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CSI2101 : Section A
Assignment 1

Exercise #50:
This barber could not exist because if he shaves people that
don’t shave themselves then he would shave himself however this would create a paradox.

Exercise #18:
You would invite Kanti and Jasmine in order to make the two people happy.

Exercise #12:

a) $[\neg p \land (p \lor q)] \rightarrow q$
$[(\neg p \land p) \lor (p \land q)] \rightarrow q$ [Distribution]
$[\mathbf{F} \lor (p \land q)] \rightarrow q$ []
$(p \land q) \rightarrow q$ $[(\mathbf{F} \lor q) \leftrightarrow q]$
$\neg(p \land q) \lor q$ $[(p \rightarrow q) \leftrightarrow (\neg p \lor q)]$
$\neg p \lor (\neg q \lor q)$ [Demorgan's law]
$\neg p \lor (\mathbf{T})$ $[(\neg q \lor q) \leftrightarrow \mathbf{T}]$
$\mathbf{T}$ **[Q.E.D.]**

b) $[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$
$[(\neg p \lor q) \land (\neg q \lor r)] \rightarrow (\neg p \lor r)$ [Simplification]
$\neg[(\neg p \lor q) \land (\neg q \lor r)] \lor (\neg p \lor r)$ [Simplification]
$\neg[(\neg p \lor q) \land (\neg q \lor r)] \lor \neg p \lor r$ [Transitive Relation]
$\neg(\neg p \lor q) \lor \neg(\neg q \lor r) \lor \neg p \lor r$ [Demorgan’s Law]
$(\neg(\neg p \lor q) \lor \neg p) \lor (\neg(\neg q \lor r) \lor r)$ [Transitive Relation]
$((p \land \neg q) \lor \neg p) \lor ((q \land \neg r) \lor r)$ [Demorgan’s Law]
$((p \lor \neg p) \land (\neg q \lor \neg p)) \lor ((q \lor r) \land (\neg r \lor r))$ [Distribution]
$(\mathbf{T} \land (\neg q \lor \neg p)) \lor ((q \lor r) \land \mathbf{T})$ [Simplification]
$(\neg q \lor \neg p) \lor (q \lor r)$ [Simplification]
$(\neg q \lor \neg p) \lor (q \lor r)$ [Transitive Relation]
$(\neg q \lor q) \lor (\neg p \lor r)$ [Simplification]
$\mathbf{T} \lor (\neg p \lor r)$ [Simplification]
$\mathbf{T}$ **[Q.E.D.]**

c) $[p \land (p \rightarrow q)] \rightarrow q$
$[p \land (\neg p \lor q)] \rightarrow q$ [Simplification]

$[(p \land \neg p) \lor (p \land q)] \rightarrow q$ [Distribution]
$[\mathbf{F} \lor (p \land q)] \rightarrow q$ [Negation]
$(p \land q) \rightarrow q$ [Domination Law]
$\neg(p \land q) \lor q$ [Logical Equivalence]
$(\neg p \lor \neg q) \lor q$ [De Morgan's Law]
$\neg p \lor (\neg q \lor q)$ [Transitive law]
$\neg p \lor \mathbf{T}$ [Negation]
$\mathbf{T}$ **[Q.E.D.]**

d) $[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r$
$\neg[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \lor r$
$[\neg(p \lor q) \lor \neg(p \rightarrow r) \lor \neg(q \rightarrow r)] \lor r$
$[\neg(p \lor q) \lor \neg(\neg p \lor r) \lor \neg(\neg q \lor r)] \lor r$
$(\neg p \land \neg q) \lor (p \land \neg r) \lor (q \land \neg r) \lor r$
$(\neg p \land \neg q) \lor (p \lor q) \lor (\neg r \lor r)$
$(\neg p \land \neg q) \lor (p \lor q) \lor \mathbf{T}$
$\mathbf{T}$

Exercise #24:

For this exercise I will focus on the right side of the equation.

$(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$

$\neg p \lor (q \land r)$ [Simplification]
$(\neg p \lor q) \land (\neg p \lor r)$ [Distributive]
$(p \rightarrow q) \land (p \rightarrow r) \equiv (p \rightarrow q) \land (p \rightarrow r)$ **[Q.E.D.]**

Exercise #10:

a) $\exists x\, (C(x) \land D(x) \land F(x))$
b) $\forall x\, (C(x) \land D(x) \land F(x))$
c) $\exists x\, (C(x) \land \neg D(x) \land F(x))$
d) $\neg \exists x\, (C(x) \land D(x) \land F(x))$
e) $\exists x\, \exists y\, \exists z\, (C(x) \land D(y) \land F(z))$

Exercise #42:

Let A(x,y) be the statement "user *x* has access to mailbox *y*"
Let S(x) be the statement "x is a system mailbox"
Let F(x) be the statement "the file system x is unlocked"

a) $\forall x \; \exists y \; A(x,y)$
b) $\exists x \; \forall y \; (S(x) \rightarrow A(x,y))$

Exercise #20:

Exercise #32:

Exercise #46: