INTRODUCTION TO STARS

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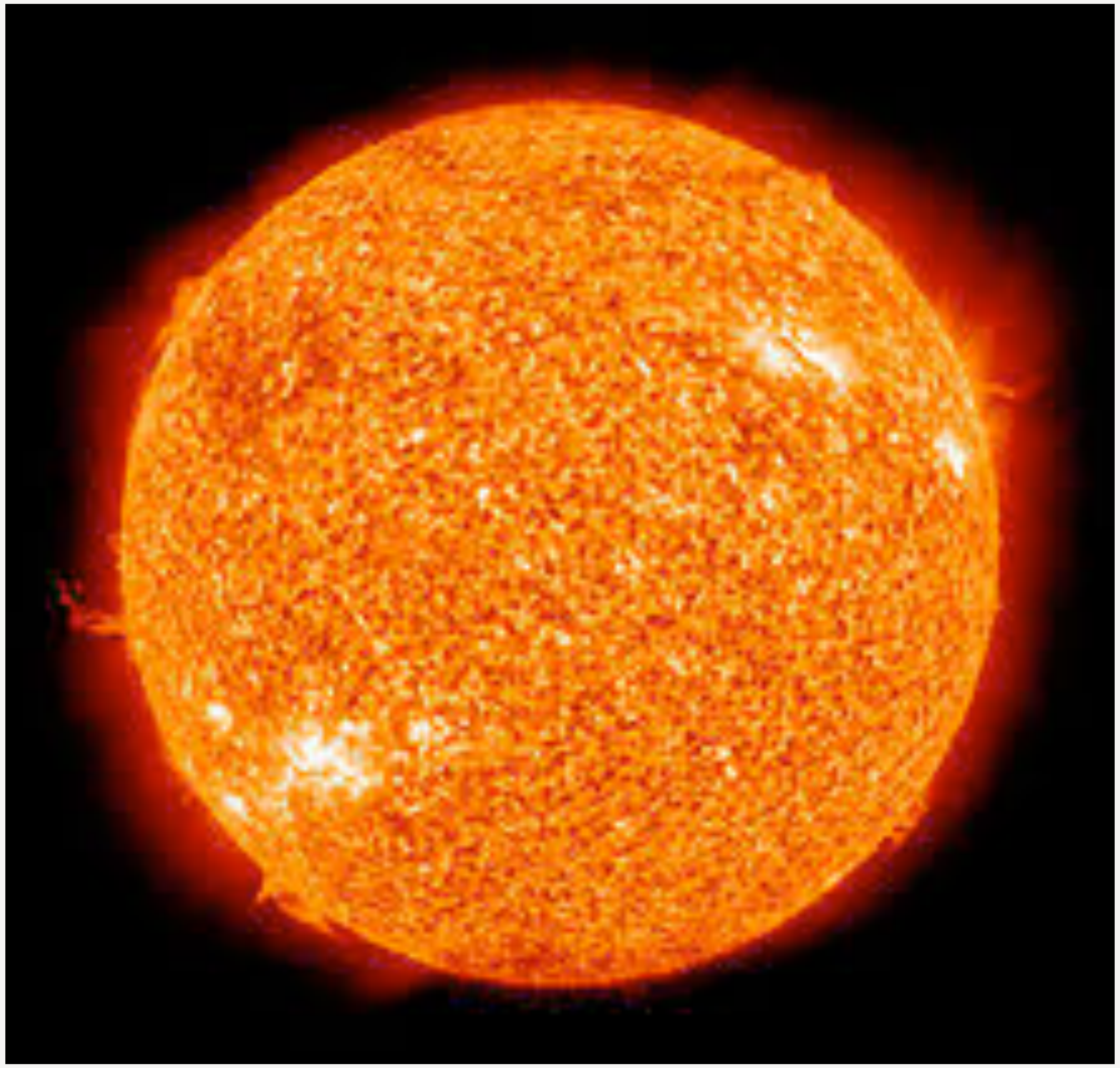
SUN: THE NEAREST STAR



Solar constant: 1.38 x10^6 erg s^{-1} cm^{-2}
Solar distance: 1.49 x 10^{13} cm (1 AU)
Solar radius : 6.96 x10^{10} cm
Solar luminosity: 4.0 x10^{33} erg s^{-1}
Solar Mass: 2.0 x 10^{33} grm

- Solar Luminosity is,

$$L_\odot = 4\pi R^2 \sigma T^4_{eff}$$

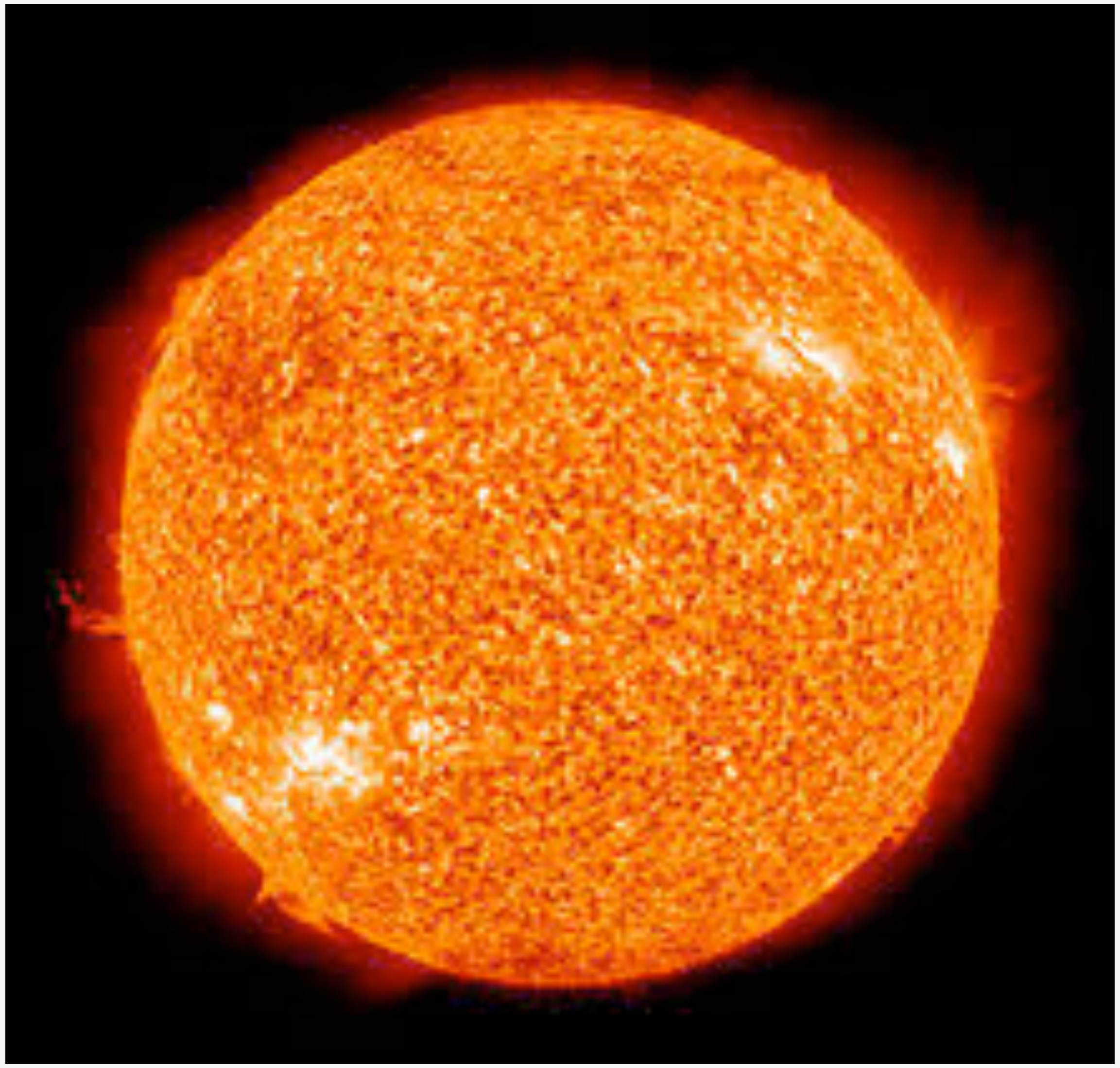
- The observed luminosity is consistent with the effective temperature of the sun to be 5700 K.
- Average density of Sun is 1.4 g cm^{-3} very close to the density of water (i.e 0.99 gm cm^{-3}).
- Total energy radiated by the sun in its lifetime,

$$\begin{aligned} E &= L_\odot \tau \\ &= (4.0 \times 10^{33}) \times (5 \times 10^9) \\ &= 6 \times 10^{50}\ ergs \end{aligned}$$

- Total energy to mass ratio is,

$$\frac{E}{M_\odot} = 3 \times 10^{17}\ erg\ gm^{-1}$$

SUN: THE ENERGETICS



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- Chemical reaction: Energy to mass ratio in any typical chemical reaction is ~ 10^{12} erg gm^{-1}. Therefore, solar radiation is not due to any chemical reaction.
- Gravitational energy: If we assume sun to be a sphere of constant density then the total potential energy available is,

$$\Phi = \int_0^{R_\odot} \frac{G}{r}\left(\frac{4}{3}\pi r^3 \rho\right)(4\pi r^2 \rho)dr$$
$$= \frac{3}{5}\frac{GM_\odot^2}{R_\odot} = 2 \times 10^{48}\ erg$$

The average potential energy per mass is 10^{15} $erg\ gm^{-1}$.

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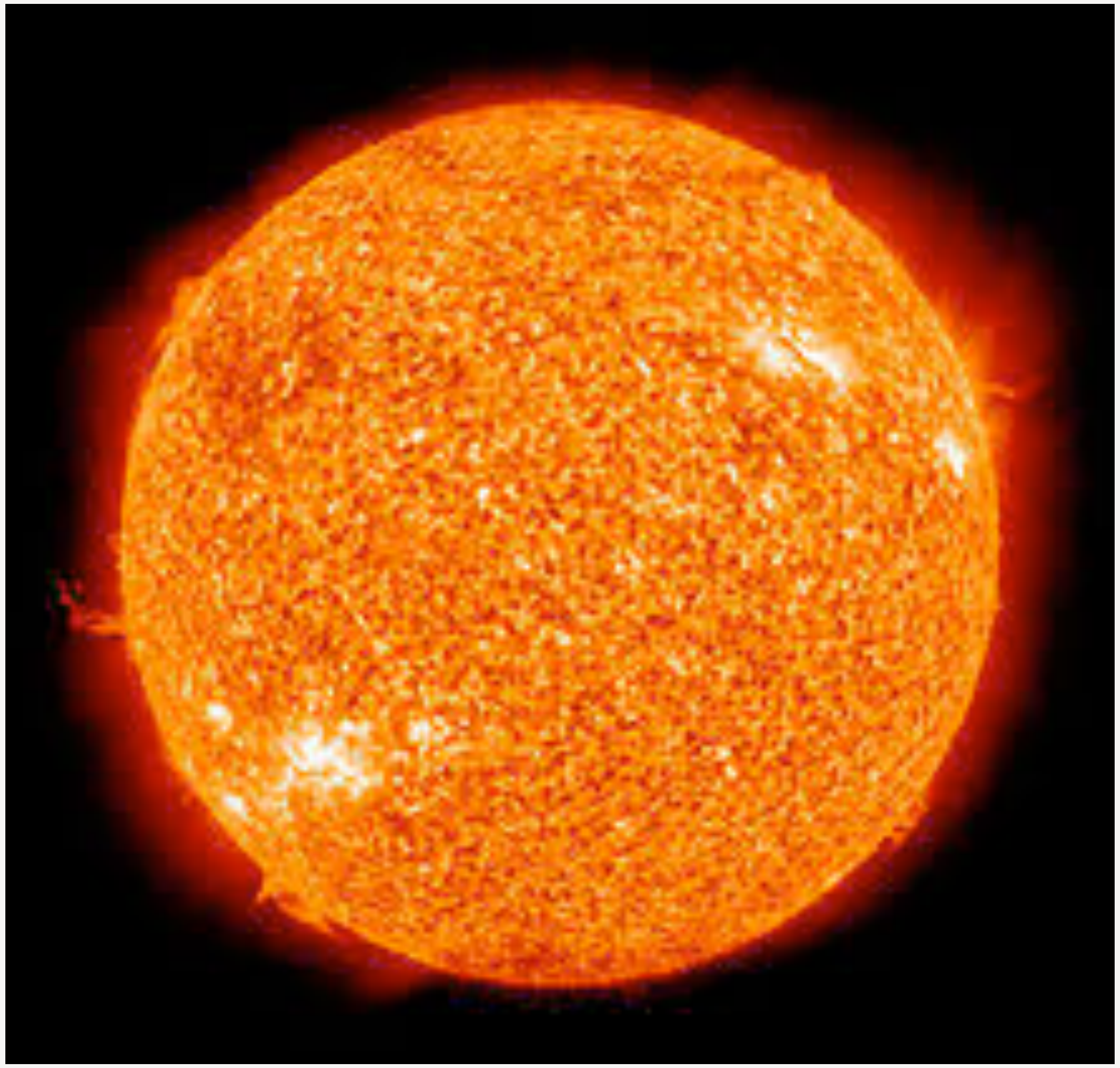

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- Nuclear reaction: Let us consider a nuclear fusion reaction in which 4 H atoms combined to give one He atom.

$$4m_H - m_{He} = 0.029 m_H$$

- There is a mass loss of 0.7% for each transfer of Hydrogen to Helium. This corresponds to an energy per mass of 6x10^{18} *erg* gm^{-1}.
- Therefore, even if a small fraction of sun's mass going through the nuclear reaction is sufficient to maintain the observed luminosity for long time.
- Total energy available,

$$E = 6 \times 10^{18} \; f_M \; 2 \times 10^{33} ergs$$

- Time taken to burn that much fuel (life time of the H burning)

$$\tau = \frac{E}{\Delta E/\Delta T} = \frac{1.2 \times 10^{52} f_M}{(3 \times 10^7) \times (4 \times 10^{33})} yrs$$

