Scales in the universe

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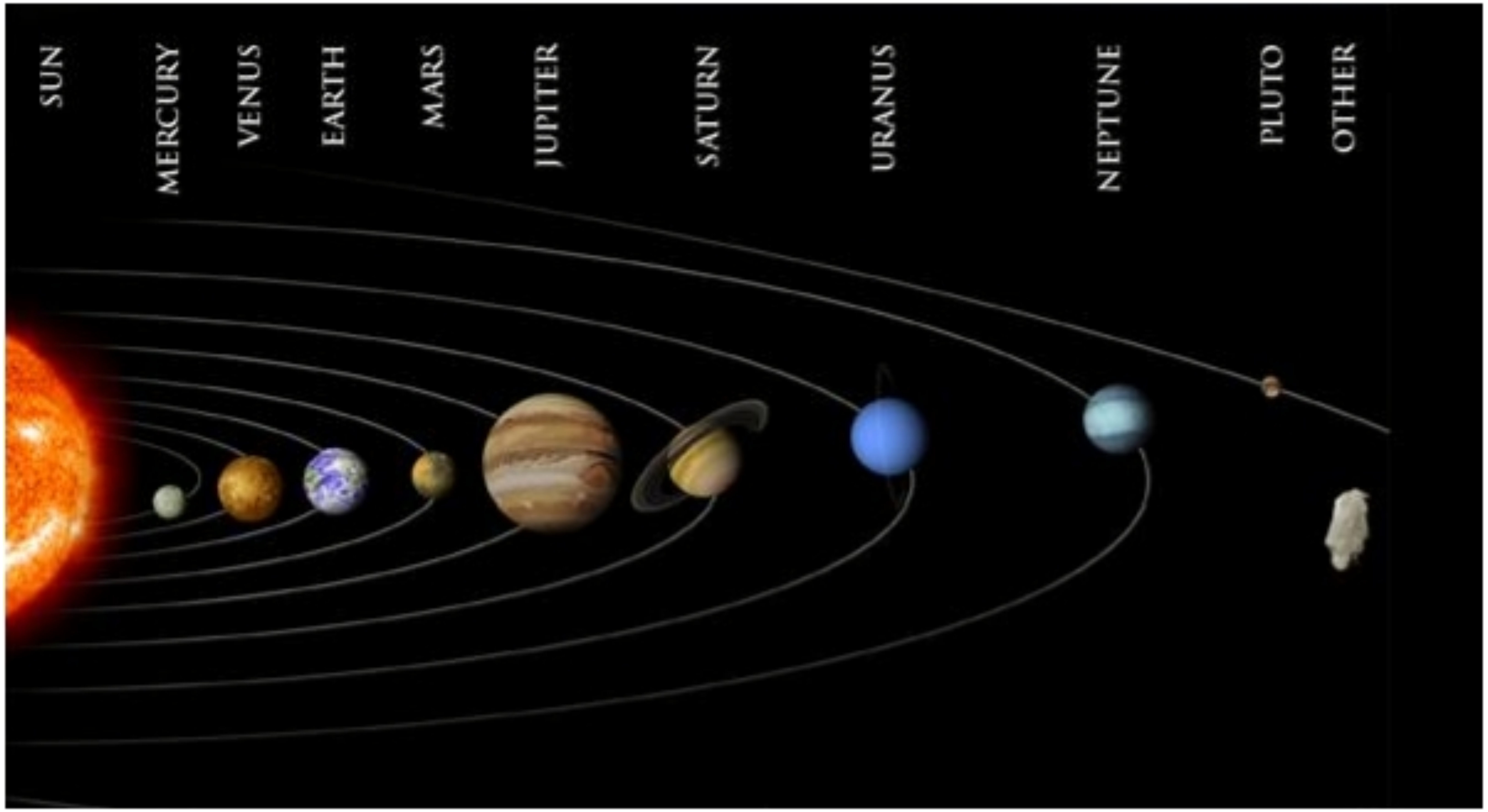
SOLAR SYSTEM:

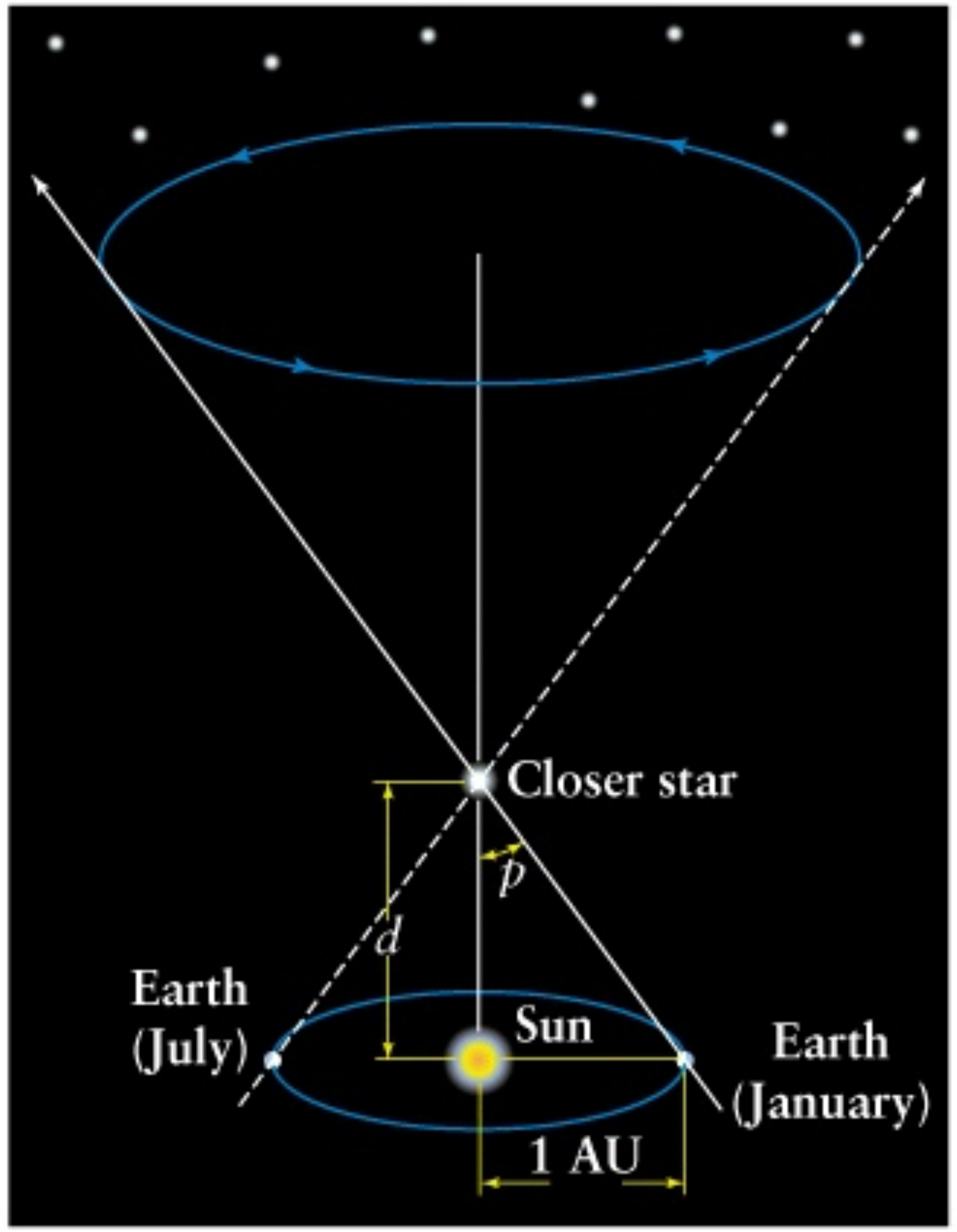


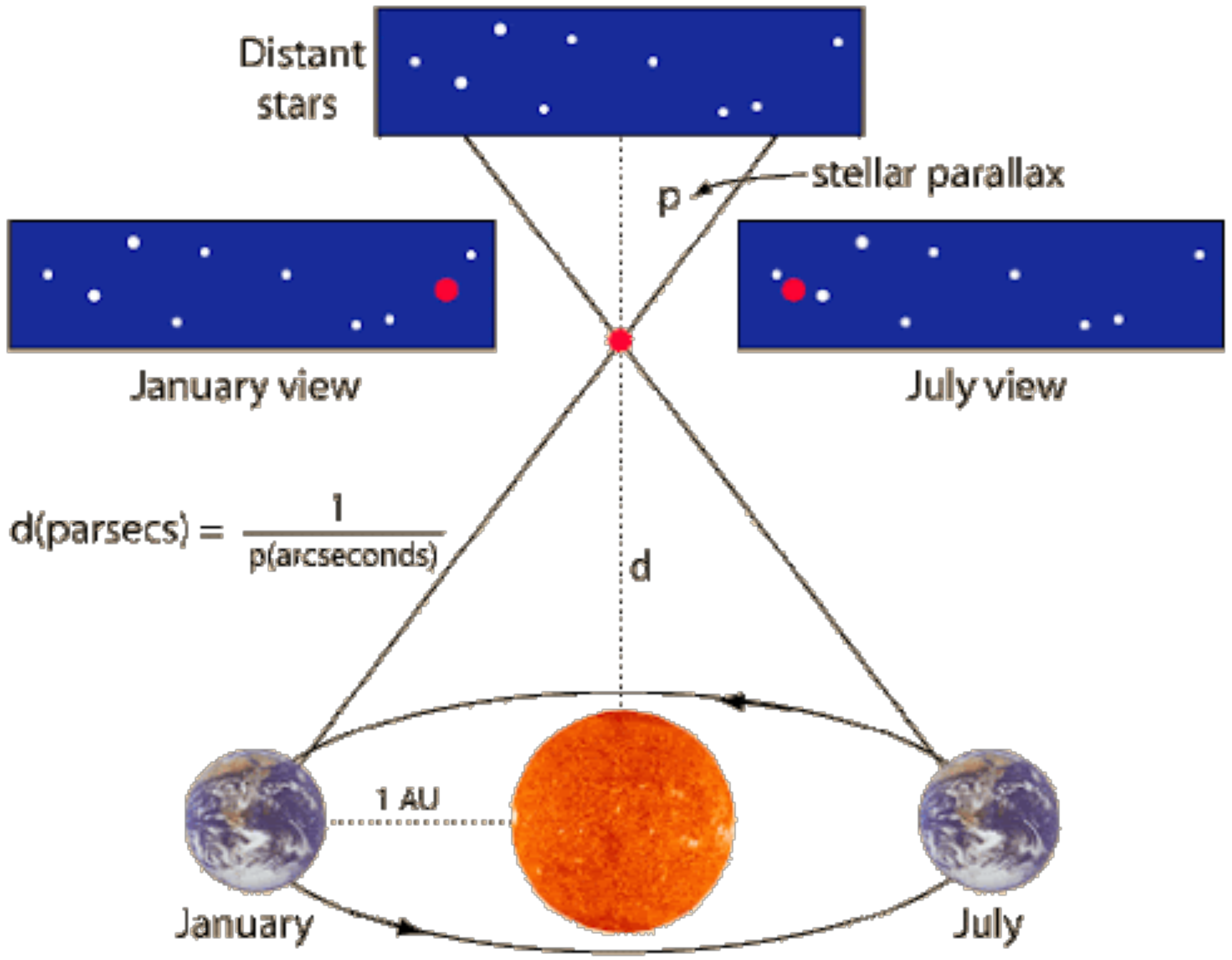
Distance to venus = 0.5$\times$ Radar travel time $\times$ c

Distance to Sun = Distance to Venus $\times$ cos(θ) = 1.5 x 10^{13} cm

$$T_p \;=\; T_e \times (R_p/R_e)^{3/2}$$

DISTANCE TO NEARBY STARS:

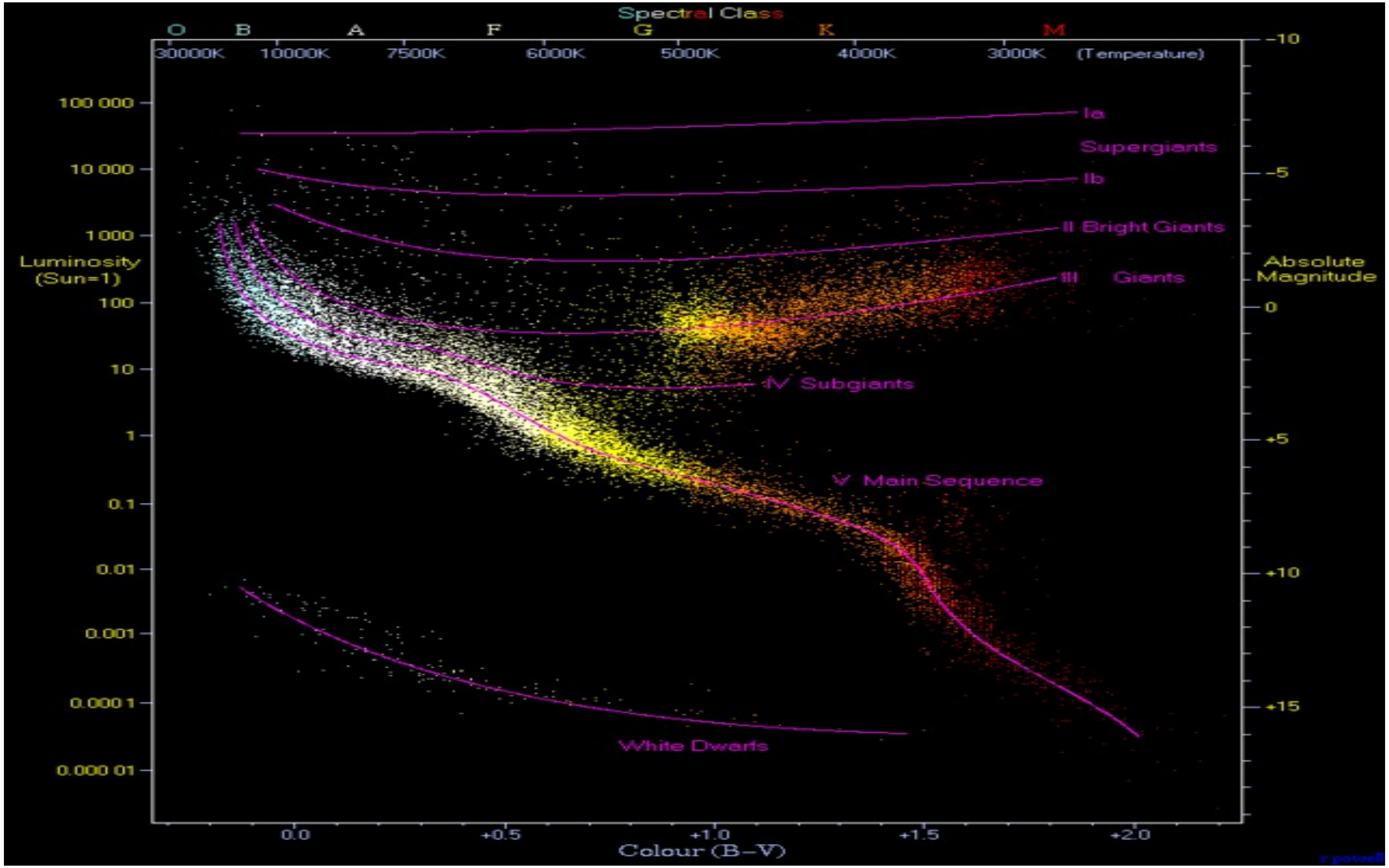




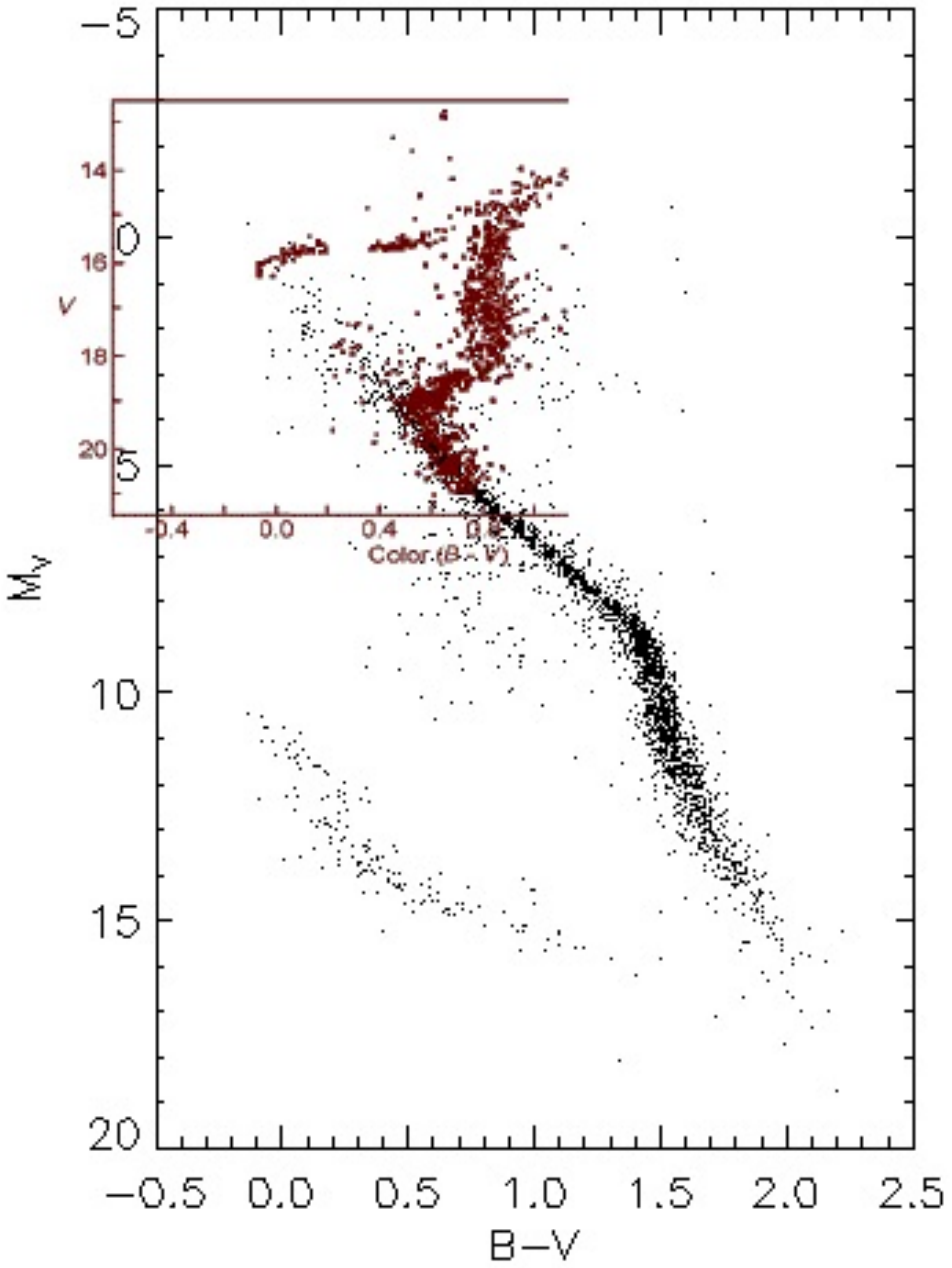
When p = 1 arcsec, d is 1 pc (i.e 3×10^{18} cm).

The Hipparcos satellite, which makes its measurements from Earth orbit, measured the parallax distances to about 120,000 stars with an accuracy of 0.001 arc seconds, and about 2.5 million stars with a lesser degree of accuracy. This gives accurate distances to stars out to several hundred light-years.

