CSI 2110 9/17/2017

Fact: if $e(n)$ is in $O(f(n))$ and $d(n)$ is in $O(g(n))$ then $e(n)+d(n) \in O(f(n)+g(n))$.

$40n^2 \in O(2n^3)$

$40n^2 \le 2n^3 \iff 20 < n \therefore \forall n > 20$

Proposition: $e(n) \in O(f(n))$ and $d(n) \in O(g(n))$ than $e(n)+d(n) \in O(f(n)+g(n))$

$l :=$ $e(n) \in O(f(n))$ and $d(n) \in O(g(n))$

Suppose l

that means $\exists c, n_0 \in N^+ \; \forall n \ge n_0 \; e(n) \le c \cdot f(n)$

and $\exists c', n_0' \in N^+ \; \forall n \ge n_0' \; d(n) \le c' \cdot g(n)$

we must show $\exists c'', n_0''$ s.t. $\forall n \ge n_0'' \; e(n)+d(n) \le c''(f(n)$

let $c'' = \max(c, c')$ & $n_0'' \max(n_0, n_0')$ $\therefore$ Original Statement Holds

Big Ω: $f(n) \in \Omega(g(n)) \iff g(n) \in O(f(n))$

Big Θ: $f(n) \in \Theta(g(n)) \iff f(n) \in O(g(n)) \cup f(n) \in \Omega(g(n))$

a) Show that $100 \log_{10} N^n \in O(n \log n)$

We must show that $100 \log_{10} N^n \le C \cdot n \log n$

$100 \cdot \log_{10} N^n = 100n \cdot \log_{10} N$

b) Show that $3n^2+n+1 \in \Theta(n^2)$

we must show that $3n^2+n+1 \in O(n^2)$ & $3n^2+n+1 \in \Omega(n^2)$

i.e. $\{n^2 \in \Omega(3n^2+n+1)\}$

we have $n^2 \leq 3n^2+n+1$

so $\forall n \geq 1$ $n^2 = 1 \cdot (3n^2+n+1)$

we also must proove that $3n^2+n+1 \leq c \cdot n^2$

$3n^2+n^2 \geq 3n^2+n+1 \ \forall n \geq 2 \quad \therefore n_0 = 2 \quad c = 4$

c) show that $3^n \notin \Theta(2^n)$, use contradiction

Suppose $\exists_{c, n_0} \in \mathbb{N}^+ . \ \forall n \geq n_0 \ \ 3^n \leq c \cdot 2^n$

$3^n = \left(\frac{3}{2} \cdot 2\right)^n = \left(\frac{3}{2}\right)^n \cdot 2^n \leq c \cdot 2^n \Rightarrow \frac{3}{2}^n = \boxed{1.5^n \leq c}$

d) let $k \in \mathbb{N}^+$ show that $\sum_{i=0}^{n} i^k \in O(n^{k+1})$

if $i \leq n$ $i^k < n^k$ $\qquad n^{k+1} = n \cdot n^k$

$1^k + 2^k + 3^k \dots + n^k \leq \underbrace{n^k + n^k \dots + n^k}_{n \text{ times}} \quad \therefore \sum_{i=0}^{n} i^k \leq n \cdot n^k$