STARS: NUCLEAR REACTIONS

- P-P chain reactions:

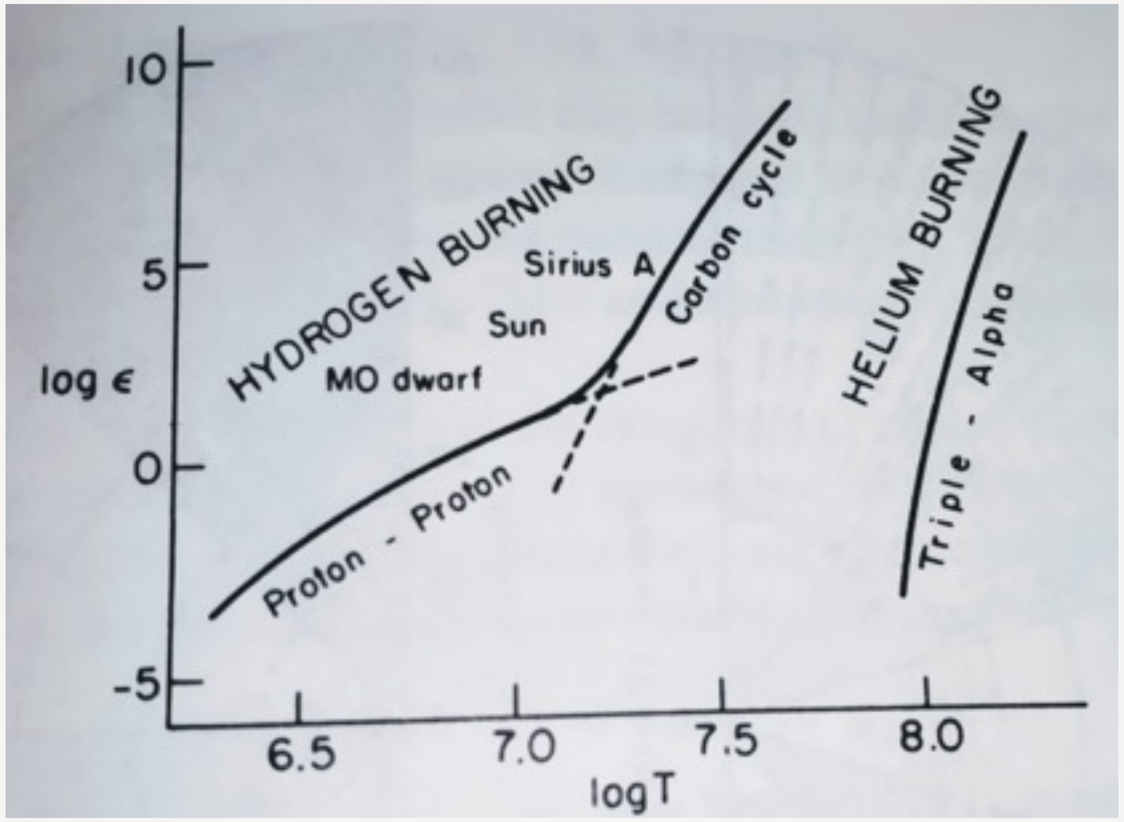
$$
\begin{aligned}
{}^1_1\mathrm{H} \;+\; {}^1_1\mathrm{H} \;&\rightarrow\; {}^2_1\mathrm{D} + \mathrm{e}^- + \bar{\nu}; \quad 1.4\ MeV,\ 1.4\times 10^{10}\ yrs\\
{}^2_1\mathrm{D} \;+\; {}^1_1\mathrm{H} \;&\rightarrow\; {}^3_2\mathrm{He} + \gamma; \quad 5.49\ MeV,\ 6\ sec\\
{}^3_2\mathrm{He} +^3_2 \mathrm{He} \;&\rightarrow\; {}^4_2\mathrm{He} + 2{}^1_1\mathrm{H}; \quad 12.85\ MeV,\ 10^6\ yr
\end{aligned}
$$

- Tripple α process:

$$
\begin{aligned}
{}^4_2\mathrm{He} +^4_2 \mathrm{He} \;&\rightarrow\; {}^8_4\mathrm{Be} + \gamma; \quad -95\ KeV,\\
{}^8_4\mathrm{Be} +^4_2 \mathrm{He} \;&\rightarrow\; {}^{12}_6\mathrm{C} + \gamma; \quad 74\ MeV
\end{aligned}
$$

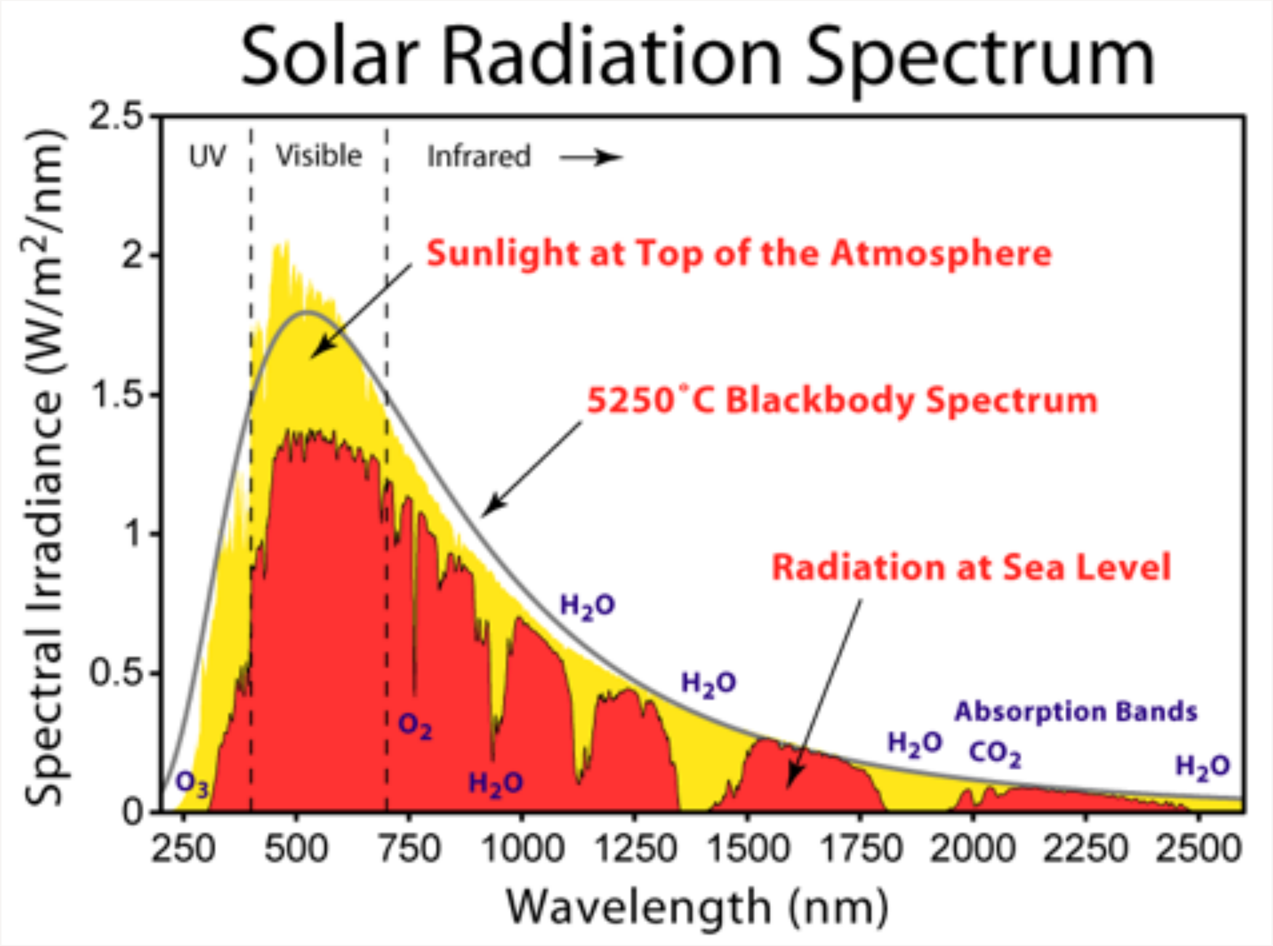
- CNO Cycle:

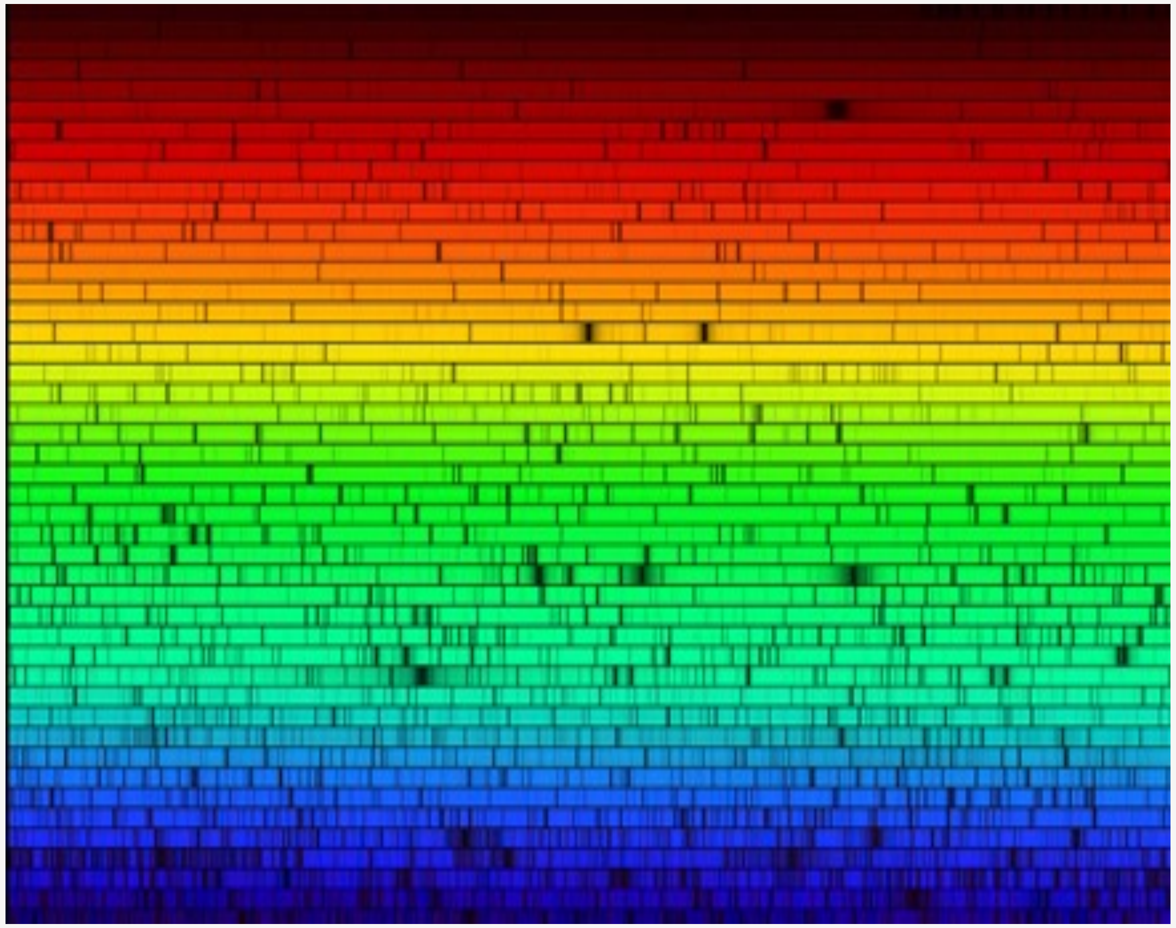
	MeV	15×10^6 K
$^{12}C + {}^1H \rightarrow {}^{13}N + \gamma$	1.94	$\sim 10^6$ yr
$^{13}N \rightarrow {}^{13}C + e^+ + \nu$	2.22	15 min
$^{13}C + {}^1H \rightarrow {}^{14}N + \gamma$	7.55	2×10^5 yr
$^{14}N + {}^1H \rightarrow {}^{15}O + \gamma$	7.29	2×10^8 yr
$^{15}O \rightarrow {}^{15}N + e^+ + \nu$	2.76	3 min
$^{15}N + {}^1H \rightarrow {}^{12}C + {}^4He$	4.97	10^4 yr
$^{15}N + {}^1H \rightarrow {}^{16}O + \gamma$	12.1	4×10^{-4} of $^{15}N(P,\alpha)^{12}C$ rate
$^{16}O + {}^1H \rightarrow {}^{17}F + \gamma$	0.60	2×10^{10} yr
$^{17}F \rightarrow {}^{17}O + e^+ + \nu$	2.76	1.5 min
$^{17}O + {}^1H \rightarrow {}^{14}N + {}^4He$	1.19	2×10^{10} yr



