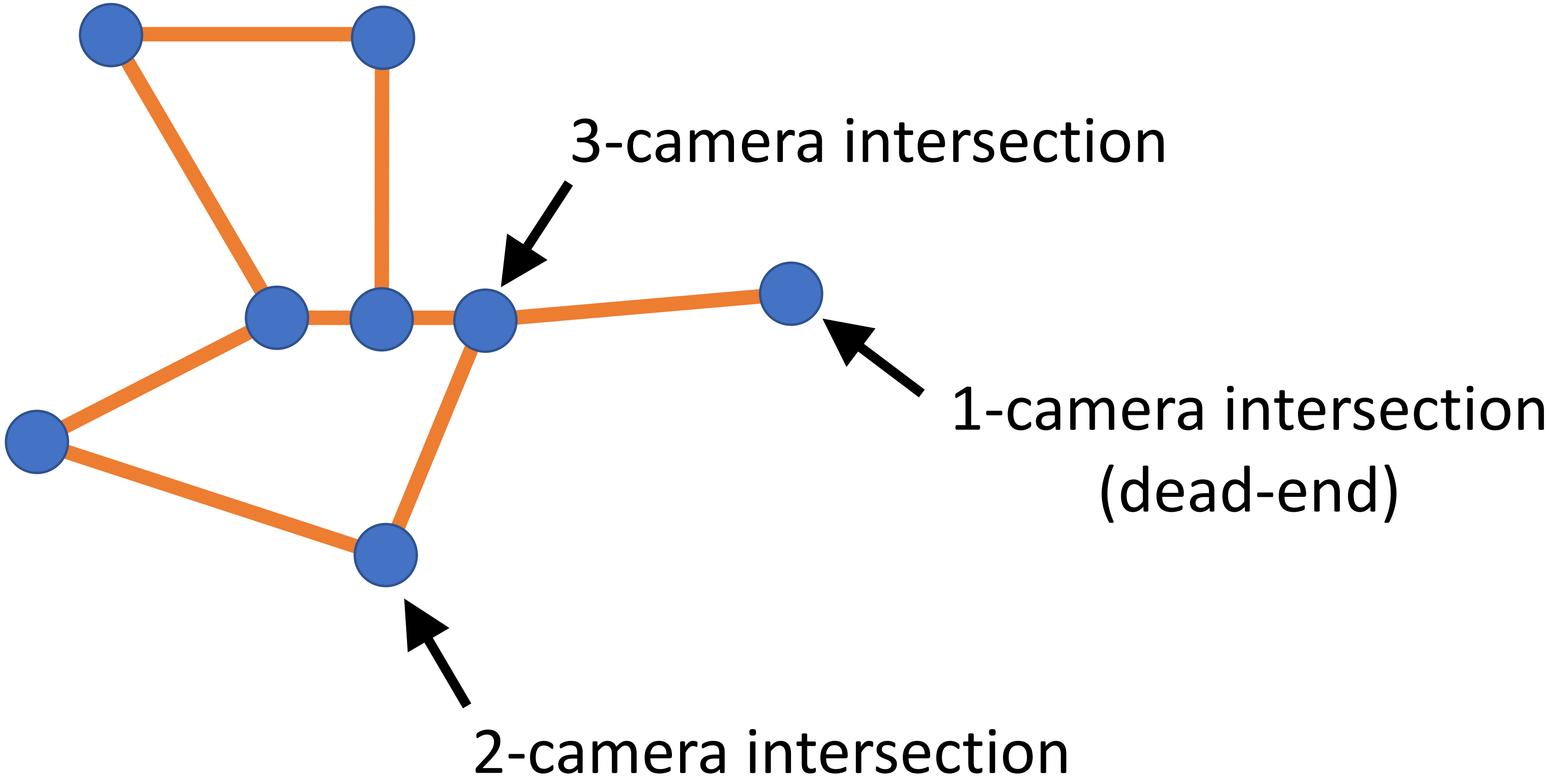


Let’s use our mathematical model of a road network to answer some questions.

Mathematical Model

Each intersection requires a camera to monitor each road segment.