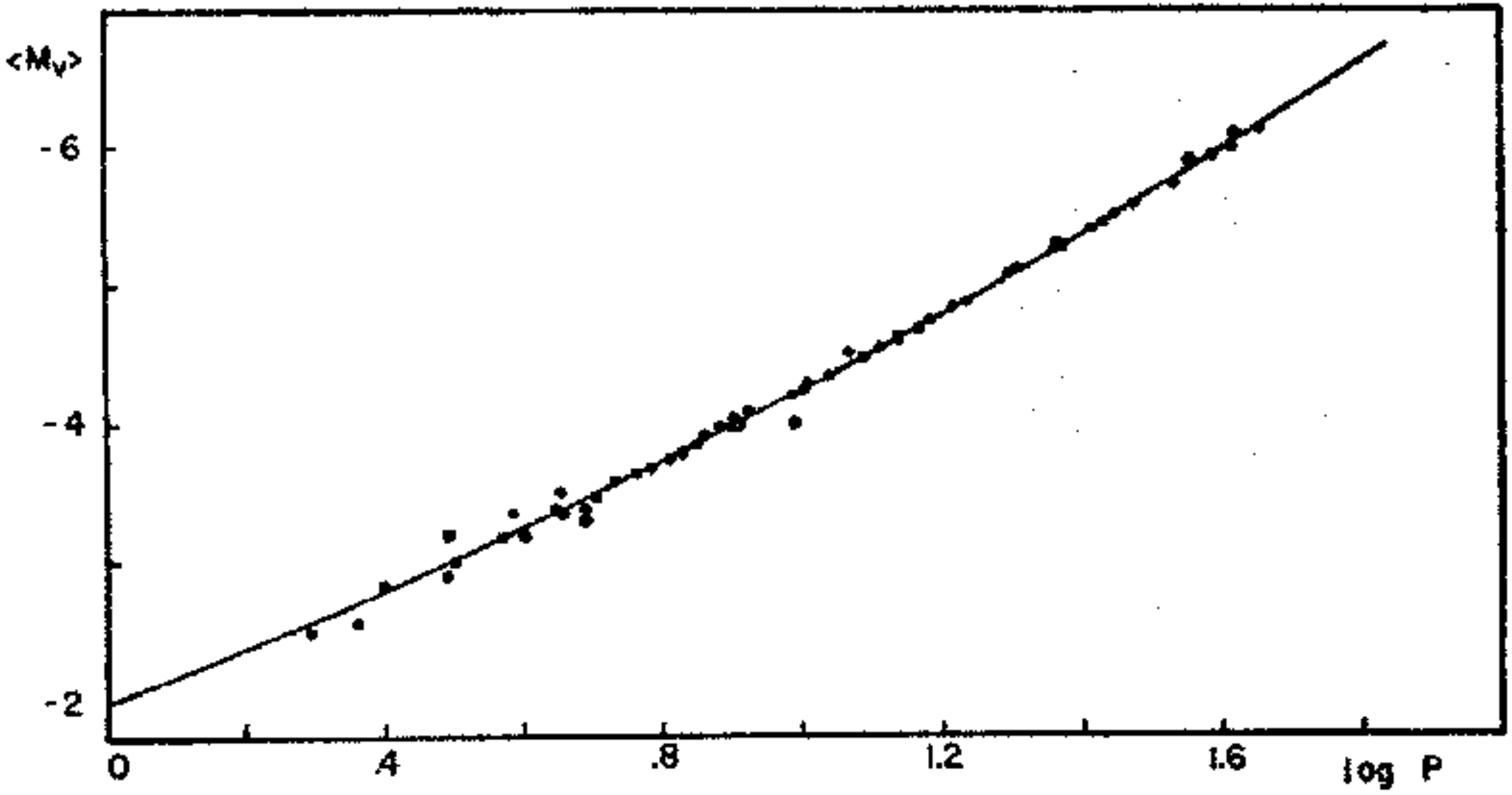
Astronomical sources: Stars



Main-sequence fitting: Cluster distance

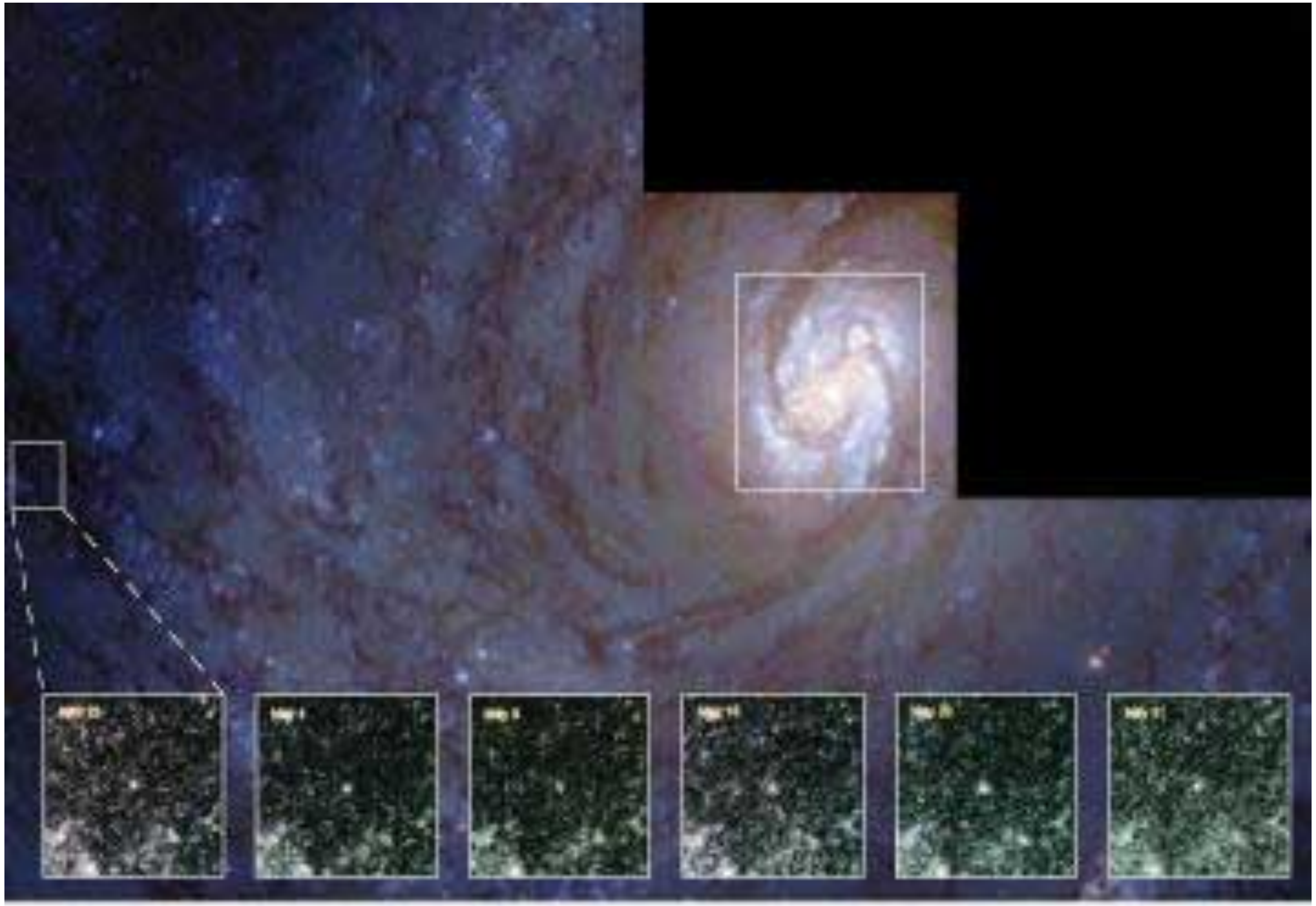


CEPHEID VARIABLES: (T,L) RELATIONSHIP



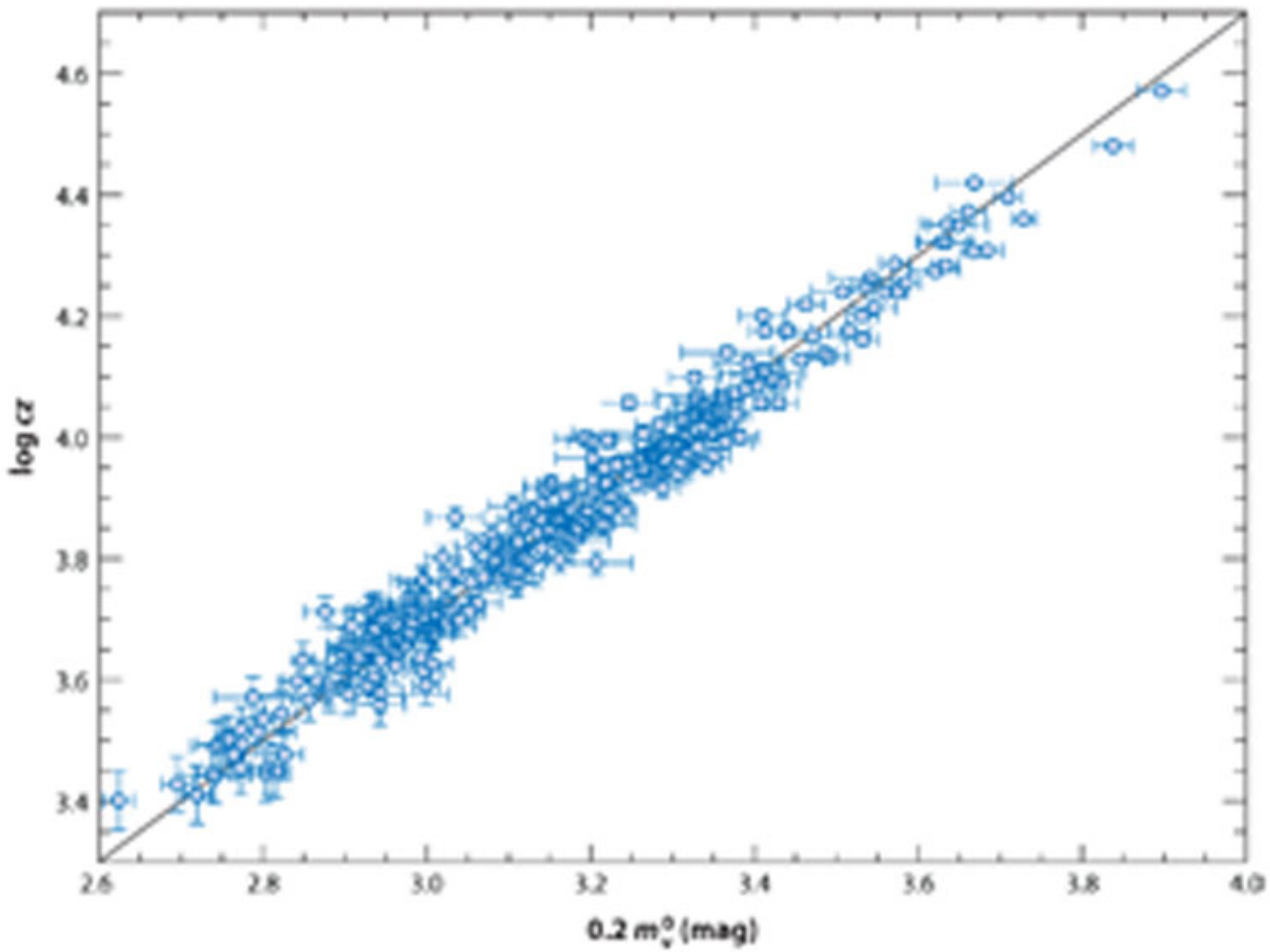
$$M_v = -[2.76(log10(P) - 1.0)] - 4.16$$