Ref: Astrophysical concepts: Martin Harwit

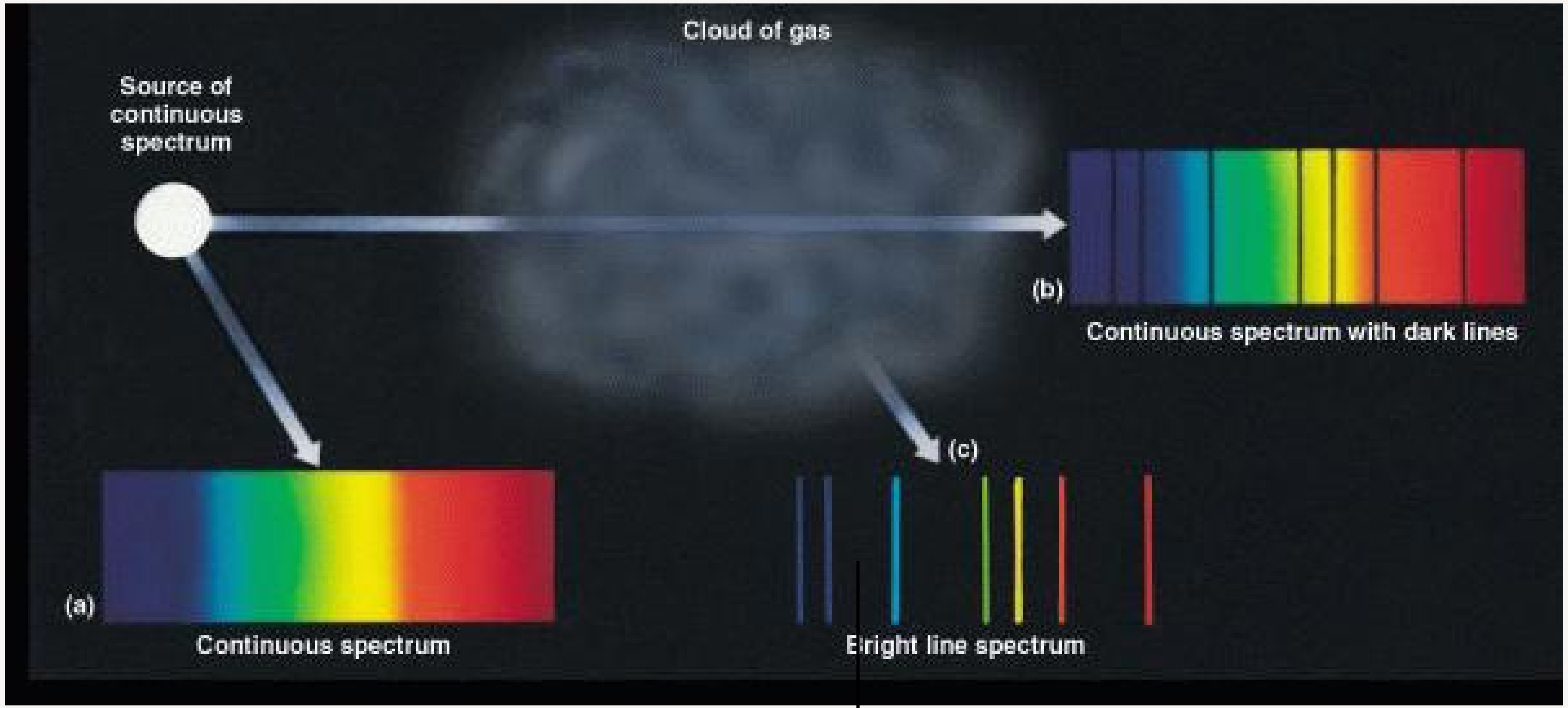
SUN: OPTICAL SPECTRUM





- Continuum fit to the full solar spectrum confirms the surface temperature is 5250 deg C. Nuclear reaction can not take pace at this temperature and the average density of 1 gm cm^{-3}.
- Solar spectrum as well as spectrum of other stars taken at higher spectral resolutions clearly show absorption lines. This confirms stars may have multiple layers of gas with different temperatures.

BASIC RADIATIVE TRANSPORT



$$\frac{dI_\nu}{dS} = -\kappa_\nu I_\nu + j_\nu$$

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + \int_0^\tau \frac{j_\nu}{\kappa_\nu} e^{-\tau_\nu} d\nu$$

$$= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau})$$

When τ is very large then,

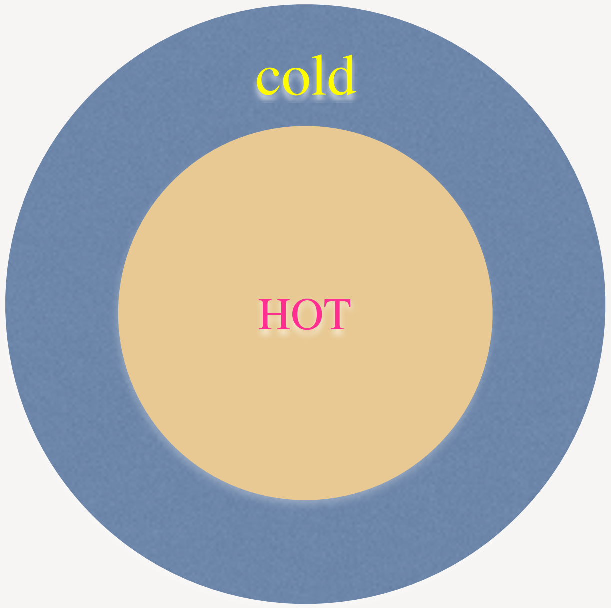
$$I_\nu = S_\nu$$

When τ_ν is very small,

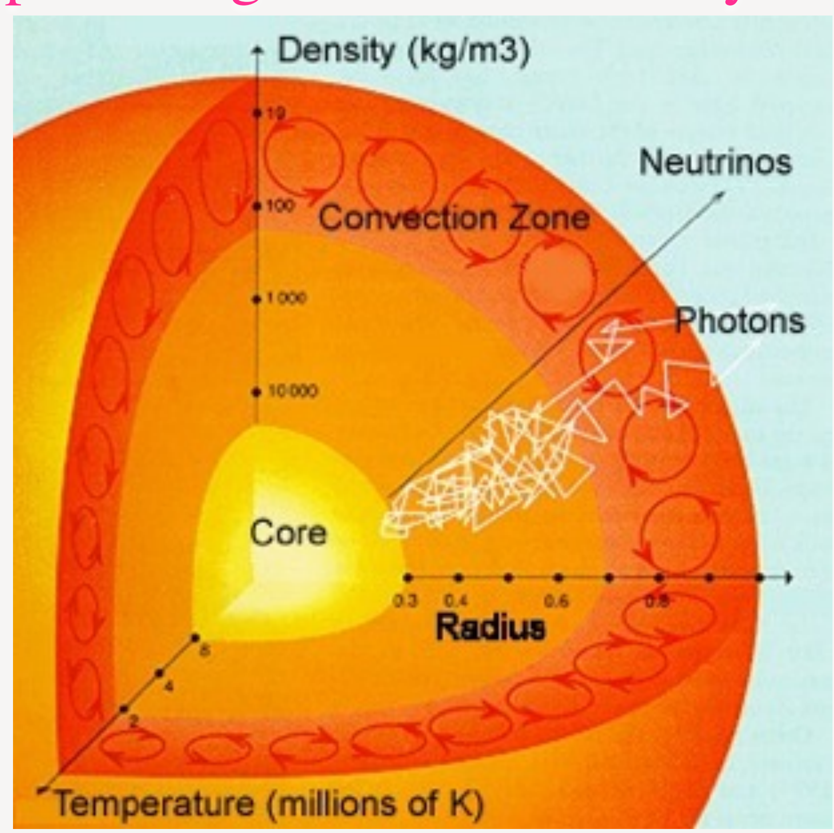
$$I_\nu = I_\nu(0) + \tau_\nu[S_\nu - I_\nu(0)]$$

Kirchoff's law

STRUCTURE OF THE STAR:



You need some force to keep gas with temperature gradients in a steady state.

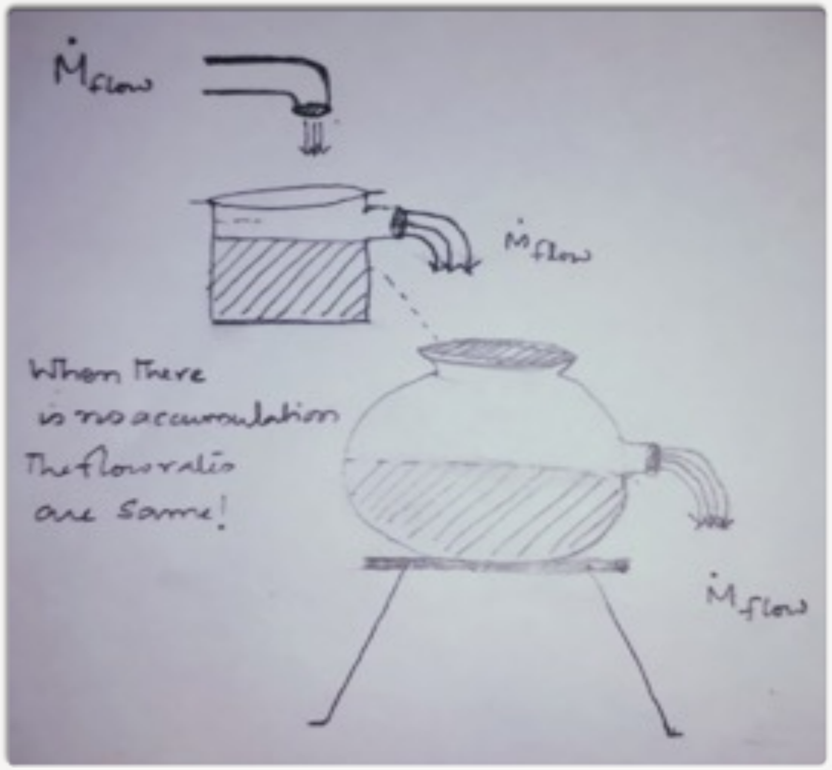


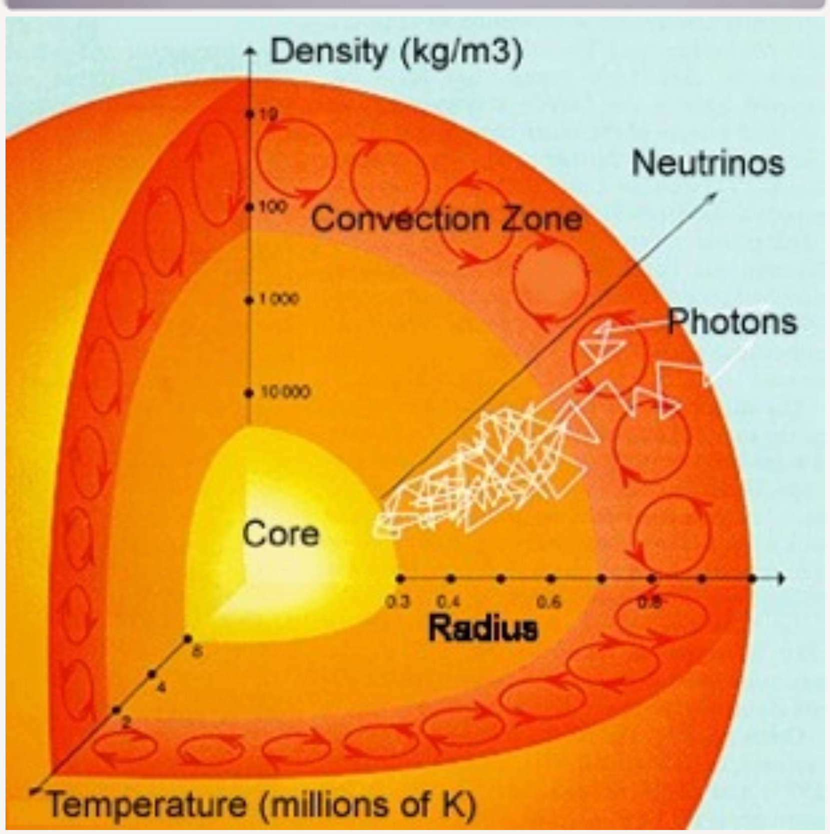
- Hydrostatic equilibrium:

$$
\begin{aligned}
\frac{dp}{dr} &= \rho\frac{d\phi}{dr} \\
\frac{dp}{dr} &= \frac{-GM(<r)}{r^2}\rho \\
\frac{P_s - P_c}{R} &= \frac{-GM}{R^2}\rho \\
P_c &= \frac{GM\rho}{R} = \left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\rho^{4/3}
\end{aligned}
$$

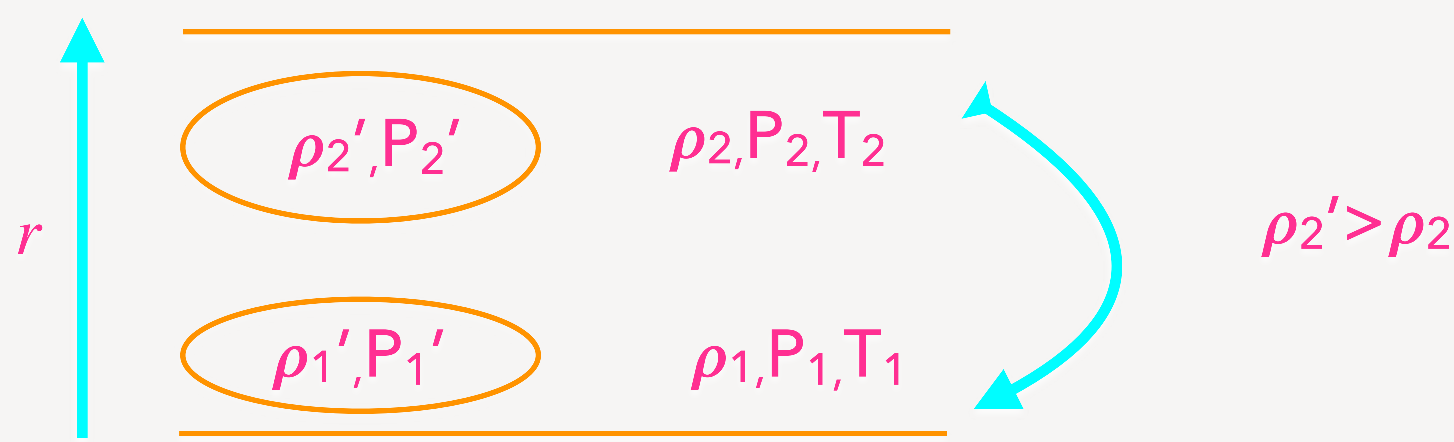
- If we know the equation of state of the matter then we will be able to replace P_c by ρ.
- So in principle with proper choice of density, temperature and pressure gradients one will be able to form a stable star.
- We require T = few 10^3 K (to satisfy the observations) at the surface and $>10^6$ K at the centre (to trigger the nuclear reaction).

TRANSPORT OF ENERGY:

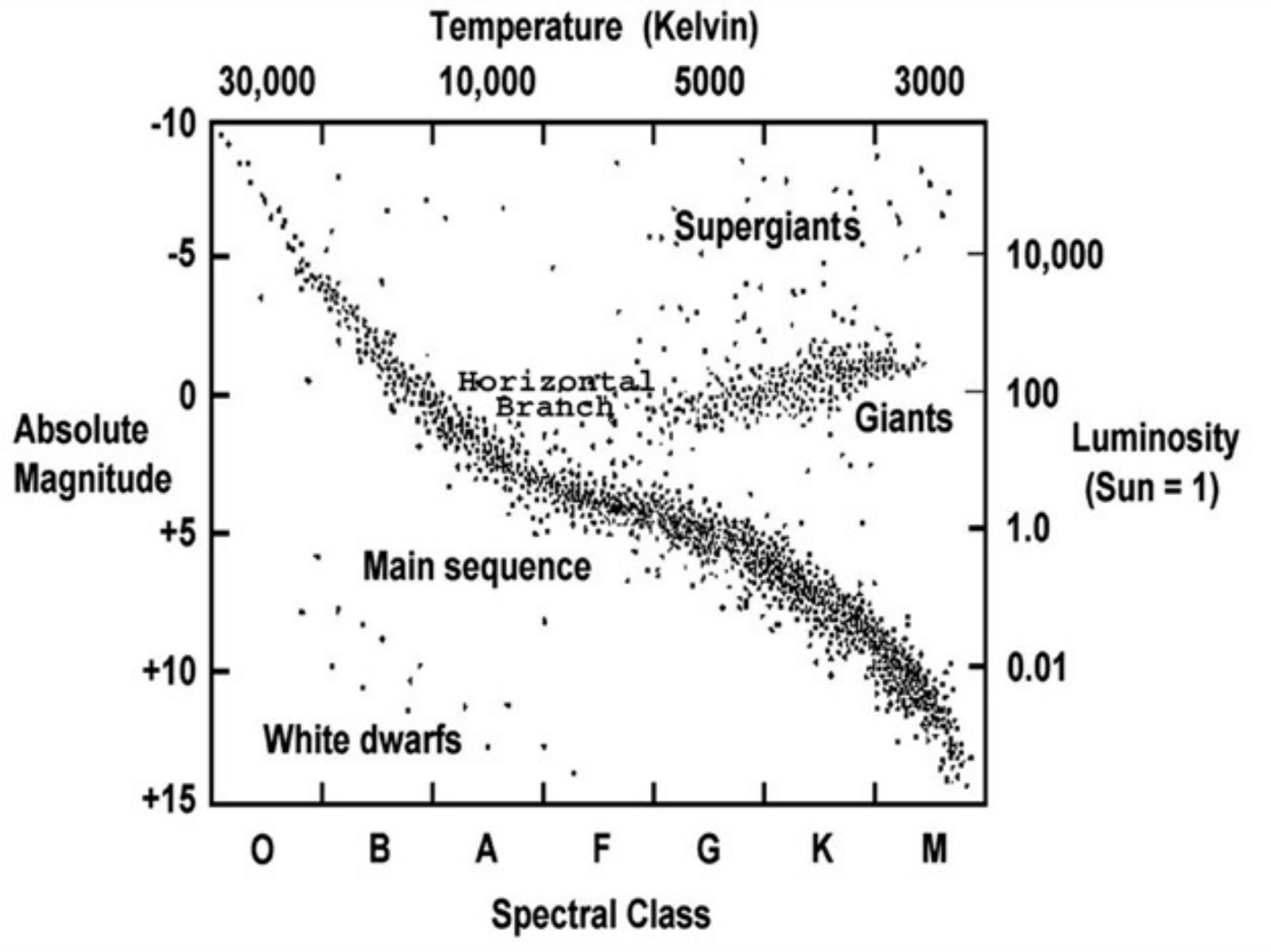




- Requirements: The energy per unit time generated at the centres of stars should be equal to the observed luminosity and there is no accumulation of energy anywhere while transporting energy.
- Possible modes:
  - (i) Transport of radiation: But the radiation can not free propagate. It has to closely interact with matter so that energy can be exchanged. We will have (i) scattering, (ii) free-free, (iii) bound-free and (iv) bound bound transitions that depends on local density and temperature.
  - (ii) Convection:

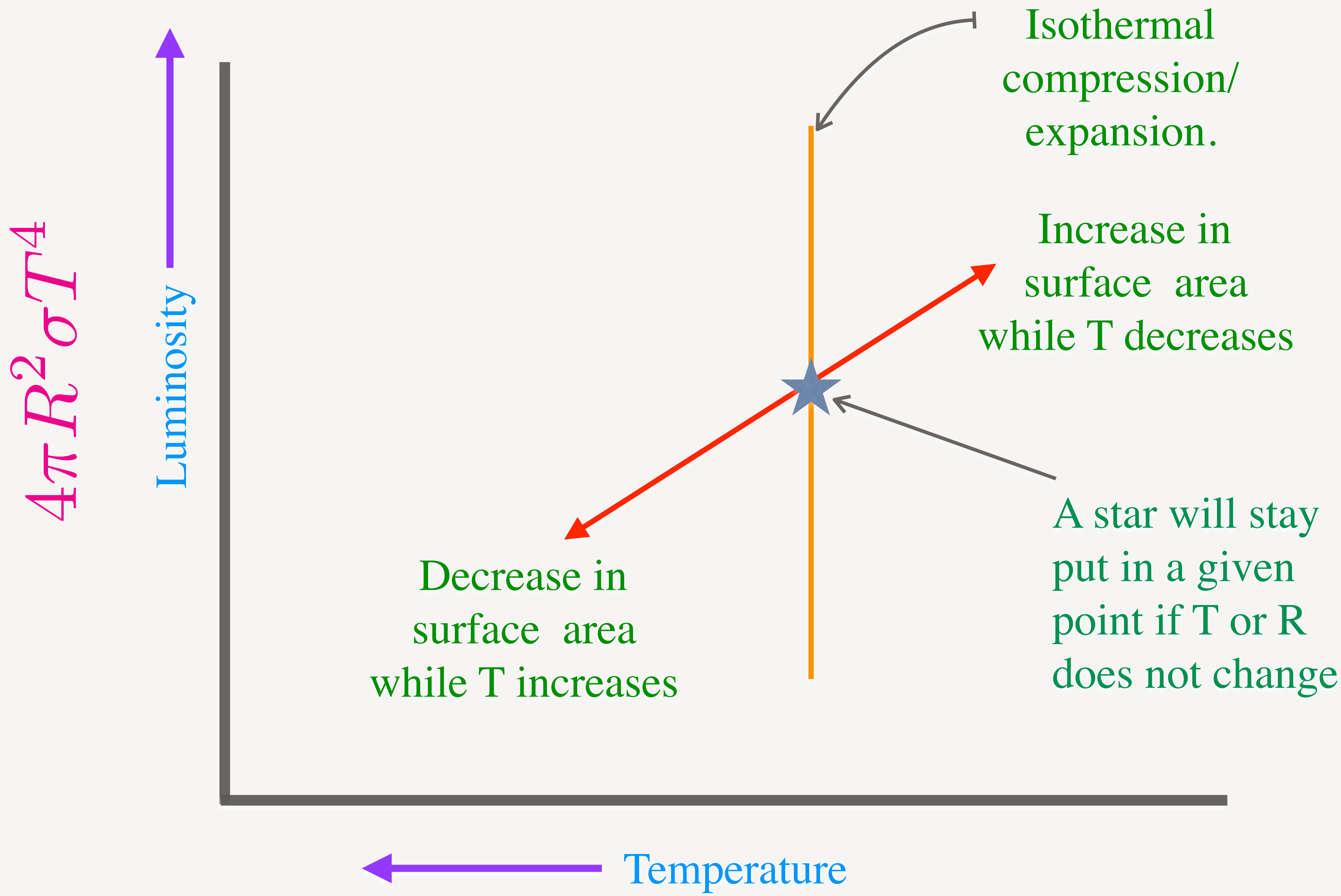


HERTZPRUNG-RUSSELL (HR) DIAGRAM

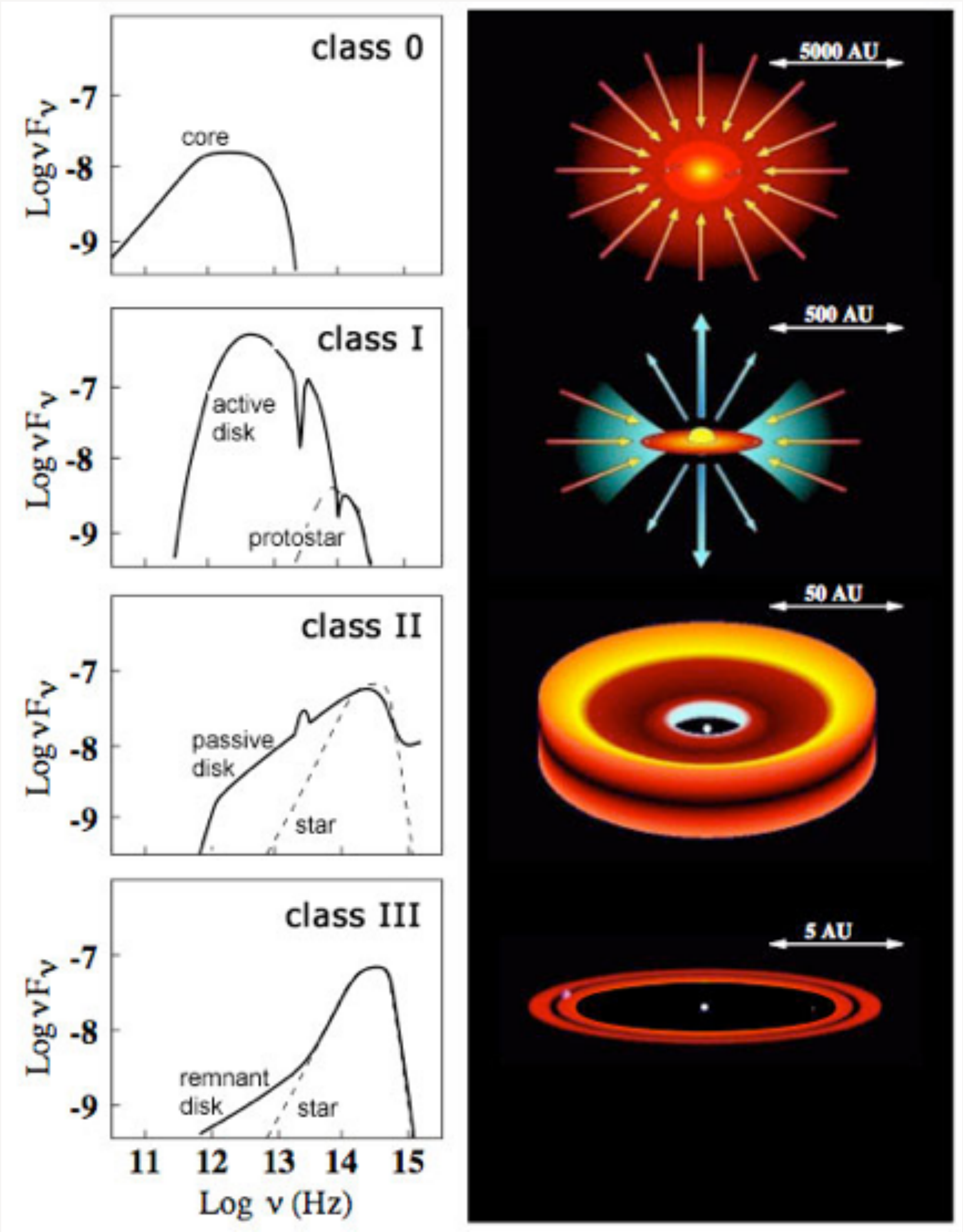


Trend in the population could be an evolutionary sequence. The region occupied by a large number of stars is the parameter range prevailing in individual stars for most part of its life time.

EVOLUTION IN THE HR DIAGRAM



FORMATION OF STARS: HAYASHI TRACKS



- Requirement for forming stars from gas (Jeans instability): Free fall time is shorter than sound crossing time.

$$\frac{1}{\sqrt{G\rho}} < \frac{R_J}{C_S}; \quad C_S^2 = \frac{\gamma P}{\rho} = \frac{\gamma KT}{m_H}$$

- Jeans mass M_J is then,

$$\begin{aligned} M_J &= \frac{4}{3}\pi R_J^3 \rho_0 \quad \text{with} \quad R_J = \frac{C_s}{\sqrt{G\rho}} \\ &= \frac{4}{3}\pi G^{-3/2} C_S^3 \rho_0^{-1/2} \\ &= \frac{4}{3}\pi G^{-3/2} \left(\frac{\gamma KT}{m_H}\right)^{3/2} \rho_0^{-1/2} \end{aligned}$$

- If mass of the gas cloud is more than M_J then it will lead to a collapse. Collapse can be maintained till the gas can cool all the energy gained.
- Cooling generally means energy in the form of radiation leaving the system.

