Nondeterminism

CSCI 338

DFA vs NFA

DFA:

??????

DFA vs NFA

DFA: Model of a computer that determines (accept or reject) if a string has a specific format.

DFA vs NFA

NFA

~~DFA~~: Model of a computer that determines (accept or reject) if a string has a specific format.

With some more bells and whistles.

DFA vs NFA

Deterministic Finite Automaton (DFA):

Nondeterministic Finite Automaton (NFA):

DFA vs NFA

Deterministic Finite Automaton (DFA):

- Every state has exactly one transition for every character ($e \in \Sigma$).

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DFA vs NFA

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- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.

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- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.
- Allowed to have transitions that happen without input.

DFA vs NFA

Deterministic Finite Automaton (DFA):

- Every state has exactly one transition for every character ($e \in \Sigma$).
- For the same input, everyone takes the same path to the same final state.

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- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.
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DFA vs NFA

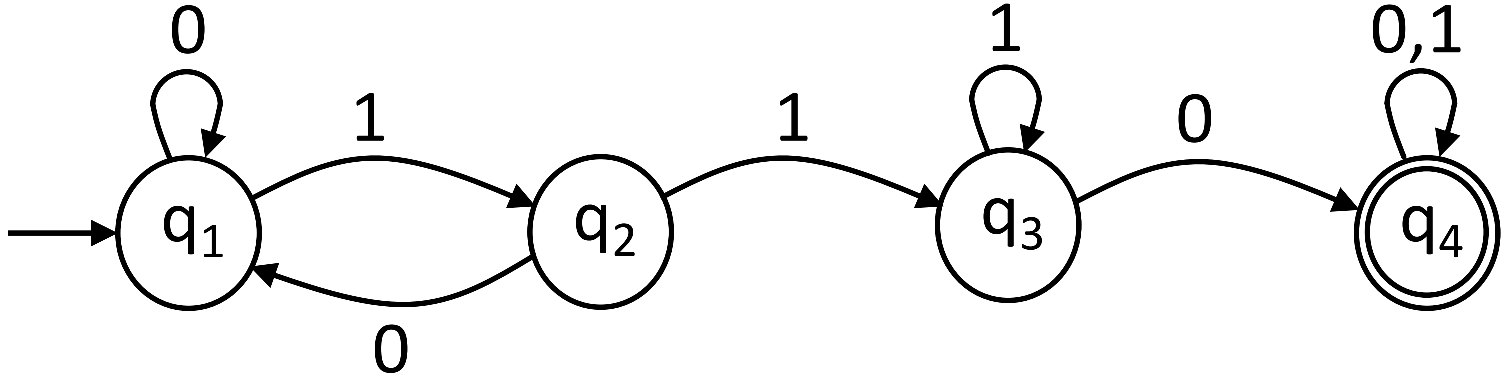
Deterministic Finite Automaton (DFA):

- Every state has exactly one transition for every character ($e \in \Sigma$).
- For the same input, everyone takes the same path to the same final state.

Nondeterministic Finite Automaton (NFA):

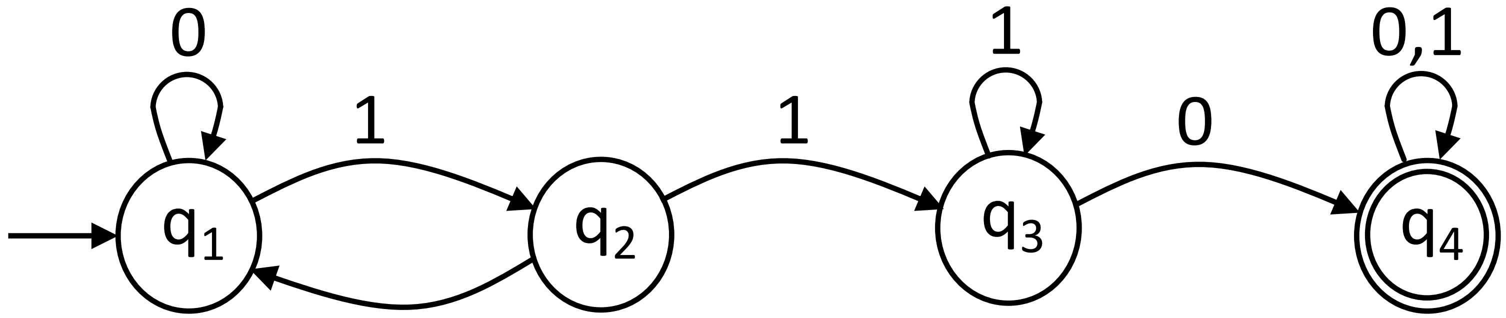
- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.
- Allowed to have transitions that happen without input.
- Processing strings is…different.

DFA Processing



$\{\omega\text{: } \omega$ contains the substring $110\}$

DFA Processing



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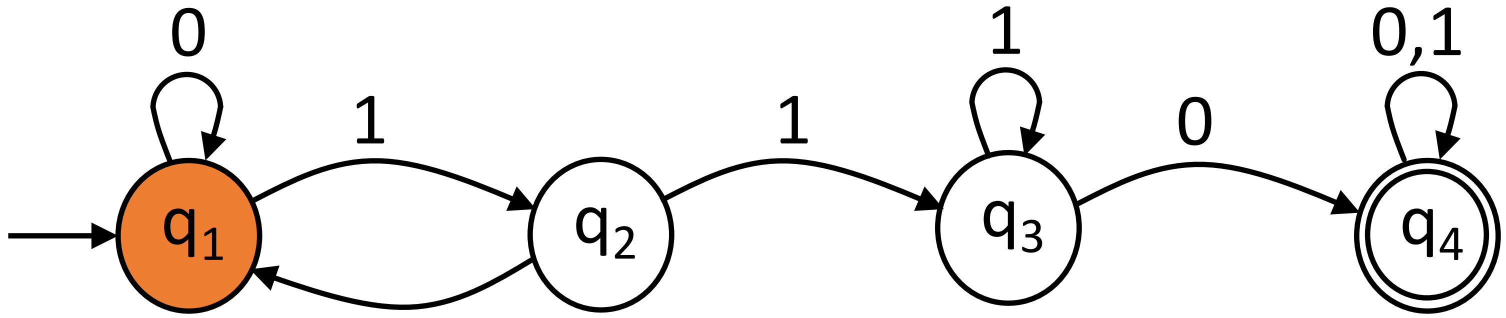
$$\omega = 1110$$

DFA Processing

q_1



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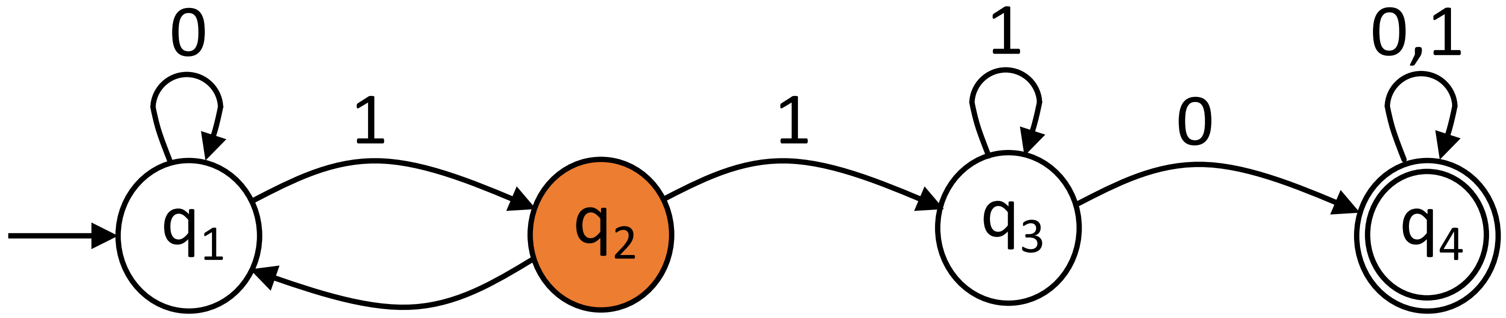


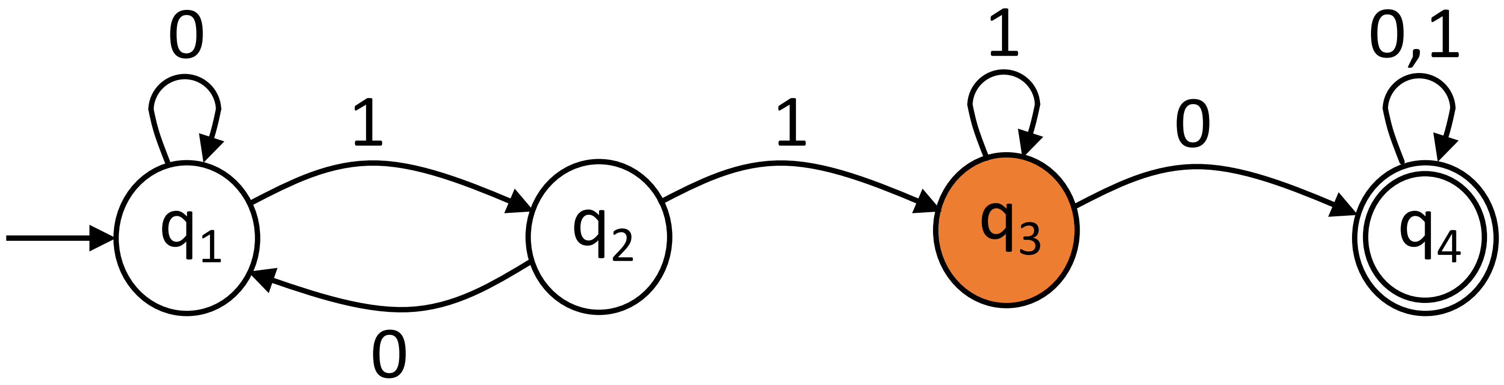
$\{\omega\colon\ \omega$ contains the substring $110\}$

$$q_1 \xrightarrow{} q_2 \quad 1$$

$$\omega = 1110$$

DFA Processing

$$\begin{tikzcd}
q_1 \arrow[d, "1"] \\
q_2 \arrow[d, "1"] \\
q_3
\end{tikzcd}$$



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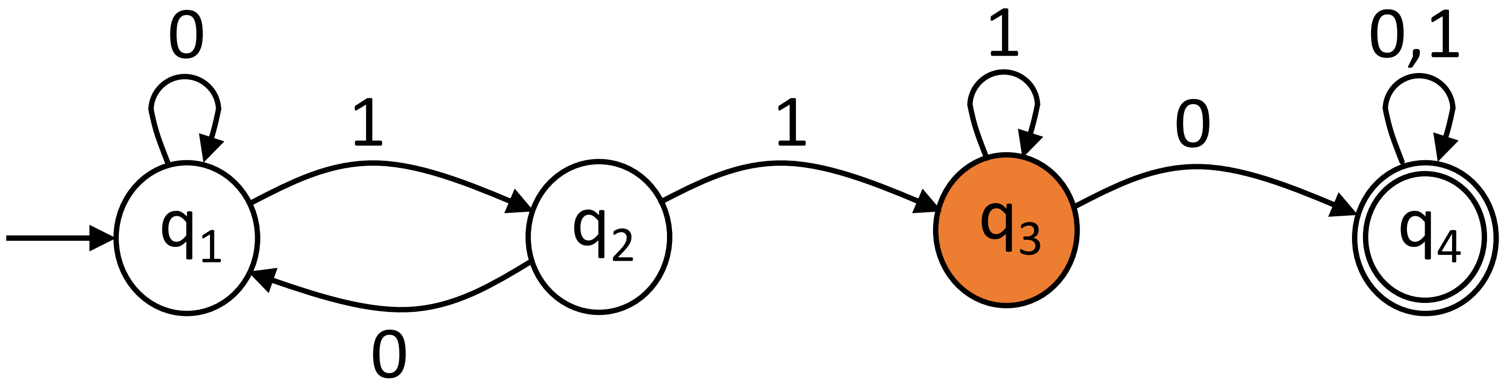
DFA Processing

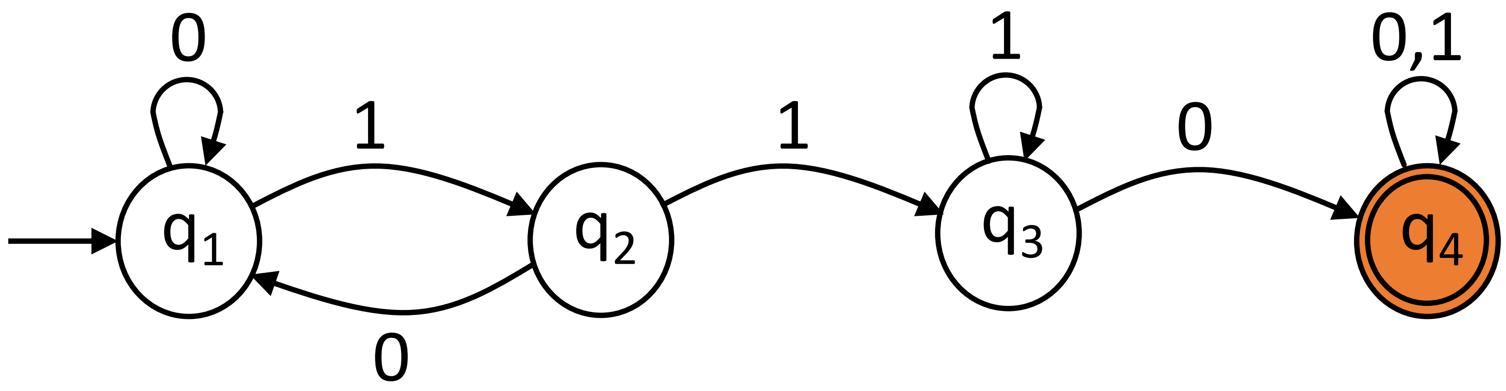


$\{\omega\text{: } \omega \text{ contains the substring } 110\}$

q_1
↓ 1
q_2
↓ 1
q_3
↓ 1
q_3

$\omega = 1110$

DFA Processing



$\{\omega\text{: } \omega \text{ contains the substring } 110\}$

$$\begin{tikzcd}
q_1 \arrow[d, "1"] \\
q_2 \arrow[d, "1"] \\
q_3 \arrow[d, "1"] \\
q_3 \arrow[d, "0"] \\
q_4
\end{tikzcd}$$

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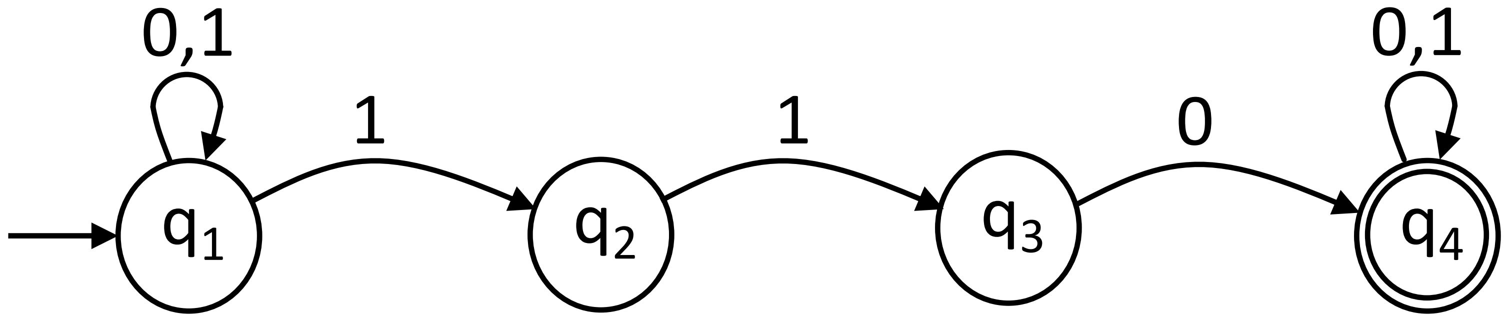
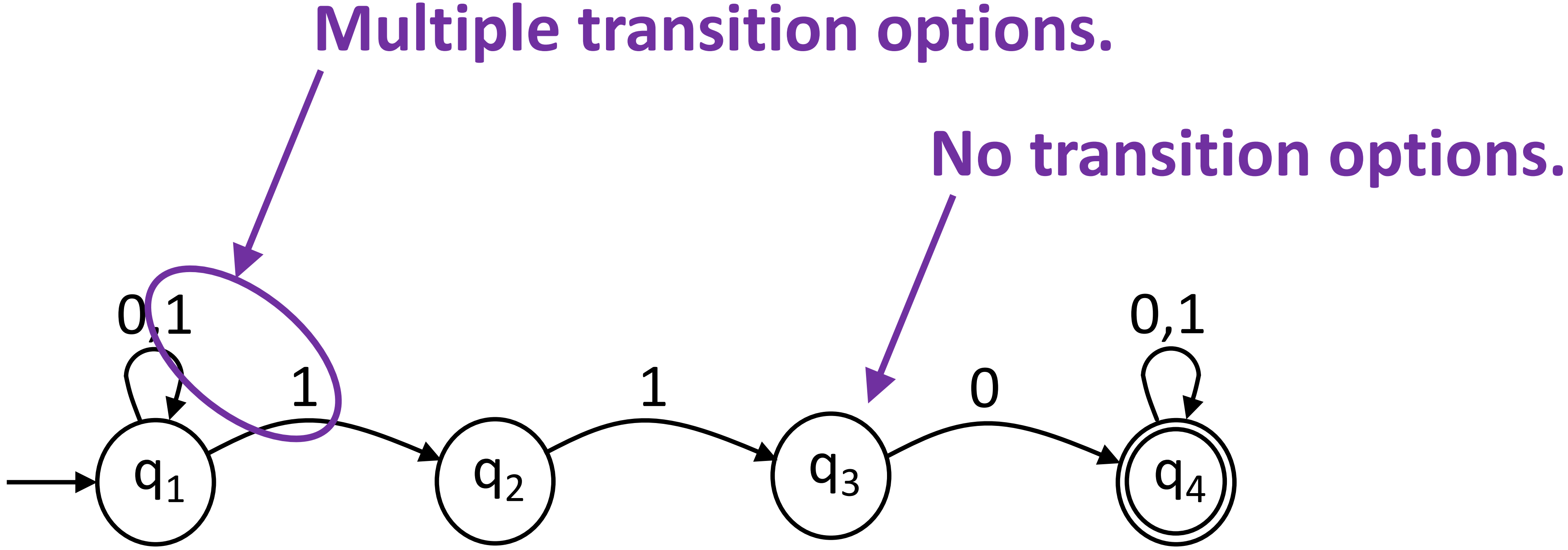
NFA Processing



$\{\omega$: ω contains the substring $110\}$

NFA Processing



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NFA Processing

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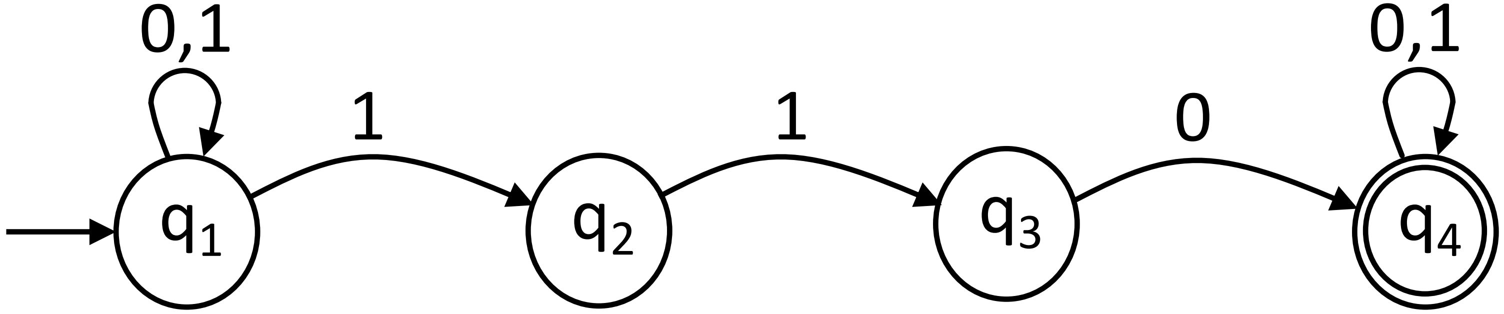

