ELEC344 Assignment 1

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1 Question 1
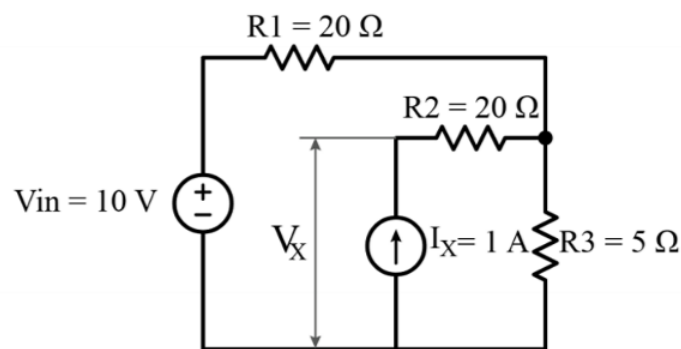
Use this circuit to solve the parts of question 1:

Part a:

Solve the circuit and find the voltage V_X.
Using KVL over the following two loops we get the two equations:

$$V_{in} - i_1 R_1 - R_3(i_1 - i_x) = 0$$
$$V_x - R_2 i_x - R_3(i_1 - i_x) = 0$$

Substituting and solving the for i_1 in (1) we get $i_1 = 0.2$ and consequently use it to solve (2):

$$10 - 20i_1 - 5i_1 - 5(1) = 0$$
$$V_x - (20)(1) - 5(0.2) - 5(1) = 0$$

Solving the second equation we arrive that $V_x = 26V$

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Part b:

Add a capacitor of 100 μF in parallel with R_3 and find the equivalent RC circuit (Thevinin). Calculate the voltage at node A ($V_A = V_{TH}$). Use KCL:

$$i_1 + i_2 - i_3 = \frac{V_{in} - V_{TH}}{R_1} + i_x - \frac{V_{TH}}{R_3} = 0$$

Rearranging this equation we get that:

$$V_{TH} = (\frac{1}{R_3} + \frac{1}{R_1})^{-1} * (\frac{V_{in}}{R_1} + i_x) = (1/20 + 1/5)^{-1} * (10/20 + 1)$$

Therefore, $V_{TH} = 6V$

Now solving for R_{TH} we set the current source as open and short our voltage source. As a result we get that the resistance seen by the load is $R_1 || R_2$.

$$R_{TH} = R_1 || R_2$$
$$= \frac{R_1 R_2}{R_1 + R_2}$$
$$= \frac{20 * 5}{20 + 5}$$
$$R_{TH} = 4\Omega$$

Our Thevinin circuit has the attributes: $R_{TH} = 4\Omega$ and $V_{TH} = 6V$

Part c:

Find the expression for the capacitor voltage and calculate the time constant for the charging capacitor.

Apply boundary conditions assuming that: for $t = 0^+ \rightarrow V_c = 0$ and for $t = \infty \rightarrow V_C = 6V$

$$V_s = V_R(t) + V_c(t)$$
$$V_c(t) = V_s(t) - V_R(t)$$
$$V_c(t) = V_s - i_R(t)R$$
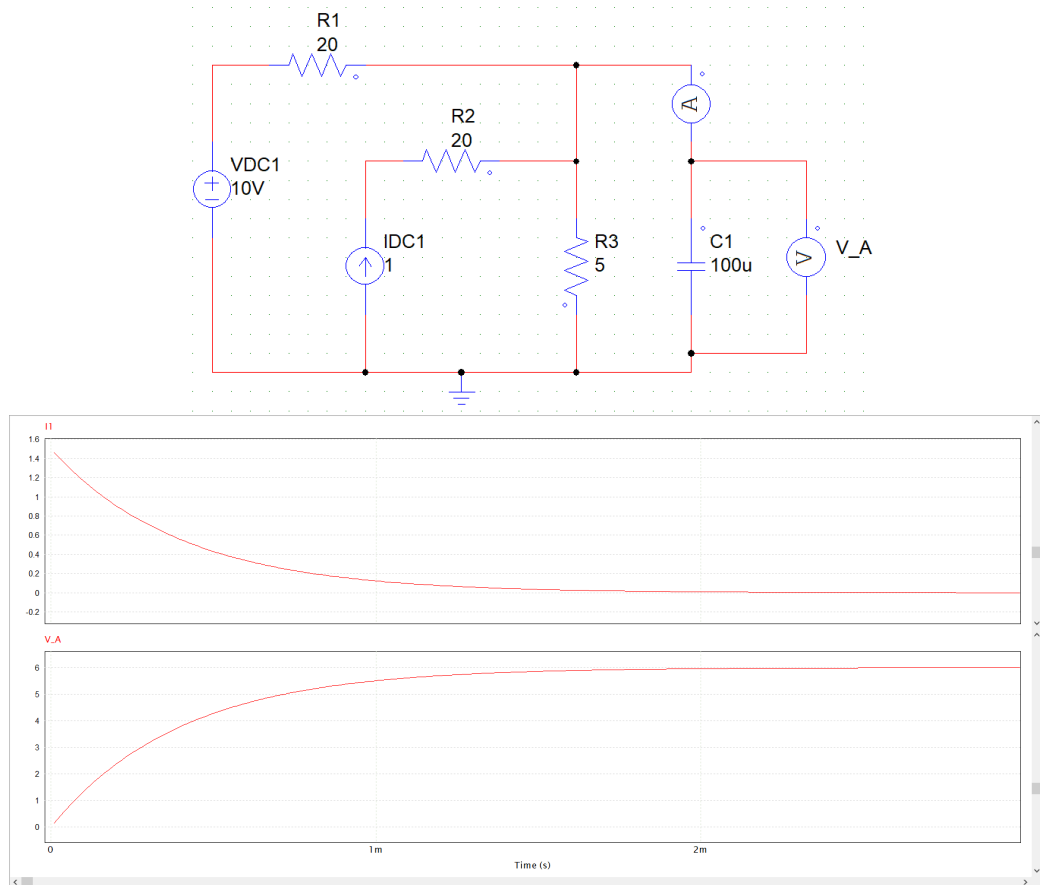
We now calculate $i_R(t) = i_c(t) = C\frac{dV_c(t)}{dt}$

$$i_c(t) = C\frac{d}{dt}(V_s - i(t)R)$$
$$= C(0 - \frac{di}{dt}R)$$
$$= -RC\frac{di}{dt}$$

integrating we get: $i_c(t) = A * e^{-t/RC} = \frac{V}{R}e^{-t/RC}$

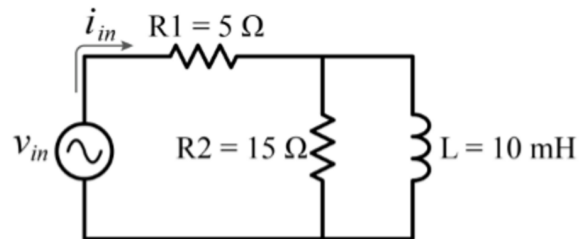Let $RC = \tau$ a time constant then $\tau = 4 * 100 * 10^{-6} = 0.4$ milliseconds

Part d:

The PSIM circuit I used and the relevant waveform during the capacitor charging transient was:

2 Question 2

Using the following circuit with an input power supply $V_{in} = \sqrt{2} * 110 \sin\left(\omega t\right)$ and $f = 60Hz$

Part a:

Calculate magnitude and phase of the input current.

