CSC 212: Data Structures and Abstractions

Introduction to Analysis of Algorithms

Marco Alvarez

Department of Computer Science and Statistics
University of Rhode Island

Fall 2020

THINK BIG WE DO℠

Quick notes

- Course-related communication
  - avoid emails please (use **Piazza** instead)
- Lab sessions
  - use your laptops (coding is an essential part of labs)
  - read lab instructions carefully
  - important to submit solutions before 2p (attendance)

Analysis of Algorithms

Problem, algorithm and program

- **Problem** is a task to be performed
  - best thought in terms of (well-defined) <u>inputs</u> and <u>outputs</u>
  - problem definition does not impose constraints on how the problem is solved but often includes resource constraints

- **Algorithm** is a sequence of steps followed to solve a problem
  - it must be correct and composed of a finite number of concrete steps
  - there can be no ambiguity
  - it must terminate

- **Program** is a representation of an algorithm in some programming language

Analysis of algorithms

Algorithm

"Any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output."

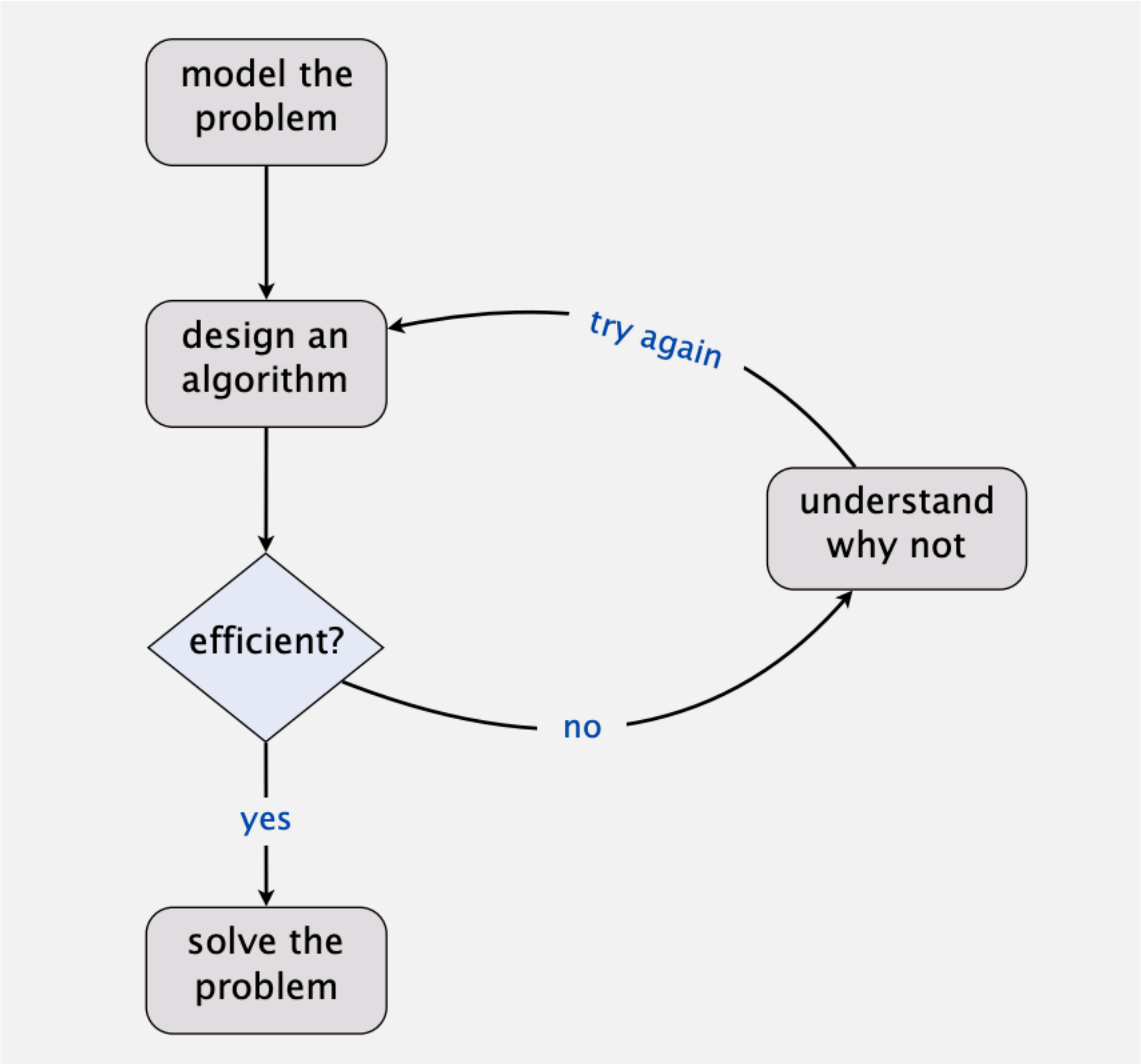
[Cormen et al., Introduction to Algorithms, 3rd. Ed.]