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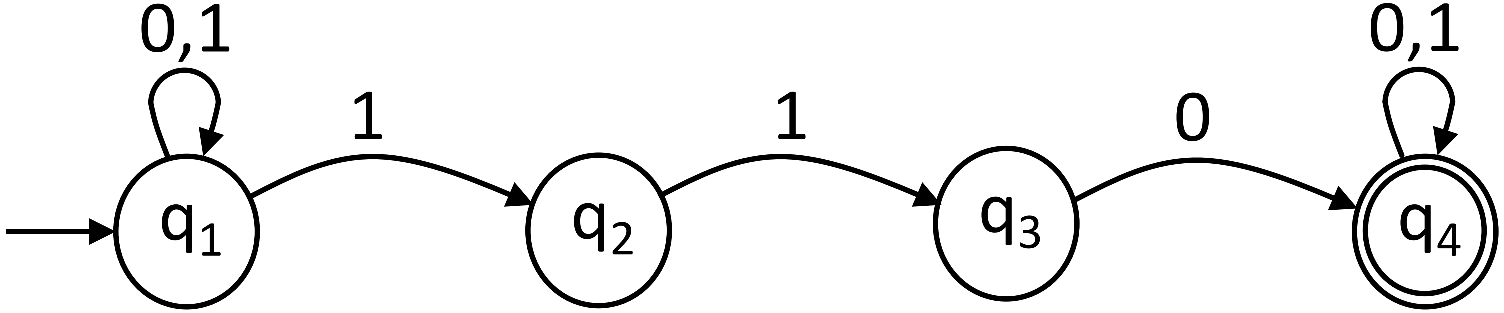

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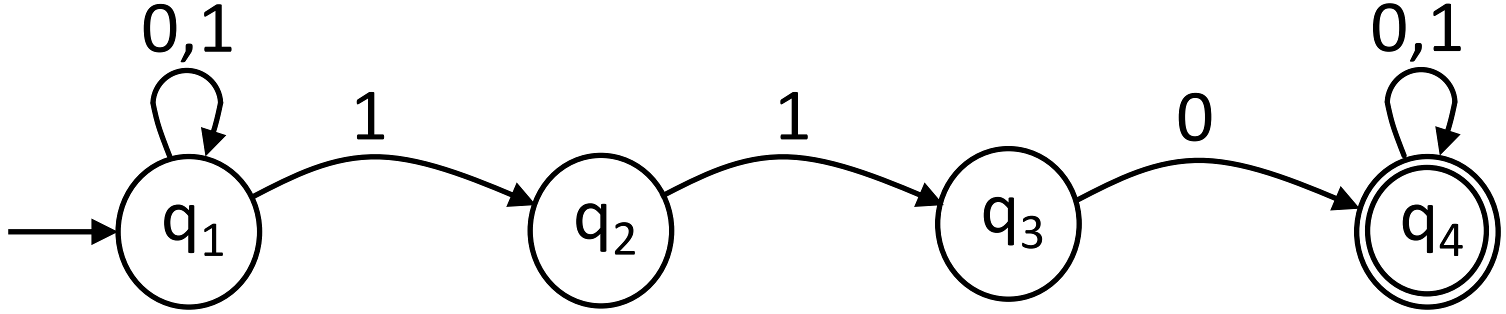

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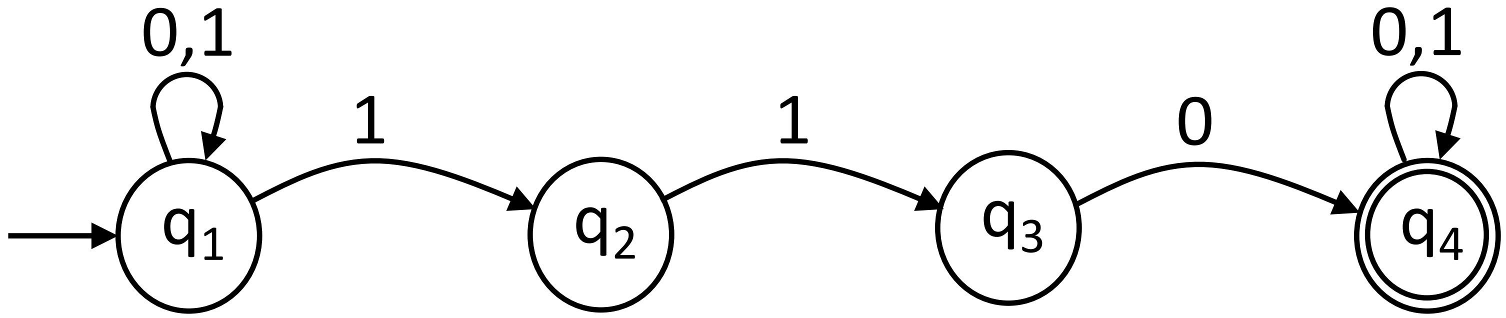

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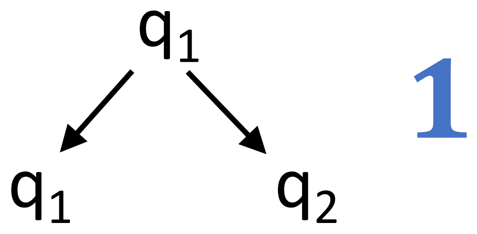

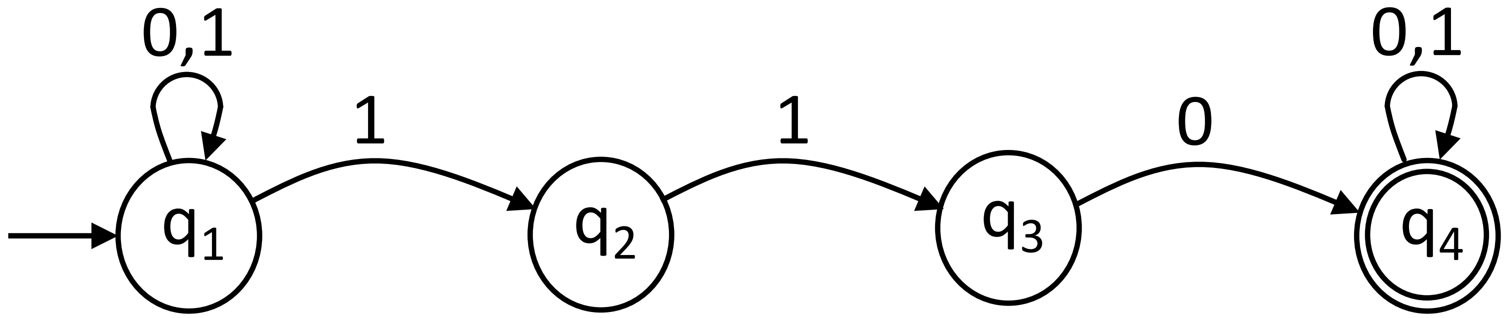

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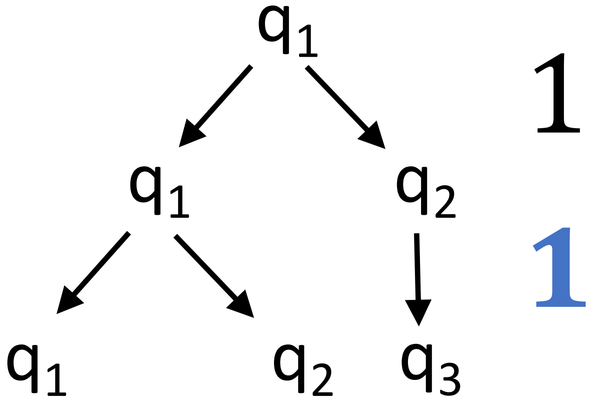

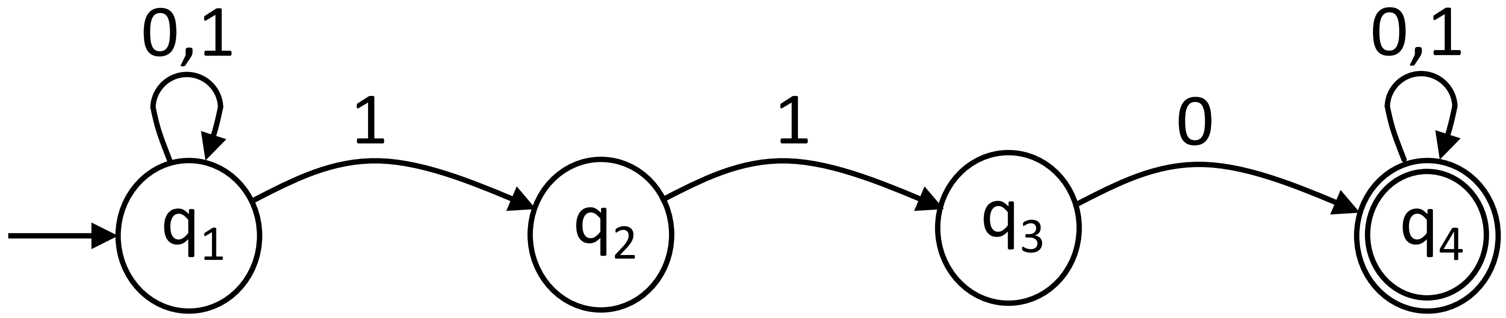

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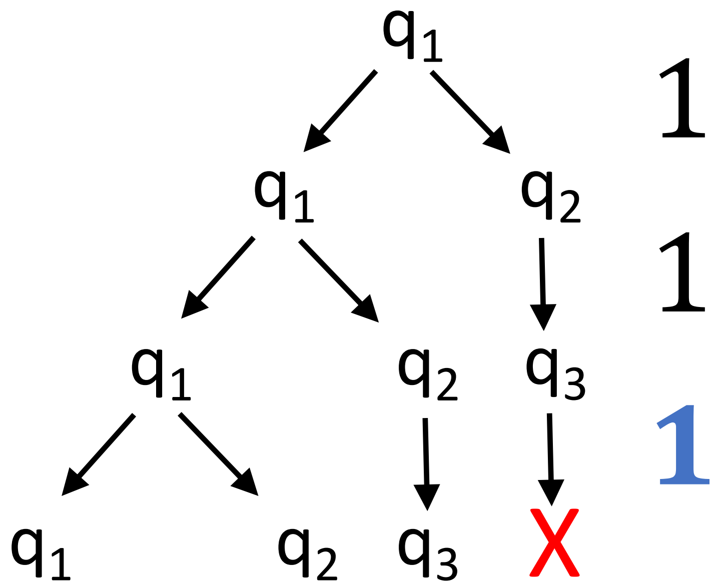

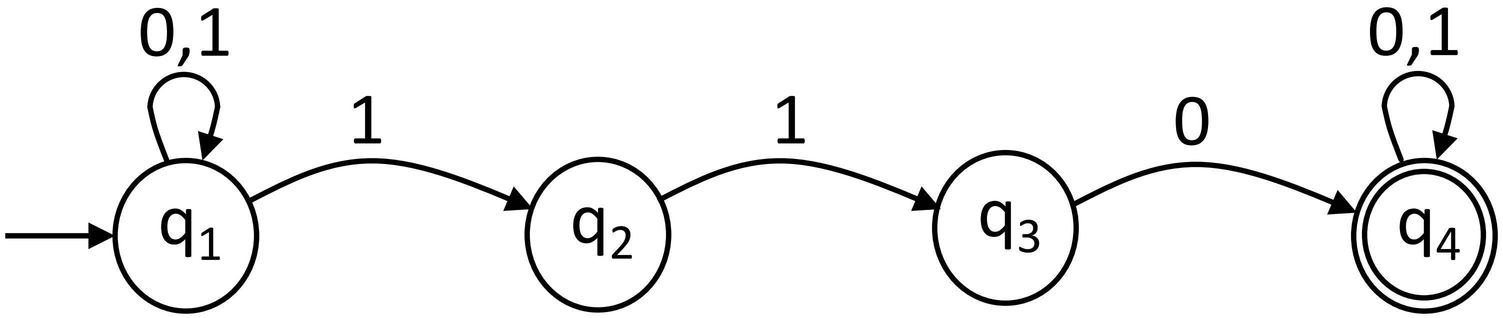

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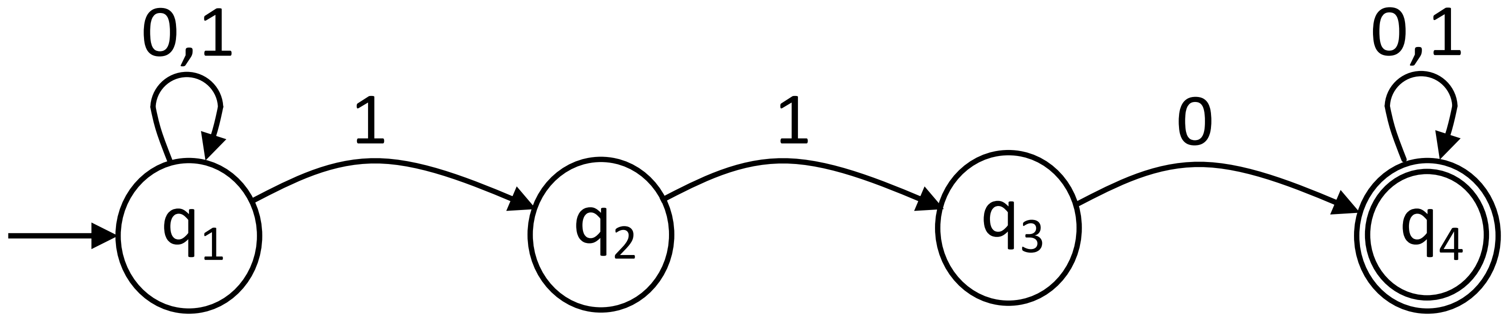

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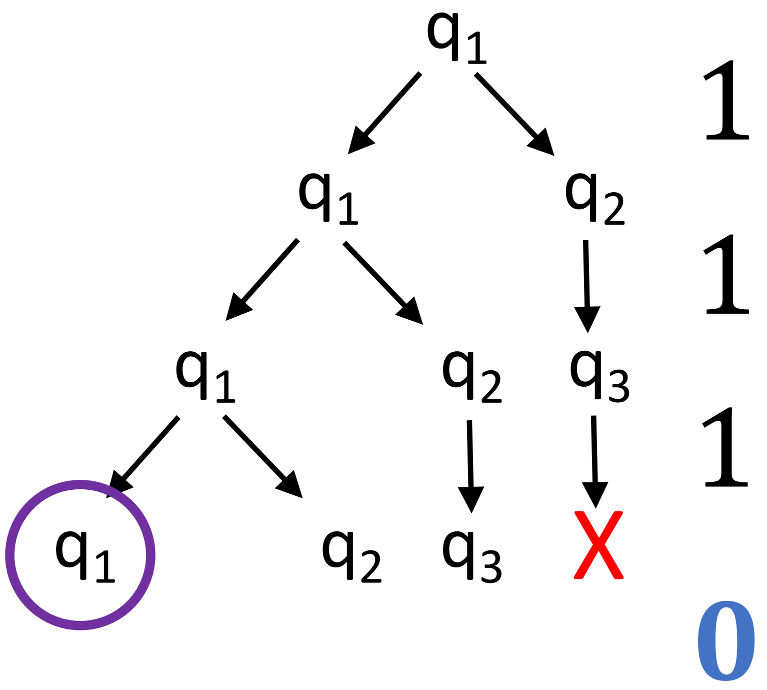

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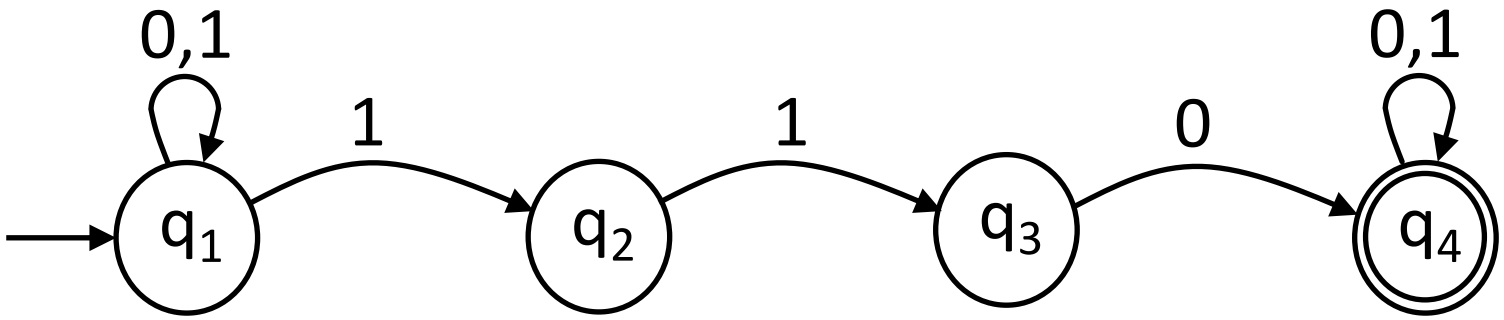

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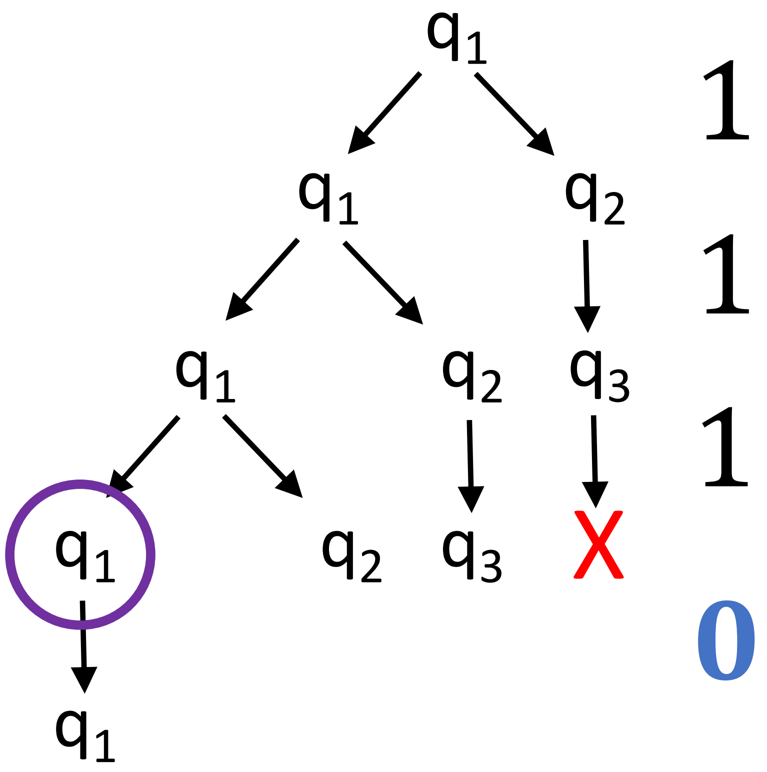

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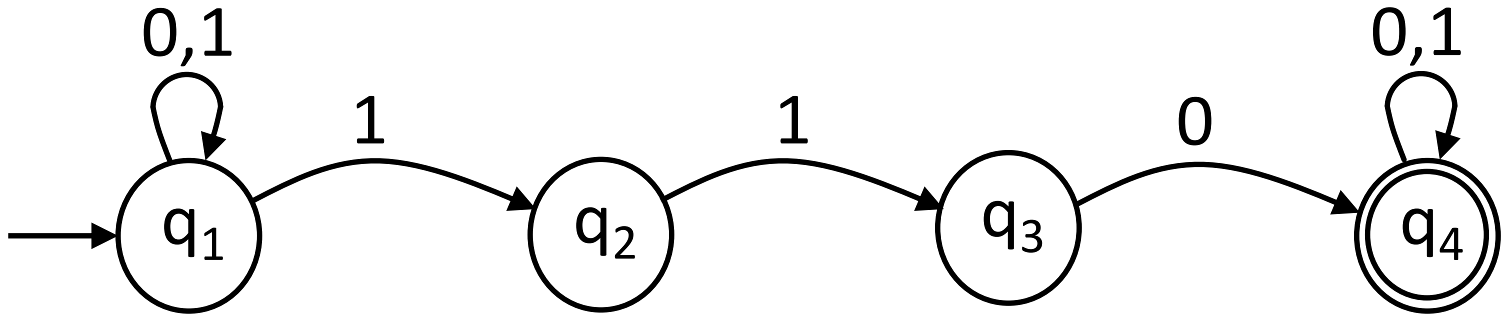

