CSC 212: Data Structures and Abstractions

Big O Notation

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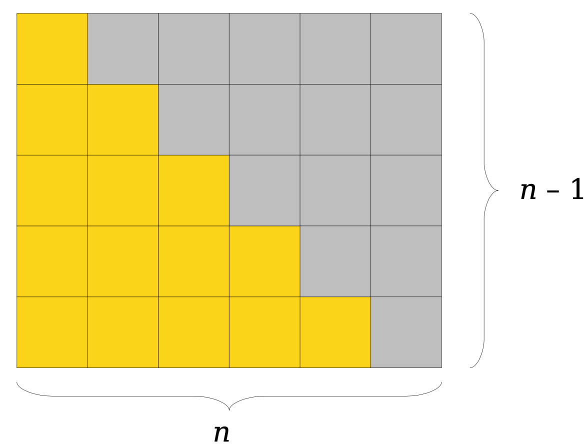
THINK BIG WE DO℠

The story so far …

- Can measure actual runtime to compare algorithms
  - however, runtime is noisy (highly sensitive to HW/SW and implementation details)
- Can count instructions to compare algorithms
  - can define T(n), which depends on the input size
  - for large inputs, our focus should be on the dominant terms of T(n)
  - we will now see formal ways for this approach

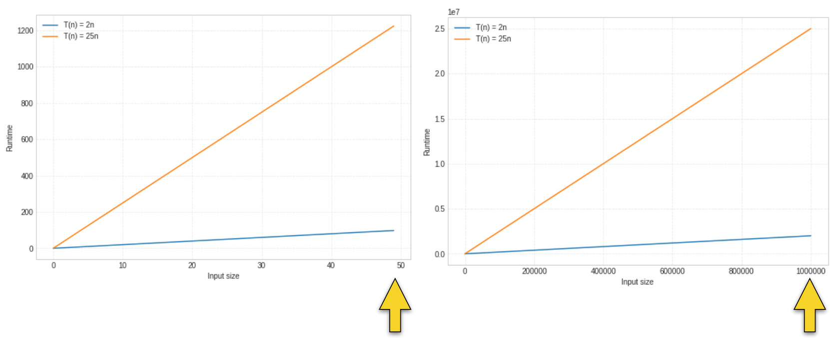
$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + (n-1) + n$$

$$\sum_{i=1}^{n-1} i = 1 + 2 + 3 + \ldots + (n-2) + (n-1)$$

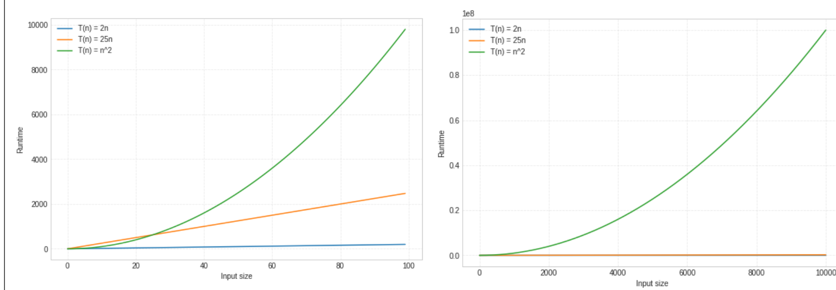


Should we consider these the same?