Cepehid variables: M100

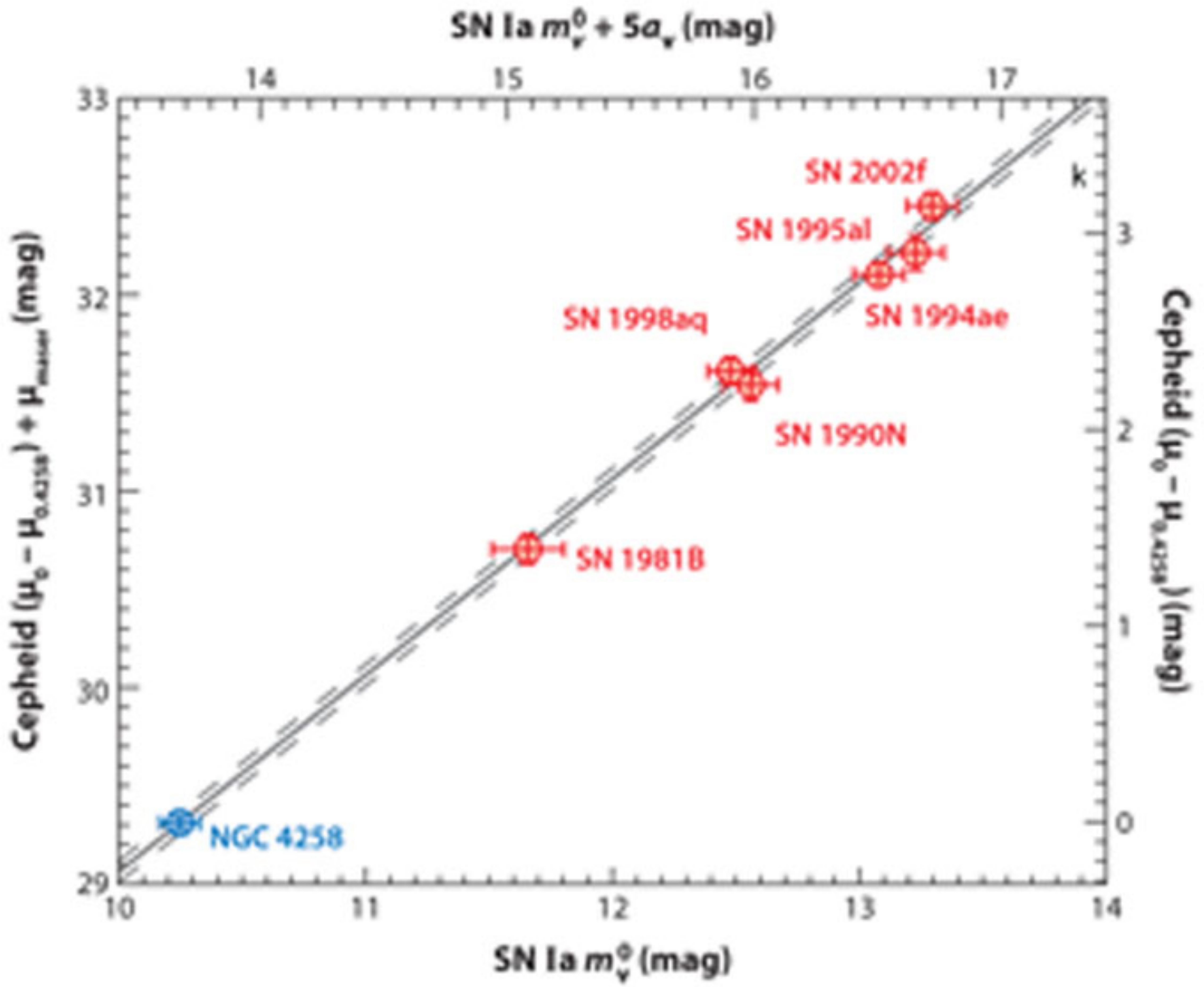


$$M_v = -[2.76(log10(P) - 1.0)] - 4.16$$

SNe calibration

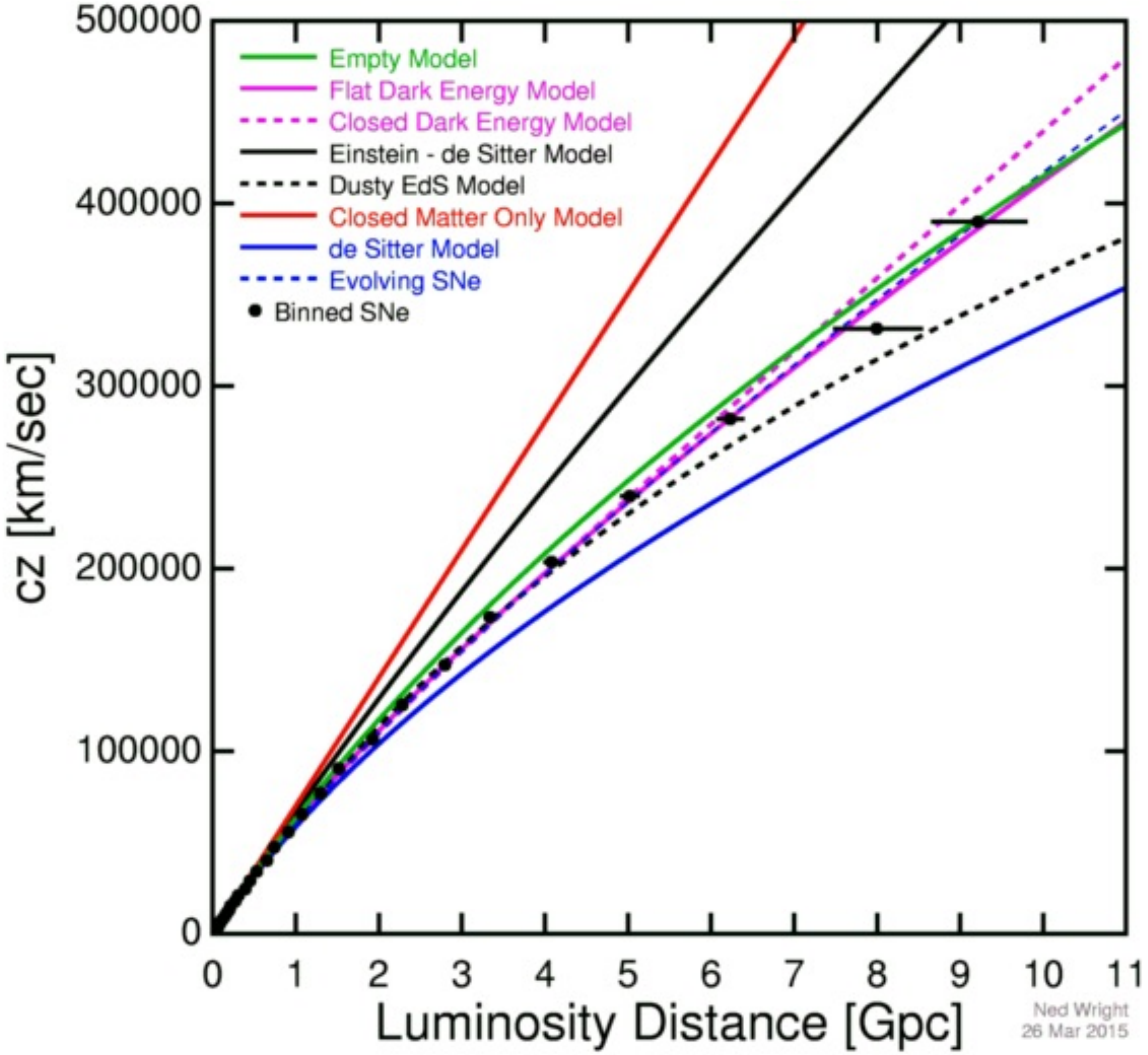


CEPHEID-SNE CALIBRATION



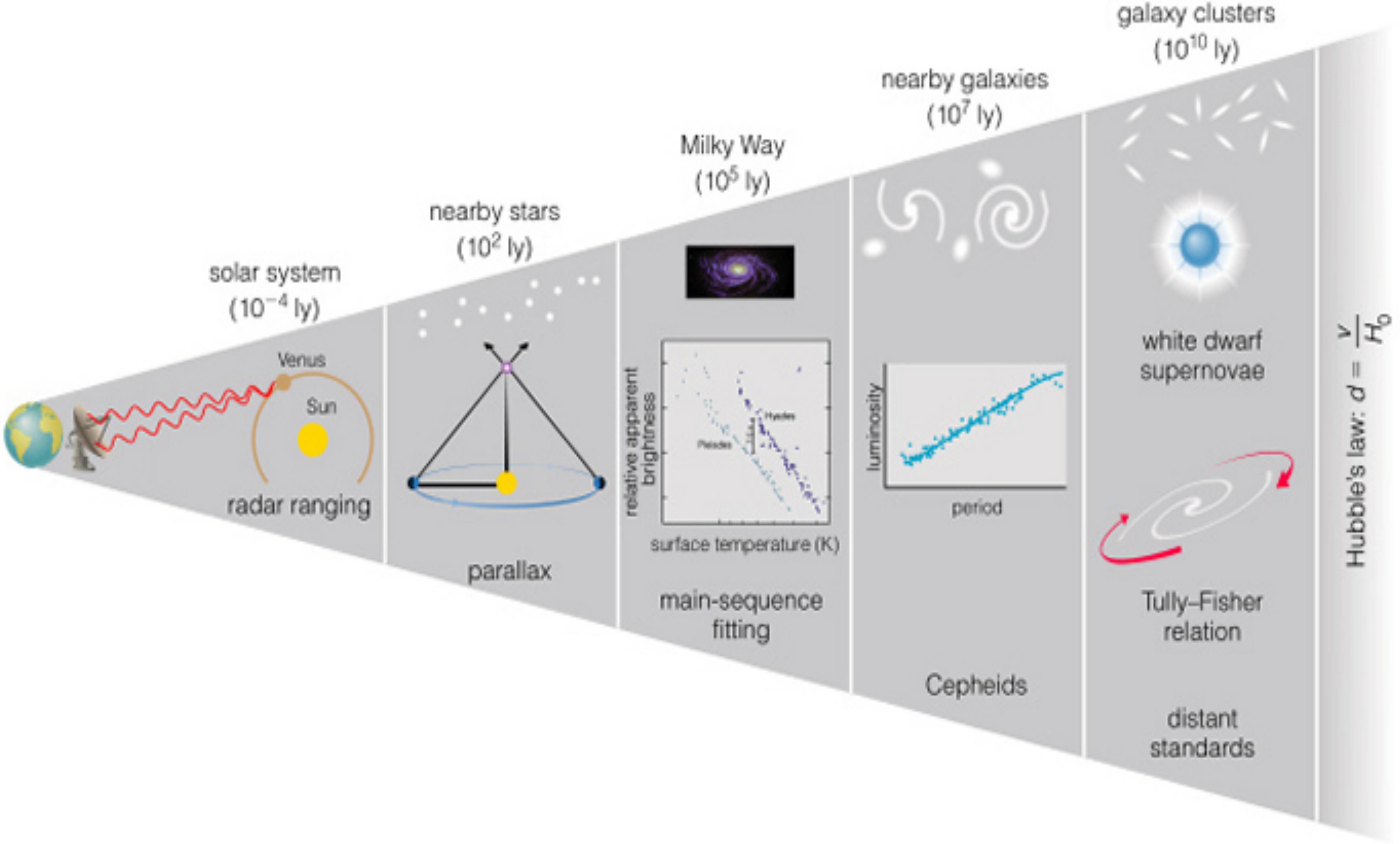
$$M_v = -[2.76(log10(P) - 1.0)] - 4.16$$