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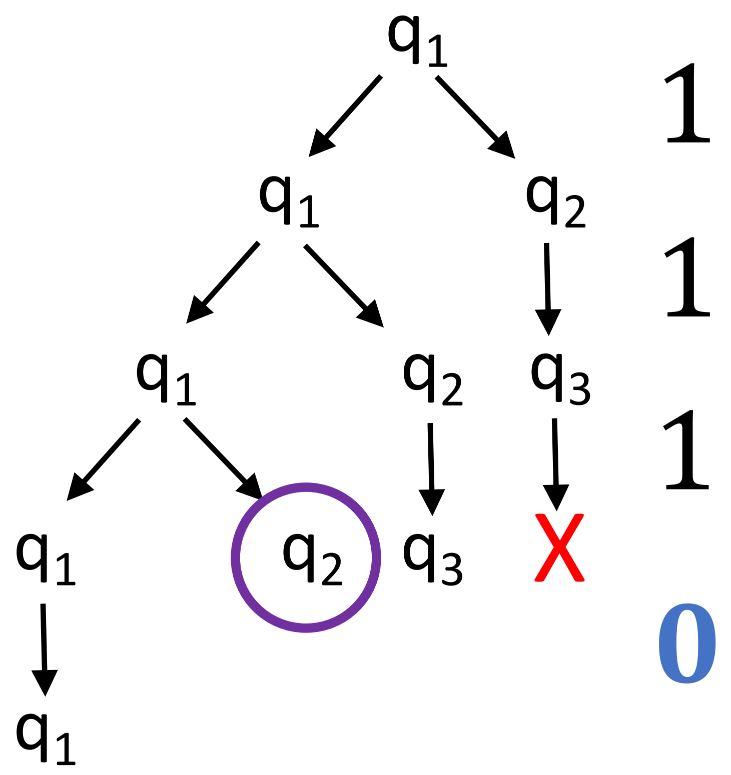

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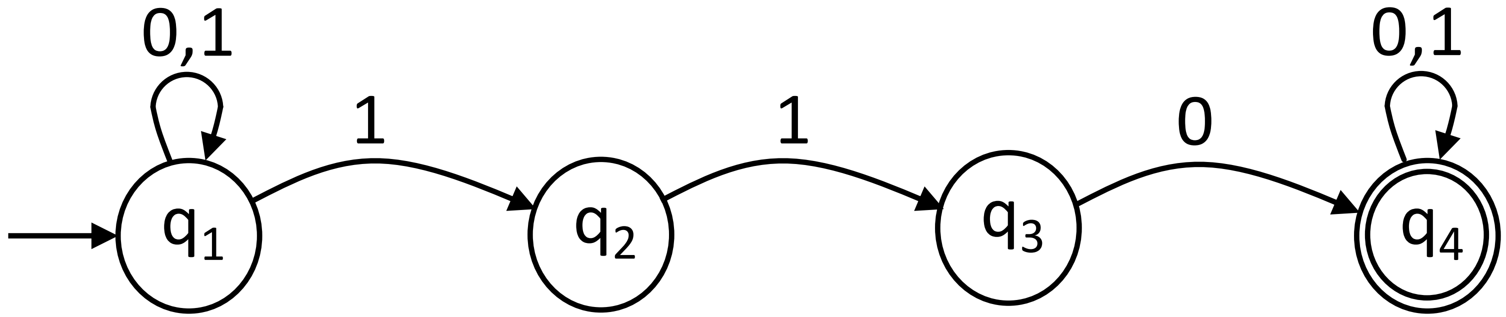

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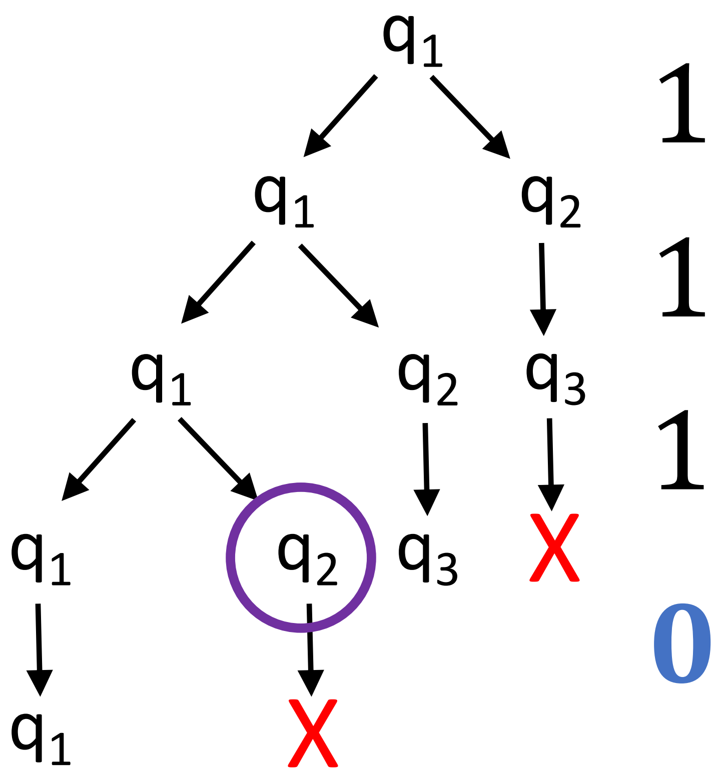

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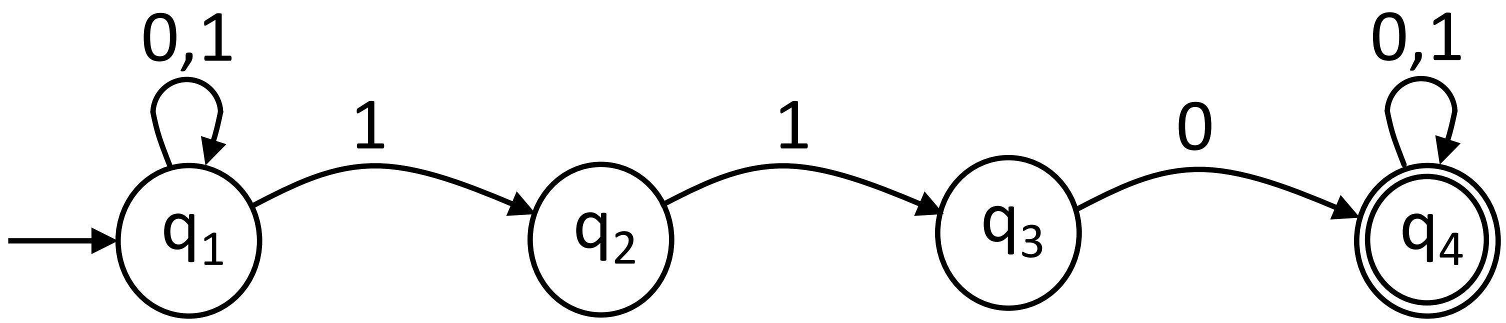

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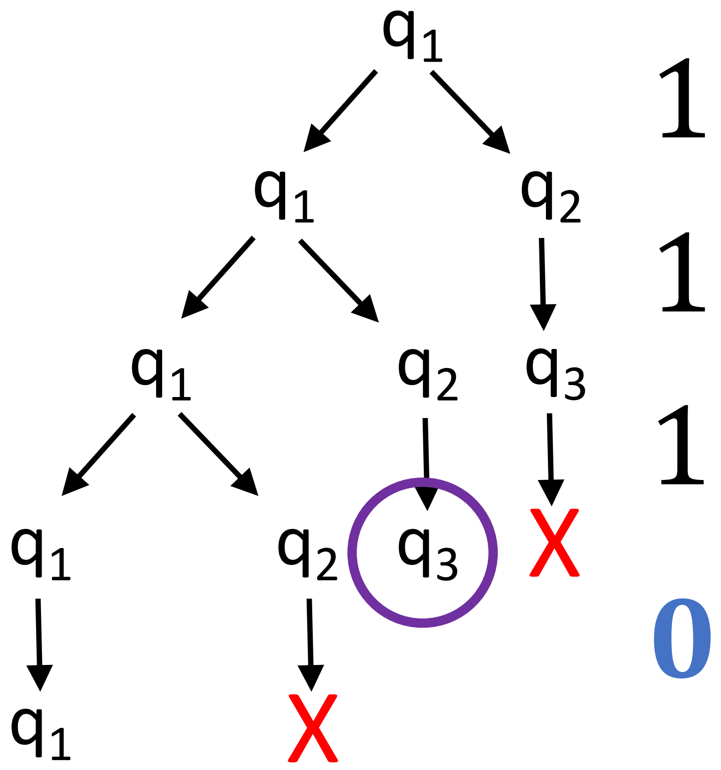

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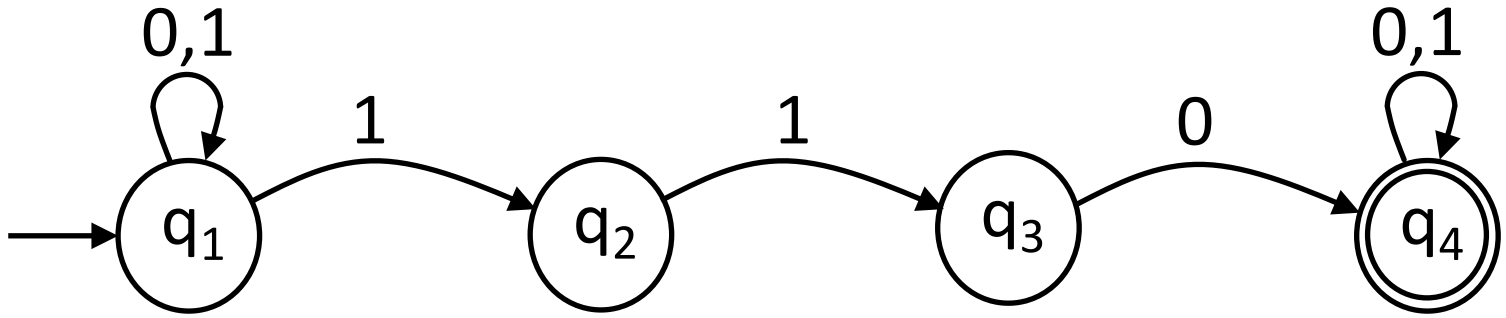

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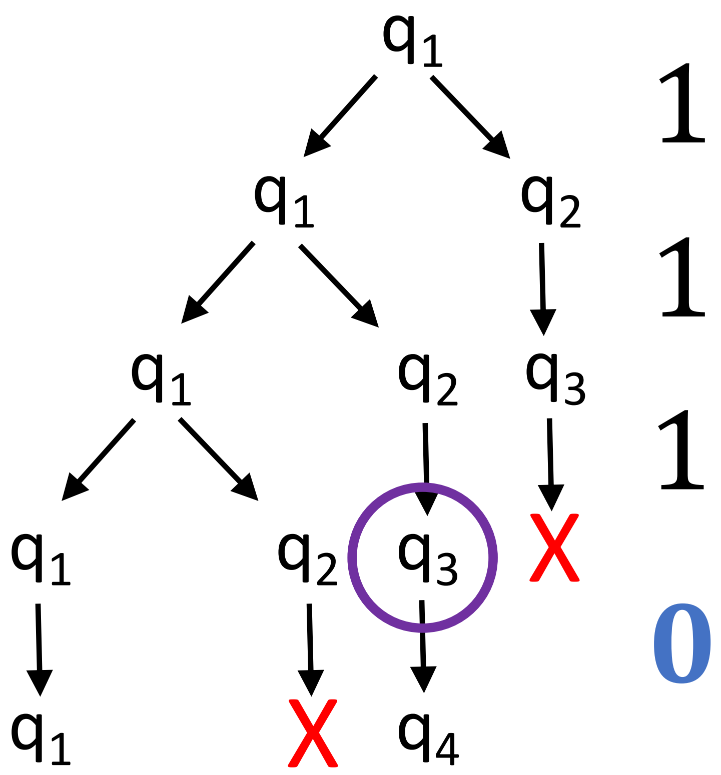

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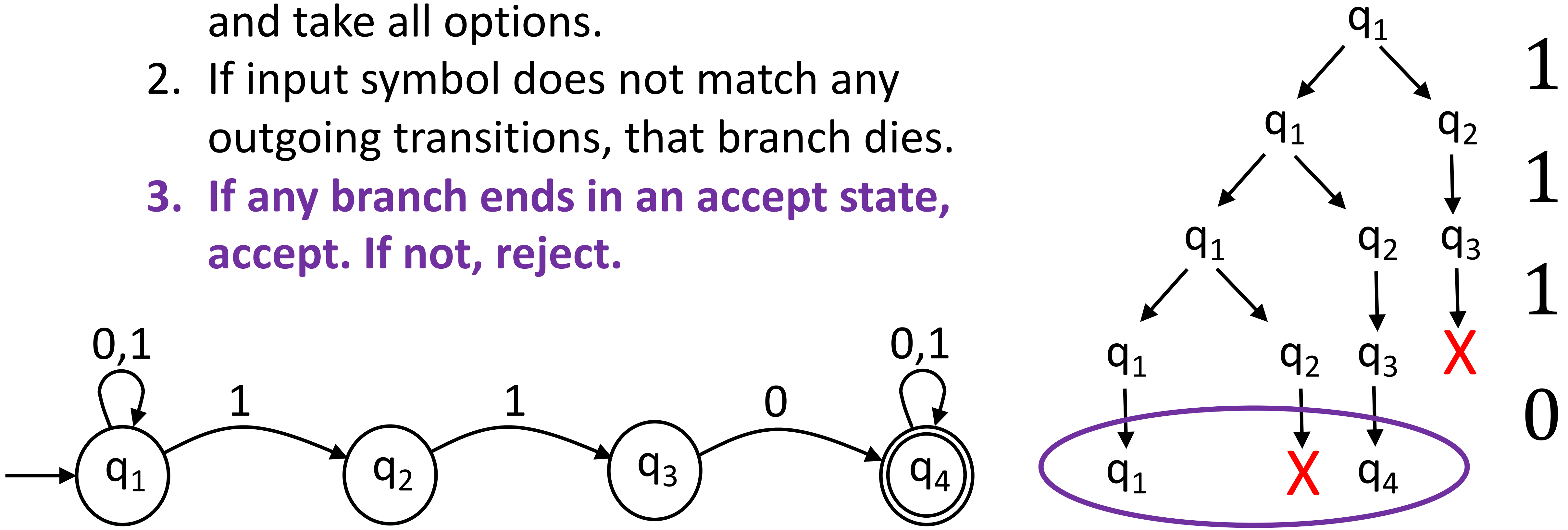


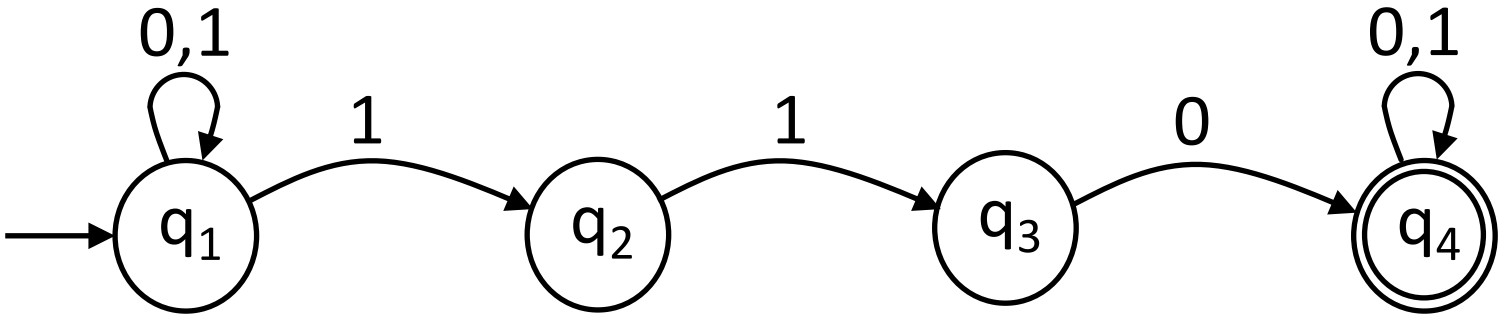
{ω: ω contains the substring 110}

$\omega = 1110$

NFA Processing

1. If a "decision" is encountered, split
   ar
2. If
   ou
3. If
   accept. If not, reject.

Alternate approach: If it is possible to end on an accept state, accept.



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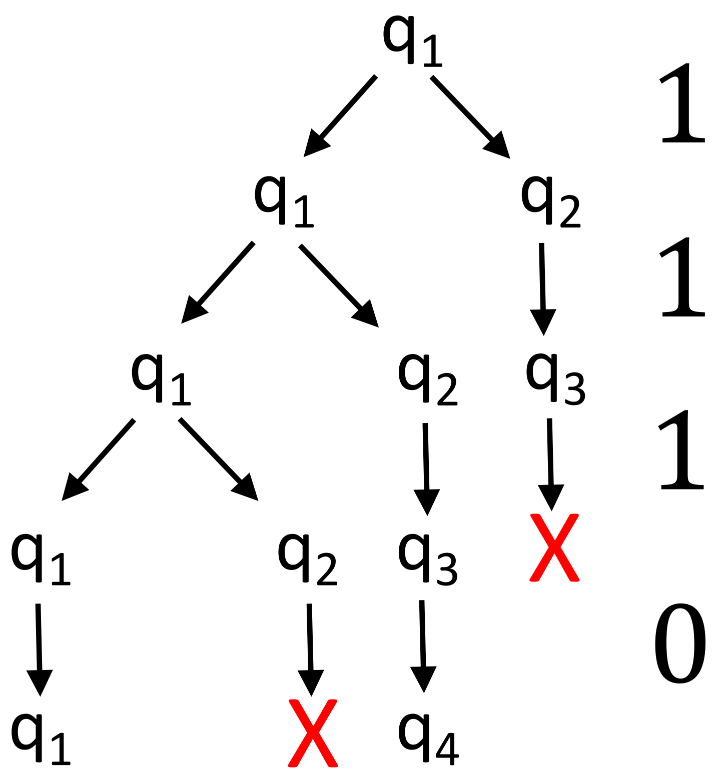

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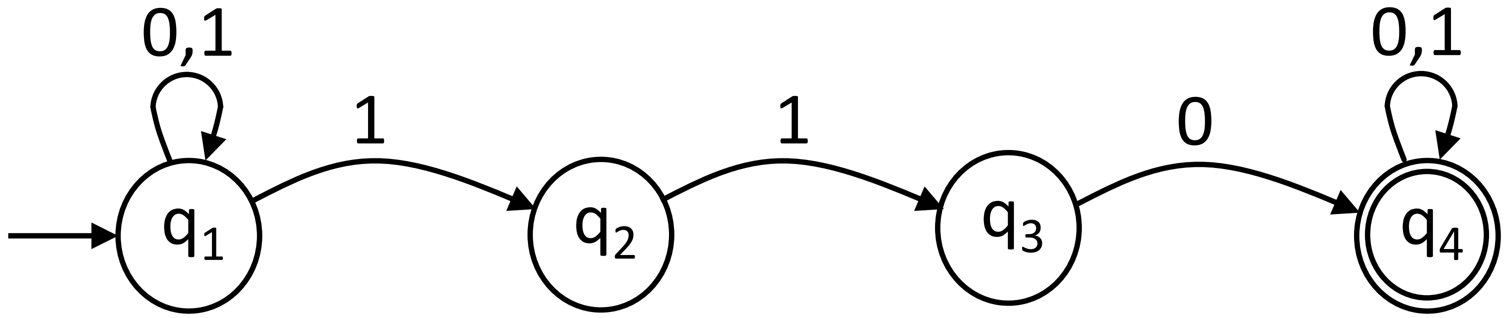

$\{\omega\text{: } \omega$ contains the substring $110\}$

$$\omega = 101$$

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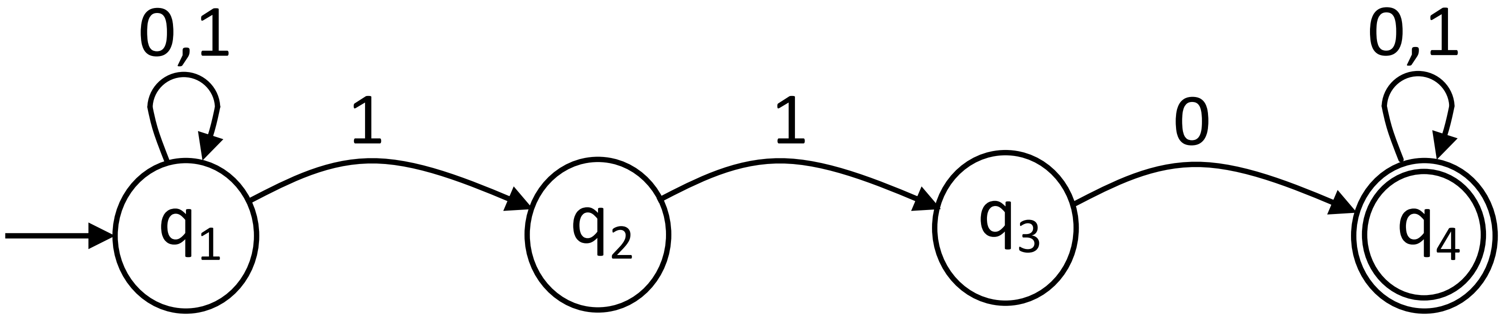