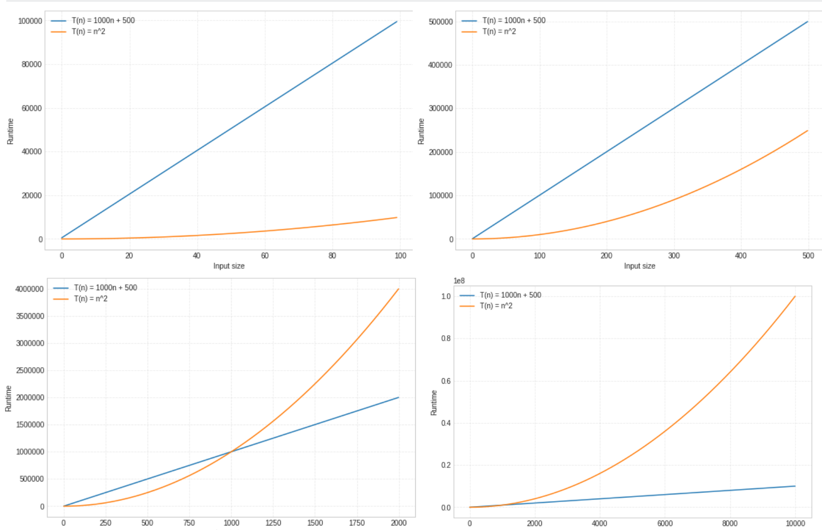
Comparing two algorithms with $T(n) = 2n$ and $T(n) = 25n$ respectively



What about now?

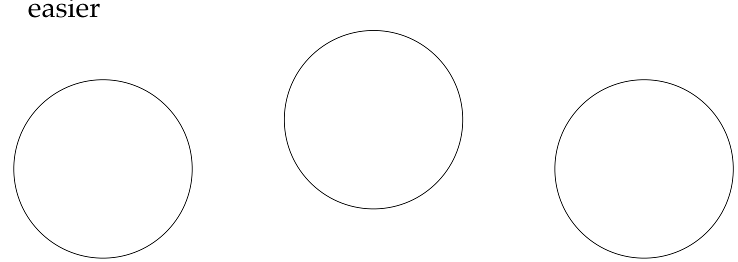


Now consider this example …



Bottom line …

- We are trying to compare **T(n)** functions, but we also care about large values of **n**
- Can we properly define '<=' for functions?
  - we can group functions into **'sets'** and make our lives easier

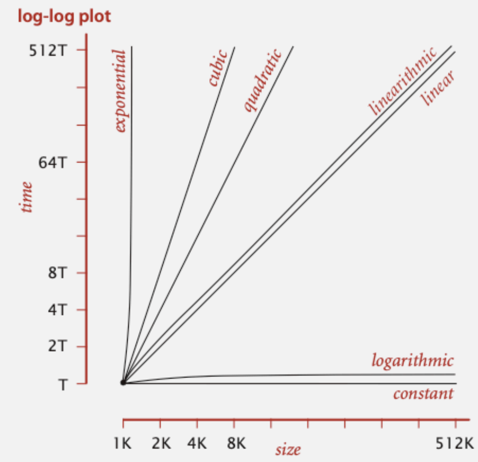


Asymptotic Analysis

- Refers to the study of an algorithm as the input size “gets big” or reaches a limit (in the calculus sense)
- **Growth rate**
  - rate at which the cost of an algorithm grows as the size of its input grows

$$c_1 n \qquad c_2 n^2$$

Common sets of functions



Typical orders of growth

Faster growth rate … slower algorithm

Algorithm A is better than algorithm B if for large values of n, $T_A(n)$ grows slower than $T_B(n)$

