Terrain Simulation Using a Model of Stream Erosion

Alex D. Kelley [1]
Michael C. Malin [2]
Gregory M. Nielson[1]

[1] Department of Computer Science [2] Department of Geology
Arizona State University
Tempe, Arizona 85287

Abstract

The major process affecting the configuration and evolution of terrain is erosion by flowing water. Landscapes thus reflect the branching patterns of river and stream networks. The network patterns contain information that is characteristic of the landscape's topographic features. It is therefore possible to create an approximation to natural terrain by simulating the erosion of stream networks on an initially uneroded surface. Empirical models of stream erosion were used as a basis for the model presented here. Stream networks of various sizes and shapes are created by the model from a small number of initial parameters. The eroded surface is represented as a surface under tension, using the tension parameter to shape the profiles of valleys created by the stream networks. The model can be used to generate terrain databases for flight simulation and computer animation applications.

CR Categories and Subject Descriptors: I.3.3 [**Computer Graphics**]: Picture/Image Generation - Display Algorithms; I.3.5 [**Computer Graphics**]: Computational Geometry and Object Modeling - curve, surface and object representations; geometric algorithms; modeling packages; I.3.7 [**Computer Graphics**]: Three-dimensional Graphics and Realism - animation; color, shading, texture.

Additional Keywords and Phrases: Drainage Network Simulation, Erosion Models, Surfaces Under Tension, Database Amplification, Structural Models.

1. Introduction

During the past decade, considerable progress has been made toward developing efficient models for generating approximations to natural terrain. However, models that are both realistic and efficient have not been perfected. Models used in real-time applications (e.g., flight simulation) often sacrifice realism for efficiency, and the most realistic models may take many hours to compute a single scene.

Fractal techniques [8] are considered by many to be the most efficient method for creating realistic-appearing terrain.

Their efficiency stems from their ability to generate complex detail by "amplifying" a small database of structural or statistical primitives. In the case of terrain, this information may be derived directly from digital elevation maps of actual topography. In such cases, however, care must be taken to insure proper sampling of the geographic data: too little "seed" information will result in a conspicuous self-similarity, characterized by unrealistically complex and irregular terrain. In natural landscapes, different erosional and weathering processes shape the surface at different scales, thereby restricting self-similarity to a finite range in scale.

This paper describes an alternative approach from which images of realistic-looking terrain may be produced while retaining a degree of database amplification as present in fractal models. Its basic tenet is that greater realism can be achieved from somewhat more deterministic approximations of the relief of natural landscapes. As relief is primarily created through water erosion, the model developed here creates topographic structure by tracking the "negative space" formed by a *drainage system* comprised of a stream and its tributaries. Such systems are sometimes called *stream networks, channel networks* or *drainage networks*. The model shares an "amplifying" quality with fractals, in that tributaries may be added to a stream at a variety of scales. By increasing or decreasing the number of tributaries, terrain may be modeled at variable degrees of detail.

The primary sources of information for this work are empirical erosion models used in geomorphology. These models provide simple equations for simulating the features of drainage systems. The stream networks thus created then provide a coarse framework for surface fitting, which is accomplished by triangulation and interpolation using a bivariate analog of the spline under tension [19]. The tension parameters, useful for controlling the shape of the modeled terrain, are selected based upon the values of certain features of the stream network.

An additional goal of this model is to create a numerical system that can be used as a test bed for examining the physics by which erosion occurs on the earth and other planets [13]. Given the complexity of such problems, the amount of data produced by numerical simulations are nearly uninterpretable using conventional methods of analysis. By employing computer graphics, it is possible to synthesize these data into visual form, thus taking advantage of the visual bandwidth into the human brain.

