Brennan McDonald - 8195614
CSI2110 Assignment 1
Professor: Lucia Moura

Question 1:

1. **(n+1)(n+8)**

    a. $(n+1)(n+8) = n^2+9n+8$
    $n^2+9n+8 \leq n^2+9n^2+8n^2$
    $n^2+9n^2+8n^2 = 15n^2$
    Since $15n^2 \leq C \cdot n^2$ for all $n > 1$, $(n+1)(n+8)$ is $O(n^2)$

    b. $(n+1)(n+8) = n^2+9n+8$
    We need to prove that $n^2+9n+8 > C \cdot n^2$ for all n with some constant C
    Picking $C = 5$ we find that for all $n > 3$ the equation $n^2+9n+8 > C \cdot n^2$ holds
    Therefore $(n+1)(n+8)$ is in $\Omega(n^2)$

    c. Since $(n+1)(n+8)$ is in both $O(n^2)$ and $\Omega(n^2)$ is it therefore in $\Theta(n^3)$

2. **n^2 +log n**

    a. Since $log\ n \leq n^2$ for all n we can rewrite $n^2 + l\ og\ n$ as $n^2 + n^2$ OR $2n^2$
    Since $2n^2 \leq 2 \cdot n^2$ for all n > 1
    $n^2 + log\ n$ is $O(n^2)$

    b. Assume $n^2 + log\ n$ is $\Omega(n^2)$
    Therefore there must exist a C where $n^2 + log\ n > C \cdot n^2$ if true for all n
    Dividing both sides by n^2 gives us $1 + log\ n/n^2 > C$
    Since the limit as n approaches infinity of $log\ n/n^2$ is 0
    Since we can pick any C > 1 that disproves this inequality
    Therefore $n^2 + log\ n$ is not in $\Omega(n^2)$

    c. Since n^2 +log n is not in $\Omega(n^2)$, it is therefore not in $\Theta(n^2)$

3. **(n+8) log n**

    a. $(n+8)\ log\ n = n\ log\ n + 8\ log\ n$
    Since $n\ log\ n > 8\ log\ n$ for all $n > 8$, $2n\ log\ n > n\ log\ n + 8\ log\ n$ for all $n > 8$
    By definition of big oh ,the equation $2n\ log\ n < C \cdot n^2$ for all n must hold

Dividing both sides by 2n we get *$\log n < C \cdot n/2$*
This holds true for all $n > 4$ while $C = 1$
Therefore *$(n+8) \log n$* is in $O(n^2)$

4. **$10n^3$**
   a. Assume $10n^3$ is in $O(n^2)$
   Therefore there must exist a C and n where $10n^3 \leq C \cdot n^2$ for all n
   Therefore $n \leq C/10$ should hold for all n
   However this doesnt hold for $n = C/10 + 1$
   Therefore $10n^3$ is not in $O(n^2)$

   b. Since $10n^3 > C \cdot n^2$ where $C = 1$ for all $n > 1$, $10n^3$ is $\Omega(n^2)$

   c. Since $10n^3$ is not in $O(n^2)$ and it is in $\Omega(n^2)$, it is therefore not in $\Theta(n^2)$

5.
6. **4n+5**
   a. Since $5n^2 > 5$ for all N and $4n^2 > 4n$ for all n, we can rewrite 4n+5 as $4n^2+5n^2$
   $4n^2+5n^2 = 9n^2$
   $9n^2 \leq C \cdot n^2$ for all $n > 1$ where $C = 9$
   Therefore $4n+5$ is in $O(n^2)$

   b. Proof by contradiction:
   Assume $4n+5$ is in $\Omega(n^2)$
   Therefore we assume $4n+5 > C \cdot n^2$
   Let $C_2 = C - 5,$
   Since for $4n > C_2 \cdot n^2$ there is no C_2 where 4n is greater for all $n > n_o$
   We have found a contradiction.
   Therefore $4n+5$ is in not in $\Omega(n^2)$

   c. Since $4n+5$ is not in $\Omega(n^2)$ we can state that 4n+5 is not in $\Theta(n^2)$

<u>Question 2:</u>

A. Worst case, the algorithm performs (n * (n - 1))/2 comparisons

B. The Big-Oh of this algorithm would be $O(n^2)$.
$n \cdot (n - 1) = n^2 - n$, since $n < n^2$ for all n we can rewrite the equation as $2n^2$

Therefore since $2n^2 > 2 \cdot n^2$ for all $n > 0$ we can say $n \cdot (n - 1)$ is in $O(n^2)$

C.

```
1. x=0; y=1;
2. int minDifference = abs(Age[0]-Age[1]);
3. for (i=0; i≤n-1; i++) {
5.     int difference = abs(Age[i]-Age[i+1]);
6.     if (difference < minDifference) {
7.             minDifference=difference;
8.             x=i;
9.             y=j;
10.    }
11. }
12. return x, y;
```

D. Due to the fact that the for loop will always perform n iterations, we can say that the algorithm is in O(n)

<u>Question 3:</u>

A)
Given array A of N integers

```
for(int i = 0; i ≤ N; i++){
   int cSum = 0;
   for(int j = 0; j ≤ N; j++){
      if(A[j] < A[i])
         cSum++;
   }
   If(cSum == (N-1)/2)
      return A[i];
}
```

B)
Best Case = O(n)

This is the case where the first number is the median, total values visited is N + 1 which is O(n) complexity

Worst Case = $O(n^2)$
This is the case where the last number is the median, total values checked is N * N which is $O(n^2)$ complexity