Use Gaussian Elimination to solve Linear Systems:

Apply Row Operations to the rows of the whole augmented matrix until the coefficient matrix is in RREF.
From the coefficient matrix in its RREF:

1) Decide if the system is consistent

   If it is consistent:

2) Assign parameters to the non-leading variables

3) Solve for leading variables in terms of parameters

Note: All decisions in the algorithm will only depend on the coefficient matrix
Definition: The "Rank" of a matrix A, denoted by rank(A) is the number of leading ones (pivots) determine the REF of A

Example:
Matrix:
$[1\ 2\ 3] = A = rank(a)$

Note: The rank does not change when we do elementary row operations and it can not exceed the number of columns or rows of A
Notation: [A | b], where a is the coefficient matrix and b is the constants matrix
We can determine if a system is consistent or not by comparing the ranks of A & [A | B]
suppose we reduce A in [A | b] to its RREF:

Rank(A) <= Rank([A|b]) <= Rank(A)+1

Summary

1) The system is inconsistent iff Rank(A) < Rank([A | b])

2) The system has a unique solution iff Rank(A) == Rank([A | b]) **AND** rank(A) = # of columns of A

3) The system has infinitely many solutions iff Rank(A) = Rank([A | b]) **AND** Rank(A) < # columns of A

Applications:

I) Network and Traffic Flow Problem:
Goal: Model the internal flow of a network - We dont expect a unique solution

```
300>   a      < x1       b 100>
--------------------------------------------
          | ^                 |
In:       | x2            x3|    Out
600       |                 \/  |    600
----------        x4 >        ----------------
300 > d|                     | c 200>
          |                   |
          |                   |
```

Nodes	In	Out
a	300+x2	x1
b	x1	x3+100
c	x3+x4	200 + 300
d	300	x2 + x4

X1 - X2 = 300
X1 - X3 = 100
X3 + X4 = 500
X2 + X4 = 300

Matrix [A | b]:

1 -1 0 0 | 300
1 0 -1 0 | 100
0 0 1 1 | 500
0 1 0 1 | 300

Reduce to REF / RREF:

X1	X2	X3	X4		Const.
1	0	0	1	|	600
0	1	0	1	|	300
0	0	1	1	|	500
0	0	0	0	|	0

<u>X4 does not have a leading 1</u>

X4 = S

X1 + S = 600
X2 + S = 300
X3 + S = 500

X1		600 - S
X2	_	300 - S
X3	—	500 - S
X4		S

Assuming Xi >= 0

0 <= S <= 300 where $S \varepsilon R$

II) Testing scenarios

Find all possible values of k, such that the following system has either:
(a) No solutions
(b) A Unique Solution
(c) infinitely many solutions

kx + y + z = 1
x + ky + z = 1
x + y + kz = 1

k	1	1	|	1	R1
1	k	1	|	1	R2
1	1	k	|	1	R3

R1 <-> R2

1	k	1	|	1	R2
k	1	1	|	1	R1
1	1	k	|	1	R3

-kR1 + R2 -> R2
-R1 + R3 -> R3

1	k	1	|	1	R2
0	$1-k^2$	1-k	|	1-k	R1
0	1-k	k-1	|	0	R3

1	k	1	| 1		R2
0	$1-k^2$	1-k	| 1-k		R1
0	1-k	k-1	| 0		R3

Cases:
1)
If K = 1:

X1 X2 X3 => X2 & X3 are non-leadings

2)
K != 1
K - 1 != 0

III) Solving vector equations

Does the set {(1,2,3),(4,5,6),(7,8,9)} Span R^3

Do there exist scalars a1, a2, a3 such that
a1(1,2,3) + a2(4,5,6) + a3(7,8,9) = (x,y,z) in R^3

a1 + 4a2 + 7a3 = x
2a1 + 5a2 + 8a3 = y
3a1 + 6a2 + 9a3 = z

Augmented Matrix

1	4	7	x
2	5	8	y
3	5	8	z

After Row Reduction:

1	4	7	x
0	1	2	-(y-2x)/3
0	0	0	x-2y+z

For the system to be consistent we have to get rid of the homogeneous equation (x - 2y + z)
Since they can only span a plane, They do NOT have the ability to span R^3 A.K.A. They are LD.