

# **Solving Combinatorial Optimization problems with Quantum inspired Evolutionary Algorithm Tuned using a Novel Heuristic Method**

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## **Abstract:**

Quantum inspired Evolutionary Algorithms were proposed more than a decade ago and have been employed for solving a wide range of difficult search and optimization problems. A number of changes have been proposed to improve performance of canonical QEA. However, canonical QEA is one of the few evolutionary algorithms, which uses a search operator with relatively large number of parameters. It is well known that performance of evolutionary algorithms is dependent on specific value of parameters for a given problem. The advantage of having large number of parameters in an operator is that the search process can be made more powerful even with a single operator without requiring a combination of other operators for exploration and exploitation. However, the tuning of operators with large number of parameters is complex and computationally expensive. This paper proposes a novel heuristic method for tuning parameters of canonical QEA. The tuned QEA outperforms canonical QEA on a class of discrete combinatorial optimization problems which, validates the design of the proposed parameter tuning framework. The proposed framework can be used for tuning other algorithms with both large and small number of tunable parameters.

**Keywords:** Meta-heuristic, Orthogonal Arrays, Design of Experiments, Taguchi based method.

Quantum inspired Evolutionary Algorithms (QEA) are population based meta-heuristics that draw inspiration from quantum mechanical principles to improve search and optimization capabilities of Evolutionary Algorithms (EAs). QEA has been applied to solve a wide variety of problems ranging from Automatic Color Detection [1], Image Segmentation [2], Bandwidth [3], Circuit testing [4], Software Testing [5], Economic Dispatch [6], [7], Engineering Design Optimization [8], design of digital filters [9] and process optimization [10] etc.

The potential advantages of parallelism offered by quantum computing [11] and simultaneous evaluation of all possible represented states, have led to the development of approaches for integrating some aspects of quantum computing with evolutionary computation [12]. Most hybridizations have focused on designing algorithms that would run on conventional computers and not on quantum computers and are most appropriately classified as “quantum inspired”. The first such attempt was made by Narayan and Moore [13] which used quantum parallel world interpretation to define a quantum inspired genetic algorithm to be run on a classical computer, subsequently, a number of other hybridizations also have been proposed. Han and Kim [14] proposed a popular model of quantum inspired evolutionary algorithm (QEA), that used a Q-bit as the smallest unit of information and a Q-bit individual as a string of Q-bits rather than binary, numeric or symbolic representations. Results of experiments showed that QEA performed well, even with a small population, without suffering from premature convergence as compared to the conventional genetic algorithm. Experimental studies have also been reported by Han and Kim to identify suitable parameter settings for the algorithm and enhancements have been proposed with new termination criterion and operators [15].

The QEA proposed by Han and Kim [14] is termed as Canonical QEA [14] as the basic structure proposed for QEA has remained unaltered in subsequent modifications [16]. The canonical QEA has three main constituents which differentiates it from other class of Evolutionary Algorithms. These are Q-bit representation, Measurement Operator for generating binary string from Q-bit string and Rotation Gate as Variation Operator to update the Q-bits. Further, it also has an island or coarse grained population model [17].

There have been many attempts to further improve canonical QEA by incorporating crossover and mutation operators in Li et al. [18]. The crossover and mutation operators are termed as quantum crossover and quantum mutation operators as they vary the Q-bit individuals instead of binary strings. Moreover, attempts have also been made by Zhang et al. [16] to modify canonical QEA by using a catastrophe operator and a novel update method for Q-gates. Further, hybridization of QEA has been attempted with Canonical Genetic Algorithm (CGAs) [19], immune algorithms [20] and particle swarm optimization (PSO) [21] to improve their performance. A number of modifications have also been proposed to improve the applicability and performance of canonical QEA, which range from modifications of existing operators like in case of real observation QEA [6], population structure as in case of vQEA [22] and variation operator [23] to introducing new variation operators like Quantum Crossover [24], Quantum Mutation [25] and Neighborhood operators [23], [26].

Most of the efforts on improving canonical QEA have focused on Variation operators i.e. either by modifying the existing rotation gate [26] or by introducing new type of operators through hybridization [21].

Canonical QEA has a well-designed Variation operator with eight rotation angle parameters to explore the genotype space comprehensively by obtaining suitable feedback from the objective function value. The rotation angles in variation operator, the migration condition and local neighborhood are taken to be design parameters and have to be chosen appropriately for the problem at hand. It is felt that these parameters may have a strong influence on the performance of a QEA, but studies on these have not been extensively reported in the literature. There are eleven design parameters in canonical QEA viz., eight rotation angles, population size, group size and global migration and they require fine tuning to improve their performance on specific problems. This has motivated investigation into parameter tuning of QEA and an attempt has been made to improve the performance of canonical QEA by effective and efficient tuning of parameters in Rotation Gate as well as other important parameters like number of groups, migration period and population size.

The motivation for this approach is twofold; first, the rotation gate has eight parameters, which are problem specific, so it is inherently difficult to set eight parameters by any ad-hoc mechanism. Second, the rotation gate with its eight parameters appears to be a powerful variation operator, at least in principle, which may provide for good search capability in wide variety of problems. Further, the rotation gate is mostly left unexplored in majority of efforts. These employ the same set of parameter values as suggested by Han and Kim [14] more than a decade ago.

Han and Kim [14] had intuitively assigned the value of eight rotation angles and then performed experiments by choosing three levels  $0$ ,  $+0.005\pi$ , and  $-0.005\pi$  for eight rotation angle parameters on the knapsack problem of size 100. Though, the number of experiments were factorial i.e.  $3^8$ , however, the rest of the parameters like population size and migration periods, etc. were kept constant. After having verified the effect of rotation angles, the other parameters like population size and migration periods etc. were studied independently. One of the reasons for the ad-hoc tuning of parameters is the relatively large number of parameters, which need to be tuned.

Parameter Tuning is an important part in the designing process of an Evolutionary Algorithm as it affects efficacy of the search process. It is said that most of the effort in designing an EA for solving a set of problems is spent in parameter tuning [27]. It is a difficult optimization problem in itself as it is usually poorly structured, ill-defined and complex in nature [27], [28]. Further, the best set of parameter values can be guaranteed to be found only after exhaustive search in the entire parameter-space,

however, such a strategy may not be practically feasible due to the huge amount of resources and time involved. Parameter tuning is essentially a problem of Design of Experiments (DOE) and analysis of results obtained from the experiments to arrive at the best parameter vector. Traditionally, there are four well known strategies for design of experiments viz., Ad-hoc, Factorial, Fractional Factorial and Random design of experiments [29]. In Ad-hoc experimentation, the design of experiments is guided by intuition and is subsequently verified by limited set of experiments. This technique has received wide popularity in EA due to two specific reasons. First, EAs are designed to give good solutions quickly for difficult optimization problems, thus if an EA can give better solutions than the existing ones, then it is considered as successful and acceptable. Therefore, there is no compulsion to study the behavior of EA in the entire parameter space as long as EA can find better solutions. Secondly, the structure of EA and the problem being solved, usually give some insight into the possible parameter values to an experienced EA designer. Thus, limited experiments using ad-hoc approach has been the most popular method amongst EA designers. However, there is lot of subjectivity in ad-hoc approach and as has been shown that a well-designed parameter tuning method can perform better than the claimed best EA [30], so parameter tuning should be done using a structured approach [27].

A number of attempts have been made for designing methods for parameter tuning in EAs [27], which have been developed to provide good parameter values within reasonable cost. These methods have been categorized as Sampling Methods, Model Based Methods, Screening Methods and Meta-Evolutionary Methods in [27]. They have been further divided into non-iterative and iterative sub categories, with iterative versions outperforming non-iterative versions. Similarly, they have also been categorized as Single Stage and Multi-Stage version, with multi-stage being more effective. These methods tend to use different designs of experiments during different stages, thereby leading to a hybrid design of experiments.

Sampling methods use fractional factorial design of experiments to reduce the number of experiments as in case of the Taguchi's Orthogonal Arrays [31]. Statistical analysis is performed on the result of the experiments to compute the parameter vector that may give the best result. However, there is no guarantee of finding the optimal set of parameters as the emphasis is on reducing the effort. These methods require finding a set of levels for each parameter, which can be done in an ad-hoc manner or by adding an initialization stage to automatically find the levels with which they begin sampling [32], [33]. Further, they have been augmented by iterating the sampling process to refine the parameter vectors [32]. Calibra [34] uses full factorial design of experiments in its initial stage and fractional factorial design of experiment in its later stage.

Model based Methods generally use Sampling methods as a starting point to construct a model of the search process of EA [35]. The model helps in predicting the utility of the parameter vector in solving the problem. However, they appear more amenable in studying the nature of EA in terms of its characteristics like tuning ability and robustness to changes in problem specification. Single stage methods are generally unable to find the best parameter vector. The iterative multi stage methods [36] are computationally expensive and they rely heavily on the accuracy of the model. It is well known that there is always a trade-off between computational effort and accuracy of the model in such an effort. Further, if the objective is to find the best set of parameter values for solving a set of problems, then first finding an accurate model of a Stochastic process and subsequently using it to predict the best parameter vector appears to be indirect and errors often accumulate.

Screening methods try to reduce the number of experiments by using statistical measures to test only a small subset from a large set of parameter vectors [37], [38]. F-Race and its improved version, iterated F-Race, use the non-parametric Friedman test as a family-wise test, i.e. it checks whether there is evidence that at least one of the configurations is significantly different from the rest [39]. However, the implementation problem with such methods is due to the stochastic nature of EA. That is only after running the EA with a parameter vector for sufficient number of times with different set of random numbers, can any conclusion be drawn about its effectiveness. There are EAs which initially focus on exploration of the search space and only at later stages they exploit the good regions. Moreover, the EA instance with competing parameter vectors often performs differently with different set of random numbers, especially in case of difficult problems, therefore a fair comparison can be made only after each EA instance has been run to completion (i.e. same termination criteria) for about 30 to 50 times with different set of random numbers. Thus the idea of screening methods, which appear good in concept, would either not do just comparisons or not be able to reduce the effort, especially in case of difficult problems. However, there is reported evidence in favor of such methods and application of non-parametric statistical testing is a good approach to eliminate non-performers.

Meta Evolutionary Algorithms have also been used for parameter tuning as the nature of problem of finding high utility parameter vectors is similar to the problems solved by Evolutionary Algorithms. They have been quite successful as reported in [30]. However, the very idea of using Meta EA runs into the proverbial "Chicken or the Egg Causality Dilemma" i.e. an untuned EA is being used for parameter tuning of another EA or an EA tuned by some other mechanism is being used for parameter tuning of another EA. Thus, conceptually, it is a two level method, which is going to require double the effort in parameter tuning. However, examples of Meta EA like REVAC [40] have been successful in practice.

The generic process of solving the parameter tuning problem by the above discussed methods, in general, involves the following steps:

- a) Selection of a set of Parameter Vectors
- b) Experiment with selected Set of Parameter Vectors
- c) Analysis of the Experimental Result
- d) If Non Iterative OR Stopping\_Criteria met, Output the Best Vector Found

- e) If Multi\_Stage & Stage\_Transition\_Criteria met, Change Selection and / or the Analysis process
- f) If Iterative, Selection of a new set of Parameter Vectors based on Analysis of the Experimental Result
- g) Go to (b)

The general conclusions that can be drawn from the reported efforts are that random initial selection of parameter set is more effective than a fixed set, Iterative and multi stage methods are better than non-iterative and single stage methods. An interesting observation is that iterative parameter tuning methods can be cast into meta-heuristic framework. The methods differ in statistical analysis and subsequent selection of parameter vectors. Further, there is no guarantee in any of the methods for arriving at the best parameter vector. All of them try to find a good parameter vector with respect to a given set of objectives while consuming minimum possible resources.

This paper proposes a novel parameter tuning method which integrates Taguchi's method in the meta-heuristic framework and explicitly divides the search for a parameter vector into exploration and exploitation stages. It does not require tuning of Meta EA parameters as Taguchi's method is being used for selection, analysis and variations. Further, it does not complicate the statistical analysis by trying to build regression models as it has a clear objective of finding the parameter vector that gives best results with respect to a set of well-defined objectives. It uses a modification of well-known Taguchi's method in selection of parameter vectors and statistical analysis and it can quickly find the main effect of parameters for the chosen levels. Therefore, the proposed method needs less effort to find a good parameter vector. It is a type of iterative Sampling method, which overcomes the limitation of existing sampling methods like Calibra [34] by explicitly dividing the search into two phases of exploration and exploitation and employing multiple levels of parameters and Taguchi's method in the first phase of iterative exploration instead of using full factorial design with just two levels in Calibra [34]. Further, it reduces effort during exploitation phase by testing around the best parameter vector. Of course, the proposed method does not guarantee finding the optimal parameter vector, but it can find near optimal parameter vectors by searching in a methodical way and consuming much less effort.

The objective of parameter tuning in this paper is to find the best set of parameter vector values for solving a class of problems while spending only a reasonable amount of resources. This would then corroborate the fact that EAs find good solutions (not necessarily optimal) in reasonable amount of time and an EA can be tuned to find good solutions for a particular class of problems. In this paper, the focus is also on finding near optimal solution in reasonable amount of time with the robustness to the randomness essential in stochastic search process of QEA. Robustness to changes in problem specification is handled indirectly by running the algorithm on the several problems of same as well as different class. The parameter set for the same set of problems may be taken to be same where as it can be different for a different set of problems. However, they can all be arrived at by following the same procedure for parameter tuning.

The paper is further organized as follows. Section 2 discusses QEA's basic structure. Parameter Tuning method is proposed in Section 3 along with a discussion on Orthogonal Arrays. Section 4 presents the experimental testing and analysis of the proposed Parameter Tuning method on QEA. Conclusions are drawn in Section 5 giving directions for future research endeavors.

## I. QUANTUM-INSPIRED EVOLUTIONARY ALGORITHMS

Canonical QEA maintains a population of individuals in quantum bits or Q-bits. A Q-bit coded individual can probabilistically represent a linear superposition of states in the search space. Thus it has better characteristics of population diversity than other representations [14]. A Q-bit is represented as follows:

$$q_i = \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (1)$$

where  $|\alpha_i|^2$  is probability of Q-bit,  $q_i$  to be in state 0,  $|0\rangle$  and  $|\beta_i|^2$  is probability of Q-bit,  $q_i$  to be in state 1,  $|1\rangle$  and

$$|\alpha_i|^2 + |\beta_i|^2 = 1. \quad (2)$$

$\alpha_i$  and  $\beta_i$  are real numbers for QEA implementations in this paper. Each individual is represented by a set of Q-bits in a string as:

$$Q(t) = \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \\ \beta_1 & \beta_2 & \dots & \beta_n \end{bmatrix} \quad (3)$$

such that  $|\alpha_i|^2 + |\beta_i|^2 = 1$ , where  $i = 1$  to  $n$ .

Measurement is the process of generating binary strings from the Q-bit string,  $Q$ . To observe the Q-bit string ( $Q$ ), a string consisting of random numbers between 0 and 1 ( $R$ ) is generated. The  $i^{\text{th}}$  element of binary string,  $b_i$ , is set to zero if the  $i^{\text{th}}$

random number,  $r_i$ , is less than  $|\alpha_i|^2$  and one otherwise. In every iteration, it is possible to generate more than one solution strings from the Q by generating a new string of Random numbers as given above. The fitness values of each of these strings can be computed and the solution with the best fitness is identified.

A quantum gate or Q-Gate is utilized for updating the elements of a Q-bit string so that they move towards the best solution. Thus, there is a higher probability of generating solution strings, which are similar to the best solution in subsequent iterations. One such Q-Gate is Rotation gate, which is unitary in nature and updates the Q-bit as follows:

$$\begin{bmatrix} \alpha_i^{t+1} \\ \beta_i^{t+1} \end{bmatrix} = \begin{bmatrix} \cos(\Delta\theta_i) & -\sin(\Delta\theta_i) \\ \sin(\Delta\theta_i) & \cos(\Delta\theta_i) \end{bmatrix} \begin{bmatrix} \alpha_i^t \\ \beta_i^t \end{bmatrix} \quad (4)$$

where  $\alpha_i^{t+1}$  and  $\beta_i^{t+1}$  denote probabilities of  $i^{\text{th}}$  Q-bit in  $(t+1)^{\text{th}}$  iteration.  $\Delta\theta_i$  is the angle of rotation, which is depicted in Fig. 1.

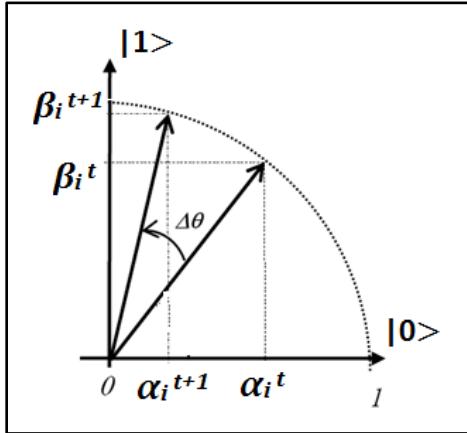


Fig 1. Effect of Quantum Rotation Operator on Q-bit

The quantum Rotation gate also requires an attractor [22] towards which the Q-bit would be rotated. It further takes into account the relative current fitness level of the individual and the attractor and also their binary bit values for determining the magnitude and direction of rotation. The magnitude of rotation is a tunable parameter and is selected from a set of eight rotation angles viz.,  $\theta_1, \theta_2, \dots, \theta_8$ . The value of the rotation angles are problem dependent and require tuning [14]. The selection of eight rotation angles viz.,  $\theta_1, \theta_2, \dots, \theta_8$  is made from the lookup Table 1.

The  $\Delta\theta_i$  has been made a function of the  $i^{\text{th}}$  bit,  $b_i$  of the best solution  $B_j$ , found so far till  $j^{\text{th}}$  iteration, the  $i^{\text{th}}$  bit  $x_i$  of the current binary solution and the condition that the Q-bit,  $q_i, |q_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$ , associated with  $x_i$  should rotate towards the corresponding basis state  $|0\rangle$  or  $|1\rangle$  to increase the probability of  $q_i$  so that  $x_i$  in the next iteration has better probability of collapsing to  $b_i$ . Let us consider the following example, if  $b_i$  and  $x_i$  are 0 and 1, respectively, and if objective function value of the best solution,  $f(B)$  is better than the objection function value of the current solution,  $f(X)$ , (i.e.  $f(X) < f(B)$  for maximization problems and  $f(X) > f(B)$  for minimization problems), then:

- (i) If the Q-bit is located in the first or the third quadrant in Fig. 2, the value of  $\Delta\theta_i$  is set to a negative value so that the probability of  $q_i$  to collapse to the state  $|0\rangle$  is increased.
- (ii) If the Q-bit is located in the second or the fourth quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a positive value so that the probability of  $q_i$  to collapse to the state  $|0\rangle$  is increased.

If  $b_i$  and  $x_i$  are 1 and 0, respectively, and if  $f(B)$  is better than  $f(X)$ , then:

- (i) If the Q-bit is located in the first or the third quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a positive value so that the probability of  $q_i$  to collapse to the state  $|1\rangle$  is increased.
- (ii) if the Q-bit is located in the second or the fourth quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a negative value so that the probability of  $q_i$  to collapse to the state  $|1\rangle$  is increased.

Han and Kim had suggested that if it is ambiguous to select a positive or a negative number for the values of the angle parameters, it is recommended to set the values to 0 [14]. However, the Fig. 2, suggests that there should be no ambiguity in selection of values for any of the possible cases listed in the Table 1. For example, the case were  $b_i$  and  $x_i$  are 0 and 0, respectively, and if  $f(B)$  is not better than  $f(X)$ , then:

- (i) If the Q-bit is located in the first or the third quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a negative value so that the probability of  $q_i$  to collapse to the state  $|0\rangle$  is increased as it is the desired state through which it has found better solution.

- (ii) If the Q-bit is located in the second or the fourth quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a positive value so that the probability of  $q_i$  to collapse to the state  $|0\rangle$  is increased as it is the desired state through which it has found better solution.

Similarly, the case were  $b_i$  and  $x_i$  are 0 and 0, respectively, and if  $f(B)$  is better than  $f(X)$ , then:

- (i) If the Q-bit is located in the first or third quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a negative value so that the probability of  $q_i$  to collapse to the state  $|0\rangle$  is increased as it is in desired state through which  $f(B)$  has better solution.  
(ii) if the Q-bit is located in the second or fourth quadrant, in Fig. 2, the value of  $\Delta\theta_i$  is set to a positive value so that the probability of  $q_i$  to collapse to the state  $|0\rangle$  is increased as it is in desired state through which  $f(B)$  has better solution.

Han and Kim had recommended [14] to set all the angles zero except  $\theta_3 = 0.01 \pi$  and  $\theta_5 = -0.01 \pi$ . The magnitude of  $\Delta\theta_i$  has an effect on the speed of convergence, but if it is too big, the solutions may diverge or converge prematurely to a local optimum. The values from  $.001 \pi$  to  $.05 \pi$  are recommended for the magnitude of  $\Delta\theta_i$ , although they depend on the problems. The sign of  $\Delta\theta_i$  determines the direction of convergence.

TABLE I  
LOOKUP TABLE FOR ROTATION ANGLES

$x_i$	$b_i$	$f(B)$ better than $f(X)$	$\Delta\theta_i$	$\alpha_i$	$\beta_i$	Sign	Han & Kim, [Han2002]	Remarks
0	0	True	$\theta_1$	+	+	-	0 (so no change in state vector)	State vector should be rotated slightly towards $ 0\rangle$ as $b_i=0$ , so in next iteration also, $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 0\rangle$ .
				-	+	+		
				-	-	-		
				+	-	+		
0	0	False	$\theta_2$	+	+	-	0 (so no change in state vector)	State vector should be rotated slightly towards $ 0\rangle$ as with $x_i=0$ it has found a better solution so in next iteration also, $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 0\rangle$ .
				-	+	+		
				-	-	-		
				+	-	+		
0	1	True	$\theta_3$	+	+	+	0.01 $\pi$	State vector should be rotated adequately towards $ 1\rangle$ as $b_i=1$ so in next iteration, $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 1\rangle$ .
				-	+	-		
				-	-	+		
				+	-	-		
0	1	False	$\theta_4$	+	+	-	0 (so no change in state vector)	State vector should be rotated slightly towards $ 0\rangle$ as with $x_i=0$ , it has found a better solution so in the next iteration also, $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 0\rangle$ .
				-	+	+		
				-	-	-		
				+	-	+		
1	0	True	$\theta_5$	+	+	-	0.01 $\pi$	State vector should be rotated adequately towards $ 0\rangle$ as $b_i=0$ so in next iteration $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 0\rangle$ .
				-	+	+		
				-	-	-		
				+	-	+		
1	0	False	$\theta_6$	+	+	+	0 (so no change in state vector)	State vector should be rotated slightly towards $ 1\rangle$ as with $x_i=1$ , it has found a better solution so in next iteration also $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 1\rangle$ .
				-	+	-		
				-	-	+		
				+	-	-		
1	1	True	$\theta_7$	+	+	+	0 (so no change in state vector)	State vector should be rotated slightly towards $ 1\rangle$ as $b_i=1$ so in next iteration
				-	+	-		

				<table border="1"> <tr><td>-</td><td>-</td><td>+</td></tr> <tr><td>+</td><td>-</td><td>-</td></tr> </table>	-	-	+	+	-	-		also this $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 1\rangle$ .						
-	-	+																
+	-	-																
1	1	False	$\theta_8$	<table border="1"> <tr><td>+</td><td>+</td><td>+</td></tr> <tr><td>-</td><td>+</td><td>-</td></tr> <tr><td>-</td><td>-</td><td>+</td></tr> <tr><td>+</td><td>-</td><td>-</td></tr> </table>	+	+	+	-	+	-	-	-	+	+	-	-	0 (so no change in state vector)	State vector should be rotated slightly towards $ 1\rangle$ as with $x_i=1$ it has found a better solution so in the next iteration also $i^{\text{th}}$ Q-bit, $q_i$ should have an improved probability of collapsing $ 1\rangle$ .
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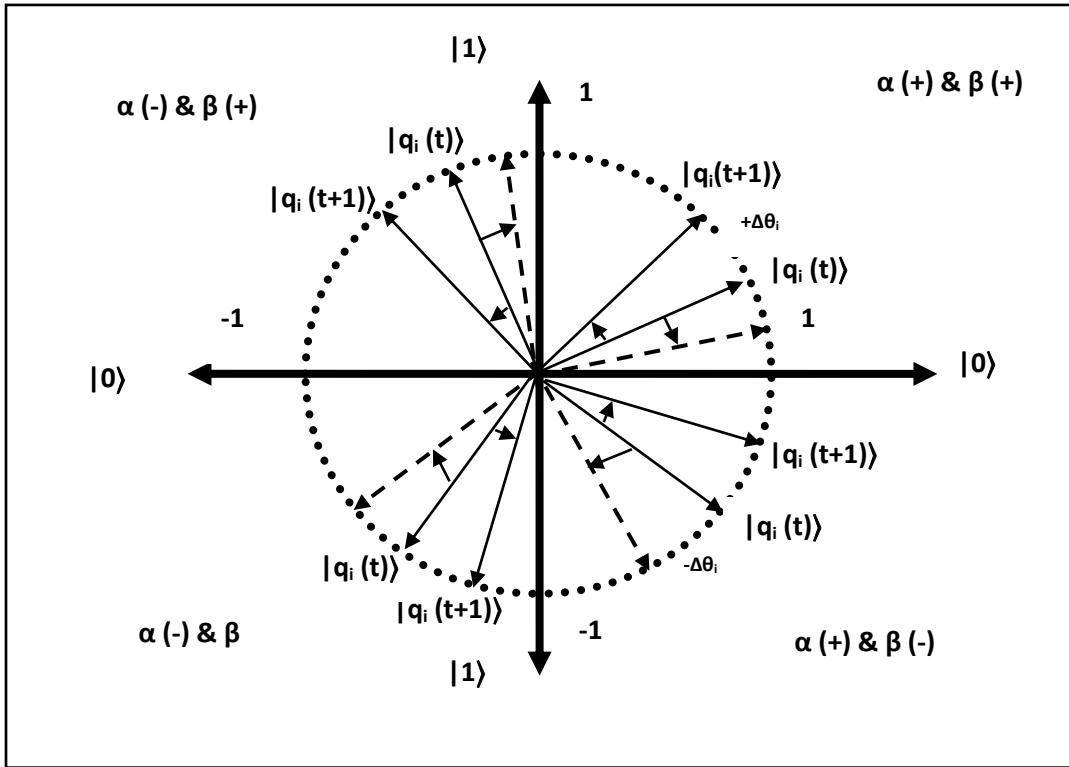


Fig 2. Four Quadrant Rotation of Q-bit

There are two methods to initially set all the Q-bits viz., the random initialization and Equal Probability initialization. In random initialization, Q-bits are assigned values between -1 to +1 by generating them randomly, while taking into account the normalization criteria described in eq. 5. The second method of initialization is done so that observation results in either 0 or 1 with equal probability by setting the values of  $\alpha_i$  and  $\beta_i$  to 0.707.

$$|Q\rangle = \sum_{k=1}^{2^m} \frac{1}{\sqrt{2^m}} |X_k\rangle \quad (5)$$

The termination condition is usually based on the number of generations, number of fitness evaluations and convergence of search or a combination of them. A measure of diversity in population can be made out of real-valued Q-bit strings. If the Q-bits have a value of  $\alpha$  as 0.707, the diversity can be considered to be highest, whereas the diversity can be considered least when the value of  $\alpha$  is near the extremes, i.e. 0 or 1. Hence level of convergence can be considered as number of Q-bits which have reached very close to 0 or 1. If this number is equal to number of elements in  $Q$ , then the chances of the measurement process generating diverse solutions becomes very low and it may be said that the search has converged. In this work, the stopping criterion is taken as the completion of a maximum number of generations.

The structure of the canonical Quantum-inspired Evolutionary Algorithm is shown by flow chart in Fig. 3 and it works as follows [14]:

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a) t = 0; Population Size = N, Group Size = GS;
b) initialize Q1(t)...QN(t), divide into M (= N/GS) groups (QG1... QGM);
c) make P1(t)...PN(t) by observing the states of Q1(t)...QN(t) respectively;
d) if repair required then repair Pi(t), i = 1 .. N ;
e) evaluate P1(t)...PN(t) & store in OP1(t) .. OPN(t);
f) store the global, Local and individual best solutions into GB(t), LBj(t) & IBi(t)
   respectively, i = 1 .. N, j = 1 .. M;

while (termination condition is not met) {

g) t = t + 1;
for each individuali i = 1 .. N {
h) determine Attractor Ai(t) based on migration condition:
   Global: Ai(t) = GB(t-1);
   Local: Ai(t) = LBj(t-1) where ith individual belongs to jth Group;
   No-Migration: Ai(t) = IBi(t-1);
i) apply Q-gate(s) on Qi(t-1) to update to Qi(t);
j) make Pi(t) by observing the states of Qi(t);
k) if repair required then repair Pi(t);
l) evaluate Pi(t) & store result into OPi(t);
m) store better solution among IBi(t-1) and OPi(t) into IBi(t);
n) store the better solution among LBi(t-1) and Best_individual_in_Groupj(t) into
   NBi(t), where ith individual belongs to jth Group ;
o) store the global best solution GB(t-1) among IBi(t) into GB(t); }}
```

In step a), initialize the population size N, size of the Group to GS. In step b), the qubit register Q(t) containing Q-bit strings Q<sub>1</sub>(t) ... Q<sub>N</sub>(t) are initialized randomly and divided into M groups (QG<sub>1</sub>... QG<sub>M</sub>), where no. of groups M = N/GS. In step c), the binary solutions represented by P<sub>1</sub>(t) ... P<sub>N</sub>(t) are constructed by measuring the states of Q<sub>1</sub>(t) ... Q<sub>N</sub>(t) respectively. In step d), if repairing is required in binary solutions P<sub>i</sub>(t) then repairing is performed. In step e) binary solution is evaluated to give a measure of its fitness OP<sub>i</sub>(t), where OP<sub>i</sub>(t) represents the objective function value. In step f), the initial global, neighborhood and individual best solutions are then selected among the binary solutions OP<sub>i</sub>(t), and stored into GB(t), LB<sub>i</sub>(t), IB<sub>i</sub>(t) respectively, Local best solution is determined from the individuals in the Group. In step h), the attractor A<sub>i</sub>(t) for the i<sup>th</sup> individual is determined according to the migration strategy. If the migration is global, then global best is assigned as the attractor, whereas if the migration is local then local group best is assigned as the attractor and if no migration is there then individual best becomes the attractor. In step i), update Q<sub>i</sub>(t-1) to Q<sub>i</sub>(t) using Q-Gates, which is quantum rotation gate described earlier in Section 2. In step j), the binary solutions in P<sub>i</sub>(t) are formed by measuring the states of Q<sub>i</sub>(t) as in step c). In step k), if the repair is required then it is performed as in step d) and in step l), each binary solution is evaluated for the fitness as in step e). In step m), n) and o), the global, local and individual best solutions are selected and stored into GB(t), LB<sub>i</sub>(t) and IB<sub>i</sub>(t) respectively based on a comparison between previous and current best solutions.

In the Fig. 3, flow chart depicts the working of canonical QEA, three different lines are used to identify information flow of Q-bit string, Binary String and the fitness function. The broken / dash & dot line indicates that information flowing is the Q-bit string, whereas dot line indicates that information flowing is binary string. The solid line shows that information flowing is objective function value. QG<sub>1</sub> to QG<sub>M</sub> are M groups in which N individuals are assigned. Measurement operator creates N binary bit strings from N individual Q-bit strings. The rest is as explained in the algorithm.

## II. PROPOSED PARAMETER TUNING METHOD

The proposed parameter tuning method has been developed by integrating Taguchi method into metaheuristic framework. It is an iterative multistage technique that utilizes the strength of Taguchi's method [31] while avoiding its well-known disadvantage of aliasing in presence of interaction between parameters. The metaheuristic framework is required for exploring the parameter space as Taguchi's method can perform local optimization only and that too if there are no interactions amongst parameters. If there are interactions amongst parameters then Taguchi's method fails due to aliasing but metaheuristic framework helps in taking the search forward by selecting the parameter set with the best result in the experiments performed so far i.e. an elitist selection is made and the search proceeds to the next iteration or the stage depending on the prevailing conditions.

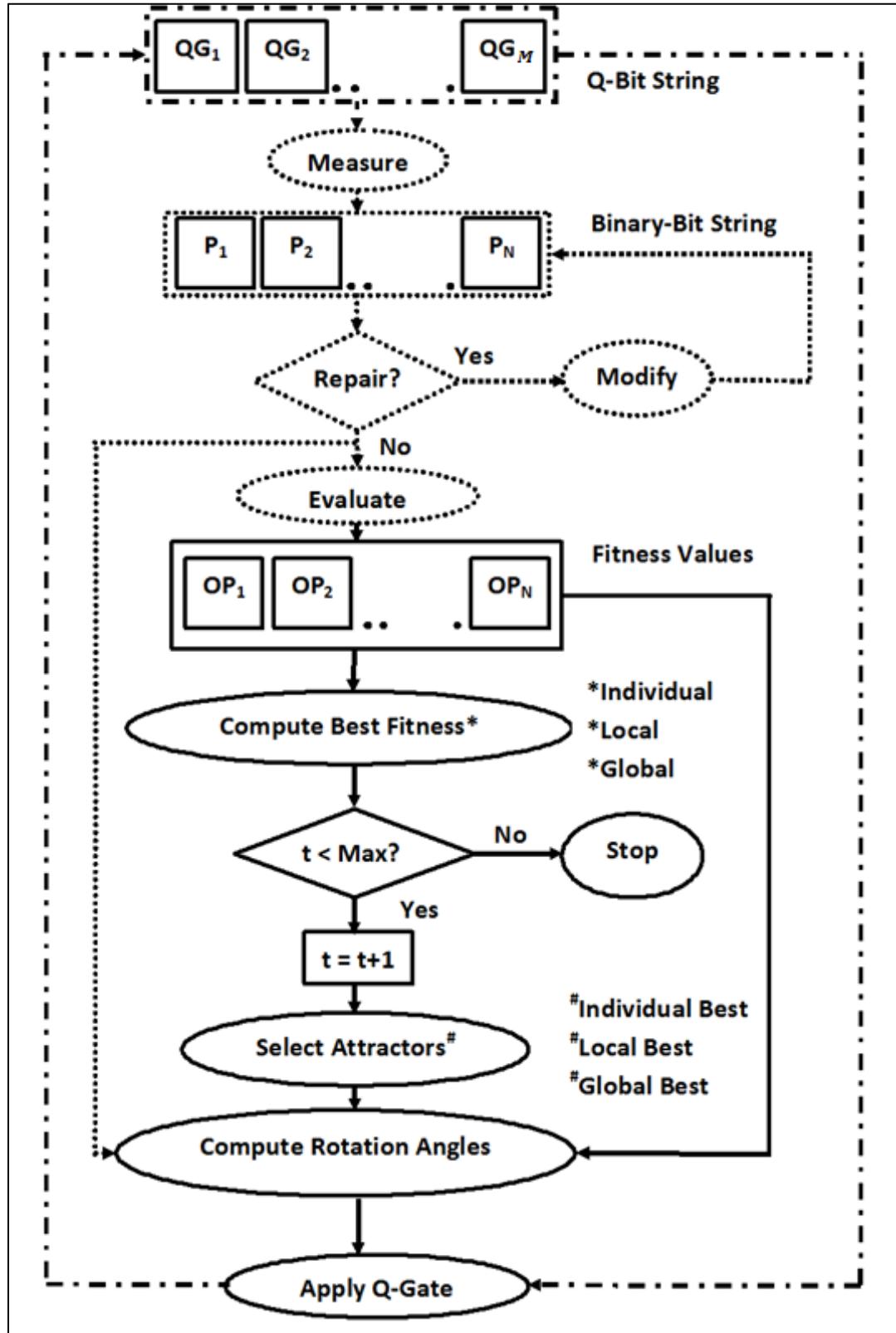


Fig 3. Flow Chart of QEA

The following notation is introduced in order to describe the proposed parameter tuning method:

- N1: Maximum Number of iterations that can be run during Stage 1.
- N2: Maximum Number of iterations that can be run during Stage 2.
- NWI1: Maximum Number of consequent iterations that can be run without observing any improvement in objective function value during Stage 1.
- NWI2: Maximum Number of consequent iterations that can be run without observing any improvement in objective function value during Stage 2.
- NP: Number of parameters to be tuned.
- UL<sub>j</sub>: Upper limit on the value of parameter j (for j = 1...NP).
- LL<sub>j</sub>: Lower limit on the value of parameter j (for j = 1...NP).
- NL1: Number of levels in Stage 1 of each parameter.
- NL2: Number of levels in Stage 2 of each parameter.
- OA1: Orthogonal Array for Stage 1.
- OA2: Orthogonal Array Stage 2.
- PVA<sub>i</sub>: Parameter Vector obtained from the Analysis of results of Experiments designed by OA at i<sup>th</sup> iteration.
- PVB<sub>i</sub>: Parameter Vector which performed best in Experiments designed by OA at i<sup>th</sup> iteration.
- OFV\_PVA<sub>i</sub>: Objective Function Value with PVA<sub>i</sub> at the i<sup>th</sup> iteration.
- OFV\_PVB<sub>i</sub>: Objective Function Value with PVB<sub>i</sub> at the i<sup>th</sup> iteration.
- PIVOT<sub>i</sub>: Best performing parameter vector known till i<sup>th</sup> iteration.
- OFV\_PIVOT<sub>i</sub>: Objective Function Value with Best performing parameter vector known till i<sup>th</sup> iteration.
- i: Iteration Counter.
- I: No. of iterations since no improvement in OFV\_PIVOT<sub>i</sub> has been observed.

Framework of proposed tuning method is as follows:

1. Set N1, N2, NWI1, NWI2, NP, NL1, NL2, OA1 & OA2;
2. For each tuned parameter, Set LL<sub>j</sub> and UL<sub>j</sub>, where j = 1 to NP;
3. Initialize I = 0; i = 0;  
/\* EXPLORATION STAGE \*/
4. For each parameter, Randomly Distributed 'NL1' levels between LL<sub>j</sub> and UL<sub>j</sub>;  
Do {
5. Perform experiments according to OA1.
6. Compute PVA<sub>i</sub> using Taguchi Method.

```

7. Compute OFV_PVAi.
8. Determine PVBi and OFV_PVBi.
9. If i = 0 then
    If OFV_PVBi better than OFV_PVAi then
        OFV_PIVOTi = OFV_PVBi
        PIVOTi = PVBi
        I=0;
    Else
        OFV_PIVOTi = OFV_PVAi
        PIVOTi = PVAi
        I=0;
Else If OFV_PVBi better than OFV_PVAi and OFV_PIVOTi-1 then
    OFV_PIVOTi = OFV_PVBi;
    PIVOTi = PVBi;
    I=0;
Else if OFV_PVAi better than OFV_PIVOTi-1 then
    OFV_PIVOTi = OFV_PVAi;
    PIVOTi = PVAi;
    I=0;
Else
    PIVOTi = PIVOTi-1;
    I++;
10. Use the PIVOTi to generate randomly ('NL1' - 1) other levels. (Assign (NL1 - 1)
    / 2 levels between LLj & PIVOTj,i and (NL1 - 1) / 2 levels between ULj & PIVOTj,i for
    the jth parameter in the PIVOTi vector) to be used in the next iteration, j = 1 .. NP;
11. ++i;
} While (i != N1 && I < NWI1)
/* EXPLOITATION STAGE */
12. PIVOT-1 = PIVOTi; OFV_PIVOT-1 = OFV_PIVOTi; I = 0; i = 0;
13. Use the PIVOTi-1 to generate randomly ('NL2' - 1) other levels. Assign the (NL2 -
    1)/2 levels between (PIVOTj,i-1 - 0.1*(PIVOTj,i-1 - LLj)) & PIVOTj,i-1 and other (NL2 -
    1)/2 levels between PIVOTj,i-1 & PIVOTj,i-1 + 0.1*(ULj - PIVOTj,i-1)), j = 1 .. NP.
Do {
14. Perform experiments according to OA2.
15. Compute PVAi using Taguchi Method.
16. Compute OFV_PVAi.
17. Determine PVBi and OFV_PVBi.
18. If OFV_PVBi better than OFV_PVAi and OFV_PIVOTi-1 then
    OFV_PIVOTi = OFV_PVBi;
    PIVOTi = PVBi;
    I=0;
Else if OFV_PVAi better than OFV_PIVOTi-1 then
    OFV_PIVOTi = OFV_PVAi;
    PIVOTi = PVAi;
    I=0;
Else
    OFV_PIVOTi = OFV_PIVOTi-1;
    PIVOTi = PIVOTi-1;
    ++I;
19. If (I == 0)
    Generate uniformly distributed ('NL2' - 2) other levels between PIVOTj,i and
    PIVOTj,i-1, j=1..NP;
    Else
        Use the PIVOTj,i to generate randomly ('NL2' - 1) other levels, j = 1..NP;

```

```

Assign the (NL2 - 1)/2 levels between (PIVOTj,i - (0.1/i)*(PIVOTj,i - LLj)) &
PIVOTj,i and other (NL2 - 1)/2 levels between PIVOTj,i & PIVOTj,i +
(0.1/i)*(ULj - PIVOTj,i)), j = 1..NP;