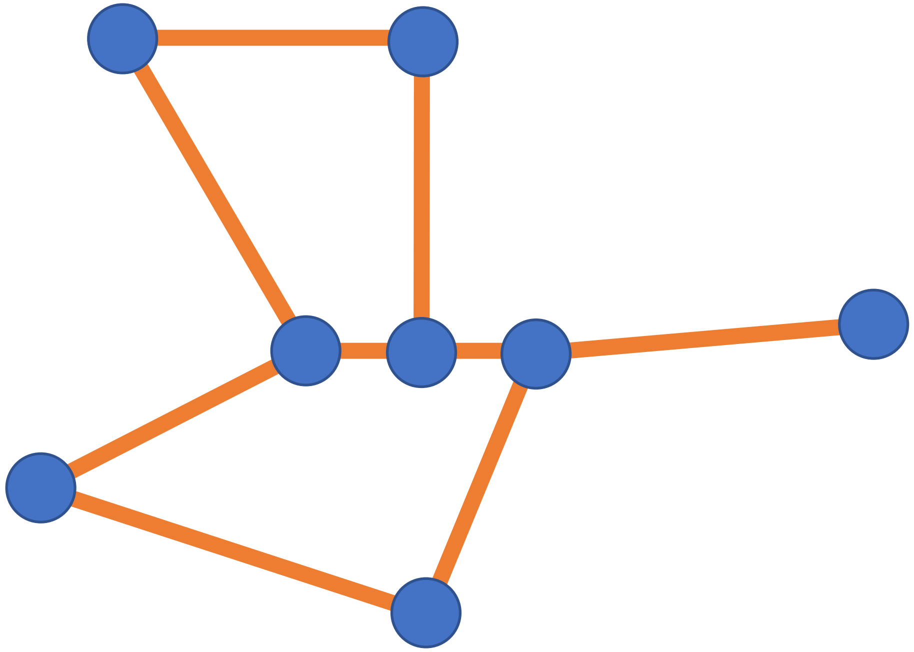


3-camera intersection

1-camera intersection
(dead-end)

2-camera intersection

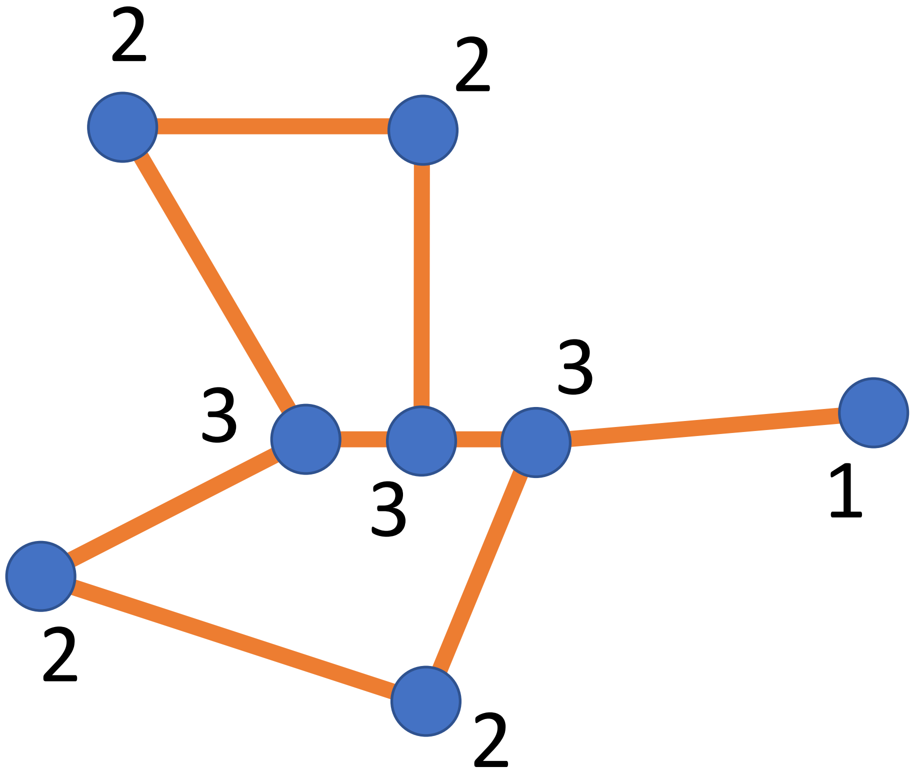
Mathematical Model



Each intersection requires a camera to monitor each road segment.

Can we build a road network so that the number of cameras we need is odd?

Mathematical Model



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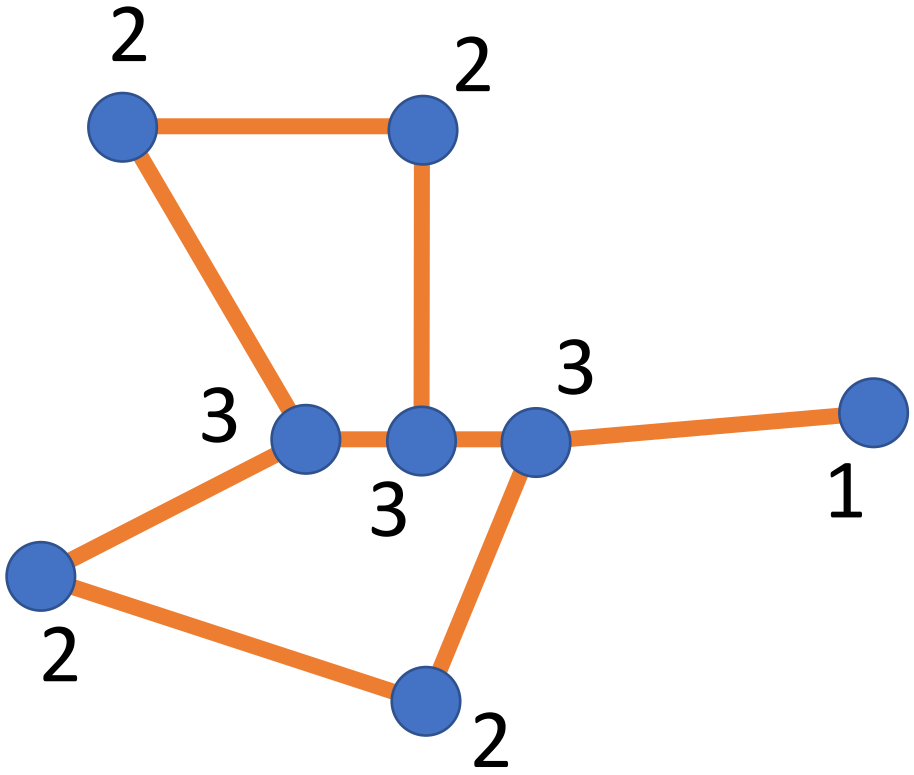