Calculate the circuits equivalent impedance Z_{eq}:

$$Z_{eq} = Z_{R1} + Z_{RL} = Z_{R1} + (Z_{R2}||Z_L) = R_1 + (R_2||(j\omega L))$$
$$Z_{eq} = 5 + (15||(j(120\pi)(10 * 10^{-3})))$$
$$Z_{eq} = 5.89119 + j3.54594$$

Note: To calculate between polar and rectangular form:

$$Magnitude = \sqrt{(real)^2 + (imaginary)^2} \text{ and } Phase = \arctan\frac{imaginary}{real}$$

Then calculate the input current using $i_{in} = V_{in}/Z_{eq}$:

$$i_{in} = \frac{\sqrt{2} * 110 * \sin(120\pi t)}{5.89119 + j3.54594} = \frac{\sqrt{2} * 100\underline{/0°}}{6.87603\underline{/31.0469°}}$$

$$i_{in} = 20.5673\underline{/-31.0439°}$$

As we can see the input currents magnitude and phase are: $20.6A$ and $-31.0°$ respectively

Part b:

Adding a capacitor in parallel with the inductor, calculate the value of the capacitor to obtain 0 degree phase shift between the input current and voltage

To have a zero phase shift between our current and voltage they must be in phase. This means that our capacitance load must incur a phase shift equal but opposite to the phase imposed by our inductor. $Z_L + Z_C = 0$
Our Z_L phase is:

$$Z_L = -Z_C \rightarrow j(\omega L) = \frac{-1}{j(\omega C)} \rightarrow \omega L = \omega C$$

Now solve for the capacitance C:

$$C = \frac{1}{\omega^2 * L} = \frac{1}{(120\pi)^2(10 * 10^{-3})}$$

$$C = 7.03619 * 10^{-4}$$

Therefore to achieve zero phase shift between our input voltage and current you would need to add a $703.6\mu F$ capacitor in parallel to the inductor.

3   Question 3

Use the shown ideal 3-phase rectifier with the following parameters to solve each part

$$v_a = \sqrt{2} * 120 \sin(\omega t) \qquad \omega = 2\pi f$$
$$v_b = \sqrt{2} * 120 \sin(\omega t + 120°) \qquad f = 60Hz$$
$$v_c = \sqrt{2} * 120 \sin(\omega t - 120°) \qquad R = 10\Omega$$
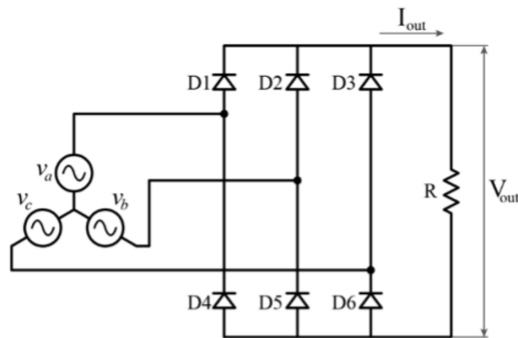

Fig 3 - Three Phase Rectifier

Part a:

Calculate the amplitude of the output voltage and current.

The output voltage will be equivalent to the line to line voltage of the 3 phase voltage. Using the derivation from tutorial we know that $V_{L-L} = \frac{V_{phase)}}{\sqrt{3}}$. We also know that $I_{out} = V_{out}/R$. Therefore:

$$V_{out} = \sqrt{3} * \sqrt{2} * 120 = 293.94V$$

$$I_{out} = \frac{V_{out}}{10} = 29.39A$$

Part b:

Simulate the circuit in Fig. 2 using PSIM and plot the output voltage, output current and current in each diode.
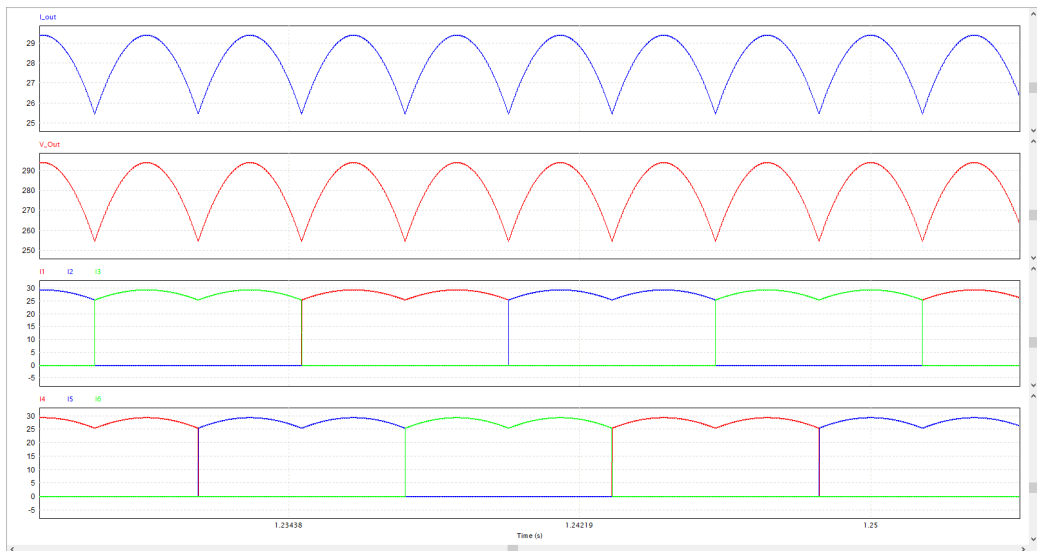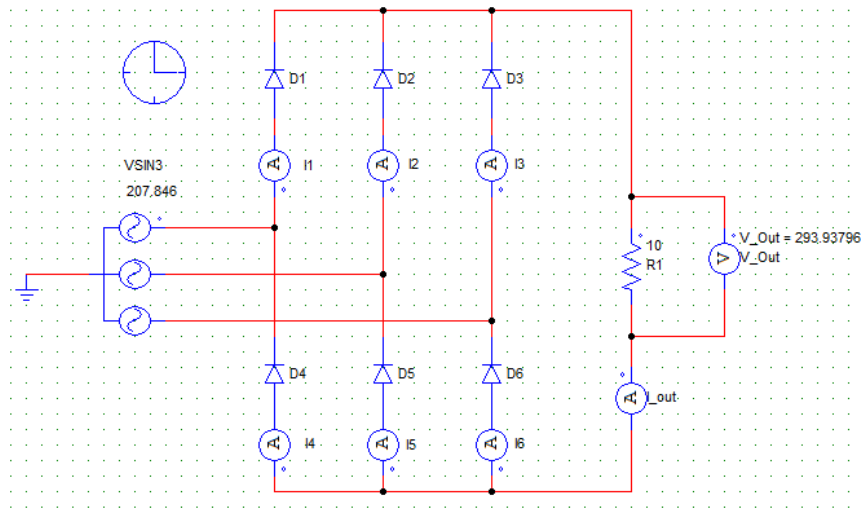
Part c:

Explain the observations from the waveforms. What diodes are conducting at each portion of time? What is the conduction angle for the diodes and explain.

From the plots one observation to make is that pairs of diodes are on at the same time. The pairs of being on are in sequence and are (D1-5, D1-6, D2-6, D2-4, D3-4, D3-5). Each pair of diodes are on for a $\frac{\pi}{3}$ or 60° time step $\rightarrow t = \frac{60}{2\pi60} = 0.1592s$ while each individual diode is on for $\frac{2\pi}{3}$ or 120° time step $\rightarrow t = 0.3183s$.
The active diodes are dependent on the phase voltage that is most positive and most negative at that time. Also given our frequency (60Hz) and phase shift (120°) the change between each voltage wave is continuous and activites different diodes each time in a cycle