Amount of resources necessary to execute an algorithm?

- ✓ **Time Complexity** (running time)
- ✓ **Space Complexity** (memory)

`Resources typically depend on` **`input size`**

Developing a usable algorithm



[COS 226 lectures, Princeton University]

Why analysis of algorithms?

- Classify algorithms / problems
- Predict performance / resources
- Provide guarantees
- Understand underlying principles of problems
- and …

UBER

GitHub

Google

Microsoft®
Research

airbnb

NVIDIA®

amazon

Akamai
FASTER FORWARD

EA™

Palantir

salesforce.org

ACTIVISION®

intel®

DeepMind

IBM
Research

cloudera

Linked in®

Analyzing computational cost

Empirical Analysis

- **Run** algorithm
- Measure actual time

Mathematical Model

- **Analyze** algorithm
- Develop Model

Empirical analysis (timing)

- Implement algorithm
- Run on different input sizes
- Record actual running times
- Calculate hypothesis
- Predict and validate

Timing Algorithms

Example 1

$$e$$

… mathematical constant that is the base of the natural logarithm. It is approximately equal to 2.71828.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Leonhard Euler (1707–1783) was a Swiss mathematician, physicist, astronomer, geo grapher, logician and engineer who made important and influential discoveries in many branches of mathematics.

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

Algorithm 1

```
long double euler1(int n) {
    long double sum = 0;
    long double fact;
    for (int i = 0 ; i <= n ; i ++) {
        fact = 1;
        for (int j = 2 ; j <= i ; j++) {
            fact *= j;
        }
        sum += (1.0 / fact);
    }
    return sum;
}
```

Algorithm 2

```
long double euler2(int n) {
    long double sum = 0;
    long double fact = 1;
    for (int i = 0 ; i <= n ; i++) {
        sum += (1.0 / fact);
        fact *= (i+1);
    }
    return sum;
}
```

Which is more efficient?

```
long double euler1(int n) {
    long double sum = 0;
    long double fact;
    for (int i = 0 ; i <= n ; i ++) {
        fact = 1;
        for (int j = 2 ; j <= i ; j++) {
            fact *= j;
        }
        sum += (1.0 / fact);
    }
    return sum;
}
```

```
long double euler2(int n) {
    long double sum = 0;
    long double fact = 1;
    for (int i = 0 ; i <= n ; i++) {
        sum += (1.0 / fact);
        fact *= (i+1);
    }
    return sum;
}
```

Example 2

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

0 1 1 2 3 5 8 13 21 34 ...

