Max 3-SAT Approximation Writeup

## Strategy

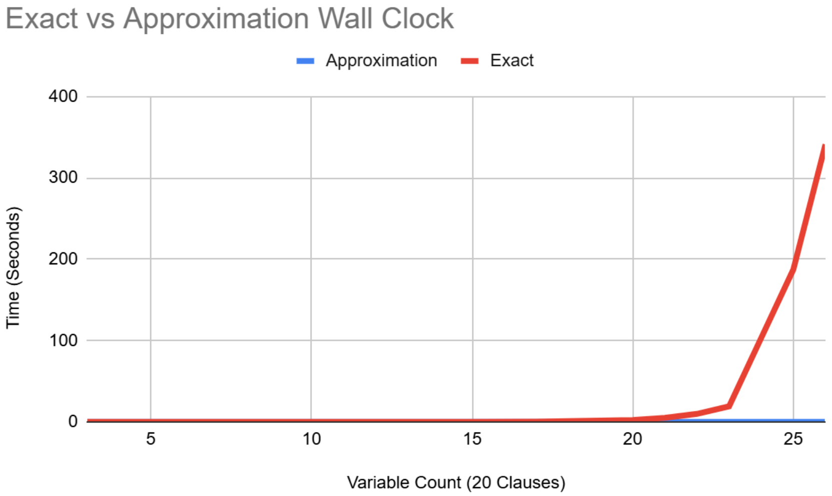
1. Randomization - Assign random Boolean values to each variable.
2. Attempt to Improve - Iteratively flip the truth value of each variable one at a time. After each flip, compute the new number of satisfied clauses. Retain the flip if it improves the solution; otherwise, revert the flip.
3. Anytime - The algorithm attempts to improve the solution incrementally over multiple iterations. It stops either when no further improvement is made after 5 consecutive iterations or when all clauses are satisfied (best_count == m).

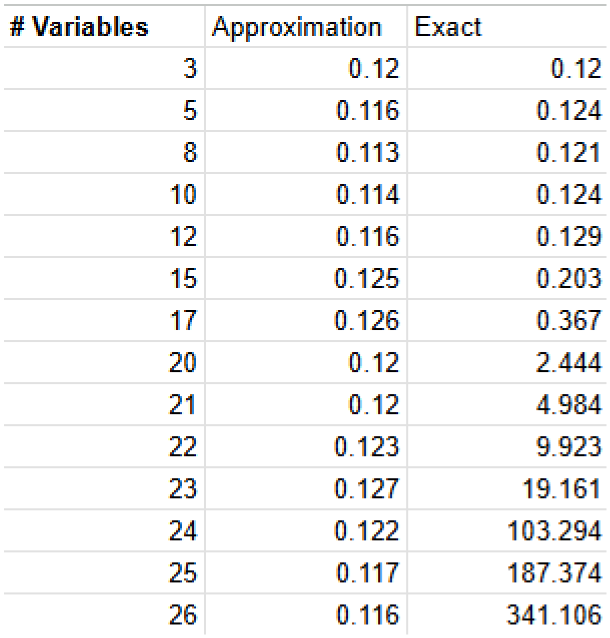
## Analytical Runtime

1. Randomly assigning truth values to each variable; Takes O(n) where n is the number of variables
2. satisfied_clauses iterates over each clause and each literal in the clause; Iterating over each clause takes O(m) where m is the number of clauses and each literal takes constant time O(1) since each clause can only have 3 literals
3. max_3sat_approx has two loops:
    a. Outer loop: Runs up to 5 iterations: Outer loop runs in constant time O(1)
    b. Inner loop: Iterates over each variable and uses satisfied_clauses; Iterating over each variable takes O(n) and since satisfied_clauses takes O(m), the inner loop takes O(n * m)
4. Runtime: O(n * m)

## Wall Clock

- As the input size of m (variables) increases, the time the exact solution takes grows exponentially, while the approximation grows linearly.





| # Variables | Approximation | Exact |
|---:|---:|---:|
| 3 | 0.12 | 0.12 |
| 5 | 0.116 | 0.124 |
| 8 | 0.113 | 0.121 |
| 10 | 0.114 | 0.124 |
| 12 | 0.116 | 0.129 |
| 15 | 0.125 | 0.203 |
| 17 | 0.126 | 0.367 |
| 20 | 0.12 | 2.444 |
| 21 | 0.12 | 4.984 |
| 22 | 0.123 | 9.923 |
| 23 | 0.127 | 19.161 |
| 24 | 0.122 | 103.294 |
| 25 | 0.117 | 187.374 |
| 26 | 0.116 | 341.106 |

**Non Optimal**

- As the number of clauses m increases, the approximation algorithm becomes less optimal.
    - Highly Constrained Instances: When clauses are heavily interdependent (e.g., many overlapping variables among cl