2. Background

2.1. Basic Terminology

The following is a glossary of mostly geological terms, used throughout the remainder of the paper:

baselevel -- the level below which a land surface cannot be reduced by running water.

constant of channel maintenance -- the minimum area necessary to support the development of a stream.

divide angle -- the angle (measured in the horizontal plane) between a drainage divide and an adjacent stream.

drainage area -- the amount of surface area draining into an individual section of channel.

drainage basin -- the area occupied by a stream network defined by the ridge crests that surround the network.

drainage density -- the total length of channel per unit area. This parameter is the reciprocal of the constant of channel maintenance.

drainage divide -- a boundary between streams, separating the area drained by these streams; this is usually a ridge.

drainage polygon -- a polygon representing a portion of the surface area drained by one side of a section of channel.

exterior link -- a section of tributary channel that extends from a source of water to a junction with another stream.

interior link -- a section of channel between two tributaries.

junction -- the point of confluence of two streams.

junction angle -- the angle (measured in the horizontal plane) formed by the confluence of two streams.

link -- a section of channel extending between two tributaries or between a source and its first junction with a stream.

link gradient -- the change in elevation between two junctions, divided by the horizontal distance between them.

longitudinal profile -- the change in elevation as a function of position along a stream.

main trunk stream -- the stream to which all water collected by the tributaries is funnelled.

outlet link -- a link through which all water is discharged from the network. This link is the lowest end of the network.

sources -- the farthest points upstream in a drainage system.

valley sidewall slope -- the slope measured from the edge of a channel to a brink where the slope begins to taper off.

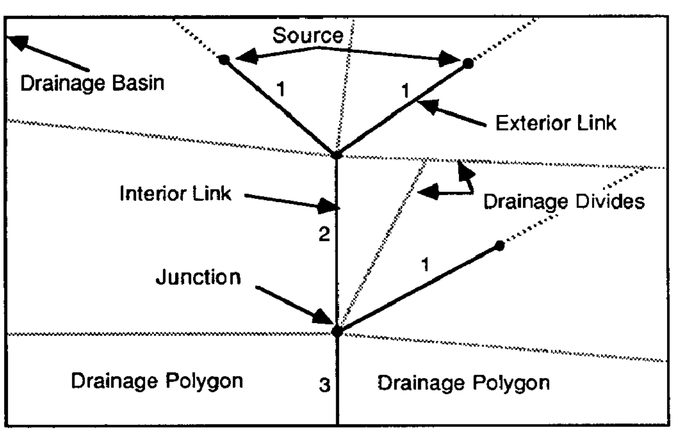

Figure 1.
Schematic Diagram Illustrating Terminology
Introduced in Section 2.1.

Several of these terms are illustrated in Figure 1. The links are numbered according to *Shreve Ordering* [23]. These numbers reflect the cumulative increase in the amount of water as links come together. Exterior links are assigned a magnitude of 1. When links of magnitude n and m come together the resulting link has magnitude (n + m). Research in geomorphology has shown that there are relationships between stream order and many of the properties of drainage systems [20]. Several empirical models of planimetric features employed in the present simulation are derived from these relationships.

2.2. Previous work

Stream network simulations have been developed by researchers in geomorpholgy over the last two decades. The majority are two dimensional, falling into two broad categories herein termed the stream convergence and headward growth models. A survey published by Abrahams [1] is an excellent starting point for the reader interested in learning more about drainage network research.

In stream convergence models [15,22], a stream is initiated by randomly choosing a source location within a grid. Its growth and direction is controlled by successive random moves into adjacent grid areas. It continues to grow until it joins another stream or goes outside the grid. Stream convergence models produce various statistics that are similar to those of natural networks, but they do not simulate their physical appearance. Streams often wander excessively and drainage basins exhibit highly irregular geometries.

Headward growth models [11,5] better simulate the physical appearance of natural networks and often produce statistics closer to reality than stream convergence models. The growth of a stream is initiated through headward random walks on a grid representing uneroded area. Branching of an original stream occurs upon reaching a pre-defined length. If a stream threatens to cross an existing stream, its growth is terminated.