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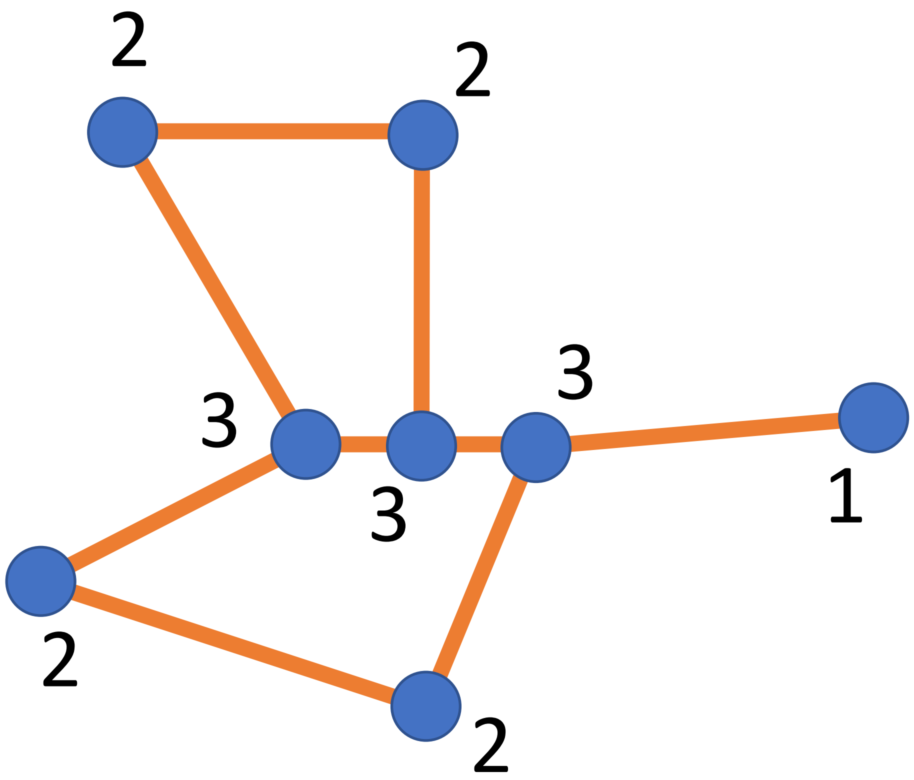
Can we build a road network so that the number of cameras we need is odd?

2+2+3+3+3+2+2+1 = 18, which is even.

Mathematical Model



Each intersection requires a camera to monitor each road segment.

Can we build a road network so that the number of cameras we need is odd?

2+2+3+3+3+2+2+1 = 18, which is even. Therefore, we cannot build a road network so that the number of cameras needed is odd.

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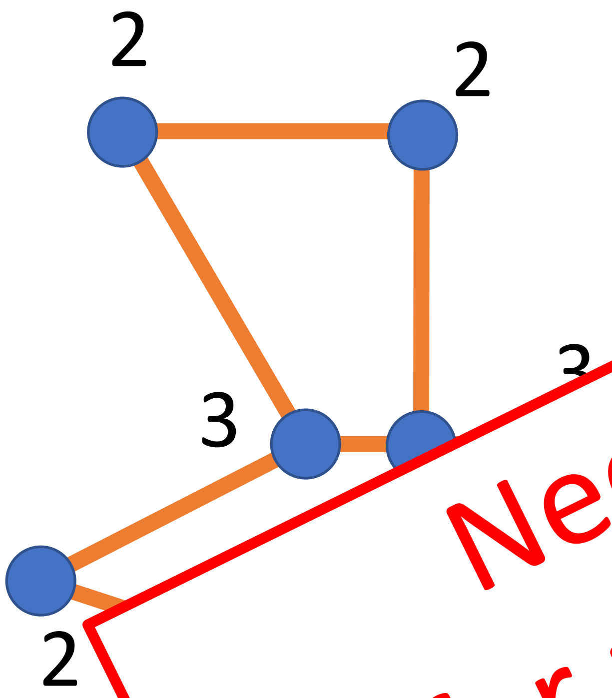