20.    ++i;
} while (i != N2 && I < NWI2)
21.    OUTPUT PIVOTi-1 as Parameter Value.

```

The proposed method has two explicit iterative stages i.e. exploration and exploitation. First of all initialization of parameters controlling the parameter tuning method like N1, N2, NWI1, NWI2, NP, NL1, NL2, OA1 & OA2 are specified. In step 2, lower and upper limits of all the parameters being tuned are specified. In exploration stage, first of all, each parameter is randomly initialized to NL1 different levels within their respective range. In step 5), the experiments are performed according to orthogonal array OA1, i.e. OA1 is used for determining the parameter values of the EAs being tuned on a specific problem, for performing the experiments. The results are analyzed by using Taguchi's method for finding the parameter vector, PVA<sub>i</sub> that gives the best result for the given problem. Analysis of the result is performed by using the best objective function value obtained from the individual experiment (i.e. thirty runs minimum for each experiment) as the search is for finding the PVA<sub>i</sub>. In step 7), the EA being tuned is executed using PVA<sub>i</sub> to find the OFV\_PVA<sub>i</sub>. In step 8), the PVB<sub>i</sub> and OFV\_PVB<sub>i</sub> are determined from experiments performed using OA1 for cross checking Taguchi's method and is also the elitist selection for implementing meta-heuristic framework, if Taguchi's method fails. In step 9), OFV\_PVA<sub>i</sub> and OFV\_PVB<sub>i</sub> are compared and the better of the two becomes PIVOT<sub>i</sub>. In step 10), PIVOT<sub>i</sub> is used for generating new levels for parameters of EA being tuned for further experiments in exploration stage, till the termination criteria of maximum number of iterations N1 or maximum number of iteration since no improvement is observed in PIVOT<sub>i</sub>, NWI1 is met.

The exploitation stage begins with the selection of NL2 levels for every parameter, out of which, one of the level is taken as the best PIVOT<sub>i</sub> from the exploration stage. In step 14), the experiments are performed according to orthogonal array OA2, i.e. OA2 is used for determining the parameter values of the EA being tuned on a specific problem, for performing the experiments. The results are analyzed by using Taguchi's method for finding the set of parameter vector, PVA<sub>i</sub> that gives the best result for the given problem. Analysis of the result is performed by using the best objective function value obtained from the individual experiment (i.e. thirty runs minimum for each experiment) as the search is for finding the PVA<sub>i</sub>. In step 16), the EA being tuned is executed using PVA<sub>i</sub> to find the OFV\_PVA<sub>i</sub>. In step 17), the PVB<sub>i</sub> and OFV\_PVB<sub>i</sub> are determined from experiments performed using OA2 for cross checking Taguchi's method and is also the elitist selection for implementing meta-heuristic framework, if Taguchi's method fails. In step 18), OFV\_PVA<sub>i</sub> and OFV\_PVB<sub>i</sub> are compared and the better of the two becomes PIVOT<sub>i</sub>. In step 19), if there has been improvement in PIVOT<sub>i</sub> from previous iteration then PIVOT<sub>i</sub> and PIVOT<sub>i-1</sub> is used for generating new levels otherwise, new levels are generated randomly in the vicinity of PIVOT<sub>i</sub>, by perturbing it on the either side by (10% / i) of its Euclidian distance from the extremes randomly, for the parameters of EA being tuned for further experiments in exploitation stage, till the termination criteria of maximum number of iterations N2 or maximum number of iteration since no improvement is observed in PIVOT<sub>i</sub>, NWI2 is met.

In this work, N1 = 3, N2 = 3, NWI1 = 2, NWI2 = 2, NL1 = 5, NL2 = 3, OA1 is OA(50, 2<sup>1</sup> x 5<sup>11</sup>, 2) i.e. **L-50 Table** [41], [42] with fifty experiments, one parameter with two levels and eleven parameters with five levels each and its strength is 2. OA2 is OA(27, 3<sup>13</sup>, 2) i.e. **L-27 Table** [41], [42] with 27 experiments, 13 parameters with three levels each and its strength is 2. The rest of the parameters of proposed parameter tuning method are problem dependent.

### III. TESTING

The canonical QEA has been fine-tuned by the proposed parameter tuning method on a suite of benchmark test problems, which have been used in many studies. This serves two purposes, firstly, it validates the proposed parameter tuning framework and secondly, it helps in further improving the performance of canonical QEA.

The process followed for testing involves fine-tuning the eleven parameters of canonical QEA with a single instance of the problem and subsequently using the same parameter values to solve some instances of the problem. Further, a comparison is made with the canonical QEA [14] to study the efficacy of the proposed technique. The computational time required in parameter tuning depends on the maximum number of function evaluation in each run, the number of runs in each experiment, number of experiment to be performed in each iteration (OA1 for exploration and OA2 for exploitation stage) and the number of iterations used in Exploration Stage (N1) and Exploitation Stage (N2). The maximum number of function evaluations may be limited to five hundred thousand. It can be more or less depending on the problem and can be determined with some random

experiments. One can start with a certain value and then change it according to the performance. If the optimal is known then, the run can be terminated upon reaching the optima, thus saving the computational resource. The number of independent runs should be minimum thirty as QEA are stochastic in nature. The number of experiments is a function of the orthogonal array selected for a particular stage. L50 with fifty experiments, five levels for eleven parameters and two levels for one parameter has been used for Exploration stage and L27 with twenty seven experiments and three levels for eleven parameters has been used for Exploitation Stage. In our experience, three to five iterations were sufficient in exploration stage. The exploitation stage, at times, ended in a single iteration but required up to five iterations in some other cases. Thus, the availability of computational resources and complexity of problem should help in deciding the effort to be consumed in Exploration and Exploitation Stage. Further, experiments can be designed with larger size of orthogonal Arrays to accommodate more number of parameters as well as more number of levels. Thus, the proposed framework can be used for a very quick parameter tuning as well as for a relative exhaustive parameter tuning also.

In this work,  $N1 = 3$ ,  $N2 = 3$ ,  $NWI1 = 2$ ,  $NWI2 = 2$ ,  $NL1 = 5$ ,  $NL2 = 3$ ,  $OA1$  is  $OA(50, 2^1 \times 5^{11}, 2)$  i.e. **L-50 Table** [41], [42] with fifty experiments, one parameter with two levels and eleven parameters with five levels each and its strength is 2.  $OA2$  is  $OA(27, 3^{13}, 2)$  i.e. **L-27 Table** [41], [42] with 27 experiments, 13 parameters with three levels each and its strength is 2. The rest of the parameters of proposed Tuning method are problem dependent.

The robustness of the algorithm to changes in parameter values [27] have been studied from the data collected during the tuning process. It has been studied both for large and small variations in parameter values [27]. The data for the first case i.e. large variation has been collected from the first iteration of the Exploration Stage. The solutions of all the experiments in the first iteration of the exploration stage have been used for computing the deviation from the average solution of the QEA. This measure can statistically indicate the robustness of the QEA in the entire parameter space for a given problem through sampling. The data for the second case i.e. small variation has been collected from the first iteration of the Exploitation Stage. The solutions of all the experiments in the first iteration of the exploitation stage have been used for computing the deviation from the best solution of the QEA. This measure can statistically indicate the robustness of the QEA against minor variations in the parameters for a given problem through sampling.

The problem suite used for testing has Massively Multimodal Deceptive Problems (MMDP), COUNTSAT Problems, Knapsack problems and P-Peaks Problems. The eleven parameters of QEA i.e. eight rotation angles ( $\theta_1$  to  $\theta_8$ ), migration period, group size / No. of groups (i.e. population = No. of groups \* group size) and population size have been fine-tuned using the proposed framework. Subsequently a comparison has been made between the fine-tuned QEA and the canonical QEA with parameters given in Table 2 as they have been widely used in literature [14], [16]. The stopping criterion is same for both the QEAs, which is maximum number of function evaluations.

#### A. Massively Multimodal Deceptive Problem (MMDP) (MMDP) [43], [44]

MMDP with size  $K = 40$  was used for parameter tuning of canonical QEA. The initial range of values for parameters in QEA used for tuning is given in Table 3. The parameter range for magnitude of rotation angles ( $\theta_1, \theta_2, \theta_4, \theta_6, \theta_7$  &  $\theta_8$ ) is 0 to  $0.05\pi$  as they change by small magnitude as compared to  $\theta_3$  &  $\theta_5$ , whose range is from 0 to  $0.5\pi$ , which is very large as compared to the range suggested by [14]. The direction of rotation depends on the sign of  $\alpha$ ,  $\beta$  and relative fitness as per Table 1. The range for population size is 5 to 200, and covers the values for similar parameter for most studies in Evolutionary Algorithms. The range for no. of groups is 1 to 20, which is twice big as the value suggested in [14]. The range for global migration is 1 to 500, which is again five times the value suggested by [14]. The change in value of each parameter during the tuning process is depicted in Table 4 and shown in Fig. 4 to Fig. 14. There were three rounds for exploration stage and one round for exploitation stage. It can be observed that after the third round, all the thirty independent runs of QEA of the Tuning experiment had reached the optimal, so it was decided to stop further exploration and start the exploitation so as to further search within the vicinity of the Best Parameter Set found so far and improve the convergence rate. After the first round of the exploitation stage, the convergence could be achieved within twenty generations as shown in Fig. 15, so it was decided to stop further tuning.

TABLE 2  
PARAMETER SETTING FOR QEA

Parameters	Canonical QEA
$\theta_1$	0
$\theta_2$	0
$\theta_3$	$0.01\pi$
$\theta_4$	0
$\theta_5$	$0.01\pi$
$\theta_6$	0
$\theta_7$	0
$\theta_8$	0

Population Size	50
Group size / No. of Groups	5 / 10
Global Migration Period (Generations)	100

TABLE 3  
INITIAL RANGE OF PARAMETERS OF CANONICAL QEA

Parameter	<b>θ1</b> (* π)	<b>θ2</b> (* π)	<b>θ3</b> (* π)	<b>θ4</b> (* π)	<b>θ5</b> (* π)	<b>θ6</b> (* π)	<b>θ7</b> (* π)	<b>θ8</b> (* π)	Pop size	No. of Groups	Global Migration
Lower Limit	0	0	0	0	0	0	0	0	5	1	1
Upper Limit	0.05	0.05	0.5	0.05	0.5	0.05	0.05	0.05	200	20	500

TABLE 4  
BEST PARAMETER VECTOR (PIVOT) DURING TUNING PROCESS

Iter. No.	Best Pivot Parameter Value										Av. OFV
	<b>θ1</b> (* π)	<b>θ2</b> (* π)	<b>θ3</b> (* π)	<b>θ4</b> (* π)	<b>θ5</b> (* π)	<b>θ6</b> (* π)	<b>θ7</b> (* π)	<b>θ8</b> (* π)	Pop. Size	No. of Grp.	
Explor. – 1	0.001	0.032	0.066	0.021	0.176	0.027	0.034	0.02	36	6	137
Explor. – 2	0.0002	0.032	0.276	0.0431	0.019	0.0069	0.034	0.033	120	3	5
Explor. – 3	0.00004	0.0282	0.223	0.0484	0.0022	0.018	0.035	0.033	33	3	3
Expltt. – 4	<b>0.000147</b>	<b>0.0282</b>	<b>0.205</b>	<b>0.0485</b>	<b>0.002</b>	<b>0.0205</b>	<b>0.035</b>	<b>0.033</b>	<b>28</b>	<b>4</b>	<b>6</b>
											<b>40.0</b>

The parameter value for  $\theta_1$  initially decreased during exploration and then increased a little during exploitation stage. The parameter value for  $\theta_2$  initially remained constant then decreased during exploration and then remained constant during exploitation stage. The parameter value for  $\theta_3$  initially increased and then decreased during exploration and then again decreased a little during exploitation stage. The parameter value for  $\theta_4$  initially increased during exploration and then increased minutely during exploitation stage. The parameter value for  $\theta_5$  initially decreased during exploration and then decreased a little during exploitation stage. The parameter value for  $\theta_6$  initially decreased and then increased during exploration and then increased a little during exploitation stage. The parameter value for  $\theta_7$  initially remained constant and then increased minutely during exploration and then remained constant during exploitation stage. The parameter value for  $\theta_8$  initially increased and then remained constant during exploration and then remained during exploitation stage. The parameter value for Population Size initially increased then decreased during exploration and then decreased slightly during exploitation stage. The parameter value for no. of groups initially decreased during exploration and then increased a little during exploitation stage. The range of parameter value for Global Migration had to be reset to 0 to 10 as the optimal was reached by Best performing experiment in twenty iterations. Thus, the Fig. 15 is a semi log graph, which shows that parameter value for global migration initially decreased during exploration and subsequently increased during exploitation stage. The final value of each parameter is given in Table 4 in the last row. The convergence graph given in Fig. 16 shows fast convergence to optimal within ten generations.

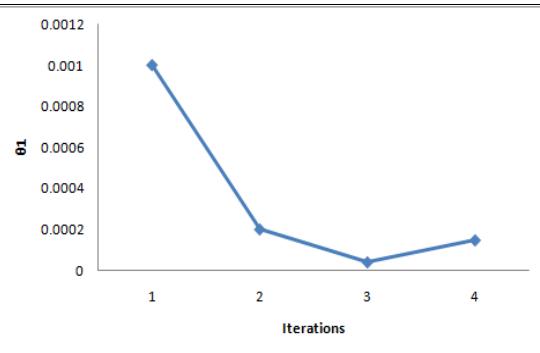


Fig 4: Change in  $\theta_1$  value during Tuning Process

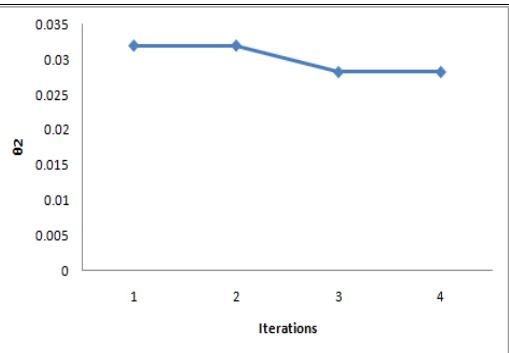


Fig 5. Change in  $\theta_2$  value during Tuning Process

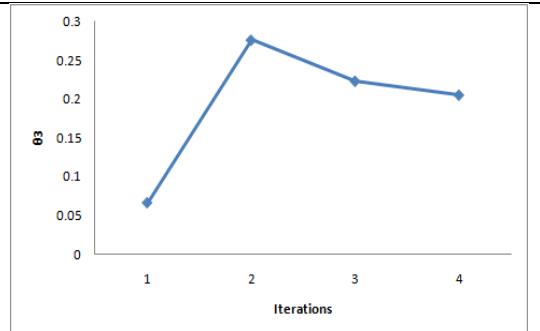


Fig 6. Change in  $\theta_3$  value during Tuning Process

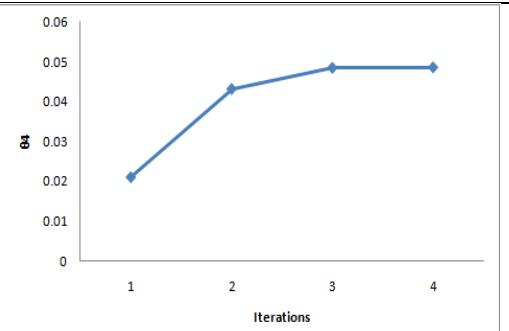


Fig 7. Change in  $\theta_4$  value during Tuning Process

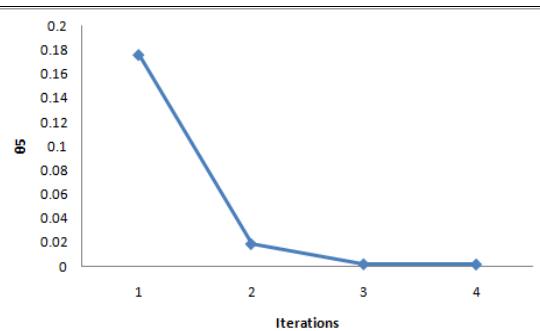


Fig 8. Change in  $\theta_5$  value during Tuning Process

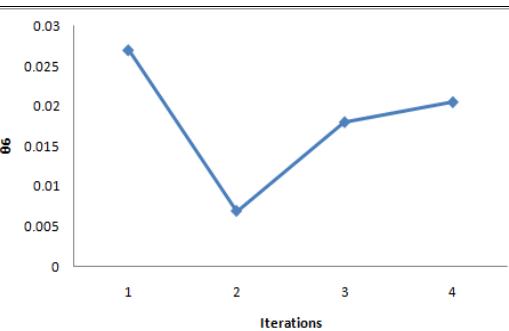


Fig 9. Change in  $\theta_6$  value during Tuning Process

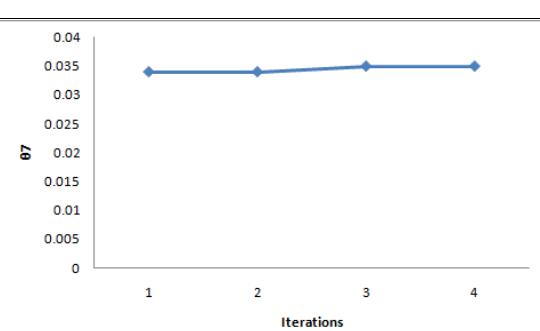


Fig 10. Change in  $\theta_7$  value during Tuning Process

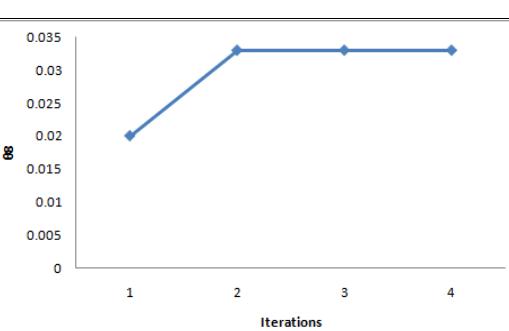


Fig 11. Change in  $\theta_8$  value during Tuning Process

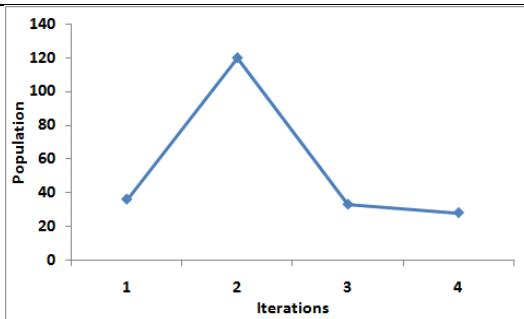


Fig 12. Change in Population size during Tuning Process

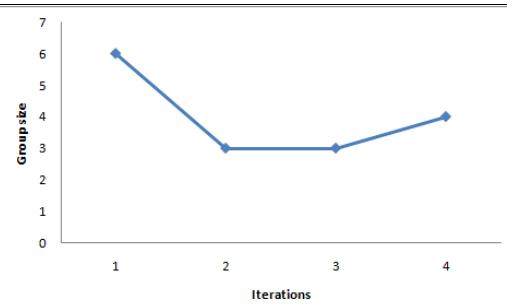


Fig 13. Change in No. of Groups value during Tuning Process

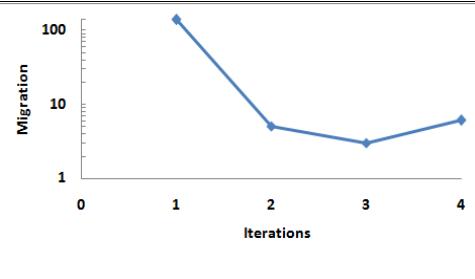


Fig 14. Change in Migration value during Tuning Process

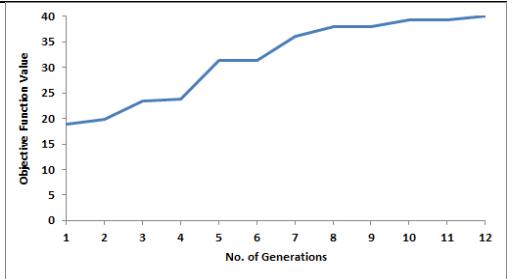


Fig 15. Convergence Graph of TCQEA

The deviation from optimal for large variation in parameter setting, which is computed from the average results of fifty experiments from the first iteration of exploration stage is 7.54 whereas the deviation from optimal for small variation in parameter setting, which is computed from the average results of twenty seven experiments from the first iteration of exploitation stage is zero. Therefore, the tuned QEA is robust to small variation in parameter set, however, it is relatively unstable for large variation in parameter set, thus justifying the effort put in tuning the parameter set of QEA for MMDP. Thus, the deviation from the known optimal at the end of the iteration can be used for deciding the need for continuing the search in that stage. However, if the Optimal is unknown then this strategy cannot be applied with the same confidence, but in case of real world problems, often a best known solution is available, so in such cases, it may be used in place of the Optimal.

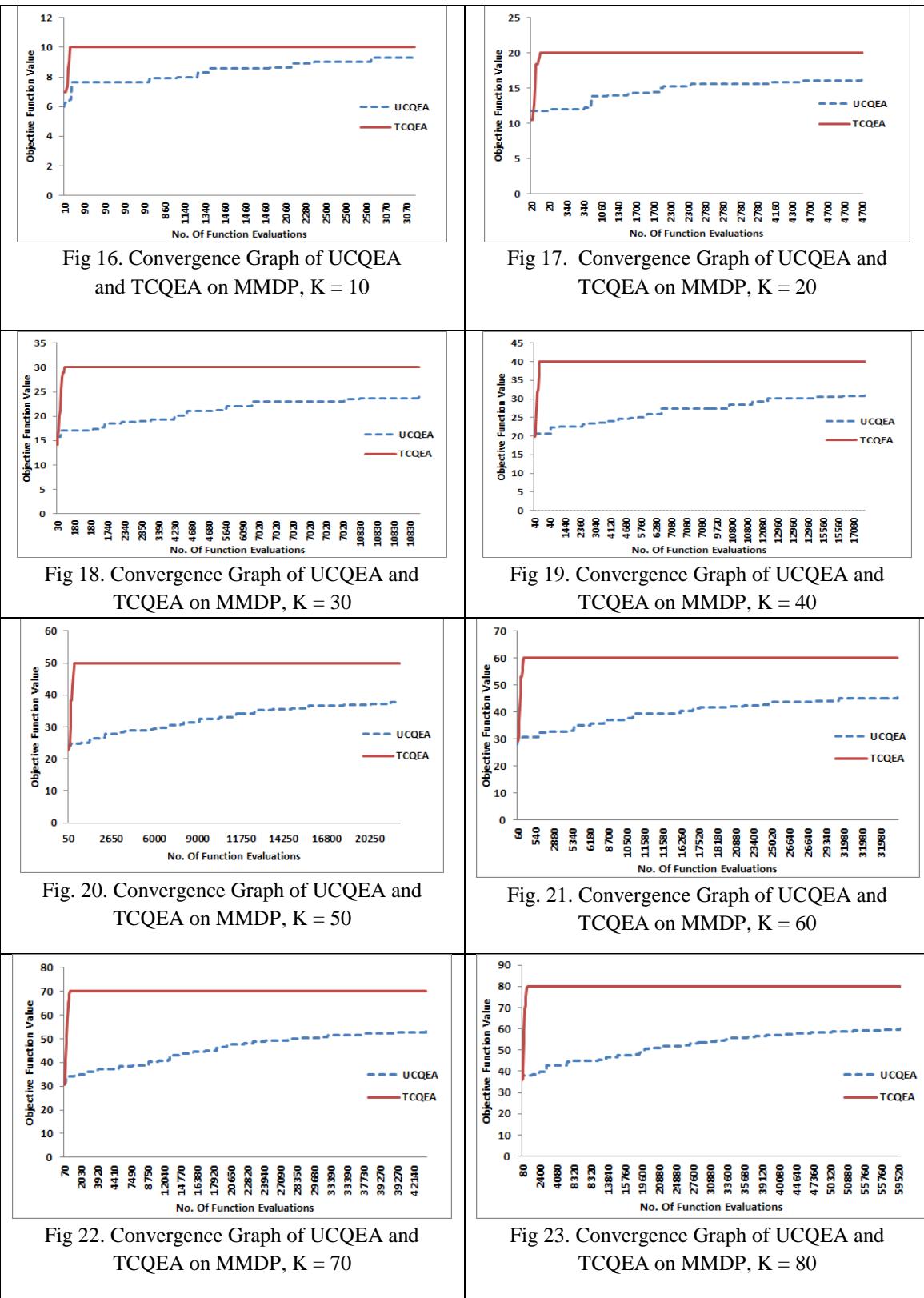
A comparative study was done between parameter Tuned QEA (TCQEA) and Canonical QEA (UCQEA) with Parameters given in Table 2 on MMDP with a set of twenty problems of size k varying from 10 to 200 at an interval of 10. The results are given in Table 5. Thirty independent runs were made on each problem size and the comparison has been made on Best, Median, Worst, Mean, percentage of runs in which optimal was achieved i.e. percentage of Success runs, and Average number of function evaluations (NFE). The Canonical QEA was not able to reach optimal value any runs for any of the twenty problems whereas the Tuned-QEA was able to reach 100% success in nineteen out of twenty problems. It has 93.3% of success run in the last problem of the benchmark suite. The performance of Tuned QEA was also good on speed of convergence as indicated by average NFE and the convergence graphs shown in figure, which also compared the speed of convergence of Tuned QEA to Canonical QEA.

The convergence graphs have been plotted between objective function value and number of generations for both Tuned-QEA (TCQEA) and Canonical QEA (UCQEA) for all the problem instances of MMDP for the median run as shown in Fig. 17 to Fig. 36. The convergence graph clearly establishes the superiority of Tuning as the Tuned QEA is able to reach the optimal in less than twenty generations in all the graphs whereas the Canonical QEA is not able to reach the optimal even in two thousand generations. The distance from optimal has increased as the size of the problem has increased in case of Canonical QEA but Tuned QEA has performed consistently well for this class of problems.

TABLE 5  
COMPARATIVE STUDY BETWEEN TCQEA AND UCQEA ON MMDP INSTANCES

Problem	Algo	Best	Median	Worst	Mean	% Success Runs	Average NFE
K=10	UCQEA	9.64	9.28	8.56	9.17	0.00	500050
	TCQEA	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>10.00</b>	<b>100.00</b>	<b>182</b>
K=20	UCQEA	18.56	16.59	15.33	16.67	0.00	500050

	TCQEA	<b>20.00</b>	<b>20.00</b>	<b>20.00</b>	<b>20.00</b>	<b>100.00</b>	<b>234</b>
K=30	UCQEA	25.69	23.89	22.45	24.06	0.00	500050
	TCQEA	<b>30.00</b>	<b>30.00</b>	<b>30.00</b>	<b>30.00</b>	<b>100.00</b>	<b>259</b>
K=40	UCQEA	33.53	31.01	29.58	31.21	0.00	500050
	TCQEA	<b>40.00</b>	<b>40.00</b>	<b>40.00</b>	<b>40.00</b>	<b>100.00</b>	<b>280</b>
K=50	UCQEA	42.09	38.49	35.98	38.94	0.00	500050
	TCQEA	<b>50.00</b>	<b>50.00</b>	<b>50.00</b>	<b>50.00</b>	<b>100.00</b>	<b>297</b>
K=60	UCQEA	48.14	45.26	43.47	45.56	0.00	500050
	TCQEA	<b>60.00</b>	<b>60.00</b>	<b>60.00</b>	<b>60.00</b>	<b>100.00</b>	<b>280</b>
K=70	UCQEA	57.42	53.11	49.15	53.26	0.00	500050
	TCQEA	<b>70.00</b>	<b>70.00</b>	<b>70.00</b>	<b>70.00</b>	<b>100.00</b>	<b>345</b>
K=80	UCQEA	63.83	59.87	57.00	60.15	0.00	500050
	TCQEA	<b>80.00</b>	<b>80.00</b>	<b>80.00</b>	<b>80.00</b>	<b>100.00</b>	<b>360</b>
K=90	UCQEA	72.39	67.36	65.20	67.64	0.00	500050
	TCQEA	<b>90.00</b>	<b>90.00</b>	<b>90.00</b>	<b>90.00</b>	<b>100.00</b>	<b>389</b>
K=100	UCQEA	78.79	74.84	71.61	75.50	0.00	500050
	TCQEA	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>	<b>415</b>
K=110	UCQEA	87.36	81.79	79.45	82.16	0.00	500050
	TCQEA	<b>110.00</b>	<b>110.00</b>	<b>110.00</b>	<b>110.00</b>	<b>100.00</b>	<b>441</b>
K=120	UCQEA	93.40	90.35	85.50	90.18	0.00	500050
	TCQEA	<b>120.00</b>	<b>120.00</b>	<b>120.00</b>	<b>120.00</b>	<b>100.00</b>	<b>469</b>
K=130	UCQEA	100.17	96.93	93.34	96.93	0.00	500050
	TCQEA	<b>130.00</b>	<b>130.00</b>	<b>130.00</b>	<b>130.00</b>	<b>100.00</b>	<b>495</b>
K=140	UCQEA	106.57	103.70	100.10	103.84	0.00	500050
	TCQEA	<b>140.00</b>	<b>140.00</b>	<b>140.00</b>	<b>140.00</b>	<b>100.00</b>	<b>553</b>
K=150	UCQEA	115.86	112.44	107.59	111.95	0.00	500050
	TCQEA	<b>150.00</b>	<b>150.00</b>	<b>150.00</b>	<b>150.00</b>	<b>100.00</b>	<b>658</b>
K=160	UCQEA	123.70	119.03	113.63	119.18	0.00	500050
	TCQEA	<b>160.00</b>	<b>160.00</b>	<b>160.00</b>	<b>160.00</b>	<b>100.00</b>	<b>713</b>
K=170	UCQEA	130.82	125.97	122.56	126.13	0.00	500050
	TCQEA	<b>170.00</b>	<b>170.00</b>	<b>170.00</b>	<b>170.00</b>	<b>100.00</b>	<b>873</b>
K=180	UCQEA	137.95	133.28	128.96	133.25	0.00	500050
	TCQEA	<b>180.00</b>	<b>180.00</b>	<b>180.00</b>	<b>180.00</b>	<b>100.00</b>	<b>921</b>
K=190	UCQEA	144.35	140.22	136.45	140.16	0.00	500050
	TCQEA	<b>190.00</b>	<b>190.00</b>	<b>190.00</b>	<b>190.00</b>	<b>100.00</b>	<b>1139</b>
K=200	UCQEA	151.84	147.88	145.01	147.99	0.00	500050
	TCQEA	<b>200.00</b>	<b>200.00</b>	<b>199.00</b>	<b>199.96</b>	<b>93.30</b>	<b>3338</b>



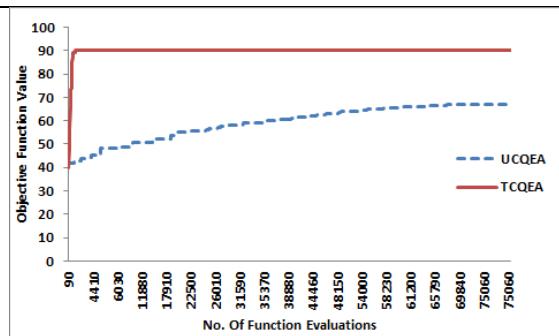


Fig 24. Convergence Graph of UCQEA and TCQEA on MMDP, K = 90

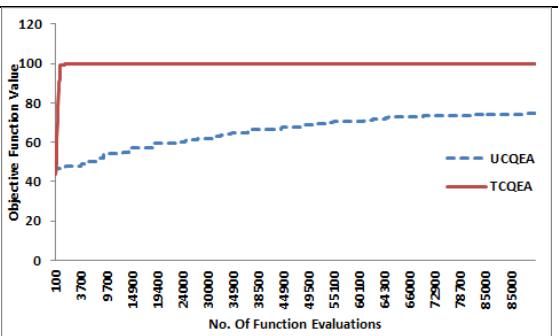


Fig 25. Convergence Graph of UCQEA and TCQEA on MMDP, K = 100

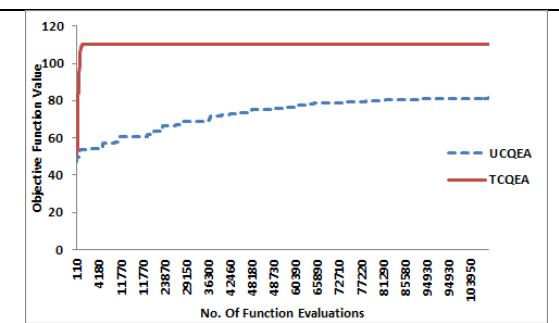


Fig 26. Convergence Graph of UCQEA and TCQEA on MMDP, K = 110

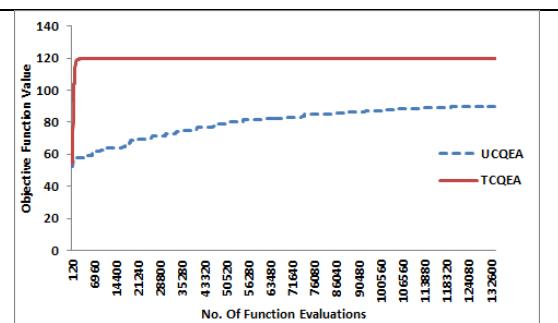


Fig 27. Convergence Graph of UCQEA and TCQEA on MMDP, K = 120

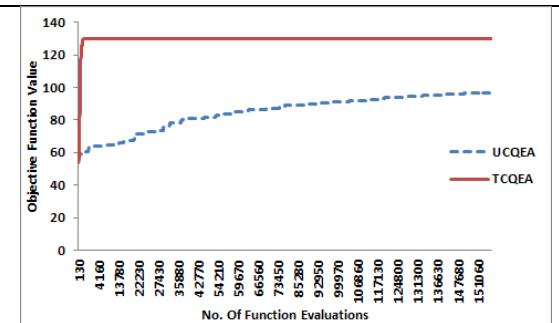


Fig 28. Convergence Graph of UCQEA and TCQEA on MMDP, K = 130

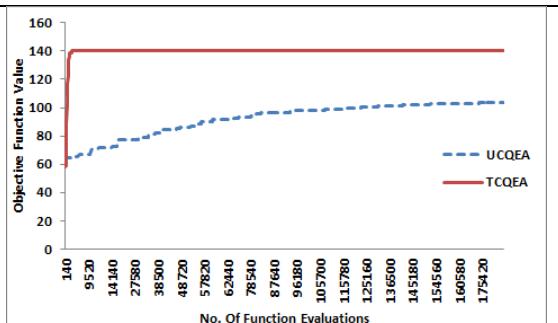


Fig 29. Convergence Graph of UCQEA and TCQEA on MMDP, K = 140

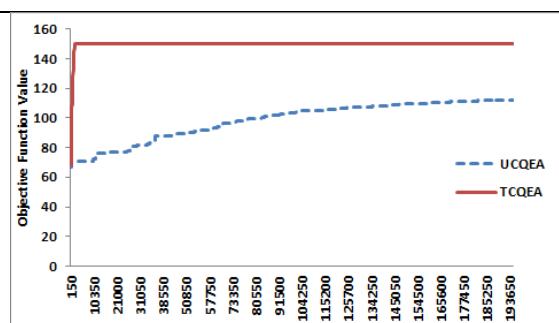


Fig 30. Convergence Graph of UCQEA and TCQEA on MMDP, K = 150

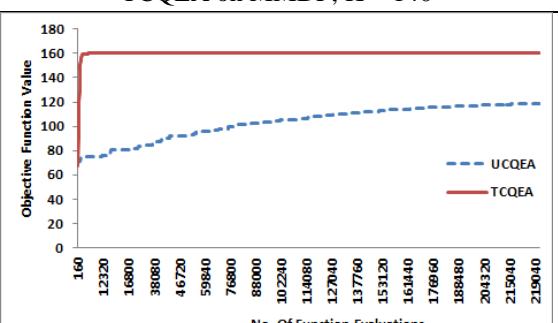


Fig 31. Convergence Graph of UCQEA and TCQEA on MMDP, K = 160

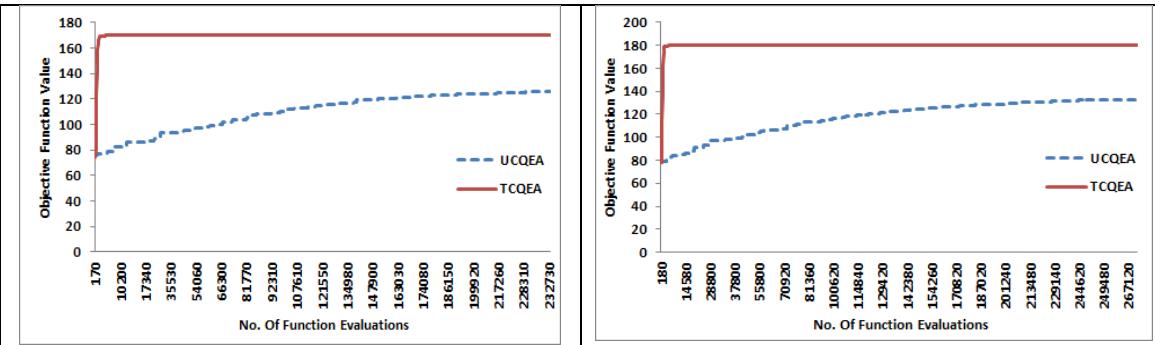


Fig 32. Convergence Graph of UCQEA and TCQEA on MMDP, K = 170

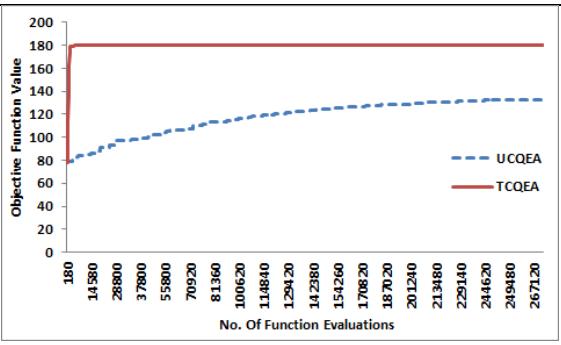


Fig 33. Convergence Graph of UCQEA and TCQEA on MMDP, K = 180

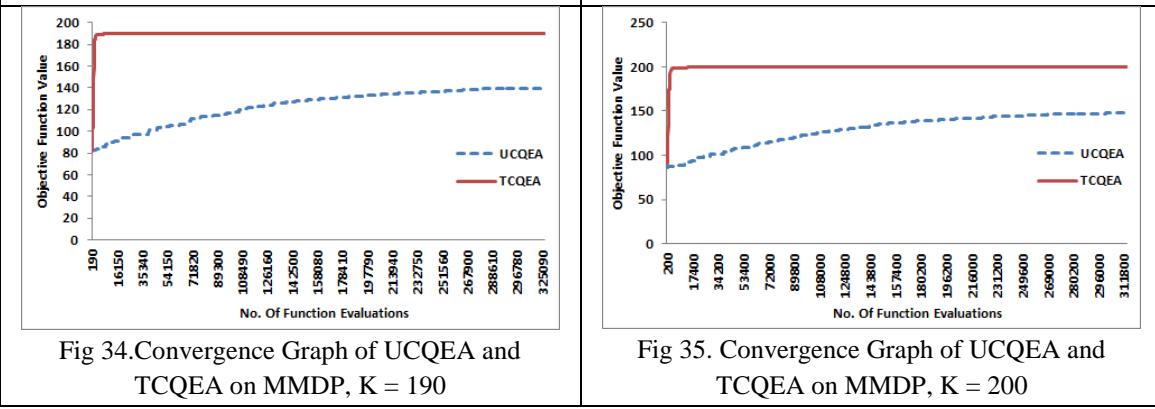


Fig 34. Convergence Graph of UCQEA and TCQEA on MMDP, K = 190

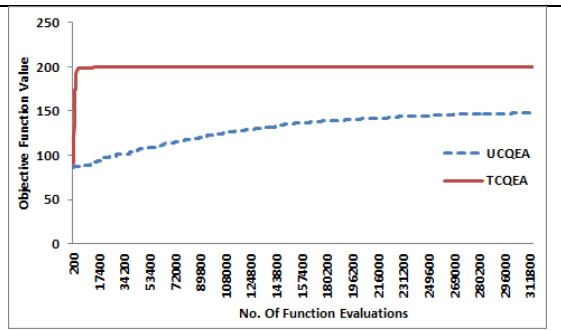


Fig 35. Convergence Graph of UCQEA and TCQEA on MMDP, K = 200

The performance of Tuned QEA is superior to Canonical QEA for all instances of MMDP used in this work as indicated by the Table 5 and Fig. 16 to 35. This indicates the success of the proposed tuning method for tuning QEA on problems like MMDP.

In order to confirm the findings in Table 5, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average NFE of Tuned QEA and Canonical QEA on all the instances of MMDP at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 6, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 60 at significance level of  $\alpha = 5\%$ . This indicates the success of the proposed tuning method for tuning QEA on problems like MMDP.

Table 6: Wilcoxon's Signed Rank Test on MMDP Instances (OFV)

	TCQEA	UCQEA	Wilcoxon's Signed Rank Test Results		
			$\Sigma(+)$	$\Sigma(-)$	n
1	X <sub>1</sub>	X <sub>2</sub>	210	0	20
2	10	9.17			
3	20	16.67			
4	30	24.06			
5	40	31.1			
6	50	38.5			
7	60	45.56			
8	70	53.26			
9	80	60.15			
10	90	67.64			
11	100	75.5			
12	110	82.16			

Null Hypothesis:  $H_0: \mu_1 \leq \mu_2$       Test Statistic:  $T = 0$       Critical Value: At an  $\alpha$  of 5%

12	120	90.18
13	130	96.93
14	140	103.84
15	150	111.95
16	160	119.18
17	170	126.13
18	180	133.25
19	190	140.16
20	200	147.99

In case of comparison on average NFE, the null hypothesis for comparison was average NFE of Tuned QEA  $\mu_1$  is greater than or equal to the average NFE of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 7, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 60 at significance level of  $\alpha = 5\%$ . This indicates the success of the proposed tuning method for tuning QEA on problems like MMDP.

**Table 7: Wilcoxon's Signed Rank Test on MMDP Instances (NFE)**

	TCQEA	UCQEA			Test Statistic	Critical Value
	$X_1$	$X_2$			$T$	At an $\alpha$ of 5%
1	182	500050				
2	234	500050				
3	259	500050				
4	280	500050				
5	297	500050				
6	280	500050				
7	345	500050				
8	360	500050				
9	389	500050				
10	415	500050				
11	441	500050				
12	469	500050				
13	495	500050				
14	553	500050				
15	658	500050				
16	713	500050				
17	873	500050				
18	921	500050				
19	1139	500050				
20	3338	500050				

$\Sigma(+)$  0      Null Hypothesis       $T$       At an  $\alpha$  of 5%  
 $\Sigma(-)$  210       $H_0: \mu_1 \geq \mu_2$       0      60  
 $n$  20

The same set of parameters has been also used for solving instances of a well-known problems known as COUNTSAT.

#### B. COUNTSAT problem [43]

It is an instance of the MAXSAT problem. In COUNTSAT, the value of a given solution is the number of satisfied clauses (among all the possible Horn clauses of three variables) by an input composed by  $n$  boolean variables. It is easy to check that the optimum is obtained when the value of all the variables is 1 i.e.  $s = n$ .

In this study we consider the instance of  $n = 20$  to 1000 variables, and thus, the value of the optimal solution varies from 6860 to 997003000.

$$f_{\text{COUNTSAT}}(s) = s + n \cdot (n - 1) \cdot (n - 2) - 2 \cdot (n - 2) \cdot \binom{s}{2} + 6 \cdot \binom{s}{3}$$

$$\text{For } (n=20) = s + 6840 - 18 \cdot s \cdot (s - 1) + s \cdot (s - 1) \cdot (s - 2) \quad (7)$$

$$\text{For } (n=1000) = s + 997002000 - 998.s.(s-1) + s.(s-1).(s-2)$$

A comparative study performed between parameter Tuned QEA (TCQEA) and Canonical QEA (UCQEA) with Parameters given in Table 2 on COUNTSAT with a set of 21 problems of size 20 to 1000. The results are given in Table 8. Thirty independent runs were made on each problem size and the comparison has been made on Best, Median, Worst, Mean, percentage of runs in which optimal was achieved i.e. percentage of Success runs, and Average number of function evaluations (NFE). The Canonical QEA was able to reach optimal value till problem size 700, but was not able to find the optimal for rest of the problem instances whereas the Tuned QEA was able to reach 100% success in all the problem instances. The performance of Tuned QEA was also good on speed of convergence as indicated by average NFE and the convergence graphs shown in fig. 36 to 56, which also compared the speed of convergence of Tuned QEA to Canonical QEA.

The convergence graphs have been plotted between objective function value and number of generations for both Tuned-QEA (TCQEA) and Canonical QEA (UCQEA) for all the problem instances of COUNTSAT for the median run. The convergence graph clearly establishes the superiority of Tuning as the Tuned QEA is able to reach the optimal in less than 30 generations in all the graphs whereas the Canonical QEA is much slower and takes much larger number of generations to reach near the optimal, which increase with the size of the problem. The distance from optimal has increased as the size of the problem has increased in case of Canonical QEA but Tuned QEA has performed consistently well for this class of problems.

TABLE 8  
COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON COUNTSAT PROBLEM INSTANCES

Prob.	Algo	Best	Worst	Average	Median	% Success Runs	Std	Avg. NFE
K=20	UCQEA	6860	6860	6860	6860	100	0	1152