An alternate type of model simulates erosion of the entire landscape using transport equations for the removal of solid material [14]. These models show greater promise in understanding the geomorphic processes and mechanisms that control the evolution of natural drainage systems. The principal difficulty with this approach is the selection of appropriate transport equations for the channel and hillslope subsystems.

3. Modeling the Drainage Network

Caution must be exercised in comparing the model presented here with the simulation models described above, as an approach directly analogous to these other works has not employed. Rather, the model presented below is structural [9] in the sense that a three-dimensional skeleton of terrain is developed. Aside from fitting points representing the stream network, parameters controlling the appearance of the surface (e.g., texture, tension, etc.) may be freely adjusted. Structural models have been previously used as a basis for modeling plants and trees [3,24].

The model presented herein incorporates empirical methods for determining tributary arrangement along principal streams, interior and exterior link lengths, drainage density, stream junction angles, drainage divides, longitudinal profile, and valley sidewall slope.

3.1. Drainage Network Initialization

The data structure used to describe the drainage network is a tree, where each node represents an individual channel link and the surface area that contributes water to it. The components of each node are used to store the link's endpoints, length, Shreve order, drainage polygons, and drainage area. The tree is constructed by recursively adding sub-branches, hereinafter referred to as *tributary links.*

To initialize the root of the tree, points projected onto the horizontal plane that describe the initial outline of the drainage basin and position of the main trunk stream are specified by the user. An example of an initial drainage system is shown in Figure 2. The procedure for computing the elevations at these data points is described at the end of Section 3.2. To initialize the left and right drainage polygons, the drainage basin is partitioned along the line segment representing the maintrunk stream. Since the maintrunk stream is initially represented by a single link, the Shreve order assigned to this root node is 1.

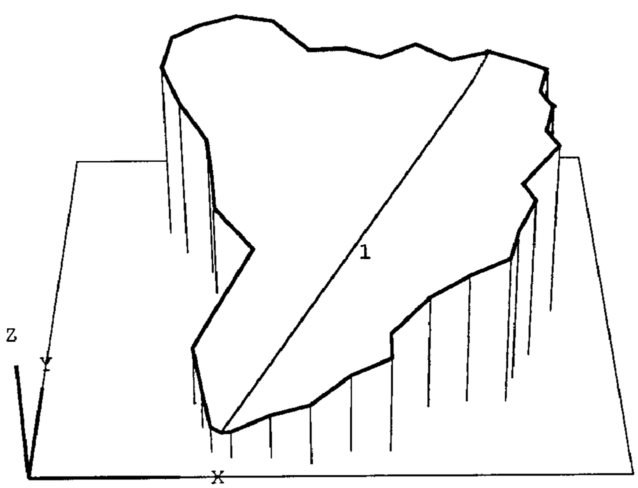

Figure 2.
An Initial Drainage System.

3.2. The Addition of Tributaries

A recursive algorithm is used to generate additional links within the drainage network. The addition of a tributary link occurs when the channel maintenance of a candidate link is greater than the mean value specified for the entire drainage basin. The constant of channel maintenance is defined by the quantity:

$$C = A / L \tag{1}$$

where A is the total drainage area and L is the total length of channel. On a regional scale, surface material is the main factor controlling this parameter [1]. For example, in humid-temperate climates, typical values range between 0.33 and 0.25 for resistant materials (e.g., sandstone). In contrast, if the surface material is weak (e.g., clay), the same climate produces values that range from 0.00077 - 0.00091 [21].

Adding a new tributary link results in the insertion of two additional nodes into the tree. The second node, herein termed the *upstream link,* is created by subdividing the the original, or *parent link,* into two sublinks at the point of junction (the shortened parent link is then called the *downstream link*). Initializing the branch nodes as well as modifying the parameters within the parent node involves the computation of Shreve orders, junction position, tributary arrangement, junction angle and drainage divides.