HUBBLE DIAGRAM: SNE 1A

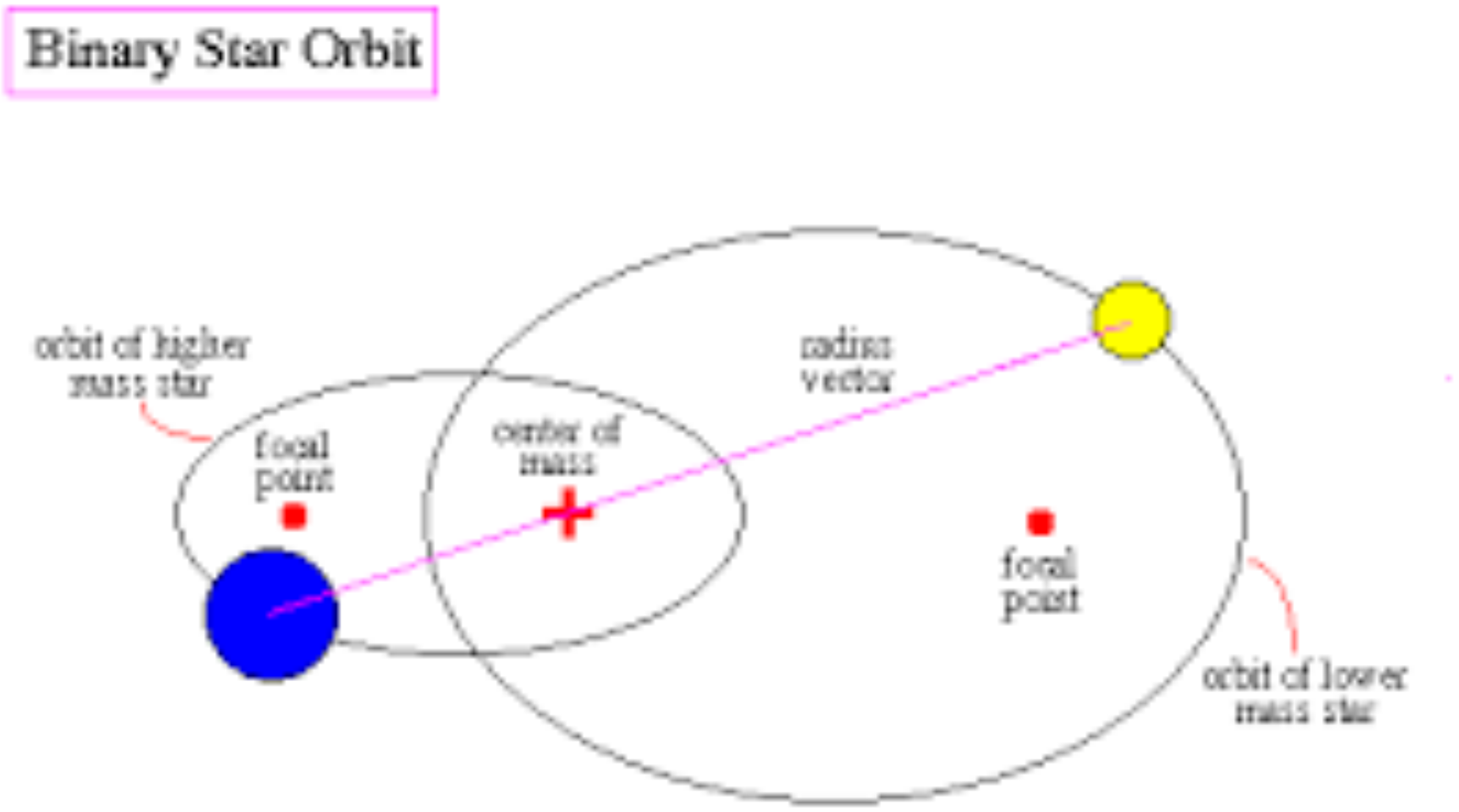


$a(z=0)/a(z) = (1. + z)$ and a(z) depends on the constituent of the universe.