4   Question 4

The circuit below is used to control the brightness of LEDs by using PWM modulating with a digital microcontroller. LEDs work as rectifier diodes and its brightness is proportional to the average current that passes through the diode (i_d).
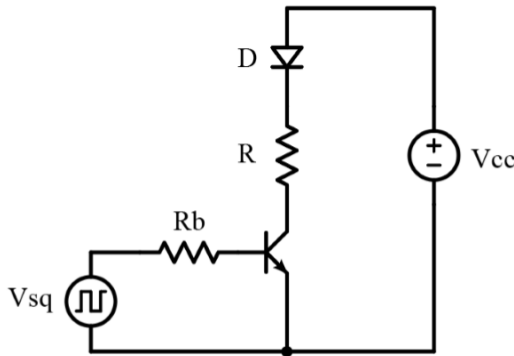
LED (D):

$i_{D(max)} = 200mA$

$V_F = 2V$

$R_D = 1\Omega$

Transistor:

$v_{CE(sat)} = 2V$

$v_{BE(sat)} = 0.7V$

$i_b \geq \dfrac{i_c}{100}$

$R_C = 0.1\Omega$

Part A:

For $V_{CC} = 5V$ calculate:

- Value of resistor R necessary for full brightness when the transistor is continuously ON

- Maximum value of the resistor R_b that enables such condition

- Average diode power (P_D), resistor power (P_R), transistor power (P_T), and source power (P_{VCC}) for 50% brightness

To calculate the resistor for full brightness assume the maximum current for i_D and V_{CE} is saturated. As a result we can perform KVL on the right sided loop.

$$V_{CC} - V_F - (i_D * R_D) - (i_D * R) - V_{CE} = 0$$

Substitute the values in and solving for R we get:

$$R = 5(5 - 2 - (0.2 * 1) - 2) = 4\Omega \tag{1}$$

To calculate the maximum value of R_B we can perform KVL on the left sided loop taking the minimum current $i_B = 200mA/100 = 2mA$

$$V_{sq} - V_{BE} - (i_b * R_B) = 0$$

Substitute the values in and solving for R_b we get:

$$R_b = (5 - 0.7) * (2 * 10^{-3})^{-1} = 2.15k\Omega \tag{2}$$

Assuming that 50 % brightness is when V_{sq} is on 50 % of the time then the transistor power can be calculated using $V_{CE} * i_C$. We can also calculate the resistor power individually and identify the source power using the source voltage and current in the wire. As a result, each

$$P_T = v_{CE} * i_C * (1/2) = 2V * 200mA * (1/2) = 0.2W$$
$$P_R = R * i_d^2 * (1/2) = 4 * 0.2^2 * (1/2) = 0.08W$$
$$P_{V_{CC}} = V_{CC} * i_D * (1/2) = 5 * 0.2 * (1/2) = 0.5W$$

Using these values we can finally calculate the power from the diode using the conservation of power. $\Sigma(P) = 0$

$$P_D = P_{V_C} - P_R - P_T = 0.5 - 0.08 - 0.2 = 0.22W \tag{3}$$

Part B:

Repeat the calculations performed in (part A) but for $V_{CC} = 12V$
Use the same approach as on part a of this question

$$V_{CC} - V_F - (i_D * R_D) - (i_D * R) - V_{CE} = 0$$

Substitute the values in and solving for R we get:

$$R = 5(12 - 2 - (0.2 * 1) - 2) = 39\Omega \tag{4}$$

The maximum value of R_b remains unchanged from part A.

$$R_b = (5 - 0.7) * (2 * 10^{-3})^{-1} = 2.15k\Omega \tag{5}$$

Use the same approach of manipulating the conservation of power to calculate P_D.

$$P_T = v_{CE} * i_C * (1/2) = 2V * 200mA * (1/2) = 0.2W$$
$$P_R = R * i_d^2 * (1/2) = 39 * 0.2^2 * (1/2) = 0.78W$$
$$P_{V_{CC}} = V_{CC} * i_D * (1/2) = 12 * 0.2 * (1/2) = 1.39$$

$$P_D = P_{V_C} - P_R - P_T = 0.5 - 0.08 - 0.2 = 0.22W \tag{6}$$

As a result we can conclude that the power of the diode doesn't change. This is that when V_{CC} changed the power of the diode remains the same.