The Shreve order at each of the three links is computed in the following manner. As all tributaries are initially exterior links, their magnitudes are 1. The upstream link inherits the original Shreve order of the parent link. The downstream link receives water from both the upstream link and the magnitude 1 tributary. Thus its magnitude is the original (ancestral) Shreve order plus 1. Stream orders throughout the network increment with the contribution of each new tributary, as appropriate.

The following equation determines the junction position where the tributary link enters the parent link:

$$\text{Junction} = \text{MeanJunction} + \text{Rand}(\,) * \text{DeltaJunction} \tag{2}$$

where Rand( ) is a procedure returning a pseudorandom number uniformly distributed between -1.0 and +1.0. Each link in the network has "parametric length" 1. Therefore, the values of MeanJunction and DeltaJunction are set to insure that Junction is greater than 0.0 and less than or equal to 1.0. For example, if MeanJunction is 0.5 and DeltaJunction is 0.0, the junction position will subdivide the parent link into two sublinks of equal length.

The placement of junctions has a direct effect on the resulting length of interior and exterior links. A small MeanJunction is likely to result in smaller interior links than exterior links. Conversely, if MeanJunction is large, exterior links are likely to be smaller than interior links. DeltaJunction and the pseudorandom number are used to provide a stochastic perturbation on the resulting link lengths. Investigation into the ratios of exterior link lengths to interior link lengths have found them to vary considerably between regions, although the ratio is usually greater than 1 [1].

The decision on which side of the parent link to place the tributary link is based on field observations of tributary arrangements in natural networks [7]. These observations show that in the lower reaches of a stream, the initial tributaries have a greater probability of occurring on the obtuse or outer side of the stream. Along the middle reaches of the stream, a tributary is more likely to develop on the side opposite the next tributary encountered downstream. Both of these observations are attributed to space filling constraints imposed on tributary development.

In order to model these empirical relationships, a stream is partitioned into three reaches (lower, middle, and upper) of equal length. Each junction's position is evaluated with respect to this partitioning and the tributary arrangement in the lower and middle reaches are biased accordingly. In the upper reaches of the stream, the model assigns both sides an equal probability, although some evidence has shown that the shape of the basin at the upstream end will tend to promote tributary development on one side over the other [1].

The tributary and upstream links are next assigned a junction angle, which is estimated using the Howard geometric model [12]:

$$\text{Junction Angle} = E_1 + E_2 \tag{3}$$

where

$$\cos E_1 = S_3/S_1 \tag{4}$$

$$\cos E_2 = S_3/S_2 \tag{5}$$

The entrance angles E_1 and E_2 (Figure 3) are projected onto the horizontal plane. S_1, S_2 and S_3 are the stream gradients (slope tangents) of the tributary, upstream and downstream links, respectively. The magnitude of each of the three links is used to estimate the stream gradients, derived using a simplified form of Flint's [6] equation:

$$S = p(2u - 1)^q \tag{6}$$

where S is the gradient of a link of magnitude u, p is the mean link length of exterior links, and q is a negative exponent that is specific to a particular network. In the networks he studied, Flint found q to range between -0.37 and -0.837. Other field studies have shown q to have an average value of -0.6.

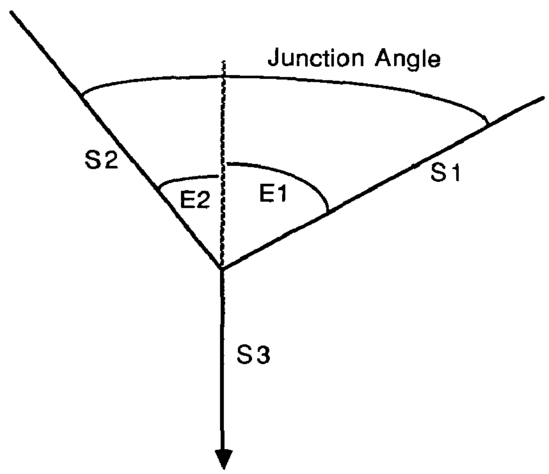

Figure 3.
Schematic Diagram of Howard's Junction Angle Model.

The orientation of the tributary and upstream links are set according to the junction angle. The source point of the tributary is computed by estimating the length of this exterior link. This is performed using Equation 2, replacing MeanJunction and DeltaJunction with MeanLength and DeltaLength, respectively. Similarly, the value of these parameters are set so that the exterior link length is greater than 0.0 and less than or equal to 1.0. If the result is 1.0, the tributary link will extend to a corresponding edge of the drainage polygon.

The drainage polygons of upstream, tributary, and downstream links are initialized by partitioning the original drainage polygons belonging to the parent link. As shown in Figure 1, the dotted lines represent the three drainage divides that separate the area drained by each of the three links connected at a junction. The orientation of each drainage divide is computed using a model that relates divide angles to the link and valley sidewall slopes [2]. Each of the three divides is extended from the junction to a corresponding edge of the left or right drainage polygon.

The planimetric features discussed thus far are computed by projecting the drainage system onto the horizontal plane. The third dimension is now added by computing the elevation at each data point. The height at the source and junctions are computed by approximating the longitudinal profile of individual streams. The remaining elevations at the ridge crests are computed using a model for valley sidewall slopes.

The longitudinal profile of a stream is determined using Equation 6 to approximate the gradients (slope tangents) at each link in the stream. First, the baselevel of the junction at the end of the outlet link is assigned to a specified elevation. Then, working upstream, the gradient at each link is computed and the elevation at each junction set according to the following scheme: If z_1 is the elevation at junction j_1 and z_2 the elevation at the next upstream junction j_2,

$$z_2 = z_1 + S * L \tag{7}$$

where S is the link gradient computed in Equation 6 and L is the projected horizontal distance from j_1 to j_2. Starting from the outlet link, the initial value of z_1 is the baselevel elevation. To set the elevation of the upstream junction at the next link in the stream, z_1 is assigned the value of z_2 from the previous link. The resulting curve of each stream exhibits the characteristic concave upward profile [17].

Valley sidewalls are represented by the left and right drainage polygons associated with each link in the network. As shown in Figure 2, a stream link occupies the lower edge shared by the left and right drainage polygons. Excluding the endpoints of this edge, the height at the remaining vertices of the drainage polygons are computed using a model for valley sidewall slopes. The slope angle (measured from the stream link to each vertex) is determined from Equation 2, replacing MeanJunction with MeanValley Sidewall Slope and DeltaJunction with DeltaValley Sidewall Slope. In this case, these parameters are set to insure that the angle of elevation ranges between zero and ninety degrees.

4. Modeling the Surface

The result from applying the methods described in Section 3 is a three-dimensional polygonal representation of the drainage basin, shown in Figure 4. To complete the description of the modeled terrain, a surface under tension is constructed from the polygonal representation using a method of scattered data interpolation. The tension parameter is used to control the shape of the valley sidewall profiles.

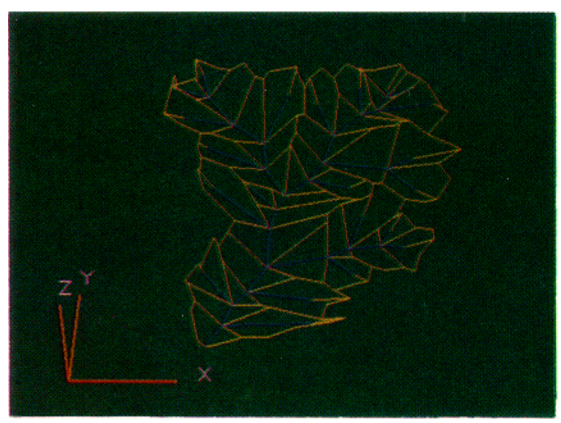

Figure 4.
Polygonal Representation of Modeled Terrain.

