Regular Expressions
CSCI 338

String Construction

Accept | Reject

0^*: Zero or more 0’s

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's		

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
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$(0^*1)^*$: ?		

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ε?

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ε?

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	ε	

$1?$

String Construction

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0^*: Zero or more 0’s	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01’s	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1$	

$$1?$$

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
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$(0^*1)^*$: ?	$\varepsilon, 1$	

$$10?$$

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1$	10

$$10?$$

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1$	10

$111?$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111$	10

$$111?$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111$	10

$$000?$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111$	$10, 000$

$$000?$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111$	$10, 000$

$$01001?$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001$	$10, 000$

$$01001?$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001$	$10, 000$

$$101?$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001, 101$	$10, 000$

$101?$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001, 101$	$10, 000$

$$0001110110001$$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001, 101$	$10, 000$

$$0001\,1\,1\,01\,1\,0001$$

Accept if string can be broken into sequences of 0^*1

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001, 101$	$10, 000$

$$0001\,1\,1\,01\,1\,0001$$

$$0001110111000$$

Accept if string can be broken into sequences of 0^*1

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: ?	$\varepsilon, 1, 111, 01001, 101$	$10, 000$

$0001\,1\,1\,01\,1\,0001$

$0001110111\boxed{000}$

Accept if string can be broken into sequences of 0^*1

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: Doesn't end with 0	$\varepsilon, 1, 111, 01001, 101$	$10, 000$

$$0001\,1\,1\,01\,1\,0001$$

$$0001110111\,000$$

Accept if string can be broken into sequences of 0^*1

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: Doesn't end with 0	$\varepsilon, 1, 111, 01001, 101$	$10, 000$
0^+: One or more 0's	$0, 0000$	$\varepsilon, 1$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: Doesn't end with 0	$\varepsilon, 1, 111, 01001, 101$	$10, 000$
0^+: One or more 0's	$0, 0000$	$\varepsilon, 1$
$(001^+)^*$	$001, 0011, 0010011, \varepsilon$	$1, 00$

String Construction

	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: Doesn't end with 0	$\varepsilon, 1, 111, 01001, 101$	$10, 000$
0^+: One or more 0's	$0, 0000$	$\varepsilon, 1$
$(001^+)^*$	$001, 0011, 0010011, \varepsilon$	$1, 00$
$1^*(001^+)^*$	$\varepsilon, 1, 1001, 10010011, 001$	$00, 101$

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	Accept	Reject
0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: Doesn't end with 0	$\varepsilon, 1, 111, 01001, 101$	$10, 000$
0^+: One or more 0's	$0, 0000$	$\varepsilon, 1$
$(001^+)^*$	$001, 0011, 0010011, \varepsilon$	$1, 00$
$1^*(001^+)^*$	$\varepsilon, 1, 1001, 10010011, 001$	$00, 101$
$(0 \cup 1)$: A single 0 or 1	$1, 0$	$\varepsilon, 00, 101$

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
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0^+: One or more 0's	$0, 0000$	$\varepsilon, 1$
$(001^+)^*$	$001, 0011, 0010011, \varepsilon$	$1, 00$
$1^*(001^+)^*$	$\varepsilon, 1, 1001, 10010011, 001$	$00, 101$
$(0 \cup 1)$: A single 0 or 1	$1, 0$	$\varepsilon, 00, 101$

$(0 \cup 1)0^*$: A 0 or 1 followed by zero or more 0s.

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$: Doesn't end with 0	$\varepsilon, 1, 111, 01001, 101$	$10, 000$
0^+: One or more 0's	$0, 0000$	$\varepsilon, 1$
$(001^+)^*$	$001, 0011, 0010011, \varepsilon$	$1, 00$
$1^*(001^+)^*$	$\varepsilon, 1, 1001, 10010011, 001$	$00, 101$
$(0 \cup 1)$: A single 0 or 1	$1, 0$	$\varepsilon, 00, 101$

$(0 \cup 1)0^*$: A 0 or 1 followed by zero or more 0s.

$(0 \cup 1)^*$: A string with any number of 0s and 1s.

String Construction

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0^*: Zero or more 0's	$0, 0000, \varepsilon$	$1, 0001, 1000$
$(01)^*$: Zero or more 01's	$01, 010101, \varepsilon$	$10, 001, 01010$
$(0^*1)^*$:		$0, 000$
0^+: One		$\varepsilon, 1$
$(001^+)^*$		$1, 00$
$1^*(001^+$		$0, 101$
$(0 \cup 1)$:		$00, 101$

$(0 \cup 1)0^*$: A 0 or 1 followed by zero or more 0s.

$(0 \cup 1)^*$: A string with any number of 0s and 1s.

Regular Expressions:

- **Algebraic formula that specifies a pattern of strings (i.e. a language).**
- **Used for text matching/searching**

Regular Expressions

Rules for building regular expressions (regex):

1. Each $e \in \Sigma$ is a regex

Regular Expressions

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2. $\{\varepsilon\}$ is a regex ← Language with one string: The empty string.

Regular Expressions

Rules for building regular expressions (regex):

1. Each $e \in \Sigma$ is a regex
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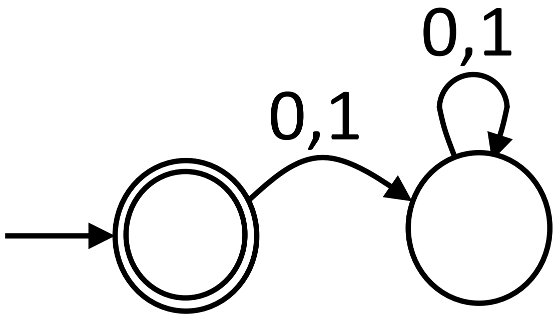
3. $\emptyset$ is a regex ← Language with no strings.

Regular Expressions

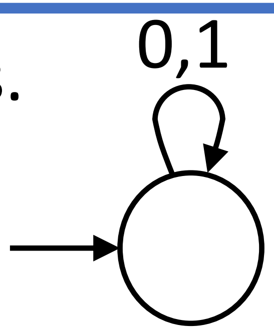
Rules for building regular expressions (regex):

1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex
3. $\emptyset$ is a regex

Language with one string: The empty string.



Language with no strings.



Regular Expressions

Rules for building regular expressions (regex):

1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex ← Language with one string: The empty string.
3. $\emptyset$ is a regex ← Language with no strings.
4. $(R_1 \cup R_2)$ is a regex

R_1 and R_2 are regexs

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5. $(R_1 \circ R_2)$ is a regex

R_1 and R_2 are regexs

Regular Expressions

Rules for building regular expressions (regex):

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2. $\{\varepsilon\}$ is a regex ⟵ Language with one string: The empty string.
3. $\emptyset$ is a regex ⟵ Language with no strings.
4. $(R_1 \cup R_2)$ is a regex
5. $(R_1 \circ R_2)$ is a regex
6. $R_1{}^*$ is a regex

R_1 and R_2 are regexs

Regular Expressions

Regular Expression notation:

- R^* (i.e. zero or more strings from R)
  e.g. 1^* includes: $1, 11111111, \varepsilon$
- $RR = R \circ R$ (i.e. two strings from R concatenated)
  e.g. 1^*0 includes: $10, 111111110, 0$
- $R^+ = RR^*$ (i.e. at least one string from R)
  e.g. 1^+ includes: $1, 11111111$, but not ε

Order of operations:

- Parentheses, star (and plus), concatenation, union.

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 =$?

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 =$?

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\text{: } w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w\text{: } w$ ends in $1\}$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w : w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w : w \text{ ends in } 1\}$
- $\{w : w \text{ contains a single } 1\} =$?

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w\colon w \text{ ends in } 1\}$
- $\{w\colon w \text{ contains a single } 1\} = 0^*10^*$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in } 1\}$
- $\{w: w \text{ contains a single } 1\} = 0^*10^*$
- $\{w: w \text{ contains at least one } 1\} = ?$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w\colon w$ ends in $1\}$
- $\{w\colon w$ contains a single $1\} = 0^*10^*$
- $\{w\colon w$ contains at least one $1\} = \Sigma^*1\Sigma^*$ or $(0 \cup 1)^*1(0 \cup 1)^*$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w\colon w$ ends in $1\}$
- $\{w\colon w$ contains a single $1\} = 0^*10^*$
- $\{w\colon w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* =$?

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Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w : w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w : w \text{ ends in } 1\}$
- $\{w : w \text{ contains a single } 1\} = 0^*10^*$
- $\{w : w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w : w \text{ has even length}\}$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w: w$ ends in $1\}$
- $\{w: w$ contains a single $1\} = 0^*10^*$
- $\{w: w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w$ has even length$\}$
- $\{w$: every 0 is followed by at least one $1\} =$?

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in } 1\}$
- $\{w: w \text{ contains a single } 1\} = 0^*10^*$
- $\{w: w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w: w$ ends in $1\}$
- $\{w: w$ contains a single $1\} = 0^*10^*$
- $\{w: w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w$ has even length$\}$
- $\{w$: every 0 is followed by at least one $1\} = 1^*(01^+)^*$
- $1^*\emptyset =$?

By definition, $A \circ B = \{xy: x \in A, y \in B\}$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w\colon w \text{ ends in } 1\}$
- $\{w\colon w \text{ contains a single } 1\} = 0^*10^*$
- $\{w\colon w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w\colon w \text{ has even length}\}$
- $\{w\colon \text{every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$

By definition, $A \circ B = \{xy\colon x \in A, y \in B\}$
Since there is no element in $\emptyset$, there cannot be any xy such that $y \in \emptyset$.

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\!: w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w\!: w$ ends in $1\}$
- $\{w\!: w$ contains a single $1\} = 0^*10^*$
- $\{w\!: w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w\!: w$ has even length$\}$
- $\{w$: every 0 is followed by at least one $1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon =$?

By definition, $A \circ B = \{xy\!: x \in A, y \in B\}$

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Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w : w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w : w$ ends in $1\}$
- $\{w : w$ contains a single $1\} = 0^*10^*$
- $\{w : w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w : w$ has even length$\}$
- $\{w$: every 0 is followed by at least one $1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$

By definition, $A \circ B = \{xy : x \in A, y \in B\}$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w\colon w$ ends in $1\}$
- $\{w\colon w$ contains a single $1\} = 0^*10^*$
- $\{w\colon w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w\colon w$ has even length$\}$
- $\{w\colon$ every 0 is followed by at least one $1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* =$?

By definition, $A^* = \{x_1x_2 \ldots x_k\colon k \geq 0, x_i \in A\}$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\colon w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w\colon w \text{ ends in } 1\}$
- $\{w\colon w \text{ contains a single } 1\} = 0^*10^*$
- $\{w\colon w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w\colon w \text{ has even length}\}$
- $\{w\colon \text{every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$

By definition, $A^* = \{x_1x_2 \ldots x_k\colon k \geq 0, x_i \in A\}$
Thus, it can append 0 elements of $\emptyset$ and get
the empty string ε.

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$
- $(1 \cup 0)^*1 = \{w: w$ ends in $1\}$
- $\{w: w$ contains a single $1\} = 0^*10^*$
- $\{w: w$ contains at least one $1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w$ has even length$\}$
- $\{w$: every 0 is followed by at least one $1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$ $\qquad\qquad$ $\emptyset^+ = ?$

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w\!: w \text{ contains} \geq 0 \text{ 1s, then} \geq 0 \text{ 0s, then a } 1\}$
- $(1 \cup 0)^*1 = \{w\!: w \text{ ends in } 1\}$
- $\{w\!: w \text{ contains a single } 1\} = 0^*10^*$
- $\{w\!: w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w\!: w \text{ has even length}\}$
- $\{w\!: \text{every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$ $\qquad\qquad$ $\emptyset^+ = \emptyset$