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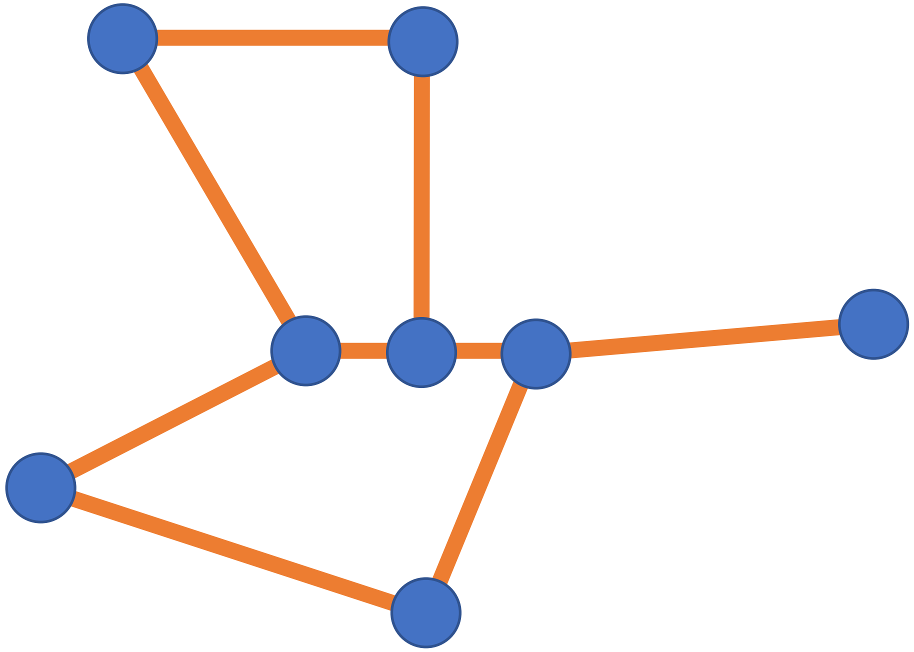
Need to prove in general

I.e. for **any** graph, not one example!

Mathematical Model



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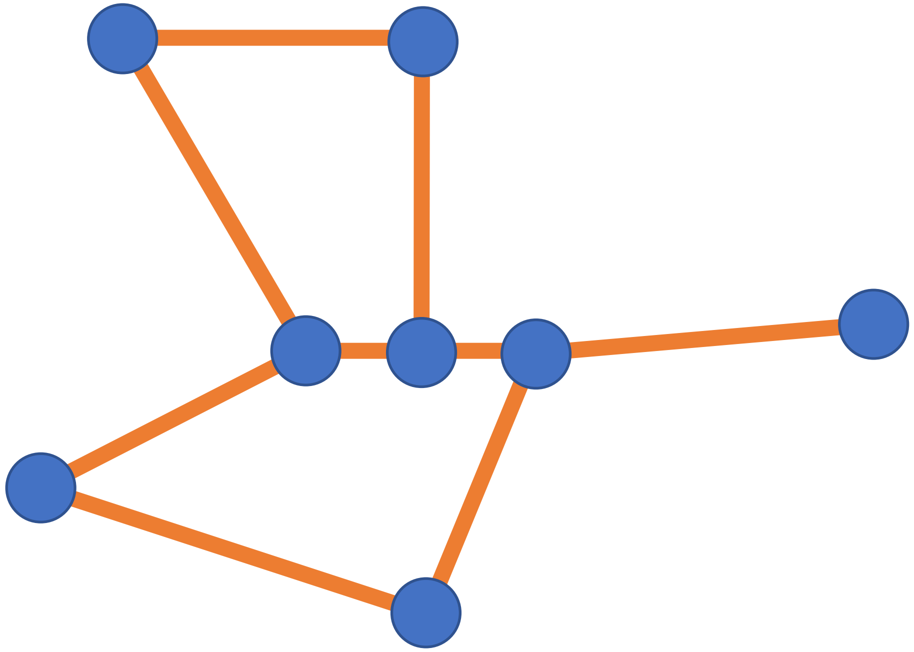

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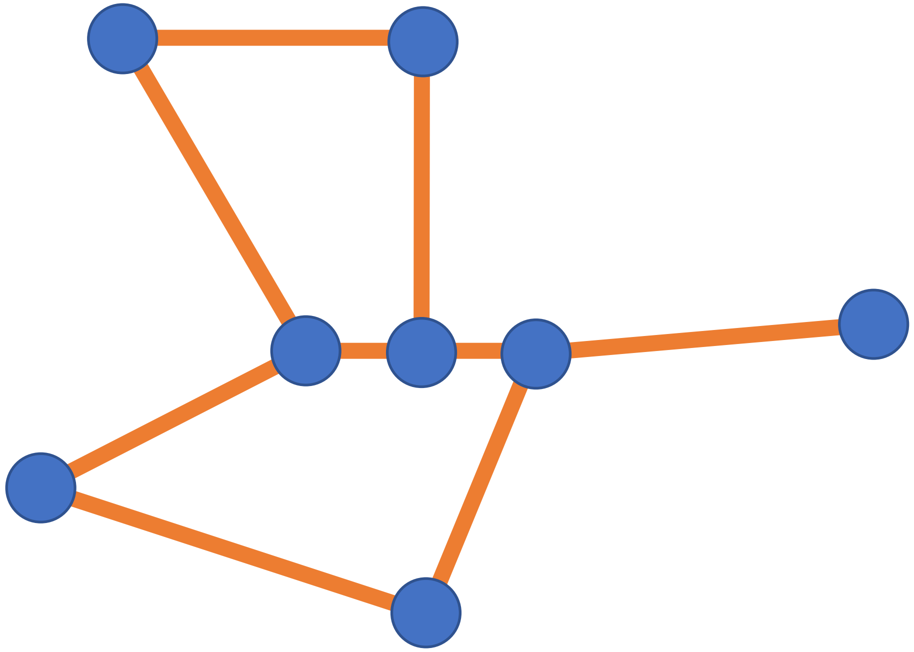
Can we build a road network so that the number of cameras we need is odd?