INITIAL MASS FUNCTION (IMF)

- Salpeter IMF

$$\xi(M) = 0.03(\frac{M}{M_{\odot}})^{-1.35}$$

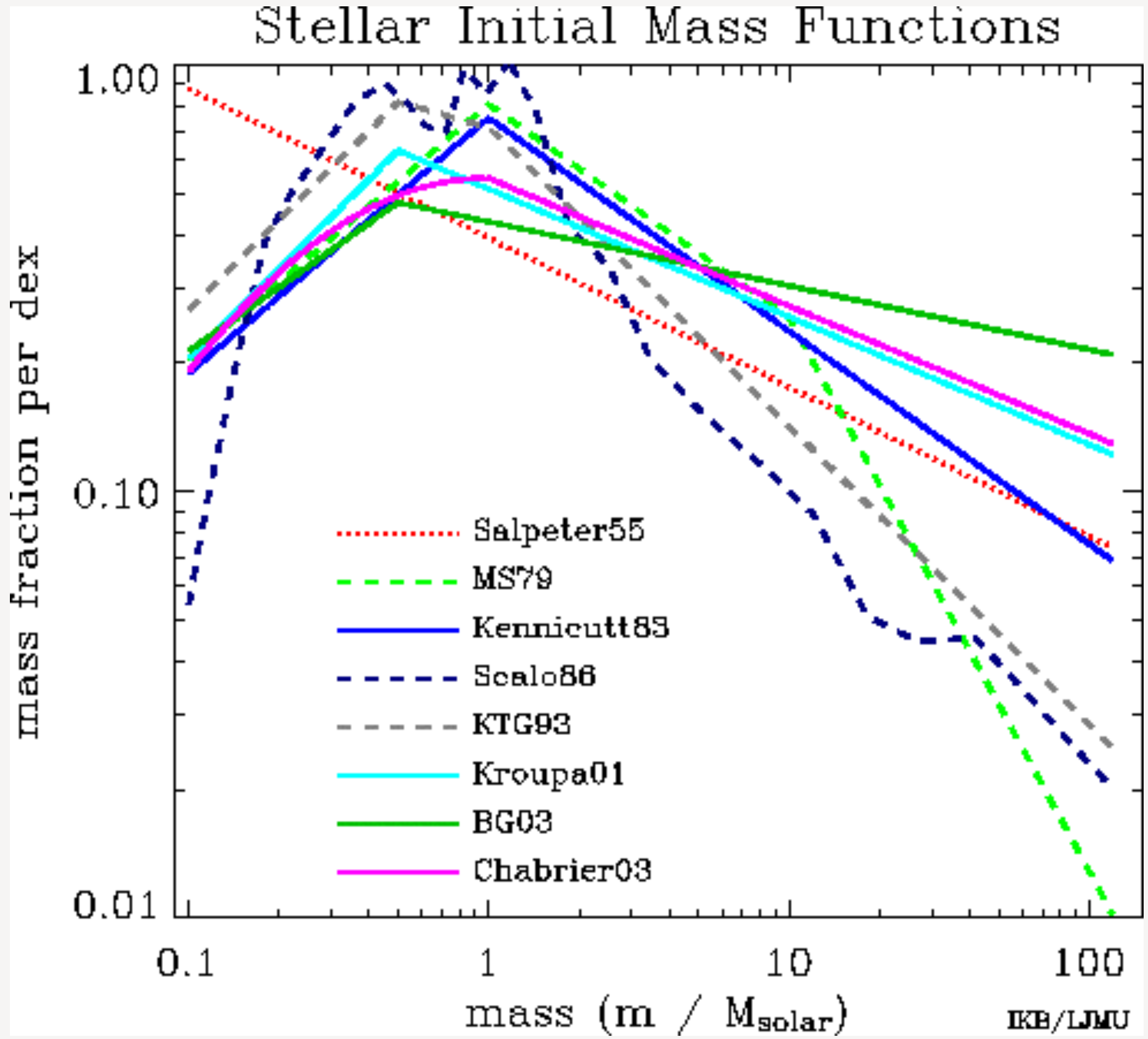
- Scalo IMF

- Chabrier IMF

TABLE 1
Disk IMF and PDMF for Single Objects

Parameter	IMF	PDMF
$m \leq 1.0\ M_{\odot}$, $\xi(\log m) = A \exp[-(\log m - \log m_c)^2/2\sigma^2]$		
A	$0.158^{+0.051}_{-0.046}$	$0.158^{+0.051}_{-0.046}$
m_c	$0.079^{-0.016}_{+0.021}$	$0.079^{-0.016}_{+0.021}$
σ	$0.69^{-0.01}_{+0.05}$	$0.69^{-0.01}_{+0.05}$
$m > 1.0\ M_{\odot}$, $\xi(\log m) = Am^{-x}$		
A	4.43×10^{-2}	
x	1.3 ± 0.3	
$0 \leq \log m \leq 0.54$:		
A		4.4×10^{-2}
x		4.37
$0.54 \leq \log m \leq 1.26$:		
A		1.5×10^{-2}
x		3.53
$1.26 \leq \log m \leq 1.80$:		
A		2.5×10^{-4}
x		2.11

Note.—For unresolved binary systems, the coefficients are given by eq. (18). The normalization coefficient A is in $(\log M_{\odot})^{-1}$ pc^{-3}.



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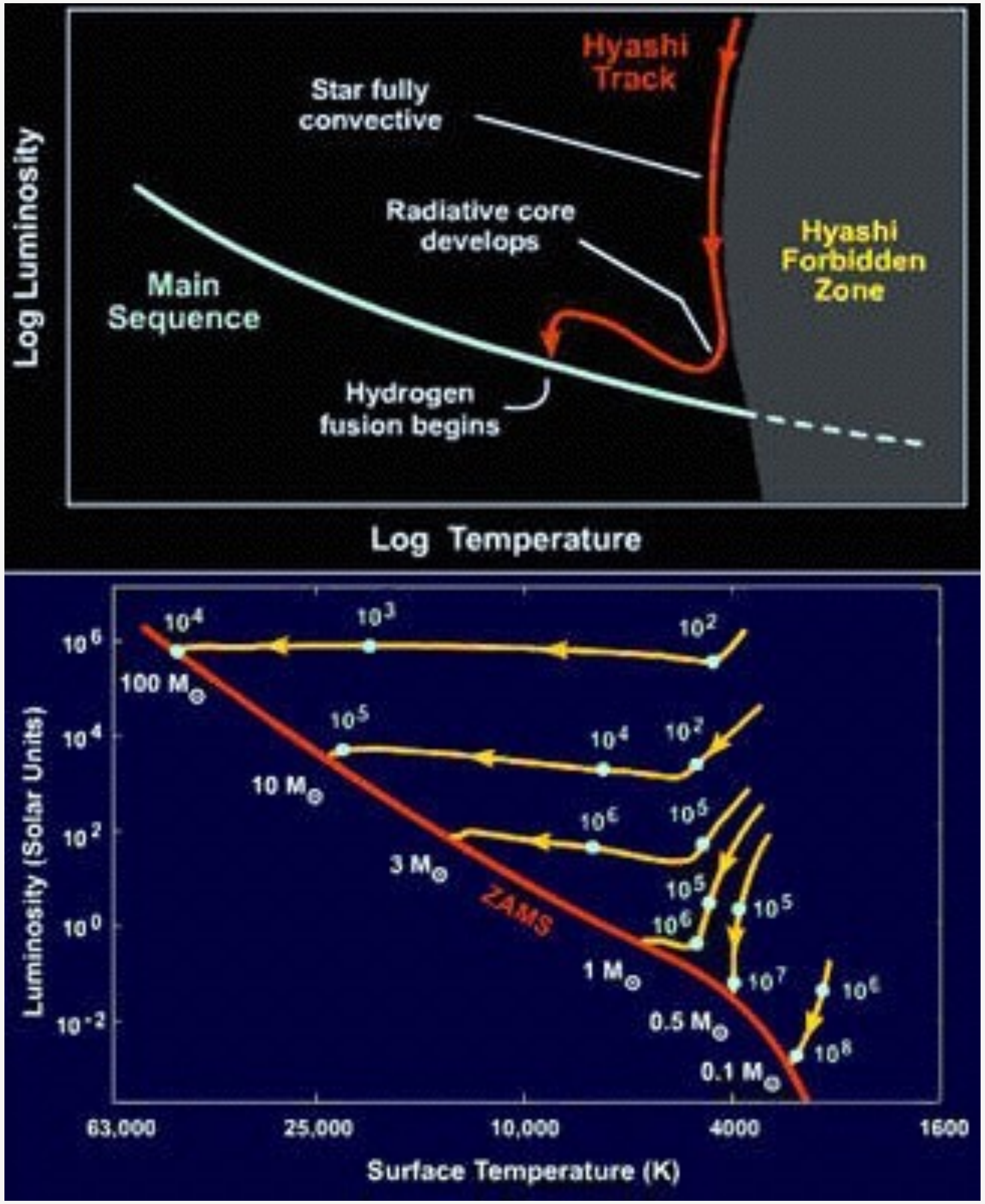

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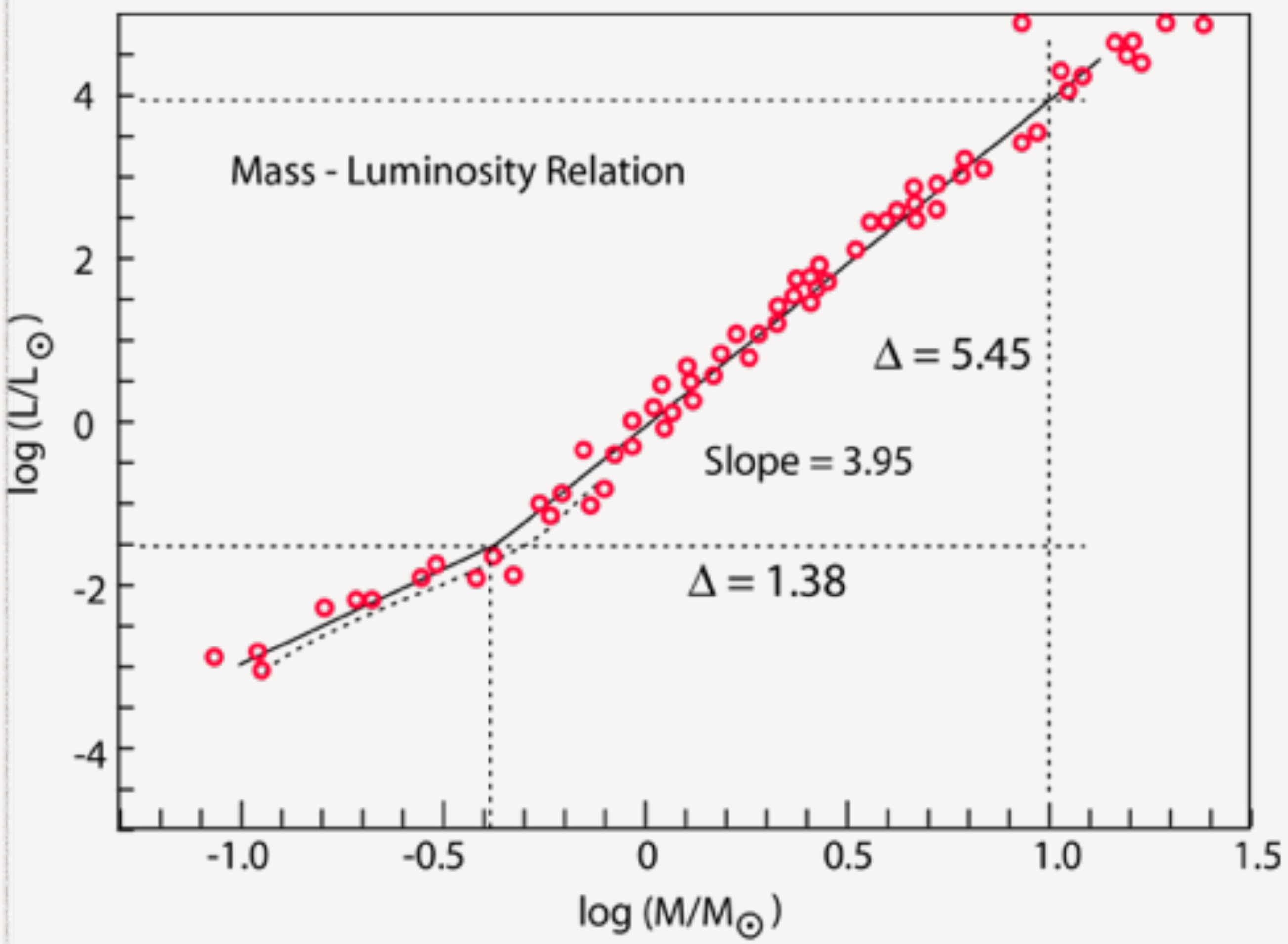
LIFE OF STARS: MAIN SEQUENCE



$$L = L_\odot \left(\frac{M}{M_\odot}\right)^{3.5}$$

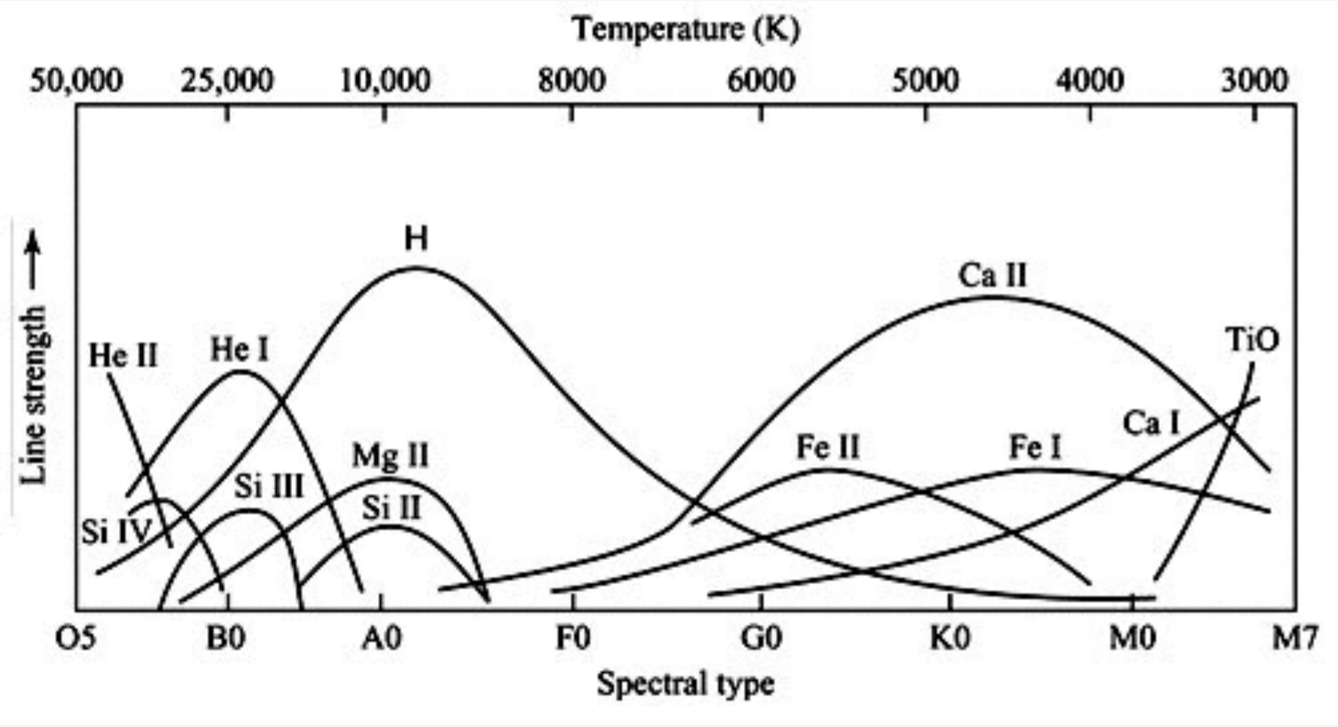
- Stars from most of their time in the so called "Main sequence".
- It is found that there is a tight relationship between the "Luminosity of the star" and its "Mass".
- It is believed that the main source of energy is the Hydrogen burning when the star is in the main sequence.
- As we say before,