	TCQEA	<b>6860</b>	<b>6860</b>	<b>6860</b>	<b>6860</b>	<b>100</b>	<b>0</b>	<b>106</b>
K=50	UCQEA	117650	117650	117650	117650	100	0	4907
	TCQEA	<b>117650</b>	<b>117650</b>	<b>117650</b>	<b>117650</b>	<b>100</b>	<b>0</b>	<b>177</b>
K=100	UCQEA	970300	970300	970300	970300	100	0	10462
	TCQEA	<b>970300</b>	<b>970300</b>	<b>970300</b>	<b>970300</b>	<b>100</b>	<b>0</b>	<b>226</b>
K=150	UCQEA	3307950	3307801	3307925	3307950	83	56	20863
	TCQEA	<b>3307950</b>	<b>3307950</b>	<b>3307950</b>	<b>3307950</b>	<b>100</b>	<b>0</b>	<b>255</b>
K=200	UCQEA	7880600	7880401	7880580	7880600	90	61	22300
	TCQEA	<b>7880600</b>	<b>7880600</b>	<b>7880600</b>	<b>7880600</b>	<b>100</b>	<b>0</b>	<b>298</b>
K=250	UCQEA	15438250	15435052	15438102	15438250	80	584	29220
	TCQEA	<b>15438250</b>	<b>15438250</b>	<b>15438250</b>	<b>15438250</b>	<b>100</b>	<b>0</b>	<b>327</b>
K=300	UCQEA	26730900	26670765	26727928	26730900	87	11303	30092
	TCQEA	<b>26730900</b>	<b>26730900</b>	<b>26730900</b>	<b>26730900</b>	<b>100</b>	<b>0</b>	<b>347</b>
K=350	UCQEA	42508550	42297626	42487362	42508550	80	52963	34312
	TCQEA	<b>42508550</b>	<b>42508550</b>	<b>42508550</b>	<b>42508550</b>	<b>100</b>	<b>0</b>	<b>364</b>
K=400	UCQEA	63521200	63258981	63512459	63521200	97	47874	34483
	TCQEA	<b>63521200</b>	<b>63521200</b>	<b>63521200</b>	<b>63521200</b>	<b>100</b>	<b>0</b>	<b>382</b>
K=450	UCQEA	90518850	89462956	90414627	90518850	83	259748	39618

	TCQEA	<b>90518850</b>	<b>90518850</b>	<b>90518850</b>	<b>90518850</b>	<b>100</b>	<b>0</b>	<b>401</b>
K=500	UCQEA	124251500	121775681	123726361	124251500	70	762261	44297
	TCQEA	<b>124251500</b>	<b>124251500</b>	<b>124251500</b>	<b>124251500</b>	<b>100</b>	<b>0</b>	<b>431</b>
K=550	UCQEA	165469150	163088041	165117481	165469150	67	681032	46058
	TCQEA	<b>165469150</b>	<b>165469150</b>	<b>165469150</b>	<b>165469150</b>	<b>100</b>	<b>0</b>	<b>432</b>
K=600	UCQEA	214921800	209881401	213598979	214385692	37	1524388	48732
	TCQEA	<b>214921800</b>	<b>214921800</b>	<b>214921800</b>	<b>214921800</b>	<b>100</b>	<b>0</b>	<b>468</b>
K=650	UCQEA	273359450	267304186	271874229	272729917	33	1881364	49330
	TCQEA	<b>273359450</b>	<b>273359450</b>	<b>273359450</b>	<b>273359450</b>	<b>100</b>	<b>0</b>	<b>478</b>
K=700	UCQEA	341532100	324622561	338459233	340076761	3	3598957	50003
	TCQEA	<b>341532100</b>	<b>341532100</b>	<b>341532100</b>	<b>341532100</b>	<b>100</b>	<b>0</b>	<b>515</b>
K=750	UCQEA	419072236	406621801	416371450	417418345	0	3027881	50050
	TCQEA	<b>420189750</b>	<b>420189750</b>	<b>420189750</b>	<b>420189750</b>	<b>100</b>	<b>0</b>	<b>535</b>
K=800	UCQEA	508810386	498371881	504276211	504764499	0	2502890	50050
	TCQEA	<b>510082400</b>	<b>510082400</b>	<b>510082400</b>	<b>510082400</b>	<b>100</b>	<b>0</b>	<b>588</b>
K=850	UCQEA	607691092	584540601	601343410	603543166	0	6311747	50050
	TCQEA	<b>611960050</b>	<b>611960050</b>	<b>611960050</b>	<b>611960050</b>	<b>100</b>	<b>0</b>	<b>647</b>
K=900	UCQEA	717888937	698472201	712273067	714460559	0	5259800	50050
	TCQEA	<b>726572700</b>	<b>726572700</b>	<b>726572700</b>	<b>726572700</b>	<b>100</b>	<b>0</b>	<b>644</b>
K=950	UCQEA	843268921	793964756	834497691	838219561	0	10389589	50050
	TCQEA	<b>854670350</b>	<b>854670350</b>	<b>854670350</b>	<b>854670350</b>	<b>100</b>	<b>0</b>	<b>843</b>
K=1000	UCQEA	979662826	948748605	969462614	972362722	0	8750511	50050
	TCQEA	<b>997003000</b>	<b>997003000</b>	<b>997003000</b>	<b>997003000</b>	<b>100</b>	<b>0</b>	<b>881</b>

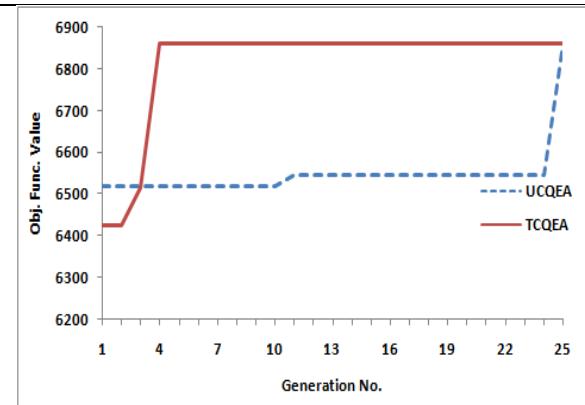


Fig 36. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 20

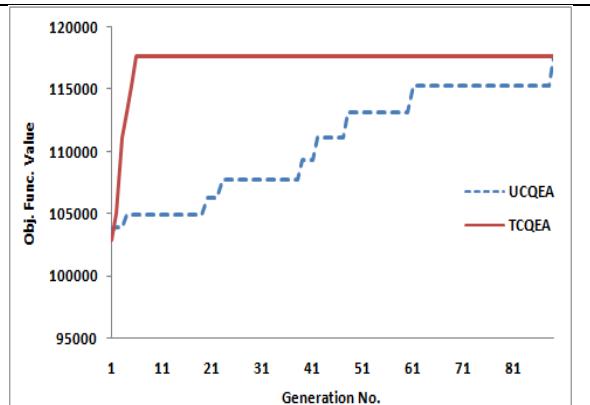


Fig 37. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 50

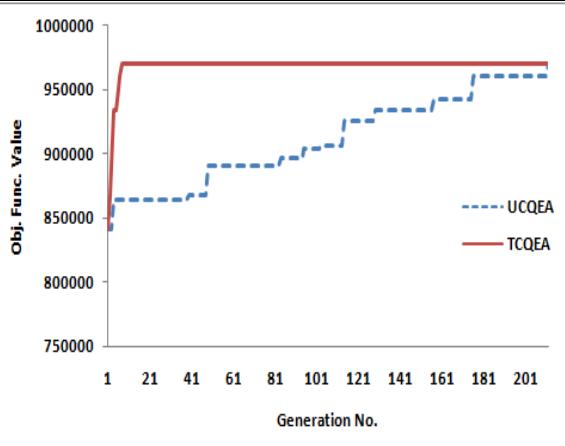


Fig 38. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 100

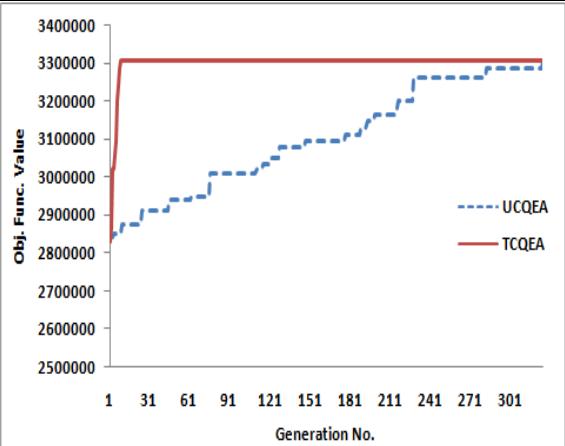


Fig 39. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 150

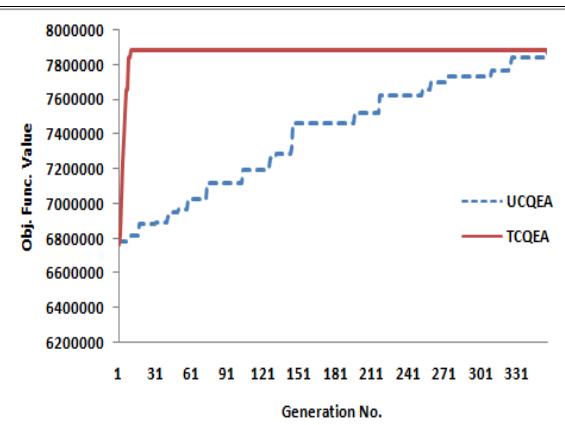


Fig 40. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 200

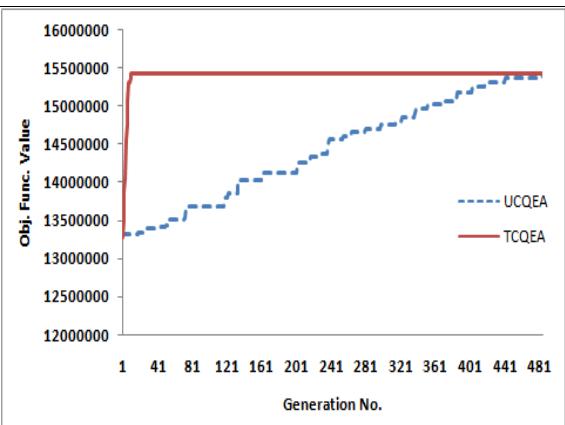


Fig 41. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 250

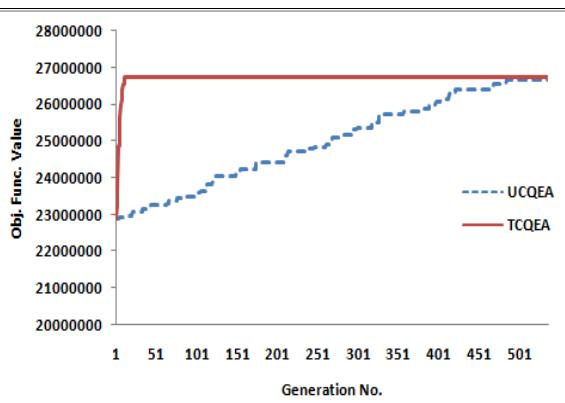


Fig 42. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 300

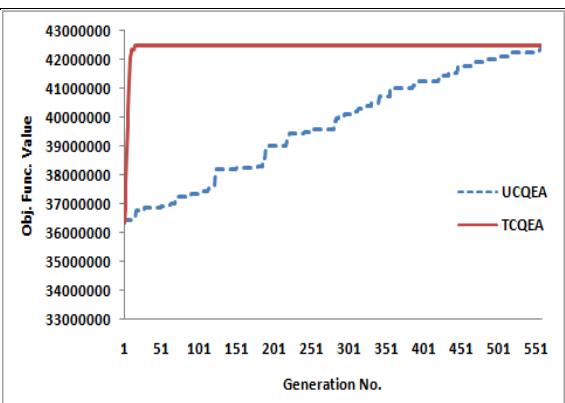


Fig 43. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 350

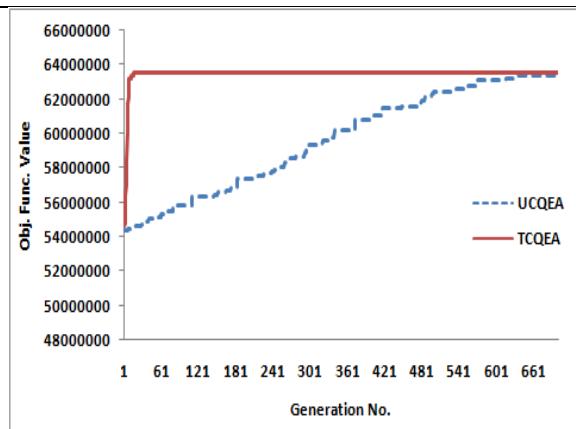


Fig 44. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 400

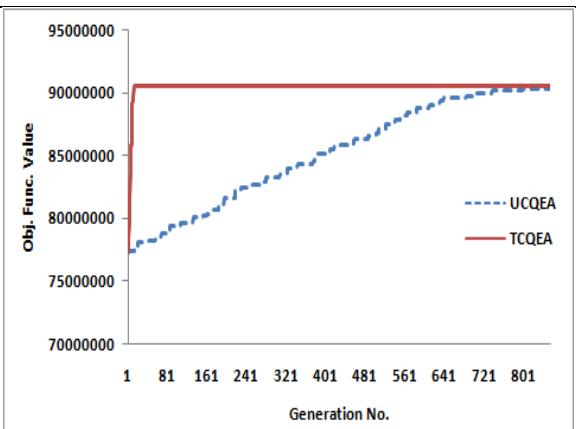


Fig 45. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 450

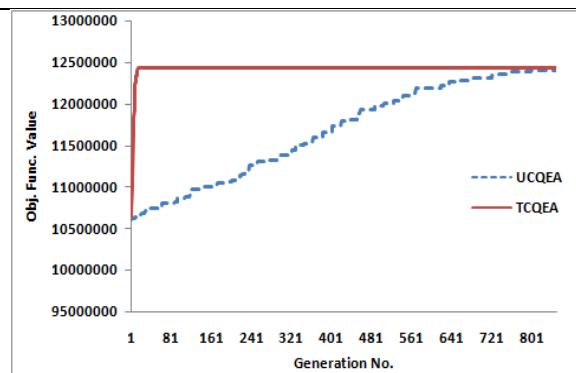


Fig 46. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 500

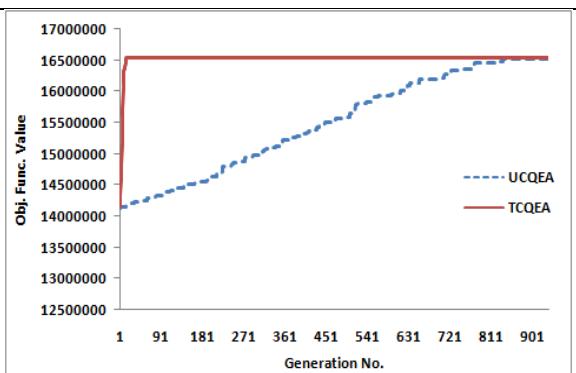


Fig 47. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 550

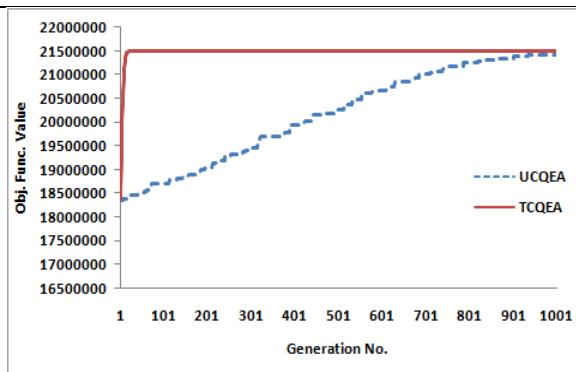


Fig 48. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 600

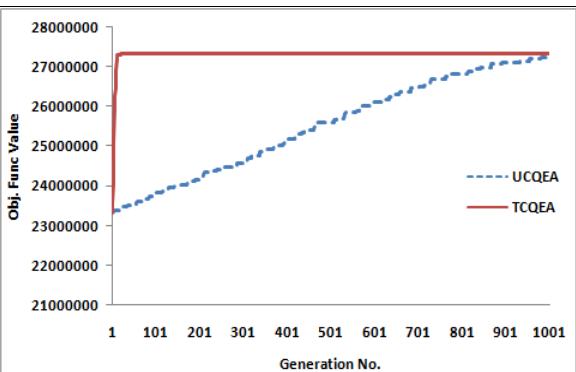


Fig 49. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 650

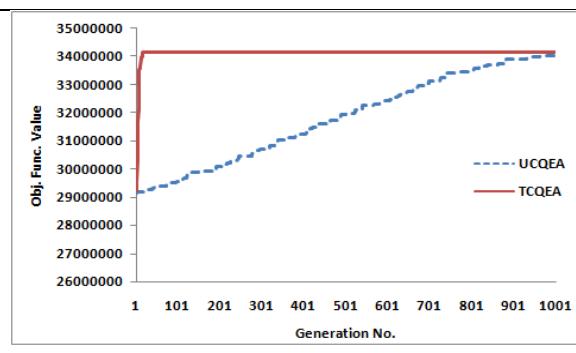


Fig 50. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 700

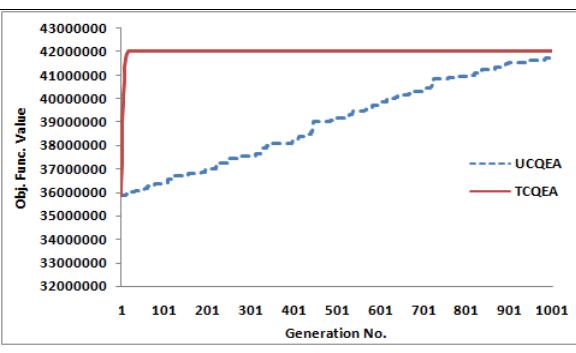
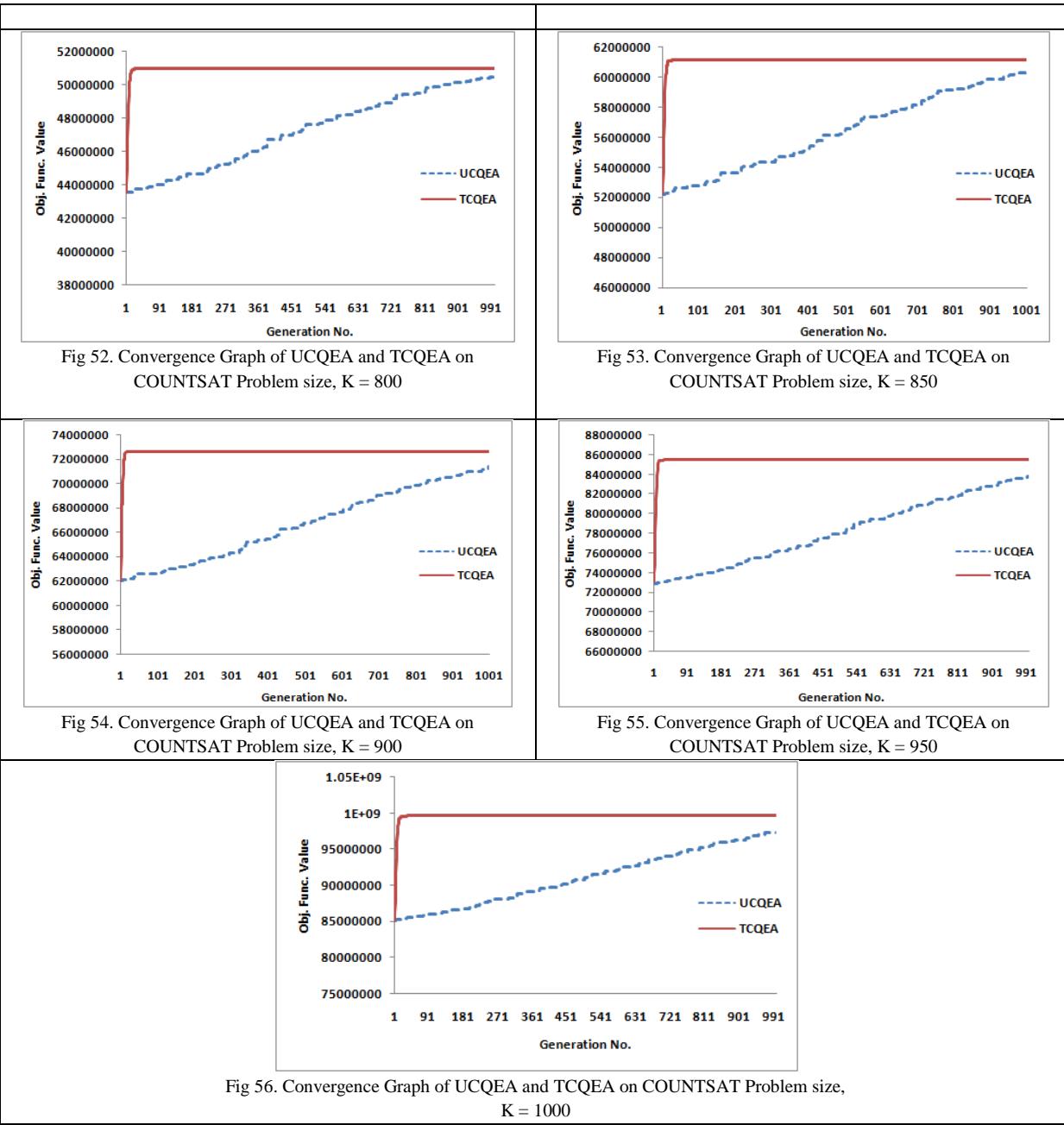


Fig 51. Convergence Graph of UCQEA and TCQEA on COUNTSAT Problem size, K = 750



The performance of Tuned QEA is superior to Canonical QEA for all instances of COUNTSAT Problem used in this work as indicated by the Table 8 and Fig. 36 to 56. This indicates the success of the proposed tuning method for problems like COUNTSAT problem.

In order to confirm the findings in Table 8, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average NFE of Tuned QEA and Canonical QEA on all the instances of COUNTSAT at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 9, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 47 at significance level of  $\alpha = 5\%$ . This indicates the success of the proposed tuning method for tuning QEA on problems like COUNTSAT.

In case of comparison on average NFE, the null hypothesis for comparison was average NFE of Tuned QEA  $\mu_1$  is greater than or equal to the average NFE of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $\mu_1 < \mu_2$ . The result of test is presented in Table 10, which shows that null hypothesis can be safely rejected as Wilcoxon's

Signed Rank test statistic is zero and less than the critical value of 68 at significance level of  $\alpha = 5\%$ . This indicates the success of the proposed tuning method for tuning QEA on problems like COUNTSAT.

**Table 9: Wilcoxon's Signed Rank Test on COUNTSAT Instances (OFV)**

	TCQEA	UCQEA			Test Statistic	Critical Value
	<b>X<sub>1</sub></b>	<b>X<sub>2</sub></b>				
1	6860	6860				
2	117650	117650				
3	970300	970300				
4	3307950	3307925				
5	7880600	7880580				
6	15438250	15438102				
7	26730900	26727928				
8	42508550	42487362				
9	63521200	63512459				
10	90518850	90414627				
11	124251500	123726361				
12	165469150	165117481				
13	214921800	213598979				
14	273359450	271874229				
15	341532100	338459233				
16	420189750	416371450				
17	510082400	504276211				
18	611960050	601343410				
19	726572700	712273067				
20	854670350	834497691				
21	997003000	969462614				

$\Sigma(+)$ 

171
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 $\Sigma(-)$ 

0
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 $n$ 

18
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Null Hypothesis  
 $H_0: \mu_1 \leq \mu_2$

T
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At an $\alpha$ of 5%
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0
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47
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**Table 10: Wilcoxon's Signed Rank Test on COUNTSAT Instances (NFE)**

	TCQEA	UCQEA			Test Statistic	Critical Value
	<b>X<sub>1</sub></b>	<b>X<sub>2</sub></b>				
1	106	1152				
2	177	4907				
3	226	10462				
4	255	20863				
5	298	22300				
6	327	29220				
7	347	30092				
8	364	34312				
9	382	34483				
10	401	39618				
11	431	44297				
12	432	46058				
13	468	48732				
14	478	49330				
15	515	50003				
16	535	50050				
17	588	50050				
18	647	50050				

$\Sigma(+)$ 

0
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 $\Sigma(-)$ 

231
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 $N$ 

21
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Null Hypothesis  
 $H_0: \mu_1 \geq \mu_2$

T
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At an $\alpha$ of 5%
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0
---

68
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19	644	50050
20	843	50050
21	881	50050

Therefore, the same set of parameter has been used for solving instances of two well-known benchmark problems viz., MMDP and COUNTSAT problem. However, it was found that the parameter vector did not perform well on 0-1 Knapsack problem instances, so it was decided to again tune the QEA using the proposed method for 0-1 knapsack problem.

### C. 0-1 Knapsack problems

The 0-1 knapsack problem is a profit maximization problem, in which there are n items of different profit and weight available for selection. The selection is made to maximize the profit while keeping the weight of the selected items below the capacity of the knapsack. It is formulated as follows:

Given a set of n items and a knapsack of Capacity C, select a subset of the items to maximize the profit  $f(x)$ :

$$f(x) = \sum p_i x_i \quad (8)$$

subject to the condition

$$\sum w_i x_i < C \quad (9)$$

where  $x_i = (x_1 \dots x_n)$ ,  $x_i$  is 0 or 1,  $p_i$  is the profit of item  $i$ ,  $w_i$  is the weight of item  $i$ . If the  $i$ th item is selected for the knapsack,  $x_i = 1$ , else  $x_i = 0$ .

Eleven groups of randomly generated instances of (KP) which have been constructed to test the canonical and Tuned-QEA. In all instances the weights are uniformly distributed in a given interval. The profits are expressed as a function of the weights, yielding the specific properties of each group [46].

- i. **Uncorrelated data instances:** The profits,  $p_j$  and weights,  $w_j$  of the items are chosen randomly in [1, 1000] so there is no correlation between the profit and weight of an item. These instances represent situations where it can be safely assumed that there is no correlation between weight and profits of the items and are generally easy to solve, as there is a large variation between the profits and weights.
- ii. **Weakly correlated instances:** The weights  $w_j$  of the items are chosen randomly in [1, 1000] and the profits  $p_j$  are function of  $w_j$  i.e.  $p_j$  lies in  $[w_j - 100, w_j + 100]$  such that  $p_j \geq 1$ . Weakly correlated instances have relatively high correlation between the profit and weight of an item as the profit differs from the weight by only a few percent. These instances are quite realistic in management, as the return of an investment is mostly proportional to the sum invested with some random variations.
- iii. **Strongly correlated instances:** The weights  $w_j$  of the items are distributed in [1, 1000] and profit  $p_j = w_j + 100$ . These instances correspond to real-life situations where the return is proportional to the investment plus some fixed charge for each project. The strongly correlated instances are mostly hard to solve as they are *ill-conditioned* and Sorting is not of much help.
- iv. **Inverse strongly correlated instances:** The profits  $p_j$  of the items are distributed in [1, 1000] and weight  $w_j = p_j + 100$ . These instances are similar to strongly correlated instances, however, the fixed charge is negative.
- v. **Almost strongly correlated instances:** The weights  $w_j$  of the items are distributed in [1, 1000] and the profits  $p_j$  in  $[w_j + 98, w_j + 102]$ . These instances are type of fixed-charge problem with some randomness and have the properties of both strongly and weakly correlated instances.
- vi. **Subset sum instances:** The weights,  $w_j$ , of the items are randomly distributed in [1, 1000] and the profit  $p_j = w_j$ . These instances represent situation in which the profit of each item is equal (or proportional) to the weight so the goal is to obtain a filled knapsack.
- vii. **Uncorrelated instances with similar weights:** The weights of the items,  $w_j$ , are distributed uniformly in [100 000, 100 100] and the profits,  $p_j$  in [1, 1000]. These instances are similar to uncorrelated data instances but all the items have similar weights with large difference in profits.
- viii. **Spanner instances** span ( $v, m$ ): These instances are constructed from spanner set i.e. all the items are multiples of a very small set of items. The spanner instances span ( $v, m$ ) are defined by the size,  $v$ , of the spanner set, the multiplier limit,  $m$ , and the distribution (uncorrelated, weakly correlated, strongly correlated, etc.) of the items in the spanner set. The instances used in this work are generated as follows: A set of  $v=2$  items is generated with weights in the interval [1, 1000], and profits according to the strongly correlated distribution. The items  $(pk, wk)$  in the spanner set are normalized by dividing the profits and weights with  $m+1$ . The  $n$  items are then constructed, by repeatedly choosing an item  $(pk, wk)$  from the spanner set, and a multiplier,  $a$ , randomly generated in the interval [1, 10]. The constructed item has profit and weight  $(a*pk, a*wk)$ . Computational experiments have showed that the instances became harder to solve for smaller spanner sets [46], so the instances with strongly correlated span (2, 10) have been used in this work.

- ix. **multiple strongly correlated instances** mstr(  $k_1, k_2, d$  ) : They are constructed as a combination of two sets of strongly correlated instances, which have profits  $p_j := w_j + k_i$  where  $k_i, i = 1, 2$  is different for the two sets. The multiple strongly correlated instances mstr(  $k_1, k_2, d$  ) have been generated in this work as follows: The weights of the  $n$  items are randomly distributed in [1, 1000]. If the weight  $w_j$  is divisible by  $d=6$ , then we set the profit  $p_j := w_j + k_1$  otherwise set it to  $p_j := w_j + k_2$ . The weights  $w_j$  in the first group (i.e. where  $p_j = w_j + k_1$ ) will all be multiples of  $d$ , so that using only these weights can at most use  $d[c/d]$  of the capacity, therefore, in order to obtain a completely filled knapsack, some of the items from the second distribution will also be included. Computational experiments have shown that very difficult instances could be obtained with the Parameters mstr(300, 200, 6) [46].
- x. **profit ceiling instances** pceil( $d$ ) : These instances have profits of all items as multiples of a given parameter  $d$ . The weights of the  $n$  items are randomly distributed in [1, 1000], and their profits are set to  $p_j = d[w_j/d]$ . The parameter  $d$  has been experimentally chosen as  $d=3$ , as this resulted in sufficiently difficult instances [46].
- xi. **circle instances** circle( $d$ ) : These instances have the profit of their items as function of the weights form an arc of a circle (actually an ellipsis). The weights are uniformly distributed in [1, 1000] and for each weight  $w_i$  the corresponding profit is chosen as  $p_i = d\sqrt{2000^2 - (w_i - 2000)^2}$ . Experimental results have showed in [Pis2004] that difficult instances appeared by choosing  $d=2/3$  which was chosen for testing in this work.

The set of parameter vector that was used for successfully solving instances of two well-known benchmark problems viz., MMDP and COUNTSAT problem, did not perform well on 0-1 Knapsack problem instances, so it was decided to again tune the QEA using the proposed method on Strongly Correlated instance with weight of the items,  $w_i$ , uniformly distributed between [1, 1000] with  $p_i = w_i + 100$  and capacity,  $C$ , as half the total weight of all the available 1000 items.

The initial range of values for parameters in QEA used for tuning is given in Table 11. The parameter range for magnitude of rotation angles for ( $\theta_1$  to  $\theta_8$ , except  $\theta_3$  &  $\theta_5$ ) is 0.0 to 0.001 $\pi$ , and for  $\theta_3$  &  $\theta_5$  is 0.0 to 0.05  $\pi$ , which is large as compared to the range suggested by [14]. The direction of rotation depends on the sign of  $\alpha$ ,  $\beta$  and relative fitness as per Table 1. The range for population size is 5 to 100. The range for no. of groups is 1 to 10. The range for global migration is 1 to 200, which is two times the value suggested by [14]. The maximum number of generations was limited to ten thousand. The change in value of each parameter during the tuning process is depicted in Table 12 and shown in Fig. 57 to 67. There were four rounds of exploration and three rounds of exploitation. It can be observed that between round two and round three of the exploration stages, there was slight decrease in the best objective function value. On further, exploration in round four, there was only slight increase in the best objective function value, so it was decided to stop further exploration and start the exploitation so as to further search within the vicinity of the Best parameter values found so far. After the third round of the exploitation stage, there was no improvement in the best objective function value, so it was decided to stop further tuning.

The parameter value for  $\theta_1$  and  $\theta_2$  has changed during exploration stage but remained constant during exploitation stage. The parameter value for  $\theta_3$  changed during exploration and first two round of exploitation stage but did not change in the last round. The parameter value for  $\theta_4$  initially remained unchanged during first two rounds of exploration and then changed during last round of exploration and first round of exploitation stage, but remained unchanged in the last two rounds of exploitation. The parameter value for  $\theta_5$  initially remained unchanged then decreased a little before increasing during exploration. It kept decreasing during exploitation stage. The parameter value for  $\theta_6$  initially remained unchanged, then kept changing during exploration and exploitation stage. The parameter value for  $\theta_7$  remained almost unchanged. The parameter value for  $\theta_8$  decreased during initial part of exploration and then increased before becoming constant during exploitation stage. The parameter value for Population Size kept changing during exploration and initial round of exploitation stage, but did not change in the last round. The parameter value for no. of groups initially increased and then remained unchanged during exploration and then increased in first round of exploitation stage. The parameter value for migration changed during exploration and remained unchanged during first two rounds of exploitation stage, but increased in the last round. The final value of each parameter is given in Table 12 in the last row. The convergence graph given in Fig. 68 shows fast convergence to near optimal within 1500 generations.

TABLE 11  
INITIAL RANGE OF PARAMETERS

Parameter	$\theta_1$ (* $\pi$ )	$\theta_2$ (* $\pi$ )	$\theta_3$ (* $\pi$ )	$\theta_4$ (* $\pi$ )	$\theta_5$ (* $\pi$ )	$\theta_6$ (* $\pi$ )	$\theta_7$ (* $\pi$ )	$\theta_8$ (* $\pi$ )	Pop size	No. of Groups	Global Migration
Lower Limit	0	0	0	0	0	0	0	0	5	1	1
Upper Limit	0.001	0.001	0.05	0.001	0.05	0.001	0.001	0.001	100	10	200

TABLE 12  
BEST PARAMETER VECTOR (PIVOT) DURING TUNING PROCESS

Iter. No.	Best Pivot Parameter Value											Best OFV
	$\theta_1$ ( $*$ $\pi$ )	$\theta_2$ ( $*$ $\pi$ )	$\theta_3$ ( $*$ $\pi$ )	$\theta_4$ ( $*$ $\pi$ )	$\theta_5$ ( $*$ $\pi$ )	$\theta_6$ ( $*$ $\pi$ )	$\theta_7$ ( $*$ $\pi$ )	$\theta_8$ ( $*$ $\pi$ )	Pop. Size	No. of Grp	Glb. Mig.	
Explor. – 1	0.0009	0.0005	0.0438	0.0006	0.0112	0.0007	0.0005	0.0007	20	3	55	314869.9892
Explor. – 2	0.0002	0.0007	0.0404	0.0006	0.0112	0.0007	0.0005	0.0005	77	7	132	315169.9709
Explor. – 3	0.0004	0.0004	0.0473	0.0006	0.0079	0.0004	0.0005	0.0003	52	8	134	315169.9404
Explor. – 4	0.00058	0.0004	0.02475	0.0003	0.0273	0.00073	0.00045	0.00088	80	8	117	315169.9770
Exploit. – 5	0.00036	0.00026	0.01949	0.00042	0.02108	0.00086	0.00070	0.00088	68	8	119	315369.9891
Exploit. – 6	0.00036	0.00026	0.01423	0.0003	0.01485	0.00073	0.00070	0.00088	80	10	119	315469.9830
Exploit. – 7	0.00035	0.00026	0.01423	0.0003	0.01405	0.00070	0.00067	0.00088	80	10	197	315469.9830

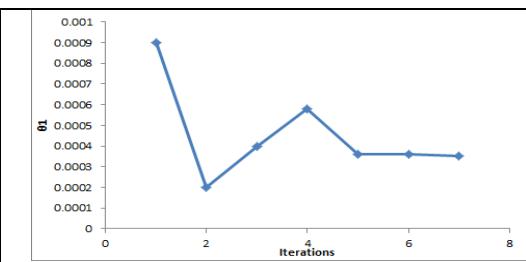


Fig 57. Change in  $\theta_1$  value during Tuning Process

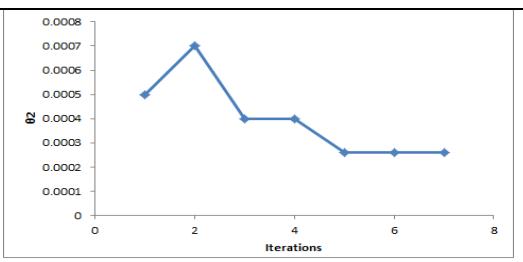


Fig 58. Change in  $\theta_2$  value during Tuning Process

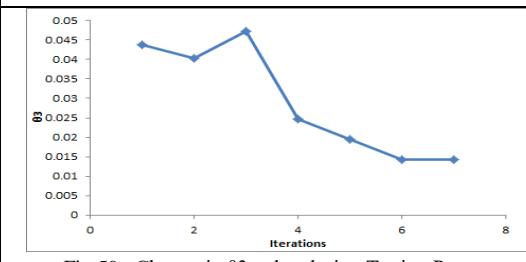


Fig 59. Change in  $\theta_3$  value during Tuning Process

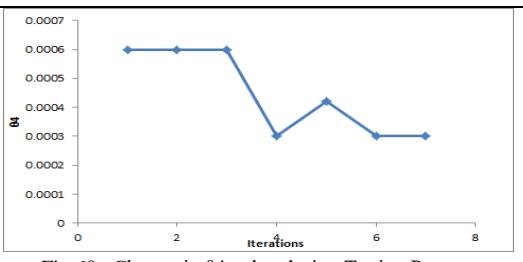
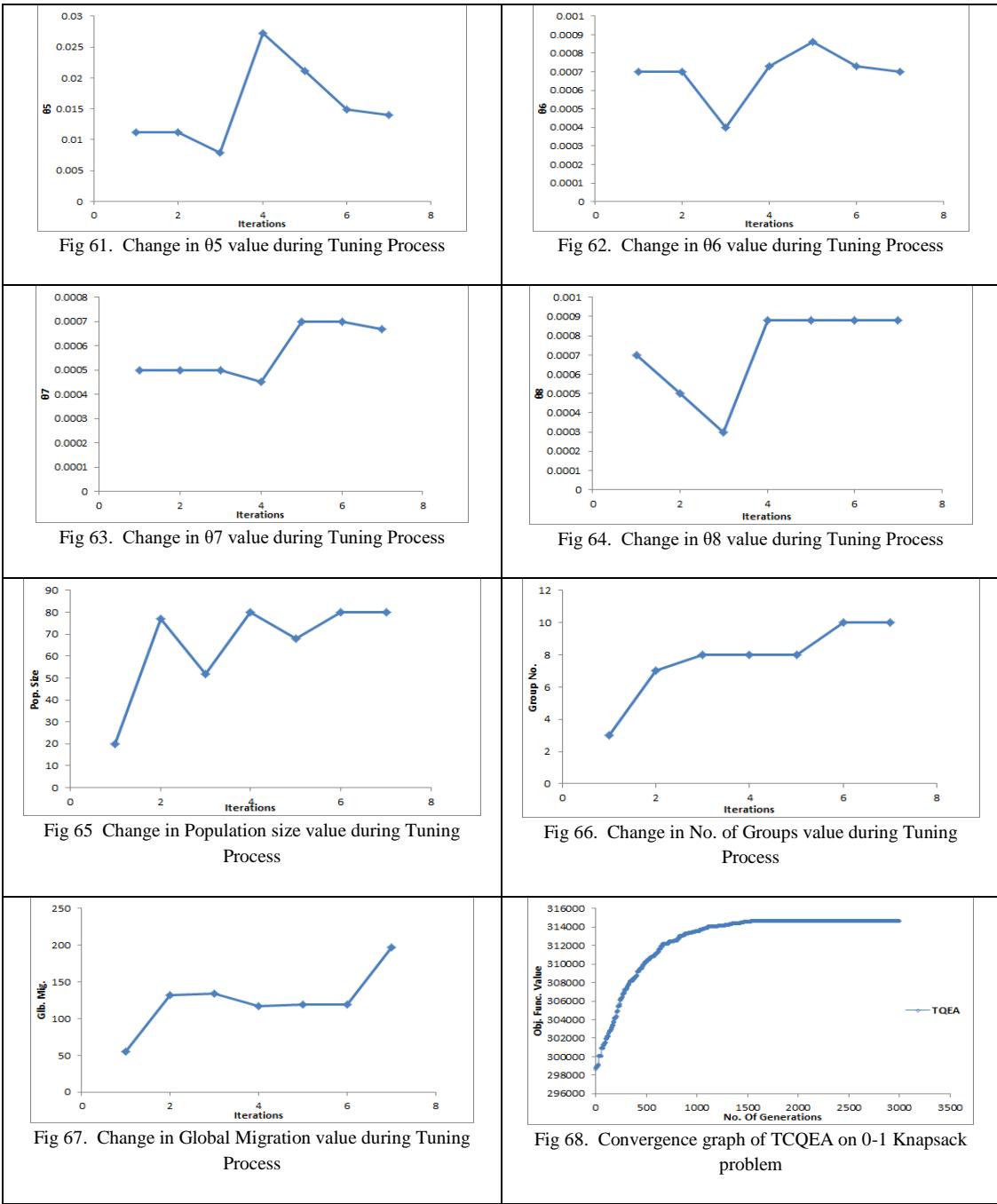


Fig 60. Change in  $\theta_4$  value during Tuning Process



The deviation from best objective function value found so far, for large variation in parameter setting, which is computed from the best results of fifty runs from the first iteration of exploration stage is **737.35** whereas the deviation from optimal for small variation in parameter setting, which is computed from the best results of twenty seven runs from the first iteration of exploitation stage is **263.6**. Therefore, the tuned QEA is relatively robust to small variations in parameter vector, however, it is quite unstable for large variation in parameter set, thus, justifying the effort put in tuning the parameter set of QEA for 0-1 Knapsack problem.

A comparative study was performed between parameter Tuned QEA (with population size as fifty and no. of groups as ten) and Canonical QEA with Parameters given in Table 2 on eleven instances of 0-1 knapsack problem instances. The population size of both the algorithms, Tuned QEA and Canonical QEA, was made equal to make the comparison fair between them. The results are given in Tables 13 to 111. Seven different problems were randomly created by varying the number of items for each of the eleven instances from 100 to 10,000. Each of these seven problems further had five instances each, generated by changing the capacity of the knapsack from 1% to 50% of the total capacity of all the items. Therefore, 35 problem instances were solved for each of the eleven instances, so total number of problems instances solved with same set of parameters is 385.

Thirty independent runs were made on each problem and the comparison has been made on Best, Worst, Average, Median of objective function value and Average number of generations. The maximum number of generations was limited to one thousand. The Tuned QEA (TCQEA) has either been able to match the performance of Canonical QEA (UCQEA) or beat it in all the 385 instances, when compared on the objective function value. The canonical QEA was able to match Tuned QEA only in small size knapsack problems on objective function value, but when compared on average generations, the Tuned-QEA was found to be much faster than canonical QEA, therefore either canonical QEA loses or is found to be relatively inefficient as compared to tuned QEA.

#### **1) Uncorrelated Data Instances:**

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Tables 13 to 19. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and the capacity of knapsack is 1% of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in all the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 69 to 74, which also compared the speed of convergence of UCQEA and TCQEA.

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for problem instances having No. of Items as 200 and 5000 and capacity as 1%, 5% and 10% of the total weight of items available for selection. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

TABLE 13

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	507.89	499.36	502.31	502.29	<b>1.80</b>	191866.67	<b>517.85</b>	<b>502.41</b>	<b>510.97</b>	<b>512.95</b>	4.60	<b>137082.00</b>
5%	<b>2496.59</b>	2461.61	2482.80	2481.55	7.60	186953.33	2491.60	<b>2475.98</b>	<b>2483.80</b>	<b>2485.43</b>	<b>5.78</b>	<b>138791.40</b>
10%	<b>4938.22</b>	4898.21	4920.25	4920.50	9.69	<b>142001.67</b>	4938.19	<b>4903.21</b>	<b>4925.10</b>	<b>4927.70</b>	<b>8.36</b>	188205.60
20%	9786.41	<b>9756.41</b>	9772.07	<b>9776.25</b>	8.54	<b>194318.33</b>	<b>9789.48</b>	9756.20	<b>9772.62</b>	9771.91	<b>8.27</b>	198214.50
50%	24261.10	<b>24236.34</b>	24252.62	<b>24255.87</b>	6.38	196908.33	<b>24266.15</b>	24231.35	<b>24253.74</b>	24255.28	<b>6.34</b>	<b>171847.50</b>

TABLE 14

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	<b>1102.11</b>	1053.54	1077.07	1078.13	11.57	343616.67	1098.55	<b>1063.50</b>	<b>1084.50</b>	<b>1083.49</b>	<b>9.59</b>	<b>212731.20</b>
5%	5223.04	5172.83	5201.03	5200.33	<b>10.85</b>	<b>230640.00</b>	<b>5228.04</b>	<b>5173.01</b>	<b>5205.08</b>	<b>5207.76</b>	15.35	241008.90
10%	10331.10	10271.12	10305.53	10305.94	<b>13.39</b>	<b>247538.33</b>	<b>10340.70</b>	<b>10276.11</b>	<b>10311.11</b>	<b>10311.08</b>	14.05	305507.40
20%	20482.24	20432.21	20464.63	<b>20467.20</b>	<b>11.41</b>	298075.00	<b>20497.26</b>	<b>20437.15</b>	<b>20466.52</b>	20464.51	13.05	<b>272794.50</b>
50%	50850.74	50810.73	50834.47	50835.44	<b>9.76</b>	269708.33	<b>50855.76</b>	<b>50815.72</b>	<b>50837.51</b>	<b>50840.51</b>	9.99	<b>224403.30</b>

TABLE 15

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	2631.25	<b>2571.23</b>	2602.28	2603.73	<b>16.63</b>	390638.33	<b>2655.76</b>	2556.16	<b>2608.91</b>	<b>2611.28</b>	24.55	<b>326587.80</b>
5%	<b>12706.48</b>	<b>12616.53</b>	<b>12657.26</b>	12658.99	<b>21.45</b>	<b>416428.33</b>	12701.51	12601.45	12656.91	<b>12660.85</b>	24.26	<b>371804.40</b>
10%	25143.04	25048.00	25104.36	25103.00	22.75	425583.33	<b>25158.05</b>	<b>25078.03</b>	<b>25113.44</b>	<b>25110.47</b>	<b>21.75</b>	<b>420499.20</b>
20%	<b>49946.07</b>	<b>49841.10</b>	<b>49895.12</b>	<b>49899.64</b>	<b>22.34</b>	448365.00	49931.11	49840.91	49887.68	49885.62	26.49	<b>428679.90</b>
50%	123990.07	123925.31	<b>123962.55</b>	<b>123965.24</b>	<b>14.77</b>	432330.00	<b>124000.31</b>	<b>123935.23</b>	123961.90	123961.07	17.16	<b>362910.90</b>

TABLE 16

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	5233.84	<b>5133.33</b>	5185.33	5180.89	<b>25.93</b>	<b>450305.00</b>	<b>5248.25</b>	5121.15	<b>5193.84</b>	<b>5197.73</b>	31.68	405474.30
5%	<b>25446.03</b>	<b>25277.00</b>	<b>25359.71</b>	<b>25356.99</b>	30.71	474241.67	25376.97	25256.99	25333.62	25341.06	<b>28.18</b>	<b>466111.80</b>
10%	<b>50408.93</b>	50263.95	<b>50351.14</b>	50348.74	<b>34.69</b>	<b>478390.00</b>	50400.43	<b>50273.48</b>	50341.67	<b>50348.97</b>	34.80	478783.80
20%	100173.00	<b>100047.85</b>	<b>100112.08</b>	<b>100111.67</b>	<b>34.56</b>	491388.33	<b>100182.87</b>	100022.97	100109.90	100110.41	39.47	<b>466085.40</b>
50%	248944.97	248869.64	<b>248905.32</b>	<b>248912.39</b>	<b>21.40</b>	479845.00	<b>248954.75</b>	248834.98	248890.56	248887.31	30.75	<b>441361.80</b>

