CSC 212: Data Structures and Abstractions Basic Sorting Algorithms

Marco Alvarez

Department of Computer Science and Statistics University of Rhode Island

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Announcements

- Programming #1
 - 4 24 / 57 / 78
- · Programming #2
 - √ will be posted today (maze generation)
 - focus on classes and dynamic arrays (std::vector)
- If C++ is still a significant issue ...
 - ✓ consider taking this class next semester

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Looking for interships/jobs?

https://careers.google.com/how-we-hire/interview/

https://www.amazon.jobs/en/landing_pages/softwaredevelopment-topics

https://www.facebook.com/careers/life/preparing-for-yoursoftware-engineering-interview-at-facebook/ Worst-case, Average-case, Best-case

Warming up: Analyze this code

```
unsigned int argmin(const std::vector<int> &values) {
   unsigned int length = values.size();
   assert(length > 0);
   unsigned int idx = 0;
   int current = values[0];
   for (unsigned int i = 1 ; i < length ; i ++) {
      if (values[i] < current) {
        current = values[i];
      idx = i;
      }
   }
  return idx;
}</pre>
```

T(n) = ? based on number of comparisons

Warming up: Analyze this code

```
bool argk(const std::vector<int> &values, int k, unsigned int &idx) {
   unsigned int length = values.size();
   for (unsigned int i = 0; i < length; i ++) {
      if (values[i] == k) {
        idx = i;
        return true;
      }
   }
   return false;
}</pre>
```

T(n) = ? based on number of comparisons

Different types of analysis

Worst-case: maximum time of algorithm on any input

Average-case: expected time of algorithm over all inputs

Best-case: minimum time of algorithm on some (optimal) input

Different types of analysis

- While asymptotic analysis describes T(n) as n approaches infinity ...
 - ✓ asymptotic notation: big O, big Omega, big Theta
- Case analysis looks into the different input types
 - ✓ best-case, worst-case, average-case

These analysis types are orthogonal to each other

Worst-case, Average-case, Best-case

- Ex: factorial of a number (iterative algorithm)
- Ex: sequential search (return first occurrence)
- Ex: sequential search (return last occurrence)

Basic Sorting Algorithms

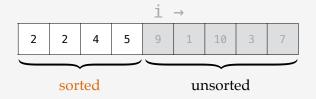
Sorting

- Given **n** elements that can be compared according to a **total order** relation
 - we want to rearrange them in non-increasing/non-decreasing order
 - √ for example (non-decreasing):
 - **input:** sequence of items $A = [k_0, k_1, ..., k_{n-1}]$
 - **output**: permutation of A $B \mid B[0] \le B[1] \le ...B[n-1]$

Central problem in computer science

Insertion Sort

- · Array is divided into sorted and unsorted parts
- ✓ algorithm scans array from left to right
- Invariants
 - ✓ elements in **sorted** are in ascending order
 - ✓ elements in **unsorted** have not been seen



Insertion Sort Demo

```
void insertionsort(int *A, unsigned int n) {
   int temp;
   unsigned int i, j;
   // grows the left part (sorted)
   for (i = 0; i < n; i ++) {
        // inserts A[j] in sorted part
        for (j = i; j > 0; j --) {
        if (A[j] < A[j-1]) {
            temp = A[j];
            A[j] = A[j-1];
            A[j-1] = temp;
        }
        else
            break;
    }
}
Number of comparisons? Number of exchanges?</pre>
```

Analysis — Insertion Sort (comparisons)

- Running time depends on the input
- Worst-case?
 - √ input reverse sorted
- Best-case?
 - √ input already sorted
- · Average-case?
 - \checkmark expect every element to move O(n/2) times

Partially sorted arrays

• An **inversion** is a pair of keys that are out of order

| 1 | 3 | 4 | 5 | 2 | 6 | 10 | 15 | 7 |
|---|---|---|---|---|---|----|----|---|
| | | | | | | | | |

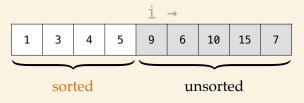
"array is **partially sorted** if the number of pairs that are out-of-order is O(n)"

For partially-sorted arrays, insertion sort runs in linear time.



Selection Sort

- · Array is divided into **sorted** and **unsorted** parts
 - ✓ algorithm scans array from left to right
- · Invariants
 - delements in sorted are fixed and in ascending order
 - no element in unsorted is smaller than any element in sorted



Selection Sort Demo

```
void selectionsort(int *A, unsigned int n) {
    int temp;
    unsigned int i, j, min;
    // grows the left part (sorted)
    for (i = 0; i < n; i ++) {
        min = i;
        // find min in unsorted part
        for (j = i+1; j < n; j ++) {
            if (A[j] < A[min]) {
                min = j;
        // swap A[i] and A[min]
        temp = A[i];
        A[i] = A[min];
        A[min] = temp;
}
Number of comparisons? Number of exchanges?
```

Analysis — Selection Sort (comparisons)

- Worst-case?
- · Best-case?
- · Average-case?
- Running time is quadratic
 - √ insensitive to the input (quadratic in all cases)
 - √ linear number of exchanges (minimal data movement)

Summary

| | Best-Case | Average-Case | Worst-Case |
|----------------|-----------|--------------|------------|
| Selection Sort | J (, | θ(n²) | θ(n²) |
| Insertion Sort | θ(n) | θ(n²) | θ(n²) |