4.1. Surface Fitting

In general terms, a scattered data method accepts independent data values (x_i, y_i), $i = 1,\dots,n$ and associated dependent values f_i, $i=1,\dots,n$ and produces a surface $S(x,y)$ such that $S(x_i,y_i) = f_i$, $i=1,\dots,n$. A variety of scattered data interpolation methods are discussed in Franke [10]. For this application, it is important that the fitting technique have certain properties. First, the influence of the data should be local in that a change in one data value, (x_k,y_k), f_k should only affect the surface in the neighborhood of that point and the change should have essentially no impact at greater distance. It is also important that the method be computationally efficient and be able to handle large data sets. Both these properties are usually shared by a class of scattered data methods which attempt to mimic the classical univariate spline by using a piecewise defined surface with some smoothing interconnecting conditions between surface segments. A decomposition of the domain into triangles is the most common approach.

The method selected here is a surface under tension technique proposed by Nielson and Franke [19]. There are three stages to their method. First, the domain is triangulated using the data points (x_i,y_i), $i=1,...,n$ as vertices. Next, the gradient of the surface, S, is estimated and finally, a triangular surface patch is used to define $S(x,y)$ over each triangle. The first stage is performed by triangulating each drainage polygon in the network. Edges inferred by the links and divides of the drainage network remain edges in the triangulation. The gradients are estimated by solving the problem of minimizing the quantity:

$$\sum_{ij \in E} \int_{e_{ij}} \left[\frac{\partial^2 S}{\partial e_{ij}^2}\right]^2 + v_{ij} \int_{e_{ij}} \left[\frac{\partial S}{\partial e_{ij}}\right]^2 \qquad (8)$$

where E is a list of all the edges in the triangulation. This quantity is similar to the norm that is used to characterize the univariate spline under tension [4]. The solution here is only defined over a domain consisting of the edges of the triangulation and is referred to as a *minimum norm network*. The parameters v_{ij} are tension parameters and can be used to adjust the shape of the network. As the tension parameter associated with an edge is increased, the arc of the network related to this edge converges to a straight line segment. In order to extend the surface to the entire domain, a triangular surface patch is used over each triangle. This surface patch is chosen so that it will match all of the boundary information provided by the minimum norm network and also reflect the affects that the tension parameters v_{ij} have on the network.

A wireframe representation of the modeled terrain is shown in Figure 5. This surface under tension was evaluated at evenly spaced locations within a 50 x 50 grid and transformed into triangles. Points outside the boundary of the surface were assigned a zero elevation value. The tension value assigned to the endpoints of the stream links was 5.0, while the remainder of the data points along the ridge crests were assigned a tension value of 0.0. This resulted in a convex curved profile at the valley sidewalls. More analytical treatment of the use of tension in simulating valley sidewall profiles will be eventually incorporated into the model.

To simulate surface roughness features, each sampled elevation may be displaced by a small random perturbation. Additional elevations for the terrain shown in Figure 5 were evaluated along the boundary edges of each stream in the network. The width of a stream is computed using a model which relates channel width to the order of the individual links [20].

5. Rendering the Surface

A fully rendered image of simulated terrain is shown in Figure 6. Perspective views of the terrain surface can be generated using conventional perspective and hidden point removal techniques. In the implementation used here, hidden surface removal is performed using a depth sort algorithm [18].

Surface colors were selected from tables which approximate the color of foliage, snow, water and rock types. The choice of color assigned to each triangle is based on its location, elevation, and gradient. For example, steep triangles (measured by the direction of the normal) are assigned the color of a rock type. The surface color is modulated using a simple diffuse lighting model with no specular component.