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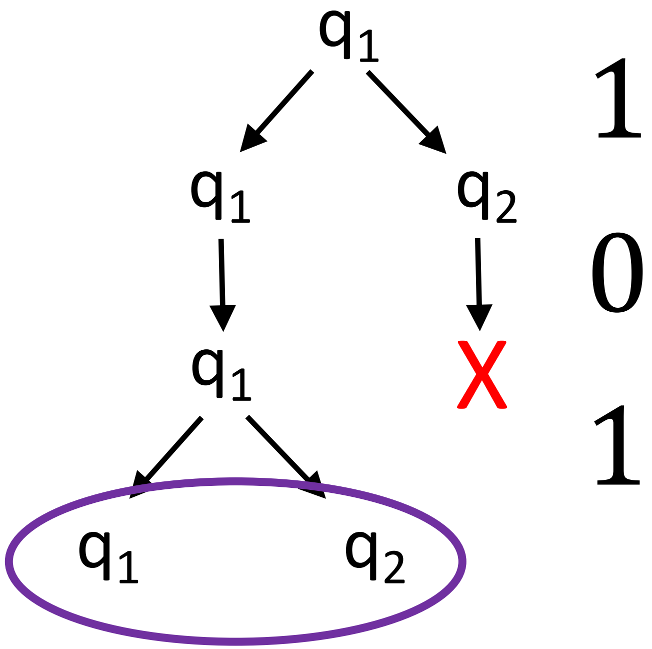
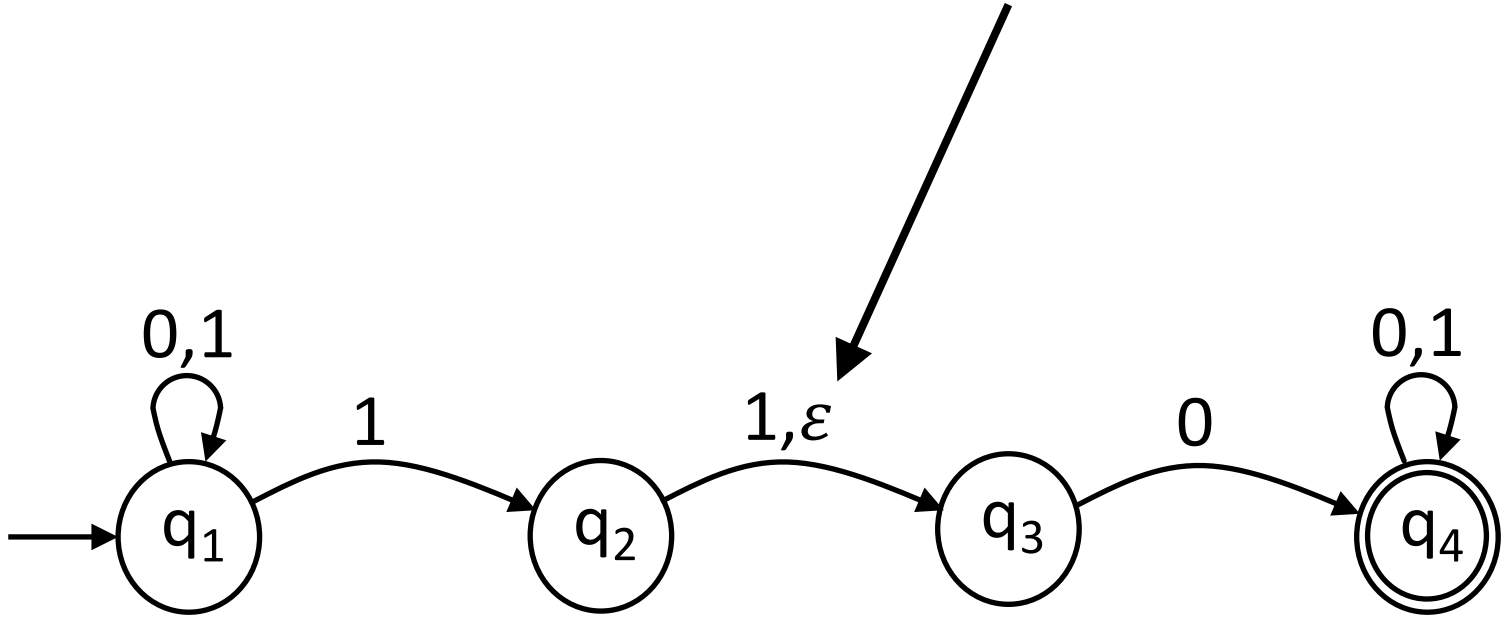


$\omega = 101$

ε-Transitions

ε-transitions can be taken without consuming any characters from the string being processed.



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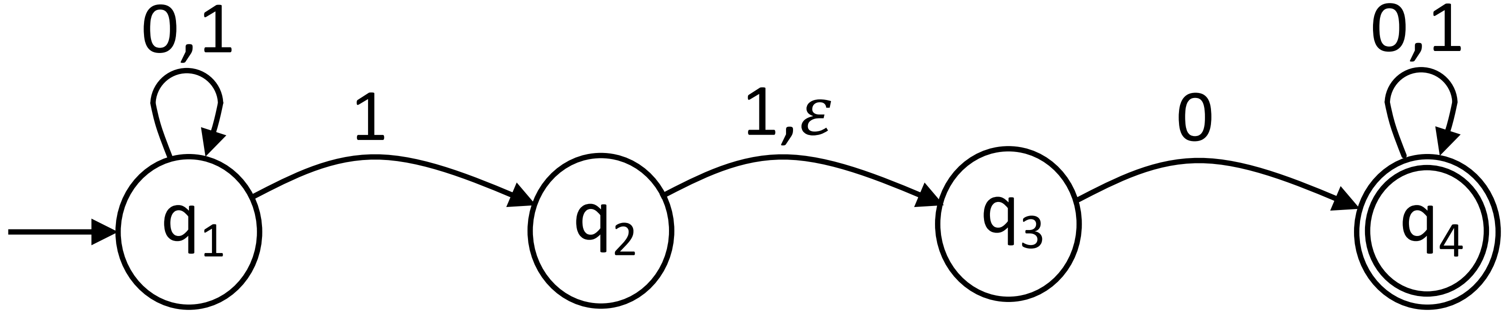

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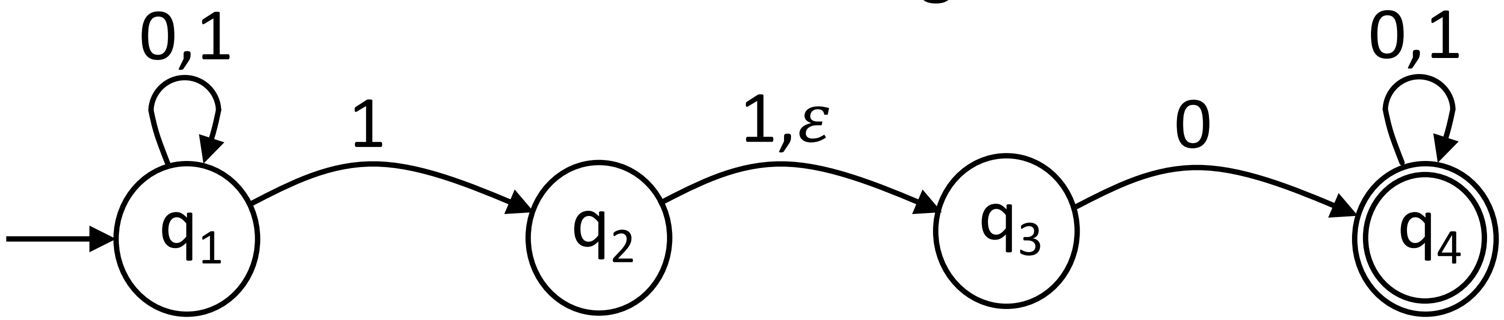

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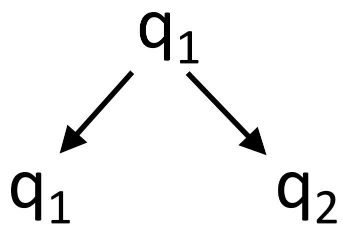

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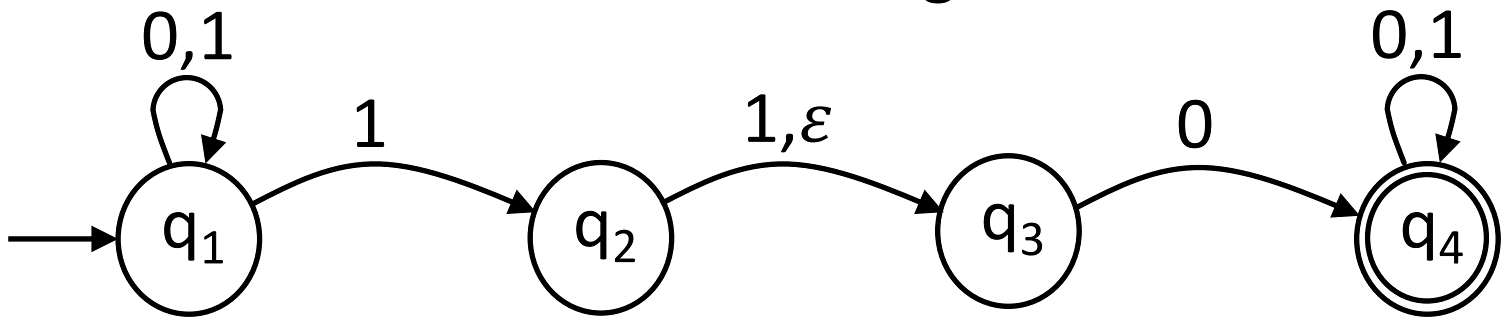

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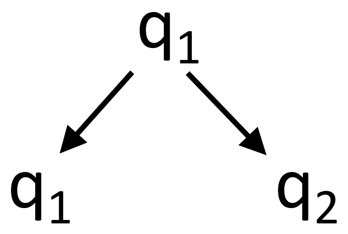
ε-Transitions



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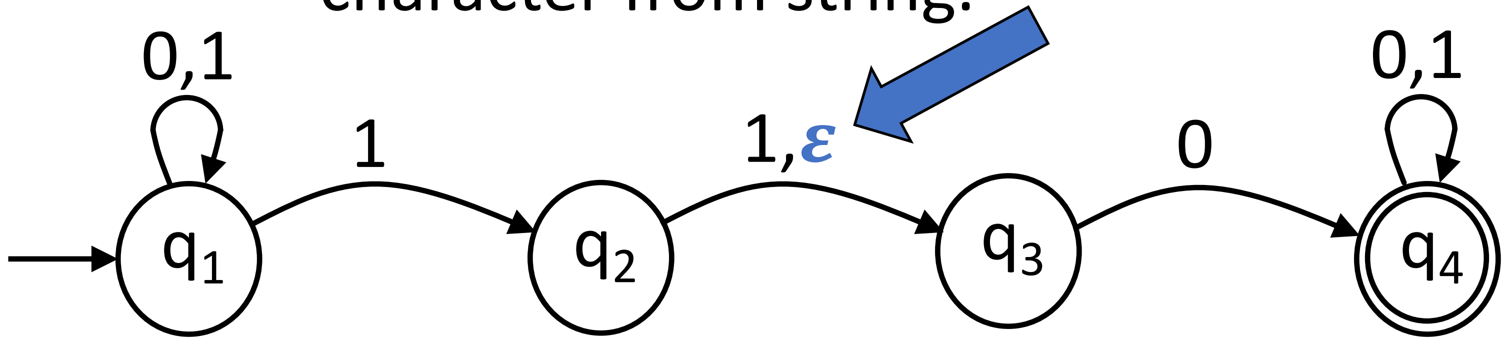

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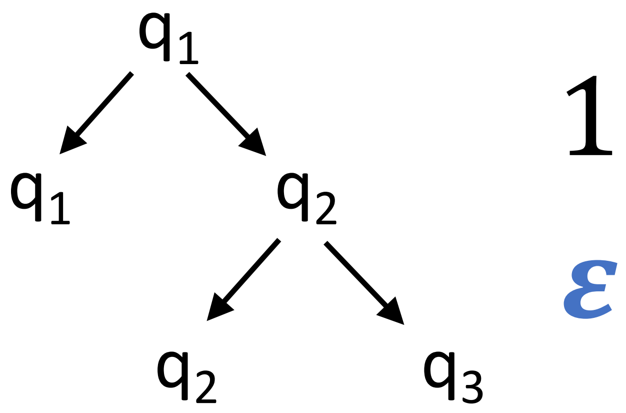

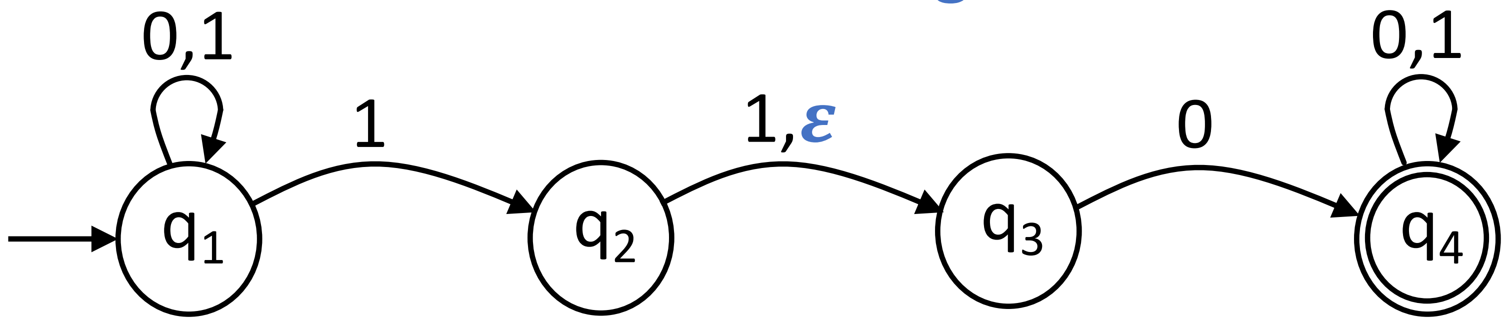

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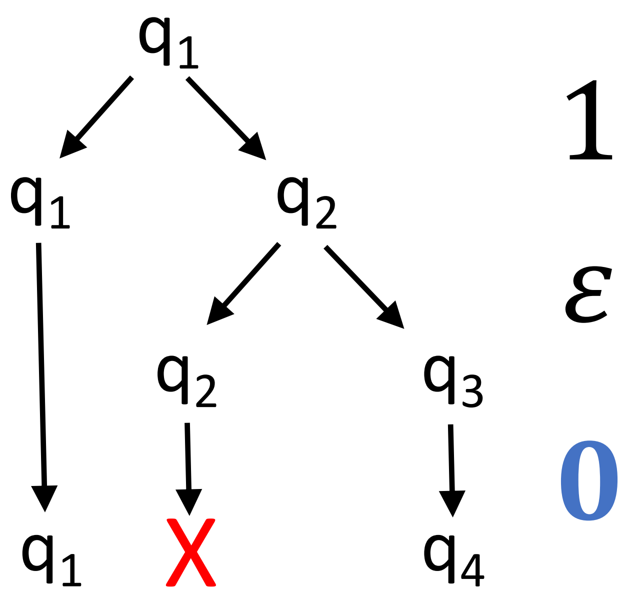

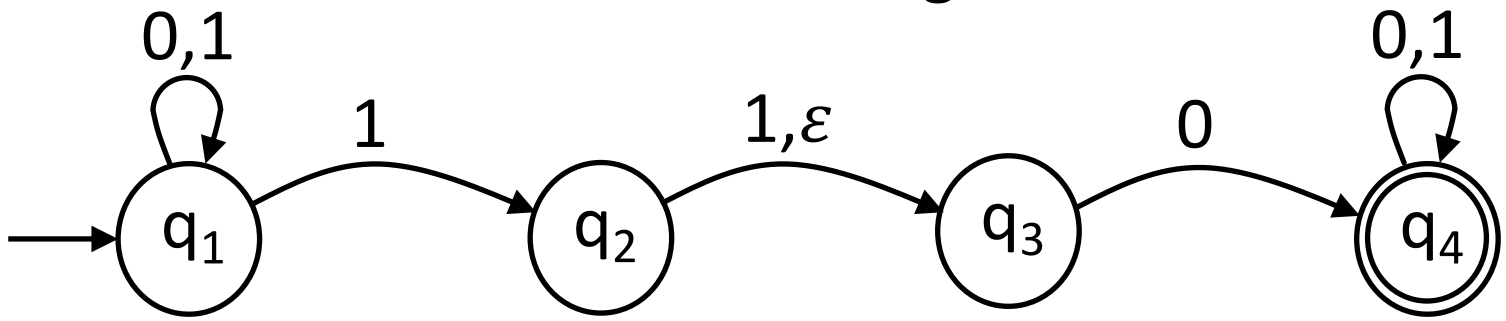

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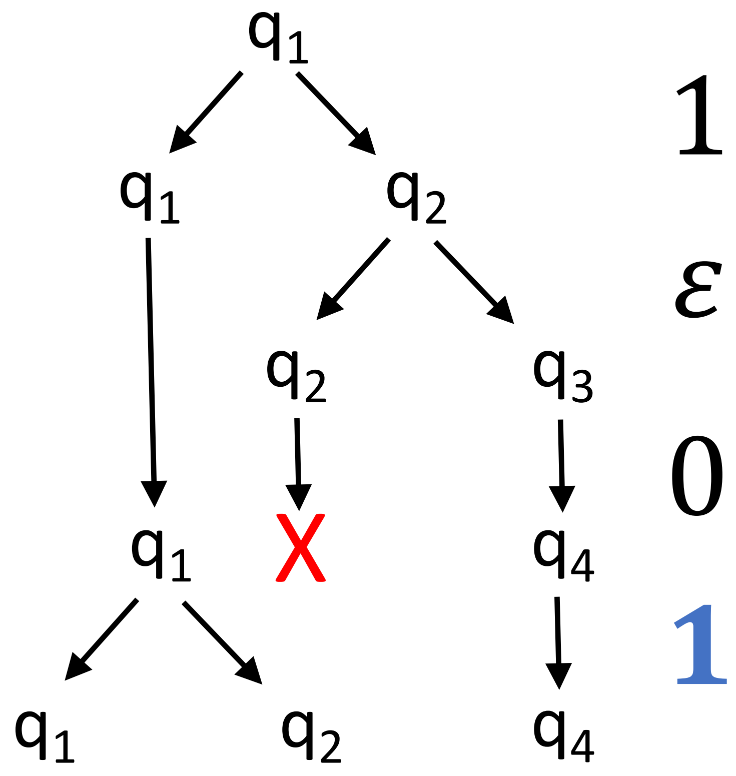

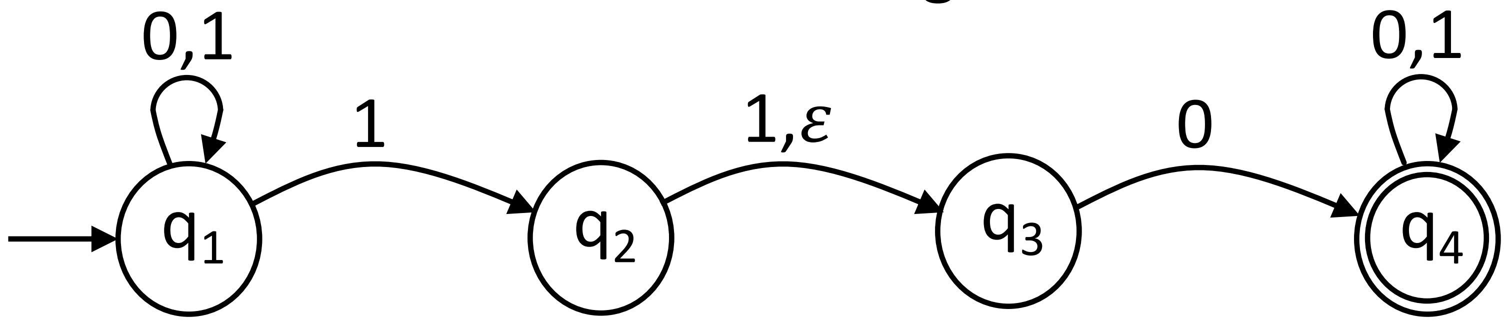

$\omega = 101$

ε-Transitions

1. If a “decision” is encountered, split and take all options.
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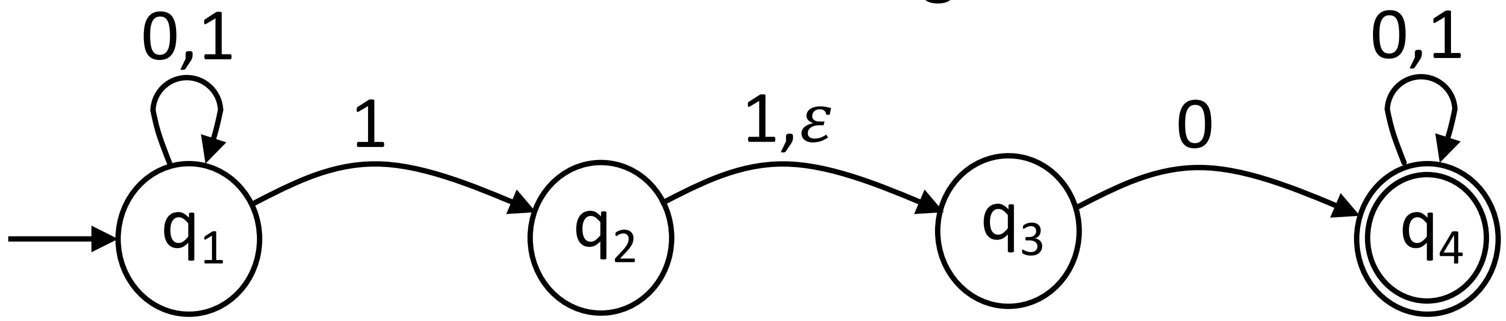