ASTRONOMICAL DISTANCE SCALES

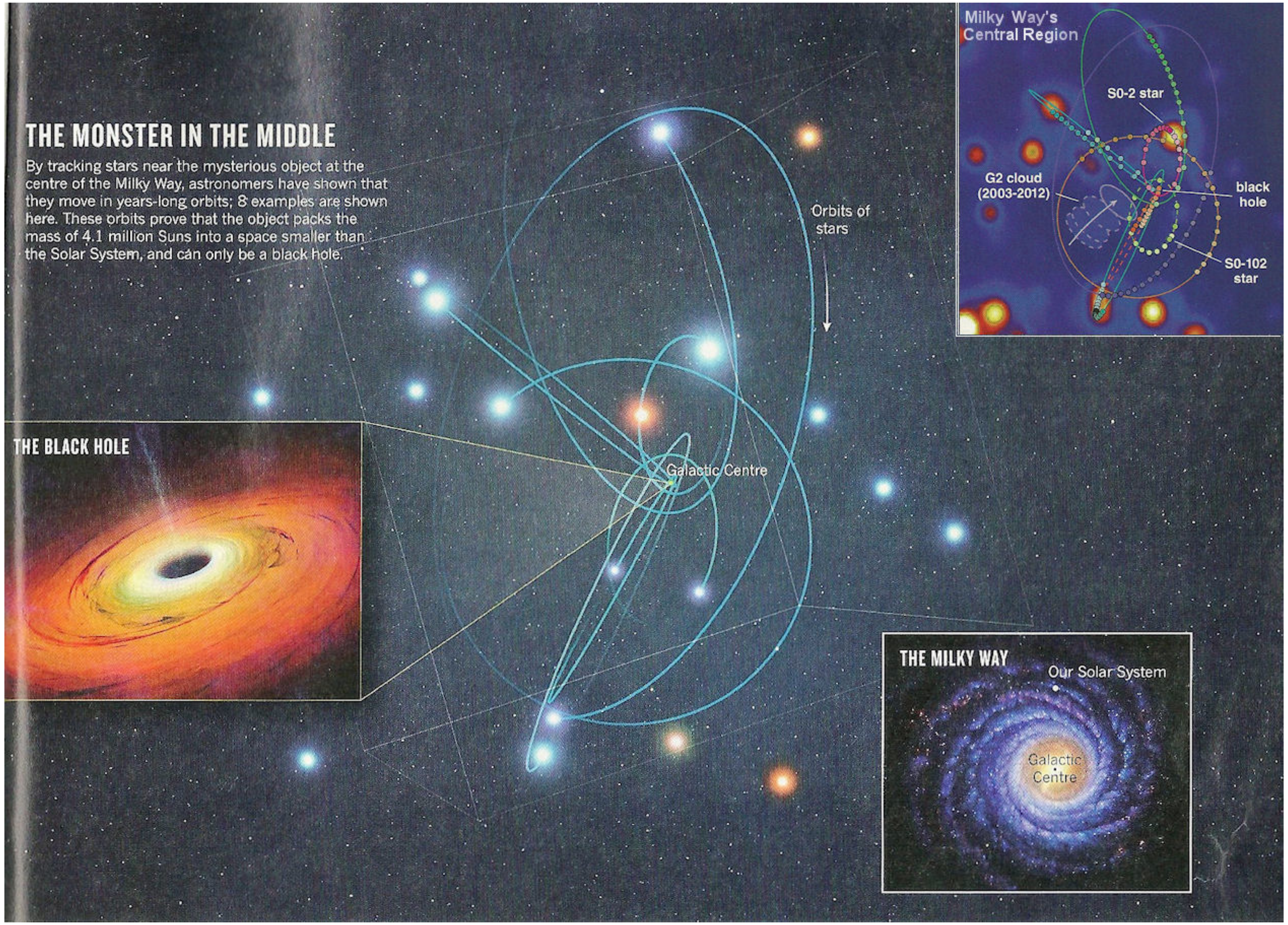


MASS ESTIMATION: BINARY STARS



$R = r_1 + r_2$, $m = m_1 + m_2$ and $P^2/a^3 \propto 1/M$

MASS ESTIMATION: STELLAR ORBITS



Some length scales to remember

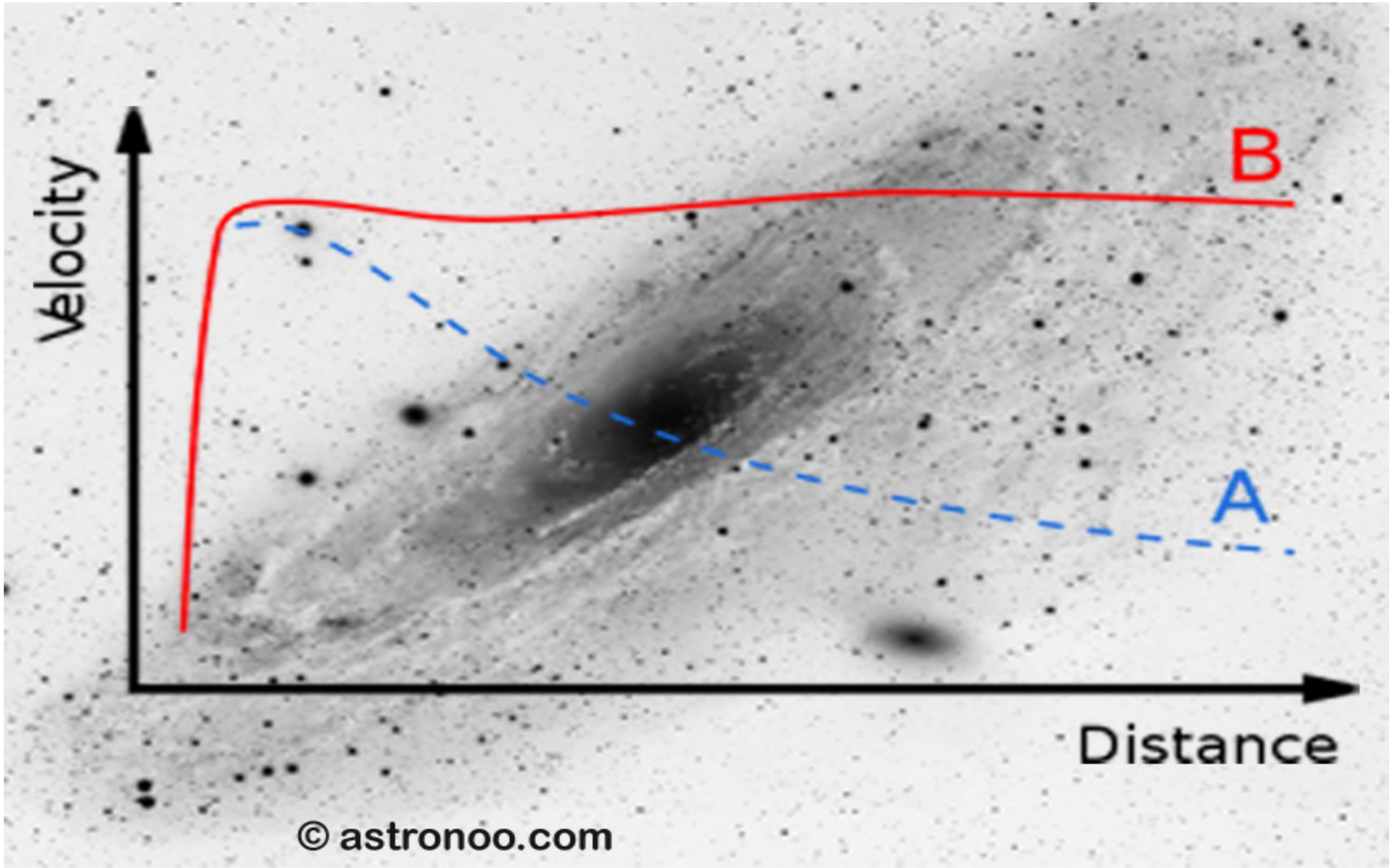
Scale of the Universe: Sizes and Distances (2026)

Astronomical Object	Typical Diameter (Size)	Distance to Nearest Neighbor
Star (Sun-like)	4.5×10^{-8} pc	**1.3 pc** (to nearest star)
Planetary System	0.0003–1 pc	**1.3 pc** (to nearest system)
Galaxy (Typical Spiral)	10–100 kpc	**0.7 – 2 Mpc** (to nearest large galaxy)
Galaxy Cluster	1–5 Mpc	**20 – 50 Mpc** (to nearest cluster)
Supercluster	50–150 Mpc	**100+ Mpc** (to nearest supercluster)

Quick Reference Guide for PPT Slides:

- **Units:**
  - **1 pc (Parsec)** ≈ 3.26 light-years ≈ 206,265 AU.
  - **1 kpc (Kiloparsec)** = 1,000 pc.
  - **1 Mpc (Megaparsec)** = 1,000,000 pc.

Mass estimation: Rotation curves



Mass estimation: Rotation curves

The fundamental relationship derived from Newton's laws is:

$$v(r)^2 = \frac{GM(r)}{r}$$

To express this in terms of the density function $\rho(r)$, we assume the mass distribution is spherically symmetric (a common initial approximation for the dark matter halo), in which case the total mass $M(r)$ enclosed within a radius r is:

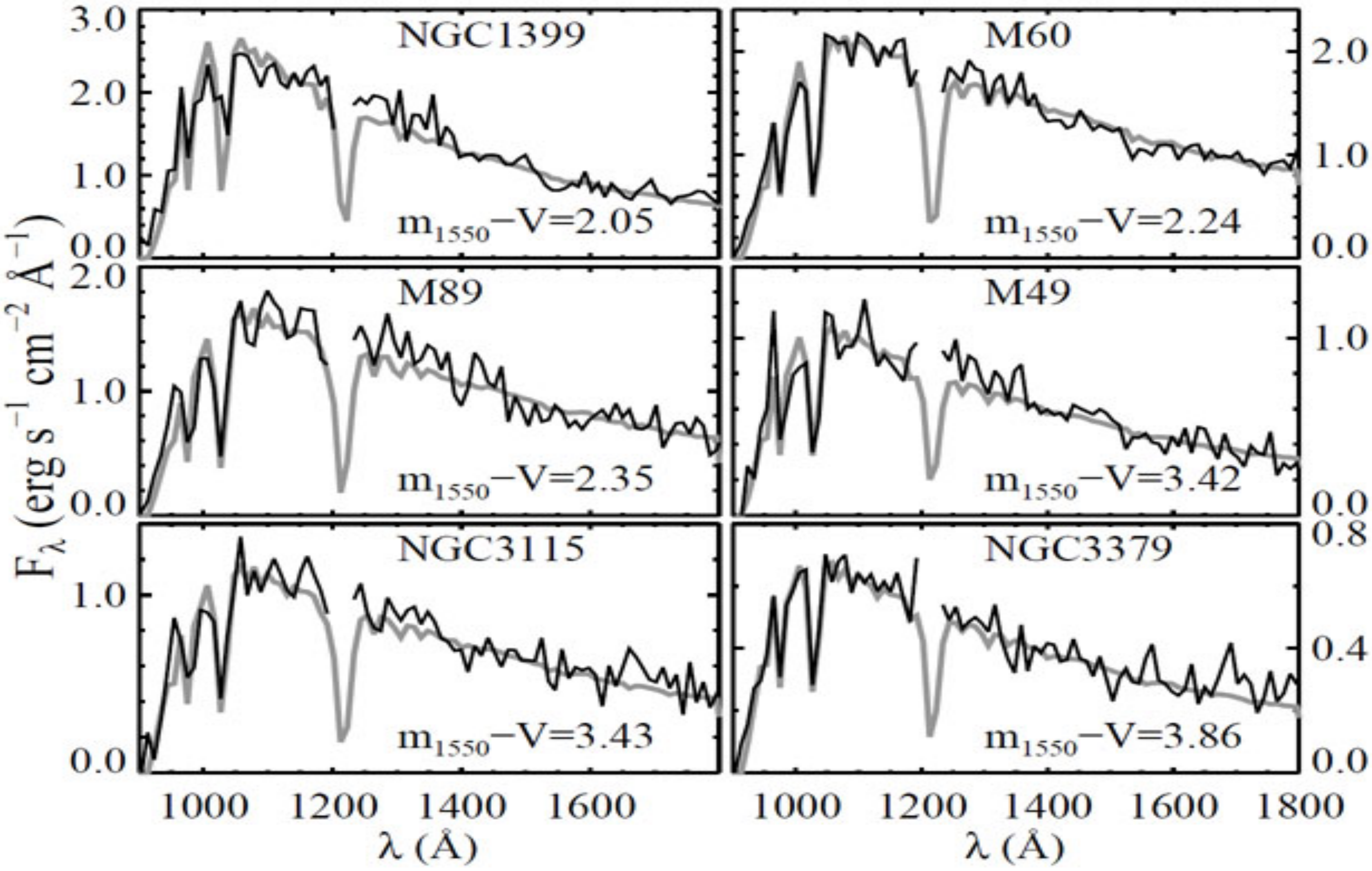
$$M(r) = \int_0^r 4\pi r'^2 \rho(r')\, dr'$$

Substituting this integral into the velocity equation gives the general form of the rotation curve equation in terms of density:

$$v(r) = \sqrt{\frac{G}{r} \int_0^r 4\pi r'^2 \rho(r')\, dr'}$$

Physicists model galactic dynamics by proposing different density profiles $\rho(r)$ for the various components (bulge, disk, and dark matter halo) and then integrating them to predict the rotation curve.

Mass estimation: Velocity dispersion



For a gravitationally bound system in equilibrium, the scalar Virial Theorem states:

$$2\langle T\rangle + \langle U\rangle = 0$$

Where $\langle T\rangle$ is the average kinetic energy and $\langle U\rangle$ is the average potential energy.

- **Kinetic Energy (T):** For a galaxy of mass M, the total kinetic energy is $T = \frac{1}{2} M\langle v^2\rangle$.

  In observational astronomy, we measure the 1D line-of-sight velocity dispersion (σ_{1D}). Assuming the system is isotropic (stars move equally in all directions), the 3D velocity is $\langle v^2\rangle = 3\sigma^2$.

  $$T = \frac{3}{2} M\sigma^2$$

- **Potential Energy (U):** The gravitational potential energy of a mass distribution can be expressed as:

  $$U = -\frac{GM^2}{R_g}$$

  where R_g is the gravitational radius, which depends on the internal mass distribution (geometry) of the galaxy.

- **Combining the Terms:** Substituting into the virial equation:

  $$2\left(\frac{3}{2} M\sigma^2\right) - \frac{GM^2}{R_g} = 0 \Longrightarrow 3M\sigma^2 = \frac{GM^2}{R_g}$$

  Solving for M gives the general virial mass estimator:

  $$M = \frac{k \cdot \sigma^2 \cdot R}{G}$$

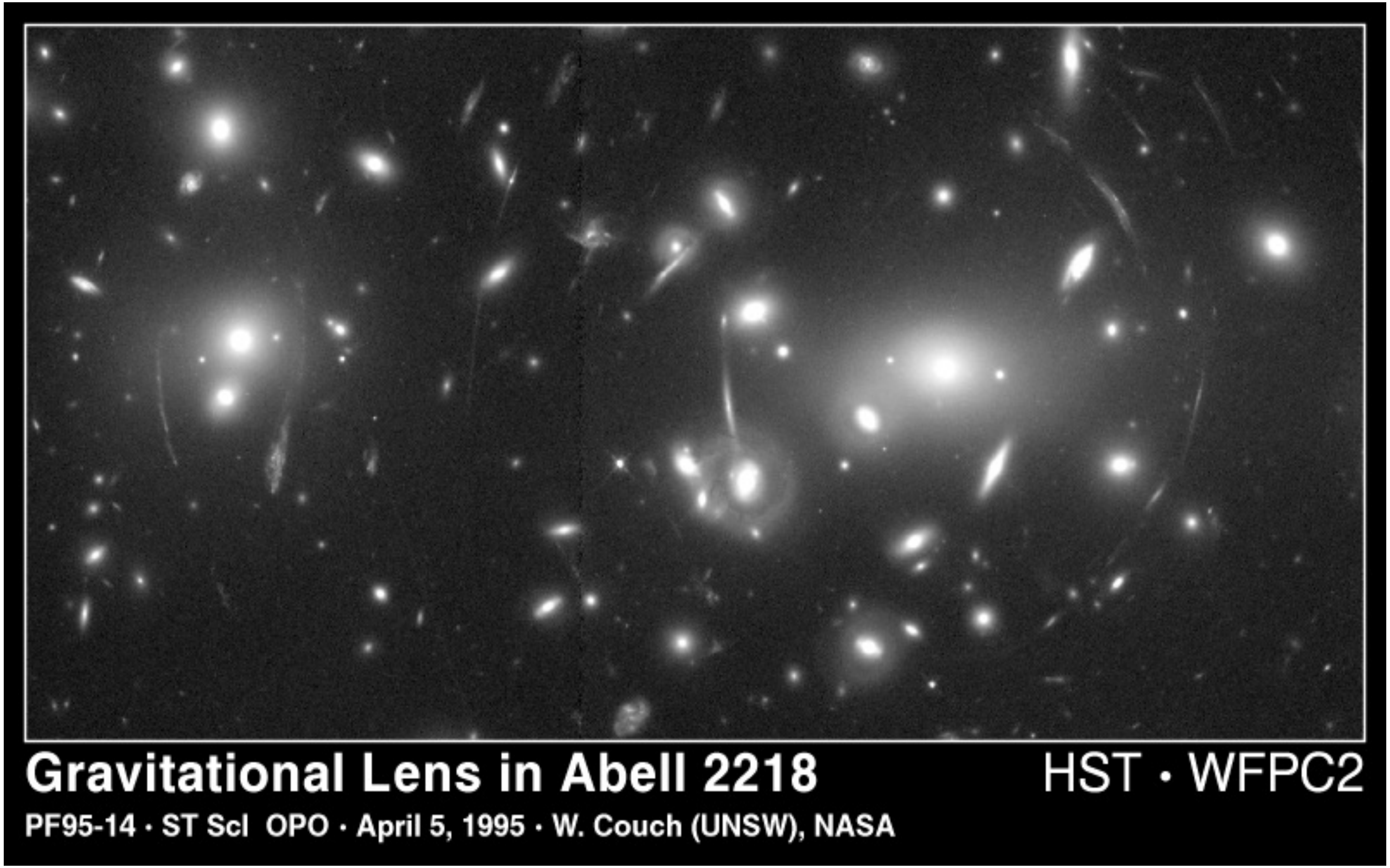
  where k is a structure constant that accounts for geometry, projection effects, and the specific radius R (e.g., half-light radius R_e) being used.

2. Typical Values of k for Different Geometries

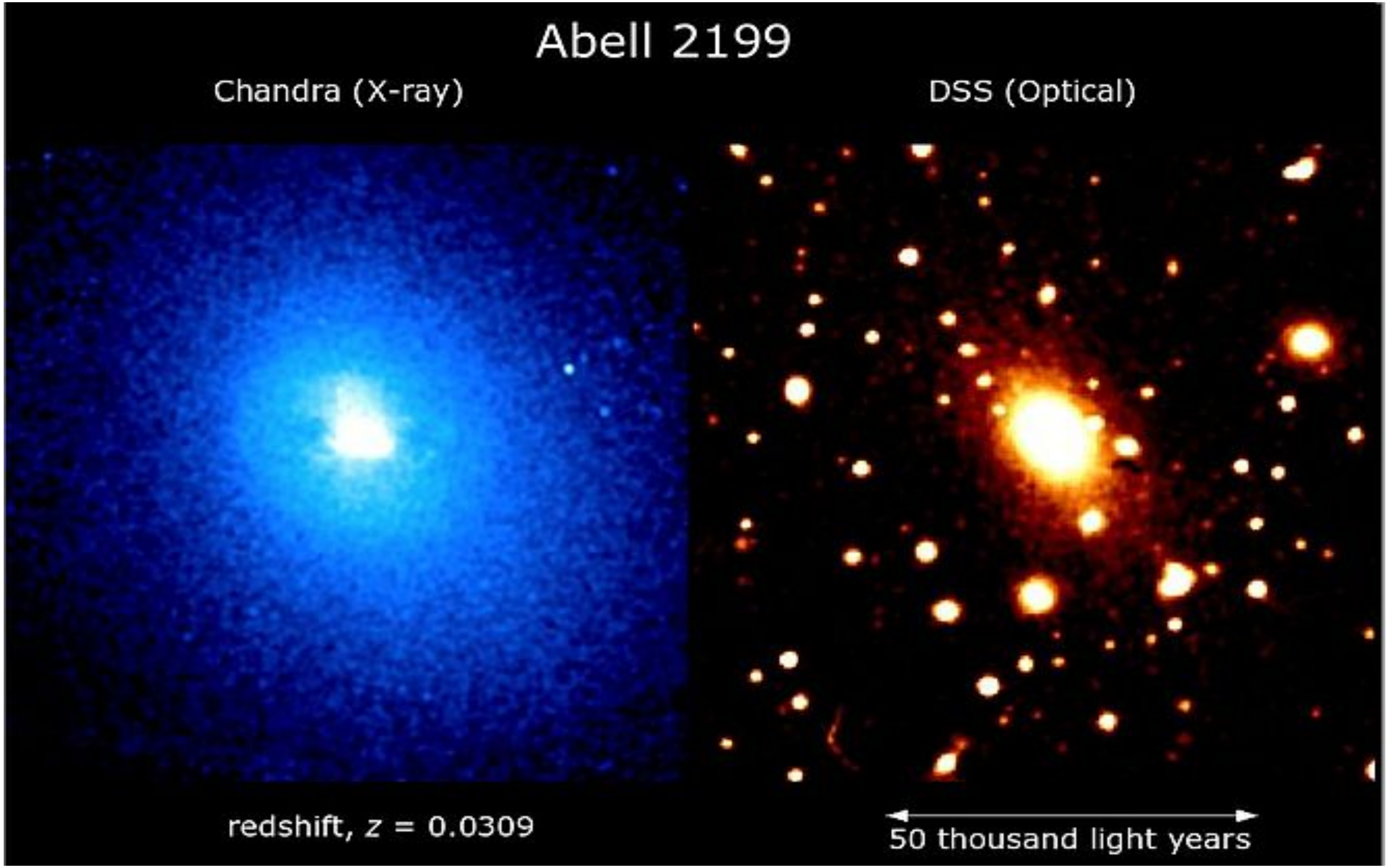
The value of k varies based on how mass is distributed and which radius is measured.

Geometry/ System	Radius Used (R)	Typical k Value	Notes
Uniform Sphere	Total Radius (R)	5.0	Derived from $U = -\frac{3}{5}\frac{GM^2}{R}$.
Elliptical Galaxy	Half-light Radius (R_e)	5.0 ± 0.1	Empirically calibrated for early-type galaxies.
Sérsic Profile	Half-light Radius (R_e)	6.0–8.0	Depends on the Sérsic index n (concentration of light).
Isothermal Sphere	Scaling Radius	3.0	Often used for galaxy clusters where $M \approx 3\sigma^2 R/G$.
Plummer Profile	Half-light Radius (R_e)	4.0	Common for globular clusters and dwarf spheroidals.

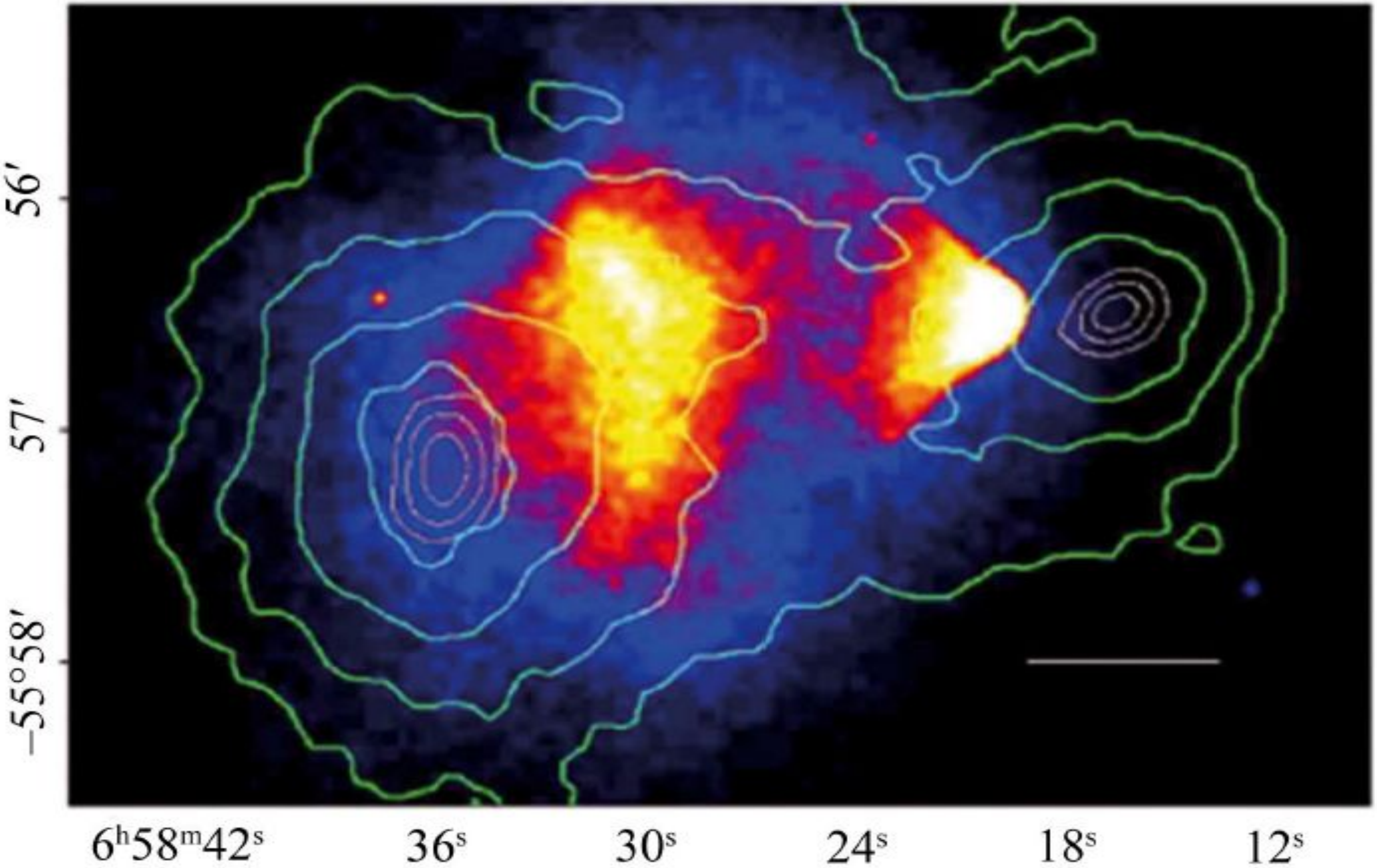
Mass estimation: gravitational lensing



Mass estimation: X-ray emission



MASS ESTIMATION: BULLOT CLUSTER!