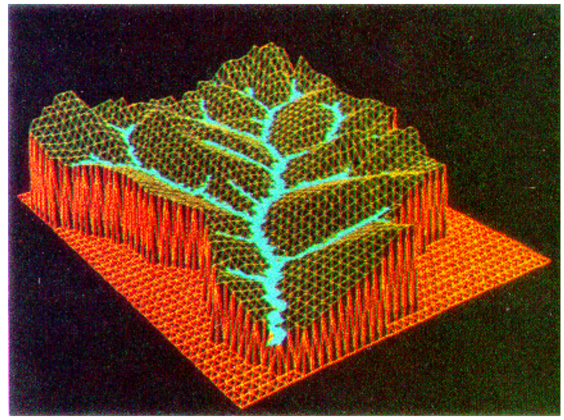

Figure 5.
Wireframe Representation of Modeled Terrain.

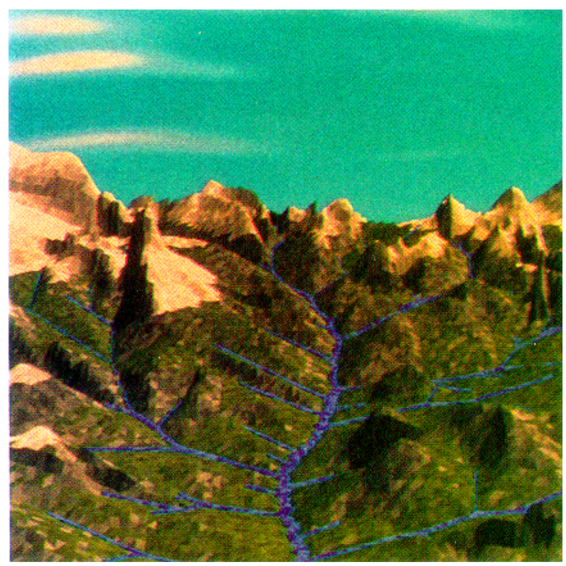

Figure 6.
Shreve Valley.

6. Implementation

The program for the model outlined in this paper is run under Berkeley 4.2 Unix on a SUN 3/160, with no floating point accelerator and 12 Mb of memory. The majority of the modeling software was written in C, except for several FORTRAN-77 subroutines used to compute the minimum norm network. The rendering programs are written in C and run under VMS 4.2 on a VAX 11/750 with floating point accelerator and 6 Mb of memory.

Modeling consists of computing the polygonal representation, triangulation, minimum norm network, and surface evaluation. Computation time for the stream network shown in Figure 6 was 8 minutes, about 90% of which was consumed by the surface evaluation and minimum norm network computations. The surface was evaluated at 26,671 locations and transformed into 41,339 triangles. This series of computations need be performed only once. Rendering the 512 x 512 x 24-bit image took 4 minutes.

7. Conclusion & Future Work

A method for modeling terrain at the scale in which fluvial processes shape its surface has been demonstrated. Terrain is modeled by simulating the erosion caused by stream networks on an initially uneroded surface. The model has an "amplifying" quality because an initial stream network evolves into a much larger network. Furthermore, very little information is required to represent the initial drainage system. Empirical models from geomorphology are used as a basis for the modeling. Planimetric attributes are parameterized and therefore do not require explicit modeling.

The model may be amplified in many ways. Although the tension parameter appears to be useful in controlling the transverse profiles of valleys, the model currently does not incorporate an explicit or physically based model of changes in hillslope profiles. Thus, the hillslopes are too simplistic.

A deficiency of the model is that it addresses only one, albeit major, mechanism for shaping landscapes. Several others contribute to the total picture. At smaller scales, various weathering processes are important factors controlling the shape and texture of rock surfaces. Stochastic subdivision methods [16] are a possible source for modeling these smaller features. The model can probably be animated to show the evolution of a landscape once these suggested areas of future work have been addressed.

Acknowledgments

ADK and MCM were supported by NASA Grant NAGW-1 and NSF Grant EAR-8313091. GMN was supported by the U.S. Department of Energy under contract DE-FG02-87ER25041 to Arizona State University and by a NATO Research Grant RG.0097/88. We wish to thank Mike Caplinger for his help in defining and implementing the rendering software used to produce Figure 6. Richard Franke wrote the original code which performed the minimum norm network computations.

