1.

$$设\sigma是锥面z=\sqrt{x^2+y^2}(0\le z\le 1),则\iint_\sigma (x^2+y^2)dS=()$$

$$A.\int_0^\pi d\theta\int_0^1 r^2\cdot rdr$$

$$B.\int_0^{2\pi} d\theta\int_0^1 r^2\cdot rdr$$

$$C.\sqrt{2}\int_0^\pi d\theta\int_0^1 r^2\cdot rdr$$

$$D.\sqrt{2}\int_0^{2\pi} d\theta\int_0^1 r^2\cdot rdr$$

$$D$$

$$\sigma在xOy面上的投影为D_{xy}:x^2+y^2\le 1,$$

$$\iint_\sigma (x^2+y^2)dS=\iint_{D_{xy}}(x^2+y^2)\sqrt{1+(z_x)^2+(z_y)^2}dxdy$$
$$=\sqrt{2}\int_0^{2\pi} d\theta\int_0^1 r^2\cdot rdr.$$

2.

$$设\sigma为曲面z=2-(x^2+y^2)在xOy平面上方部分,则\int_\sigma ds=()$$

$$A.\int_0^{2\pi} d\theta\int_0^{\sqrt{2}}\sqrt{1+4r^2}rdr$$

$$B.\int_0^{2\pi} d\theta\int_0^{\pi}\sqrt{1+4r^2}rdr$$

$$C.\int_0^{2\pi} d\theta\int_0^{2}\sqrt{1+4r^2}rdr$$

$$D.\int_0^{2\pi} d\theta\int_0^{2}(2-r^2)\sqrt{1+4r^2}rdr$$

$$A$$

$$I=\iint_{x^2+y^2\le 2}\sqrt{1+(-2x)^2+(-2y)^2}dxdy$$
$$=\int_0^{2\pi} d\theta\int_0^{\sqrt{2}}\sqrt{1+4r^2}rdr$$

3.

$$\text{设}\sigma\text{为曲面}z=2-(x^2+y^2)\text{在}xOy\text{平面上方部分,则}I=\int_\sigma zdS=()$$

$$A.\int_0^{2\pi}d\theta\int_0^{\sqrt{2}}(2-r^2)\sqrt{1+4r^2}rdr$$

$$B.\int_0^{2\pi}d\theta\int_0^{2r^2}(2-r^2)\sqrt{1+4r^2}rdr$$

$$C.\int_0^{2\pi}d\theta\int_0^{2}(2-r^2)\sqrt{1+4r^2}rdr$$

$$D.\int_0^{2\pi}d\theta\int_0^{\sqrt{2}}(2-r^2)rdr$$

$$A$$

$$I=\iint\limits_{x^2+y^2\leq 2}(2-x^2-y^2)\sqrt{1+(-2x)^2+(-2y)^2}dxdy$$
$$=\int_0^{2\pi}d\theta\int_0^{\sqrt{2}}(2-r^2)\sqrt{1+4r^2}rdr$$

4.

$$\text{设}\sigma\text{是锥面}z=\sqrt{x^2+y^2}(0leqzleq1)\text{则}\iint_\sigma x^2+y^2dS=()$$

$$A.\sqrt{2}\int_0^{2\pi}d\theta\int_0^1 r^2\cdot rdr$$

$$B.\int_0^{\pi}d\theta\int_0^1 r^2\cdot rdr$$

$$A.\sqrt{2}\int_0^{2\pi}d\theta\int_0^1 r^2\cdot rdr$$

$$D.\sqrt{2}\int_0^{\pi}d\theta\int_0^1 r^2\cdot rdr$$

$$A$$

$$\sigma\text{在}xOy\text{面上的投影为}D_{xy}:x^2+y^2\leq 1,$$

$$\iint\limits_\sigma(x^2+y^2)dS=\iint\limits_{D_{xy}}(x^2+y^2)\sqrt{1+(z_x)^2+(z_y)^2}dxdy$$
$$=\sqrt{2}\int_0^{2\pi}d\theta\int_0^1 r^2\cdot rdr$$

5.

$$设\sigma为x^2+y^2+z^2=a^2,则\oiint_{\sigma}(x^2+y^2+z^2)dS=()$$

$$A.4\pi a^4$$

$$B.4\pi a^2$$

$$C.2\pi a^4$$

$$D.\pi a^4$$

$$A$$

$$\oiint_{\sigma}(x^2+y^2+z^2)dS=a^2\oiint_{\sigma}dS=a^2\cdot 4\pi a^2=4\pi a^4$$

6.

$$设\sigma为曲面x^2+y^2+z^2=a^2,则\oiint_{\sigma}xyzdS=()$$

$$A.0$$

$$B.4\pi a^4$$

$$C.\frac{4}{3}\pi a^4$$

$$D.\frac{2}{3}\pi a^4$$

$$A$$

$$由于\sigma关于坐标面xOy,yOz,zOx均对称,$$

$$且被积函数关于x,y,z是奇函数，所以有\oiint_{\sigma}xyzdS=0$$

7.