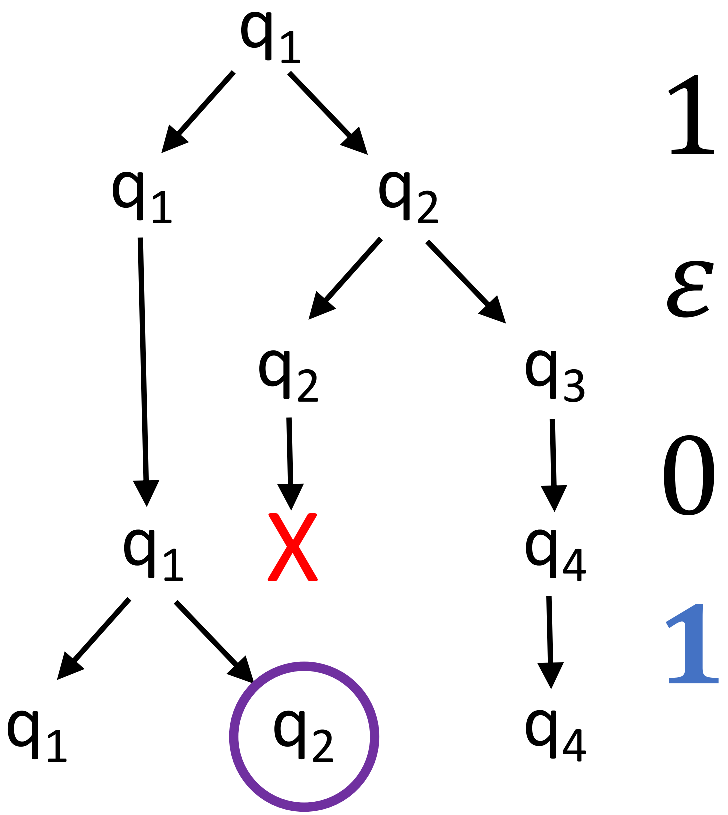

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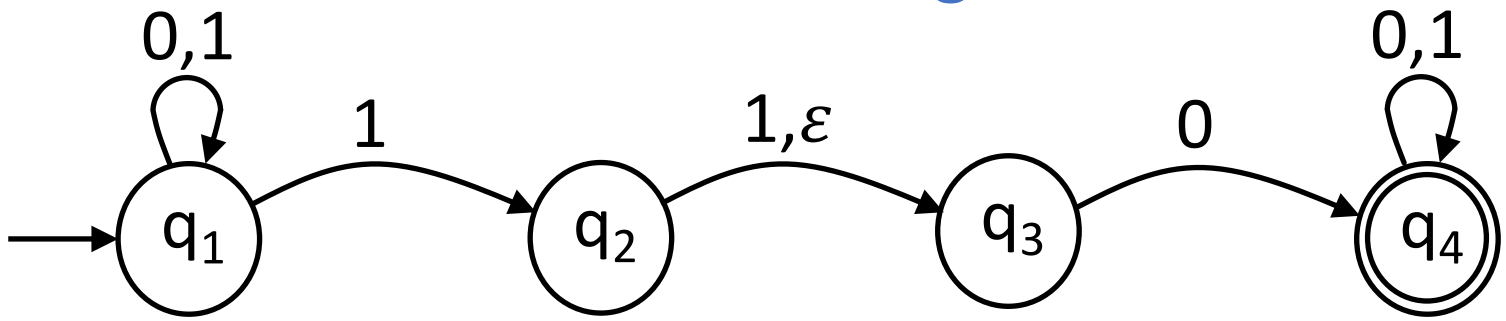

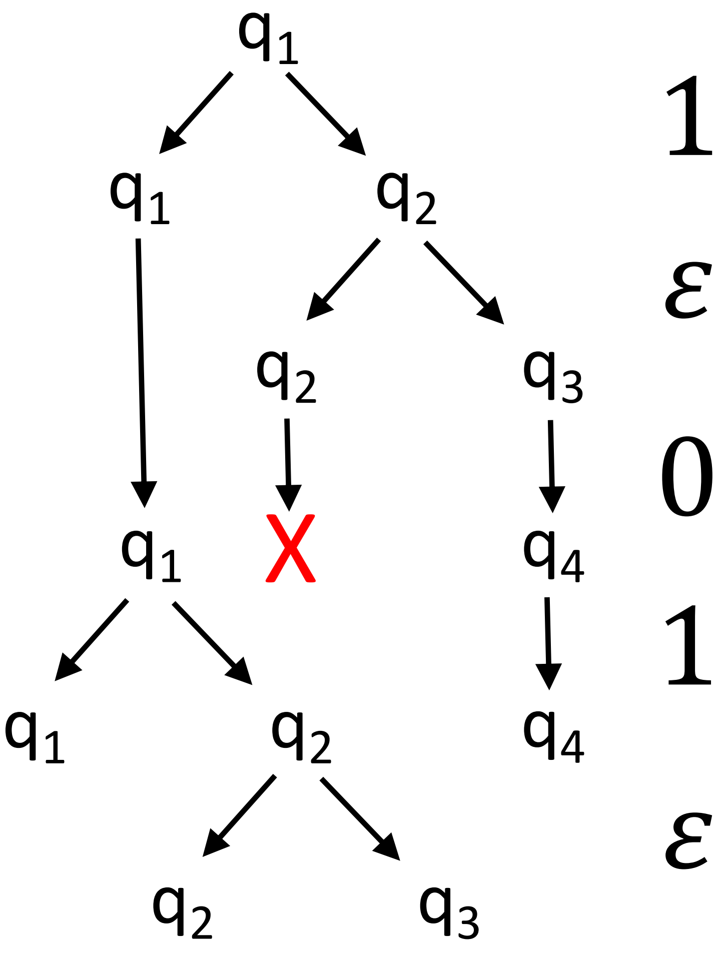

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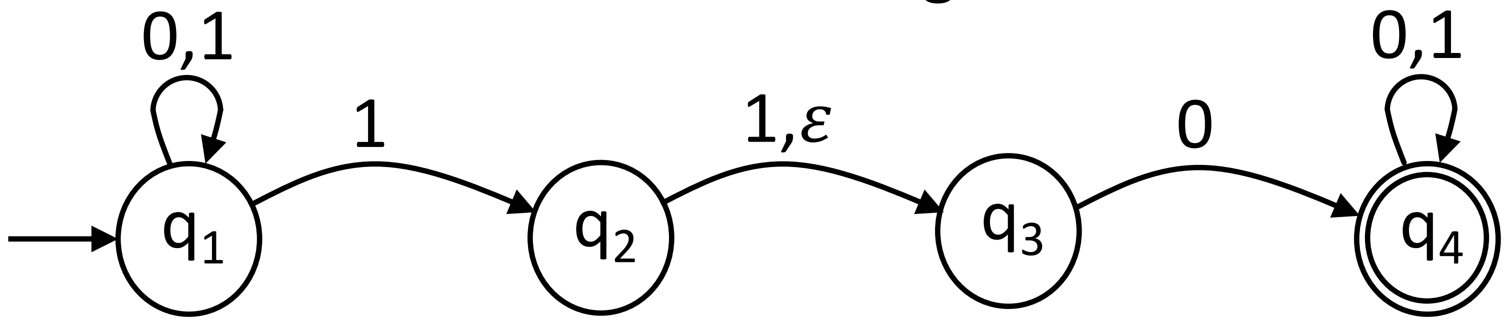

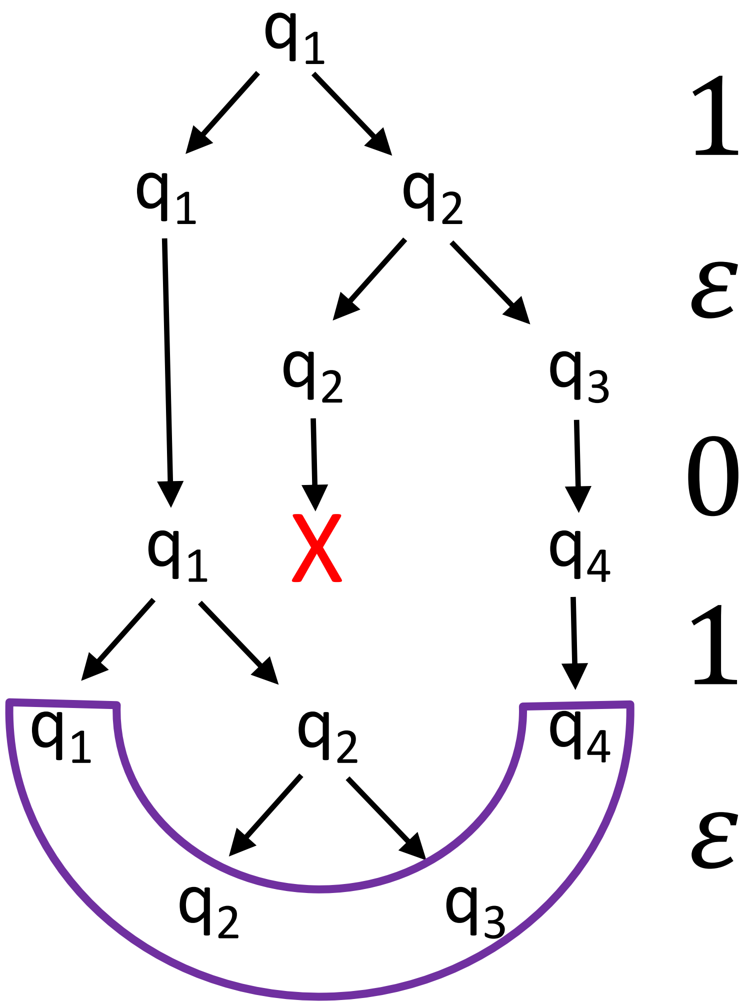

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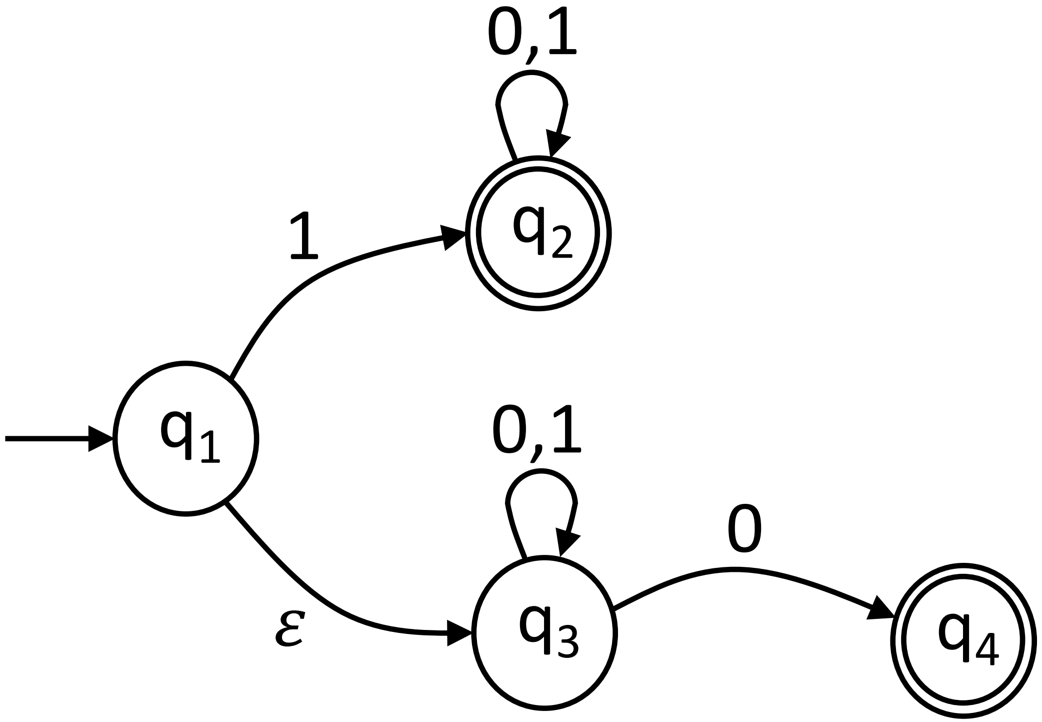
NFA Practice

What is the language of the following NFA?



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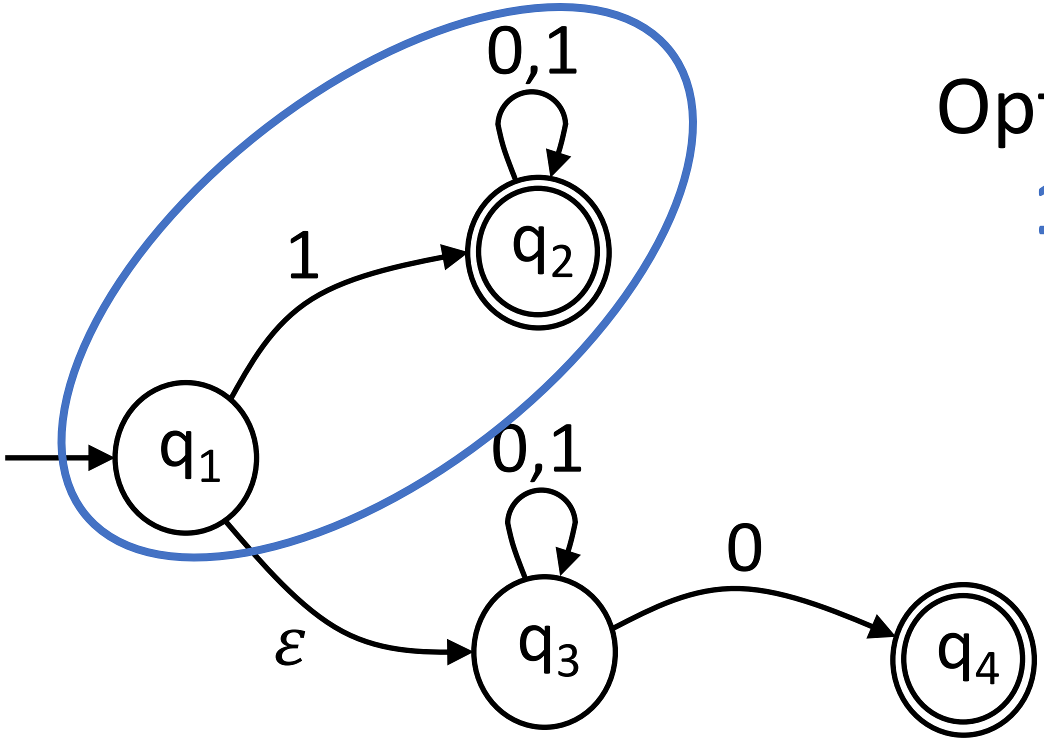

Options for string to be accepted:

1. **Start with 1, followed by anything.**

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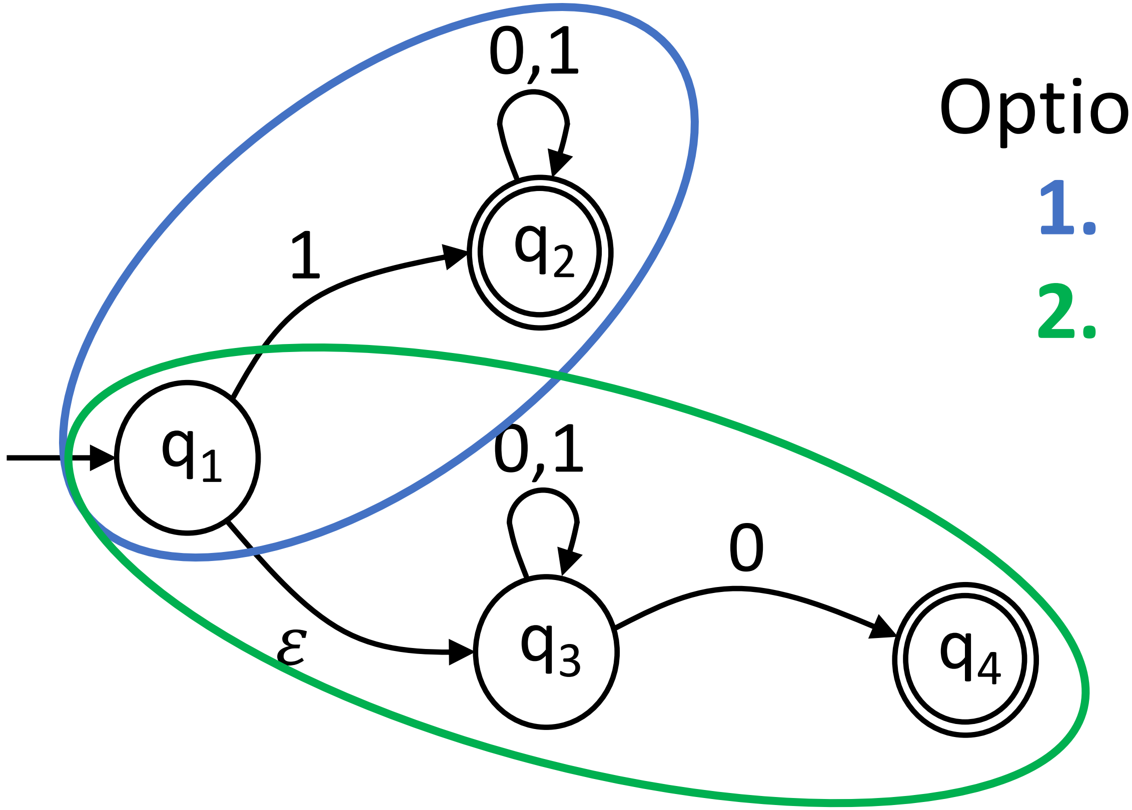

Options for string to be accepted:

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2. **Start with anything, ending with 0.**

NFA Practice

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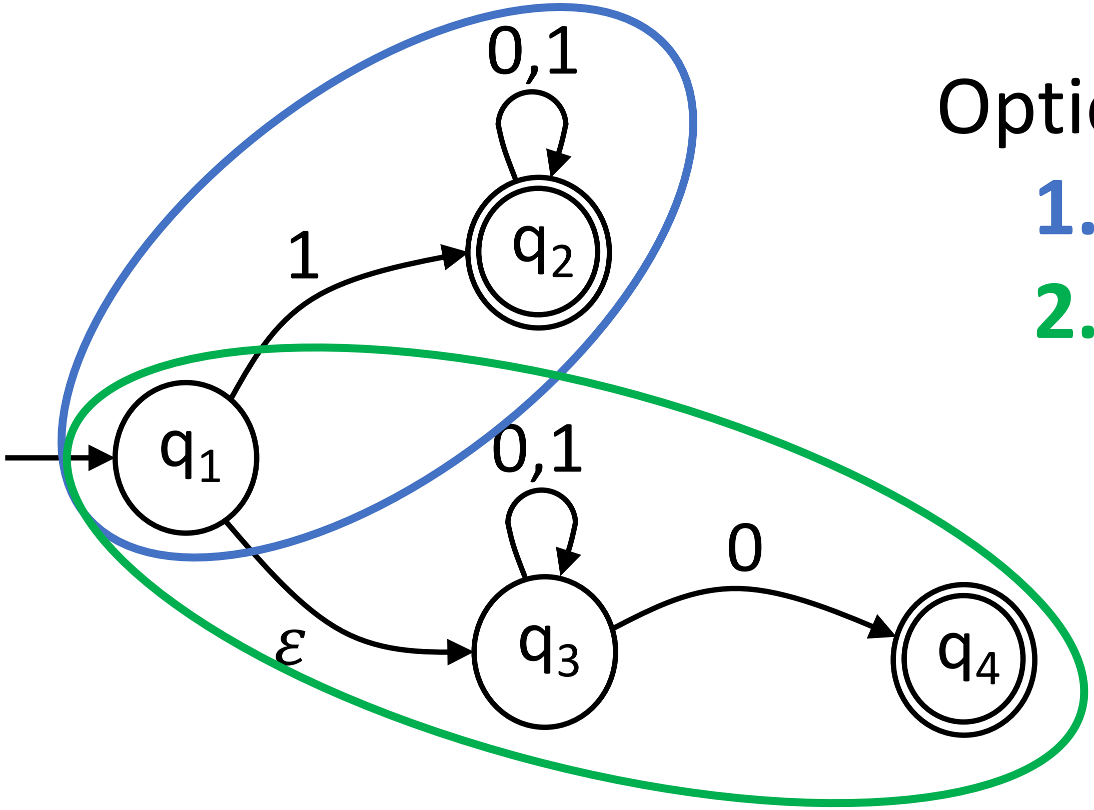

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$\{\omega\colon\ \omega$ starts with 1 or ends with $0\}$

NFA Practice

What is the NFA that accepts $\{\omega\colon\ \omega$ starts with 1 **and** ends with $0\}$?

