Notes Optimization

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1 Linear Programming Model

It is a way to represent a problem as linear model, to create this model we need:

- variables: used to describe the problem
- objective function: find the maximum or minimun of given function
- constraints: limit the variable in a range

In order to create a linear model objective functions and constraints must be **linear**. If we can describe the problem as linear system, there is a method to find the optimal solution, which is called **simplex methdo**.

$$\begin{aligned}&\text{real life problem} \longrightarrow_{\text{modeling}}\\&\text{LP Problem} \longrightarrow_{\text{simplex}}\\&\text{linear solution} \longrightarrow_{\text{decoding}}\\&\text{real life solution}\end{aligned}$$

2 Modeling

The steps of modeling are:

- idenitfy variables
- write objective function
- write constraints

The variables are a set of entities that describes the solution of the problem, given those:

- it is immediate the get the objective function
- it is immediate to check if the solution respects the constraints

Example 2.1 – The Knapsak Problem

A bunch of friends is organizing an excursion and decies to put everithing in a single knapsack with capacity 10Kg.

The knapsack may be filled with:

- chocolate (500g)
- fruit juice (1l)
- beer (0.33l)
- snadwiches (100g)
- mineral water (1l)
- cookies (500g)

Each product is given a score:

- chocolate (10)
- fruit juice (10)
- beer (6)
- snadwiches (3)
- mineral water (20)
- cookies (8)

It was decided to garantee a minimun amount of each product:

- chocolate (2)
- fruit juice (2)
- beer (6)
- snadwiches (10)
- mineral water (1)
- cookies (2)

Given the full model, we insert it into the simplex and it returns back the best optimal solution

Example: The steel plant

Example: From chicken farm to fast food.

The varaibles are the quantity of chicken going from every farm to each single shop. We can represent this a matrix with columns the farms and as for rows the quantities of chicken

Minimize and absolute value, we want to minimize the difference between the target and the value

$$\begin{aligned} &\min z = |v - t| \\ &\min z = |a| \\ &|a| = \max\{a, -a\} \\ &\min z = \max\{a, -a\} \end{aligned}$$

we introduce an auxiliary variable Y

$$\begin{aligned} &\min z = \min Y = \max\{X_1, X_2\} \\ &Y >= X_1 \\ &Y >= X_2 \\ \\ &\min Y \\ &Y >= a \\ &Y >= -a \end{aligned}$$

Example: ... We can express binary variables, $if X_1 > 0 then X_2 = 0$ and $if X_2 > 0 then X_1 = 0$. We can express them with binary variable $Y_1 = 0, 1$ $Y_2 = 0, 1$

BIG M, where M is constant, big fixed number. We want to express

$$
\begin{aligned}
&Y_1 = 0, 1 \\
&Y_2 = 0, 1 \\
&Y_1 + Y_2 <= 1 \\
&X_1 <= M_1 Y_1 \\
&X_2 <= M_2 Y_2
\end{aligned}
$$

If Y_1 is 0 X_1 is bount to 0, if $Y_1 = 1$ X_1 is virtually unbounded, the same for X_2 and Y_2. M_1 and M_2 are large constants. During a software simulation it reasonable to choose the varaibles in a range that won't be reachable.

2.1 Fixed Values

Problems where we have only a fixed amount of resources. We manage them by using several logical variable for every variable that is fixed.

$$
\begin{aligned}
Y_1 + \cdots + Y_k &= 1 \\
X &= p_1 Y_1 + \cdots + p_k Y_k
\end{aligned}
$$

The first equation imposes that only of the Y can be one. The second equation imposes that X_j can only have one of the quantities $p1, p2, \ldots, p_k$.

Example 6:

A1, ..., A5 B1, ..., B5 C1, ..., C5 D1, ..., D5 we remove the years we cannot invest in.

we derive the first constraints starting from the first year. Wich is the sum of investments less than the total money.