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350 HW 8

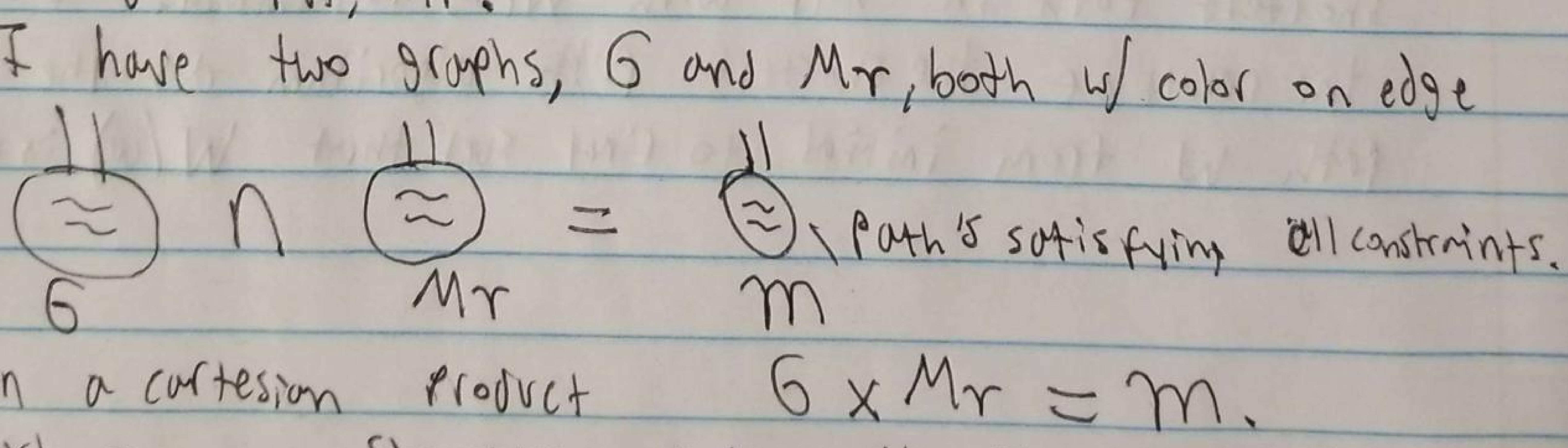
Let G be a directed graph w/ each edge given a positive number called its weight. In particular, there is a designated node in G called the initial node and there is a designated node in G called the final node. Additionally, each edge is also decorated w/ a color in $\Sigma = \{red, yellow, green\}$. Try to sketch ideas in designing efficient algorithms for the following problems.

1. For a given number k, enumerating the first i-th shortest paths, for all $1 \le i \le k$, from the initial to the final.

First, I run a loop from $i=1$ to $i=k$.
For each value of i, I run shortest path algorithm from the start node in G to node i, collecting the weights on this path and returning that number.
Thus, the algorithm will return k - i-th smallest numbers for any given number k in graph G.

2. Finding a shortest path that does not have a red edge immediately followed by a yellow edge.

First, I have r as a regular color pattern with no red edges immediately followed by a yellow edge. Thus, there is a FA, Mr.
I have two graphs, G and Mr, both w/ color on edge



I run a cartesian product $G \times Mr = m$.
Next, I run shortest path algorithm on m from start node to final node. Finally, I have a path in m and a path in G.

3. For each path w from the initial to the final, one can collect the colors on the path & therefore a color sequence $C(w)$ is obtained. Notice that, it might be the case two distinct paths w and w' coorresponds to same color sequence. [$C(w) = c(w')$. Computing the size of the set $\{C(w) : w$ is a path from the initial to the final$\}$.

First, I realize that each $C(w)$ is a graph (T) which is a subset of original graph G $(T \subseteq G)$ such that T is a tree. ~~[illegible]~~ Next, I run dijkstra's algorithm on graph T to find the weight of the shortest path from start to final node. This method will allow us to determine uniqueness of paths not only based on color sequence but on the size of the sequence.

4. For each path w from initial to final, one can multiply the weights on the path & therefore, a number $W(w)$ is obtained. Find a path w from initial to final such that $W(w)$ is minimum.

First, I begin by creating two FA's M_G and M_r, which represent the original graph and the weights of the graph multiplied by a number, respectively. I run shortest path algorithm on M_r in order to find the shortest path in M_r with $W(w)$ is minimal. I return the result of the shortest path algorithm as the desired path W from initial to final such that $W(w)$ is minimal.

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