Recursive

```
uint64_t fibR(uint16_t n) {
    if (n < 2) {
        return n;
    } else {
        return fibR(n-1) + fibR(n-2);
    }
}
```

Iterative

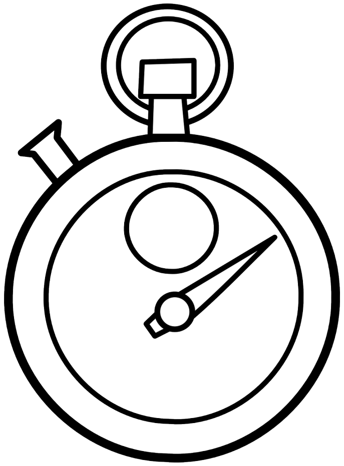
```
uint64_t fibI(uint16_t n) {
    uint64_t sum;
    uint64_t prev[] = {0, 1};

    if (n < 2) {
        return n;
    }

    for (uint16_t i = 2 ; i <= n ; i++ ) {
        sum = prev[0] + prev[1];
        prev[0] = prev[1];
        prev[1] = sum;
    }

    return sum;
}
```

Timing …



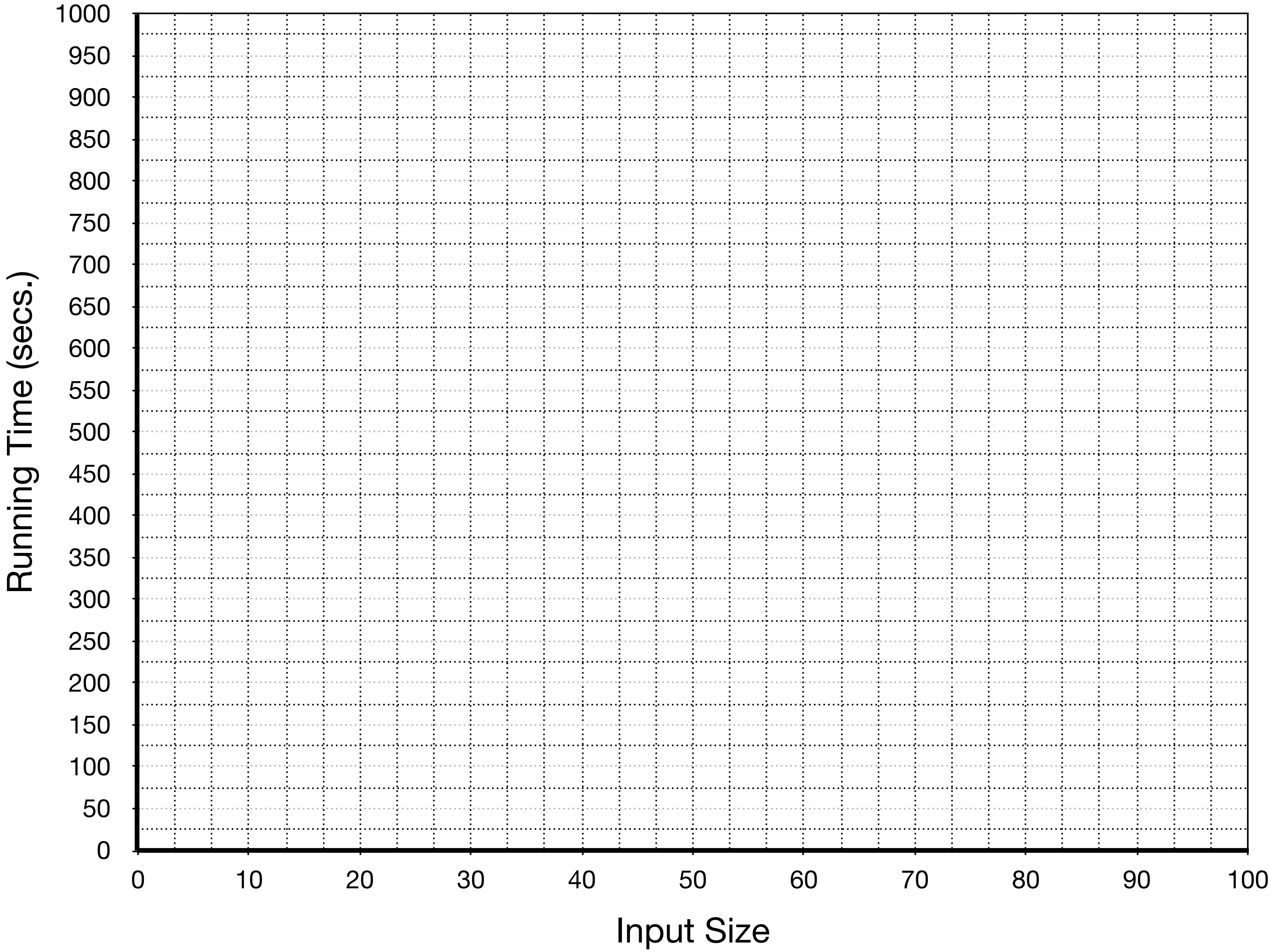
```
void time_func(uint16_t n, const char *name) {
    uint64_t val;
    Clock::time_point tic, toc;
    if (! strcmp(name, "Iter")) {
        tic = Clock::now();
        val = fib_iter(n);
        toc = Clock::now();
    }
    if (! strcmp(name, "Rec")) {
        tic = Clock::now();
        val = fib_rec(n);
        toc = Clock::now();
    }
    std::cout << name << " fib(" << n << "):\t" << std::fixed << std::setprecision(4)
<< Seconds(toc-tic).count() << " sec.\tOutput: " << val << std::endl;
}

int main(int argc, char **argv) {
    if (argc != 3) {
        std::cout << "Usage: ./fib <n> <alg>\n";
        std::cout << "\t<n>\tn-th term to be calculated\n";
        std::cout << "\t<alg>\talgorithm to be used (Rec or Iter)\n";
        return 0;
    }
    uint16_t n = (uint16_t) atoi(argv[1]);
    time_func(n, argv[2]);
}
```

```
Iter fib(1):  0.0000 sec.   Output: 1
Iter fib(2):  0.0000 sec.   Output: 1
Iter fib(3):  0.0000 sec.   Output: 2
Iter fib(4):  0.0000 sec.   Output: 3
Iter fib(5):  0.0000 sec.   Output: 5
Iter fib(6):  0.0000 sec.   Output: 8
Iter fib(7):  0.0000 sec.   Output: 13
Iter fib(8):  0.0000 sec.   Output: 21
Iter fib(9):  0.0000 sec.   Output: 34
Iter fib(10): 0.0000 sec.   Output: 55
Iter fib(11): 0.0000 sec.   Output: 89
Iter fib(12): 0.0000 sec.   Output: 144
Iter fib(13): 0.0000 sec.   Output: 233
Iter fib(14): 0.0000 sec.   Output: 377
Iter fib(15): 0.0000 sec.   Output: 610
Iter fib(16): 0.0000 sec.   Output: 987