TABLE 17

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	10480.13	<b>10341.12</b>	10404.56	10402.48	34.87	486423.33	<b>10482.77</b>	10351.11	<b>10418.22</b>	<b>10411.76</b>	<b>34.40</b>	<b>482552.40</b>
5%	<b>51157.67</b>	<b>50971.89</b>	<b>51085.41</b>	<b>51094.35</b>	<b>44.08</b>	<b>496360.00</b>	51103.69	50909.93	51037.56	51045.01	50.84	497102.10
10%	101595.65	<b>101461.34</b>	<b>101537.05</b>	<b>101543.73</b>	<b>37.11</b>	495966.67	<b>101606.22</b>	101422.20	101514.86	101518.64	57.63	<b>493762.50</b>
20%	202225.04	<b>202012.11</b>	202114.84	<b>202110.81</b>	<b>51.73</b>	<b>496485.00</b>	<b>202247.18</b>	201933.87	<b>202114.87</b>	202105.43	80.51	497577.30
50%	503210.64	<b>503031.93</b>	<b>503118.04</b>	<b>503116.78</b>	45.90	493806.67	<b>503201.96</b>	503020.75	503114.95	503114.84	<b>45.70</b>	<b>491366.70</b>

TABLE 18

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	25806.44	25670.00	25752.66	25750.26	<b>37.40</b>	497273.33	<b>25864.24</b>	<b>25678.16</b>	<b>25774.55</b>	<b>25779.08</b>	39.19	<b>495181.50</b>

5%	127217.29	<b>126962.57</b>	<b>127083.98</b>	<b>127079.93</b>	<b>74.64</b>	<b>497981.67</b>	<b>127230.99</b>	126905.62	127068.43	127065.11	90.41	498316.50
10%	253198.19	<b>252902.52</b>	253011.73	253003.24	<b>79.37</b>	498426.67	<b>253360.55</b>	252886.99	<b>253086.30</b>	<b>253059.34</b>	124.34	<b>497966.70</b>
20%	504556.96	504101.91	504405.49	504423.50	<b>99.99</b>	<b>497416.67</b>	<b>504909.64</b>	<b>504200.93</b>	<b>504612.95</b>	<b>504613.31</b>	159.22	497768.70
50%	1257566.72	1257159.23	1257350.48	1257341.09	115.53	498088.33	<b>1257654.00</b>	<b>1257256.55</b>	<b>1257471.27</b>	<b>1257488.44</b>	<b>110.49</b>	<b>497412.30</b>

TABLE 19

COMPARATIVE STUDY BETWEEN UCQEA AND TCQEA ON 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON UNCORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	51480.36	51263.46	51356.21	51351.30	<b>41.04</b>	<b>497313.33</b>	<b>51639.86</b>	<b>51337.12</b>	<b>51440.05</b>	<b>51443.58</b>	66.73	497828.10
5%	254514.46	<b>254188.82</b>	254310.69	254299.47	<b>85.19</b>	<b>497960.00</b>	<b>254659.32</b>	254149.05	<b>254420.41</b>	<b>254430.36</b>	128.50	498075.60
10%	507210.74	506788.58	506998.11	506981.74	<b>109.10</b>	<b>498386.67</b>	<b>507610.71</b>	<b>506852.40</b>	<b>507301.69</b>	<b>507325.85</b>	190.47	498613.50
20%	1012173.75	<b>1011518.17</b>	1011774.31	1011761.50	<b>152.75</b>	<b>497621.67</b>	<b>1012822.26</b>	1011501.42	<b>1012375.03</b>	<b>1012419.11</b>	313.10	497983.20
50%	2524717.33	2523997.82	2524404.46	2524434.51	<b>152.20</b>	498496.67	<b>2525302.57</b>	<b>2524211.82</b>	<b>2524793.80</b>	<b>2524818.19</b>	243.62	<b>497864.40</b>

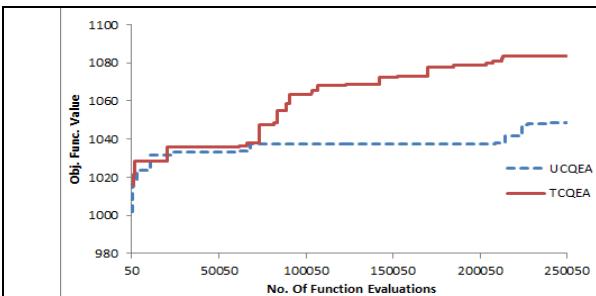


Fig 69. Convergence Graph of UCQEA and TCQEA on 0-1 Knapsack problem with Uncorrelated Data Instances having No. of Items as 200 and Capacity as 1% of Total Weight

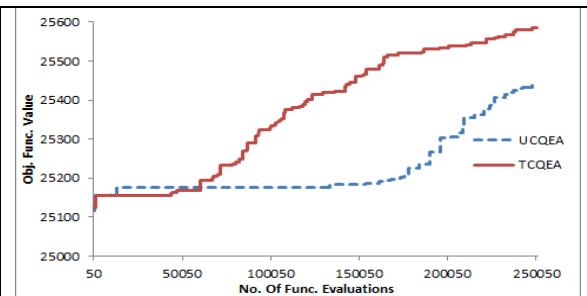


Fig 70. Convergence Graph of UCQEA and TCQEA on 0-1 Knapsack problem with Uncorrelated Data Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

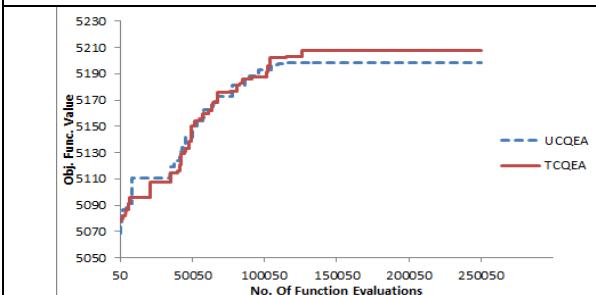


Fig 71. Convergence Graph of UCQEA and TCQEA on 0-1 Knapsack problem with Uncorrelated Data Instances having No. of Items as 200 and Capacity as 5% of Total Weight

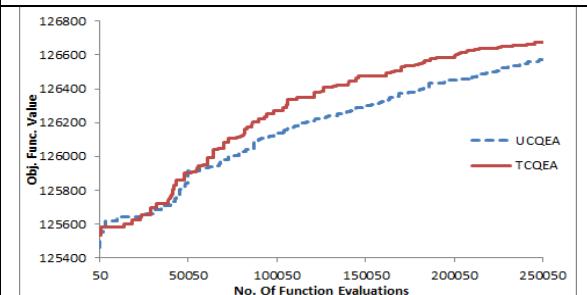


Fig 72. Convergence Graph of UCQEA and TCQEA on 0-1 Knapsack problem with Uncorrelated Data Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

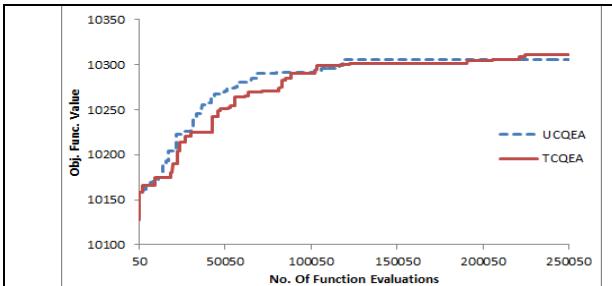


Fig 73. Convergence Graph of UCQEA and TCQEA on 0-1 Knapsack problem with Uncorrelated Data Instances having No. of Items as 200 and Capacity as 10% of Total Weight

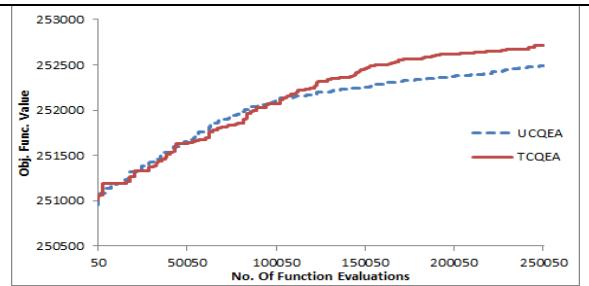


Fig 74. Convergence Graph of UCQEA and TCQEA on 0-1 Knapsack problem with Uncorrelated Data Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 13 to 19, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Uncorrelated Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 20, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 303 at significance level of  $\alpha = 5\%$ . The sample size  $n=41$  is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be rejected safely. This indicates the success of the proposed tuning method for tuning QEA on problems like Uncorrelated Knapsack problem.

**Table 20: Wilcoxon's Signed Rank Test on Uncorrelated KP Instances (OFV)**

	TCQEA	UCQEA		Test Statistic	Critical Value
1	52	52			
2	257.4	254.9			
3	506.4	499.9			
4	1001.7	986.6			
5	1983.4	1966.4			
6	2475.9	2461.8			
7	116.1	113.3			
8	539.2	523.3			
9	1056.6	1033.9			
10	2094.4	2052			
11	4146.6	4107.7			
12	5175.5	5145.6			
13	270	261.6			
14	1295	1257.5			
15	2559.5	2499.8			
16	5063.9	4981.2			
17	10058.7	9969.8			
18	12545.4	12473.3			
19	527.2	513.6			
20	2574	2507.7			
21	5098.1	4999.3			
22	10120.4	9975.7			
23	20104.5	19969.3			
24	25094.5	24975.6			

$\Sigma(+)$	861	Null Hypothesis	$T$	At an $\alpha$ of 5%
$\Sigma(-)$	0	$H_0: \mu_1 \leq \mu_2$	0	303

$n$	41
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**For Large Samples ( $n > 25$ )**

Test Statistic

$z$	-5.57857	$E[T]$	430.5
$\sigma(T)$	77.17027		

Null Hypothesis	$p$ -value	At an $\alpha$ of
$H_0: \mu_1 \leq \mu_2$	0.0000	5%
		<b>Reject</b>

25	1060.5	1024.5
26	5157.9	5052.4
27	10224.5	10083.2
28	20349	20140.5
29	40512.4	40293.9
30	50577	50393.6
31	2601.1	2537
32	12772.8	12599.7
33	25415.9	25169.4
34	50636.4	50300.5
35	100973	100620
36	126134	125816
37	5167.7	5073.3
38	25521.8	25276.7
39	50838.7	50509.4
40	101434	100970
41	202490	201959
42	252953	252497

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 21, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 778 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n = 42$  is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 1.000, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table. 20. This indicates the success of the proposed tuning method for tuning QEA on problems like Uncorrelated Knapsack instance even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 21: Wilcoxon's Signed Rank Test on Uncorrelated KP Instances (Av. Gen.)**

	TCQEA	UCQEA			Test Statistic	Critical Value
1	2.3	2.5				
2	483.3	428.3				
3	557.9	494.8				
4	548.3	541.6				
5	697	918.4				
6	710.6	922.3				
7	366.8	487				
8	533.5	514.9				
9	723.1	531.8				
10	831.6	659				
11	867.9	941.6				
12	906.9	966.8				
13	487.9	519.5				
14	817.8	431.1				
15	908.5	551.2				
16	896	524				
17	960	959.4				
			$\Sigma(+)$	778	Null Hypothesis	
			$\Sigma(-)$	125	$H_0: \mu_1 \geq \mu_2$	778
			$n$	42		319
<b>For Large Samples (<math>n &gt; 25</math>)</b>						
Test Statistic						
			$z$	-4.08245	$E[T]$	451.5
					$\sigma(T)$	79.97655907
					At an $\alpha$ of	
			Null Hypothesis	$p$ -value	5%	
			$H_0: \mu_1 \geq \mu_2$	1.000		

18	963.2	964.3
19	664.6	464.6
20	917.3	434.5
21	933.1	470.8
22	952.9	684.9
23	964.1	949.9
24	962.4	974.1
25	850.2	520.5
26	957.2	466.5
27	959.8	427.8
28	977.4	614.9
29	972.3	957.1
30	975.4	965.8
31	910.8	571.2
32	952.3	626.7
33	963.9	414
34	982.3	581
35	982	959.8
36	975.8	972.1
37	901.3	468.9
38	961.2	541.2
39	973.4	560.4
40	978.8	626.8
41	969.9	935.3
42	978.9	971

## 2) Weakly correlated instances

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Tables 22 to 28. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 or 200 and the capacity of knapsack is 1% or 5% of the total weight of all the items available for selection. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in all the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 75 to 80, which also compared the speed of convergence of UCQEA and TCQEA.

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for problem instances having No. of Items as 200 and 5000 and capacity as 1%, 5% and 10% of the total weight of items available for selection. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in most of the graphs. Generally, the difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

TABLE 22  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON WEAKLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	<b>782.1</b>	723.0	754.5	753.2	21.8	216200	<b>782.1</b>	<b>779.1</b>	<b>782.0</b>	<b>782.1</b>	0.5	<b>88998</b>
5%	<b>3328.6</b>	<b>3235.0</b>	3287.5	3290.6	25.2	101503	<b>3328.6</b>	3227.7	<b>3296.3</b>	<b>3294.7</b>	23.0	<b>77402</b>
10%	<b>6201.2</b>	6086.6	6147.9	6153.6	28.1	<b>69562</b>	<b>6201.2</b>	<b>6099.3</b>	<b>6170.9</b>	<b>6170.7</b>	23.8	100581
20%	<b>11713.5</b>	11625.7	11679.8	11681.0	<b>24.8</b>	97480	<b>11713.5</b>	<b>11631.2</b>	<b>11685.2</b>	<b>11693.9</b>	26.6	<b>93535</b>

50%	<b>27204.7</b>	27128.8	<b>27180.7</b>	<b>27182.4</b>	18.0	<b>118498</b>	<b>27204.7</b>	<b>27143.1</b>	<b>27180.0</b>	27179.3	18.6	144078
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TABLE 23

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON WEAKLY CORRELATED DATA INSTANCES

% of Total	Canonical QEA						Tuned-QEA					
	Weight	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std
1%	<b>1648.8</b>	1523.3	1588.8	1585.1	34.4	358858	<b>1648.8</b>	<b>1540.2</b>	<b>1612.6</b>	<b>1613.4</b>	<b>32.8</b>	<b>144005</b>
5%	6602.5	6361.0	6499.2	6508.2	65.5	197923	<b>6608.6</b>	<b>6457.6</b>	<b>6543.1</b>	<b>6544.9</b>	<b>44.1</b>	<b>174926</b>
10%	12245.5	12034.7	12171.6	12180.3	47.9	<b>193305</b>	<b>12261.4</b>	<b>12122.7</b>	<b>12198.2</b>	<b>12198.7</b>	<b>33.3</b>	205996
20%	23106.0	22898.1	23018.6	23018.7	51.3	<b>187658</b>	<b>23115.2</b>	<b>22948.3</b>	<b>23051.4</b>	<b>23057.4</b>	<b>37.3</b>	277952
50%	53795.7	<b>53694.2</b>	53749.7	<b>53754.8</b>	<b>25.8</b>	<b>320998</b>	<b>53811.6</b>	53653.6	<b>53753.2</b>	53753.6	32.8	329522

TABLE 24

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON WEAKLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	4372.8	4071.0	4240.6	4243.5	71.4	356395	4400.7	4151.8	4276.1	4271.0	64.2	233756
5%	16258.0	16013.3	16150.5	16148.6	59.7	336637	16331.0	15972.5	16209.7	16228.2	80.6	339768
10%	30119.9	29732.4	29938.1	29964.0	104.0	360972	30153.7	29773.8	30004.1	30021.3	81.0	372276
20%	56861.2	56346.1	56620.4	56649.1	97.9	407635	56799.8	56495.2	56658.5	56660.9	79.8	440111
50%	131718.1	131493.6	131637.2	131644.9	62.4	444695	131732.3	131539.9	131657.0	131671.0	49.5	475599

TABLE 25

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON WEAKLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	8565	8018	8387	8404	137	460040	8842	8281	8529	8497	139	425614
5%	32904	32190	32614	32650	183	485763	32996	32299	32752	32775	153	489077
10%	60978	60284	60554	60555	144	484903	60932	60412	60723	60753	124	491363
20%	114733	114055	114324	114303	137	492153	114772	114116	114388	114378	131	493660
50%	266815	266168	266467	266463	129	492188	266565	266140	266359	266362	108	494947

TABLE 26

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON WEAKLY CORRELATED DATA INSTANCES

% of Total	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen

1%	16779	15715	16345	16391	264	495787	17176	16407	16844	16855	188	494383
5%	65934	64417	65158	65138	323	497398	66129	65090	65591	65576	225	497782
10%	122227	120955	121586	121646	314	496470	122809	121756	122248	122224	290	497373
20%	231199	230263	230724	230721	216	497360	231798	230861	231351	231329	310	496917
50%	541536	540028	540727	540715	341	497678	541728	540490	540882	540827	266	498155

TABLE 27  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON WEAKLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	37315	35030	36159	36215	473	498163	39218	37691	38496	38527	387	498185
5%	156536	154255	155626	155692	555	498043	158988	156515	157849	157806	681	498115
10%	297691	295154	296500	296501	684	498063	301216	297751	299327	299361	757	499125
20%	571675	567385	569852	569887	999	498507	575293	571320	573574	573746	1047	498713
50%	1350656	1346669	1348631	1348604	1017	498580	1353792	1349705	1351542	1351552	1068	498251

TABLE 28  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON WEAKLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	68351	65701	67102	67099	657	497990	72852	70102	71579	71709	729	498350
5%	304149	300038	301944	301912	800	498317	308068	302875	306375	306446	1134	498647
10%	584452	579449	581863	581856	1385	498583	593064	584387	587999	587925	2085	498689
20%	1129281	1120745	1125317	1125765	1838	498608	1137725	1128635	1133825	1133602	2268	499208
50%	2685985	2675108	2680616	2679893	2922	498768	2691332	2679382	2685224	2685211	3002	499300

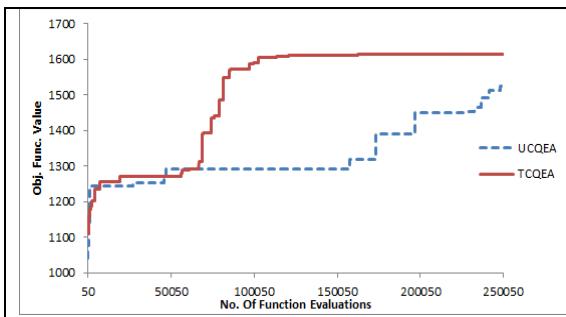


Fig 75. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Weakly correlated Data Instances having No. of Items as 200 and Capacity as 1% of Total Capacity

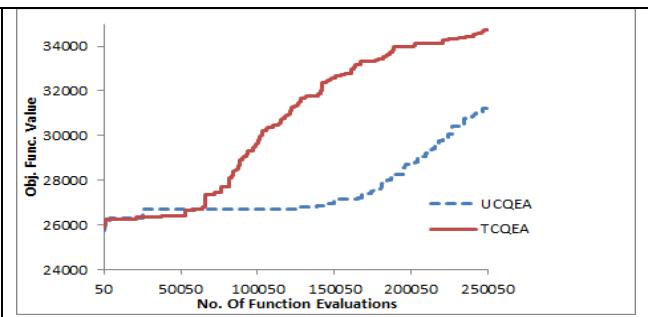


Fig 76. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Weakly correlated Data Instances having No. of Items as 5000 and Capacity as 1% of Total Capacity

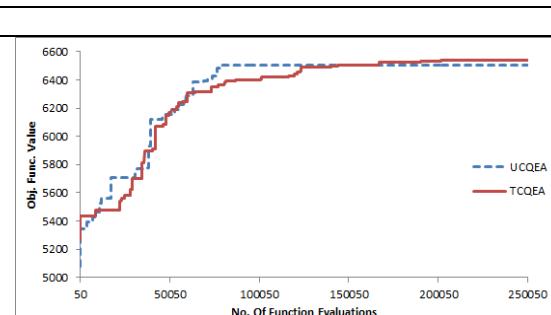


Fig 77. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Weakly correlated Data Instances having No. of Items as 200 and Capacity as 5% of Total Capacity

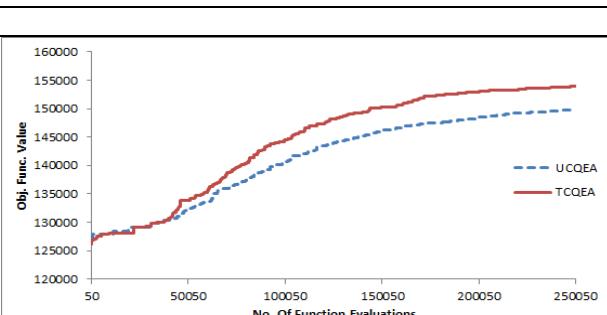


Fig 78. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Weakly correlated Data Instances having No. of Items as 5000 and Capacity as 5% of Total Capacity

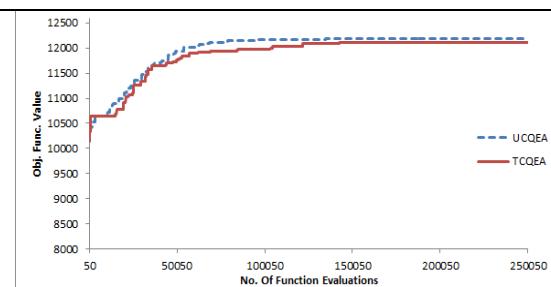


Fig 79. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Weakly correlated Data Instances having No. of Items as 200 and Capacity as 10% of Total Capacity

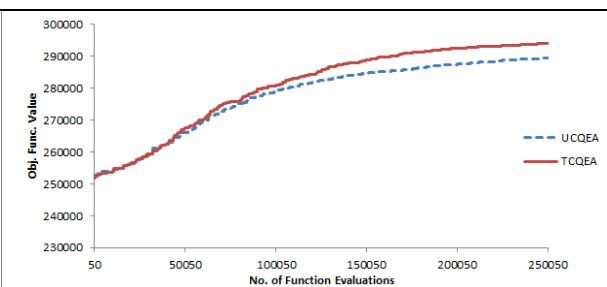


Fig 80. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Weakly correlated Data Instances having No. of Items as 5000 and Capacity as 10% of Total Capacity

In order to confirm the findings in Table 22 to 28, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Weakly Correlated Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 29, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is 37 and less than the critical value of 303 at significance level of  $\alpha = 5\%$ . The sample size  $n=41$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be rejected safely. This indicates the success of the proposed tuning method for tuning QEA on problems like Weakly Correlated Knapsack problem.

**Table 29: Wilcoxon's Signed Rank Test on Weakly Correlated KP Instances (OFV)**

	TCQEA	UCQEA			
1	X <sub>1</sub>	X <sub>2</sub>			
2	35	35			
3	424	411			
4	777	723			
5	1441	1306			
6	2669	2543			
7	3265	3218			
8	186	182			
9	838	726			
10	1560	1308			
11	2879	2390			
	5309	4824			

			Null Hypothesis	p-value	5%
12	6475	6105			
13	706	455			
14	2331	1556			
15	4070	2895			
16	7071	5488			
17	12742	11123			
18	15533	14057			
19	1295	741			
20	4429	2911			
21	7571	5530			
22	13470	10722			
23	24759	21691			
24	30210	27390			
25	2448	1311			
26	7876	5618			
27	13933	10856			
28	25546	21238			
29	47847	42886			
30	58845	54073			
31	4511	2968			
32	16971	13551			
33	31457	26474			
34	59526	52200			
35	114153	104956			
36	141200	132070			
37	7466	5693			
38	31341	26575			
39	59704	52387			
40	114723	103683			
41	221941	207979			
42	252953	260839			

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 30, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 741 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n= 42$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.9999, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table 29. This indicates the success of the proposed tuning method for tuning QEA on problems like Weakly Correlated Knapsack instance even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 30: Wilcoxon's Signed Rank Test on Weakly Corr. KP Instances (Av. Gen.)**

	TCQEA	UCQEA	$\Sigma(+)$	$\Sigma(-)$	Null Hypothesis		Test Statistic	Critical Value
					741	162		
1	2	2						
2	336	193						
3	349	582						
4	432	718						
5	562	947						
			$n$	42				

6	471	962
7	360	303
8	483	559
9	690	633
10	725	684
11	848	959
12	889	973
13	673	402
14	900	523
15	922	476
16	954	631
17	961	954
18	977	970
19	833	570
20	953	540
21	977	532
22	973	804
23	981	965
24	985	979
25	937	529
26	979	511
27	976	585
28	977	567
29	975	974
30	983	980
31	972	616
32	966	565
33	983	489
34	983	698
35	987	978
36	981	980
37	936	439
38	969	528
39	975	579
40	982	704
41	987	961
42	979	976

For Large Samples ( $n > 25$ )	
Test Statistic	
$z$	-3.61981
$E[T]$	451.5
$\sigma(T)$	79.97655907
At an $\alpha$ of	
Null Hypothesis	$p$ -value
$H_0: \mu_1 \geq \mu_2$	0.9999
	5%

### 3) *Strongly correlated instances:*

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Tables 31 to 37. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 to 500 and the capacity of knapsack is 0.1 % or 0.5% of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in all the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 81to 86, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 31

### TABLE 3.1 COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON STRONGLY CORRELATED DATA INSTANCES

Comparative Results for 0-1 Knapsack Problem with No. of Items 100 on Strongly Correlated Data Instances												
% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen

1%	1127	976	1058	1062	29	152248	1278	1273	1277	1278	1	110642
5%	4592	4291	4485	4491	74	104033	4592	4392	4527	4492	55	80540
10%	8082	7783	7959	7983	73	85902	8082	7883	7976	7983	52	85421
20%	14166	13966	14082	14066	69	71273	14167	14066	14143	14166	43	89100
50%	31016	30816	30899	30916	46	33372	31016	30816	30916	30916	26	60997

TABLE 32

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	3004	2603	2927	3003	116	297800	3004	2803	2983	3003	48	142540
5%	9618	9218	9434	9418	115	101815	9618	9418	9541	9518	77	155826
10%	16534	16136	16356	16336	92	109627	16536	16336	16419	16435	65	112768
20%	28972	28472	28749	28772	119	132833	29072	28572	28859	28872	111	107425
50%	64081	63681	63934	63931	97	63392	64180	63781	64011	63981	75	92869

TABLE 33

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	7246	6346	6893	6946	250	343017	7246	6846	7093	7146	128	238841
5%	23032	22331	22765	22732	167	245223	23331	22731	22978	22932	153	233218
10%	40063	39163	39646	39663	241	249597	40163	39363	39893	39913	174	260588
20%	71026	70326	70706	70726	185	220642	71226	70626	70976	71026	155	199947
50%	157315	156715	157052	157115	154	151160	157515	156915	157222	157215	136	169419

TABLE 34

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	14199	12813	13500	13513	359	432160	14313	13213	13820	13813	284	377418
5%	45467	44267	45017	45117	343	369815	45867	44865	45457	45467	268	339676
10%	79534	78434	79021	79034	325	391293	79834	78534	79347	79384	360	380853
20%	141768	140568	141164	141168	355	347170	142168	141168	141635	141618	237	319757
50%	315070	314070	314513	314520	263	292710	315470	314570	314999	314970	247	282200

TABLE 35

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	26636	24536	26044	26184	550	485218	27936	25936	27021	27136	488	464323
5%	90580	88478	89590	89611	561	482052	91681	88681	90344	90326	563	476966
10%	159060	155961	157833	157861	558	459438	159561	156934	158430	158411	616	460809
20%	284223	282378	283328	283343	495	474455	285023	283123	284083	284023	486	429360
50%	634507	632607	633671	633707	386	411388	635007	633407	634307	634307	509	370082

TABLE 36

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	57646	54617	55957	56030	653	498330	62425	59441	61222	61256	612	498600
5%	215226	210481	212526	212453	954	498720	218626	213111	216405	216316	1360	498703
10%	385552	381032	382674	382671	1144	498375	389252	383688	386671	386698	1361	499109
20%	701310	694992	698197	698200	1359	498918	705469	699535	701953	701512	1538	499231
50%	1583076	1579892	1581201	1581123	707	495382	1584680	1580667	1582584	1582527	1131	486334

TABLE 37

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	99496	95503	97953	98144	1046	498612	110303	105860	108394	108416	1039	499072
5%	402203	396513	399375	399436	1514	499013	410179	399941	406530	406730	2525	499521
10%	737645	730830	734270	734480	1801	499345	754851	729128	742078	743126	5181	499584
20%	1373675	1360215	1365712	1365587	3243	498960	1384688	1367158	1375294	1375172	4695	499349
50%	3159184	3149522	3156006	3156534	2444	498442	3163924	3152654	3159966	3160254	2627	499056

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs except in problems with no. of items 200 and knapsack capacity as 0.1% & 0.5%, where TCQEA matched UCQEA. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

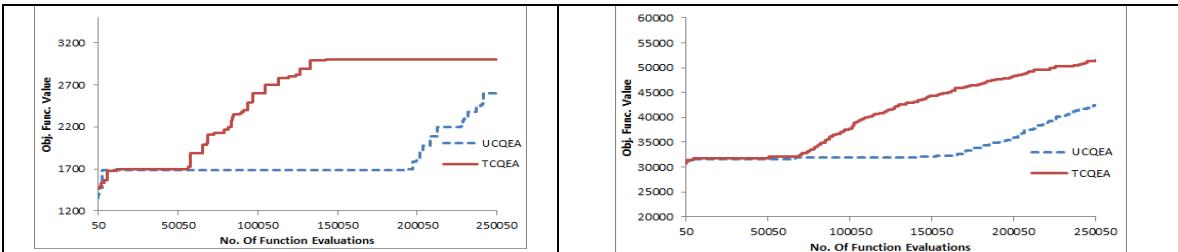


Fig 81. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Strongly correlated Data Instances having No. of Items as 200 and Capacity as 1% of Total Weight

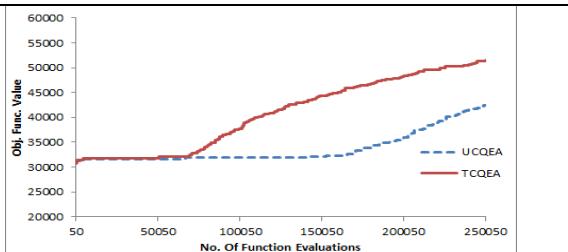


Fig 82. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

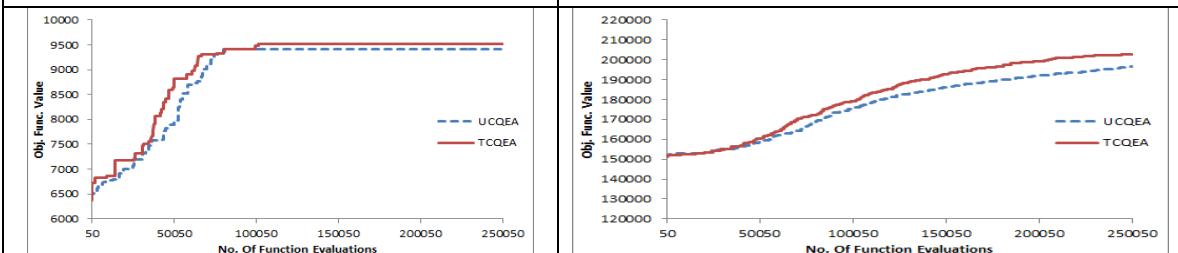


Fig 83. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Strongly correlated Data Instances having No. of Items as 200 and Capacity as 5% of Total Weight.

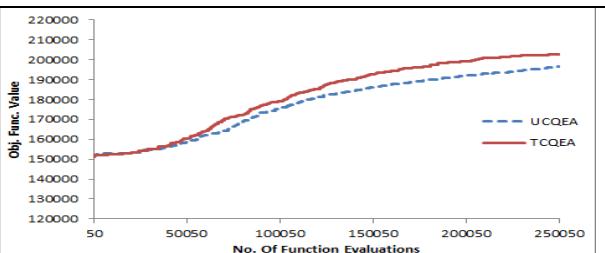


Fig 84. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

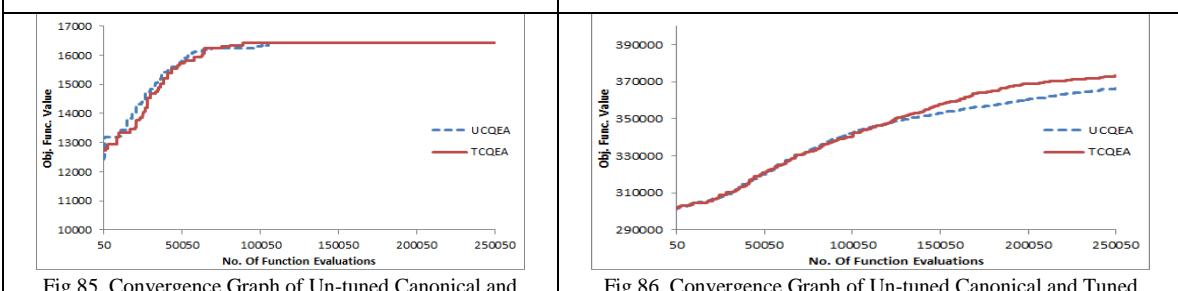


Fig 85. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Strongly correlated Data Instances having No. of Items as 200 and Capacity as 10% of Total Weight

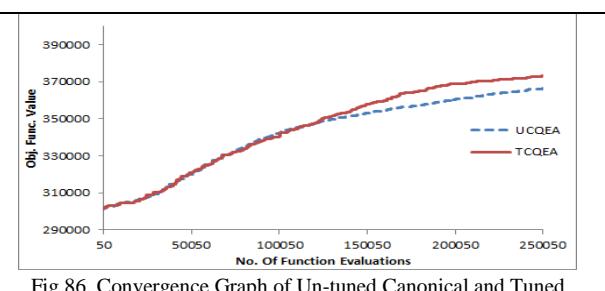


Fig 86. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 31 to 37, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Strongly Correlated Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 38, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 242 at significance level of  $\alpha = 5\%$ . The sample size  $n= 37$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be rejected safely. This indicates the success of the proposed tuning method for tuning QEA on problems like Strongly Correlated Knapsack problem.

**Table 38: Wilcoxon's Signed Rank Test on Strongly Correlated KP Instances (OFV)**

	TCQEA	UCQEA		
1	X <sub>1</sub>	X <sub>2</sub>	Test Statistic	Critical Value
	0	0		

2	12.6	12.6	$\Sigma(+)$	703	Null Hypothesis	$T$	At an $\alpha$ of 5%
3	20.3	20.3	$\Sigma(-)$	0	$H_0: \mu_1 \leq \mu_2$	0	242
4	41.6	38.6					
5	79.2	75.2	$n$	37			
6	96.2	94.9					
7	6.1	6.1					
8	25.3	24.6					
9	49.5	40.7					
10	93.2	69.7					
11	173.2	147.5					
12	208.1	190.3					
13	12.7	12.7					
14	61.3	47.3					
15	120.3	80.6					
16	226	144.7					
17	405.3	306.4					
18	488.3	396.4					
19	25.7	24					
20	126.4	78.9					
21	238.3	142.4					
22	432.9	263.5					
23	765.7	546.3					
24	909.1	708.4					
25	51.4	40.3					
26	243.5	141.7					
27	447.9	259.1					
28	796.3	493.2					
29	1378.1	1014					
30	1634.7	1297.2					
31	126	80.4					
32	548.1	318.2					
33	971.3	600.5					
34	1687.2	1164.9					
35	2966.2	2360.6					
36	3559.1	2992					
37	234.8	142.2					
38	966.4	600.8					
39	1697.4	1154.5					
40	2988.3	2258.2					
41	5372.7	4549.9					
42	6520.8	5730.4					

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 39, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 356 and more than the critical value of 303 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n= 41$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.8328, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table 38. This indicates the success of the proposed tuning method for

tuning QEA on problems like Strongly Correlated Knapsack instance even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 39: Wilcoxon's Signed Rank Test on Strongly Corr. KP Instances (Av. Gen.)**

	TCQEA	UCQEA				
1	$X_1$	$X_2$				
2	0	0				
3	61.1	83.2				
4	262.3	463.3				
5	297.9	528.1				
6	393.9	947.5				
7	404.9	951.6				
8	3.8	3.2				
9	240.4	523.3				
10	362.2	503.2				
11	455.8	607.7				
12	634.1	961.7				
13	622	979				
14	260.6	461.7				
15	426.7	493.2				
16	680.6	609.3				
17	815	616.8				
18	879.4	968.2				
19	942.2	977.1				
20	405.6	481.5				
21	687.5	513.2				
22	876.1	584.5				
23	948.1	710.1				
24	959.5	975.5				
25	977.2	986.2				
26	533	515.8				
27	825	486.5				
28	952.8	644.2				
29	985.7	652.9				
30	988.5	975.9				
31	989.9	979.2				
32	743.4	535.8				
33	959.5	419.1				
34	981.6	525.2				
35	987.8	703.3				
36	988	971.8				
37	993.4	982.7				
38	816	555.7				
39	976.1	519.7				
40	986	497.1				
41	988	741.7				
42	989.6	977.6				
	989.2	980.5				

**4) Inverse strongly correlated instances:**

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Table 40 to 46. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and 200 for different capacity of knapsack. In fact, when the capacity of knapsack was 0.1% of the total capacity and number of items 100, both the algorithm could not find even a single item for the knapsack. The performance of TCQEA and UCQEA are also similar when the capacity of knapsack is 0.1 % of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in rest of the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in figures 87 to 92, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 40  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	461	461	461	461	0	95	461	461	461	461	0	116
5%	2591	2591	2591	2591	0	23148	2591	2591	2591	2591	0	37356
10%	5183	5182	5183	5183	0	76880	5183	5181	5183	5183	0	51770
20%	10366	10364	10365	10366	0	75228	10366	10364	10366	10366	0	78180
50%	25715	25612	25707	25714	26	67112	25715	25615	25711	25714	18	83411

TABLE 41  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	1004	1004	1004	1004	0	16635	1004	1004	1004	1004	0	15302
5%	5418	5417	5418	5418	0	146885	5418	5417	5418	5418	0	76808
10%	10835	10835	10835	10835	0	96693	10835	10835	10835	10835	0	48913
20%	21671	21471	21607	21571	56	66927	21671	21571	21647	21671	43	88892
50%	53677	53477	53554	53577	57	115242	53677	53477	53564	53577	63	116157

TABLE 42  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	2646	2646	2646	2646	0	157297	2646	2646	2646	2646	0	87592
5%	13330	13230	13237	13230	25	94818	13330	13230	13250	13231	41	70600
10%	26561	26361	26474	26461	57	122140	26561	26461	26498	26461	49	95406
20%	52922	52622	52819	52822	76	150982	53022	52722	52899	52922	63	127552
50%	131206	130906	131040	131056	84	260080	131206	130907	131096	131106	80	256846

TABLE 43  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	5313	5313	5313	5313	0	237190	5313	5313	5313	5313	0	119526
5%	26665	26565	26578	26565	35	167943	26665	26565	26588	26565	43	115018
10%	53230	52930	53063	53030	92	173987	53330	52930	53160	53130	75	144847
20%	106160	105661	105949	105960	111	266106	106260	105860	106070	106060	112	260037
50%	262853	262453	262705	262734	107	406118	263053	262453	262726	262753	128	400392

TABLE 44  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	10735	10735	10735	10735	0	241557	10735	10735	10735	10735	0	115282
5%	53777	53377	53610	53577	80	268987	53777	53577	53707	53677	65	185206
10%	107353	106854	107133	107154	132	320445	107453	107054	107260	107253	117	222582
20%	214107	213508	213780	213708	181	366832	214407	213608	214057	214057	161	325496
50%	530373	529470	529968	529956	222	479302	530585	529774	530124	530074	210	479602

TABLE 45  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	26943	26744	26837	26844	52	358185	26943	26844	26884	26844	50	223945
5%	134318	133818	134011	134018	131	384282	134418	133918	134141	134118	128	264647
