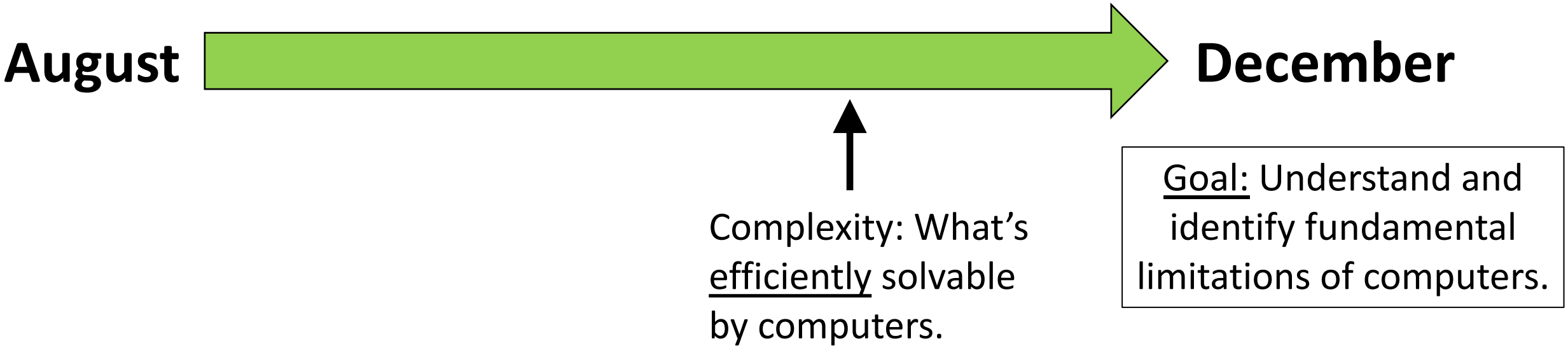
Finite Automata
CSCI 338

CSCI 338

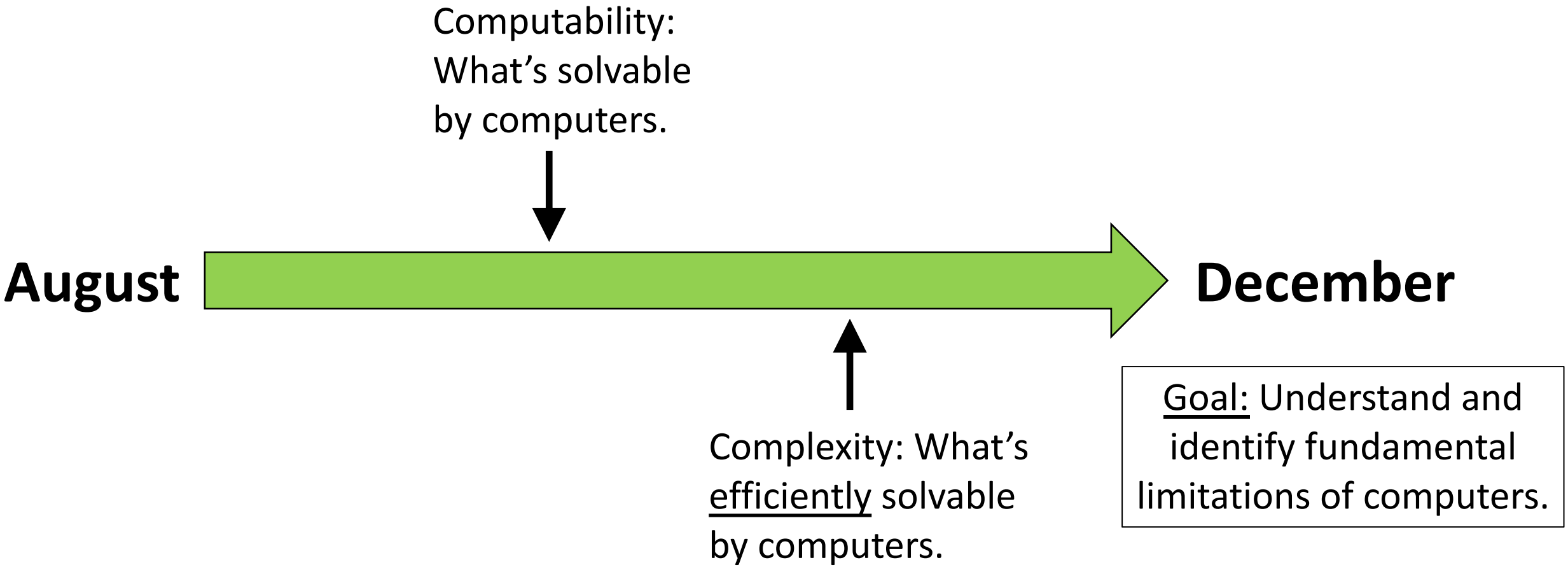
August  **December**

Goal: Understand and identify fundamental limitations of computers.

CSCI 338


August
December
Complexity: What's efficiently solvable by computers.
Goal: Understand and identify fundamental limitations of computers.

CSCI 338


Computability:
What's solvable
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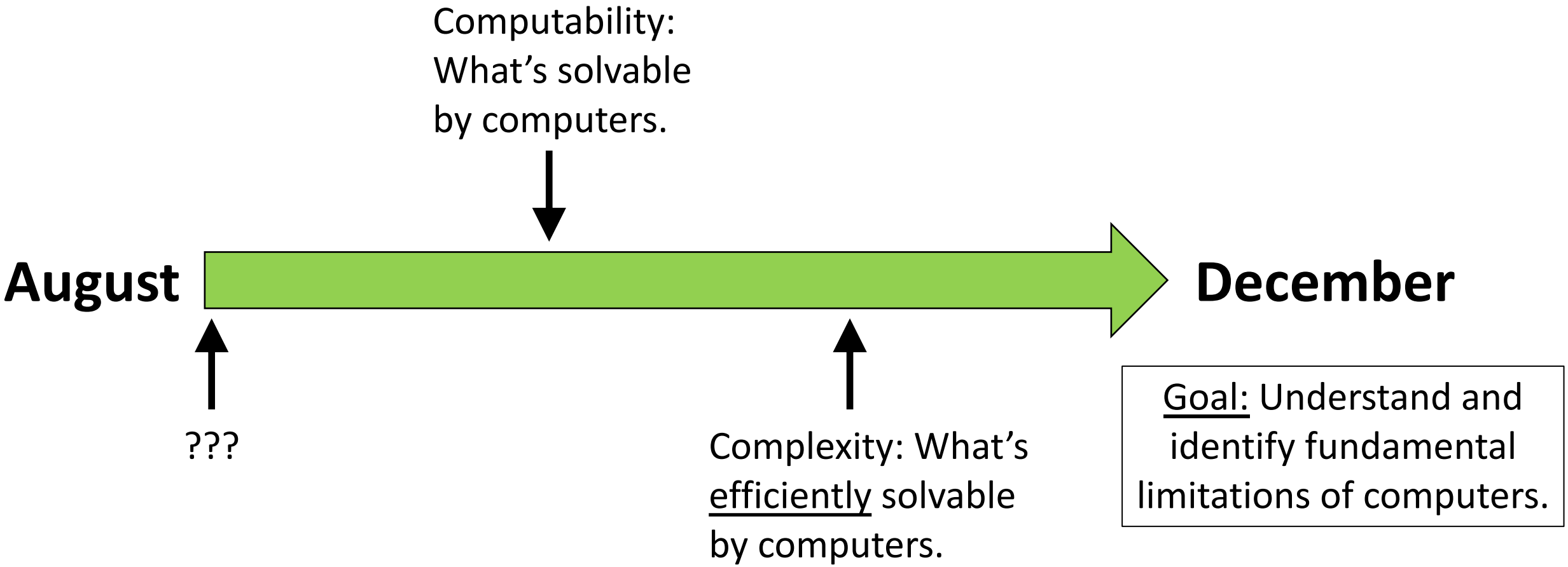
CSCI 338


Computability:
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August
December
???
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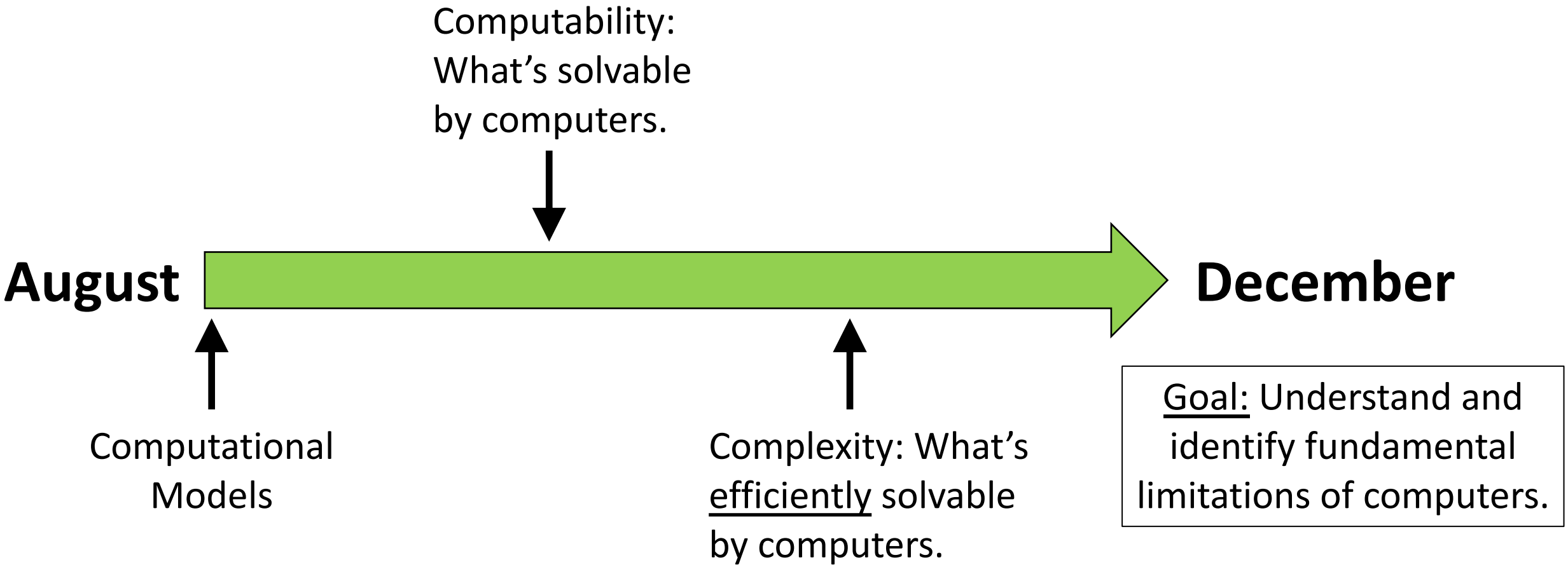
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August
December
Computational
Models
Complexity: What's
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Goal: Understand and
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Finite State Machine

Finite State Machine

Consider a car.

Finite State Machine

States:

??

Consider a car.

Finite State Machine

States:

- Moving
- Stopped

Consider a car.

Finite State Machine

States:

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Consider a car.

Actions:

??

Finite State Machine

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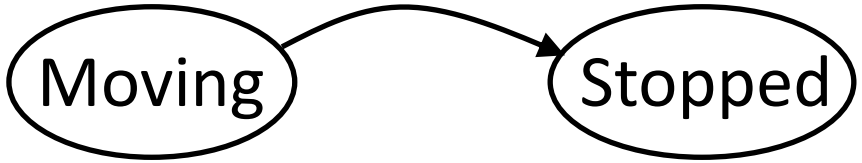

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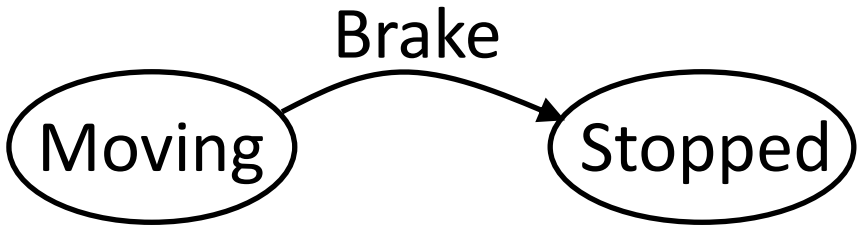

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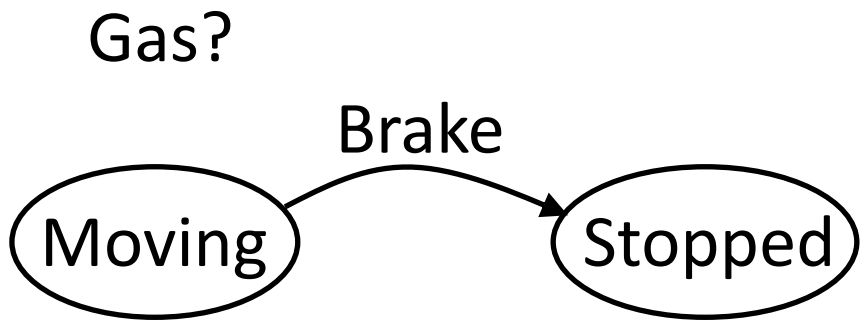

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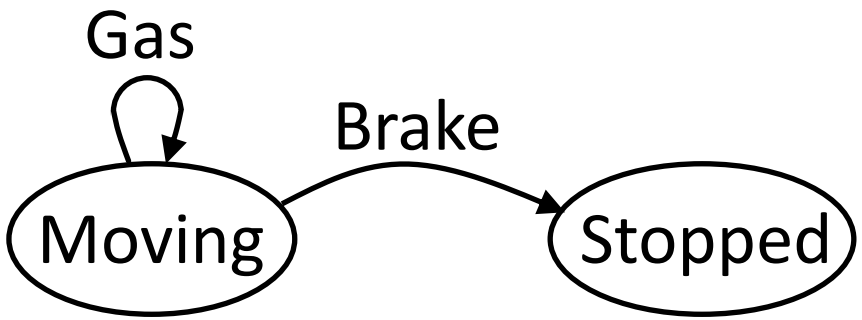

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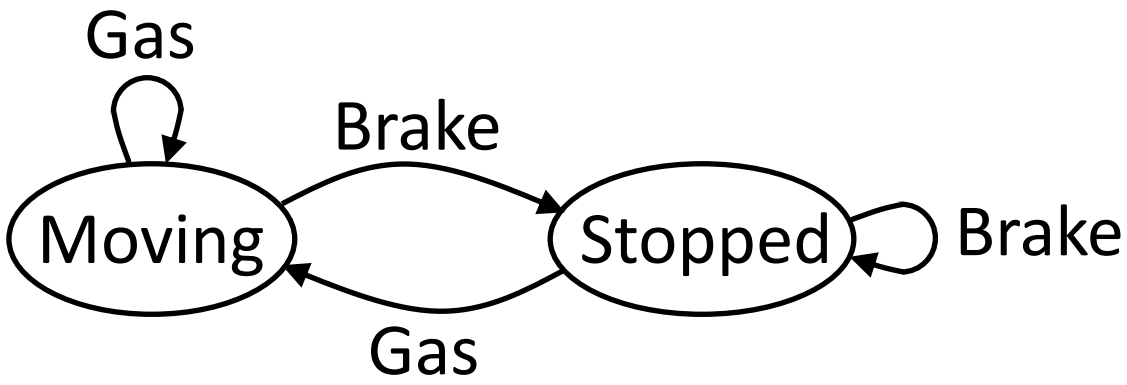

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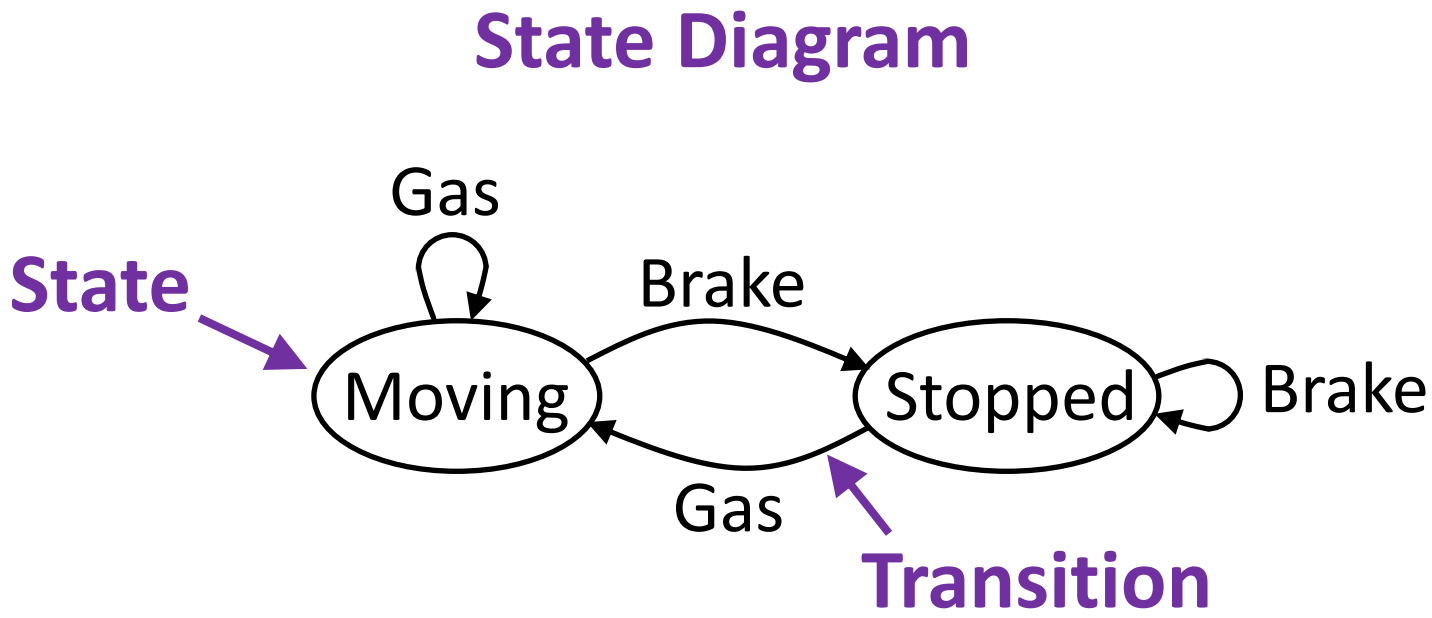

Computational Question

Is the string $\omega = 01101001$ of the form:

$\{\omega: \omega$ contains an even number of 0's and even number of 1's$\}$?

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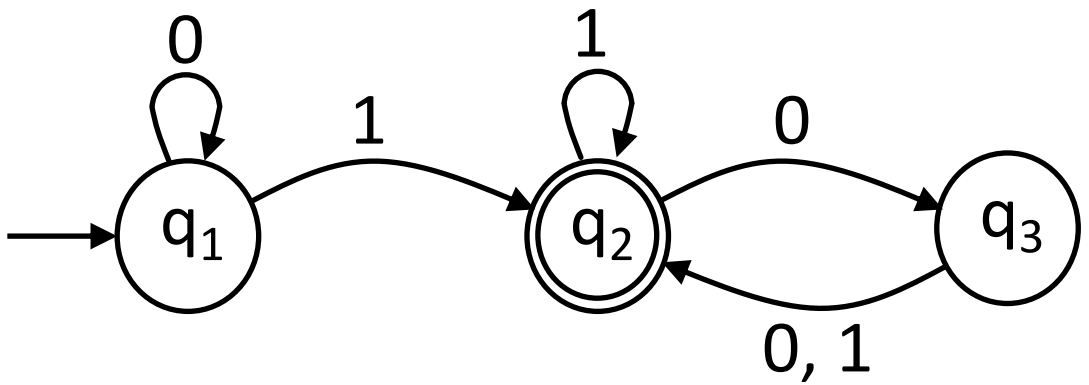
Set of strings with a specific format (pattern).

Deterministic Finite Automaton (DFA): An abstract model (machine) that determines (yes or no) if a string has a specific format.

Deterministic Finite Automaton (DFA)



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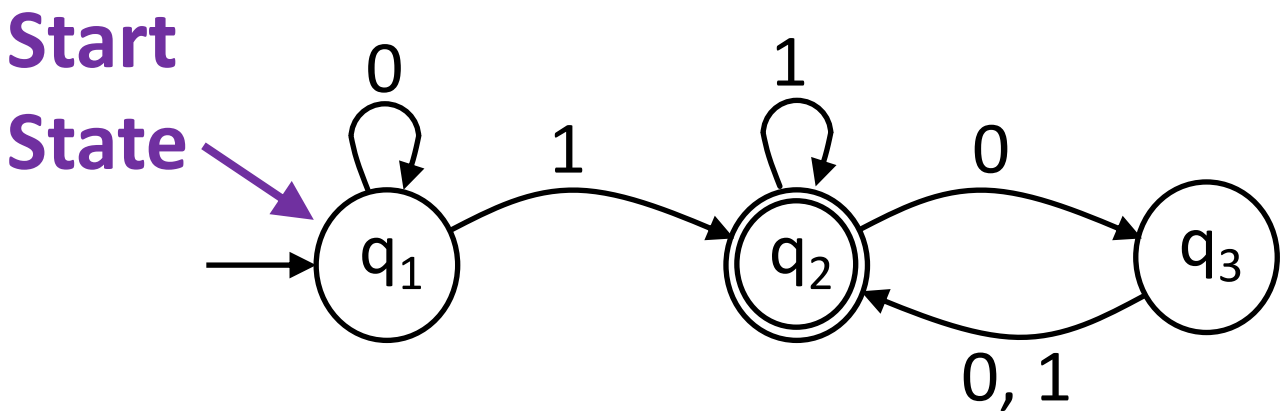

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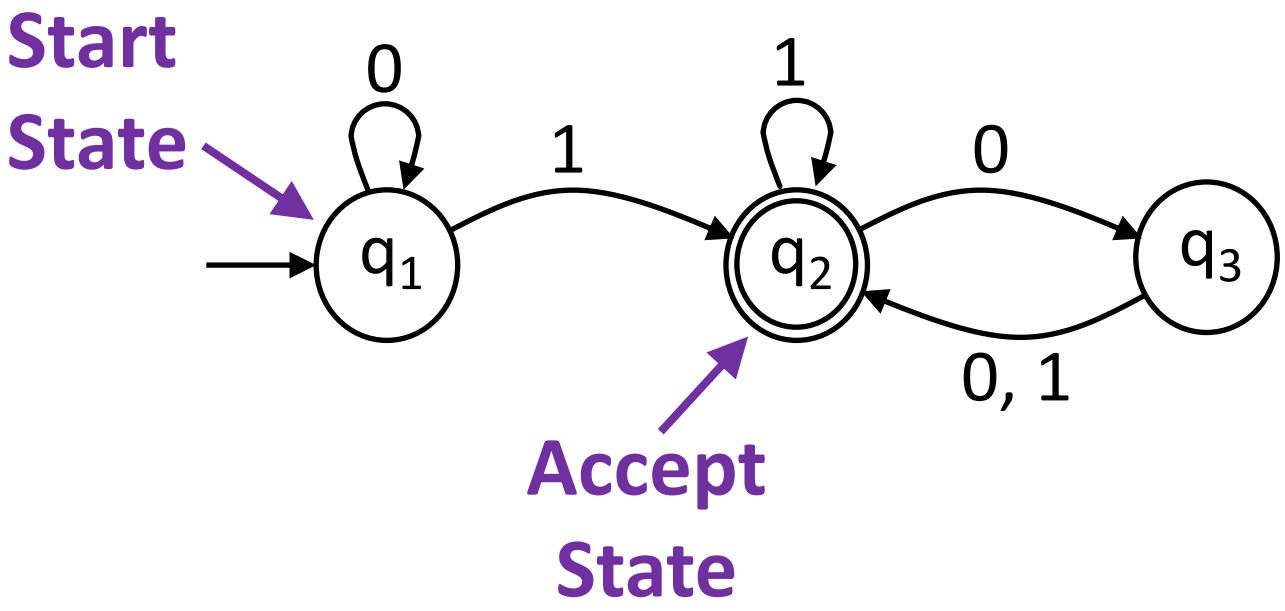

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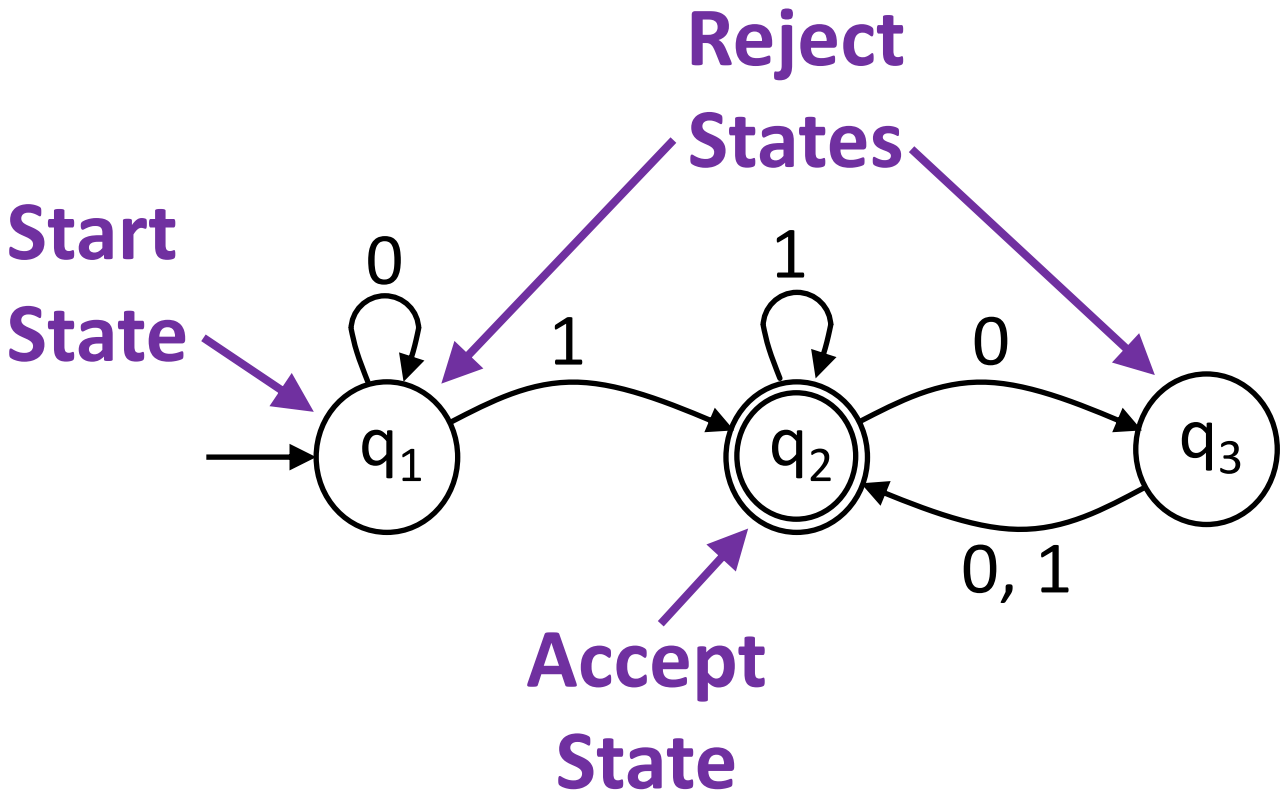

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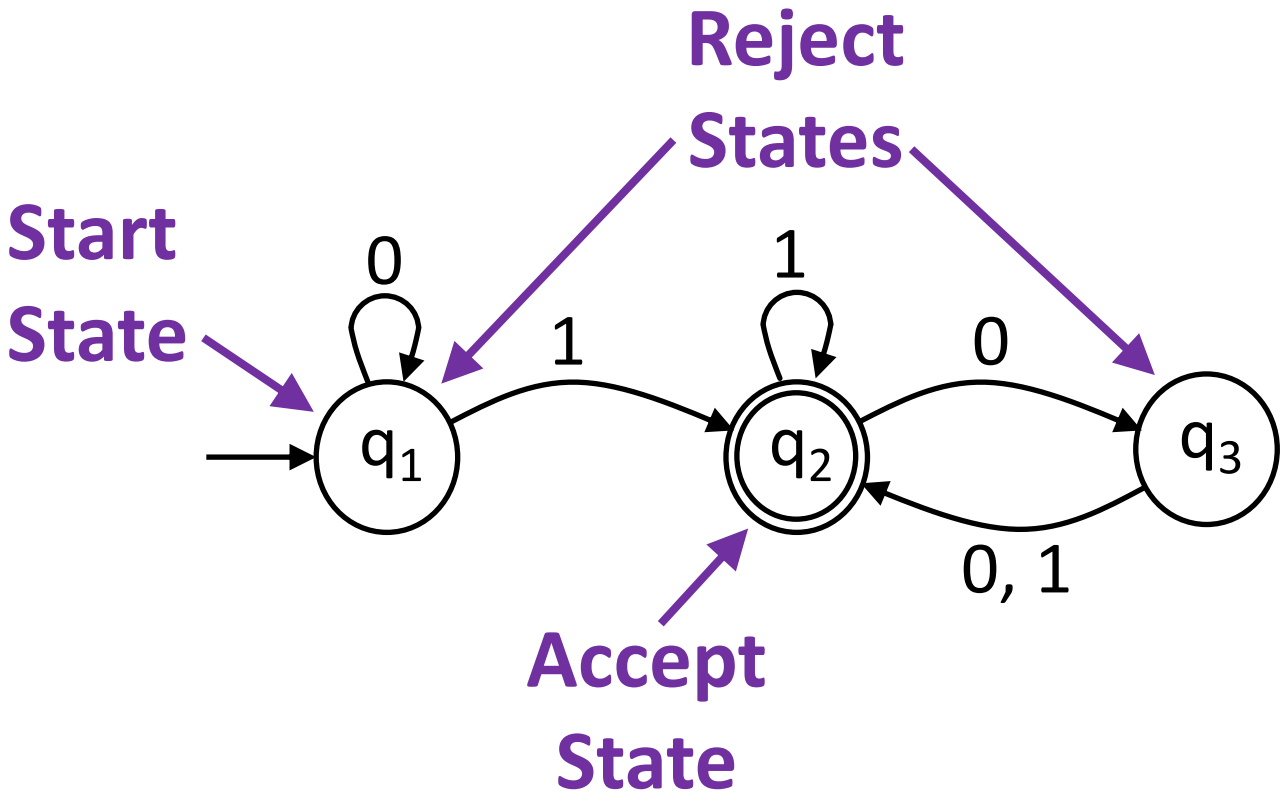

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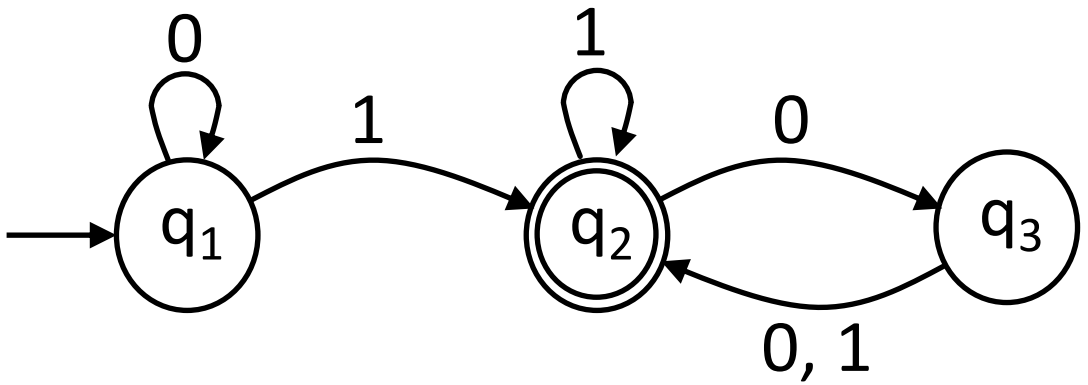

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Given string $\omega = 01101$, does this DFA accept or reject?

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DFA string processing:

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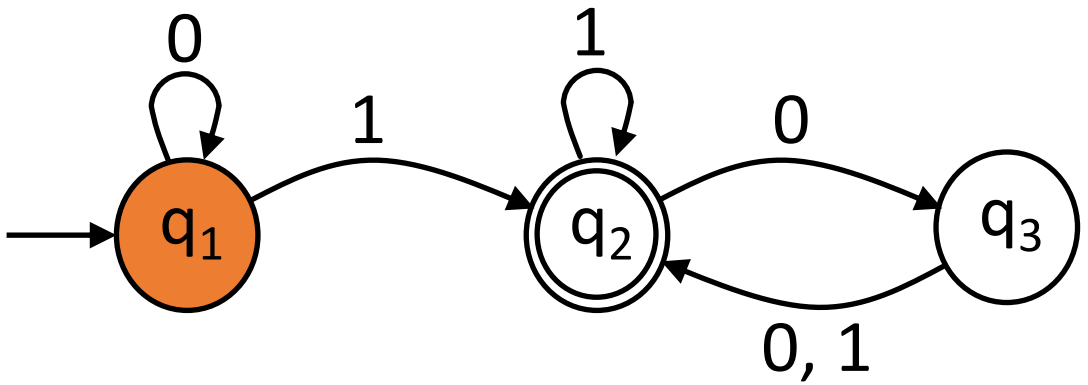

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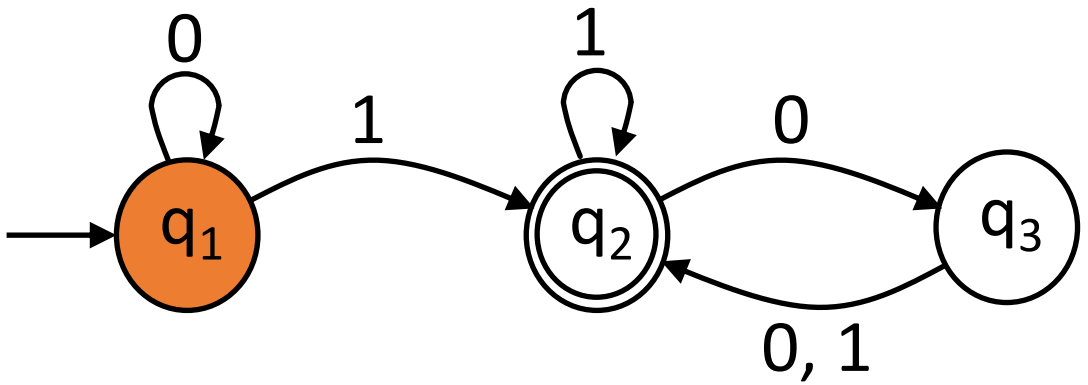

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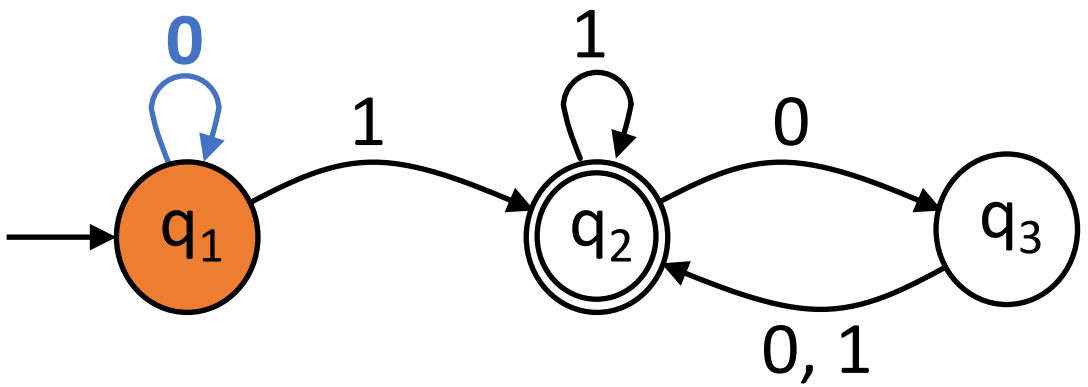

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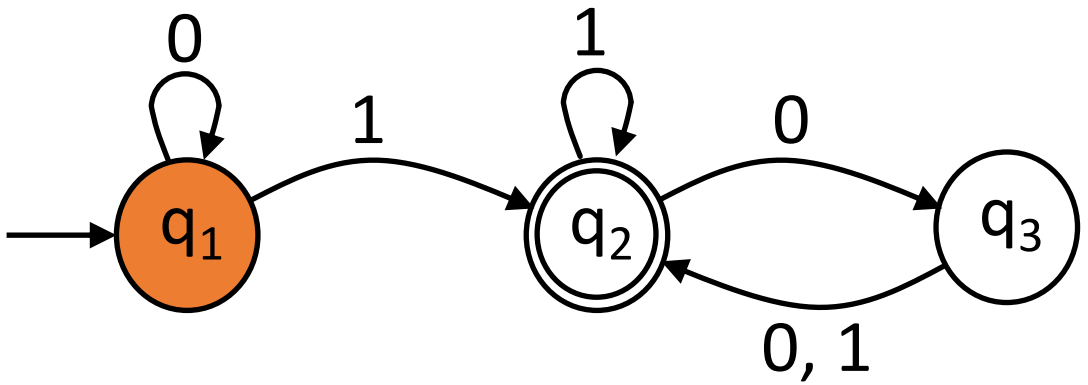

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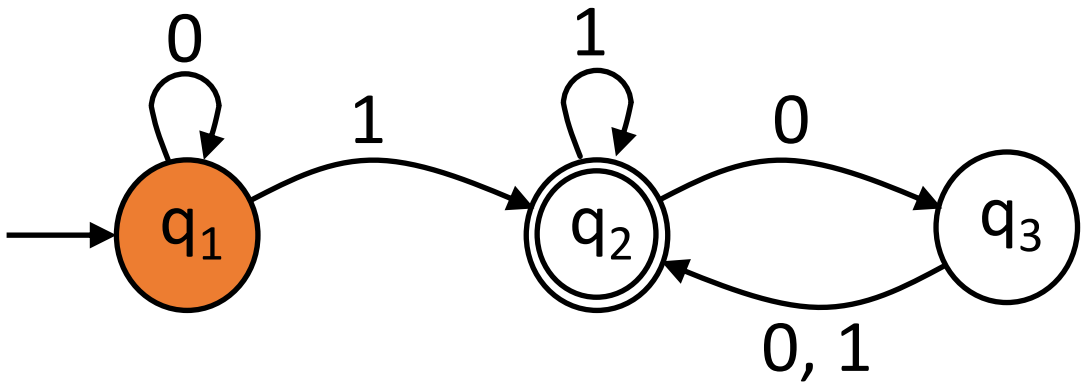

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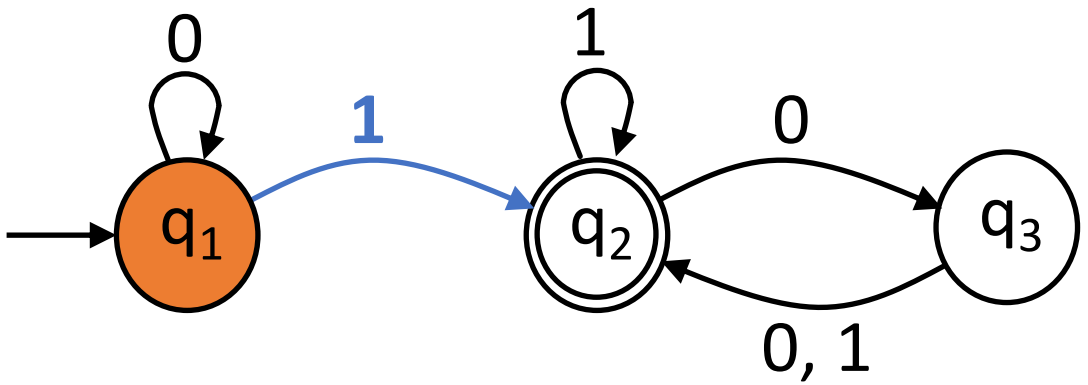

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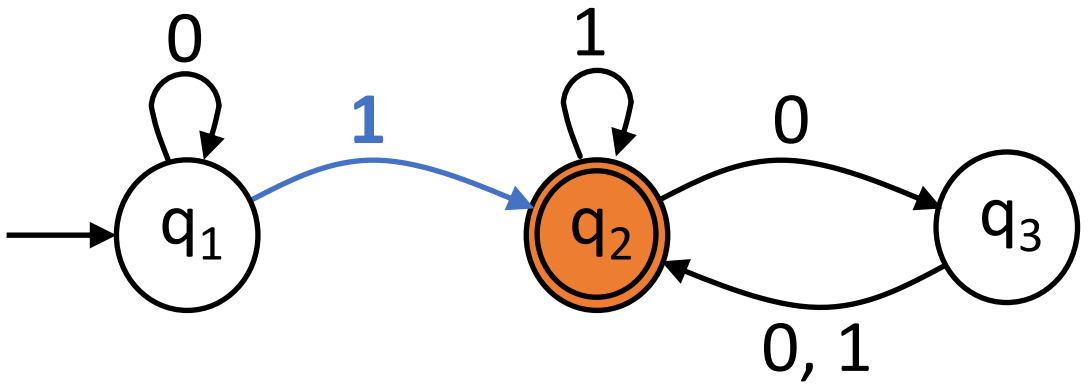

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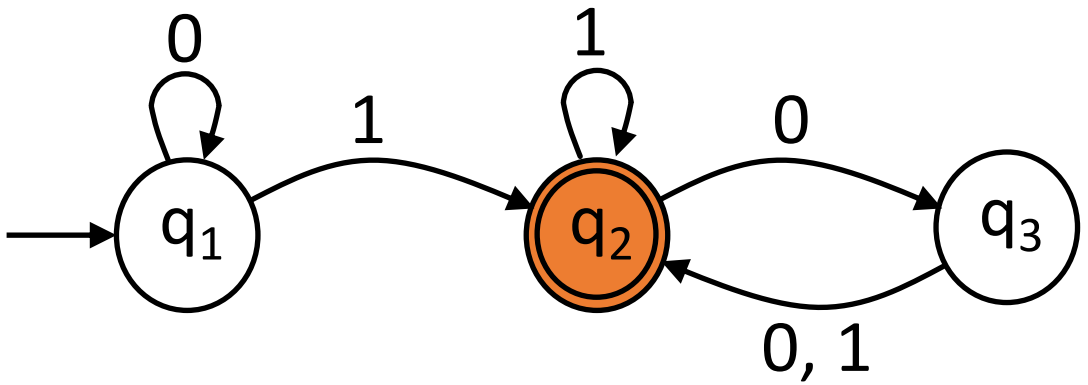

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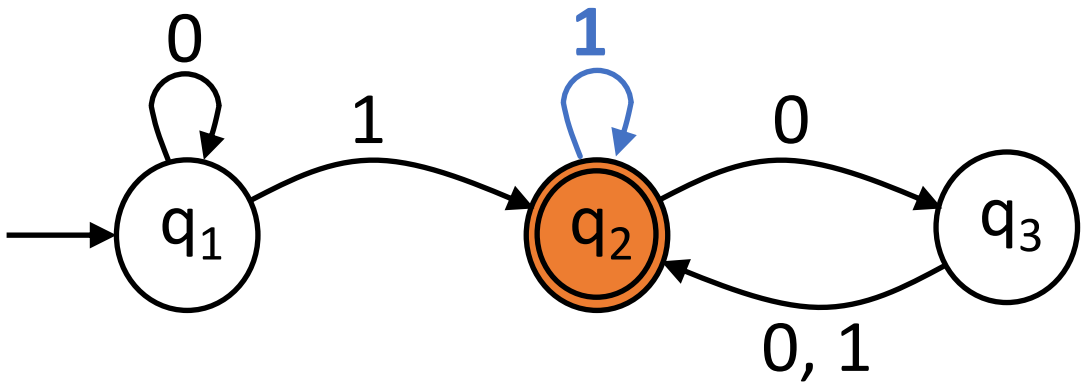

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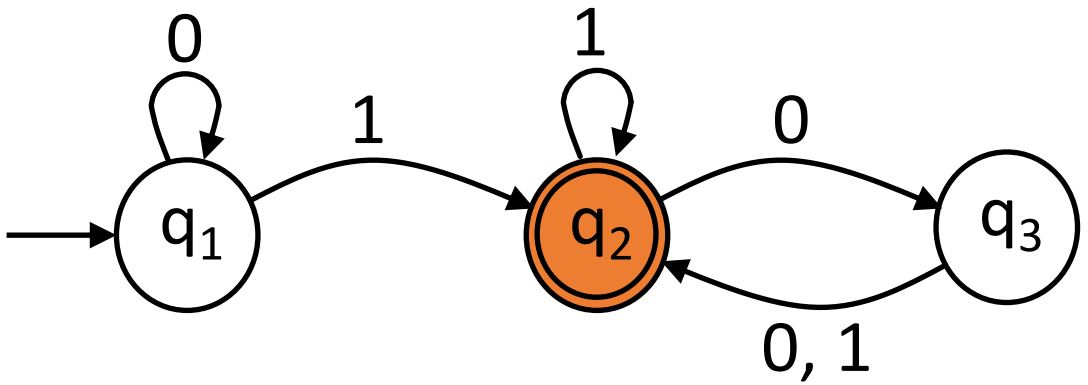

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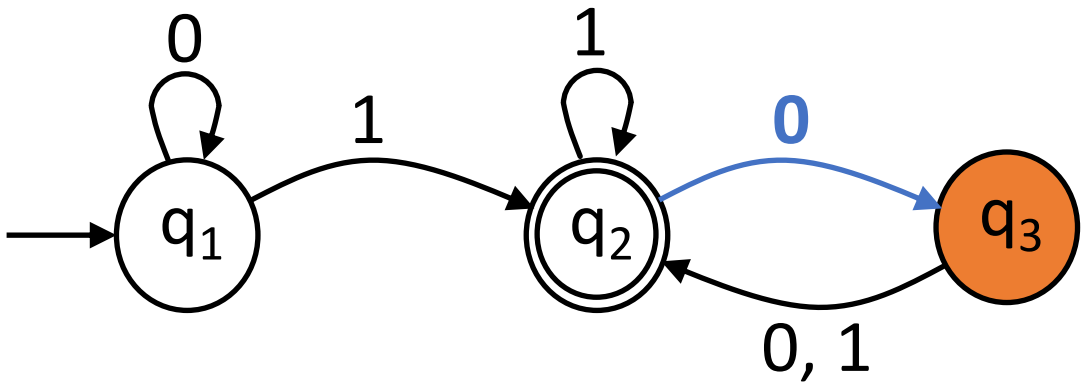

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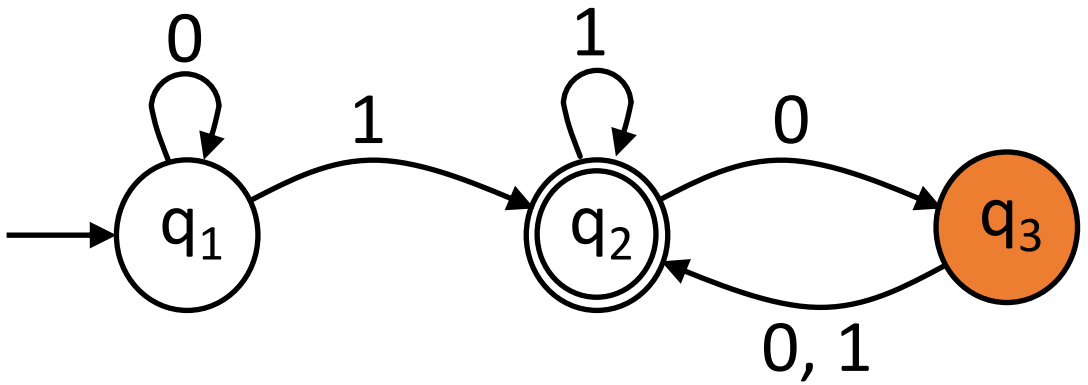

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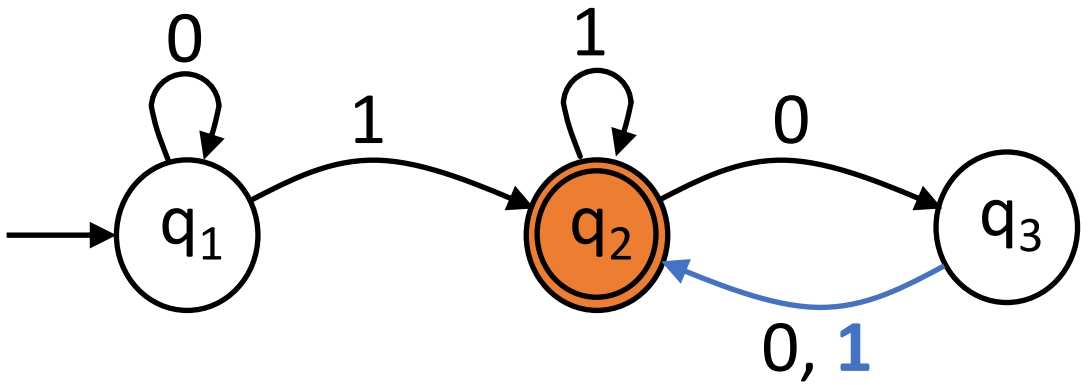

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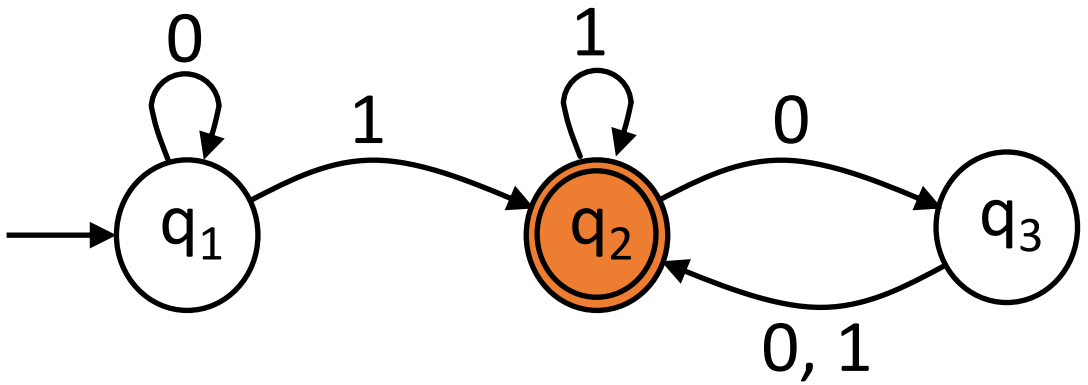

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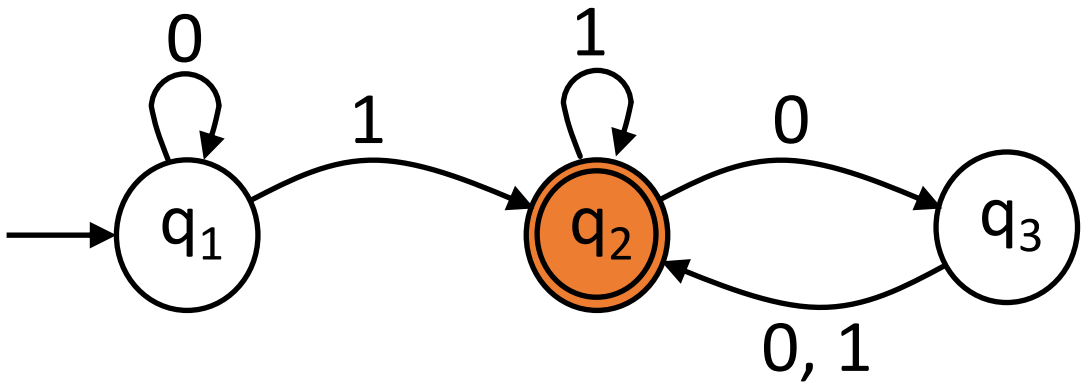

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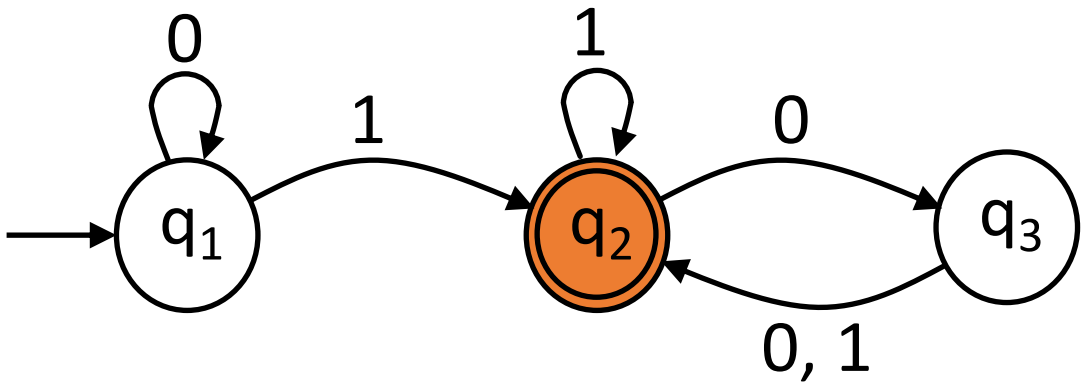

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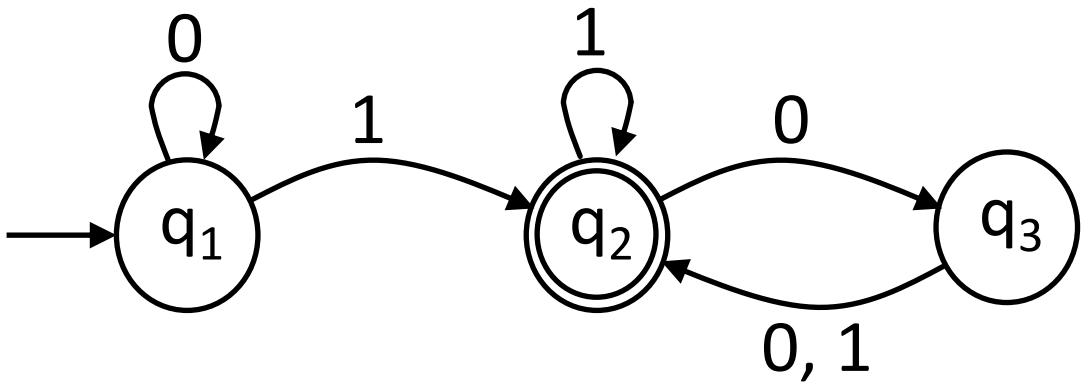

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Given string $\omega = 010$, does this DFA accept or reject?

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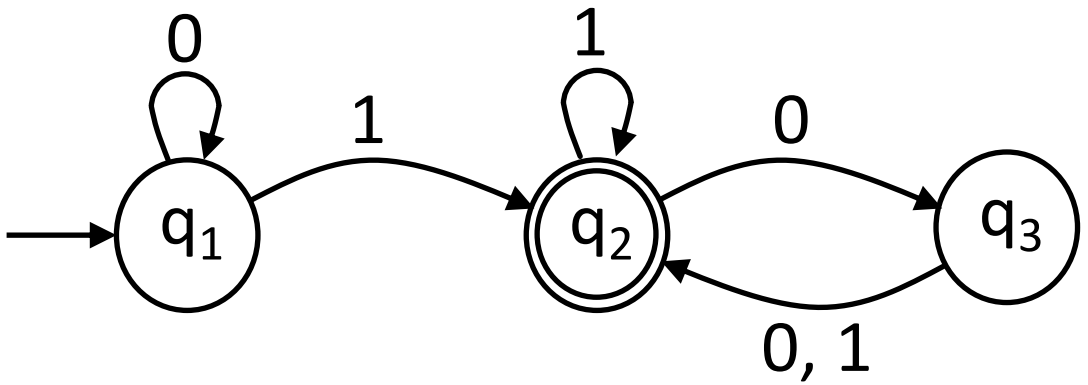

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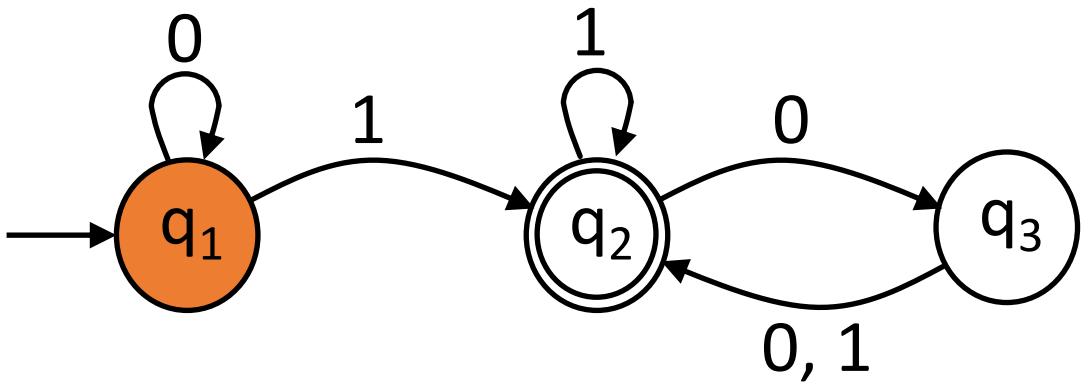

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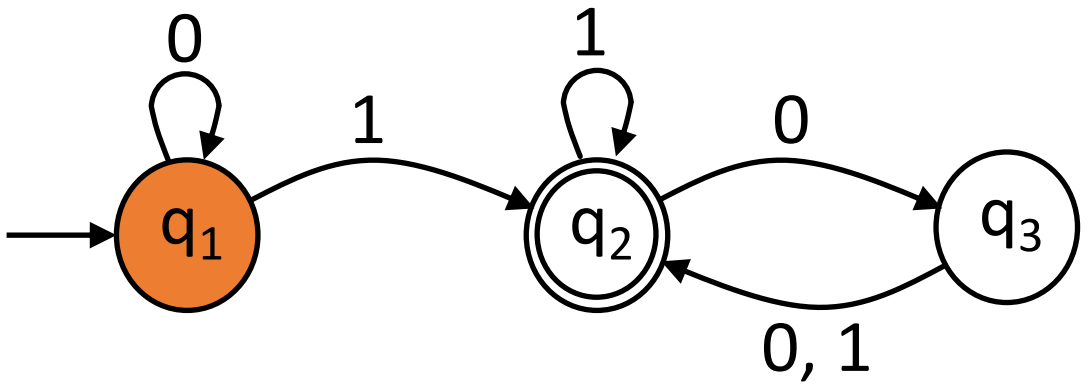

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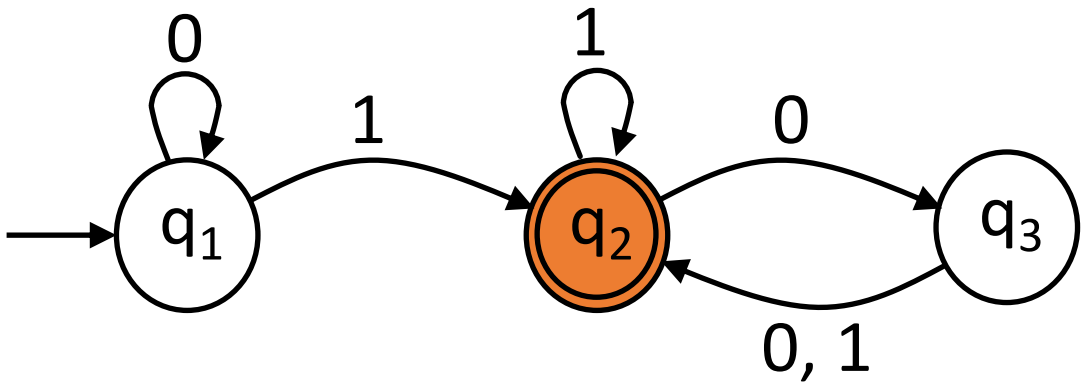

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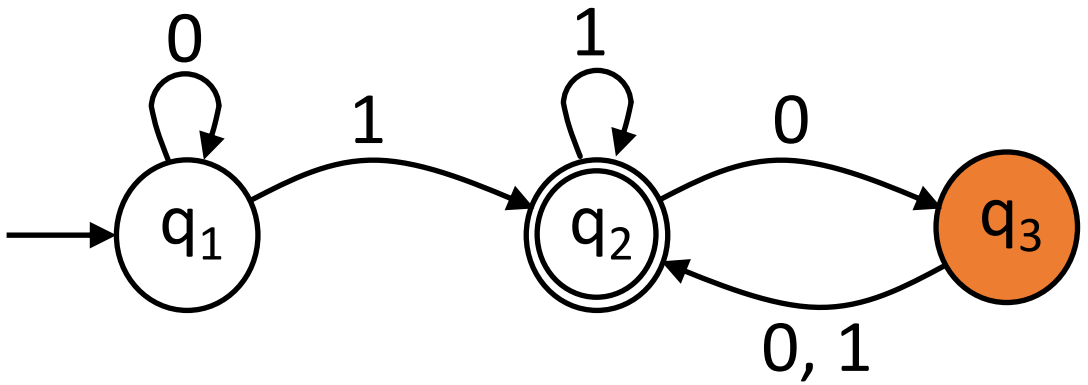

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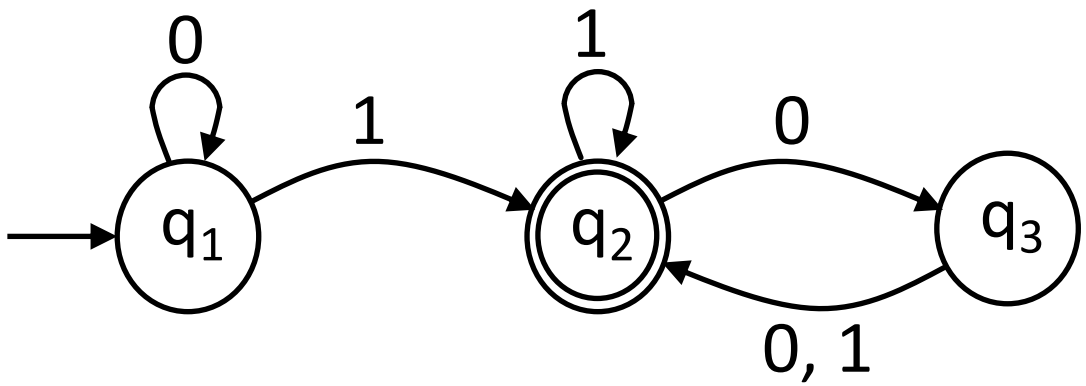

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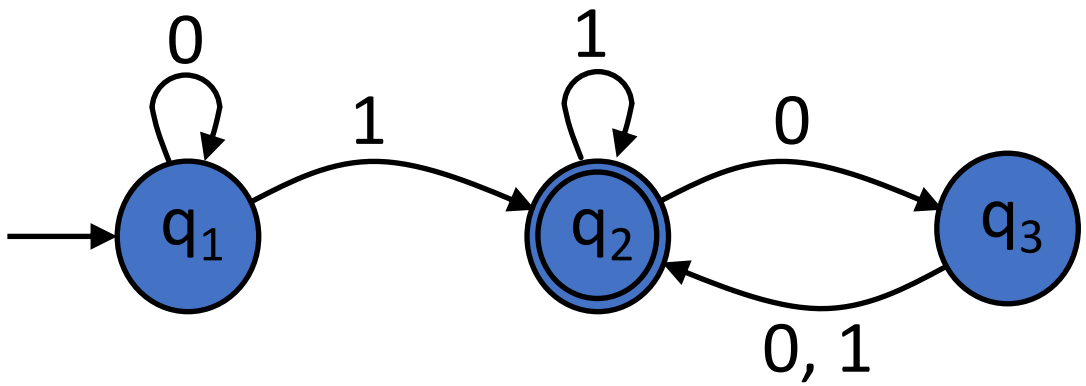

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DFAs consist of:

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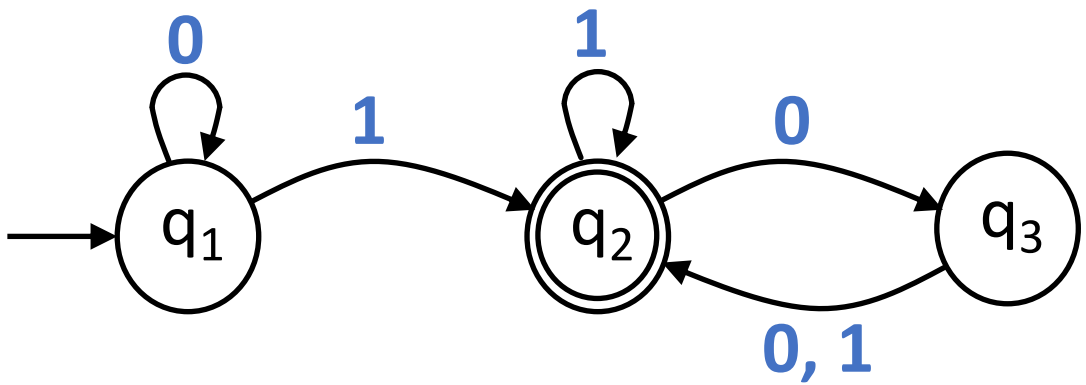
Σ consists of the transition characters (i.e. characters in the strings the DFA processes).



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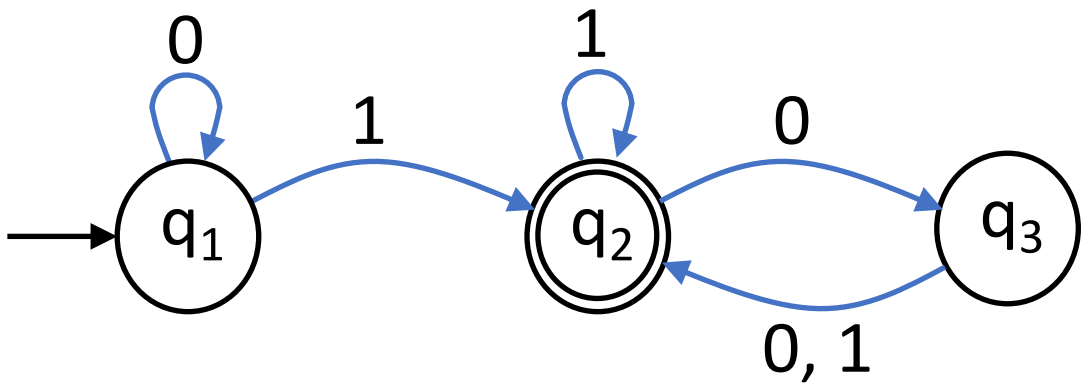

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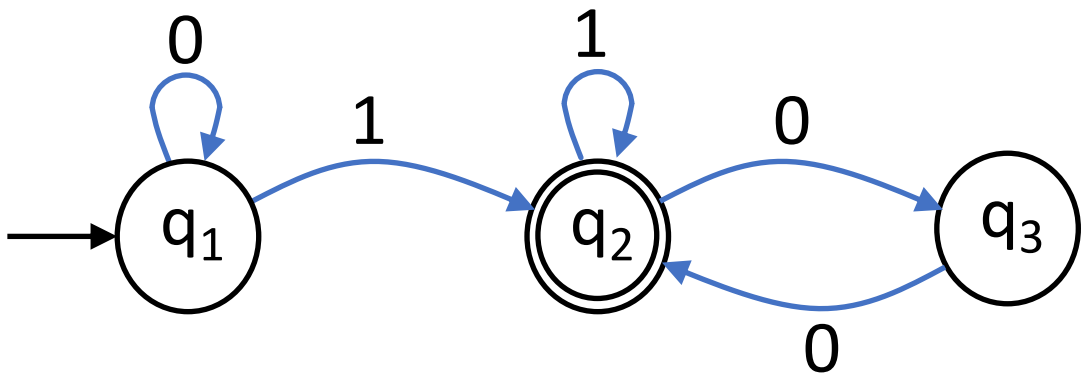

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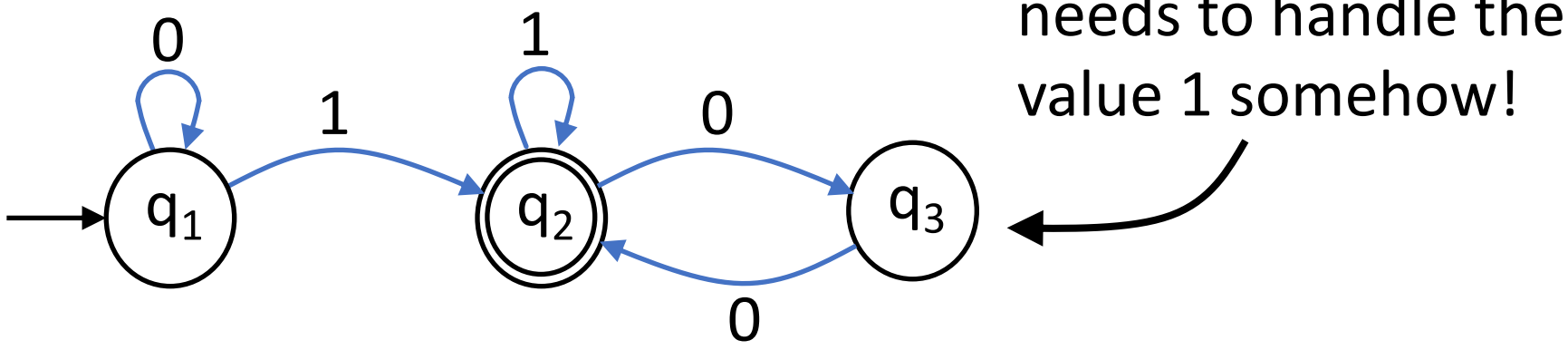

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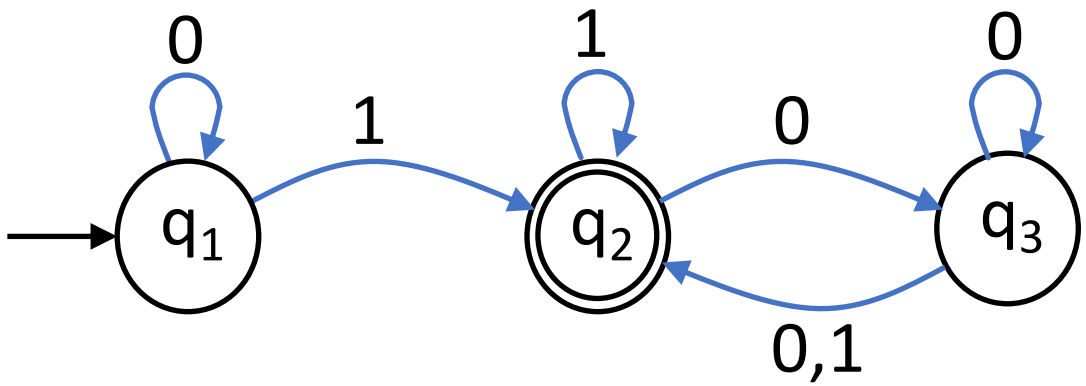

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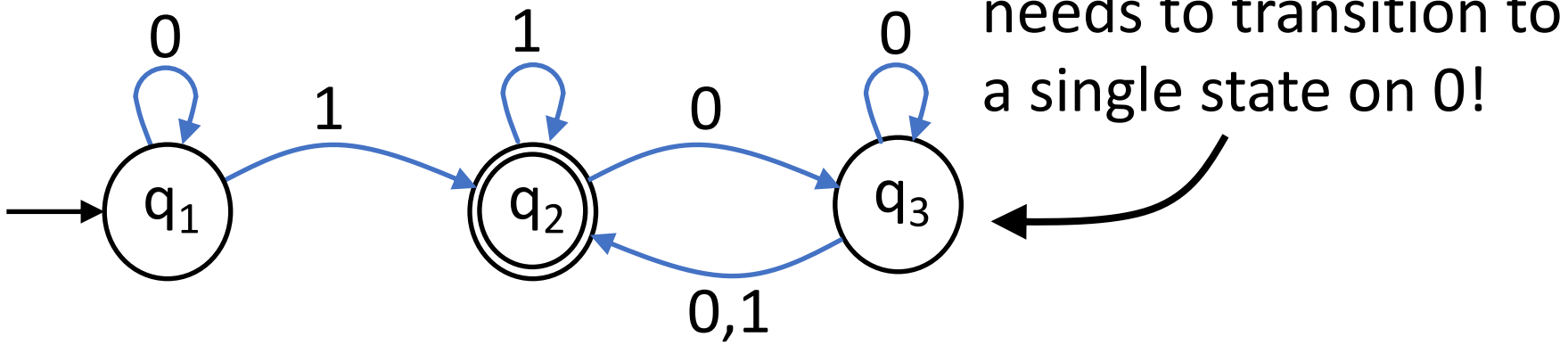

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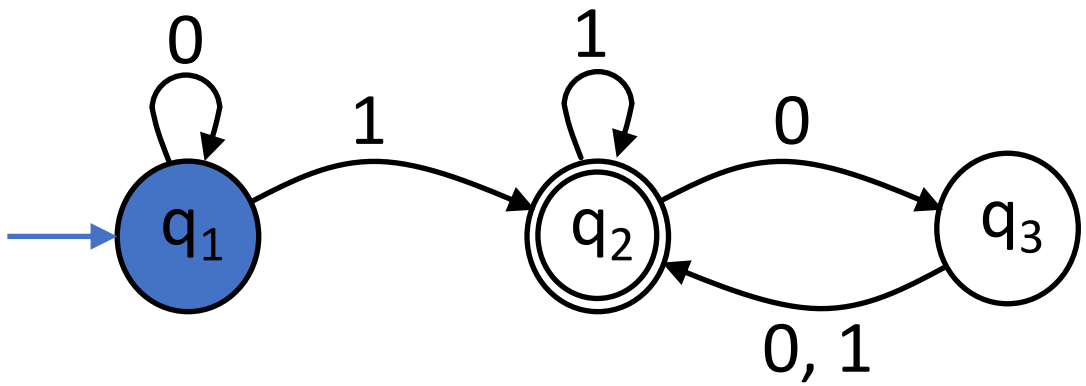
Exactly one start state needed.



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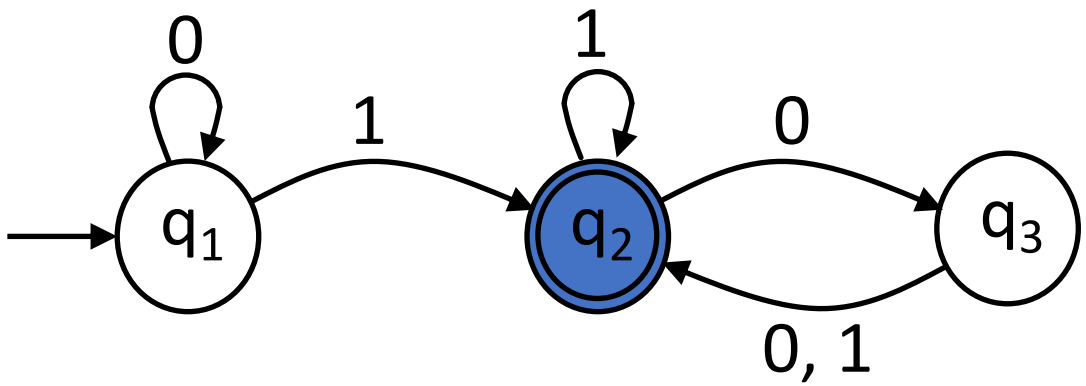
F is allowed to equal Q or be empty.



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$Q = \{q_1, q_2, q_3\}$

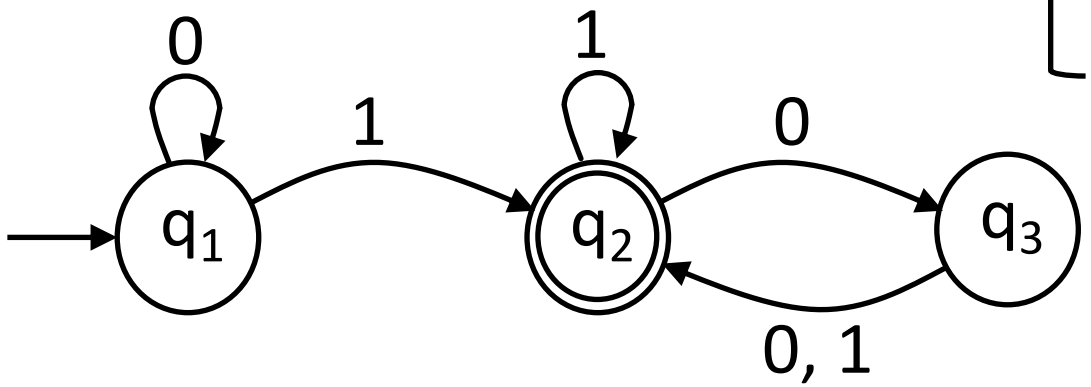
$\Sigma = \{0,1\}$

δ:

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

Start state = q_1

$F = \{q_2\}$



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Definitions:

The set of all strings A that a DFA M accepts is called its language, $L(M) = A$.

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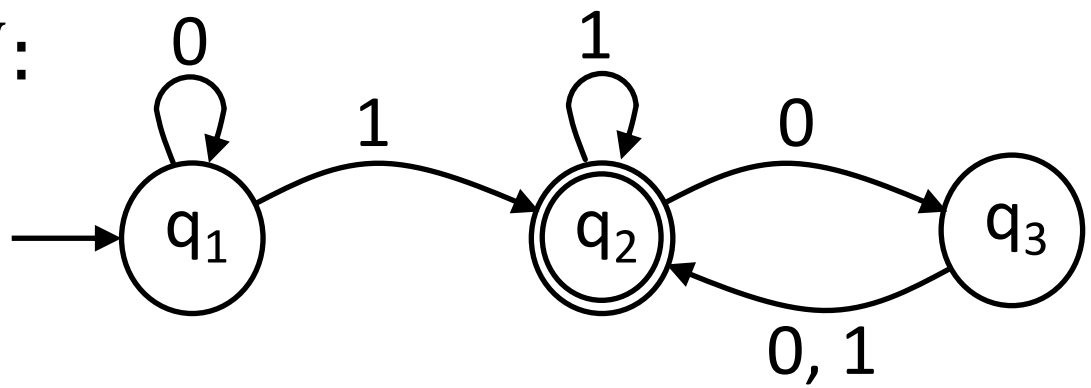

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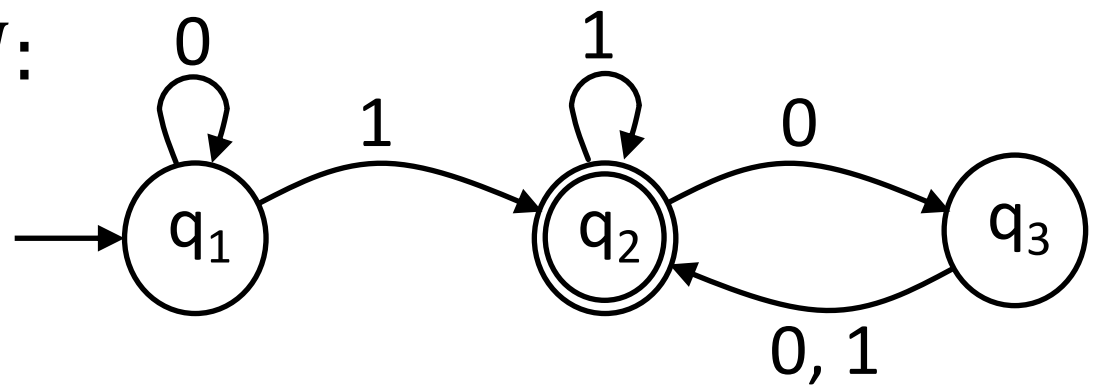

$L(M) = \{\omega: \omega$ contains at least one 1 and an even number of 0s following the final 1$\}$

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Set of regular languages are “things we can do” with DFAs.

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Prove that the following languages are regular:

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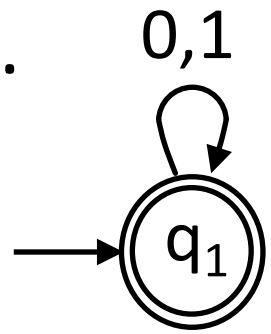

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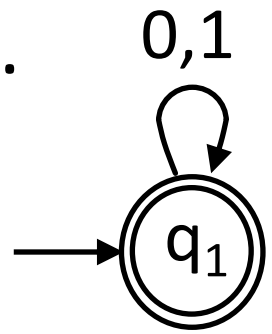

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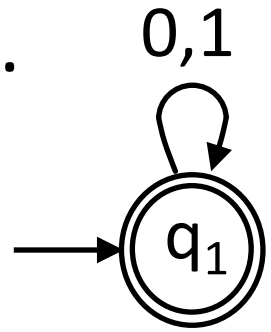

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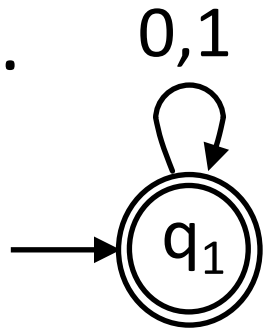

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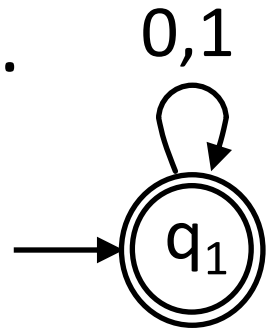

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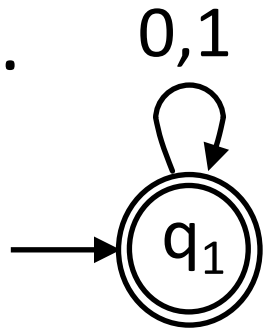

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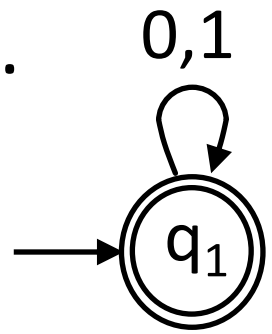

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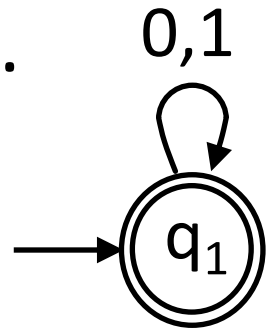

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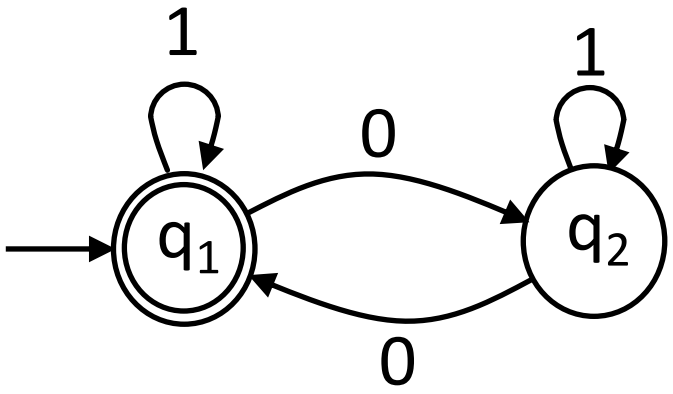

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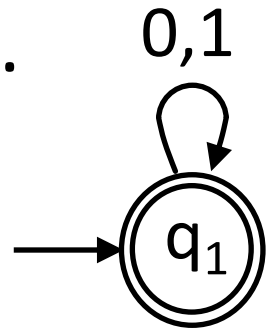

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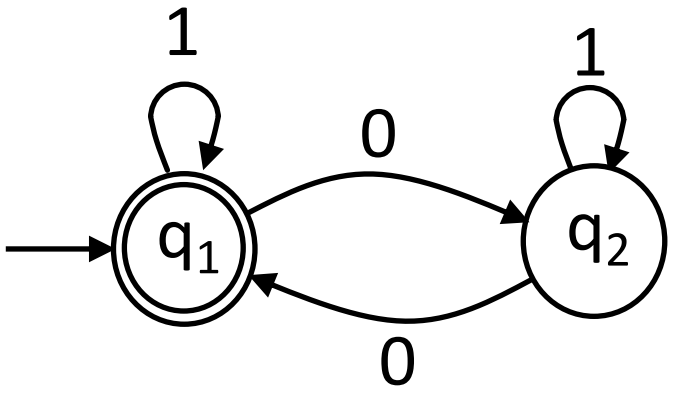

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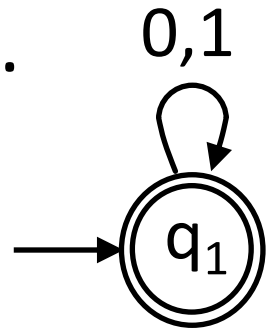

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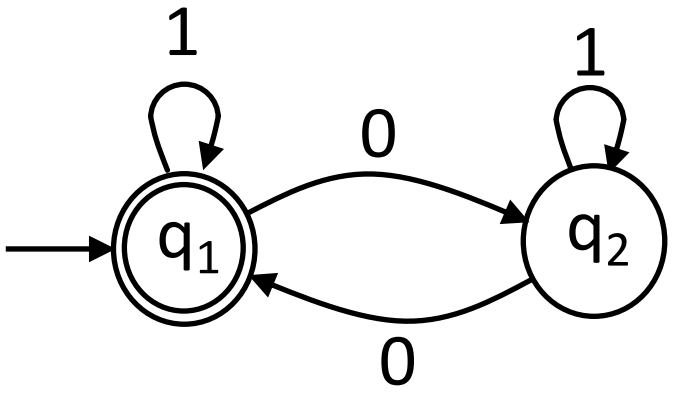

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