Iter fib(17): 0.0000 sec.   Output: 1597
Iter fib(18): 0.0000 sec.   Output: 2584
Iter fib(19): 0.0000 sec.   Output: 4181
Iter fib(20): 0.0000 sec.   Output: 6765
Iter fib(21): 0.0000 sec.   Output: 10946
Iter fib(22): 0.0000 sec.   Output: 17711
Iter fib(23): 0.0000 sec.   Output: 28657
Iter fib(24): 0.0000 sec.   Output: 46368
Iter fib(25): 0.0000 sec.   Output: 75025
Iter fib(26): 0.0000 sec.   Output: 121393
Iter fib(27): 0.0000 sec.   Output: 196418
Iter fib(28): 0.0000 sec.   Output: 317811
Iter fib(29): 0.0000 sec.   Output: 514229
Iter fib(30): 0.0000 sec.   Output: 832040
Iter fib(31): 0.0000 sec.   Output: 1346269
Iter fib(32): 0.0000 sec.   Output: 2178309
Iter fib(33): 0.0000 sec.   Output: 3524578
Iter fib(34): 0.0000 sec.   Output: 5702887
Iter fib(35): 0.0000 sec.   Output: 9227465
Iter fib(36): 0.0000 sec.   Output: 14930352
Iter fib(37): 0.0000 sec.   Output: 24157817
Iter fib(38): 0.0000 sec.   Output: 39088169
Iter fib(39): 0.0000 sec.   Output: 63245986
Iter fib(40): 0.0000 sec.   Output: 102334155
Iter fib(41): 0.0000 sec.   Output: 165580141
Iter fib(42): 0.0000 sec.   Output: 267914296
Iter fib(43): 0.0000 sec.   Output: 433494437
Iter fib(44): 0.0000 sec.   Output: 701408733
Iter fib(45): 0.0000 sec.   Output: 1134903170
Iter fib(46): 0.0000 sec.   Output: 1836311903
Iter fib(47): 0.0000 sec.   Output: 2971215073
Iter fib(48): 0.0000 sec.   Output: 4807526976
Iter fib(49): 0.0000 sec.   Output: 7778742049
```

```
Rec fib(1):   0.0000 sec.   Output: 1
Rec fib(2):   0.0000 sec.   Output: 1
Rec fib(3):   0.0000 sec.   Output: 2
Rec fib(4):   0.0000 sec.   Output: 3
Rec fib(5):   0.0000 sec.   Output: 5
Rec fib(6):   0.0000 sec.   Output: 8
Rec fib(7):   0.0000 sec.   Output: 13
Rec fib(8):   0.0000 sec.   Output: 21
Rec fib(9):   0.0000 sec.   Output: 34
Rec fib(10):  0.0000 sec.   Output: 55
Rec fib(11):  0.0000 sec.   Output: 89
Rec fib(12):  0.0000 sec.   Output: 144
Rec fib(13):  0.0000 sec.   Output: 233
Rec fib(14):  0.0000 sec.   Output: 377
Rec fib(15):  0.0000 sec.   Output: 610