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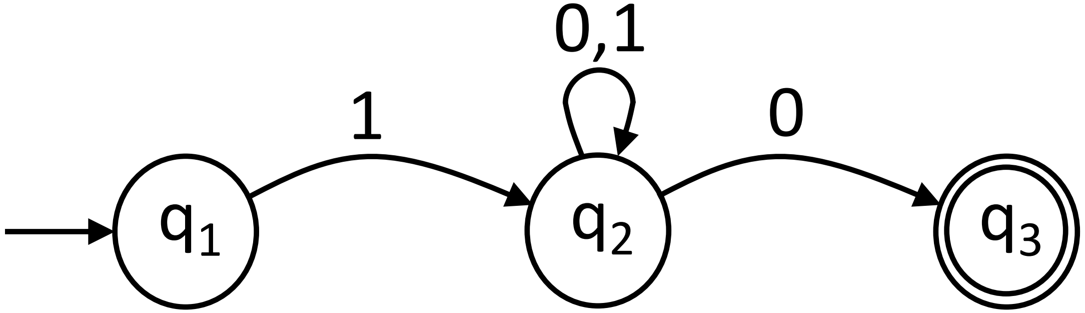

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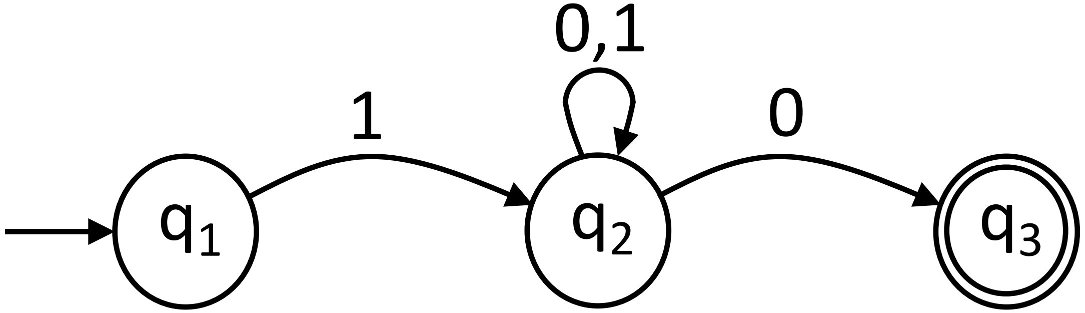

Only ω's that start with 1 get to q_2.

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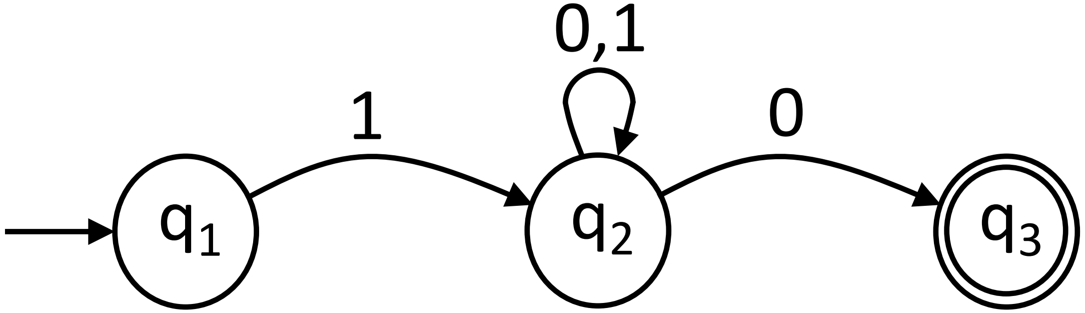
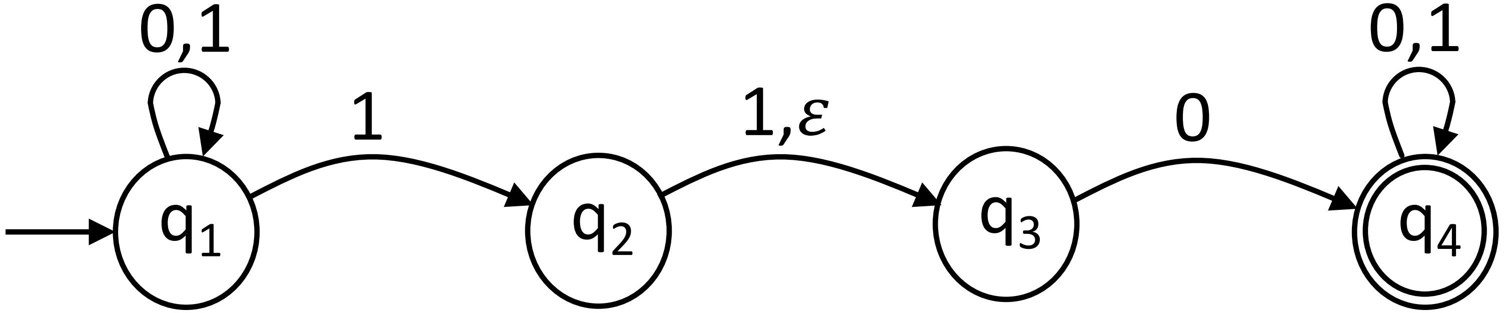


Only ω's that start with 1 get to q_2.
Any string that gets to q_2, can get to q_3 <u>and terminate</u>, if it ends with 0.

NFA Formal Definition

NFAs consist of:

1. Finite set of states, Q.
2. Finite alphabet, Σ.
3. Transition function, $\delta: Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow \mathcal{P}(Q)$.
4. Start state, $q_0 \in Q$.
5. Set of accept states, $F \subseteq Q$.



NFA Formal Definition

Power set of Q.

I.e. set of all subsets.

E.g. $Q = \{q_1, q_2\}$

$\Rightarrow \mathcal{P}(Q) = \{\emptyset, \{q_1\}, \{q_2\}, \{q_1, q_2\}\}$

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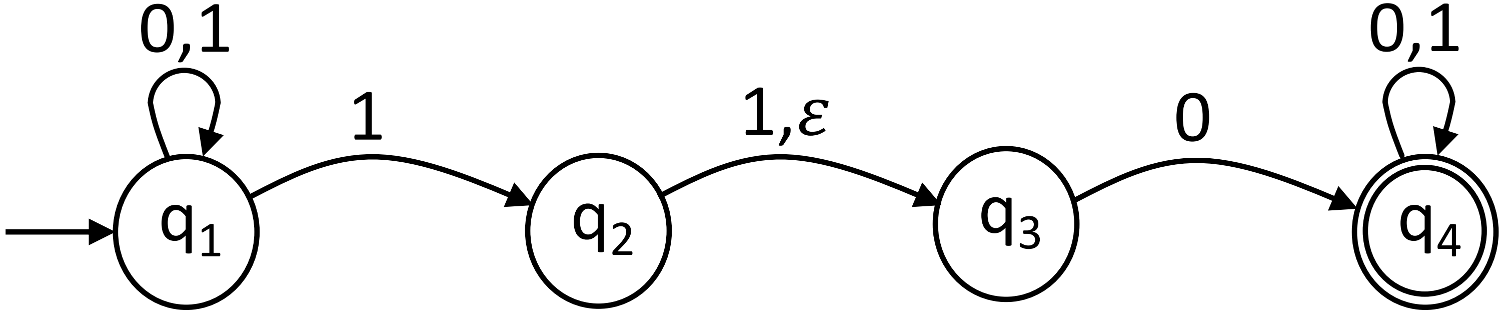

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I.e. $\exists$ 0 or more transitions for each $e \in \Sigma \cup \{\varepsilon\}$ at each state

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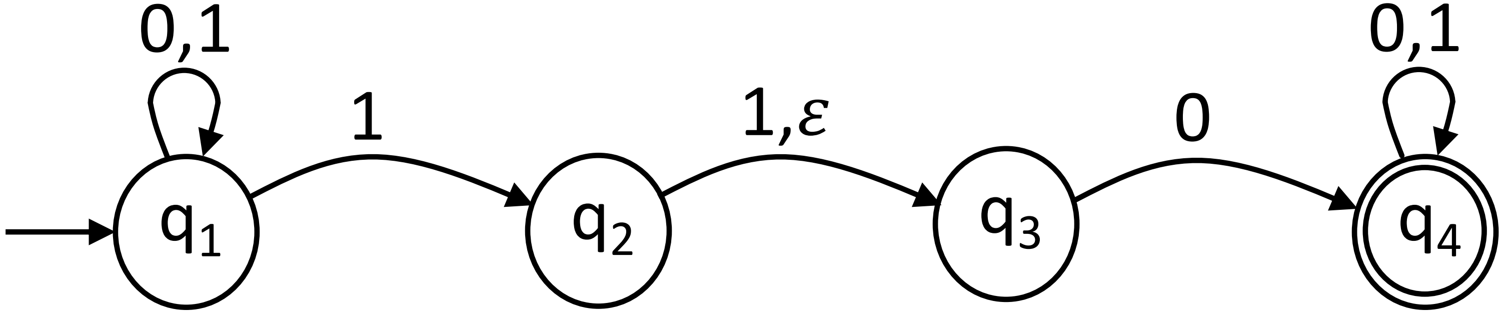
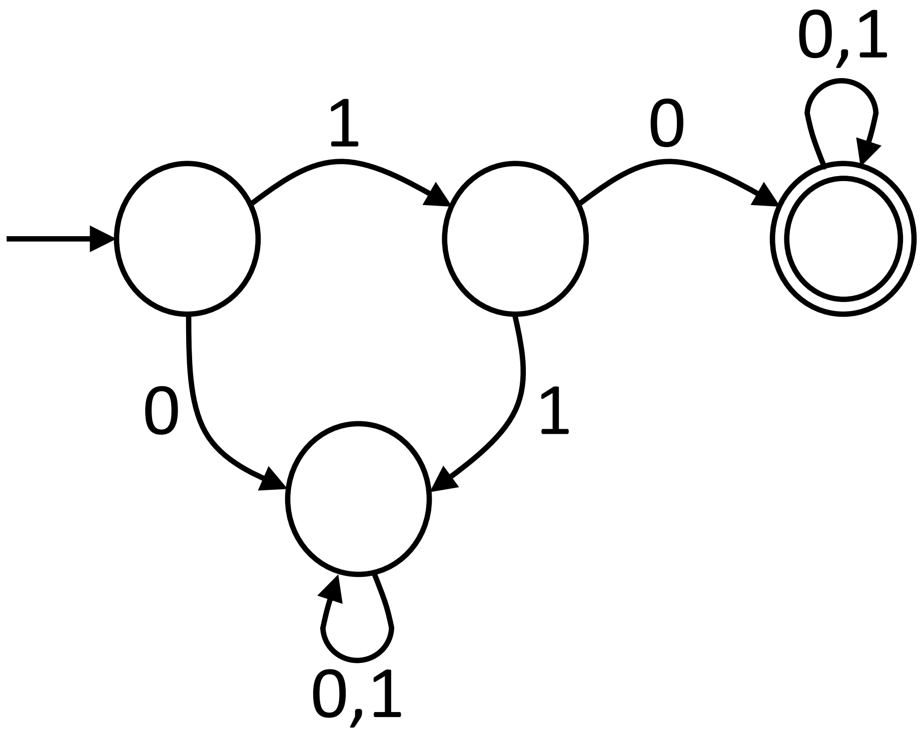


Build an NFA for the following language:

$\{\omega : \omega$ begins with sequence $10\}$.



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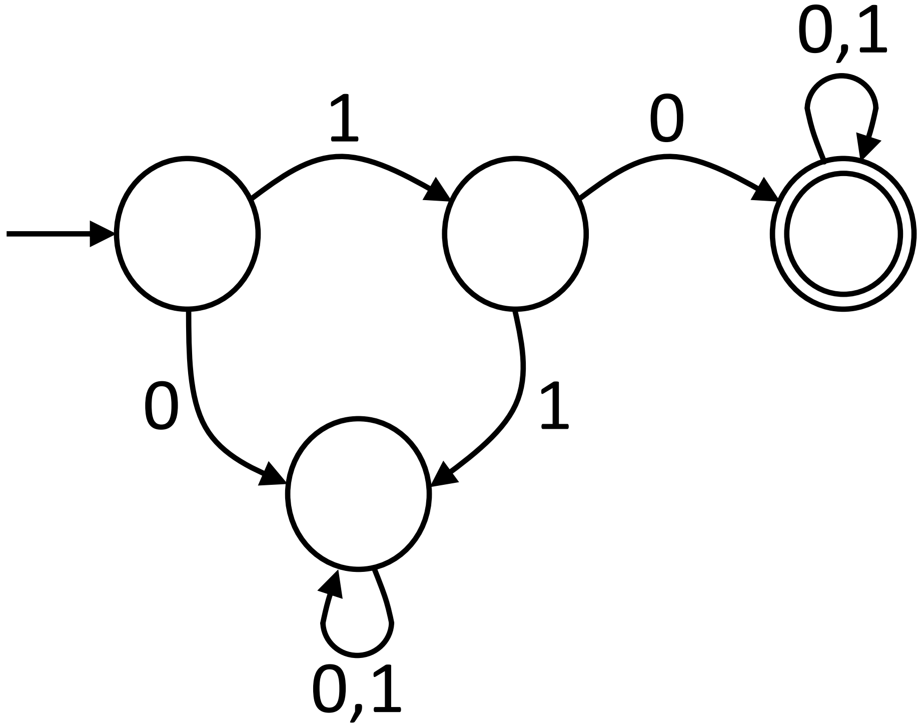

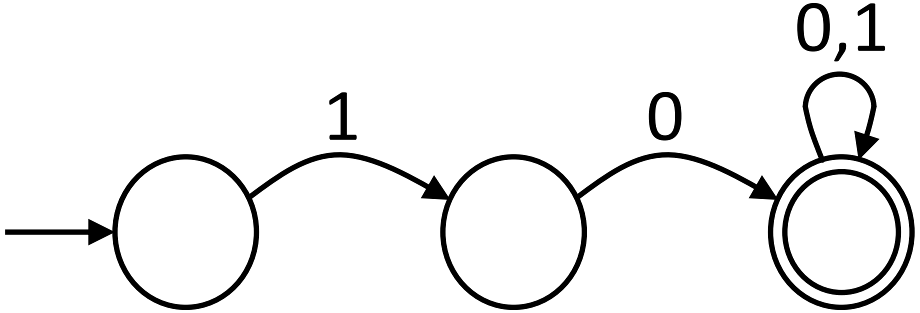

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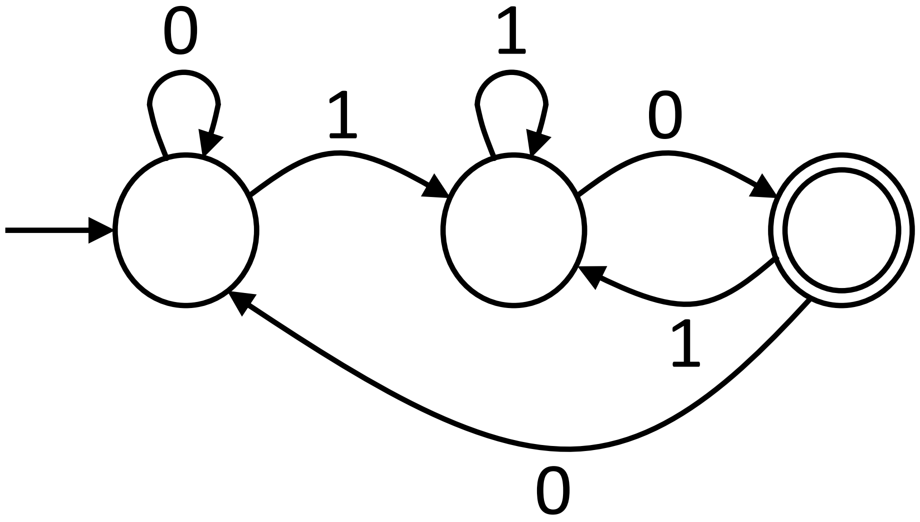

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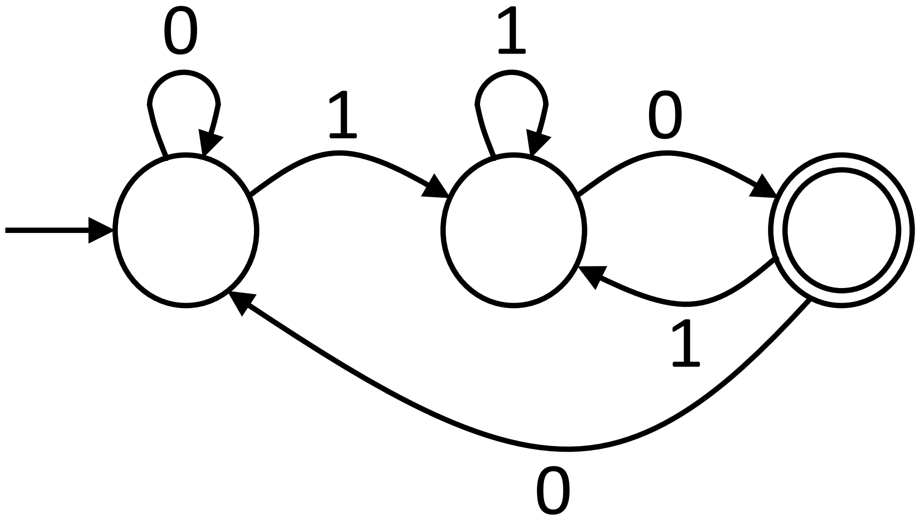

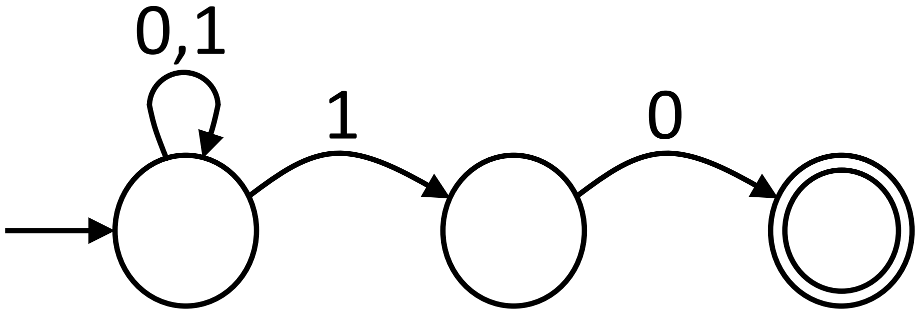

Build an NFA for the following language:

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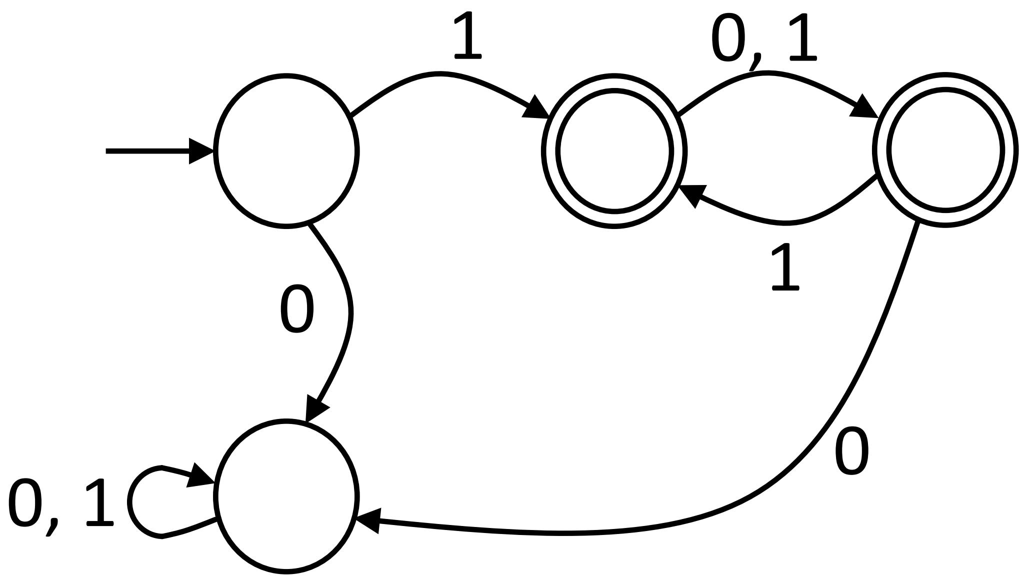

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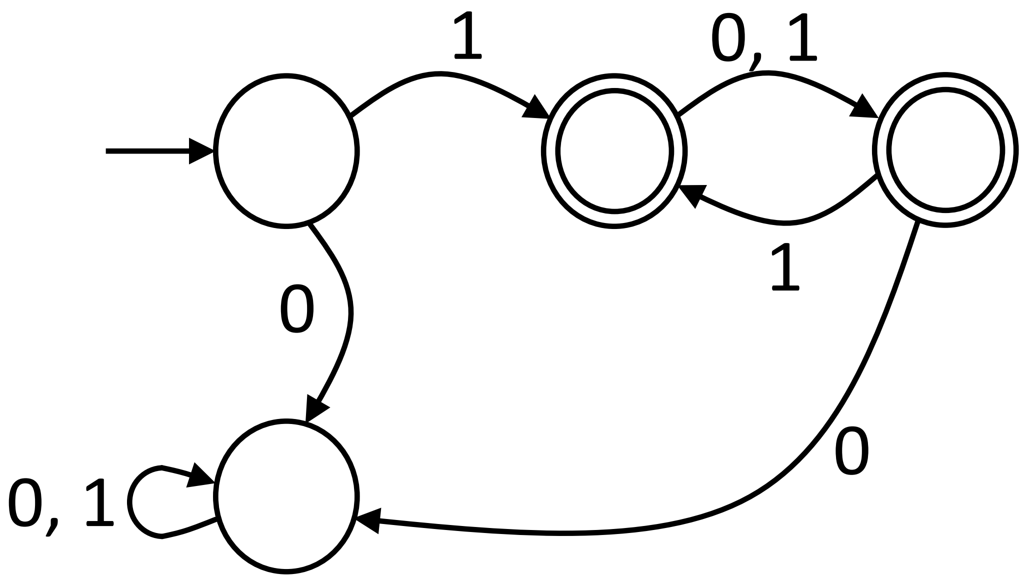
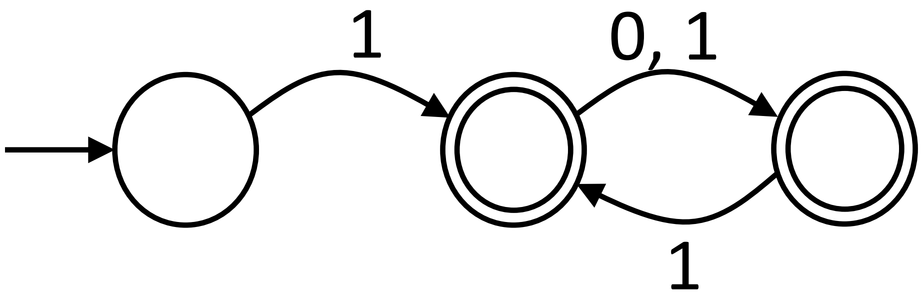
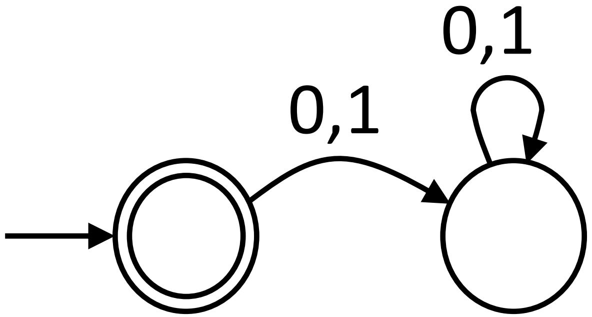




Build an NFA for the following language:

$\{\varepsilon\}$.



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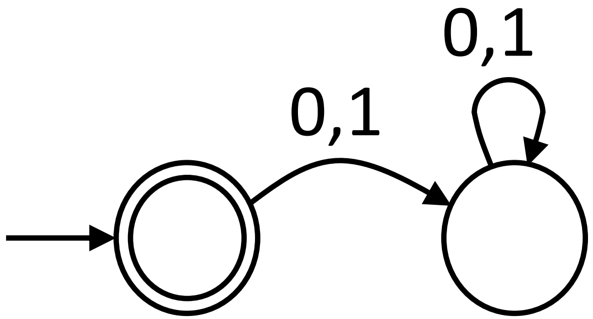

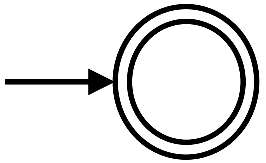

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$\{\omega : \omega$ contains an even number of 0’s$\}$.

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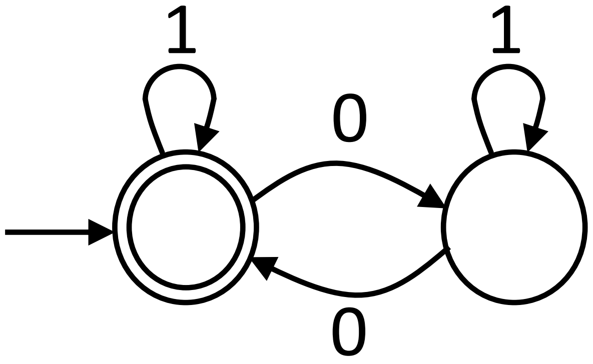

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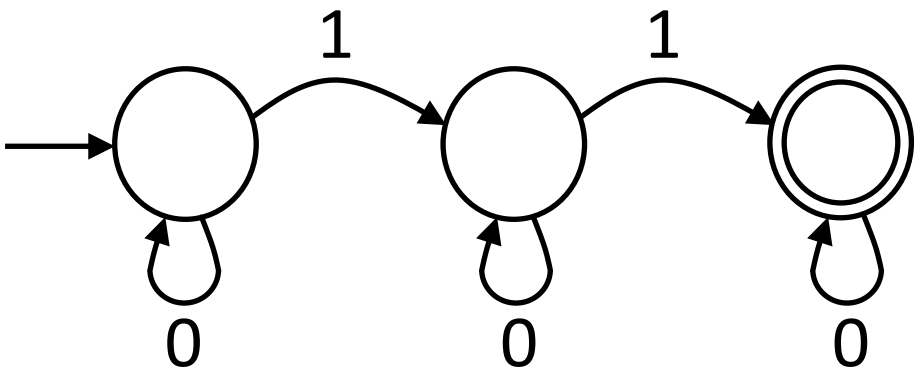
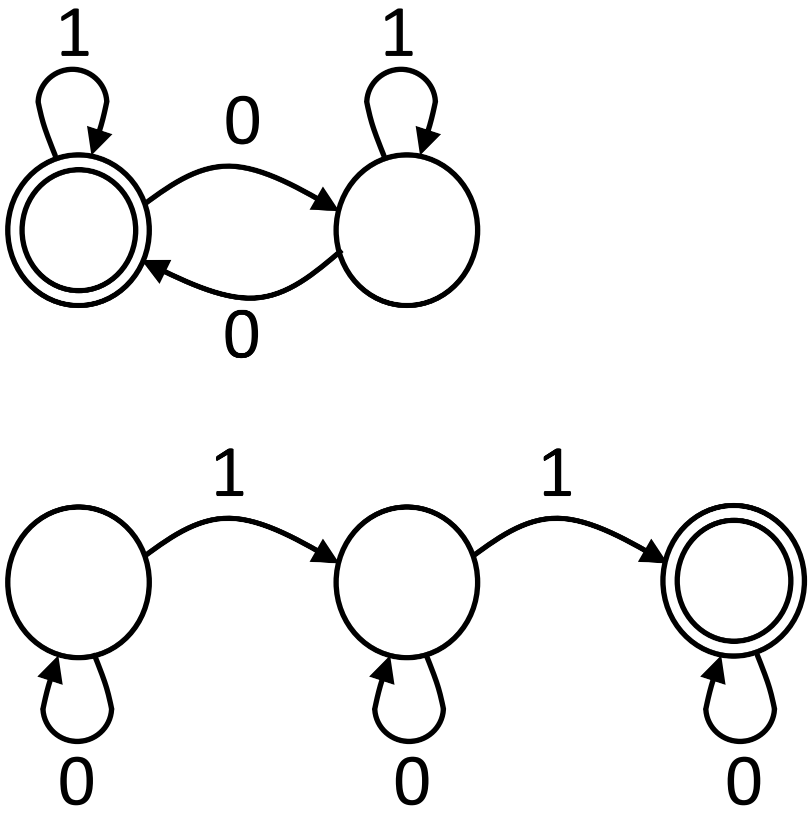


Build an NFA for the following language:

$\{\omega: \omega$ contains an even number of 0's or exactly two 1's$\}$.



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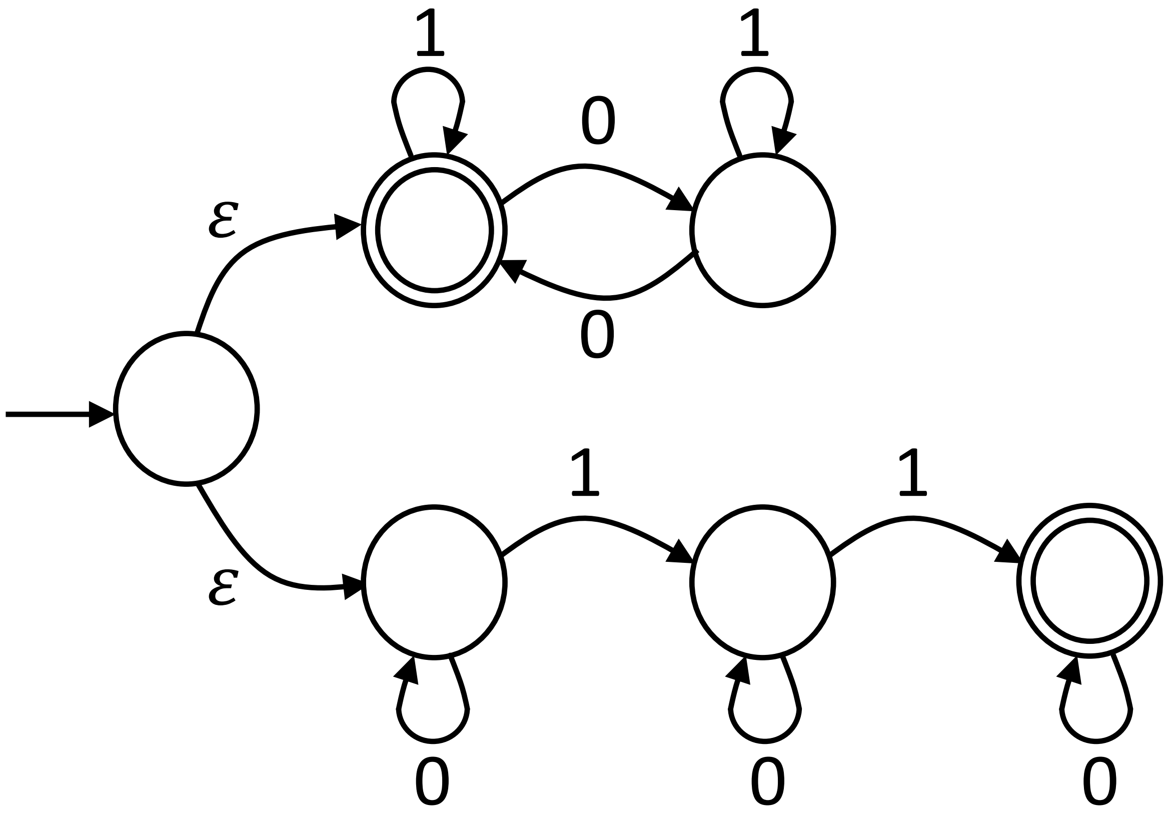
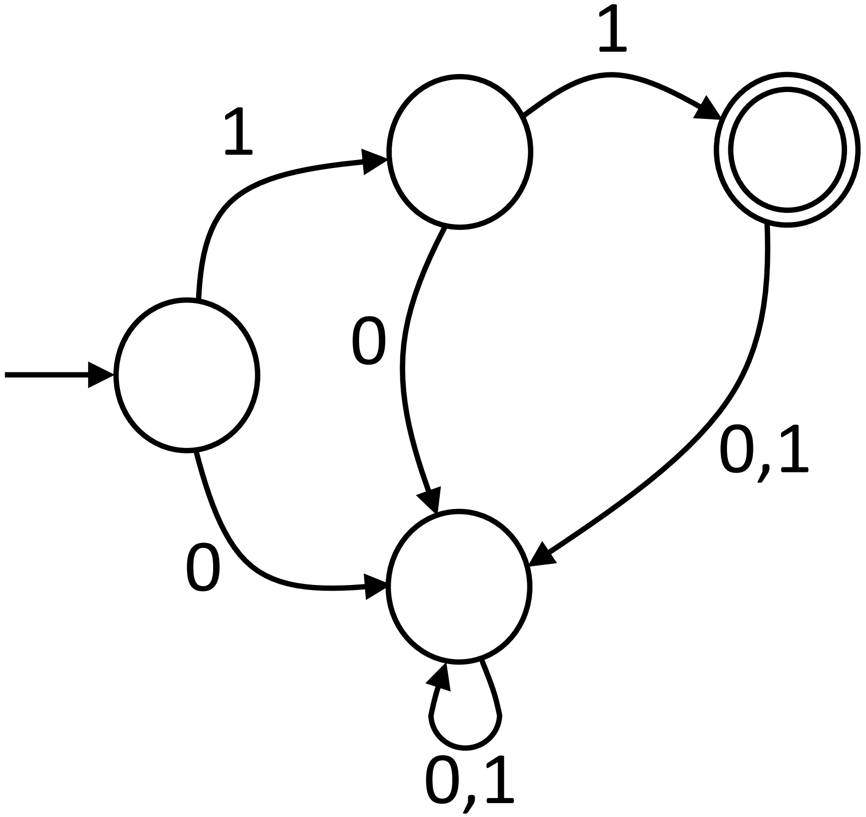


Build an NFA for the following language:

$\{11\}$.



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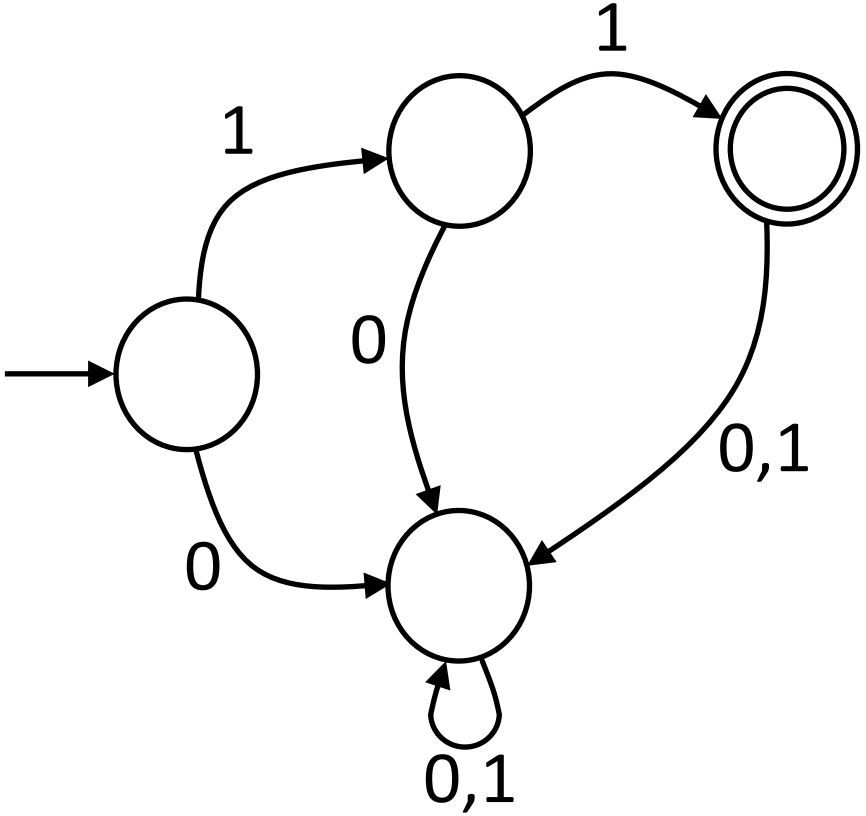
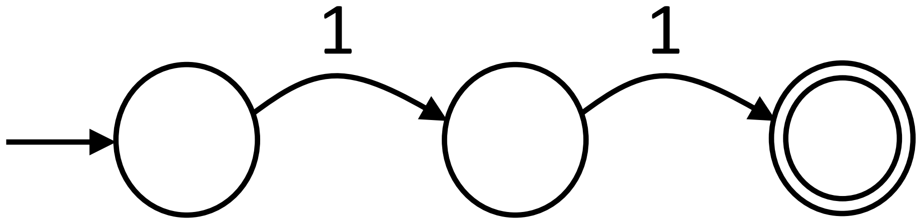
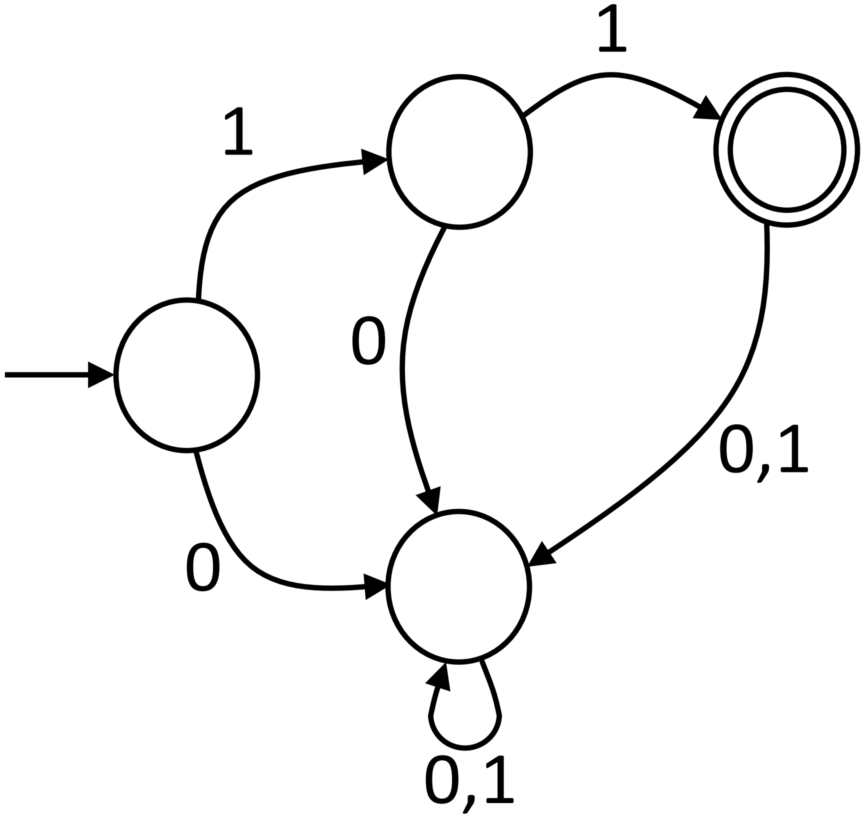
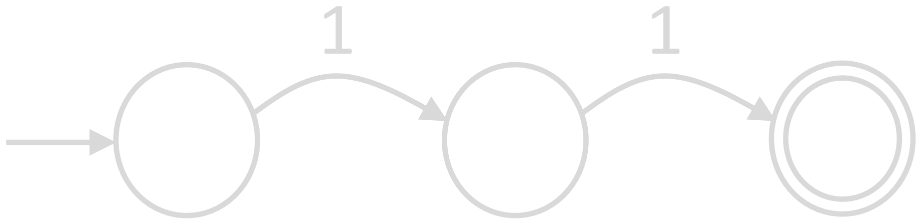




Build an NFA for the following language:

$\{\omega : \omega$ could be anything except $11\}$.





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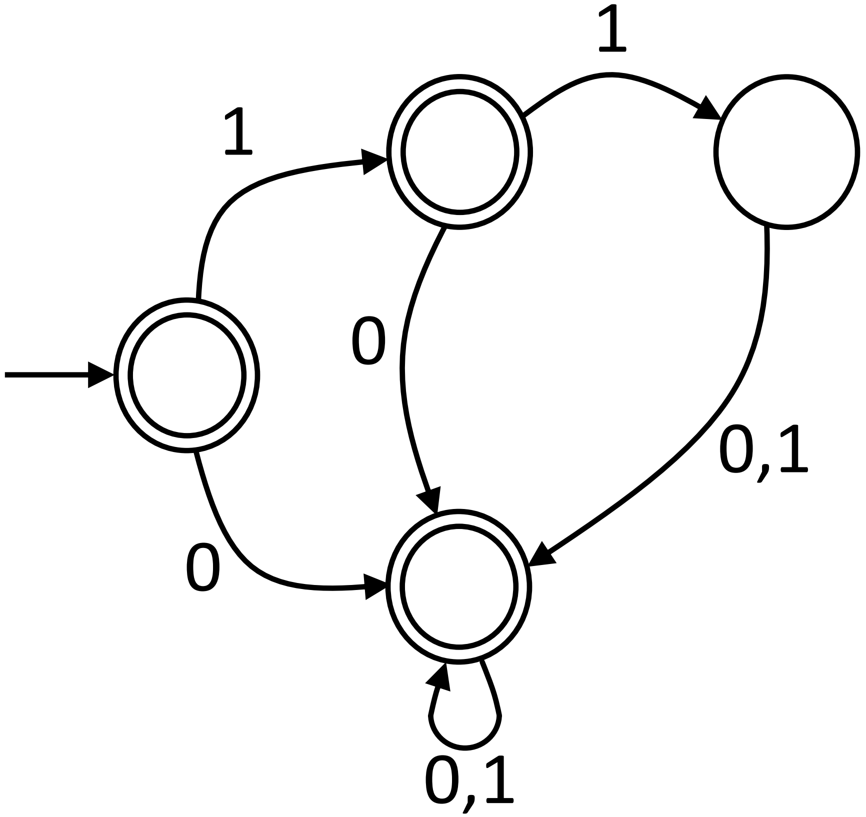

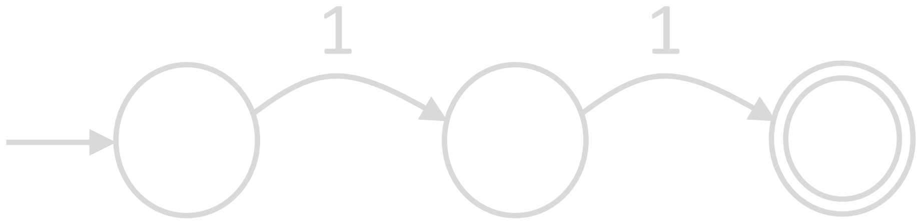

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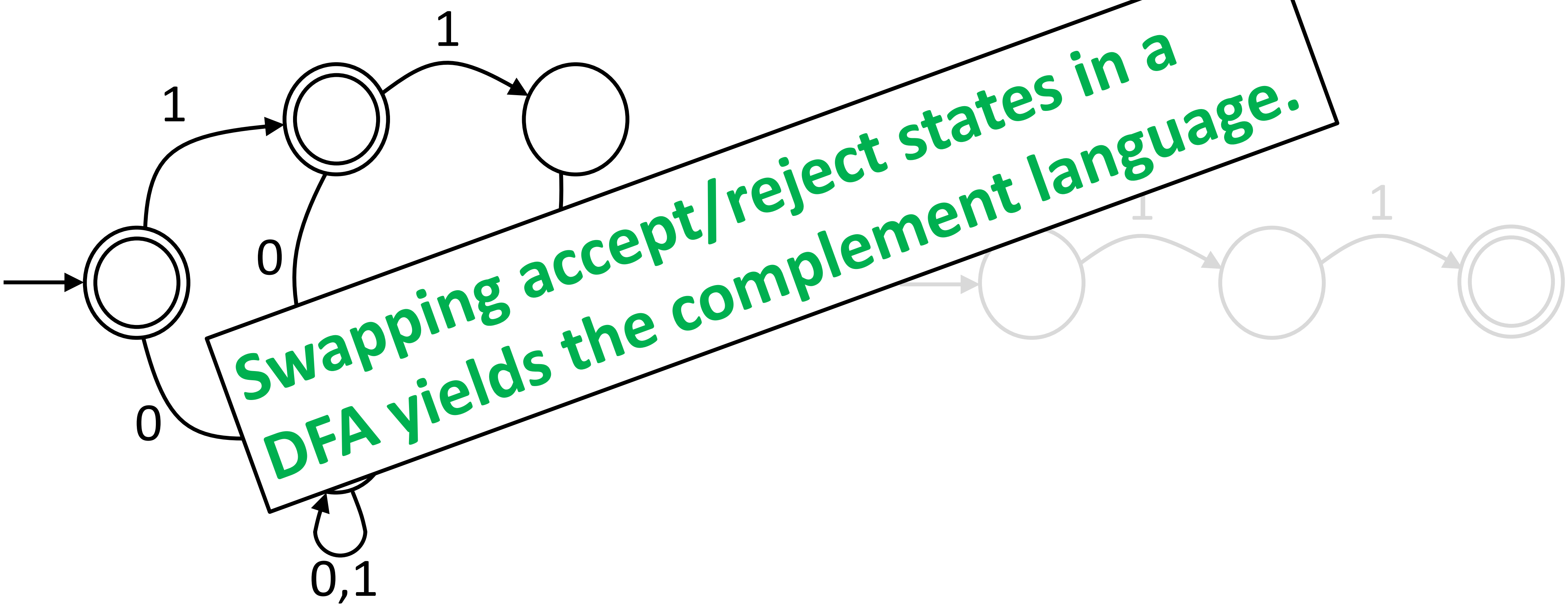

Swapping accept/reject states in a DFA yields the complement language.

Build an NFA for the following language:

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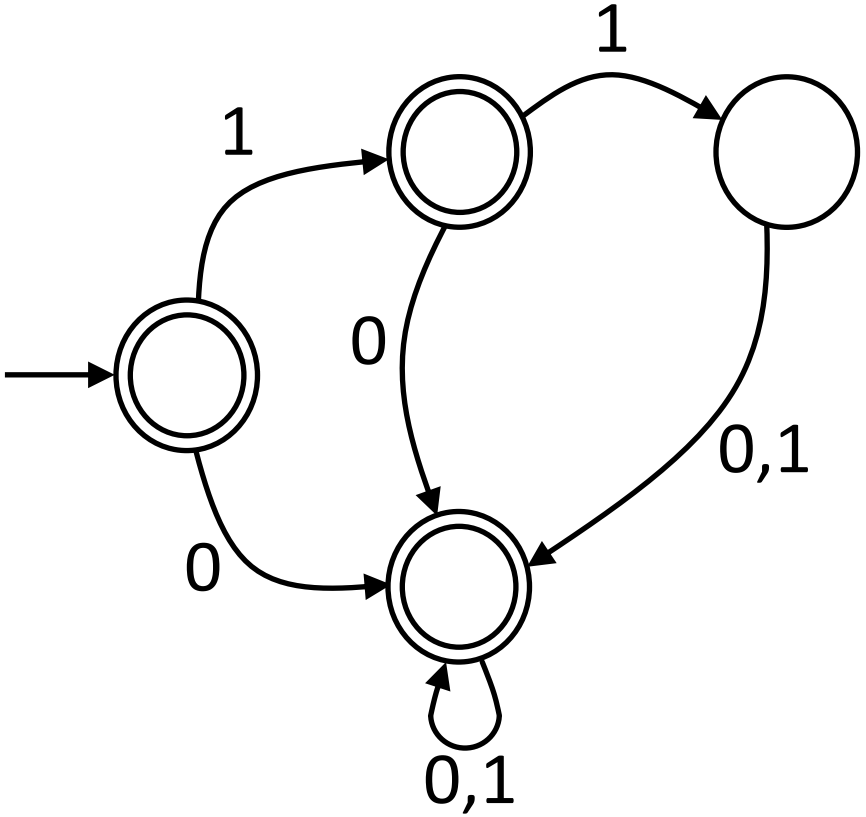

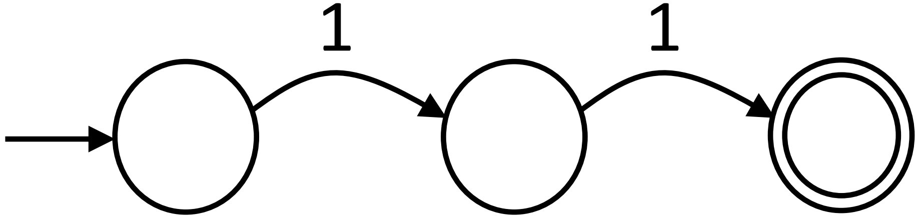

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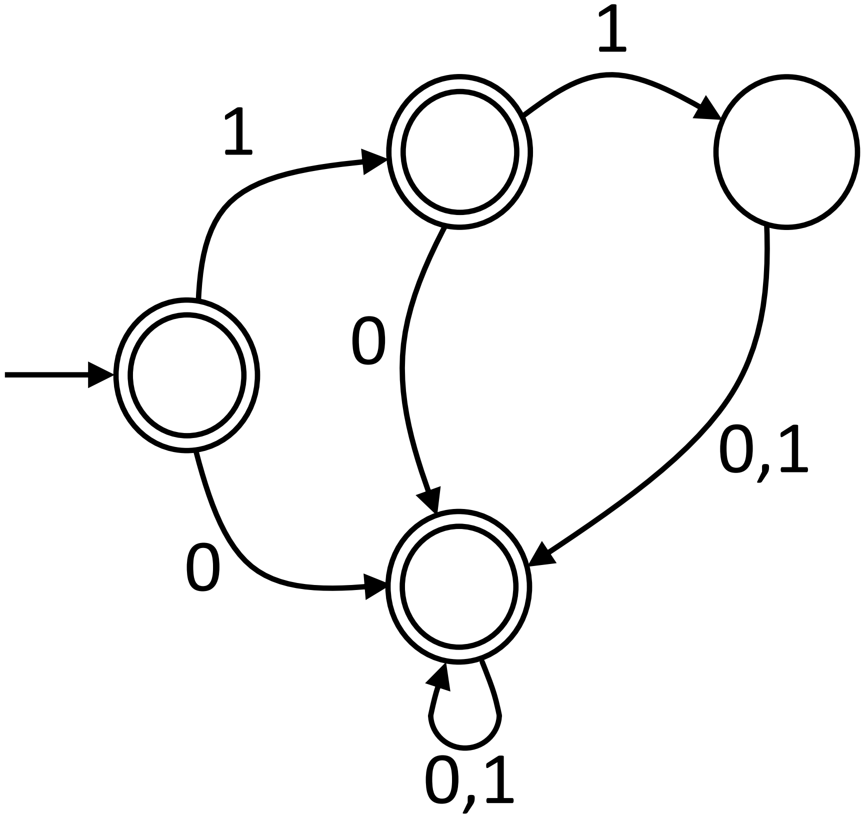

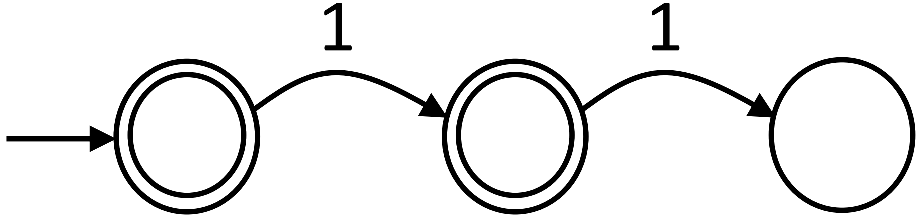

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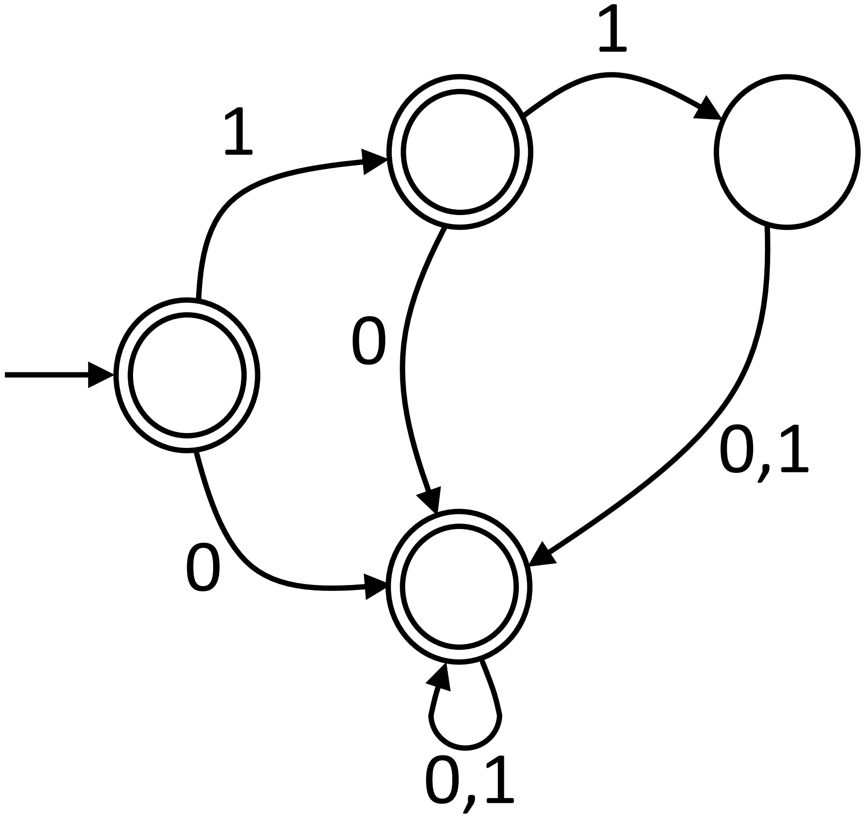

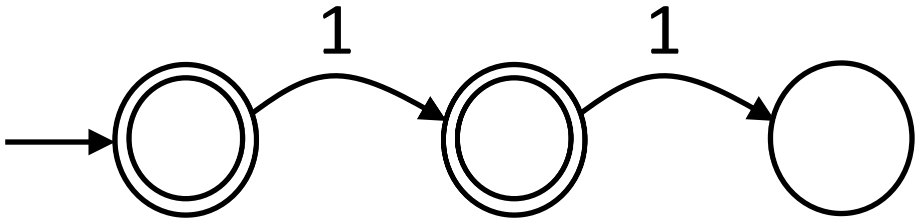

Language?

Build an NFA for the following language:

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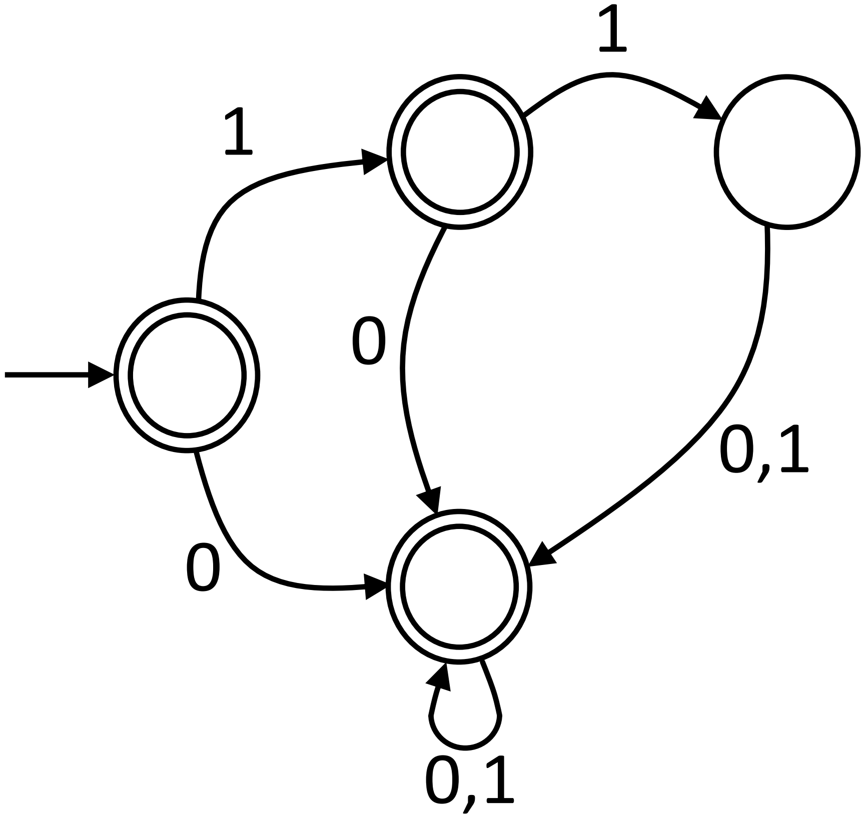

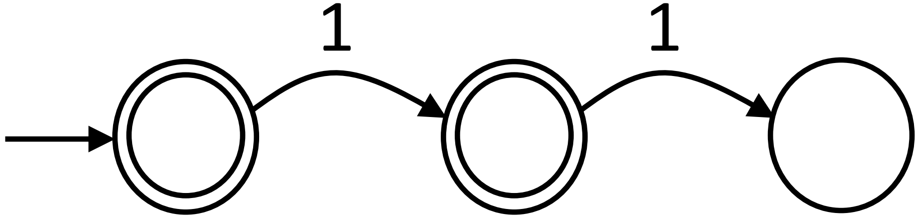

Language?

$\{\varepsilon, 1\}$

Build an NFA for the following language:
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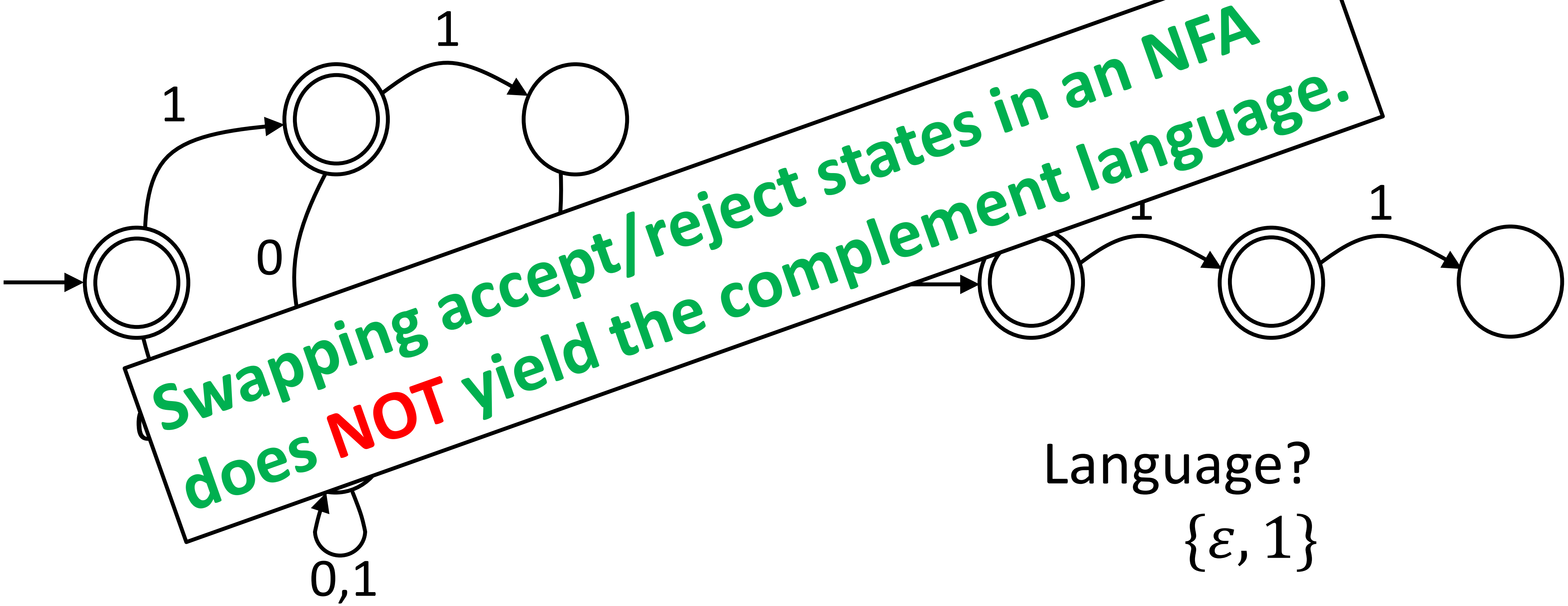

Swapping accept/reject states in an NFA does NOT yield the complement language.

Language?
$\{\varepsilon, 1\}$

Build an NFA for the following language:

$\{\omega\colon \omega$ contains the same number of 0s and $1s\}$.