References

1. Abrahams, A. "Channel Networks: A Geomorphological Perspective", *Water Resour. Res., 20,* 2 (February 1984), 161-168.

2. Abrahams, A. "Divide Angles and Their Relation to Interior Link Lengths in Natural Channel Networks", *Geographical Analysis,* 12, 2 (April 1980), 161-168.

3. Bloomenthal, J. "Modeling the Mighty Maple", *Computer Graphics 19,* 3 (July 1985), 305-311.

4. de Boor, C., *A Practical Guide to Splines,* Springer-Verlag, 1978.

5. Dunkerly, D. "Frequency Distributions of Stream Link Lengths and the Development of Channel Networks", *J. Geol. 85,* (1977), 459-470.

6. Flint, J. "Stream Gradient as a Function of Order, Magnitude, and Discharge", *Water Resour. Res., 10,* 5 (October 1974), 969-973.

7. Flint, J. "Tributary Arrangements in Fluvial Systems", *Am. J. Sci. 280,* (January 1980), 26-45.

8. Fournier, A., Fussell, D., and Carpenter, L. "Computer Rendering of Stochastic Models", *Commun. ACM , 25,* 6 (June 1982), 371-384.

9. Fournier, A. "Prolegomenon", Siggraph '87 Course Notes: The Modeling of Natural Phenomena", July 1987.

10. Franke, R., "Scattered Data Interpolation: tests of some methods, *Math. Comp.* 38 (1982), 181-200.

11. Howard, A. "Simulation of Stream Networks by Headward Growth and Branching", *Geogr. Anal., 3,* (1971), 29-50.

12. Howard, A. "Optimal Angles of Stream Junction: Geometric, Stability to Capture, and Minimum Power Criteria", *Water Resour. Res., 7,* 4 (August 1971), 863-873.

13. Kelley, A., Malin, M. "Three-Dimensional Digital Simulation of Drainage Basin Development: Modeling the Martian Valley Networks", *Reports of Planetary Geology and Geophysics Program - 1987, NASA Tech Mem.* (in press).

14. Kirkby, M. "A Two-Dimensional Simulation Model of Slope and Stream Evolution", In *Hillslope Processes,* edited by A. D. Abrahams, Allen and Unwin, Boston, 1986, 203-222.

15. Leopold, L. and Langbein, W. "The Concept of Entropy in Landscape Evolution", *Geol. Surv. Prof. Pap. 500-A,* 1962.

16. Lewis, J. "Generalized Stochastic Subdivision", *ACM Trans. Graphics, 6,* 2 (July 1987), 167-190.

17. Morisawa, M., *Streams: Their Dynamics and Morphology,* McGraw-Hill Inc. 1968.

18. Newell, M. E., Newell, R. G.and Sancha, T. L. "A Solution to the Hidden Surface Problem", *Proc. ACM Nat. Conf.,* (1972), 236-243.

19. Nielson, G. and Franke, R. "A Method for Construction of Surfaces Under Tension", *Rocky Mountain Journal of Mathematics 14,* 1 (Winter 1984), 203-221.

20. Richards, K. "A Note on Changes in Channel Geometry at Tributary Junctions", *Water Resour. Res., 16,* 1 (February 1980), 241-244.

21. Ritter, D., *Process Geomorphology,* Wm. C. Brown, Dubuque, Iowa, 1978.

22. Schenck H. "Simulation of the Evolution of Drainage-Basin Networks with a Digital Computer", *J. Geophys. Res., 68,* 20 (October 1963), 5739-5745.

23. Shreve, R., "Statistical Law of Stream Numbers", *J. Geol.,* 74, (1966), 17-37.

24. Smith, A. "Plants, Fractal, and Formal Languages", *Computer Graphics 18,* 3, (July 1984), 1-10.