CSC 212: Data Structures and Abstractions

Basic Sorting Algorithms

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THINK BIG WE DO™

Announcements

- Programming #1
  - 24 / 57 / 78
- Programming #2
  - will be posted today (maze generation)
  - focus on classes and dynamic arrays (`std::vector`)
- If programming is still a **significant** issue …
  - consider taking this class next semester and focus on addressing the issue

Looking for interships/jobs?

https://careers.google.com/how-we-hire/interview/

https://www.amazon.jobs/en/landing_pages/software-development-topics

https://www.facebook.com/careers/life/preparing-for-your-software-engineering-interview-at-facebook/

Worst-case, Average-case, Best-case

Warming up: Analyze this code

```
unsigned int argmin(const std::vector<int> &values) {
    unsigned int length = values.size();
    assert(length > 0);
    unsigned int idx = 0;
    int current = values[0];
    for (unsigned int i = 1 ; i < length ; i ++) {
        if (values[i] < current) {
            current = values[i];
            idx = i;
        }
    }
    return idx;
}
```

```
T(n) = ?
based on number of comparisons
```


Warming up: Analyze this code

```
bool argk(const std::vector<int> &values, int k, unsigned int &idx) {
    unsigned int length = values.size();
    for (unsigned int i = 0 ; i < length ; i ++) {
        if (values[i] == k) {
            idx = i;
            return true;
        }
    }
    return false;
}
```

```
T(n) = ?
based on number of comparisons
```


Different types of analysis

Worst-case: maximum time of algorithm on **any** input

Average-case: expected time of algorithm over **all** inputs

Best-case: minimum time of algorithm on **some** (optimal) input

Different types of analysis

- While **asymptotic analysis** describes $T(n)$ as n approaches infinity …
  - **asymptotic notation**: big O, big Omega, big Theta
- **Case analysis** looks into the different input types
  - best-case, worst-case, average-case

```
These analysis types are orthogonal
to each other
```

Worst-case, Average-case, Best-case

- Ex: factorial of a number (iterative algorithm)
- Ex: sequential search (return first occurrence)
- Ex: sequential search (return last occurrence)

Basic Sorting Algorithms

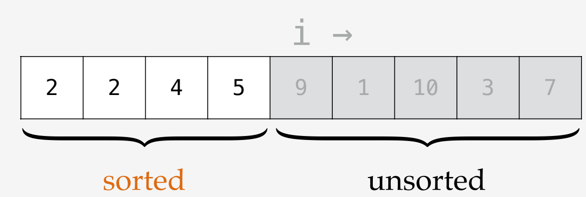
Sorting

- Given **n** elements that can be compared according to a **total order** relation
  - we want to rearrange them in non-increasing/non-decreasing order
  - for example (non-decreasing):
    - **input:** sequence of items $A = [k_0, k_1, \ldots, k_{n-1}]$
    - **output:** permutation of A $B \mid B[0] \leq B[1] \leq \ldots B[n-1]$

`Central problem in computer science`

Insertion Sort

- Array is divided into **sorted** and **unsorted** parts
  - algorithm scans array from left to right
- Invariants
  - elements in **sorted** are in ascending order
  - elements in **unsorted** have not been seen



Insertion Sort Demo

```
void insertionsort(int *A, unsigned int n) {
    int temp;
    unsigned int i, j;
    // grows the left part (sorted)
    for (i = 0 ; i < n ; i ++) {
        // inserts A[j] in sorted part
        for (j = i ; j > 0 ; j --) {
            if (A[j] < A[j-1]) {
                temp = A[j];
                A[j] = A[j-1];
                A[j-1] = temp;
            }
            else
                break;
        }
    }
}
```

Number of comparisons? **Number of exchanges?**

Analysis — Insertion Sort (comparisons)

- Running time depends on the input
- Worst-case?
  - input reverse sorted
- Best-case?
  - input already sorted
- Average-case?
  - expect every element to move O(n/2) times

Partially sorted arrays

- An **inversion** is a pair of keys that are out of order

1	3	4	5	2	6	10	15	7

“array is **partially sorted** if the number of pairs that are out-of-order is O(n)”

For partially-sorted arrays, insertion sort runs in **linear time**.

$$\Theta(n)$$

Selection Sort

- Array is divided into **sorted** and **unsorted** parts
  - algorithm scans array from left to right
- Invariants
  - elements in **sorted** are **fixed** and in ascending order
  - no element in **unsorted** is smaller than any element in **sorted**

i →

1	3	4	5	9	6	10	15	7

sorted unsorted

Selection Sort Demo

```
void selectionsort(int *A, unsigned int n) {
    int temp;
    unsigned int i, j, min;
    // grows the left part (sorted)
    for (i = 0 ; i < n ; i ++) {
        min = i;
        // find min in unsorted part
        for (j = i+1 ; j < n ; j ++) {
            if (A[j] < A[min]) {
                min = j;
            }
        }
        // swap A[i] and A[min]
        temp = A[i];
        A[i] = A[min];
        A[min] = temp;
    }
}
```

Number of comparisons? **Number of exchanges?**

Analysis — Selection Sort (comparisons)

- Worst-case?
- Best-case?
- Average-case?
- Running time is quadratic
  - insensitive to the input (quadratic in all cases)
  - linear number of exchanges (minimal data movement)

Summary

	Best-Case	Average-Case	Worst-Case
Selection Sort	$\theta(n^2)$	$\theta(n^2)$	$\theta(n^2)$
Insertion Sort	$\theta(n)$	$\theta(n^2)$	$\theta(n^2)$