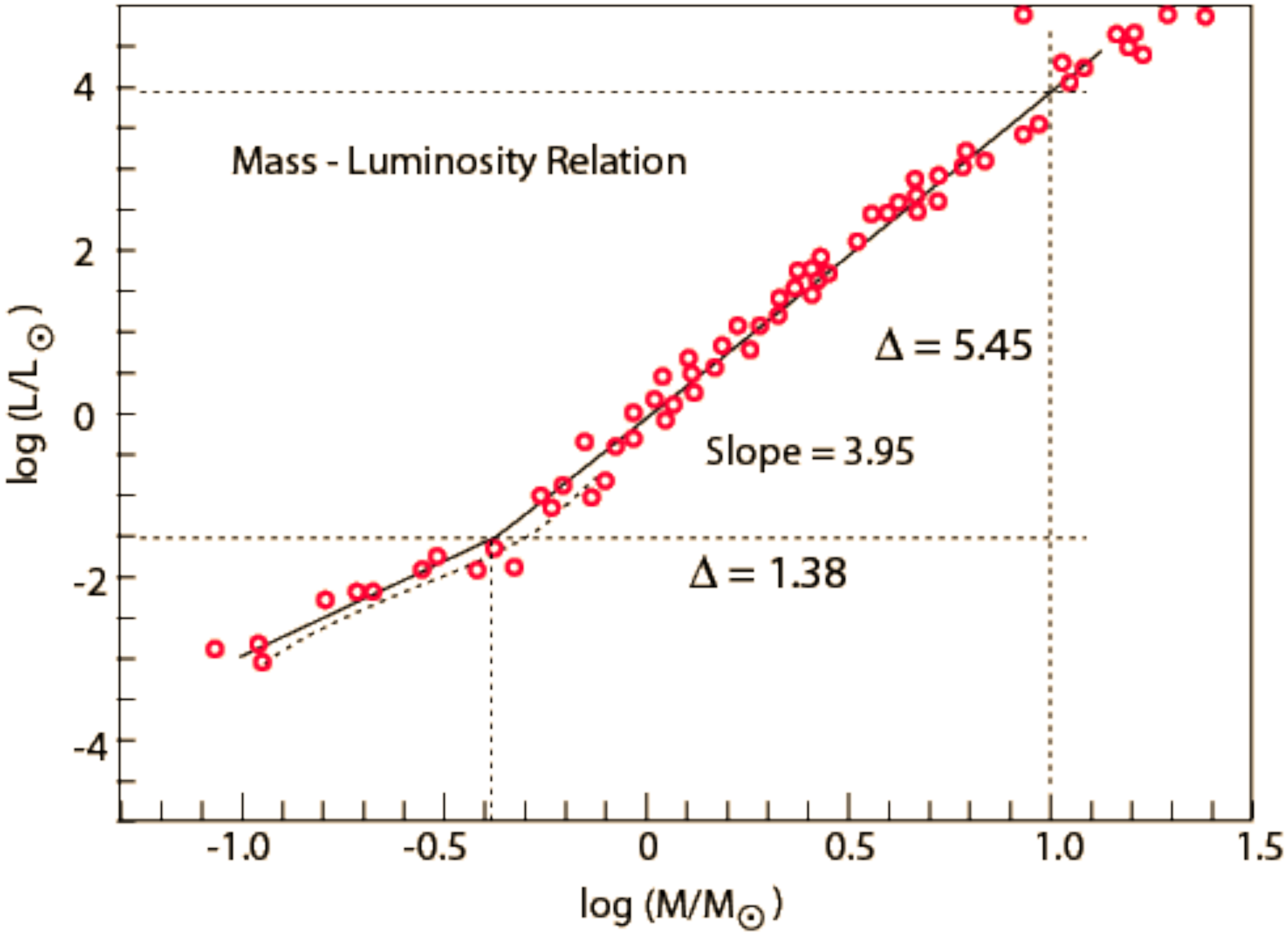


Mass estimation: Objects

Objects	Mass	Relaxation time
Stars	$10^{-2} - 10^{2}$ M$_\odot$	
Star clusters	$10^{3} - 10^{6}$ M$_\odot$	2×10^{8} yrs
Galaxies	$10^{6} - 10^{12}$ M$_\odot$	4×10^{16} yrs
Galaxy Cluters	$10^{12} - 10^{15}$ M$_\odot$	few 10^{12} yrs

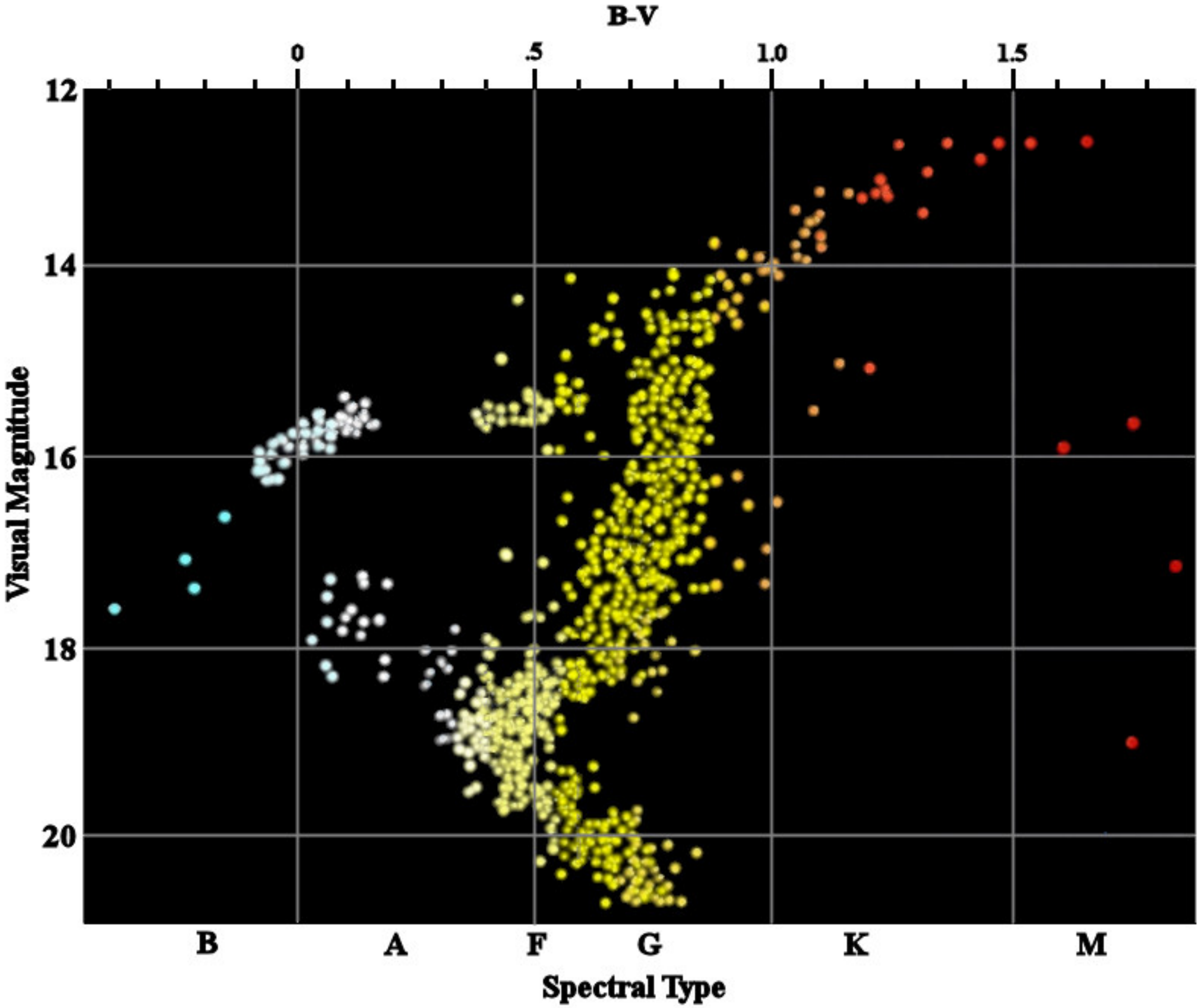
Lower mass end is decided by Jean's criterion/ Virialization
Upper mass end is decided by fragmentation/stability

MAIN-SEQUENCE LIFE TIME AND STELLAR AGE:



$L \sim fMc^2/t_{MS}$ therefore, $(\frac{t_{MS}}{t_\odot}) \propto (M/M_\odot)^{-3}$

GLOBULAR CLUSTERS: AGE OF THE UNIVERSE



Free-fall time

1. Calculation and Formula

For a spherical distribution of matter with an initial average density ρ, the free-fall time is given by the formula:

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32G\rho}}$$

- G**:** Gravitational constant.
- ρ**:** Mean density of the system.

2. Typical Timescales in Galaxies

Free-fall times vary significantly depending on the scale and density of the structure within a galaxy:

- **Galaxy-Wide Scale:** For a galaxy like the Milky Way, assuming a mass of $6 \times 10^{11} M_\odot$ and a radius of 100 kpc, the free-fall time is approximately **0.7 billion years** (7×10^8 years).
- **Giant Molecular Clouds (GMCs):** These dense regions where stars form have much shorter free-fall times, typically ranging from **10 to 15 million years** (Myr).
- **Dense Star-Forming Clumps:** In the densest parts of molecular clouds, $t_{\rm ff}$ can be as short as **5,000 to 100,000 years.**

Age of the universe

1. The Hubble Time Calculation

The simplest estimate for the age of the universe using the Hubble constant is the **Hubble Time** (t_H), which is the reciprocal of the constant:

$$t_H = \frac{1}{H_0}$$

This calculation assumes the universe has expanded at a constant rate since the Big Bang. Because the expansion rate has actually varied—decelerating initially due to gravity and then accelerating due to dark energy—this value is a useful approximation rather than an exact age.

2. Differing Estimates for 2026

Recent data as of January 2026 highlights two primary values for H_0, leading to different inferred ages:

- **Early-Universe Method (CMB):** Measurements of the Cosmic Microwave Background (CMB) by the Planck satellite and similar missions suggest a lower expansion rate of approximately **67–68 km/s/Mpc**. Using this value, the calculated age of the universe is approximately **13.8 billion years**.
- **Late-Universe Method (Local Measurements):** Observations of supernova distances and Cepheid variable stars generally yield a higher expansion rate of roughly **73–74 km/s/Mpc**. If this higher rate is correct, the implied age of the universe would be younger, roughly **12.6 to 13 billion years**.