Rec fib(16):  0.0000 sec.   Output: 987
Rec fib(17):  0.0000 sec.   Output: 1597
Rec fib(18):  0.0000 sec.   Output: 2584
Rec fib(19):  0.0001 sec.   Output: 4181
Rec fib(20):  0.0001 sec.   Output: 6765
Rec fib(21):  0.0001 sec.   Output: 10946
Rec fib(22):  0.0002 sec.   Output: 17711
Rec fib(23):  0.0004 sec.   Output: 28657
Rec fib(24):  0.0006 sec.   Output: 46368
Rec fib(25):  0.0010 sec.   Output: 75025
Rec fib(26):  0.0016 sec.   Output: 121393
Rec fib(27):  0.0026 sec.   Output: 196418
Rec fib(28):  0.0044 sec.   Output: 317811
Rec fib(29):  0.0081 sec.   Output: 514229
Rec fib(30):  0.0113 sec.   Output: 832040
Rec fib(31):  0.0190 sec.   Output: 1346269
Rec fib(32):  0.0309 sec.   Output: 2178309
Rec fib(33):  0.0513 sec.   Output: 3524578
Rec fib(34):  0.0790 sec.   Output: 5702887
Rec fib(35):  0.1345 sec.   Output: 9227465
Rec fib(36):  0.2100 sec.   Output: 14930352
Rec fib(37):  0.3293 sec.   Output: 24157817
Rec fib(38):  0.5225 sec.   Output: 39088169
Rec fib(39):  0.8442 sec.   Output: 63245986
Rec fib(40):  1.3614 sec.   Output: 102334155
Rec fib(41):  2.2176 sec.   Output: 165580141
Rec fib(42):  3.6171 sec.   Output: 267914296
Rec fib(43):  5.9064 sec.   Output: 433494437
Rec fib(44):  9.7282 sec.   Output: 701408733
Rec fib(45):  15.3014 sec.  Output: 1134903170
Rec fib(46):  24.5570 sec.  Output: 1836311903
Rec fib(47):  40.2523 sec.  Output: 2971215073
Rec fib(48):  63.8484 sec.  Output: 4807526976
Rec fib(49):  104.5104 sec. Output: 7778742049
```

Hypothesis



Running Time (secs.)
1000
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
Input Size

Limitations of empirical analysis

- Requires implementing several algorithms for the same problem
  - may be difficult and time consuming
  - implementation details also play a role (one particular algorithm may be "better written")
- Requires extensive testing
  - time consuming
  - choice of test cases might favor one of the algorithms
- Variations in HW, SW, and OS affect analysis