Each road (edge) adds ???? to the number of required cameras.

Mathematical Model



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Can we build a road network so that the number of cameras we need is odd?

Each road (edge) adds two to the number of required cameras.

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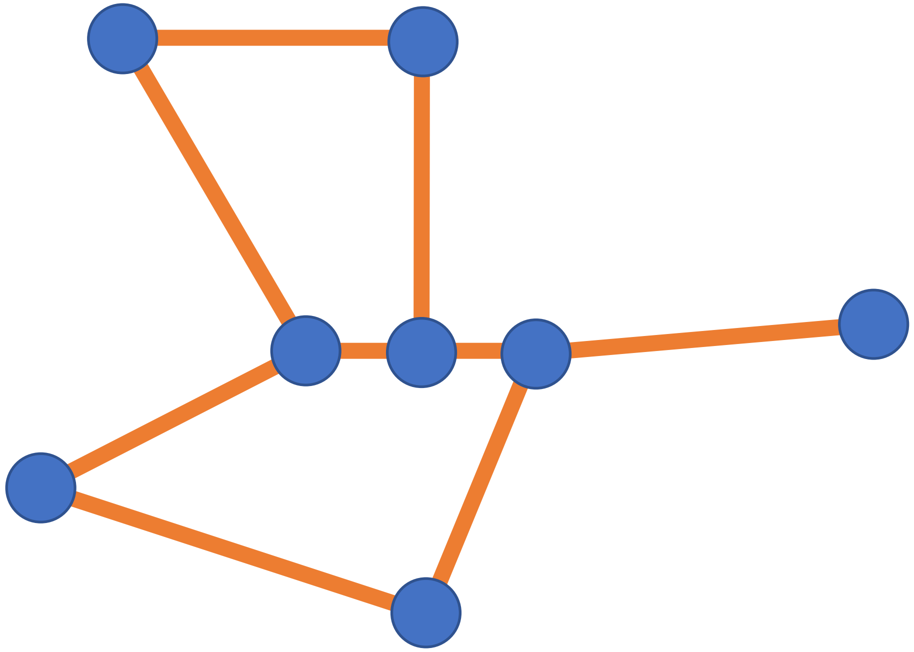

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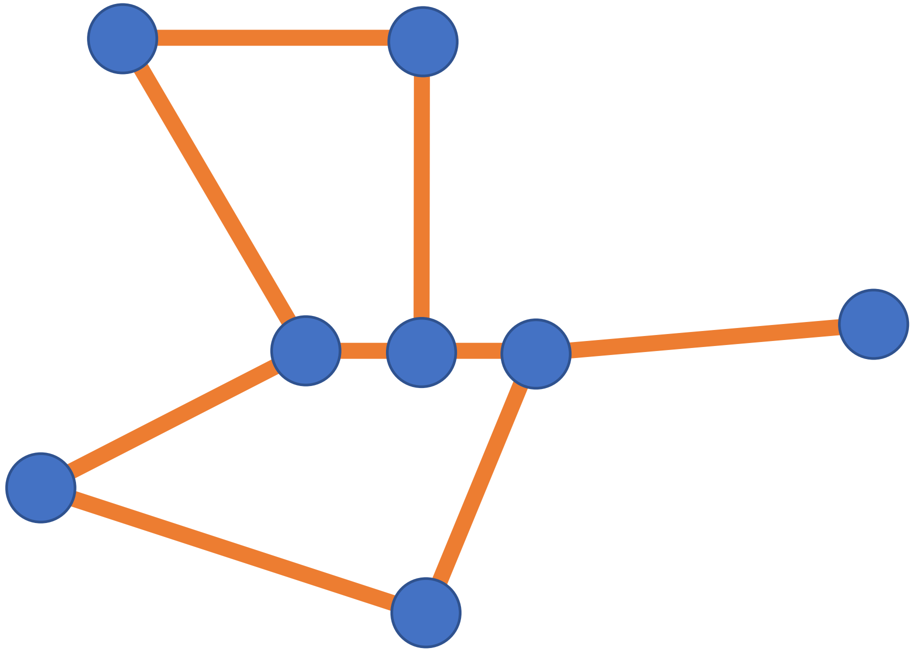
Can we build a road network so that the number of cameras we need is odd?

Each road (edge) adds two to the number of required cameras. So, if the road network (graph) has r roads (edges), the number of required cameras is ??

Mathematical Model



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Can we build a road network so that the number of cameras we need is odd?

Each road (edge) adds two to the number of required cameras. So, if the road network (graph) has r roads (edges), the number of required cameras is $2r$.

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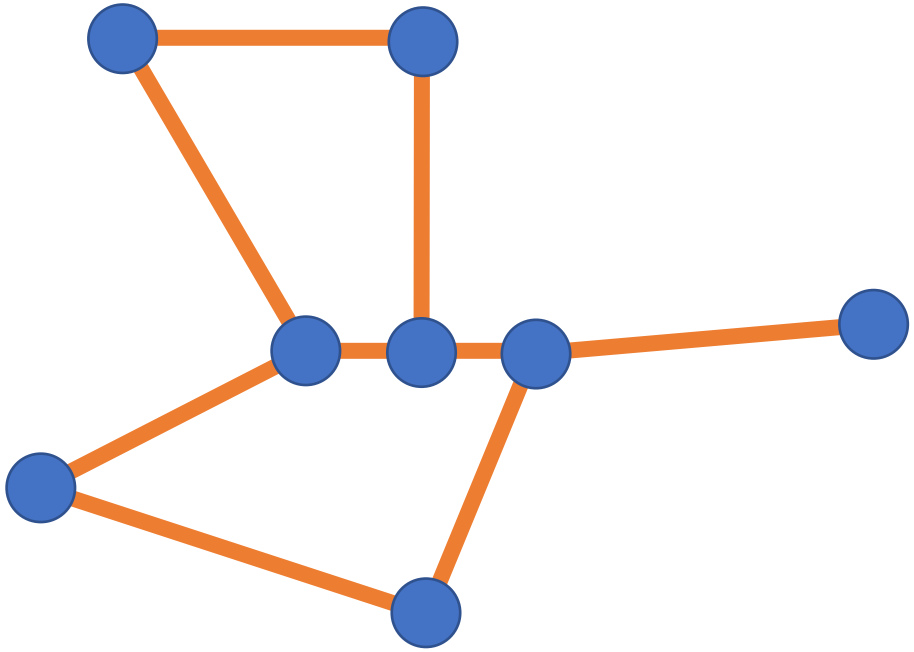

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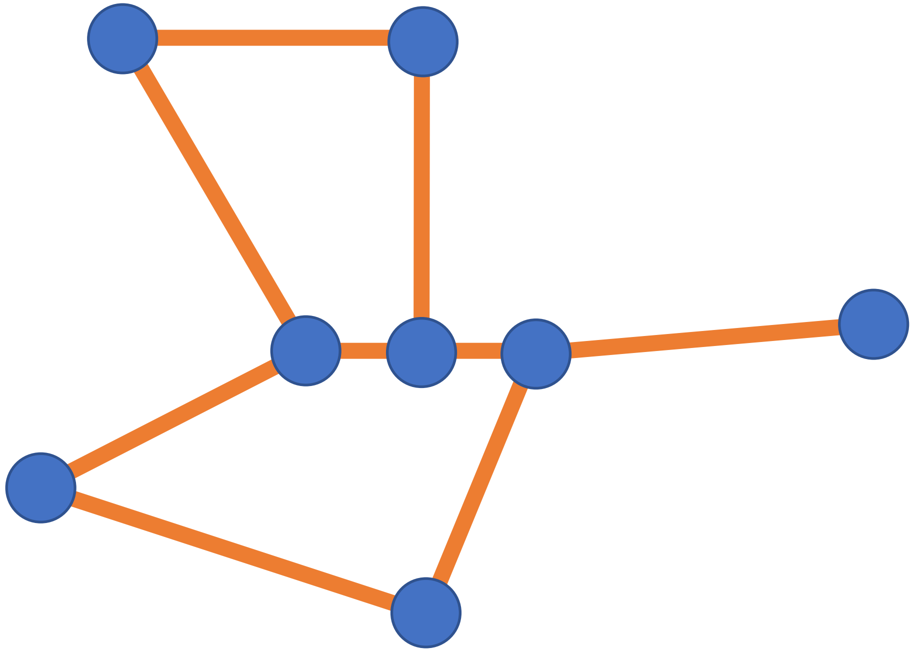

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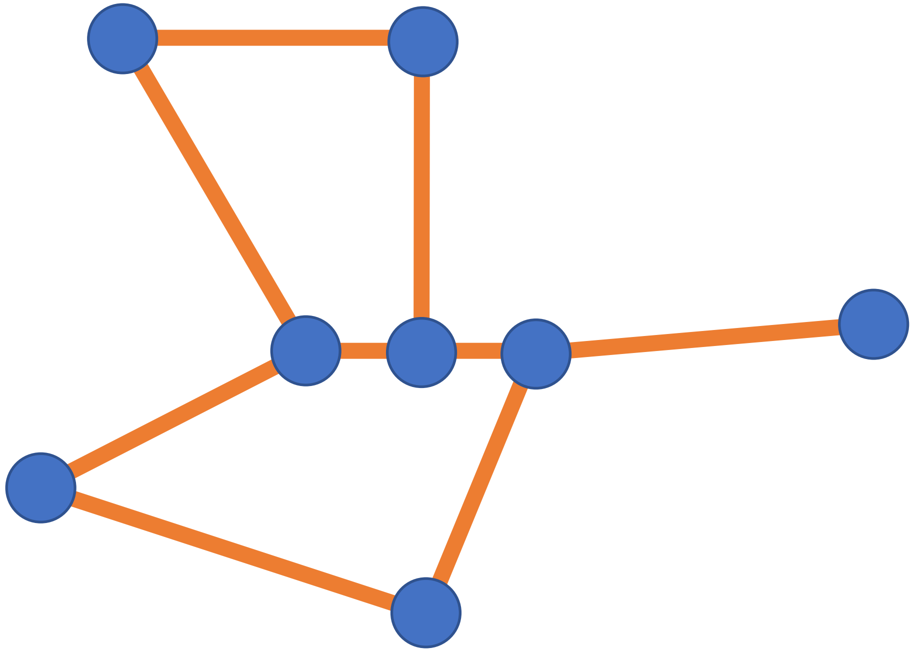

Each intersection requires a camera to monitor each road segment.

Can we build a road network so that the number of cameras we need is odd?

NO!

Each road (edge) adds two to the number of required cameras. So, if the road network (graph) has r roads (edges), the number of required cameras is $2r$, which is even.

What Did We Do?

What Did We Do?

Image stolen from Google

Step 1: Considered an ill-defined, abstract, “thing”.

What Did We Do?


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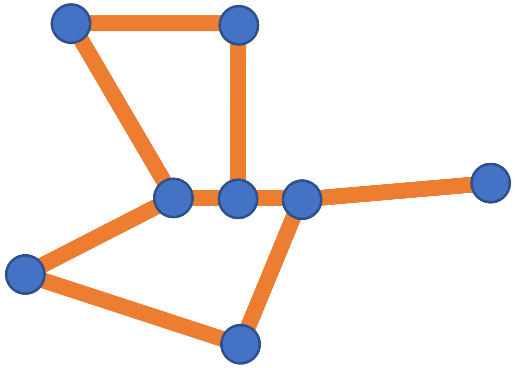

Step 1: Considered an ill-defined, abstract, “thing”.

Step 2: Built a formal model of it.

What Did We Do?

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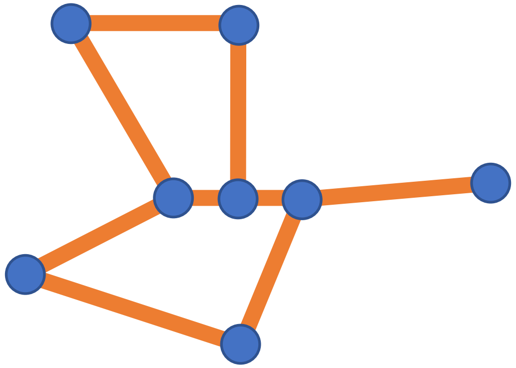

Impossible to build road network requiring an odd number of camera?

Step 1: Considered an ill-defined, abstract, “thing”.

Step 2: Built a formal model of it.

Step 3: Found limitations of the model, which translated to limitations of the “thing”.

What Did We Do?

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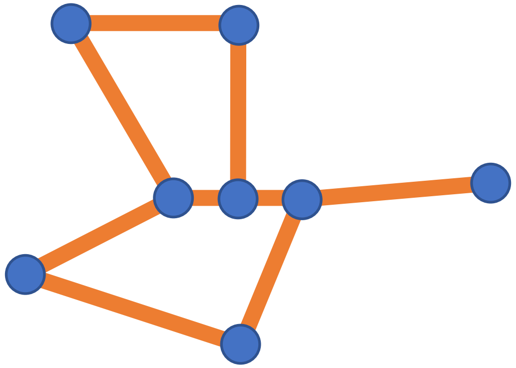

Impossible to build road network requiring an odd number of camera?

CSCI 338:

Step 1: Consider a computer.

Step 2: Build mathematical model of a computer.

Step 3: Find limitations of model, which translate to limitations of computers.

What Did We Do?

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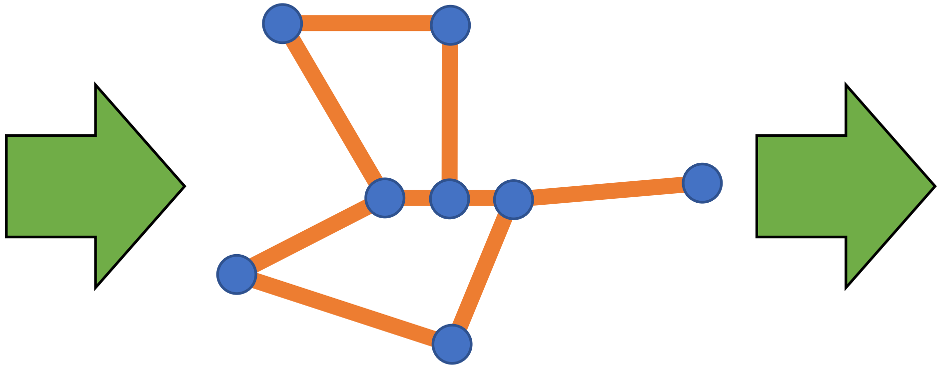

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What Will CSCI 338 Require?

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Mathematical Notation

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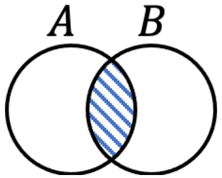
Mathematical Notation

Intersection: ∩



$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

What Will CSCI 338 Require?