10%	267837	267237	267530	267537	168	397700	268136	267237	267840	267837	226	321938
20%	534374	533076	533738	533775	283	482327	535174	533675	534458	534424	396	462921
50%	1320628	1318645	1319680	1319711	471	498748	1323057	1319198	1321242	1321151	756	497874

TABLE 46  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON INVERSE STRONGLY CORRELATED DATA INSTANCES

% of Total Capacity	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	54002	53702	53865	53902	67	416330	54102	53702	53909	53902	87	248417
5%	268910	268311	268580	268560	159	438535	269310	268511	268957	268910	227	394202

10%	536822	535823	536327	536322	229	484127	<b>537821</b>	<b>536421</b>	<b>537051</b>	<b>537035</b>	353	<b>447516</b>
20%	1068748	1066653	1067726	1067776	<b>513</b>	498317	<b>1071838</b>	<b>1068942</b>	<b>1070400</b>	<b>1070424</b>	755	<b>493779</b>
50%	2636292	2627651	2632323	2632402	<b>1950</b>	<b>498798</b>	<b>2641744</b>	<b>2632155</b>	<b>2637346</b>	<b>2637676</b>	2681	499234

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph of TCQEA & UCQEA for all the problem instances having No. of Items as 200 is almost similar with TCQEA minutely outperforming UCQEA, however, problem instances having No. of Items as 5000 establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

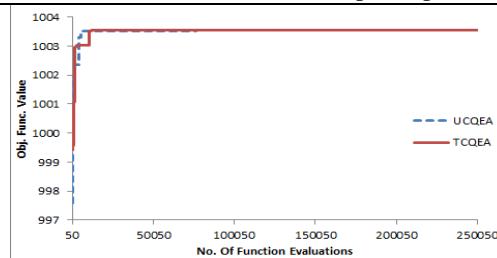


Fig 87. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Inverse Strongly correlated Data Instances having No. of Items as 200 and Capacity as 1% of Total Weight

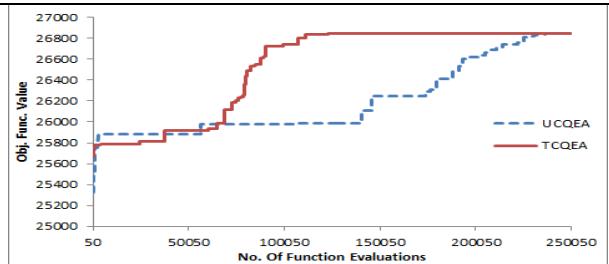


Fig 88. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Inverse Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

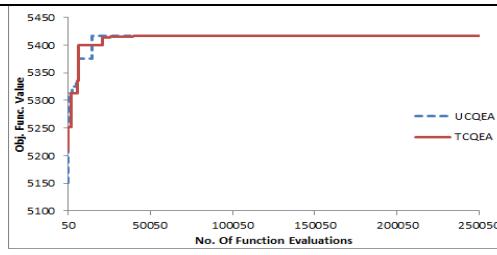


Fig 89. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 200 and Capacity as 5% of Total Weight

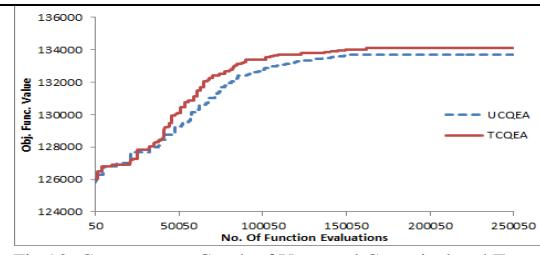


Fig 90. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

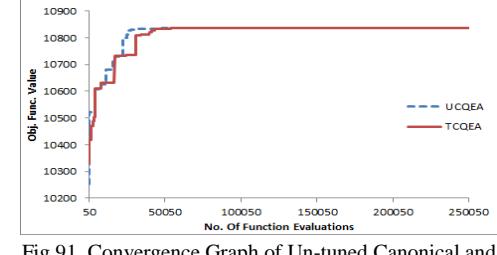


Fig 91. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Inverse Strongly correlated Data Instances having No. of Items as 200 and Capacity as 10% of Total Weight

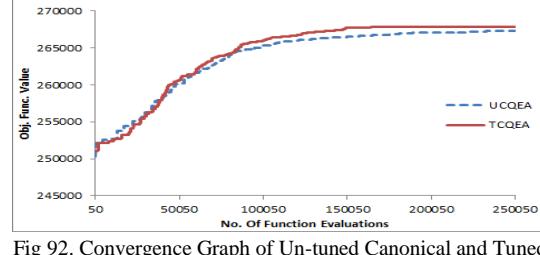


Fig 92. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Inverse Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 40 to 46, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Inverse Strongly Correlated Knapsack problem at a

significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 47, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is 2 and less than the critical value of 130 at significance level of  $\alpha = 5\%$ . The sample size  $n= 28$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Inverse Strongly Correlated Knapsack problem.

**Table 47: Wilcoxon's Signed Rank Test on Inv. Strongly Corr. KP Instances (OFV)**

	TCQEA	UCQEA			
	X <sub>1</sub>	X <sub>2</sub>			
1	0	0			
2	167.9	167.9			
3	460.7	460.7			
4	966.9	966.9			
5	2013.1	2013.2			
6	2591.2	2591.2			
7	18.01	18.01			
8	476.8	476.8			
9	1003.6	1003.6			
10	2107.1	2107.1			
11	4314	4313.8			
12	5417.4	5417.3			
13	194.26	194.26			
14	1273.1	1273.1			
15	2645.9	2645.9			
16	5292	5285.3			
17	10588	10538			
18	13237	13202			
19	488.62	488.62			
20	2656.4	2656.2			
21	5312.8	5293.7			
22	10626	10446			
23	21255	20927			
24	26575	26246			
25	999.39	999.39			
26	5367.5	5348.2			
27	10735	10538			
28	21467	20849			
29	42885	41801			
30	53598	52426			
31	2684.3	2684			
32	13422	13130			
33	26830	25934			
34	53599	51439			
35	106673	103064			
36	133103	129298			
37	5390.1	5362.4			
38	26899	26059			

		Test Statistic	Critical Value
$\Sigma(+)$	404	$T$	At an $\alpha$ of 5%
$\Sigma(-)$	2	2	130
$n$	28		
<b>For Large Samples (<math>n &gt; 25</math>)</b>			
Test Statistic		$z$	-4.57706
		$E[T]$	203
		$\sigma(T)$	43.91469003
At an $\alpha$ of			
Null Hypothesis	$p$ -value	5%	
$H_0: \mu_1 \leq \mu_2$	0.0000	<b>Reject</b>	

39	53686	51588
40	106950	102408
41	212276	205306
42	264602	257093

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 48, which shows that null hypothesis can be rejected as Wilcoxon's Signed Rank test statistic is 297 and less than the critical value of 303 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires less number of generations as compared to Canonical QEA. The sample size  $n= 41$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.0418, which is less than 0.05, so null hypothesis can be rejected. Moreover, Tuned QEA is performing significantly better than Canonical QEA as shown by Table. 47. This indicates the success of the proposed tuning method for tuning QEA on problems like Inversely Strongly Correlated Knapsack instance and it even requires less number of generations and function evaluations as compared to Canonical QEA.

**Table 48: Wilcoxon's Signed Rank Test on Strongly Corr. KP Instances (Av. Gen.)**

	TCQEA	UCQEA		Test Statistic	Critical Value
1	$X_1$	$X_2$			
2	0	0			
3	1.9	1.5			
4	1.6	1.9			
5	1.8	2.1			
6	470.9	545.8			
7	270.5	406.4			
8	4.1	3.5			
9	2.1	2.2			
10	316	237			
11	401	459			
12	254	739			
13	421	854			
14	10.2	194.26			
15	435	1273.1			
16	343	2646.1			
17	383	5292.1			
18	447	10584			
19	498	13231			
20	23.8	11.7			
21	391	448			
22	344	552			
23	582	662			
24	668	937			
25	738	962			
26	54.8	45.2			
27	343	472			
28	524	568			
29	738	637			
30	888	963			
31	900	970			

For Large Samples ( $n > 25$ )		
Test Statistic	$z$	$E[T]$
	1.729941	430.5
		$\sigma(T)$ 77.17026629
Null Hypothesis	$p$ -value	At an $\alpha$ of
$H_0: \mu_1 \geq \mu_2$	0.0418	5%
		<b>Reject</b>

31	493	560
32	629	425
33	737	483
34	924	703
35	974	966
36	966	960
37	580	535
38	749	487
39	874	504
40	949	579
41	985	950
42	983	972

### 5) *Almost strongly correlated instances:*

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Tables 49 to 55. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and the capacity of knapsack is 0.1 % of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in all the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in figures 93 to 98, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 49

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	1241	1082	1119	1096	45	236653	1500	1399	1490	1500	30	121166
5%	4896	4593	4768	4795	65	91090	4896	4793	4806	4796	30	80589
10%	8383	8182	8326	8330	57	70587	8382	8278	8347	8378	47	87707
20%	14644	14337	14569	14546	73	94975	14646	14447	14597	14640	56	99469
50%	31819	31711	31762	31721	51	95642	31823	31716	31800	31816	37	114593

TABLE 50

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	2988	2391	2761	2792	153	400905	<b>3090</b>	<b>2892</b>	<b>3034</b>	<b>2993</b>	<b>56</b>	<b>150886</b>
5%	9445	9141	9372	9437	88	172093	<b>9545</b>	<b>9337</b>	<b>9465</b>	<b>9444</b>	<b>63</b>	<b>141818</b>
10%	<b>16365</b>	16057	16206	16258	<b>78</b>	<b>131100</b>	16363	<b>16061</b>	<b>16230</b>	<b>16260</b>	88	187100
20%	28777	28384	28576	28586	<b>79</b>	<b>140243</b>	<b>28786</b>	<b>28486</b>	<b>28625</b>	<b>28594</b>	81	174474
50%	63238	62939	63110	63135	<b>72</b>	<b>146637</b>	<b>63333</b>	<b>63034</b>	<b>63131</b>	<b>63137</b>	73	174035

TABLE 51

### TABLE 31 COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

**COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON ALMOST STRONGLY CORRELATED DATA INSTANCES**

Weight	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	7622	7122	7442	7428	118	405557	7723	7225	7543	7527	120	296670
5%	23496	22679	23176	23195	187	335337	23497	22889	23247	23292	134	367419
10%	40031	39442	39742	39732	142	405793	40133	39443	39864	39841	165	408778
20%	70275	69494	69943	69976	164	409250	70382	69583	70106	70131	162	462442
50%	154330	153646	154075	154135	151	424750	154434	154022	154207	154227	111	454351

TABLE 52

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	14814	13925	14444	14465	242	489228	14918	14320	14680	14715	172	477929
5%	46432	45738	46145	46142	198	488230	46830	45724	46344	46375	242	491446
10%	79842	78948	79514	79500	232	490285	80128	79023	79666	79631	233	491915
20%	141567	140576	141042	141068	241	488678	141663	140858	141246	141251	194	493132
50%	311738	310958	311305	311352	207	490723	311944	311243	311583	311600	172	492598

TABLE 53

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	27976	26427	27150	27202	380	496143	29140	27736	28369	28347	351	496412
5%	92522	90156	91439	91421	516	496280	92522	91427	91944	91974	313	496393
10%	160354	157866	159470	159458	541	495527	161026	159116	160151	160139	443	496205
20%	285647	283651	284616	284687	453	496972	285889	284078	285220	285259	397	496911
50%	633200	631600	632440	632433	395	493363	633936	632315	633279	633318	415	494700

TABLE 54

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	57894	55665	56961	56967	621	498495	63938	60714	62249	62291	802	499019
5%	216320	212387	214464	214510	980	498615	219877	216056	217976	218161	933	499326
10%	387621	381723	384629	384410	1294	498467	391901	386206	388919	389040	1540	499208
20%	702807	698149	700531	700498	1219	498440	707706	700570	704892	705132	1707	499442

50%	1585045	<b>1581945</b>	1583469	1583527	<b>828</b>	<b>496813</b>	<b>1587196</b>	1581382	<b>1584560</b>	<b>1584633</b>	1343	497323
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TABLE 55

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON ALMOST STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	101672	96669	98592	98390	<b>1153</b>	<b>498482</b>	111412	105861	108720	<b>108706</b>	1232	499478
5%	405353	398364	401242	401271	<b>1619</b>	<b>498843</b>	411606	403020	407270	<b>407194</b>	2043	499577
10%	740740	731839	736249	736151	<b>2567</b>	499432	<b>752363</b>	<b>737389</b>	<b>744688</b>	<b>744251</b>	3824	<b>499330</b>
20%	1372606	1361099	1367754	1367855	<b>3320</b>	<b>499310</b>	1393697	1366535	<b>1378514</b>	<b>1379438</b>	5993	499551
50%	3163094	3153536	3159419	3159617	<b>2560</b>	<b>498792</b>	3170687	3159761	3164171	<b>3164275</b>	2811	499013

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

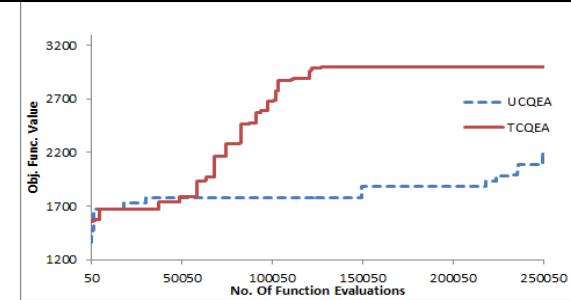


Fig 93. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 200 and Capacity as 1% of Total Weight

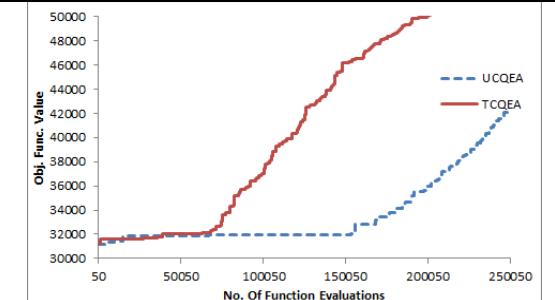


Fig 94. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

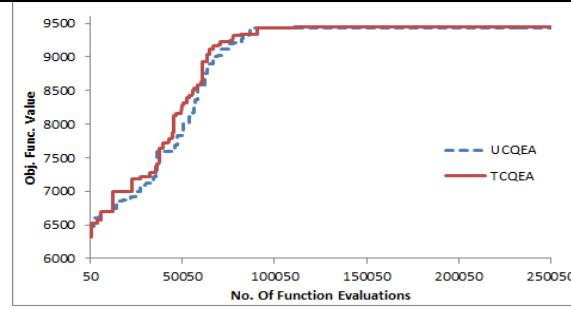


Fig 95. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 200 and Capacity as 5% of Total Weight

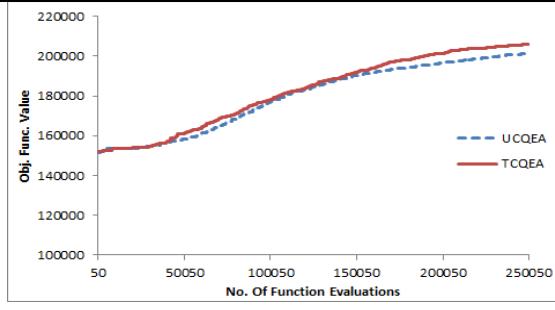


Fig 96. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

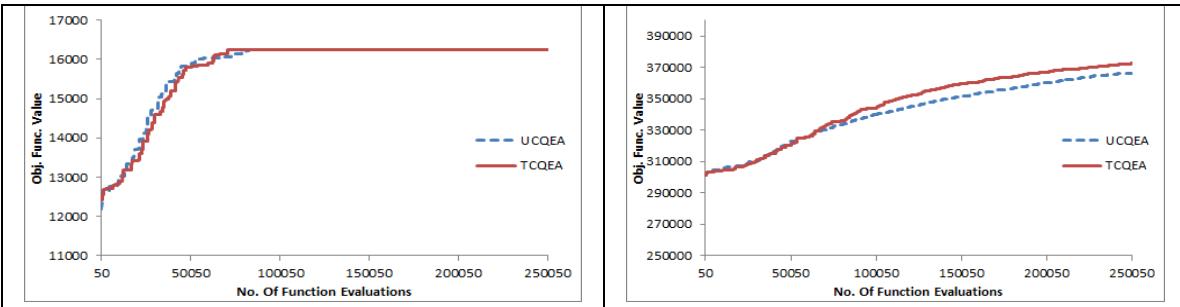


Fig 97. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 200 and Capacity as 10% of Total Weight

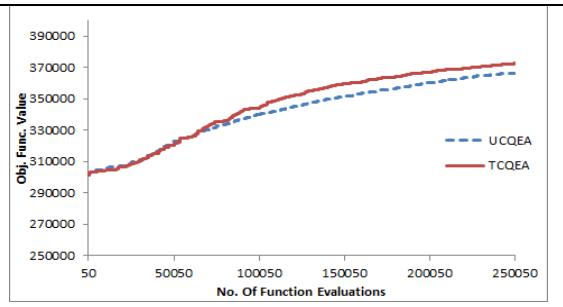


Fig 98. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Almost Strongly correlated Data Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 49 to 55, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Almost Strongly Correlated Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 56, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is 0 and less than the critical value of 303 at significance level of  $\alpha = 5\%$ . The sample size  $n=41$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Almost Strongly Correlated Knapsack problem.

**Table 56: Wilcoxon's Signed Rank Test on Al. Strongly Corr. KP Instances (OFV)**

	TCQEA	UCQEA	$\Sigma(+)$	$\Sigma(-)$	n	Test Statistic	Critical Value
	$X_1$	$X_2$					
1	246	246				Null Hypothesis	
2	856	666.7				$H_0: \mu_1 \leq \mu_2$	
3	1450	1070.7				0	303
4	2396.2	1823.9					
5	4033.1	3588.1					
6	4765.3	4576.4					
7	687.4	419.1					
8	1869.8	1062					
9	2920.3	1758.4					
10	4663.6	3127.7					
11	7825.4	6448.1					
12	9252.5	8294.7					
13	1633.3	681.7					
14	4384.7	2069.5					
15	6868.4	3666.2					
16	10953	6864.4					
17	18273.8	14091.2					
18	21657.2	18108.8					
19	2774.3	1039.1					
20	7493.2	3667.3					

For Large Samples ( $n > 25$ )		
Test Statistic		
z	-5.57857	$E[T]$
		430.5
		$\sigma(T)$
		77.17026629
At an $\alpha$ of		
Null Hypothesis	p-value	5%
$H_0: \mu_1 \leq \mu_2$	0.0000	Reject

21	11953.5	6803.3
22	19652.8	13018.2
23	33820.7	26692.8
24	40259.8	34111.4
25	4392	1753.4
26	12521.6	6870.9
27	20608.1	13149.2
28	35268.4	25637.7
29	62129.6	52046.6
30	75107.2	65762
31	7628.2	3725.3
32	24620.3	16342
33	43261.3	31842
34	77221	62615.4
35	142088	125899
36	173517	158469
37	11545.1	6939.3
38	42686.2	31809.6
39	77188.4	62492.7
40	142590	123709
41	269378	248335
42	331196	311913

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 57, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 725 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n = 42$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.9997, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table. 56. This indicates the success of the proposed tuning method for tuning QEA on problems like Almost Strongly Correlated Knapsack instance even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 57: Wilcoxon's Signed Rank Test on Al. Strongly Corr. KP Instances (Av. Gen.)**

	TCQEA	UCQEA	$\Sigma(+)$	725	Null Hypothesis		Test Statistic $T$	Critical Value At an $\alpha$ of 5%
					$\Sigma(-)$	178		
1	42.5	77						
2	275.8	431.9						
3	384.2	468.2						
4	424.9	576.7						
5	558	964.9						
6	551.1	973.5						
7	342.8	503.2						
8	590.6	495.8						
9	743.2	571.7						
10	836.7	607.5						
11	874.7	953.7						
12	765.8	979.6						
13	789.1	550.2						

For Large Samples ( $n > 25$ )		
Test Statistic		
$Z$	-3.41975	$E[T]$
		451.5
		$\sigma(T)$
		79.97655907
At an $\alpha$ of		
Null Hypothesis	$p$ -value	5%
$H_0: \mu_1 \geq \mu_2$	0.9997	

14	898.7	464.4	
15	909.8	459.9	
16	955.1	670.5	
17	949.7	972.6	
18	974	981.3	
19	890.4	482.3	
20	956.3	570.4	
21	969.5	514.6	
22	963.7	558.5	
23	985	969.1	
24	985.4	984.4	
25	958.6	471.3	
26	965.1	468.1	
27	982.2	576.7	
28	986.4	807.3	
29	980.9	973.4	
30	989.4	976.6	
31	946.3	558.8	
32	980	547.1	
33	988	412.7	
34	989.5	643.7	
35	986	964.3	
36	990.2	976.2	
37	958.5	485	
38	968	472.1	
39	987.4	534.9	
40	990.2	661.2	
41	988.5	964.2	
42	988.7	984.6	

6) *Subset sum instances:*

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Tables 58 to 64. The performance of TCQEA and UCQEA are similar in all the problem instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in figures 99 to 104, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 58  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	478	478	478	478	0	158857	478	478	478	478	0	30601
5%	2392	2392	2392	2392	0	40402	2392	2392	2392	2392	0	30743
10%	4783	4783	4783	4783	0	43490	4783	4783	4783	4783	0	48434
20%	9567	9566	9567	9567	0	62422	9567	9566	9567	9567	0	87810
50%	23916	23916	23916	23916	0	130042	23916	23916	23916	23916	0	129396

TABLE 59  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	1004	1004	1004	1004	0	107100	1004	1004	1004	1004	0	51655
5%	5018	5018	5018	5018	0	55590	5018	5018	5018	5018	0	25014
10%	10036	10036	10036	10036	0	128418	10036	10036	10036	10036	0	66551
20%	20072	20072	20072	20072	0	140185	20072	20072	20072	20072	0	117035
50%	50181	50181	50181	50181	0	146673	50181	50181	50181	50181	0	120661

TABLE 60

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	2446	2446	2446	2446	0	82852	2446	2446	2446	2446	0	40082
5%	12232	12232	12232	12232	0	148558	12232	12232	12232	12232	0	104244
10%	24463	24463	24463	24463	0	93957	24463	24463	24463	24463	0	143567
20%	48926	48926	48926	48926	0	110112	48926	48926	48926	48926	0	123281
50%	122315	122315	122315	122315	0	121532	122315	122315	122315	122315	0	151935

TABLE 61

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	4913	4913	4913	4913	0	76727	4913	4913	4913	4913	0	72567
5%	24567	24567	24567	24567	0	133743	24567	24567	24567	24567	0	136221
10%	49134	49134	49134	49134	0	200485	49134	49134	49134	49134	0	148767
20%	98268	98268	98268	98268	0	179258	98268	98268	98268	98268	0	152655
50%	245670	245670	245670	245670	0	161037	245670	245670	245670	245670	0	136112

TABLE 62

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	9936	9936	9936	9936	0	181055	9936	9936	9936	9936	0	102967
5%	49681	49681	49681	49681	0	167240	49681	49681	49681	49681	0	144395
10%	99361	99361	99361	99361	0	153303	99361	99361	99361	99361	0	191984

20%	198723	198723	198723	198723	0	165315	198723	198723	198723	198723	0	183579
50%	496807	496807	496807	496807	0	240813	496807	496807	496807	496807	0	186295

TABLE 63  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	24846	24846	24846	24846	0	227532	24846	24846	24846	24846	0	200957
5%	124228	124228	124228	124228	0	282130	124228	124228	124228	124228	0	200835
10%	248456	248456	248456	248456	0	205327	248456	248456	248456	248456	0	212048
20%	496912	496912	496912	496912	0	297685	496912	496912	496912	496912	0	225195
50%	1242280	1242280	1242280	1242280	0	188198	1242280	1242280	1242280	1242280	0	167822

TABLE 64  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON SUBSET SUM DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	49906	49906	49906	49906	0	270943	49906	49906	49906	49906	0	214365
5%	249530	249530	249530	249530	0	240035	249530	249530	249530	249530	0	221704
10%	499060	499060	499060	499060	0	259060	499060	499060	499060	499060	0	218945
20%	998119	998119	998119	998119	0	219655	998119	998119	998119	998119	0	233086
50%	2495299	2495298	2495298	2495298	0	227995	2495300	2495298	2495298	2495298	0	181688

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph shows that TCQEA is as fast as UCQEA in all the graphs.

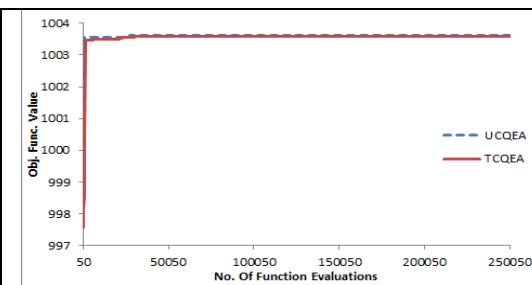


Fig 99. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Subset sum Data Instances having No. of Items as 200 and Capacity as 1% of Total Weight

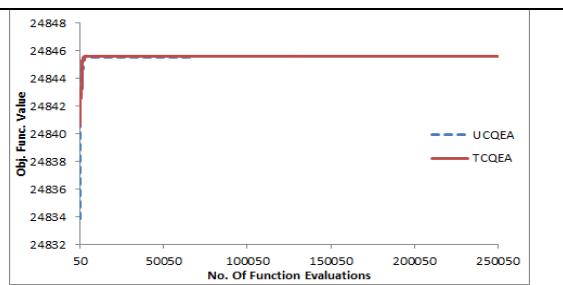


Fig 100. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Subset sum Data Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

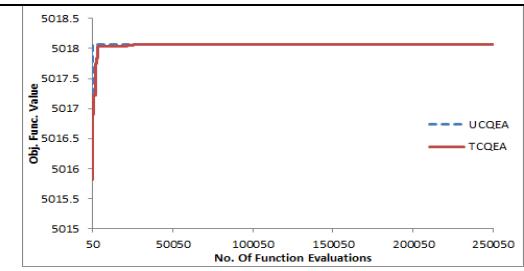


Fig 101. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Subset sum Data Instances having No. of Items as 200 and Capacity as 5% of Total Weight

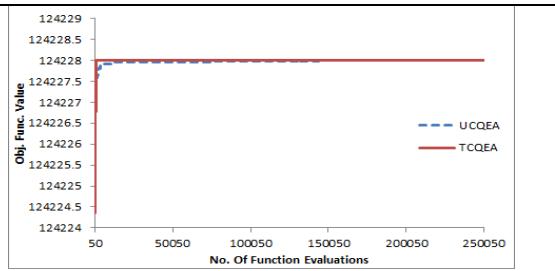


Fig 102. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Subset sum Data Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

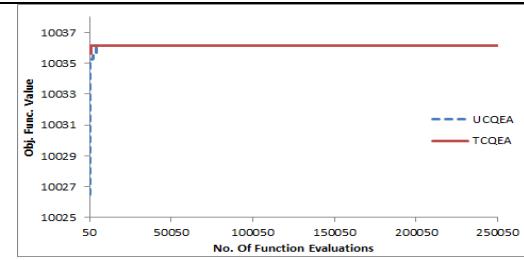


Fig 103. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Subset sum Data Instances having No. of Items as 200 and Capacity as 10% of Total Weight

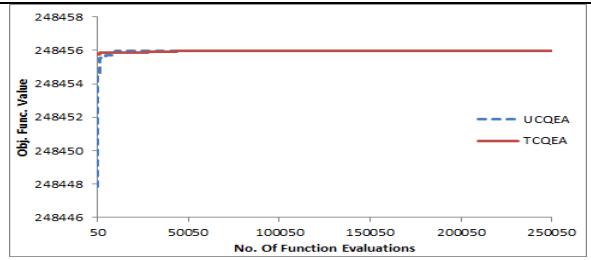


Fig 104. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with Subset sum Data Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 58 to 64, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Subset Sum Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 65, which shows that only two samples have different values so no conclusion regarding superiority of tuning method can be drawn on the basis of the available data. A comparative study on average number of generations is required to determine the relative performance of TCQEA and UCQEA.

**Table 65: Wilcoxon's Signed Rank Test on Subset Sum KP Instances (OFV)**

	TCQEA	UCQEA	$\Sigma(+)$	$\Sigma(-)$	n	Null Hypothesis	Test Statistic	Critical Value
	$X_1$	$X_2$					T	At an $\alpha$ of 5%
1	47	47						
2	239	239						
3	478.3	478.3						
4	956.6	956.6						
5	1913.3	1913.3						
6	2391.6	2391.6						
7	100.3	100.3						
8	501.7	501.7						
9	1003.6	1003.6						
10	2007.2	2007.2						
11	4014.4	4014.5						
12	5018.1	5018.1						
13	244.5	244.5						
14	1223.1	1223.1						
15	2446.3	2446.3						

16	4892.6	4892.6
17	9785.2	9785.2
18	12231.5	12231.5
19	491.3	491.3
20	2456.7	2456.7
21	4913.4	4913.4
22	9826.8	9826.8
23	19653.6	19653.6
24	24567	24567
25	993.6	993.6
26	4968.1	4968.1
27	9936.1	9936.1
28	19872.2	19872.3
29	39744.5	39744.5
30	49680.7	49680.7
31	2484.6	2484.6
32	12422.8	12422.8
33	24845.6	24845.6
34	49691.2	49691.2
35	99382.4	99382.4
36	124228	124228
37	4990.6	4990.6
38	24953	24953
39	49906	49906
40	99811.9	99811.9
41	199624	199624
42	249530	249530

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 66, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 475.5 and greater than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n= 42$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.6179, which is more than 0.05, so null hypothesis cannot be rejected. Further, if we consider the null hypothesis as  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis as  $H_1: \mu_1 > \mu_2$ , Table 66 shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 427.5 and greater than the critical value of 319 at significance level of  $\alpha = 5\%$ , so no conclusion can be drawn on the success of the proposed tuning method for tuning QEA on problems like Subset Sum Knapsack instance.

**Table 66: Wilcoxon's Signed Rank Test on Subset Sum KP Instances (Av. Gen.)**

	TCQEA	UCQEA			
1	X <sub>1</sub>	X <sub>2</sub>			
2	2.3	2.5			
2	82	101.3			
3	380.9	299.6			
4	360.3	344.2			
5	491.6	418.9			
6	406.4	418.6			
7	261.5	251.7			
8	301.4	237			
			$\Sigma(+)$	475.5	
			$\Sigma(-)$	427.5	
			n	42	
					For Large Samples ( $n > 25$ )
					Test Statistic
					Test Statistic
					Critical Value
				T	At an $\alpha$ of 5%
				475.5	319
				427.5	319

9	478.2	484.8	Z	-0.30009	E[T]	451.5
10	421.2	376.9	$\sigma(T)$	79.97655907		
At an $\alpha$ of						
11	434.8	489.6	Null Hypothesis	p-value	5%	
12	423.1	459.2	$H_0: \mu_1 \geq \mu_2$	0.6179		
13	406	383.2	$H_0: \mu_1 \leq \mu_2$	0.3821		
14	449.3	506.8				
15	517.7	530.2				
16	482	519.1				
17	476.8	587.3				
18	475.7	476.3				
19	468.6	475.4				
20	540.6	429.5				
21	489.6	440.4				
22	498.8	472.3				
23	493.5	536.9				
24	525.6	539.4				
25	530.6	492				
26	490.6	490.3				
27	515.4	509.1				
28	493.4	569.4				
29	488.7	614.5				
30	537.3	430.8				
31	537.4	413.3				
32	538.1	507				
33	528.1	542.1				
34	523.4	528.9				
35	481	427				
36	523.7	434				
37	482.6	500.3				
38	423.8	548.6				
39	508.3	405.5				
40	436.6	505.2				
41	475.2	534.2				
42	531.5	517.5				

### 7) *Uncorrelated instances with similar weights:*

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Table 67 to 73. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 for different capacity of knapsack. In fact, when the capacity of knapsack was 0.1% of the total capacity and number of items 100 / 200 / 500, both the algorithm could not find even a single item for the knapsack. The performance of TCQEA and UCQEA are also similar when the capacity of knapsack is 0.1 % of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in rest of the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in figures 105 to 110, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 67

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

% of Total	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen

1%	981	981	981	981	0	88	981	981	981	981	0	109
5%	4818	4818	4818	4818	0	43063	4818	4818	4818	4818	0	39778
10%	9522	9489	9516	9513	7	35565	9522	9513	9520	9522	4	37445
20%	18194	18194	18194	18194	0	31225	18194	18194	18194	18194	0	37947
50%	37189	37134	37178	37188	17	36655	37189	37134	37180	37188	16	48061

TABLE 68

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	1965	1965	1965	1965	0	12505	1965	1965	1965	1965	0	12817
5%	9633	9609	9624	9625	8	71378	9633	9617	9631	9633	5	62420
10%	18937	18937	18937	18937	0	58918	18937	18937	18937	18937	0	63885
20%	35749	35749	35749	35749	0	64773	35749	35749	35749	35749	0	80292
50%	73674	73667	73671	73674	4	82448	73674	73667	73673	73674	3	107765

TABLE 69

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	4969	4969	4969	4969	0	204835	4969	4969	4969	4969	0	97584
5%	24409	24395	24406	24407	4	128695	24409	24395	24406	24405	4	131086
10%	47709	47709	47709	47709	0	138478	47709	47709	47709	47709	0	154176
20%	90773	90773	90773	90773	0	199540	90773	90773	90773	90773	0	247939
50%	188641	188327	188618	188637	60	318540	188641	188583	188632	188637	16	364317

TABLE 70

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	9947	9947	9947	9947	0	248633	9947	9947	9947	9947	0	142751
5%	48865	48854	48863	48865	3	302247	48865	48854	48863	48865	3	314985
10%	95238	95238	95238	95238	0	346338	95238	95238	95238	95238	0	362713
20%	181223	181175	181214	181217	10	440893	181223	181195	181212	181213	7	453466
50%	378080	377926	378020	378022	37	471998	378067	377917	377981	377979	37	488525

TABLE 71

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	19862	19862	19862	19862	0	340618	19862	19862	19862	19862	0	211279
5%	97179	97172	97177	97178	2	464325	97179	97171	97177	97177	2	471237
10%	189378	189298	189344	189344	18	491100	189378	189271	189337	189335	23	495175
20%	359084	358628	358921	358940	101	497517	359088	358818	358961	358950	67	497142
50%	747072	746030	746617	746627	216	498212	746885	746190	746559	746538	191	497878

TABLE 72

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

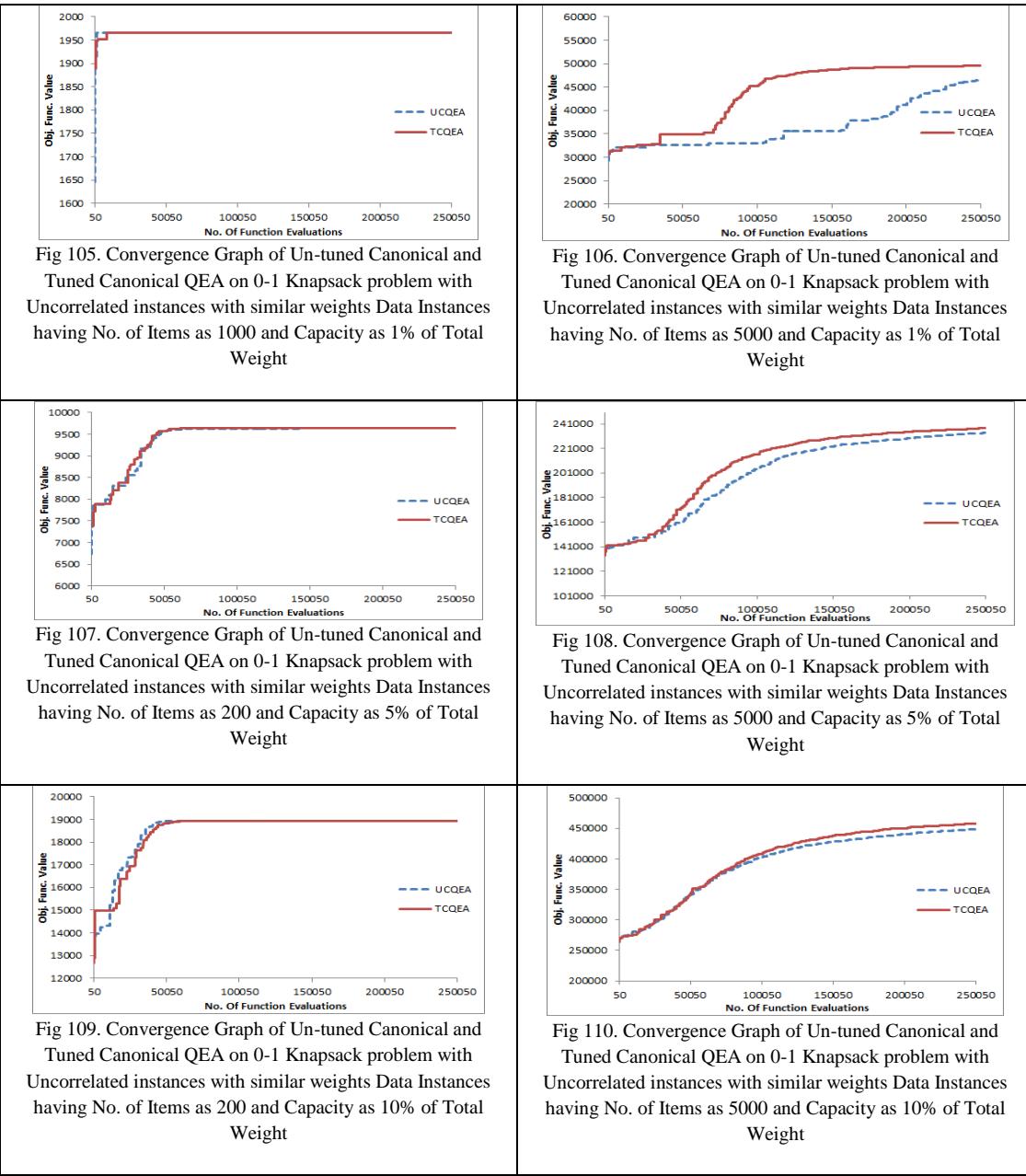
% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	49664	49585	49638	49639	16	496283	49685	49680	49683	49683	1	476035
5%	241788	240832	241409	241433	225	498203	242785	242410	242580	242564	86	498303
10%	468068	465385	466727	466778	574	498837	471688	469442	470599	470621	496	498683
20%	881986	876391	879207	879106	1283	498575	891382	883854	887490	888015	1817	499175
50%	1840683	1830433	1835985	1836343	2907	498608	1844985	1829575	1839329	1839287	3213	499326

TABLE 73

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON UNCORRELATED INSTANCES WITH SIMILAR WEIGHTS DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	98471	97677	98087	98068	161	498178	99320	99206	99270	99268	30	498142
5%	470250	466888	468767	468808	702	498295	478616	476205	477551	477732	708	499330
10%	901924	894308	898408	898562	2418	499050	923533	909934	915208	914686	2848	499076
20%	1691270	1666278	1675694	1674899	6106	499297	1714164	1668640	1699855	1700715	9838	499584
50%	3554710	3500113	3530838	3530308	11773	499440	3558266	3507462	3533864	3534393	13062	499620

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run except for 0.1% capacity (No. of items is 1000 instead of 200). The convergence graph of TCQEA & UCQEA for all the problem instances having No. of Items as 200 is almost similar, however, problem instances having No. of Items as 5000 establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.



In order to confirm the findings in Table 67 to 73, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Uncorrelated Instances with Similar Weights Data Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 74, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 175 at significance level of  $\alpha = 5\%$ . The sample size  $n= 32$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Uncorrelated Instances with Similar Weights Data Knapsack problem.

**Table 74: Wilcoxon's Signed Rank Test on UnCorr. Instances Sim. Wt. data (OFV)**

TCQEA	UCQEA

	$X_1$	$X_2$		Test Statistic	Critical Value
1	0	0	$\Sigma(+)$	528	At an $\alpha$ of 5%
2	0	0	$\Sigma(-)$	0	175
3	980.5	980.5			
4	1948.2	1948.2			
5	3854.6	3854.6			
6	4817.8	4817			
7	0	0			
8	984.4	984.4			
9	1964.9	1964.9			
10	3894.1	3813.6			
11	7728.5	7599.7			
12	9626.1	9543			
13	0	0			
14	1994.6	1993.6			
15	4969.1	4714.2			
16	9879.4	8650.4			
17	19592.4	17536			
18	24372.4	22289.3			
19	996.2	996.2			
20	4981.8	4716.3			
21	9944	8500.9			
22	19787	15549.8			
23	38954.3	31794.2			
24	48122.5	40745.8			
25	1997.5	1989.4			
26	9955.4	8457.2			
27	19803.1	15096.8			
28	38955.9	27893.6			
29	74916.3	56840.3			
30	91815.6	73367.4			
31	4993.5	4694.7			
32	24659.7	18321.1			
33	48255.7	33589.4			
34	92342.6	62912.3			
35	171550	128051			
36	207397	163230			
37	9950.6	8467.4			
38	47695.9	33495.7			
39	91580.6	62606.4			
40	171197	119073			
41	310775	241830			
42	376210	305588			

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 75, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 425.5 and more than the critical value of 256 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n= 38$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.7875, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table. 74. This indicates the success of the proposed tuning.

method for tuning QEA on problems like Uncorrelated Instances with Similar Weights Data even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 75: Wilcoxon's Signed Rank Test on Uncorr. Instances Sim. Wt. (Av. Gen.)**

	TCQEA	UCQEA				
1	X <sub>1</sub>	X <sub>2</sub>				
2	0	0				
3	0	0				
4	2	1.8				
5	35.2	67.1				
6	124	802.8				
7	158.2	851.9				
8	0	0				
9	3.6	3.8				
10	56.3	250.1				
11	181.4	693.1				
12	397	961.3				
13	441.9	974.2				
14	0	0				
15	148.8	594.7				
16	462.6	572.6				
17	721.2	572.2				
18	841.7	969.7				
19	930.4	983.4				
20	16.5	17.9				
21	587.1	591.4				
22	797.2	493.7				
23	957.6	636.8				
24	984.4	962.9				
25	985.7	984.9				
26	372.4	555.5				
27	871.6	528.4				
28	976.3	622.5				
29	982.4	595.6				
30	991.3	961.7				
31	990.1	976.2				
32	804.9	479				
33	974.3	525.9				
34	985.9	554.9				
35	988.4	594.7				
36	990.1	972.1				
37	992.8	980.8				
38	925.5	426.7				
39	981.7	510.6				
40	986.9	607.4				
41	988.5	576.7				
42	990.7	966.5				
	990.2	977.4				

8) ***Spanner instances span (v, m):***

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Table 76 to 82. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and 200 for different capacity of knapsack. In fact, when the capacity of knapsack was 0.1% of the total capacity and number of items 100, both the algorithm could not find even a single item for the knapsack. The performance of TCQEA and UCQEA are also similar when the capacity of knapsack is 0.1 % of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in rest of the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 111 to 116, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 76  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON SPANNER INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	167	167	167	167	0	103545	167	167	167	167	0	9092
5%	834	834	834	834	0	121172	834	834	834	834	0	64000
10%	1669	1669	1669	1669	0	70615	1669	1669	1669	1669	0	50513
20%	3337	3337	3337	3337	0	71262	3337	3337	3337	3337	0	98287
50%	7423	7423	7423	7423	0	71257	7423	7423	7423	7423	0	97004

TABLE 77  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON SPANNER INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	352	352	352	352	0	191482	352	352	352	352	0	73052
5%	1761	1761	1761	1761	0	116910	1761	1761	1761	1761	0	60911
10%	3522	3522	3522	3522	0	119887	3522	3522	3522	3522	0	165597
20%	7044	7044	7044	7044	0	186107	7044	7044	7044	7044	0	193393
50%	15452	15452	15452	15452	0	157733	15452	15452	15452	15452	0	174425

TABLE 78  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON SPANNER INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	853	853	853	853	0	266513	853	853	853	853	0	117064
5%	4267	4267	4267	4267	0	212348	4267	4267	4267	4267	0	164314
10%	8533	8533	8533	8533	0	226458	8533	8533	8533	8533	0	240451
20%	17067	17067	17067	17067	0	211193	17067	17067	17067	17067	0	233881
50%	37590	37583	37590	37590	2	295165	37590	37584	37590	37590	1	280137

TABLE 79  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON SPANNER INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	1707	1707	1707	1707	0	303383	1707	1707	1707	1707	0	153140
5%	8537	8537	8537	8537	0	263253	8537	8537	8537	8537	0	256153
10%	17073	17073	17073	17073	0	225295	17073	17073	17073	17073	0	257341
20%	34147	34147	34147	34147	0	264945	34147	34147	34147	34147	0	299297
50%	75217	75196	75213	75217	6	353255	75217	75187	75209	75211	9	348460

TABLE 80  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON SPANNER INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	3448	3448	3448	3448	0	315240	3448	3448	3448	3448	0	237059
5%	17241	17241	17241	17241	0	295948	17241	17241	17241	17241	0	243194
10%	34481	34481	34481	34481	0	248563	34481	34481	34481	34481	0	294050
20%	68962	68962	68962	68962	0	358537	68962	68962	68962	68962	0	364568
50%	151820	151765	151799	151801	16	445622	151820	151780	151802	151801	10	429409

TABLE 81  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON SPANNER INSTANCES

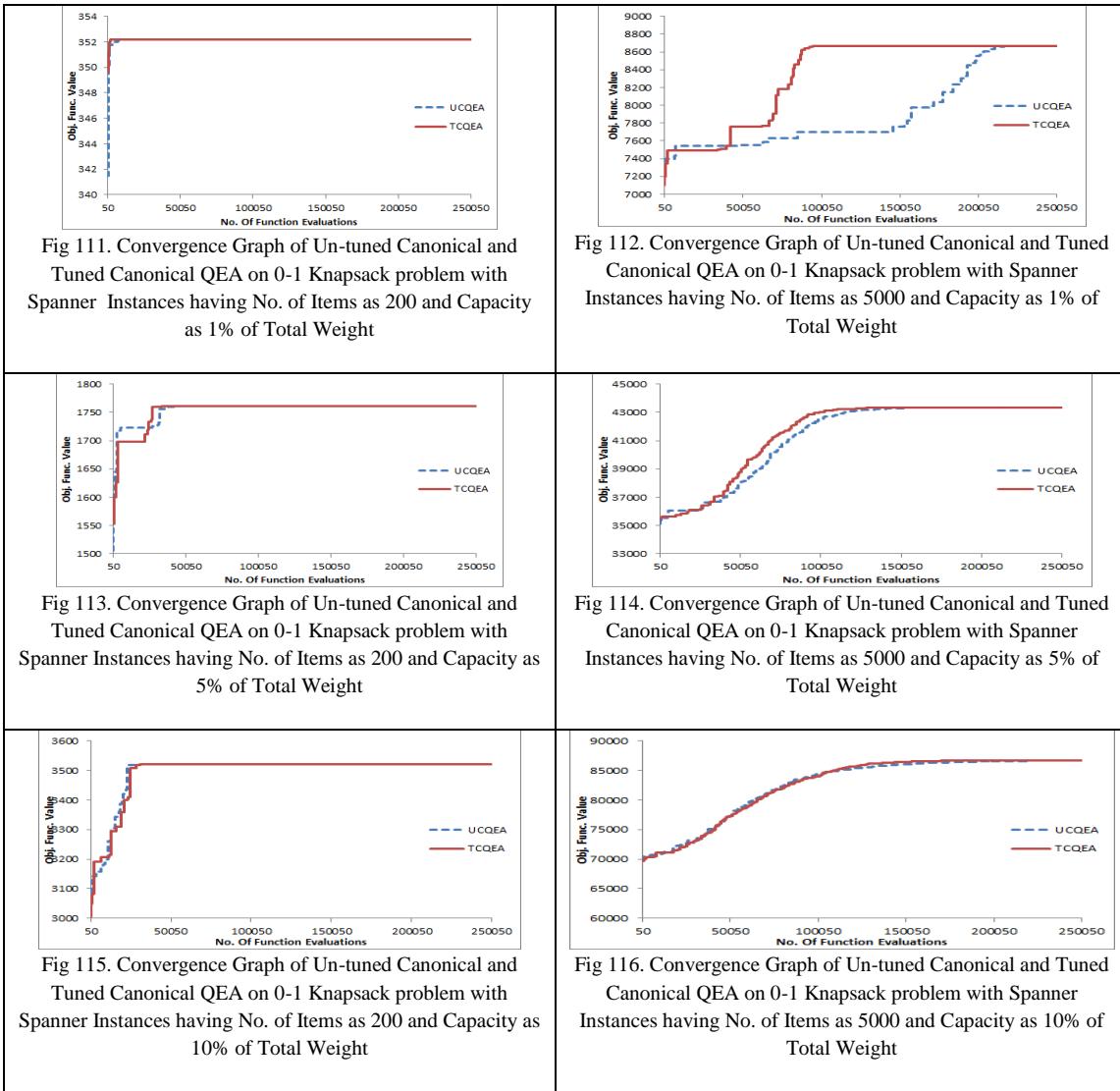
% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	8668	8668	8668	8668	0	339485	8668	8668	8668	8668	0	262291
5%	43341	43341	43341	43341	0	322480	43341	43341	43341	43341	0	321555
10%	86682	86682	86682	86682	0	363507	86682	86682	86682	86682	0	354866
20%	173216	172939	173103	173100	69	498175	173364	173207	173329	173334	34	495439
50%	379949	379352	379635	379650	137	497555	380099	379847	379956	379946	79	497330

TABLE 82  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON SPANNER INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	17439	17439	17439	17439	0	371338	17439	17439	17439	17439	0	261439
5%	87193	87193	87193	87193	0	390920	87193	87193	87193	87193	0	341263

10%	<b>174387</b>	<b>174259</b>	<b>174338</b>	<b>174348</b>	36	494885	<b>174387</b>	<b>174387</b>	<b>174387</b>	<b>174387</b>	0	<b>441052</b>
20%	342556	339872	341204	341217	<b>734</b>	<b>499162</b>	<b>346311</b>	<b>339970</b>	<b>344000</b>	<b>344132</b>	1367	499481
50%	759894	757444	758770	758725	<b>521</b>	<b>498588</b>	<b>761392</b>	<b>758344</b>	<b>760243</b>	<b>760362</b>	728	499211

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph of TCQEA & UCQEA for all the problem instances having No. of Items as 200 is almost similar with TCQEA minutely outperforming UCQEA, however, problem instances having No. of Items as 5000 establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.



In order to confirm the findings in Table 76 to 82, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Spanner Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 83, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is 0 and less than the critical value of 152 at significance level of  $\alpha = 5\%$ . The sample size

$n= 30$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Spanner Instances Knapsack problem.

**Table 83: Wilcoxon's Signed Rank Test on Spanner Instances KP data (OFV)**

	TCQEA	UCQEA			Test Statistic	Critical Value
1	$X_1$	$X_2$				
2	0	0				
3	83.4	83.4				
4	166.8	166.8				
5	333.7	333.7				
6	667.5	667.4				
7	834.3	834.3				
8	34.1	34.1				
9	176.1	176.1				
10	352.2	352.2				
11	704.4	704.2				
12	1408.8	1408.6				
13	1761	1760.9				
14	85.3	85.3				
15	426.6	426.6				
16	853.3	852.8				
17	1706.6	1684				
18	3413.3	3379.2				
19	4266.6	4258.3				
20	170.7	170.7				
21	853.6	853.2				
22	1707.3	1676				
23	3414.6	3187.4				
24	6829.3	6500.7				
25	8536.6	8284.3				
26	344.8	344.8				
27	1724	1688.5				
28	3448.1	3194.7				
29	6896.2	6153.8				
30	13792.4	12531.2				
31	17240.5	15920.9				
32	866.8	866.1				
33	4334.1	3940.3				
34	8668.1	7599.1				
35	17336.3	14820.6				
36	34476.2	29889.2				
37	42549.3	37829.9				
38	1743.8	1690.2				
39	8719.2	7619.8				
40	17438.1	14848.3				
41	34632.1	29122				
42	66178.4	58531				
	81052.5	73736.1				

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 84, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 385 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n = 42$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.2028, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table. 83. This indicates the success of the proposed tuning method for tuning QEA on problems like Spanner Instances even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 84: Wilcoxon's Signed Rank Test on Spanner Instances KP data (Av. Gen.)**

	TCQEA	UCQEA			Test Statistic $T$	Critical Value At an $\alpha$ of 5%
			$X_1$	$X_2$		
1	426.7	0				
2	510.6	2.1				
3	607.4	154.1				
4	576.7	428				
5	966.5	579.1				
6	977.4	729.9				
7	3.2	3.9				
8	398.5	416.4				
9	528.1	484.1				
10	358.7	478.2				
11	357.4	847.8				
12	377.9	893.2				
13	220.5	207.2				
14	451.4	521.7				
15	291.4	623.8				
16	371.1	673.5				
17	599.2	956.8				
18	507.1	972				
19	288.9	225.4				
20	440.4	571.1				
21	476	450.6				
22	550.7	706.5				
23	547.9	964				
24	661.1	976				
25	399.4	583.6				
26	448	579.1				
27	580.1	534.7				
28	599.6	742.9				
29	718.2	964.4				
30	753.4	982.5				
31	370.2	545.8				
32	532.2	631.4				
33	645.2	516.7				
34	802.5	702.2				
35	987.5	965				
36	988.7	983.8				
37	544.4	550.4				

38	651.9	489.5
39	849.8	590.3
40	988	649.5
41	990.8	963.4
42	989.3	980

9) *multiple strongly correlated instances mstr( k1, k2,d ) :*

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Tables 85 to 91. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and the capacity of knapsack is 0.1% of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in rest of the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 117 to 122, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 85

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	1959	1650	1790	1774	79	226247	2276	2074	2212	2276	85	102627
5%	7186	6882	7095	7086	60	91500	7186	7085	7141	7181	49	82051
10%	11975	11676	11821	11870	72	65737	11975	11675	11848	11875	58	75266
20%	19856	19653	19750	19753	66	92455	19856	19654	19766	19754	72	73187
50%	39501	39298	39423	39401	73	48160	39501	39300	39470	39500	53	77613

TABLE 86

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	5497	4899	5334	5396	139	297235	5497	5199	5404	5399	78	149351
5%	15108	14608	14871	14907	135	121588	15107	14607	14911	14908	113	145840
10%	24319	23722	23991	24021	136	127532	24321	23822	24121	24121	111	124964
20%	39753	39450	39615	39652	100	104052	39852	39353	39656	39653	106	119965
50%	81050	80650	80887	80850	103	74313	81149	80750	80936	80950	90	127925

TABLE 87

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	12836	11737	12476	12536	235	334448	12936	12136	12686	12736	201	240722
5%	36307	35308	35874	35908	242	301152	36407	35494	36063	36107	237	287097

10%	59128	58129	58629	58629	221	293907	59128	58429	58842	58878	167	299812
20%	98476	97477	97897	97877	212	240680	98476	97577	98140	98177	207	266056
50%	200437	199538	199864	199887	193	187797	200437	199838	200134	200137	161	229261

TABLE 88

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	25792	24094	25142	25193	417	458267	26390	24494	25649	25692	356	409464
5%	72419	70821	71546	71520	371	434415	72719	71220	72104	72019	423	412546
10%	117965	116167	117135	117066	439	416072	118065	116765	117568	117565	319	441646
20%	195571	194272	194964	194971	341	425442	195970	194472	195351	195420	398	430950
50%	400015	398917	399518	399616	286	413777	400415	399416	399878	399916	216	409055

TABLE 89

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	48685	<b>45797</b>	47712	47666	592	494990	<b>50383</b>	<b>48389</b>	49458	49487	474	<b>489591</b>
5%	143147	139787	141506	141618	774	495940	<b>143986</b>	<b>141587</b>	<b>142685</b>	<b>142808</b>	612	<b>494218</b>
10%	233926	231629	232718	232763	668	496428	<b>234514</b>	<b>232213</b>	<b>233545</b>	<b>233556</b>	569	<b>494314</b>
20%	391129	388333	389802	389753	702	496078	<b>391629</b>	<b>388869</b>	<b>390683</b>	<b>390821</b>	659	496102
50%	803899	801902	802993	803150	584	<b>488330</b>	<b>804899</b>	<b>802298</b>	<b>803725</b>	<b>803700</b>	639	492188

TABLE 90

## COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	100787	95661	98035	98054	1226	498928	112856	107765	110302	110402	1262	498343
5%	330892	325231	327619	327652	1525	498883	341498	331288	335872	335831	2601	498960
10%	557944	550326	554411	554773	2016	498905	566415	559850	563193	562786	1876	499257
20%	956181	947965	952023	951851	1990	499105	964102	950046	959209	959640	3412	499188
50%	1998191	1992733	1994900	1994753	1503	498720	2001816	1994017	1998843	1999038	1776	498409

TABLE 91

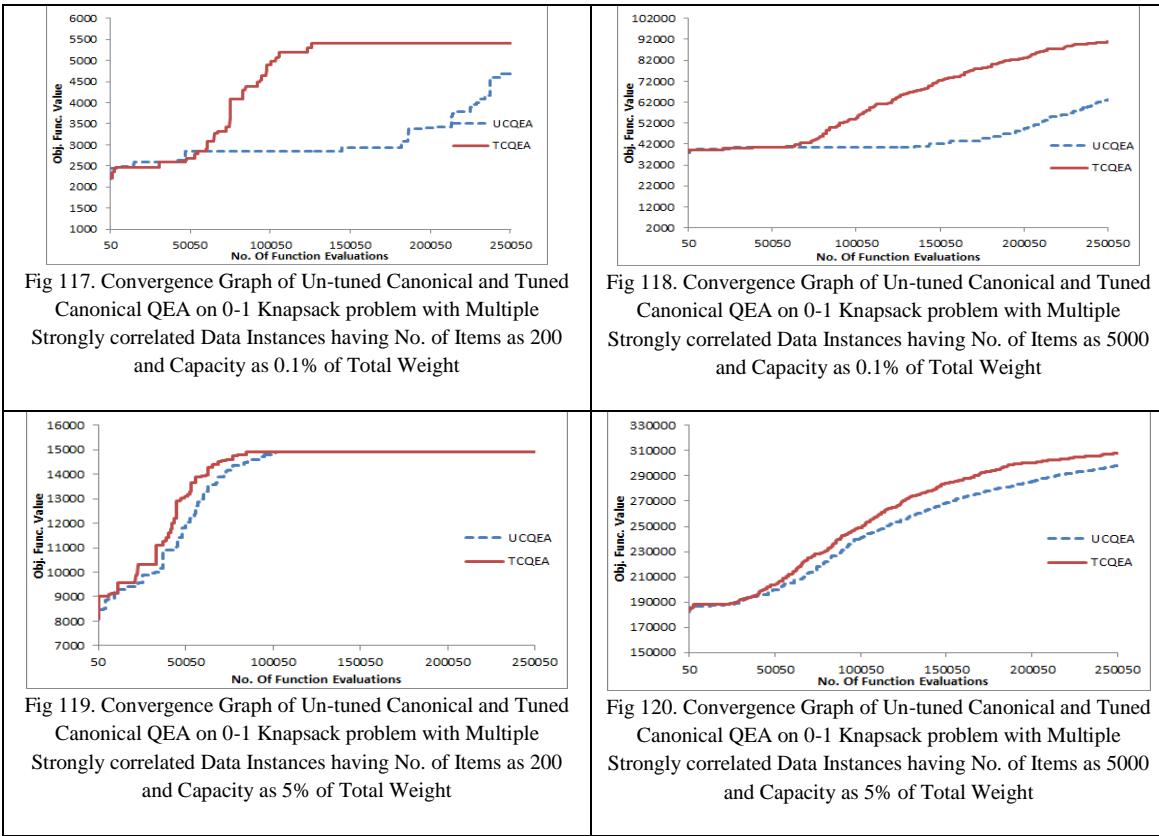
### TABLE 9

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON MULTIPLE STRONGLY CORRELATED DATA INSTANCES

% of Total	Canonical QEA	Tuned-QEA
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Weight	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	166560	157984	161856	161939	2175	499018	191838	184093	188491	188785	2000	499171
5%	600677	589762	595880	597146	2962	499280	622415	603041	611780	611599	5003	499547
10%	1046490	1026935	1036658	1035996	5043	499270	1066115	1039265	1051718	1052376	6790	499425
20%	1848462	1811336	1829781	1829247	7707	499192	1862683	1822019	1846970	1848248	9210	499544
50%	3970550	3949471	3960685	3961109	4825	499077	3982008	3949092	3964796	3965426	6382	499498

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.



In order to confirm the findings in Table 85 to 91, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Multiple Strongly Correlated Data Instances of Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 92, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 303 at significance level of  $\alpha = 5\%$ . The sample size  $n= 41$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Multiple Strongly Correlated Data Instances of Knapsack problem.

**Table 92: Wilcoxon's Signed Rank Test on Mult. Str. Corr. Data Instances (OFV)**

	TCQEA	UCQEA			Test Statistic	Critical Value
1	X <sub>1</sub>	X <sub>2</sub>				
2	246.8	246.8				
3	1168.3	1078.3				
4	2178.7	1640.2				
5	3727.7	2692.9				
6	6033.2	5347.1				
7	7058.4	6713.8				
8	1260.7	790.589				
9	3383.8	1768				
10	5353.1	2756.34				
11	8123.5	4569.51				
12	12627	9691.69				
13	14673	12460.2				
14	3295.5	1217.19				
15	7810.3	3113.42				
16	11786	5192.57				
17	18147	9453.66				
18	28521	19916				
19	33093	25818.6				
20	6128.3	1771.17				
21	13736	5194.76				
22	20492	9312.04				
23	32013	17340.1				
24	51090	36042.2				
25	60172	46525.2				
26	9130.1	2696.4				
27	21811	9357.86				
28	34095	17179.9				
29	54775	32660				
30	90465	67004.4				
31	106968	85885.3				
32	14287	5263.17				
33	40527	21049.8				
34	66048	40048.1				
35	110871	77818.5				
36	193216	157688				
37	232850	199352				
38	20137	9322.22				
39	66045	40162.1				
40	111394	77550.8				
41	195672	152154				
42	354378	306007				
	431151	385901				

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 93, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 385 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n= 42$  (distinct) is

larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.2028, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table 92. This indicates the success of the proposed tuning method for tuning QEA on problems like Multiple Strongly Correlated Data Instances, even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 93: Wilcoxon's Signed Rank Test on Mult. Str. Cor. Data Inst. (Av. Gen.)**

	TCQEA	UCQEA			Test Statistic	Critical Value
1	X <sub>1</sub>	X <sub>2</sub>			T	At an $\alpha$ of 5%
2	426.7	0				
3	510.6	2.1				
4	607.4	154.1				
5	576.7	428				
6	966.5	579.1				
7	977.4	729.9				
8	3.2	3.9				
9	398.5	416.4				
10	528.1	484.1				
11	358.7	478.2				
12	357.4	847.8				
13	377.9	893.2				
14	220.5	207.2				
15	451.4	521.7				
16	291.4	623.8				
17	371.1	673.5				
18	599.2	956.8				
19	507.1	972				
20	288.9	225.4				
21	440.4	571.1				
22	476	450.6				
23	550.7	706.5				
24	547.9	964				
25	661.1	976				
26	399.4	583.6				
27	448	579.1				
28	580.1	534.7				
29	599.6	742.9				
30	718.2	964.4				
31	753.4	982.5				
32	370.2	545.8				
33	532.2	631.4				
34	645.2	516.7				
35	802.5	702.2				
36	987.5	965				
37	988.7	983.8				
38	544.4	550.4				
39	651.9	489.5				
40	849.8	590.3				
41	988	649.5				
	990.8	963.4				

**10) profit ceiling instances pceil(d) :**

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Table 94 to 100. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and the capacity of knapsack is 0.1% to 2% of the total capacity. In case of problem instance with no. of items as 200 and the capacity of knapsack is 5% of the total capacity, the best result of UCQEA is slightly better than that of TCQEA, but TCQEA is better than UCQEA on average & median result and matches on worst result for objective function value. In case of problem instance with no. of items as 1000 and the capacity of knapsack is 0.1% of the total capacity, the best result of UCQEA is slightly better than that of TCQEA, but TCQEA is better than UCQEA on worst, average & median result. However, the performance of TCQEA is better than UCQEA in rest of the instances. The performance of TCQEA is also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 123 to 128, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 94  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	486	<b>483</b>	486	<b>486</b>	1	126227	<b>489</b>	<b>483</b>	<b>485</b>	<b>486</b>	2	<b>53635</b>
5%	<b>2421</b>	2406	2412	2412	3	133537	2418	<b>2409</b>	<b>2413</b>	<b>2415</b>	3	<b>78669</b>
10%	4824	<b>4812</b>	4819	<b>4818</b>	4	139967	<b>4827</b>	<b>4812</b>	<b>4819</b>	<b>4818</b>	4	<b>128561</b>
20%	9630	<b>9618</b>	9624	9624	<b>3</b>	180320	<b>9633</b>	9615	<b>9625</b>	<b>9627</b>	4	<b>151470</b>
50%	<b>24030</b>	24012	24021	24021	5	<b>129802</b>	24027	<b>24015</b>	<b>24023</b>	<b>24024</b>	4	193624

TABLE 95  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	<b>1026</b>	<b>1014</b>	<b>1018</b>	<b>1017</b>	<b>3</b>	252298	1023	1011	1017	<b>1017</b>	3	<b>136178</b>
5%	<b>5070</b>	5049	5061	<b>5061</b>	6	198847	<b>5070</b>	<b>5052</b>	<b>5061</b>	<b>5061</b>	5	<b>158638</b>
10%	<b>10116</b>	<b>10092</b>	<b>10105</b>	<b>10104</b>	5	256945	<b>10116</b>	<b>10092</b>	<b>10105</b>	<b>10104</b>	6	<b>194697</b>
20%	20193	20163	20180	20181	<b>7</b>	285912	<b>20199</b>	<b>20166</b>	<b>20182</b>	<b>20183</b>	8	<b>180635</b>
50%	50397	<b>50373</b>	<b>50385</b>	<b>50385</b>	6	245873	<b>50403</b>	<b>50373</b>	50385	<b>50385</b>	7	<b>233591</b>

TABLE 96  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	<b>2487</b>	<b>2466</b>	<b>2475</b>	<b>2475</b>	4	<b>327145</b>	2484	2463	2472	2472	6	<b>192981</b>
5%	<b>12345</b>	<b>12306</b>	<b>12323</b>	<b>12323</b>	9	361763	12336	12303	12317	12315	8	<b>245583</b>
10%	24627	<b>24600</b>	<b>24615</b>	<b>24614</b>	8	357690	<b>24633</b>	24591	24612	24612	9	<b>276887</b>
20%	<b>49200</b>	<b>49158</b>	<b>49179</b>	<b>49176</b>	11	424377	49197	49152	49173	49170	13	<b>276731</b>

50%	122814	122775	122794	122792	9	376148	122808	122769	122789	122790	9	289390
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TABLE 97  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	4986	4956	4969	4968	7	395845	4971	4950	4961	4959	6	243157
5%	24771	24729	24752	24753	12	412405	24753	24711	24734	24738	11	379210
10%	49473	49404	49436	49437	15	461158	49458	49383	49421	49422	16	388001
20%	98796	98736	98762	98763	14	455122	98781	98718	98749	98751	16	398188
50%	246654	246579	246619	246618	18	445543	246657	246573	246609	246608	21	346196

TABLE 98  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	10050	10017	10032	10032	9	430522	10032	9999	10014	10014	9	292951
5%	50040	49968	50012	50010	15	456985	49995	49947	49975	49977	13	414962
10%	99954	99882	99916	99918	16	473622	99906	99831	99870	99866	19	429937
20%	199689	199608	199650	199652	21	486162	199692	199563	199615	199617	28	462175
50%	498711	498624	498668	498671	20	475312	498669	498567	498634	498639	26	420826

TABLE 99  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	25077	25026	25054	25053	12	471443	25050	25002	25025	25025	13	427370
5%	125004	124929	124964	124962	20	482153	124935	124824	124885	124880	31	468877
10%	249723	249627	249688	249687	22	491412	249708	249510	249614	249614	41	480117
20%	499104	498966	499029	499025	31	494853	499101	498918	499002	499001	48	481599
50%	1246764	1246647	1246714	1246719	33	495203	1246797	1246584	1246699	1246700	63	472636

TABLE 100  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON PROFIT CEILING INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	50298	50241	50273	50271	14	479530	50277	50184	50221	50217	25	469488

5%	<b>250878</b>	<b>250788</b>	<b>250837</b>	<b>250835</b>	<b>26</b>	492728	250809	250668	250747	250748	36	<b>484641</b>
10%	501393	<b>501264</b>	<b>501315</b>	<b>501317</b>	<b>34</b>	495528	<b>501420</b>	501168	501270	501273	55	<b>492753</b>
20%	1002159	1001943	1002055	1002063	<b>60</b>	495115	<b>1002252</b>	<b>1001967</b>	<b>1002094</b>	<b>1002089</b>	73	<b>487271</b>
50%	2504061	2503740	2503870	2503869	<b>71</b>	496130	<b>2504124</b>	<b>2503761</b>	<b>2503953</b>	<b>2503961</b>	104	<b>487403</b>

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in most of the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

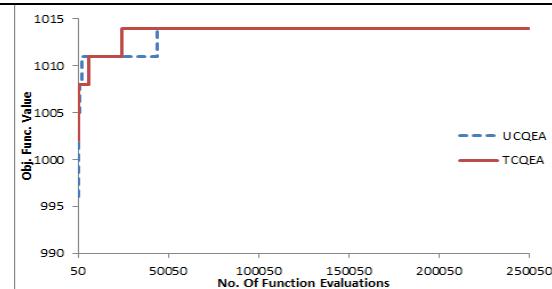


Fig 123. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with profit ceiling Instances having No. of Items as 200 and Capacity as 1% of Total Weight

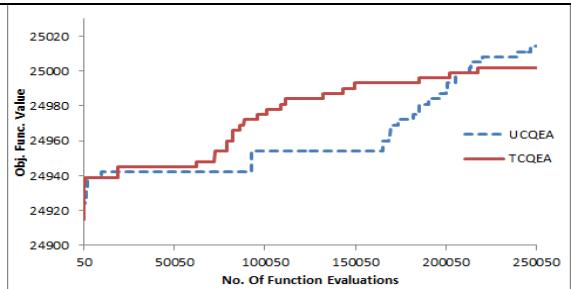


Fig 124. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with profit ceiling Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

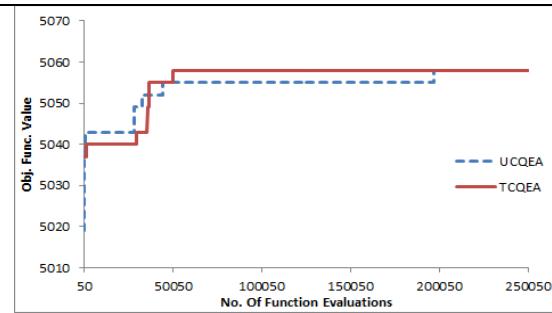


Fig 125. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with profit ceiling Instances having No. of Items as 200 and Capacity as 5% of Total Weight

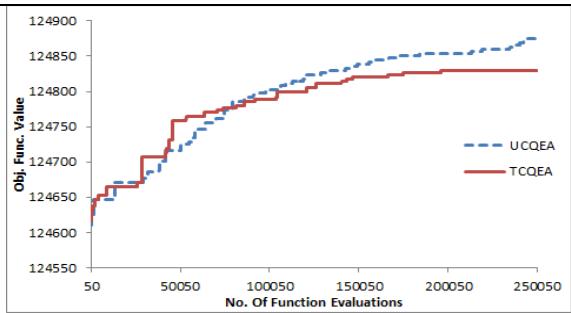


Fig 126. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with profit ceiling Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

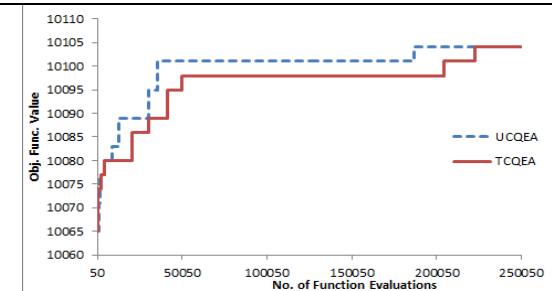


Fig 127. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with profit ceiling Instances having No. of Items as 200 and Capacity as 10% of Total Weight

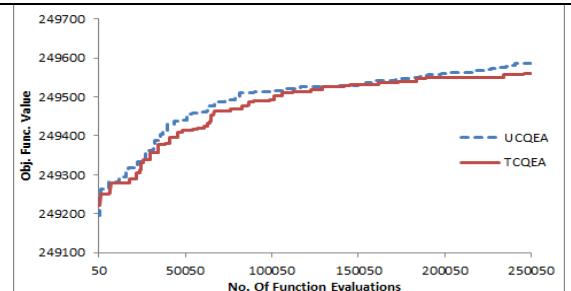


Fig 128. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with profit ceiling Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 94 to 100, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Profit Ceiling Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 101, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 287 at significance level of  $\alpha = 5\%$ . The sample size  $n= 40$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Profit Ceiling Instances of Knapsack problem.

**Table 101: Wilcoxon's Signed Rank Test on Profit Ceiling Instances (OFV)**

	TCQEA	UCQEA	$\Sigma(+)$	820	Null Hypothesis	Test Statistic	Critical Value
1	X <sub>1</sub> 48	X <sub>2</sub> 48	$\Sigma(-)$	0	$H_0: \mu_1 \leq \mu_2$	T 0	At an $\alpha$ of 5% 287
2	243	243					
3	485	484					
4	966.9	966.2					
5	1931.2	1929.1	n	40			
6	2412.2	2410.3					
7	105.2	105					
8	509.9	508.3					
9	1015.7	1013.5					
10	2026.4	2021.9					
11	4044.9	4042.3					
12	5054	5053.7					
13	250.7	249.7					
14	1237	1234					
15	2468.9	2462.5					
16	4929.1	4919.1					
17	9850.1	9839.3					
18	12307	12301.3					
19	499.6	498.4					
20	2477.8	2473.5					
21	4953.9	4941.6					
22	9894.7	9875.2					
23	19771.7	19750					
24	24706.8	24691.7					
25	1005.8	1002.9					
26	5006.8	4994.9					
27	10003.2	9983.4					
28	19991.4	19955.8					
29	39951	39918					
30	49926.4	49902.6					
31	2509.3	2501					
32	12504	12477.8					
33	24987.6	24945.2					
34	49942.4	49877.5					
35	99831.6	99766.1					
36	124779	124714					
37	5033.7	5017.2					

**For Large Samples ( $n > 25$ )**

Test Statistic

$$z \quad -5.51093 \quad E[T] \quad 410 \\ \sigma(T) \quad 74.39758061$$

At an  $\alpha$  of

Null Hypothesis	p-value	5%
$H_0: \mu_1 \leq \mu_2$	0.0000	Reject

38	25101.7	25053.3
39	50163.7	50092.5
40	100267	100167
41	200464	200339
42	250549	250438

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 102, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 638 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n = 42$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.9901, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table. 101. This indicates the success of the proposed tuning method for tuning QEA on problems like Profit Ceiling Instances, even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 102: Wilcoxon's Signed Rank Test on Profit Ceiling Instances (Av. Gen.)**

	TCQEA	UCQEA	$\Sigma(+)$	638	$\Sigma(-)$	265	n	42	Null Hypothesis	$H_0: \mu_1 \geq \mu_2$	Test Statistic	Critical Value
	X <sub>1</sub>	X <sub>2</sub>									T	At an $\alpha$ of 5%
1	2.3	2.5										
2	40.8	39.8										
3	290.2	185.1										
4	293.1	345.6										
5	412.1	529.1										
6	455.1	621.7										
7	197.8	262.8										
8	286	185.1										
9	342.1	297.2										
10	407.6	333.2										
11	449.7	771.8										
12	550.2	829.9										
13	195.8	176.8										
14	432.1	306.2										
15	444	276.3										
16	528.2	380.1										
17	739.2	889.8										
18	780	948.1										
19	244	273.2										
20	370.5	416.1										
21	727.2	340.5										
22	846.2	548.6										
23	869.1	898.4										
24	873.2	944.3										
25	448.3	247.2										
26	655.2	342.5										
27	785.4	463.2										
28	879.3	485.6										
29	900.9	931.2										
30	897.8	966.7										
31	539.9	403.1										

#### For Large Samples ( $n > 25$ )

Test Statistic

$z$

-2.33193

$E[T]$

451.5

$\sigma(T)$

79.97655907

At an  $\alpha$  of

p-value

5%

$H_0: \mu_1 \geq \mu_2$

0.9901

32	842.2	370.3
33	868.5	349.4
34	947.3	450.6
35	939.2	955.1
36	949.1	956.8
37	742.2	306.1
38	876.6	332.1
39	940.4	365.7
40	939.5	539.9
41	951.9	942.9
42	960.9	957.7

11) *circle instances circle(d) :*

The result of comparative study between Tuned QEA (TCQEA) and Canonical QEA (UCQEA) are given in Table 103 to 109. The performance of TCQEA and UCQEA are similar when the no. of items to choose is 100 and the capacity of knapsack is 0.1 % of the total capacity. However, with the increase in number of items and the capacity of knapsack, the performance of TCQEA has improved over UCQEA in all the instances. The performance of TCQEA was also good on speed of convergence as indicated by average generations and the convergence graphs shown in Fig. 129 to 134, which also compared the speed of convergence of TCQEA to UCQEA.

TABLE 103  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 100 ON CIRCLE INSTANCES

% of Total	Canonical QEA						Tuned-QEA					
	Weight	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std
1%	2174	1981	2084	2089	61	219922	2528	2215	2512	2528	61	113375
5%	9210	8955	9168	9204	78	110665	9210	8981	9198	9210	41	98789
10%	15838	15489	15727	15830	130	58628	15838	15504	15743	15830	117	73511
20%	25848	25760	25799	25800	28	65242	25848	25768	25821	25821	23	75230
50%	49049	48974	49009	49009	21	99695	49032	48979	49006	49009	15	105656

TABLE 104

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 200 ON CIRCLE INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	5802	5210	5591	5613	143	318865	5802	5407	5671	5680	82	158776