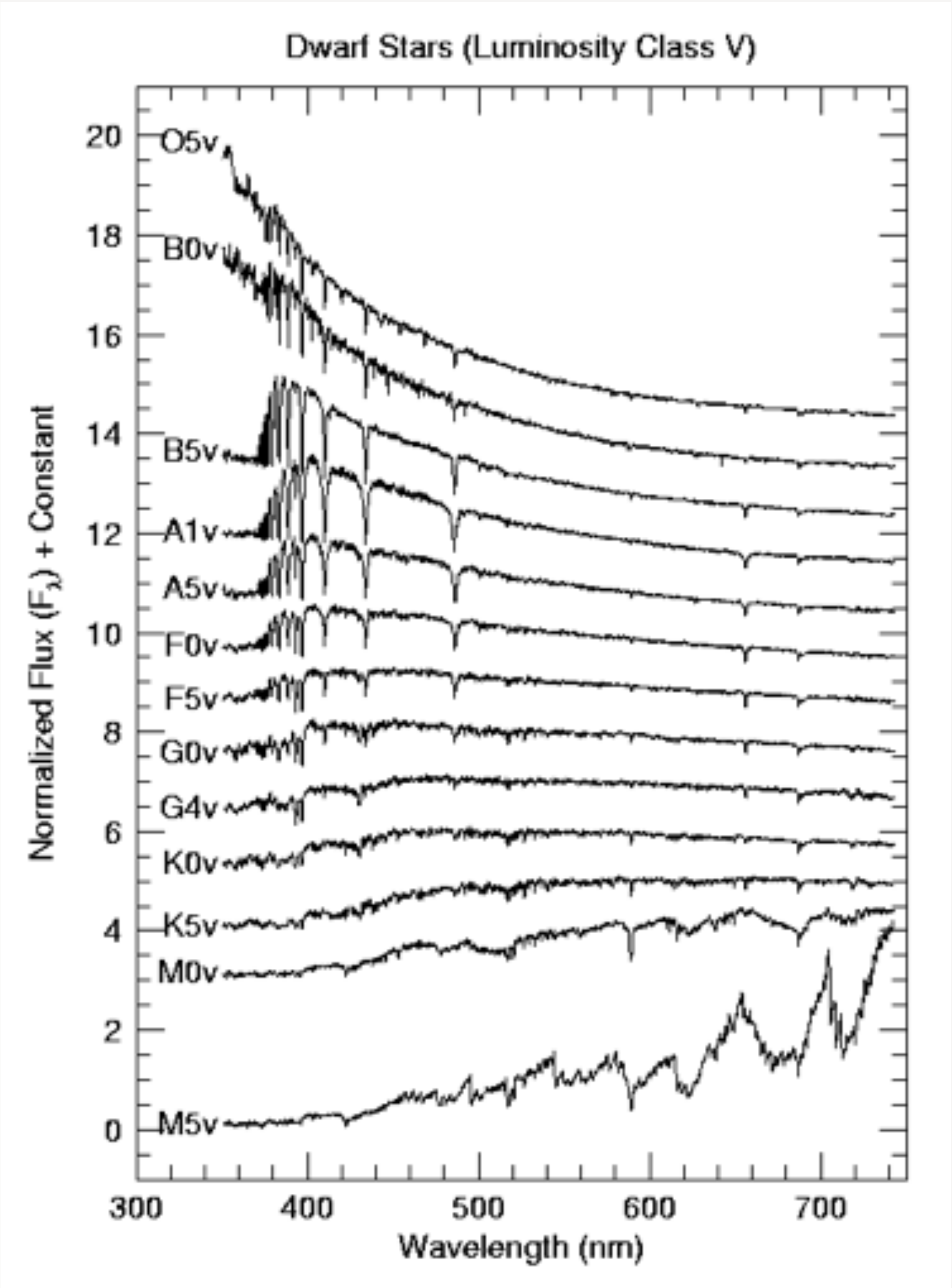
$$\begin{aligned}
\tau_{MS} &= \frac{E}{\Delta E/\Delta T} \\
&= \frac{6.4\times 10^{18}}{3\times 10^{7}} f_M \frac{M}{L}\, yr \\
&= 2.1\times 10^{11} f_M \left(\frac{M}{M_\odot}\right)\left(\frac{M_\odot}{L_\odot}\right)\left(\frac{M}{M_\odot}\right)^{-7/2} yrs \\
&\sim \left(\frac{M_\odot}{M}\right)^{-5/2} \times 10^{10}\ \ yrs
\end{aligned}$$

Massive objects spend less time in the main sequence. For example, life time of a 10 $M_\odot$ star will live only for 3 x 10^7 yrs compared to ~10^{10} yrs for sun. Therefore the main sequence turn around can be used to measure the age of the stellar population.

STAR SPECTRA

- Collisional ionisation and excitation equilibrium make spectrum deviate from BB.

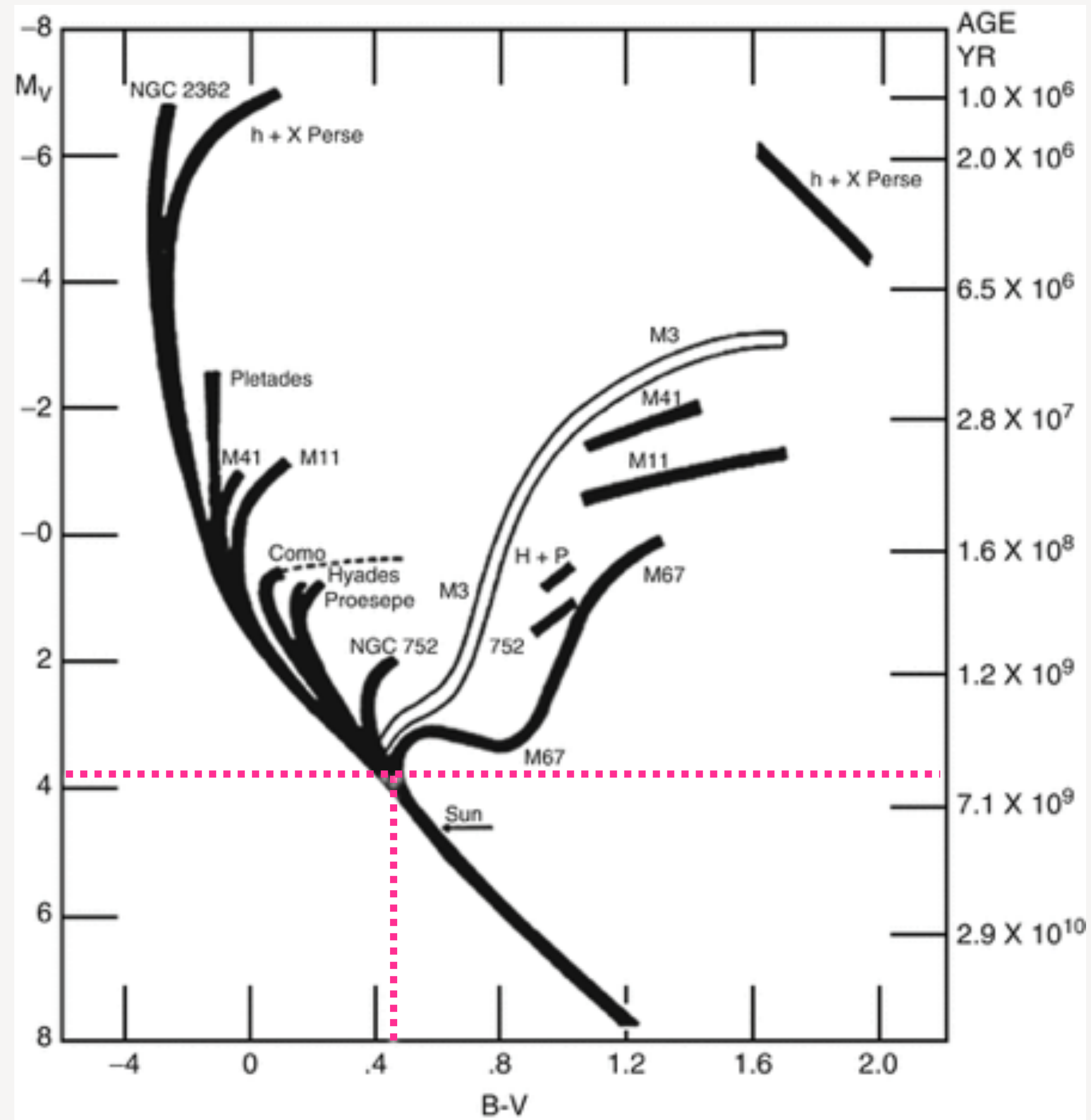




$$\frac{N_2}{N_1} = \frac{g_2}{g_1} exp(-\Delta E/KT_{ex})$$

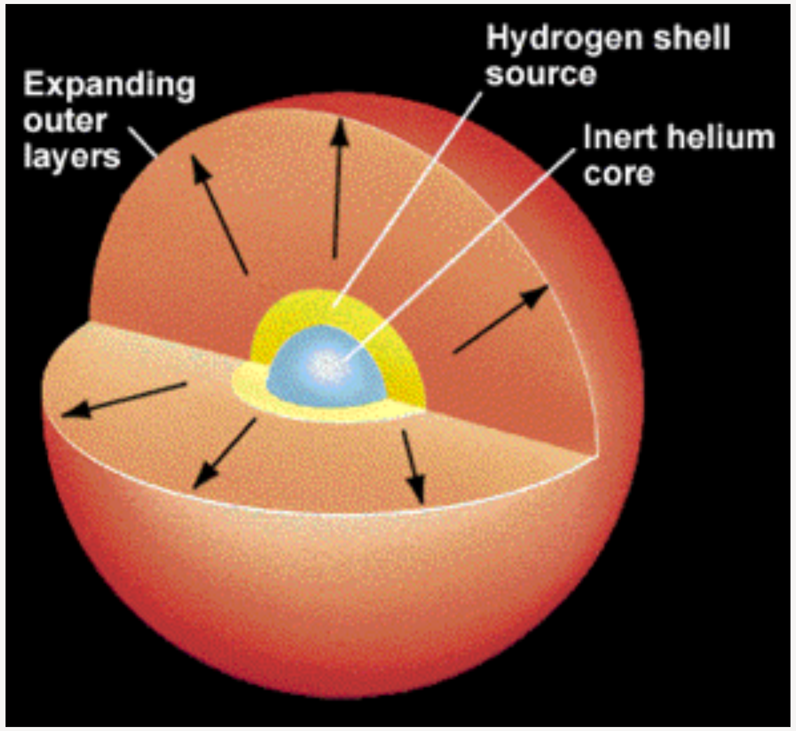
$$\frac{n(X^{(r+1)})n_e}{n(X^{(r)})} = 2\,\frac{g_{r+1,1}}{g_{r,1}}\left(\frac{2\pi m_e KT}{h^2}\right)^{3/2} exp(-\phi/KT)$$