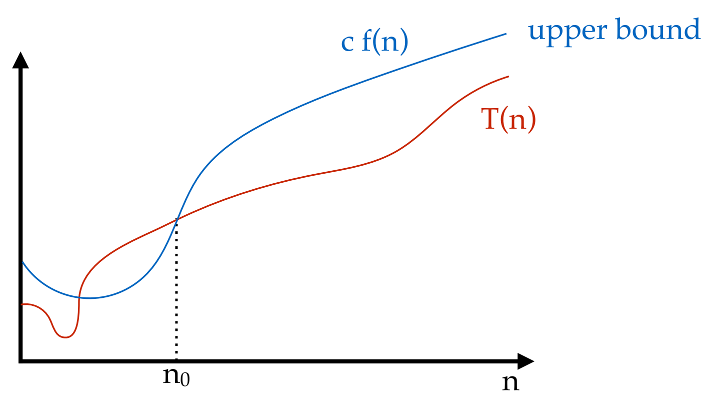
credit: COS 226, Princeton

A few examples …

order of growth	name	typical code framework	description	example
1	constant	`a = b + c;`	statement	add two numbers
$\log n$	logarithmic	`while (n > 1) { n = n/2; ... }`	divide in half	binary search
n	linear	`for (int i = 0; i < n; i++) { ... }`	single loop	find the maximum
$n \log n$	linearithmic	*see mergesort lecture*	divide and conquer	mergesort
n^2	quadratic	`for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) { ... }`	double loop	check all pairs
n^3	cubic	`for (int i = 0; i < n; i++) for (int j = 0; j < n; j++) for (int k = 0; k < n; k++) { ... }`	triple loop	check all triples
2^n	exponential	*see combinatorial search lecture*	exhaustive search	check all subsets

credit: COS 226, Princeton

Big O



$T(n)$ is $O(f(n)) \iff \exists$ positive $c, n_0 \mid 0 \le T(n) \le cf(n), \forall n \ge n_0$

set of functions

Examples

$$7n - 2 = O(n)$$

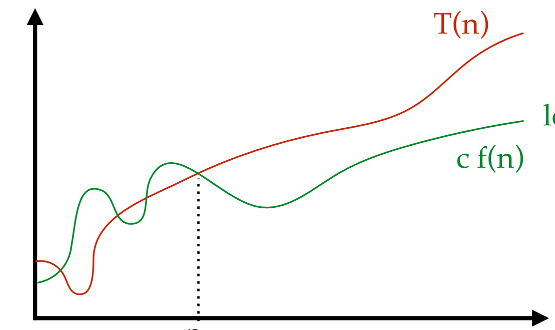
$$20n^3 + 10n \log n + 5 = O(n^3)$$

$$3 \log n + \log \log n = O(\log n)$$

$$2^{100} = O(1)$$

$T(n)$ is $O(f(n)) \iff \exists$ positive $c, n_0 \mid 0 \le T(n) \le cf(n), \forall n \ge n_0$

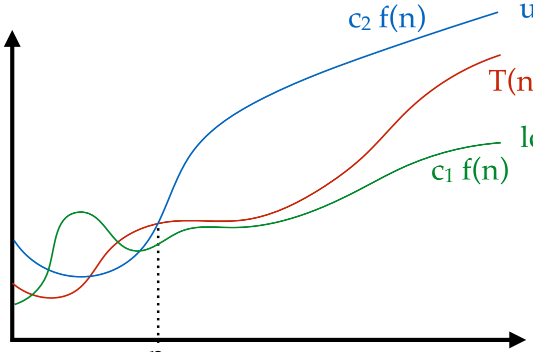
Big Omega



$T(n)$ is $\Omega(f(n)) \iff \exists$ positive $c, n_0 \mid 0 \le cf(n) \le T(n), \forall n \ge n_0$

set of functions

Big Theta



$T(n)$ is $\Theta(f(n)) \iff T(n)$ is $O(f(n))$ and $T(n)$ is $\Omega(f(n))$

Prove that …

$$3 \log n + \log \log n = \Omega(\log n)$$

$$3 \log n + \log \log n = \Theta(\log n)$$

$T(n)$ is $O(f(n)) \iff \exists$ positive $c, n_0 \mid 0 \le T(n) \le cf(n), \forall n \ge n_0$

$T(n)$ is $\Omega(f(n)) \iff \exists$ positive $c, n_0 \mid 0 \le cf(n) \le T(n), \forall n \ge n_0$

In practice you can …

"ignore constants and drop lower order terms"

True or False?

$\{n^2, n^4, 2^n, \log n, \ldots\}$

	Big O	Big Omega	Big Theta
$10^2 + 3000n + 10$			
$21 \log n$			
$500 \log n + n^4$			
$\sqrt{n} + \log n^{50}$			
$4^n + n^{5000}$			
$3000n^3 + n^{3.5}$			
$2^5 + n!$			

Asymptotic Performance

- For large values of **n**, a **Θ(n²)** algorithm always beats a **Θ(n³)** algorithm

However, we shouldn't completely ignore asymptotically slower algorithms