Mathematical
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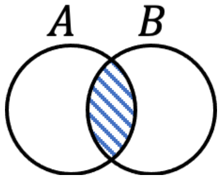

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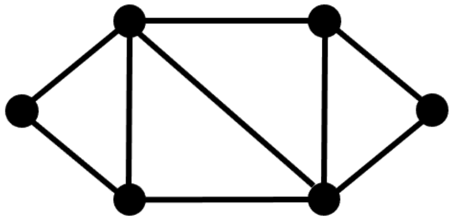
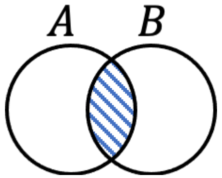


Degree of a vertex, Path, Connected Graph, Cycle, Tree

What Will CSCI 338 Require?

Mathematical Notation

Proving Things



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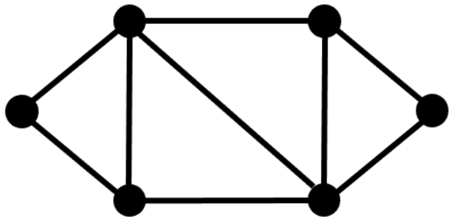

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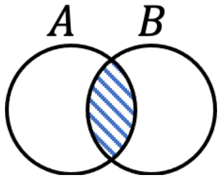
Mathematical Notation



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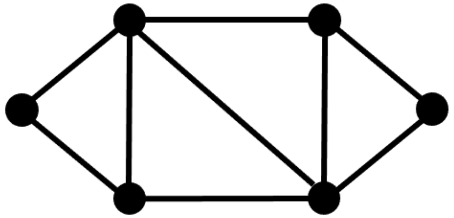

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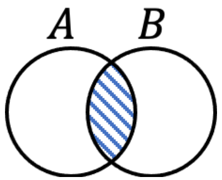
Proving Things

Common types of proof:

- Direct (proof by construction)
- Proof by contradiction
- Counterexample
- Induction

What Will CSCI 338 Require?

Mathematical Notation



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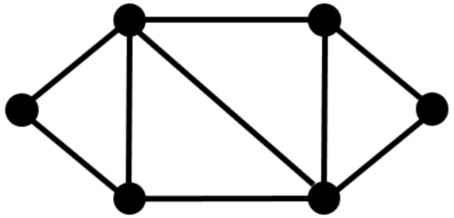

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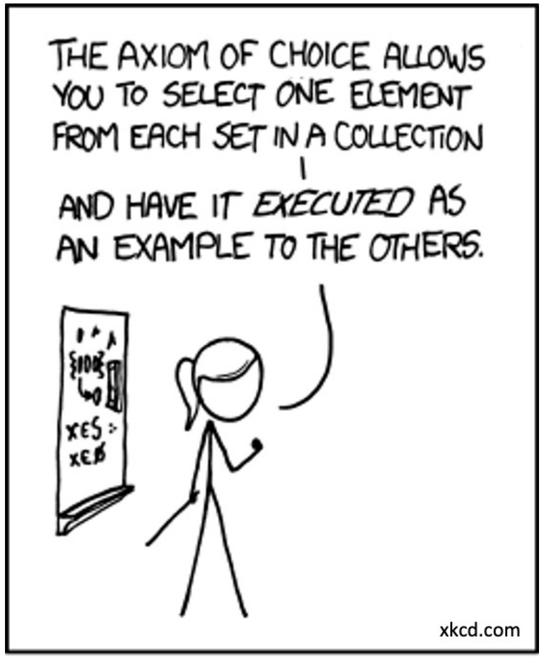


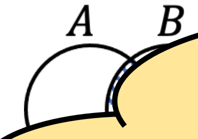
MY MATH TEACHER WAS A BIG BELIEVER IN PROOF BY INTIMIDATION.

What Will CSCI 338 Require?

Mathematical Notation

Proving Things

Curiosity



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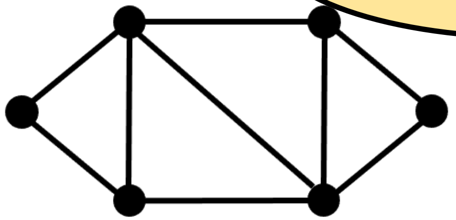

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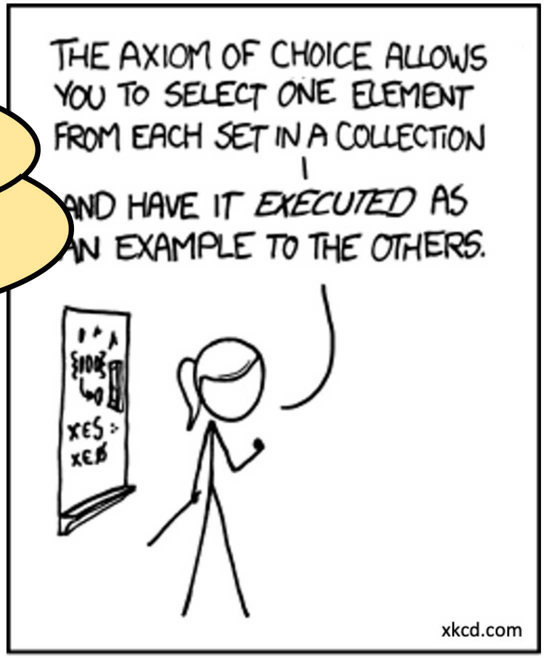

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