5%	19046	18462	18772	18774	126	138005	19047	18638	18811	18778	102	177933
10%	31587	30811	31265	31257	206	116388	31587	30923	31298	31289	150	147929
20%	51224	50534	51028	51100	158	105522	51216	50673	51091	51106	93	146698
50%	99189	98567	98875	98814	223	159023	99182	98600	99045	99177	190	190826

TABLE 105

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 500 ON CIRCLE INSTANCES

Comparative Results for 0-1 Knapsack Problem with No. of Items 500 on Circle Instances												
% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen

1%	13533	12173	13192	13331	321	358892	13713	12907	13314	13331	216	299973
5%	45669	44058	45171	45217	312	313940	45694	44359	45289	45370	320	343570
10%	76758	74937	76162	76242	370	298185	76788	75465	76321	76352	315	332571
20%	127820	126629	127195	127189	319	292023	127698	126699	127393	127478	247	302933
50%	247430	246599	247070	247109	211	266805	247701	247046	247289	247173	245	287437

TABLE 106

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 1000 ON CIRCLE INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	26811	24683	25964	26026	532	467325	26901	25126	26190	26253	492	427885
5%	90456	88152	89452	89452	517	456892	90773	88820	89845	89872	451	467795
10%	152924	150373	151642	151626	569	438443	152994	151388	152136	152067	416	452126
20%	254180	251238	253182	253279	673	439062	254559	252261	253432	253460	519	406392
50%	495096	493346	494190	494183	417	435308	495118	493283	494478	494545	402	439280

TABLE 107

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 2000 ON CIRCLE INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	49991	48344	49226	49264	426	498490	52072	49711	51015	51148	548	495257
5%	176949	172392	174911	174875	896	497640	178307	172816	176641	176779	1136	497228
10%	300645	296538	298608	298674	1067	497538	301895	298857	300197	300152	798	498399
20%	506875	503546	505441	505486	1010	496865	508014	505229	506829	507069	777	496241
50%	992783	990446	991643	991614	607	487825	993478	990992	992362	992218	811	489565

TABLE 108

COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 5000 ON CIRCLE INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	107044	102173	104741	104807	1249	498433	114531	111149	113044	112954	926	498610
5%	401861	393326	398179	398439	1937	498628	412379	404551	408455	408556	1952	499260
10%	703183	692142	697450	696994	2946	498935	718847	703895	710138	710179	4590	499036
20%	1220793	1204028	1212524	1213070	4182	498712	1235625	1219591	1228249	1228672	4297	499310

50%	2464209	2451488	2458580	2458255	<b>2723</b>	<b>498255</b>	<b>2470755</b>	<b>2458187</b>	<b>2463903</b>	<b>2463763</b>	2843	498861
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TABLE 109  
COMPARATIVE RESULTS FOR 0-1 KNAPSACK PROBLEM WITH NO. OF ITEMS 10000 ON CIRCLE INSTANCES

% of Total Weight	Canonical QEA						Tuned-QEA					
	Best	Worst	Average	Median	Std	Av. Gen	Best	Worst	Average	Median	Std	Av. Gen
1%	186565	179029	182039	181999	1776	<b>499033</b>	<b>204276</b>	<b>197290</b>	<b>200602</b>	<b>200501</b>	1693	499382
5%	733283	716838	723383	723322	<b>4120</b>	<b>499132</b>	<b>760693</b>	<b>728628</b>	<b>744025</b>	<b>744078</b>	7558	499376
10%	1298905	1282621	1292184	1292971	<b>4817</b>	<b>499083</b>	<b>1344170</b>	<b>1295281</b>	<b>1316045</b>	<b>1317789</b>	12332	499514
20%	2310887	2279802	2293063	2294553	<b>7884</b>	<b>499178</b>	<b>2356125</b>	<b>2295050</b>	<b>2324957</b>	<b>2326246</b>	13563	499445
50%	4847341	<b>4817442</b>	4833333	4836603	<b>10274</b>	<b>499165</b>	<b>4865335</b>	<b>4819671</b>	<b>4842682</b>	<b>4843552</b>	12631	499660

The convergence graphs have been plotted between objective function value and number of generations for both TCQEA and UCQEA for all the problem instances having No. of Items as 200 and 5000 for the median run. The convergence graph clearly establishes the superiority of Tuning as the TCQEA is faster than UCQEA in all the graphs. The difference in performance between TCQEA and UCQEA increases with the capacity size and number of items in the knapsack problem.

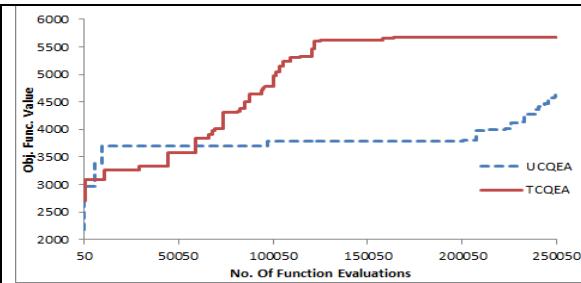


Fig 129. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with circle Instances having No. of Items as 200 and Capacity as 1% of Total Weight

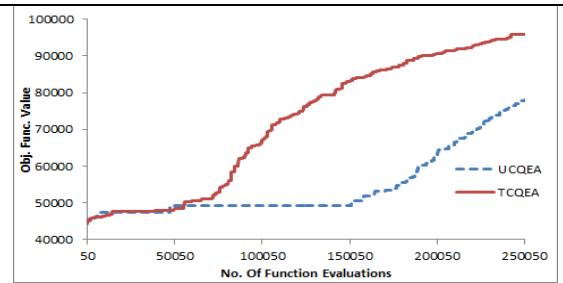


Fig 130. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with circle Instances having No. of Items as 5000 and Capacity as 1% of Total Weight

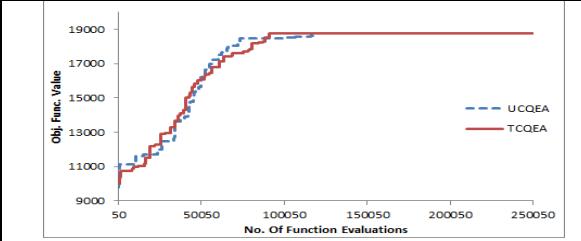


Fig 131. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with circle Instances having No. of Items as 200 and Capacity as 5% of Total Weight

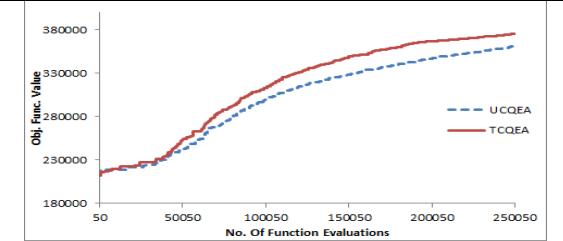


Fig 132. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with circle Instances having No. of Items as 5000 and Capacity as 5% of Total Weight

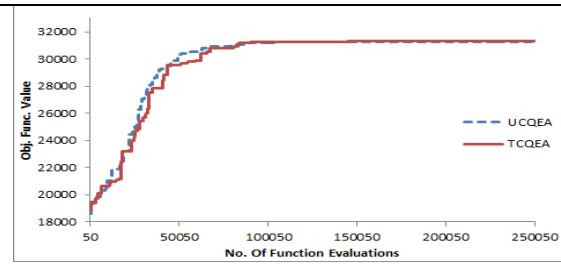


Fig 133. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with circle Instances having No. of Items as 200 and Capacity as 10% of Total Weight

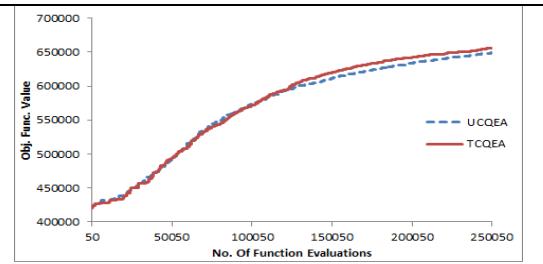


Fig 134. Convergence Graph of Un-tuned Canonical and Tuned Canonical QEA on 0-1 Knapsack problem with circle Instances having No. of Items as 5000 and Capacity as 10% of Total Weight

In order to confirm the findings in Table 103 to 109, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average number of generation (Av. Gen.) of Tuned QEA and Canonical QEA on all the instances of Circle Knapsack problem at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 110, which shows that null hypothesis can be rejected safely as Wilcoxon's Signed Rank test statistic is 0 and less than the critical value of 303 at significance level of  $\alpha = 5\%$ . The sample size  $n= 41$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.000, which is less than 0.05, so null hypothesis can be safely rejected. This indicates the success of the proposed tuning method for tuning QEA on problems like Circle Instances of Knapsack problem.

**Table 110: Wilcoxon's Signed Rank Test on Circle Instances (OFV)**

	TCQEA	UCQEA			Test Statistic	Critical Value
	$X_1$	$X_2$				
1	287.4	287.4				
2	1332.5	1200				
3	2496.3	2007.4				
4	4400.1	3378				
5	7612.7	6775				
6	9106.2	8567.3				
7	880.6	697.4				
8	3207.5	2067.2				
9	5522	3415.1				
10	9362.5	5742.4				
11	15613	12111.6				
12	18504.9	15767.1				
13	2085.2	1236.4				
14	7086.4	3923.8				
15	12341.6	6563.9				
16	20901.3	11819.6				
17	35527.8	25001.6				
18	41971.4	32429.7				
19	3937.8	1994.7				
20	13039.4	6565.1				
21	22044	11550.9				
22	37721.7	21244				
23	64260.7	44709.6				
24	76472.1	57624.2				

$\Sigma(+)$	861	Null Hypothesis	Test Statistic	Critical Value
$\Sigma(-)$	0	$H_0: \mu_1 \leq \mu_2$	$T$	At an $\alpha$ of 5%
$n$	41	0	303	

<b>For Large Samples (<math>n &gt; 25</math>)</b>		
Test Statistic	$z$	$E[T]$
	-5.57857	430.5
		$\sigma(T)$
		77.17026629
At an $\alpha$ of		
Null Hypothesis	$p$ -value	5%
$H_0: \mu_1 \leq \mu_2$	0.0000	<b>Reject</b>

25	6600.8	3283.2
26	22644.4	11612.7
27	39066.6	21192.5
28	66512.3	39654.4
29	114633	81733.4
30	136515	104687
31	13300.1	6544.4
32	46979.6	25637.3
33	81120.2	48347.5
34	139685	92543.1
35	243410	188210
36	291834	238816
37	21354.4	11603.7
38	79496.6	48200.2
39	140017	92013.3
40	245692	178704
41	439199	360713
42	528850	455649

In case of comparison on Av. Gen., the null hypothesis for comparison was Av. Gen. of Tuned QEA  $\mu_1$  is greater than or equal to the Av. Gen. of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 < \mu_2$ . The result of test is presented in Table 111, which shows that null hypothesis cannot be rejected as Wilcoxon's Signed Rank test statistic is 746 and more than the critical value of 319 at significance level of  $\alpha = 5\%$ , which indicates that Tuned QEA requires more number of generations as compared to Canonical QEA. The sample size  $n = 42$  (distinct) is larger than 25 so z statistic has also been computed under the normal distribution and p value obtained is 0.9999, which is more than 0.05, so null hypothesis cannot be rejected. However, Tuned QEA is performing significantly better than Canonical QEA as shown by Table 110. This indicates the success of the proposed tuning method for tuning QEA on problems like Circle Instances, even though it requires more number of generations and function evaluations as compared to Canonical QEA.

**Table 111: Wilcoxon's Signed Rank Test on Circle Instances (Av. Gen.)**

	TCQEA	UCQEA			Test Statistic $T$	Critical Value At an $\alpha$ of 5%
			X <sub>1</sub>	X <sub>2</sub>		
1	2.3	2.5				
2	322.4	561.2				
3	287.4	490.7				
4	392	512.6				
5	629.2	971				
6	613	965.5				
7	293.9	513.9				
8	554.4	445.5				
9	727	606.3				
10	816.4	495.4				
11	871.3	965				
12	882	971.7				
13	746.3	479.1				
14	913.1	545.6				
15	939.4	521				
16	954.9	687				
17	969.8	969.2				
18	971.3	983.6				

For Large Samples ( $n > 25$ )		
Test Statistic	$z$	$E[T]$
	-3.68233	451.5
		$\sigma(T)$ 79.97655907
At an $\alpha$ of		
Null Hypothesis	$p$ -value	5%
$H_0: \mu_1 \geq \mu_2$	0.9999	

19	870.5	484.1
20	940.5	553.5
21	979.6	494.1
22	982.8	641.7
23	979.2	977.9
24	984.9	983
25	917.9	478
26	978.1	508.2
27	984.4	599
28	987.9	629
29	988.2	972
30	988.4	978.3
31	966.1	597.4
32	985.7	447.2
33	988.4	585.6
34	984.7	752.7
35	990.2	967.7
36	991.3	982.9
37	963.2	542.6
38	979.1	527.8
39	988	587.1
40	994.1	662.7
41	993.4	964.3
42	992.9	982

#### D. P-PEAKS Problem [43], [47]

It is a multimodal problem generator, which is an easily parameterizable task with a tunable degree of difficulty. The advantage of using problem generator is that it removes the opportunity to hand-tune algorithms to a particular problem, thus, allows a large fairness while comparing the performance of different algorithms or different instances of same algorithm. It helps in evaluating the algorithms on a high number of random problem instances, so that the predictive power of the results for the problem class as a whole is very high.

The idea of P-PEAKS is to generate P random N-bit strings that represent the location of P peaks in the search space. The fitness value of a string is the hamming distance between this string and the closest peak, divided by N (as shown in Eq. 10). Using a higher (or lower) number of peaks we obtain more (or less) epistatic problems. The maximum fitness value for the problem instances is 1.0

$$f_{P-PEAKS}(\vec{x}) = \frac{1}{N} \max_{1 \leq i \leq p} \{N - \text{Hamming D}(\vec{x}, \text{Peak}_i)\} \quad (10)$$

The QEA was tuned by using the proposed framework on problem with P=100 and N=1000. The initial range of values for parameters in QEA used for tuning is given in Table 112. The parameter range for magnitude of rotation angles ( $\theta_1, \theta_2, \theta_4, \theta_6, \theta_7 & \theta_8$ ) is 0 to  $0.05\pi$  as they change by small magnitude as compared to  $\theta_3 & \theta_5$ , whose range is from 0 to  $0.5\pi$ , which is very large as compared to the range suggested by [14]. The direction of rotation depends on the sign of  $\alpha, \beta$  and relative fitness as per Table 1. The range for population size is 10 to 200, and covers the values for similar parameter for most studies in Evolutionary Algorithms. The range for group size is 1 to 20, which is four times bigger than the value suggested in [14]. The range for global migration is 1 to 500, which is again five times the value suggested by [14]. The change in value of each parameter during the tuning process is depicted in Table 113 and shown in Figures 135 to 145. Initially, there were three rounds for exploration stage and one round for exploitation stage after which optimal value was reached in every independent run, however, the average number of function evaluations was 3,45,444.5, which was considered high. It was decided to further tune the QEA for reducing the average number of function evaluation without sacrificing the quality of objective function value, so, it was decided to run the Exploration stage again with the best set of parameter vector found so far as the PIVOT. The average number of function evaluations was reduced to almost half to 1,76,830. Then again exploitation was carried out to further reduce the average number of function evaluations to 1,57,808.934 and it was decided to stop further tuning to conserve resources.

TABLE 112  
INITIAL RANGE OF PARAMETERS OF CANONICAL QEA

Parameter	$\theta_1$ ( $^*\pi$ )	$\theta_2$ ( $^*\pi$ )	$\theta_3$ ( $^*\pi$ )	$\theta_4$ ( $^*\pi$ )	$\theta_5$ ( $^*\pi$ )	$\theta_6$ ( $^*\pi$ )	$\theta_7$ ( $^*\pi$ )	$\theta_8$ ( $^*\pi$ )	Pop size	Group size	Global Migration
Lower Limit	0	0	0	0	0	0	0	0	5	1	1
Upper Limit	0.05	0.05	0.5	0.05	0.5	0.05	0.05	0.05	200	20	500

TABLE 113  
BEST PARAMETER VECTOR (PIVOT) DURING TUNING PROCESS

Iter. No.	Best Pivot Parameter Value										Av. OFV
	$\theta_1$ ( $^*\pi$ )	$\theta_2$ ( $^*\pi$ )	$\theta_3$ ( $^*\pi$ )	$\theta_4$ ( $^*\pi$ )	$\theta_5$ ( $^*\pi$ )	$\theta_6$ ( $^*\pi$ )	$\theta_7$ ( $^*\pi$ )	$\theta_8$ ( $^*\pi$ )	Pop. Size	Grp. No.	Glb. Mig.
Explor. – 1	0.0035	0.0035	0.015	0.0035	0.015	0.0035	0.0035	0.0035	72	8	200 0.93
Explor. – 2	0.0035	0	0.015	0.00175	0.018	0.0035	0.0035	0	90	10	100 0.99
Explor. – 3	0.0035	0	0.406	0.00133	0.0726	0	0.003	0	144	16	383 1.0
Exploit. – 4	0.0037	0	0.38	0.001	0.0739	0	0.009	0.008	136	8	369 1.0
Explor. – 5	0.0182	0	0.184	0.0764	0.0768	0	0.0107	0.0803	130	5	130 1.0
Exploit. - 6	0.0184	0	0.169	0.0784	0.0768	0	0.0163	0.0818	132	4	125 1.0

The parameter value for  $\theta_1$  remained constant during initial exploration & exploitation and then increased during final exploration & exploitation stages. The parameter value for  $\theta_2$  initially decreased to 0 and then remained constant in subsequent stages of exploration & exploitation. The parameter value for  $\theta_3$  increased during initial exploration and then decreased during subsequent stages. The parameter value for  $\theta_4$  decreased during initial exploration & exploitation and then increased during subsequent exploration and exploitation stages. The parameter value for  $\theta_5$  increased during initial exploration and then increased a little during subsequent stages. The parameter value for  $\theta_6$  initially remained constant and then decreased during exploration and then remained constant during subsequent stages. The parameter value for  $\theta_7$  remained constant during initial exploration and then kept on increasing during subsequent stages. The parameter value for  $\theta_8$  decreased during initial exploration and then increased during subsequent stages. The parameter value for Population Size increased during initial exploration and then decreased slightly during subsequent stages. The parameter value for group size increased during initial exploration and then decreased during subsequent stages. The parameter value for Global Migration decreased and then increased during initial exploration and then remained almost constant during initial exploitation and decreased substantially during subsequent stages. The final value of each parameter is given in Table 113 in the last row. The convergence graph given in figure 146 shows convergence to optimal within 2000 generations.

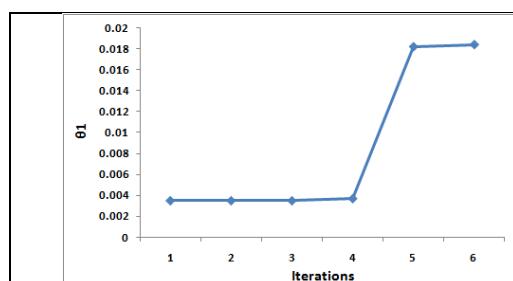


Fig 135. Change in  $\theta_1$  value during Tuning Process

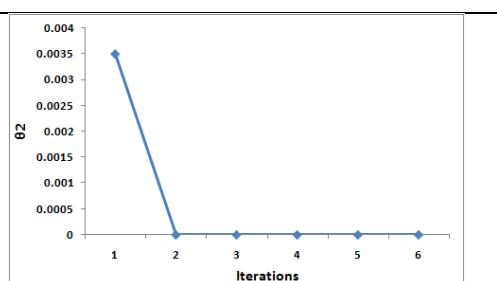


Fig 136. Change in  $\theta_2$  value during Tuning Process

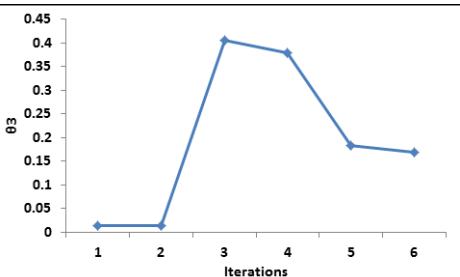


Fig 137. Change in  $\theta_3$  value during Tuning Process

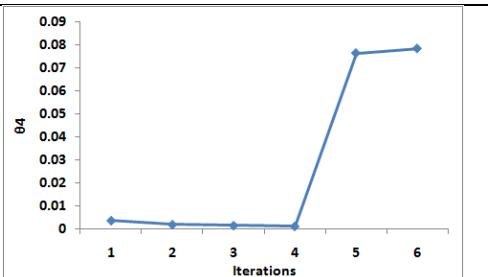


Fig 138. Change in  $\theta_4$  value during Tuning Process

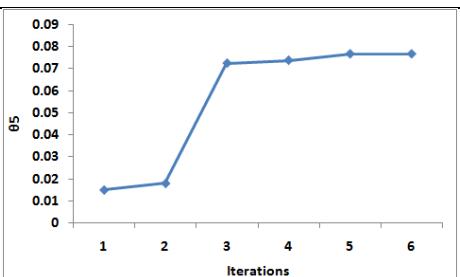


Fig 139. Change in  $\theta_5$  value during Tuning Process

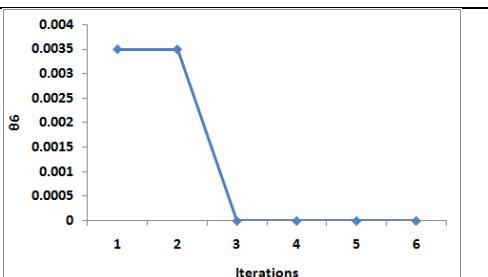


Fig 140. Change in  $\theta_6$  value during Tuning Process

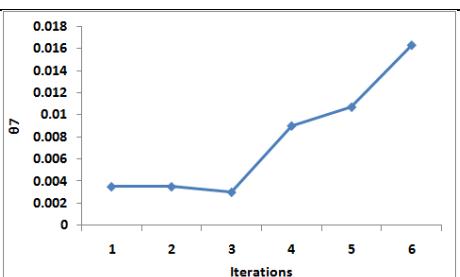


Fig 141. Change in  $\theta_7$  value during Tuning Process

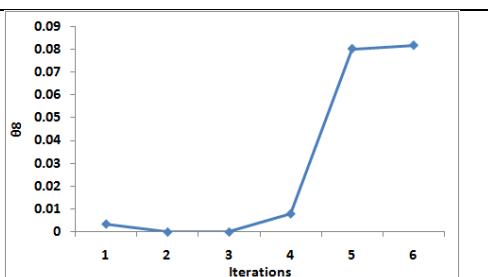


Fig 142. Change in  $\theta_8$  value during Tuning Process

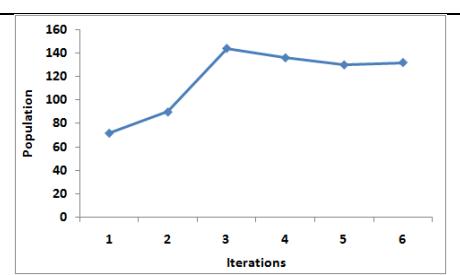


Fig 143. Change in Population size during Tuning Process

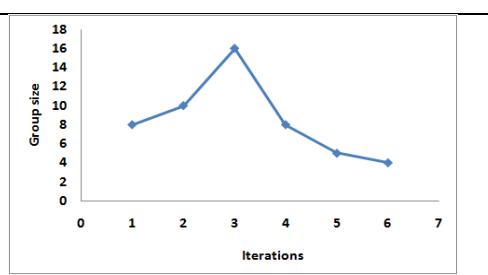


Fig 144. Change in Group Size during Tuning Process

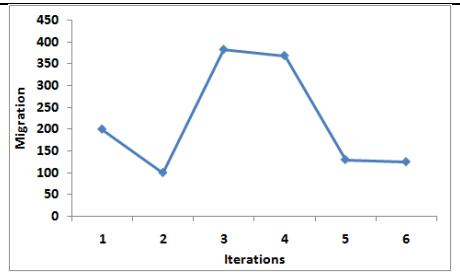


Fig 145. Change in Migration value during Tuning Process

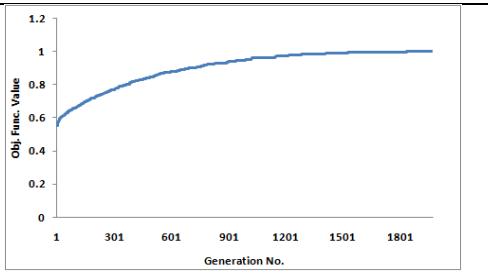


Fig 146. Convergence Graph of TCQEA

The deviation from optimal for large variation in parameter setting, which is computed from the average results of fifty runs

from the first iteration of exploration stage is 0.09 whereas the deviation from optimal for small variation in parameter setting, which is computed from the average results of twenty seven runs from the first iteration of exploitation stage is zero. Therefore, the tuned QEA is robust to small variation in parameter set, and is also stable for large variation in parameter set. Thus, the deviation from the known optimal at the end of the iteration can be used for deciding the need for continuing the search in that stage. However, if the Optimal is unknown then this strategy cannot be applied with the same confidence, but in case of real world problems, often a best known solution is available, so in such cases, it may be used in place of the Optimal.

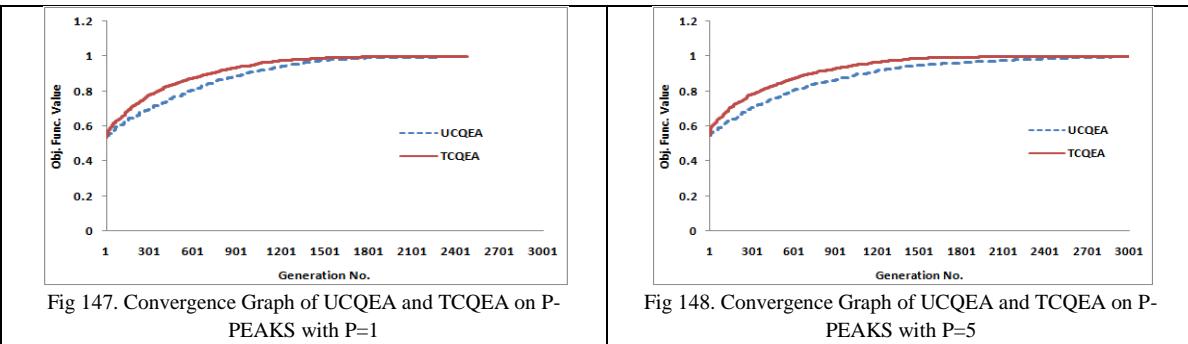
The comparative study performed between parameter Tuned QEA (TCQEA) and Canonical QEA (UCQEA) with Parameters given in Table 2 on a set of twenty one instances of P Peaks problem with  $P = 1$  to 1000 peaks of length  $N = 1000$  bits each. The results are given in Table 114. Thirty independent runs were made on each problem size and the comparison has been made on Best, Median, Worst, Mean, percentage of runs in which optimal was achieved i.e. percentage of Success runs, and Average number of function evaluations (NFE). The Canonical QEA was able to reach optimal value till problem size 700, but was not able to find the optimal for rest of the problem instances whereas the Tuned-QEA was able to reach 100% success in all the problem instances. The performance of Tuned QEA was also good on speed of convergence as indicated by average NFE and the convergence graphs shown in figures 147 to 167, which also compared the speed of convergence of Tuned QEA to Canonical QEA.

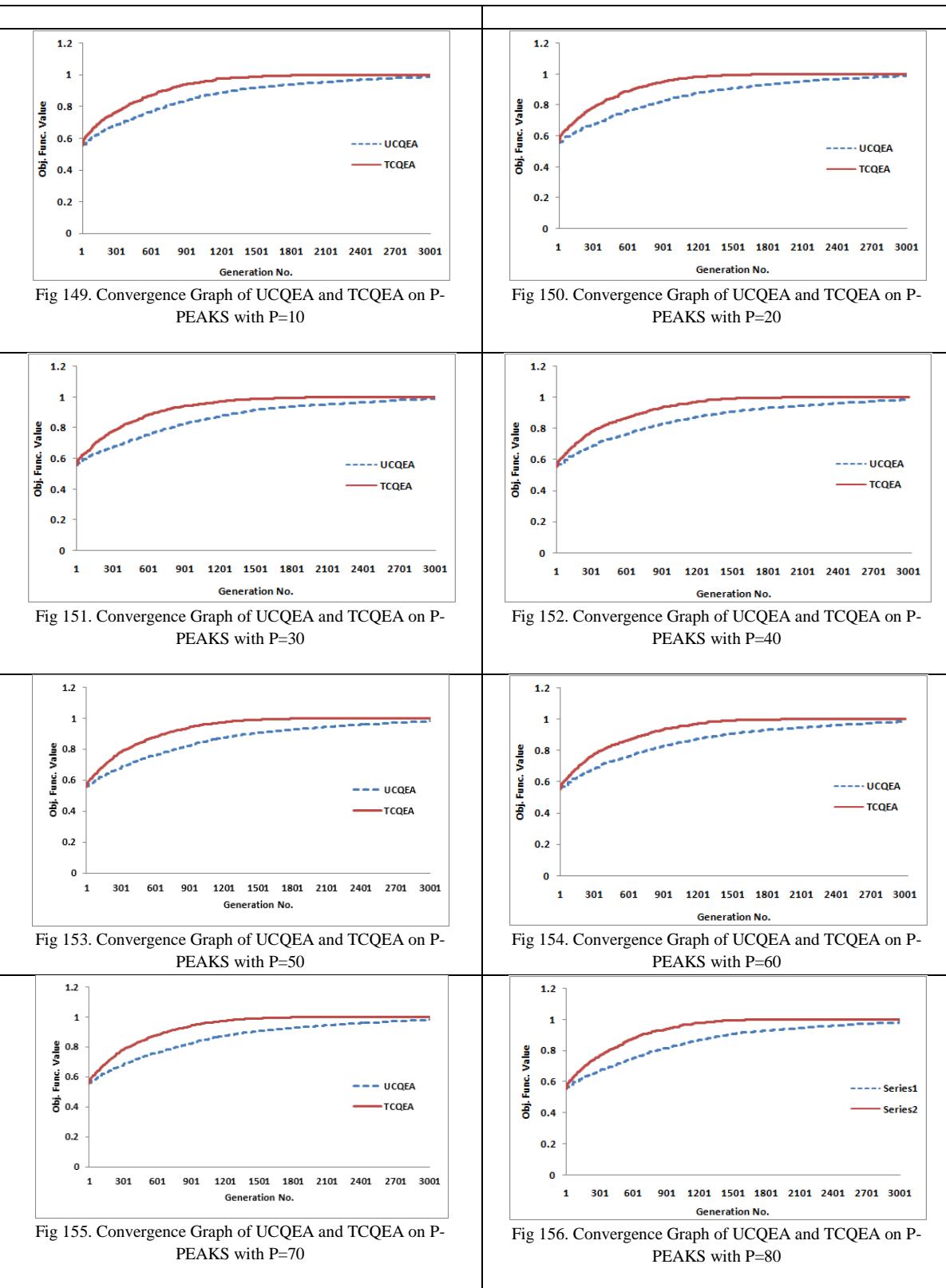
The convergence graphs have been plotted between objective function value and number of generations for both Tuned QEA and Canonical QEA for all the problem instances of P-PEAKS for the median run. The convergence graph clearly establishes the superiority of Tuning as the Tuned QEA is able to reach the near optimal in less than 2000 generations in all the graphs whereas the Canonical QEA is much slower and takes much larger number of generations to reach near the optimal.

TABLE 114  
COMPARATIVE STUDY BETWEEN TCQEA AND UCQEA ON P-PEAKS PROBLEM INSTANCES

No. Of Peaks	Algo	Best	Worst	Avg.	Median	% Success Runs	Std	Average NFE
1	UCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	115592
	TCQEA	<b>1.0000</b>	<b>1.0000</b>	<b>1.0000</b>	<b>1.0000</b>	<b>100.0</b>	<b>0.0000</b>	<b>97106</b>
5	UCQEA	1.0000	0.9750	0.9961	0.9980	33.3	0.0054	146393
	TCQEA	<b>1.0000</b>	<b>1.0000</b>	<b>1.0000</b>	<b>1.0000</b>	<b>100.0</b>	<b>0.0000</b>	<b>99634</b>
10	UCQEA	1.0000	0.9690	0.9893	0.9905	16.7	0.0095	148433
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	97085
20	UCQEA	1.0000	0.9770	0.9878	0.9870	6.7	0.0068	149448
	TCQEA	1.0000	0.9990	1.0000	1.0000	96.6	0.0002	99803
30	UCQEA	1.0000	0.9680	0.9887	0.9890	6.7	0.0075	149817
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98480
40	UCQEA	0.9990	0.9720	0.9867	0.9870	0.0	0.0072	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98674
50	UCQEA	0.9980	0.9560	0.9798	0.9810	0.0	0.0088	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	102432
60	UCQEA	1.0000	0.9700	0.9846	0.9835	3.3	0.0077	149988
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	100749
70	UCQEA	0.9960	0.9690	0.9859	0.9875	0.0	0.0074	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98008

	UCQEA	0.9990	0.9660	0.9848	0.9865	0.0	0.0088	150050
	TCQEA	1.0000	0.9990	1.0000	1.0000	96.6	0.0002	101893
90	UCQEA	0.9960	0.9580	0.9842	0.9840	0.0	0.0084	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	97146
100	UCQEA	0.9990	0.9710	0.9847	0.9840	0.0	0.0073	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	96928
200	UCQEA	0.9910	0.9630	0.9811	0.9835	0.0	0.0075	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98459
300	UCQEA	1.0000	0.9690	0.9838	0.9840	3.3	0.0068	150027
	TCQEA	1.0000	0.9990	1.0000	1.0000	96.6	0.0002	99822
400	UCQEA	0.9950	0.9600	0.9827	0.9845	0.0	0.0082	150050
	TCQEA	1.0000	0.9990	1.0000	1.0000	96.6	0.0002	99416
500	UCQEA	0.9940	0.9660	0.9816	0.9825	0.0	0.0074	150050
	TCQEA	1.0000	0.9990	1.0000	1.0000	96.6	0.0002	100646
600	UCQEA	0.9950	0.9630	0.9804	0.9810	0.0	0.0088	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98568
700	UCQEA	0.9940	0.9620	0.9802	0.9840	0.0	0.0088	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	97398
800	UCQEA	0.9940	0.9620	0.9785	0.9790	0.0	0.0081	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98970
900	UCQEA	0.9940	0.9620	0.9815	0.9840	0.0	0.0092	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	98654
1000	UCQEA	0.9940	0.9690	0.9817	0.9840	0.0	0.0058	150050
	TCQEA	1.0000	1.0000	1.0000	1.0000	100.0	0.0000	100277





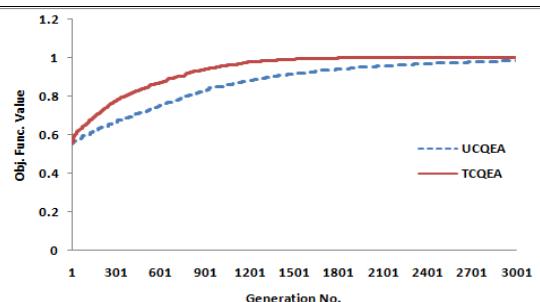


Fig 157. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=90$

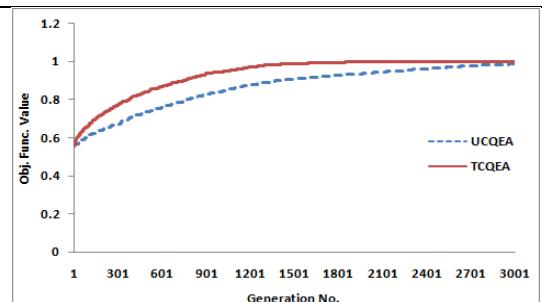


Fig 158. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=100$

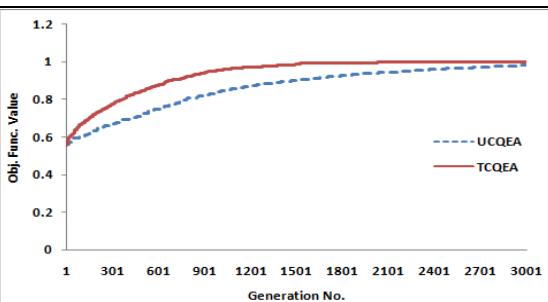


Fig 159. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=200$

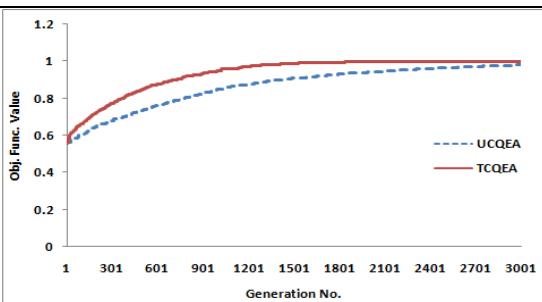


Fig 160. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=300$

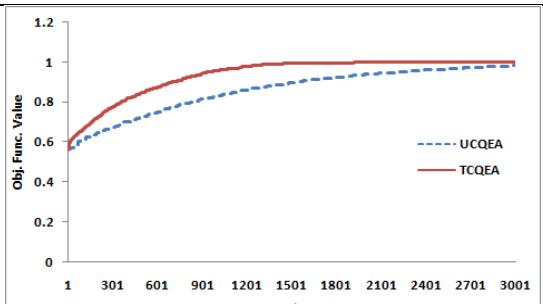


Fig 161. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=400$

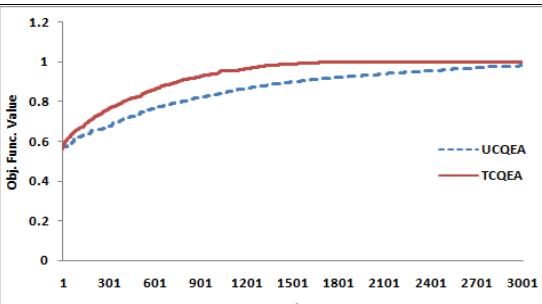


Fig 162. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=500$

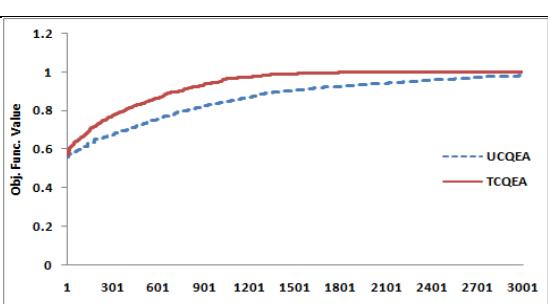


Fig 163. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=600$

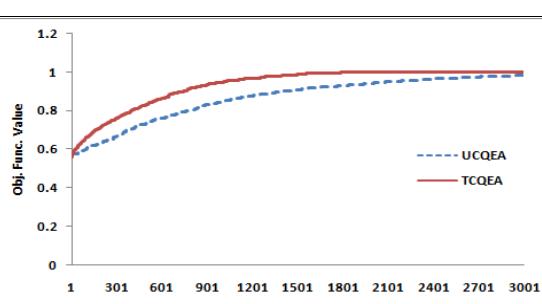


Fig 164. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=700$

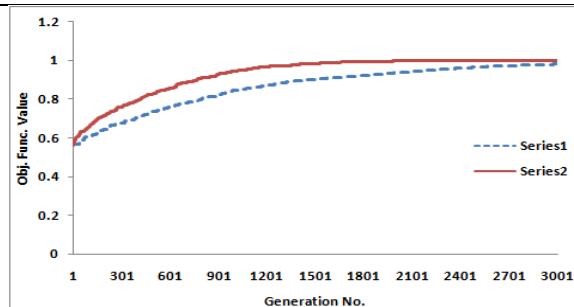


Fig 165. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=800$

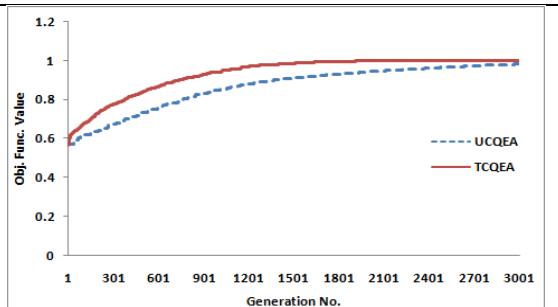


Fig 166. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=900$

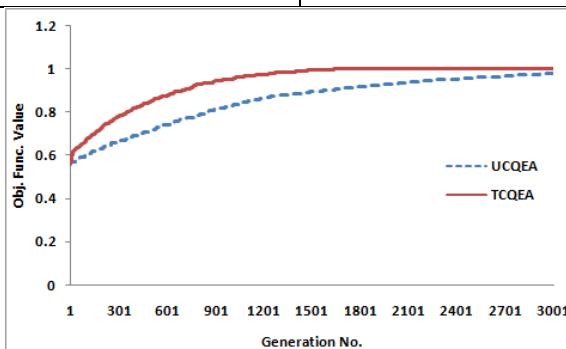


Fig 167. Convergence Graph of UCQEA and TCQEA on P-PEAKS with  $P=1000$

The performance of Tuned QEA is superior to Canonical QEA for all instances of P-PEAKS Problem used in this work as indicated by the Table 114 and Figures 147 to 167. In order to confirm the findings in Table 114, multi-problem non-parametric Wilcoxon's Signed Rank Test [Der2011] was performed on average objective function value (OFV) and average NFE of Tuned QEA and Canonical QEA on all the instances of P-PEAKS at a significance level of 5%. In case of comparison on average OFV, the null hypothesis for comparison was average OFV of Tuned QEA  $\mu_1$  is less than or equal to the average OFV of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \leq \mu_2$  and the alternate hypothesis is  $H_1: \mu_1 > \mu_2$ . The result of test is presented in Table 115, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 60 at significance level of  $\alpha = 5\%$ . This indicates the success of the proposed tuning method for tuning QEA on problems like P-PEAKS.

**Table 115: Wilcoxon's Signed Rank Test on P-PEAKS Instances (OFV)**

	TCQEA	UCQEA	$\Sigma(+)$	$\Sigma(-)$	n	Null Hypothesis	Test Statistic	Critical Value
	$X_1$	$X_2$					T	At an $\alpha$ of 5%
1	1	1	210	0	20	$H_0: \mu_1 \leq \mu_2$	0	60
2	1	0.9961						
3	1	0.9893						
4	1	0.9878						
5	1	0.9887						
6	1	0.9867						
7	1	0.9798						
8	1	0.9846						
9	1	0.9859						
10	1	0.9848						
11	1	0.9842						
12	1	0.9847						
13	1	0.9811						
14	1	0.9838						
15	1	0.9827						
16	1	0.9816						

17	1	0.9804
18	1	0.9802
19	1	0.9785
20	1	0.9815
21	1	0.9817

In case of comparison on average NFE, the null hypothesis for comparison was average NFE of Tuned QEA  $\mu_1$  is greater than or equal to the average NFE of Canonical QEA  $\mu_2$  i.e.  $H_0: \mu_1 \geq \mu_2$  and the alternate hypothesis is  $\mu_1 < \mu_2$ . The result of test is presented in Table 116, which shows that null hypothesis can be safely rejected as Wilcoxon's Signed Rank test statistic is zero and less than the critical value of 60 at significance level of  $\alpha = 5\%$ . This indicates the success of the proposed tuning method for tuning QEA on problems like P-PEAKS.

**Table 116: Wilcoxon's Signed Rank Test on P-PEAKS Instances (NFE)**

	TCQEA	UCQEA	$\Sigma(+)$	$\Sigma(-)$	n	Test Statistic	Critical Value
	$X_1$	$X_2$				T	At an $\alpha$ of 5%
1	97106	115592	0				
2	99634	146393		231			
3	97085	148433				Null Hypothesis	
4	99803	149448				$H_0: \mu_1 \geq \mu_2$	
5	98480	149817				0	60
6	98674	150050					
7	102432	150050					
8	100749	149988					
9	98008	150050					
10	101893	150050					
11	97146	150050					
12	96928	150050					
13	98459	150050					
14	99822	150027					
15	99416	150050					
16	100646	150050					
17	98568	150050					
18	97398	150050					
19	98970	150050					
20	98654	150050					
21	100277	150050					

#### IV. CONCLUSIONS

This paper focuses on working of Quantum Rotation operator used in QEA for solving combinatorial optimization problem. The Quantum Rotation operator has eight parameters, which are problem dependent and require tuning. There are several methods of parameter tuning available in literature, however, the total number of parameters to tune QEA is identified as eleven, which most of the current techniques are not capable of handling. Therefore, a new heuristic parameter tuning method was proposed for tuning algorithms like QEA with relatively large number of parameters. The proposed parameter tuning method was designed after considering the recommendation made in [27]. It is an improvement on Calibra [34] i.e. it can handle more number of parameters with more numbers of levels without using factorial number of experiments in initial stages of tuning. The proposed tuning method is a multi-stage, iterative, metaheuristic approach that uses Taguchi's approach in evaluating and generating parameter vectors for experiments. The performance of proposed tuning method has been tested by tuning QEA on four set of benchmark combinatorial problems. It required QEA to tune only on one instance of the benchmark problem, whereas in Calibra, a training set of problem instances was required to tune algorithms. After tuning, QEA was able to solve several instance of same class of problems successfully. Thus, proposed tuning method is a novel, simple and effective technique for tuning QEAs for solving combinatorial optimization problems.

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