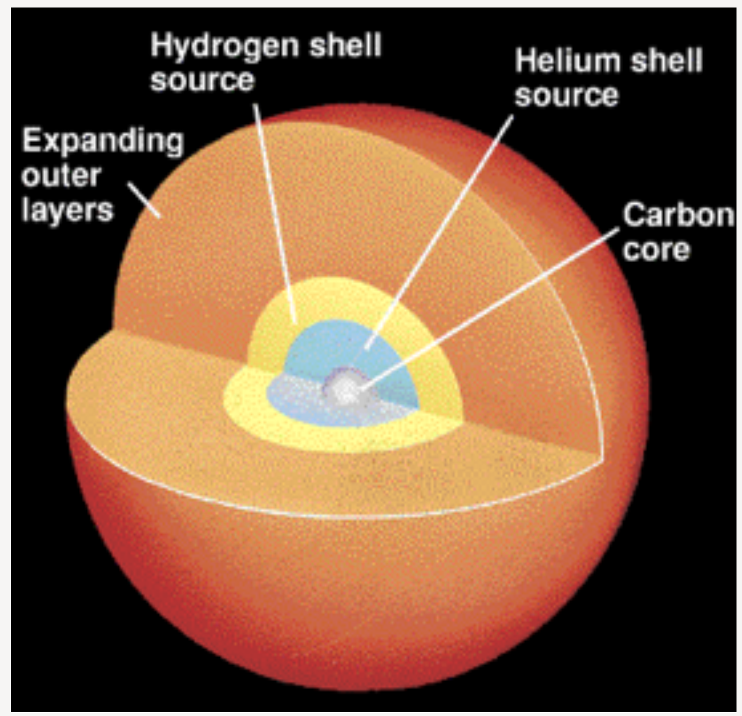
LIFE OF STARS: MAIN SEQUENCE TURNAROUND

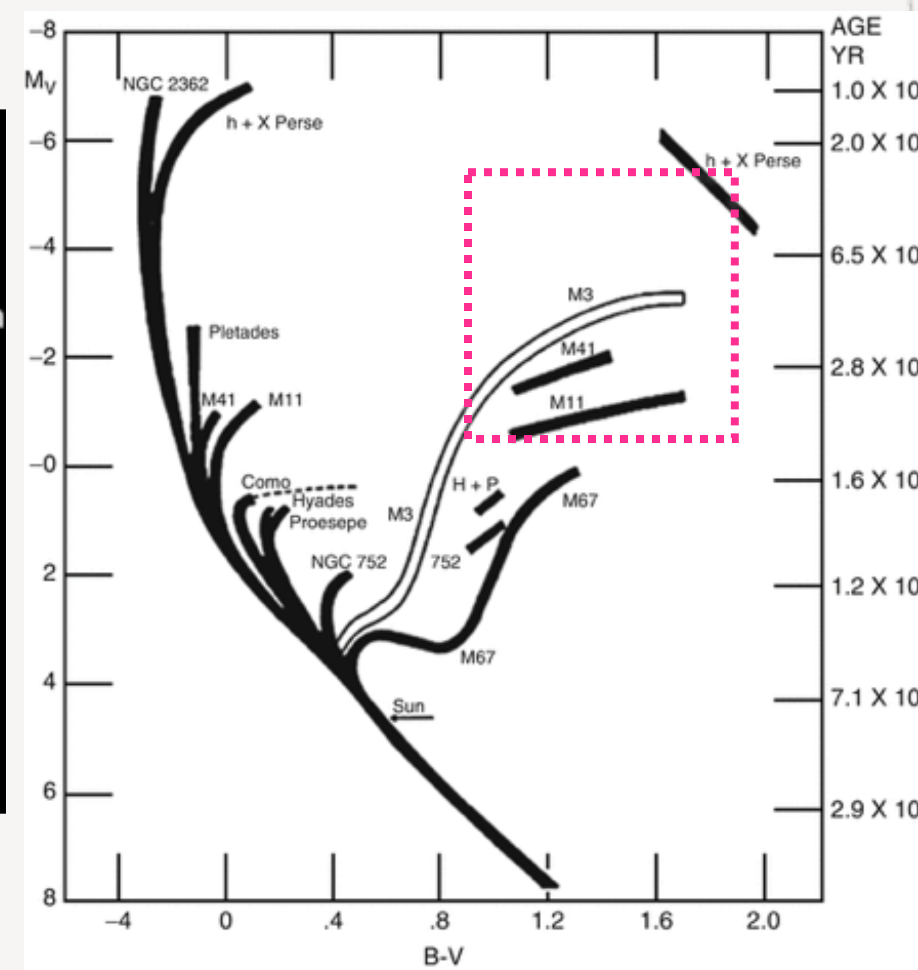


This allows us to measure the age of star cluster. By demanding no star cluster should be older than our universe itself we will be able to place a lower limit on the age of the universe

LIFE OF STARS: GIANTS

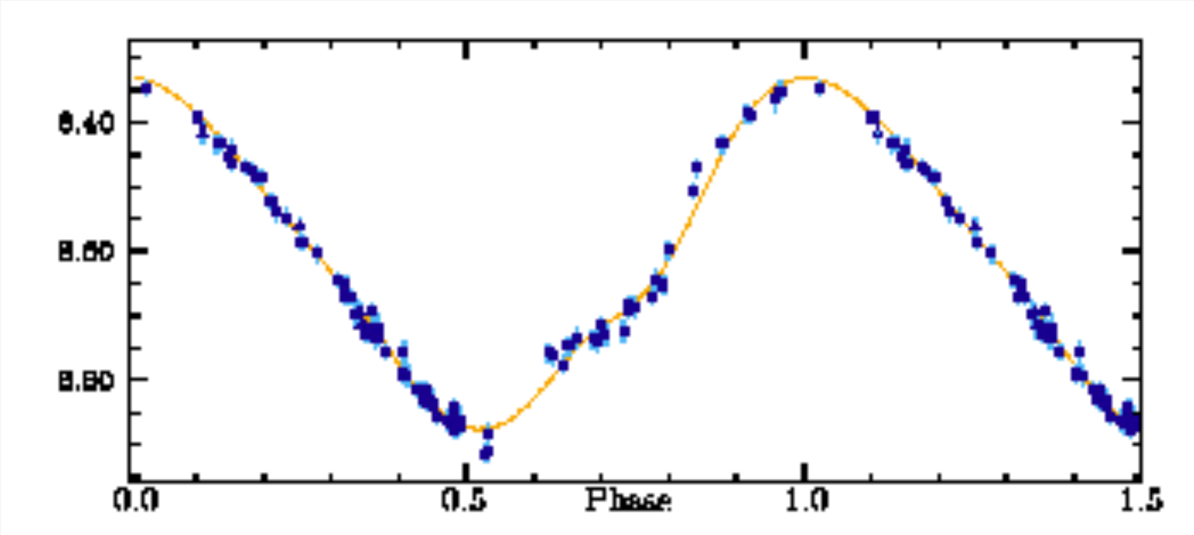


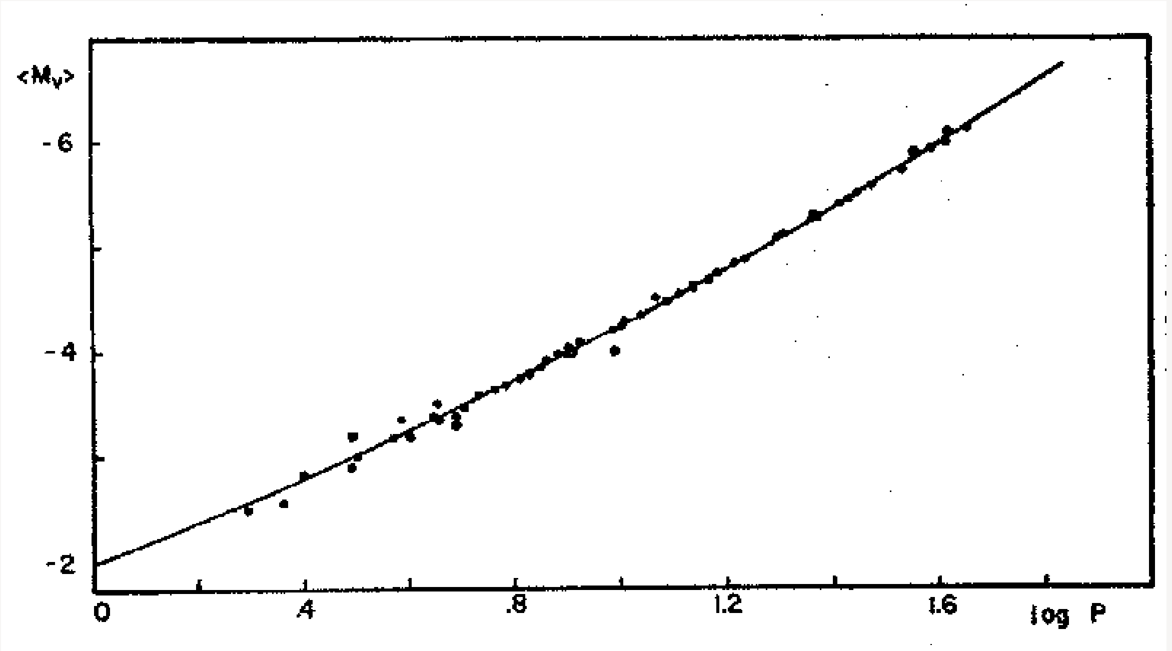


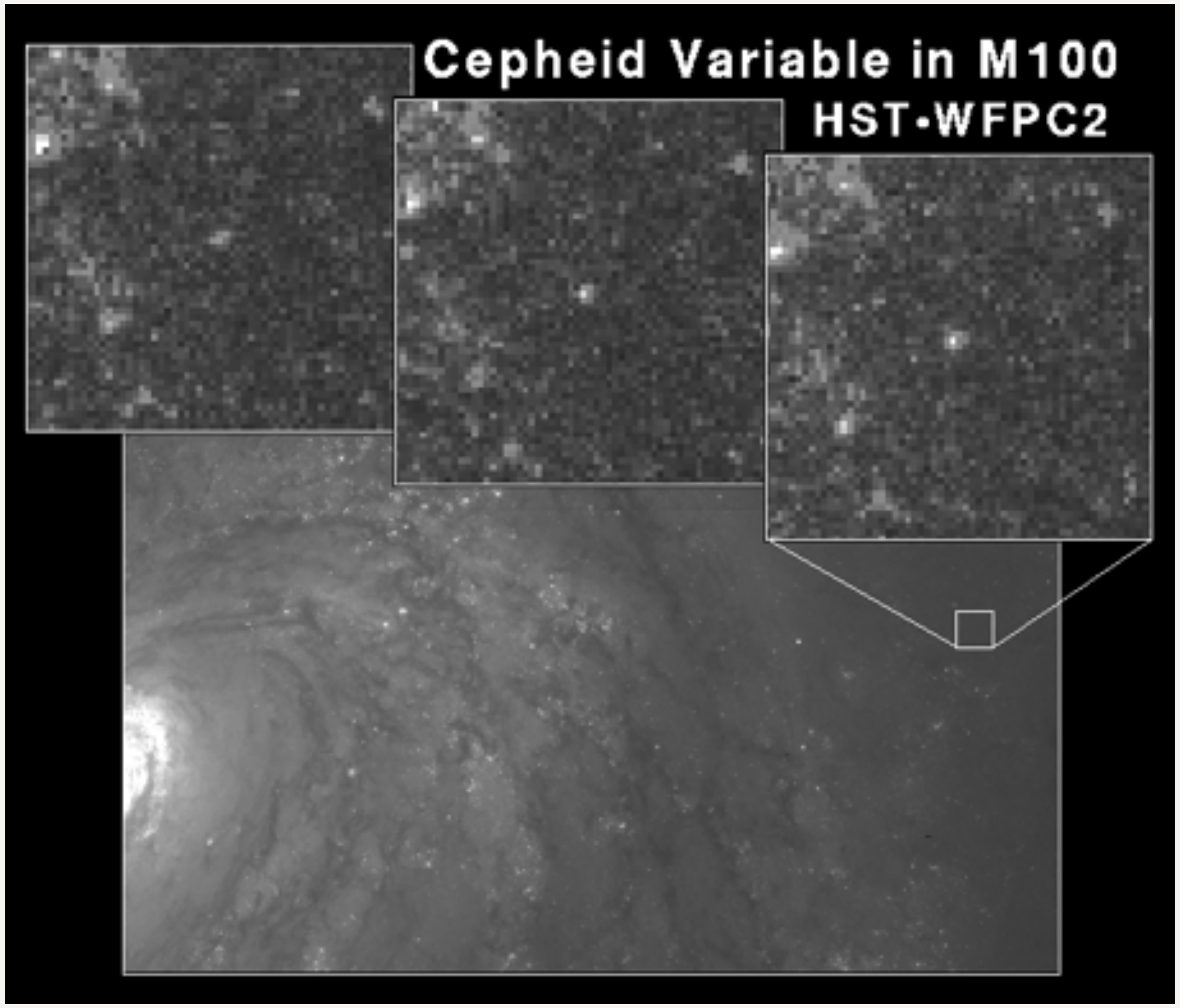


After the H burning stage, stars evolve through the HR diagram more rapidly as nuclear reactions can proceed more rapidly compared to the H-burning stages.