$$设\sigma:x^2+y^2+z^2=R^2,则\oiint_{\sigma}(x+y+z)dS=()$$

$$A.0$$

$$B.\frac{4}{3}\pi R^4$$

$$C.4\pi R^4$$

$$B.\frac{2}{3}\pi R^4$$

A

由于σ关于三个坐标面对称，

$$\oiint_{\sigma} xdS\text{关于}x\text{是奇函数，所以}\oiint_{\sigma} xdS = 0;$$

$$\text{同理}\oiint_{\sigma} ydS = 0, \oiint_{\sigma} zdS = 0$$

$$\text{所以}\oiint_{\sigma}(x+y+z)dS = 0$$

8.

设$\sigma : x^2+y^2+z^2 = a^2(zgeq0), \sigma_1$为$\sigma$在第一卦限中的部分，则下面运算正确的是()

$$A.\iint_{\sigma} zdS = 4\iint_{\sigma_1} xdS$$

$$B.\iint_{\sigma} xdS = 4\iint_{\sigma_1} xdS$$

$$C.\iint_{\sigma} ydS = 4\iint_{\sigma_1} xdS$$

$$D.\iint_{\sigma} xyzdS = 4\iint_{\sigma_1} xyzdS$$

A

$$\text{由于积分区域}\sigma\text{关于}x, y\text{的位置对称,因此可知有}\iint_{\sigma} xdS = \iint_{\sigma} ydS,$$

$$\text{又由于}\iint_{\sigma} xdS\text{被积函数为}x\text{的奇函数，}\sigma\text{关于平面}xOy\text{对称，因此}\iint_{\sigma} xdS = 0$$

$$\text{而}\iint_{\sigma} xdS \geq 0.$$

同样，对于确定的y和z, 函数xyz为x的奇函数，积分区域S关于yOz平面对称,

$$\text{因此}\iint_{\sigma} xyzdS = 0, \text{而}\iint_{\sigma} xyzdS \geq 0.$$

$$\text{积分区域}\sigma\text{关于}xOz\text{平面都对称}, \iint\limits_{\sigma} zdS\text{中被积函数为}x\text{的偶函数，}$$

$$\text{也为}y\text{的偶函数，由对称性知}\iint\limits_{\sigma} zdS = 4\iint\limits_{\sigma} xdS.$$

9.

$$\text{设}\sigma\text{为球面}x^2+y^2+z^2=a^2\text{在}z\geq h\text{的部分},0<h<a,\text{则}\int\limits_{\sigma} zdS=()$$

$$A.\int_0^{2\pi} d\theta\int_0^{\sqrt{a^2-h^2}} ardr$$

$$B.\int_0^{2\pi} d\theta\int_0^{a^2-h^2} \sqrt{a^2-r^2}rdr$$

$$C.\int_0^{2\pi} d\theta\int_0^{\sqrt{a^2-h^2}} \sqrt{a^2-r^2}rdr$$

$$A.\int_0^{2\pi} d\theta\int_{-\sqrt{a^2-h^2}}^{\sqrt{a^2-h^2}} ardr$$

$$A$$

$$\sigma\text{在}xOy\text{面上的投影为：}x^2+y^2=a^2-h^2,$$

$$z=\sqrt{a^2-x^2-y^2}$$

$$\int\limits_{\sigma} zdS = \iint\limits_{x^2+y^2\leq a^2-h^2} \sqrt{a^2-x^2-y^2}\sqrt{1+(z_x)^2+(z_y)^2}dxdy$$

$$= \iint\limits_{x^2+y^2\leq a^2-h^2} adxdy$$

10.

$$\text{已知}\sigma\text{为曲面}z=x^2+y^2\text{被平面}z=1\text{所截部分},\text{则}\iint\limits_{\sigma} |xyz|dS=()$$

$$A.2\int_0^1 r^5\sqrt{1+4r^2}dr$$

$$B.4\int_0^1 r^5\sqrt{1+4r^2}dr$$

$$C.6\int_0^1 r^5\sqrt{1+4r^2}dr$$

$$D.8\int_0^1 r^5\sqrt{1+4r^2}dr$$

$$A$$

$$\sigma\text{在平面}xOy\text{上的投影}D_{xy}:x^2+y^2=1,$$

$$\sqrt{1+(z_x)^2+(z_y)^2}=\sqrt{1+4(x^2+y^2)},$$

$$\text{利用极坐标,并由对称性,即得}$$

$$\iint\limits_{\sigma}|xyz|dS=4\int_0^{\frac{\pi}{2}}d\theta\int_0^1 r^4cos\theta sin\theta\sqrt{1+4r^2}rdr$$

$$=2\int_0^1 r^5\sqrt{1+4r^2}dr$$