LIFE OF STARS: VARIABLE STARS



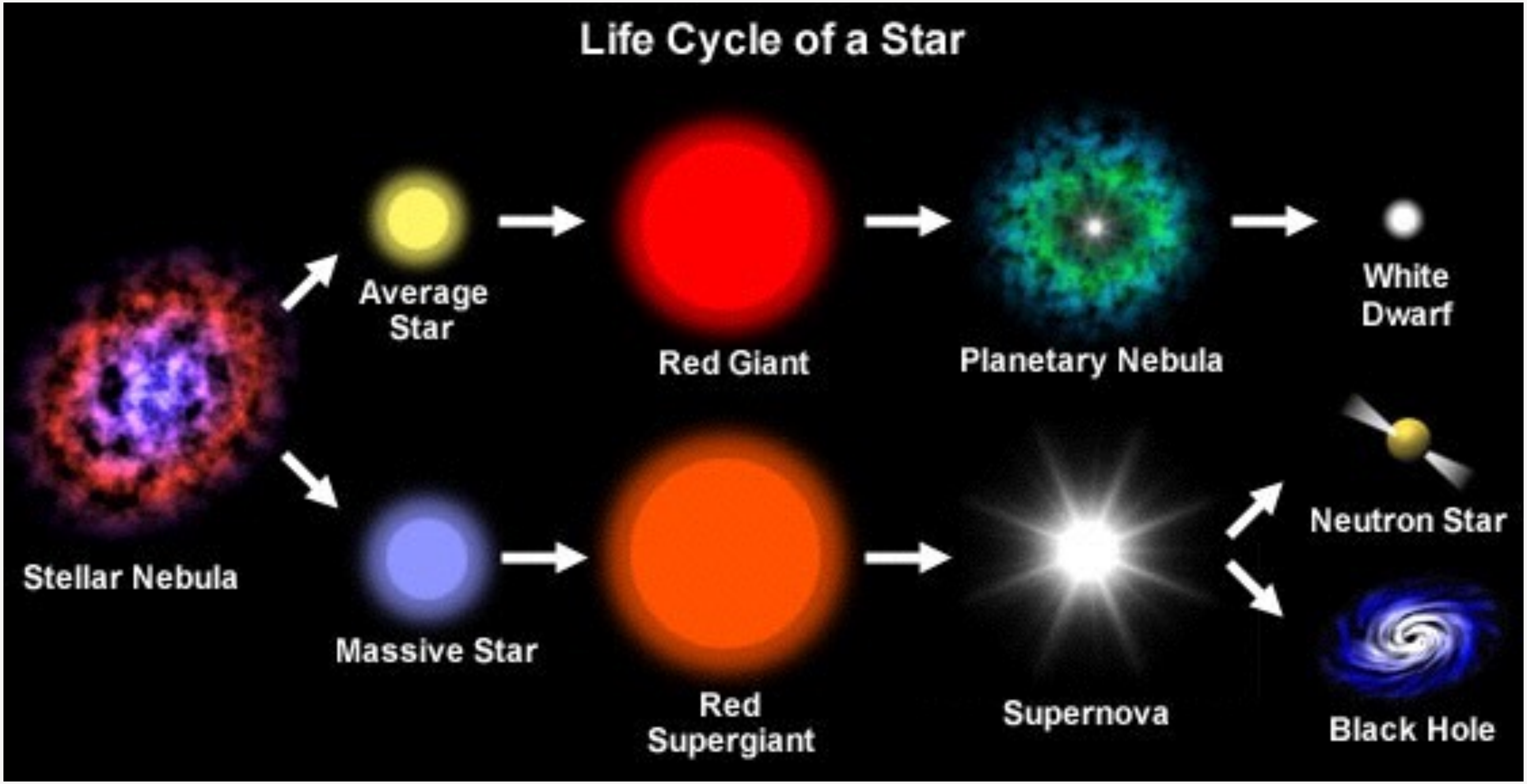




$M_v = -[2.76(log10(P) - 1.0)] - 4.16$

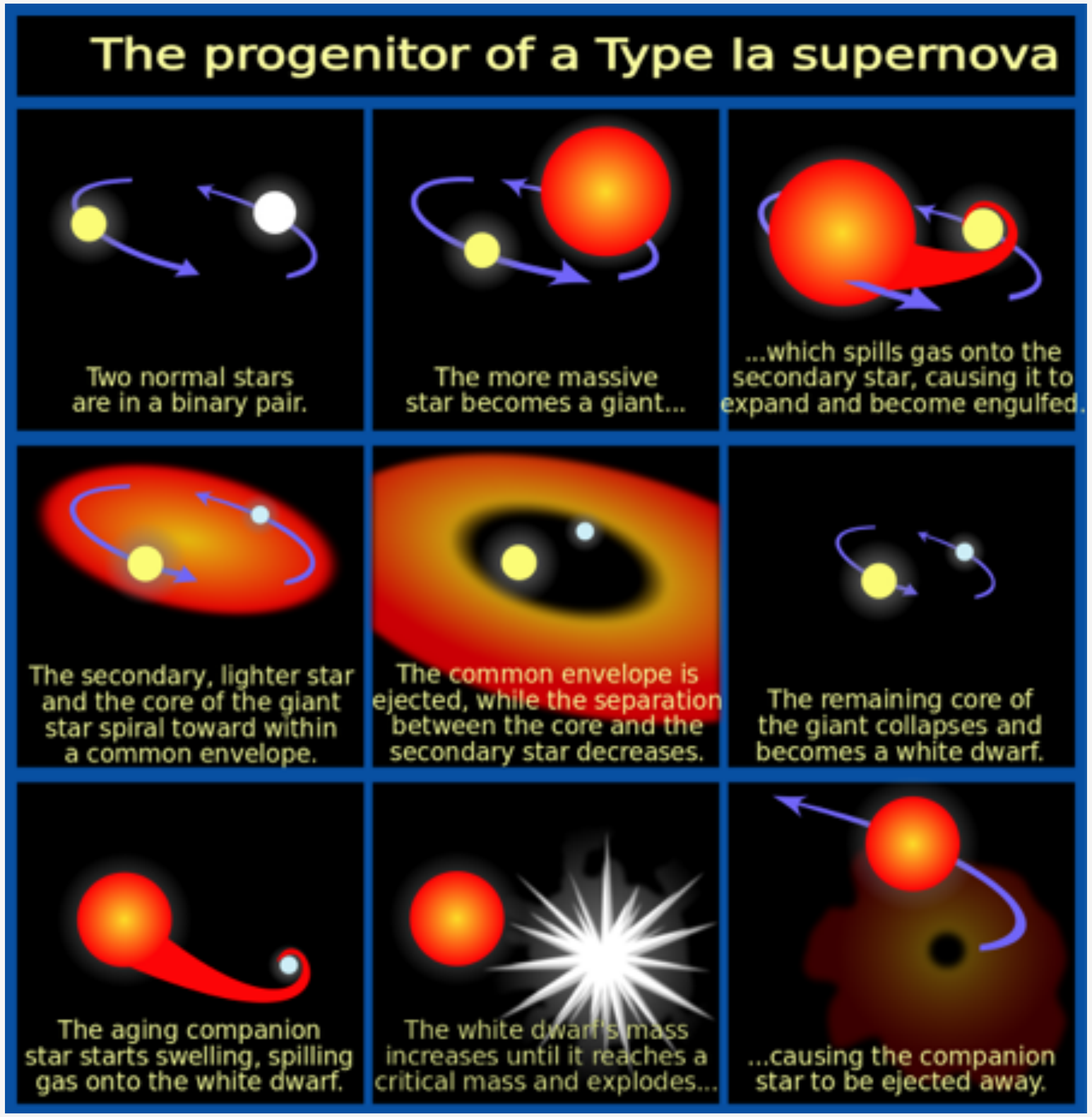
Opacity effects dominate the small scale variability. The measured period luminosity relationship allow us to measure the distances of nearby galaxies.

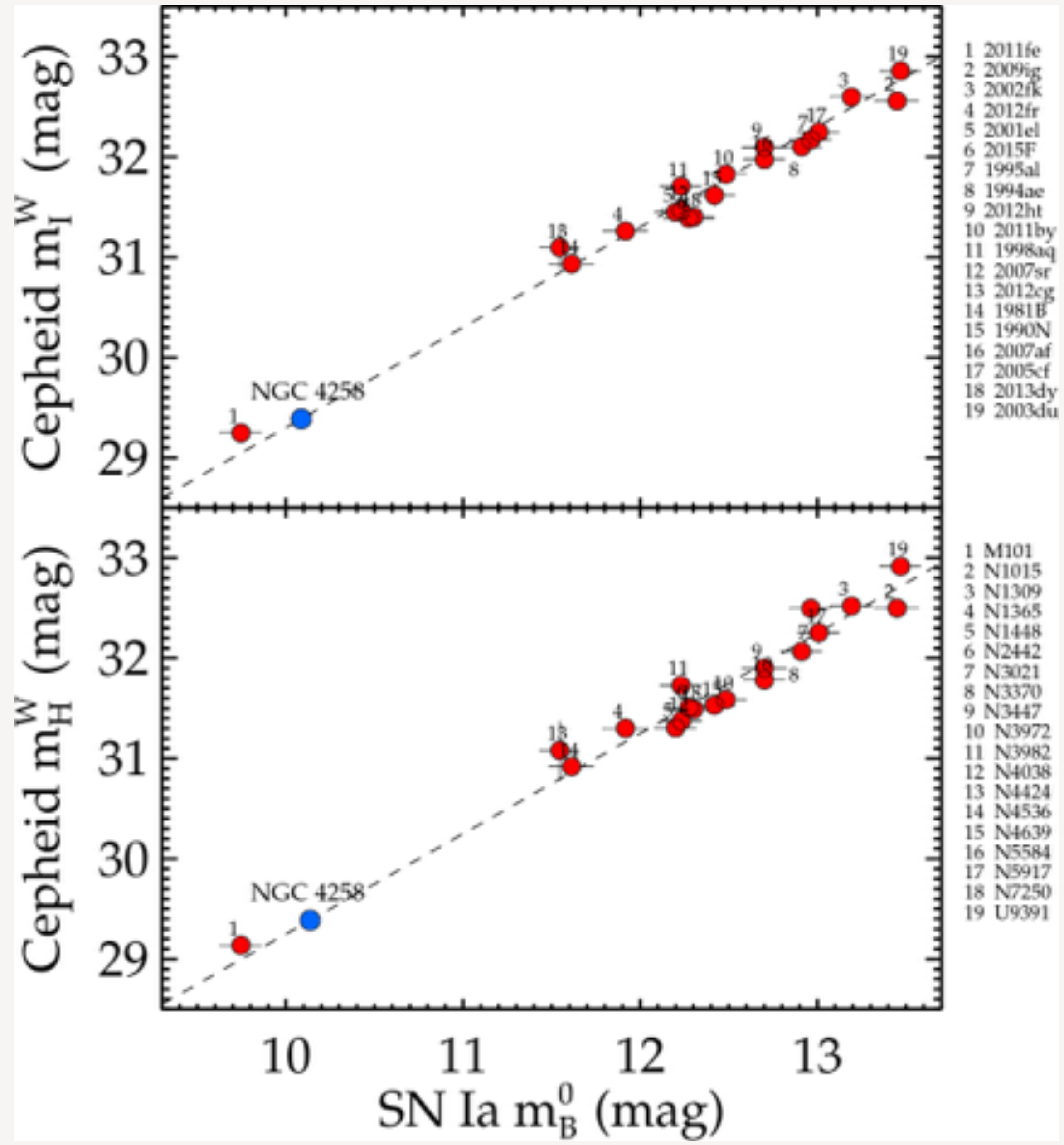
LIFE OF STARS: END STAGES



White dwarfs and neutron stars can still be modelled in terms of hydrostatic equilibrium. Only different is that we need to use appropriate equation of state for the gas. Usually correspond to EOS of degenerate electron or neutron matter.

LIFE OF STARS: END STAGES

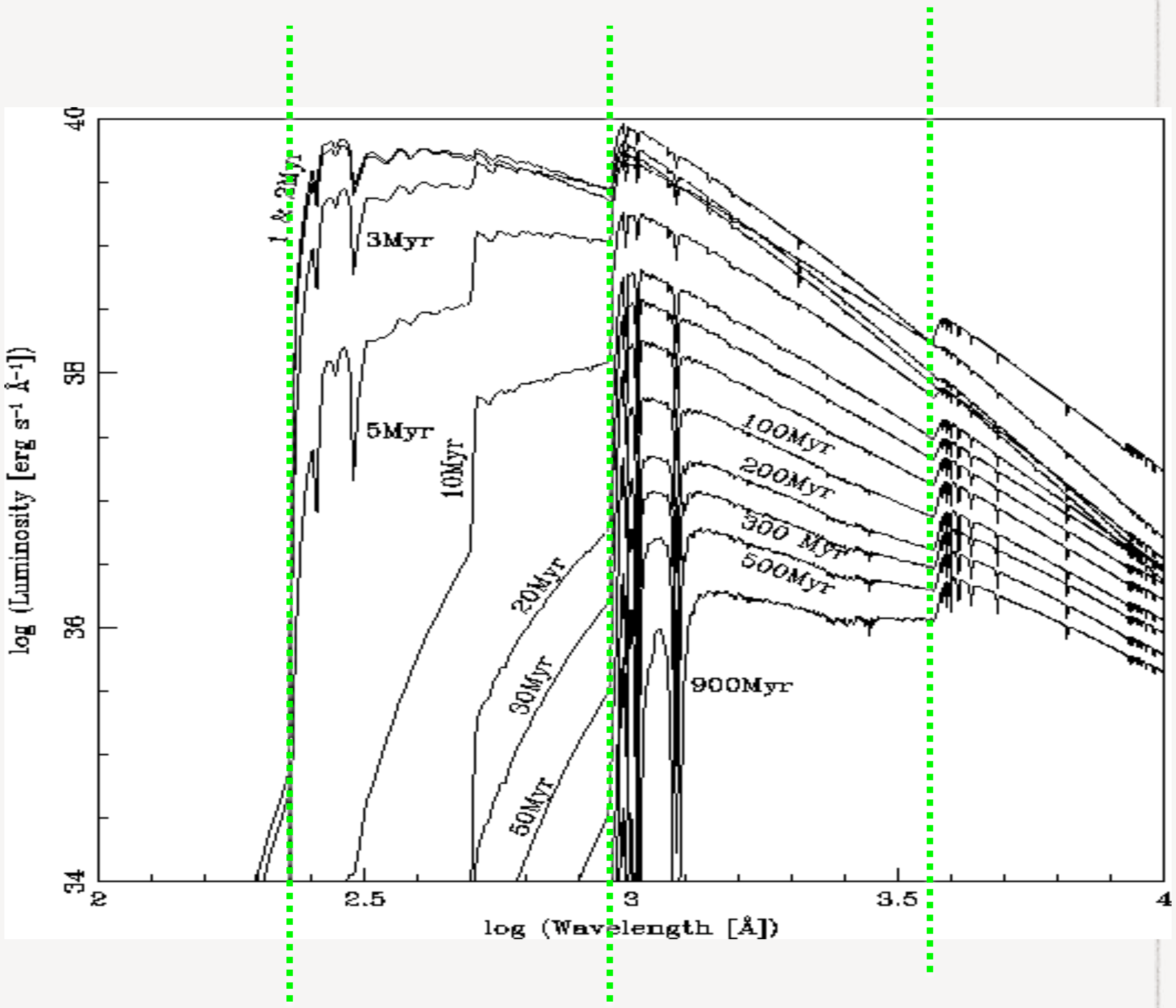




White dwarfs and neutron stars can still be modelled in terms of hydrostatic equilibrium. Only different is that we need to use appropriate equation of state for the gas. Usually correspond to EOS of degenerate electron or neutron matter.

SPECTRAL EVOLUTION OF A GROUP OF STARS

- Salpeter IMF.
- $10^6\ M_\odot$ single burst.
- $Z = 1/20\ Z_\odot$.
- "STARBURST99"
- Metallicity, Binary fraction, IMF, mass cut-offs will affect the computed spectrum.



WHY STUDY STARS

- Stars and associated stellar processes inject various forms of energy into the galaxy (in particular the medium between galaxies): Energy injection by radiation, mechanical energy injection and high energy cosmic rays.
- Stars produce heavier elements and dust and distribute to the surrounding medium.
- Stellar remnants are excellent cites to study matter at extreme densities and also GR effects.
- Stellar light are beacons to study the material between us and the stars (i.e the so called interstellar medium) that has the imprints of the present and past star formation history.
- Understanding star formation & evolution is important to understand the light coming from distant galaxies.
- Some stars are good distant indicators. This allows us to probe the distant universe.