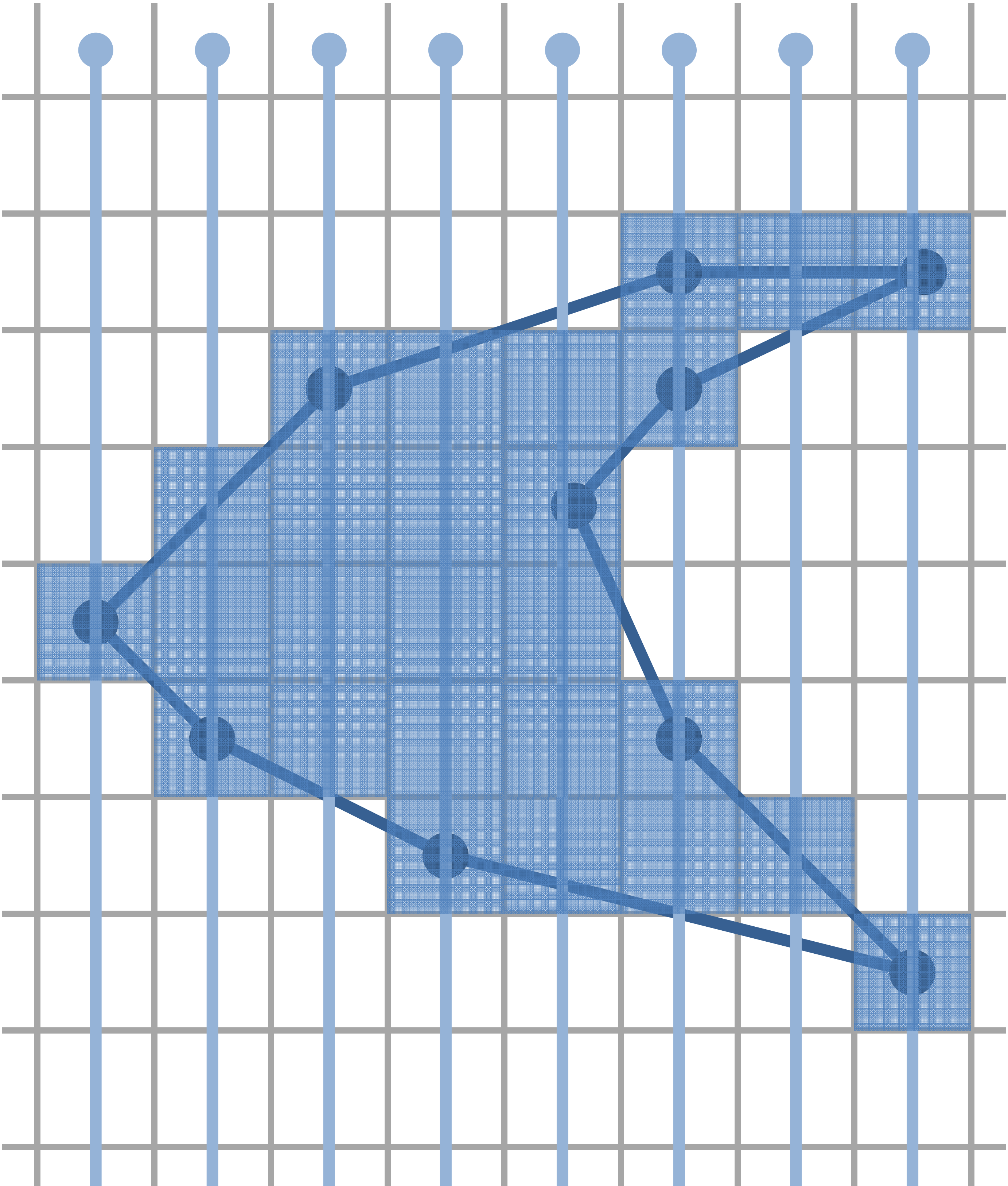
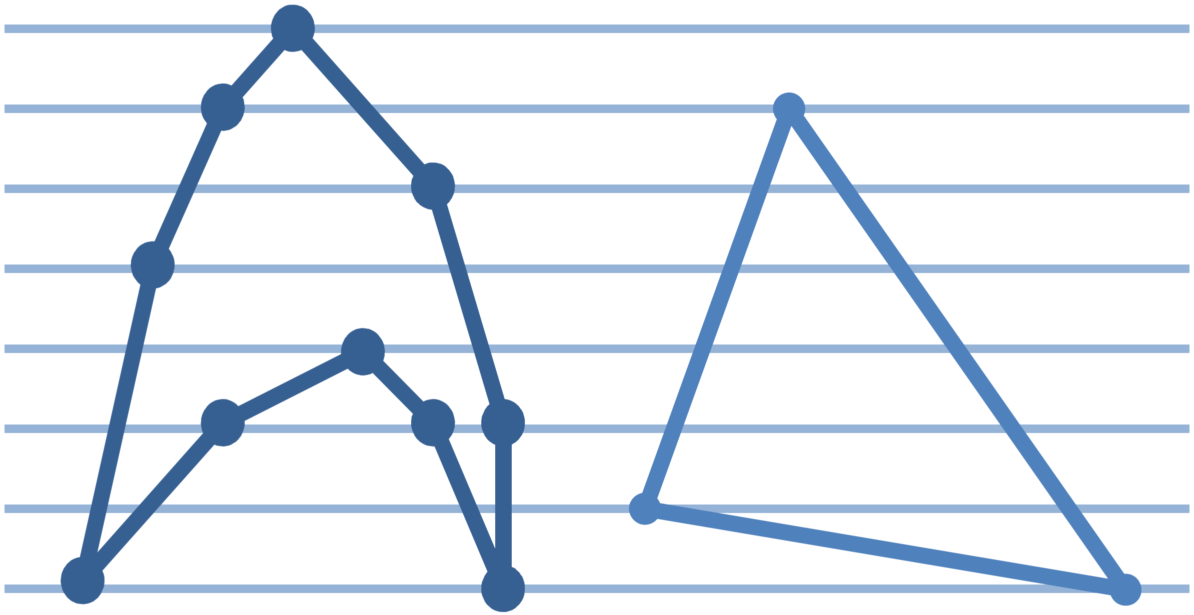
Triangle Rasterization



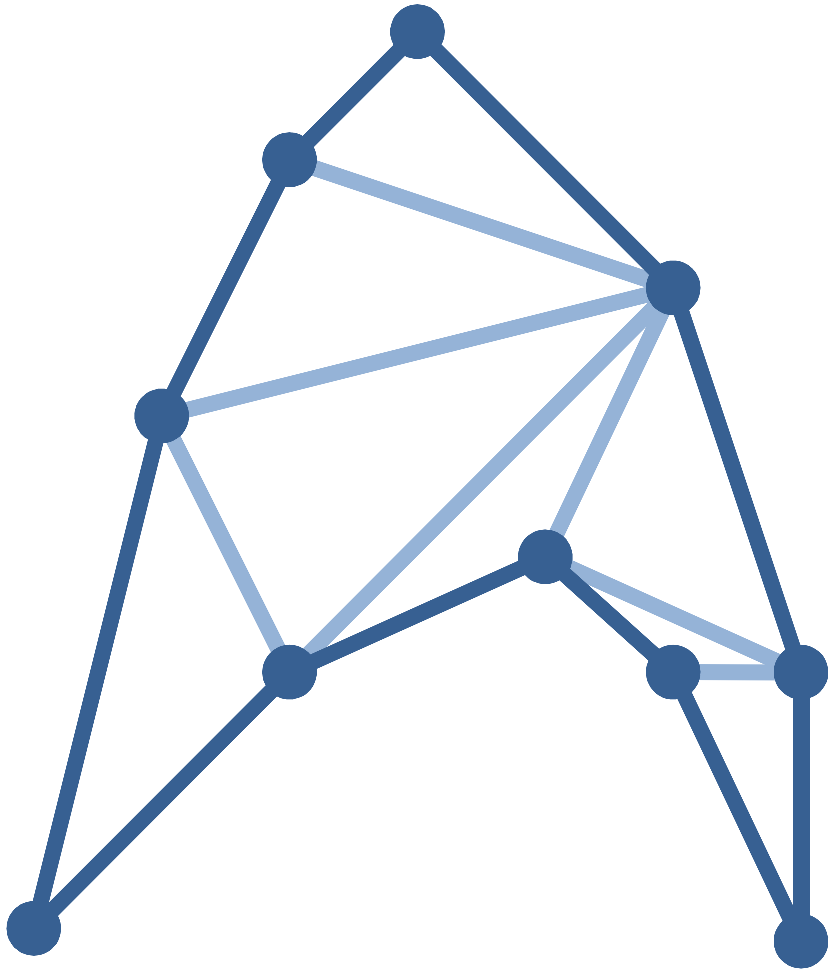
Triangles – Why?

1. Easy to specify
2. Always convex and planar
3. Going to 3D is easy
4. All polygons can be broken into triangles



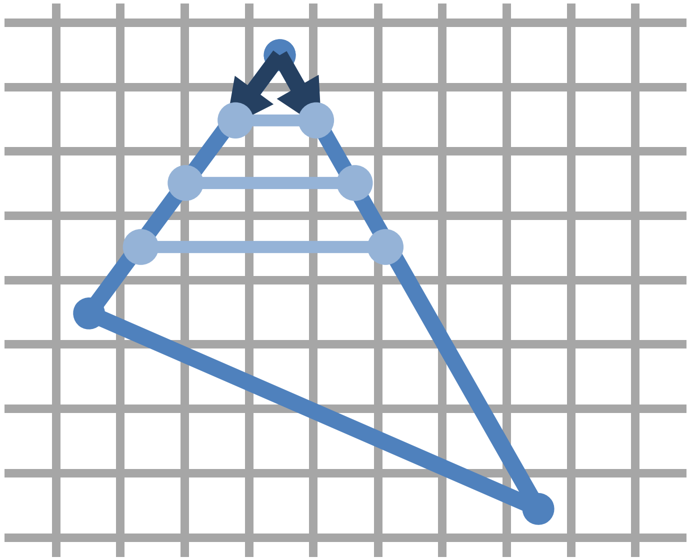
Triangulation

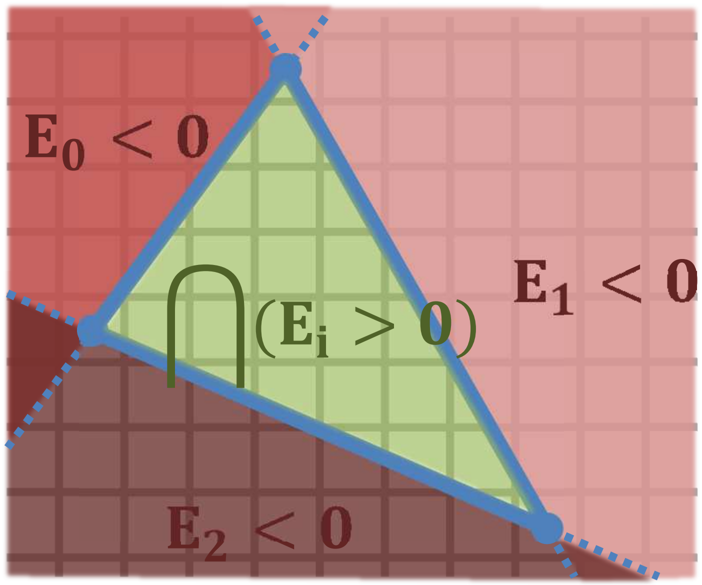
- Breaking a polygon into triangles
  - Delaunay-Triangulation



Scan Converting a Triangle

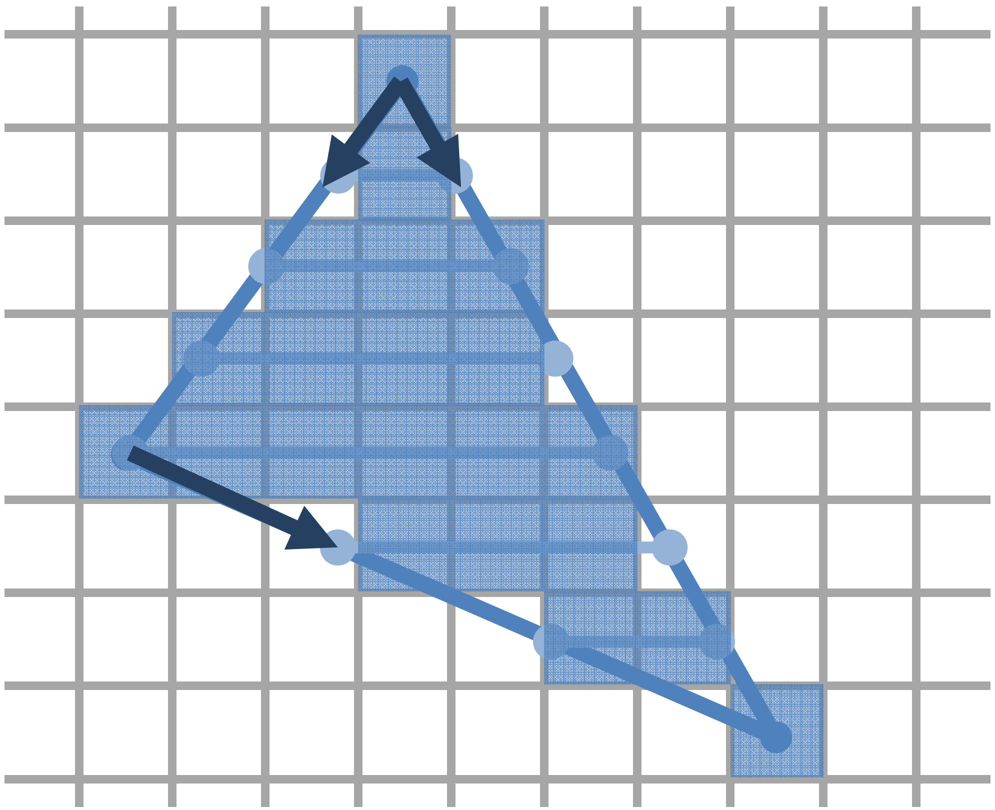
- Edge Walking
- Edge Equations





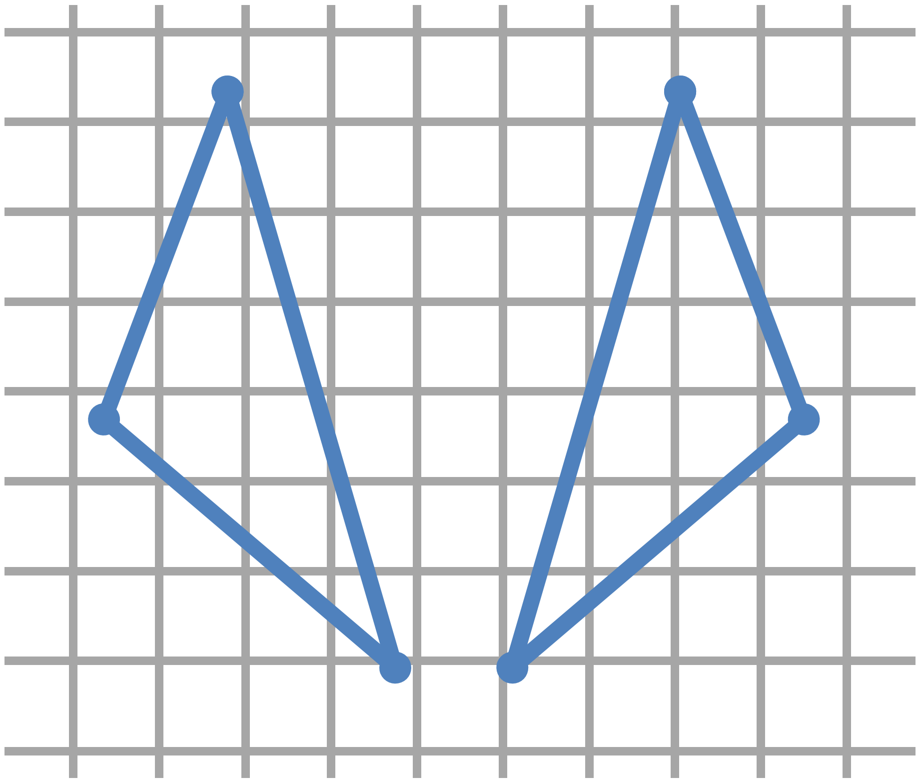
Edge Walking

1. Sort vertices in y
2. Walk down edges from extremal y-point
3. Compute spans
4. Switch in 3rd edge
5. Repeat 2 and 3
   until lowest point

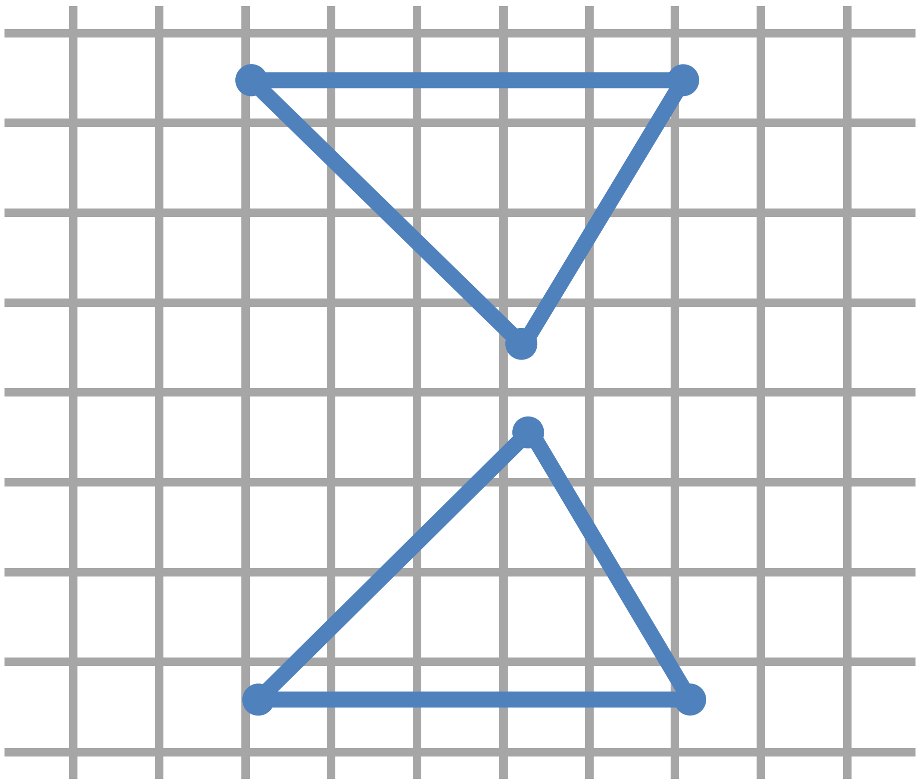


Possible Cases

- Left or right y middle point

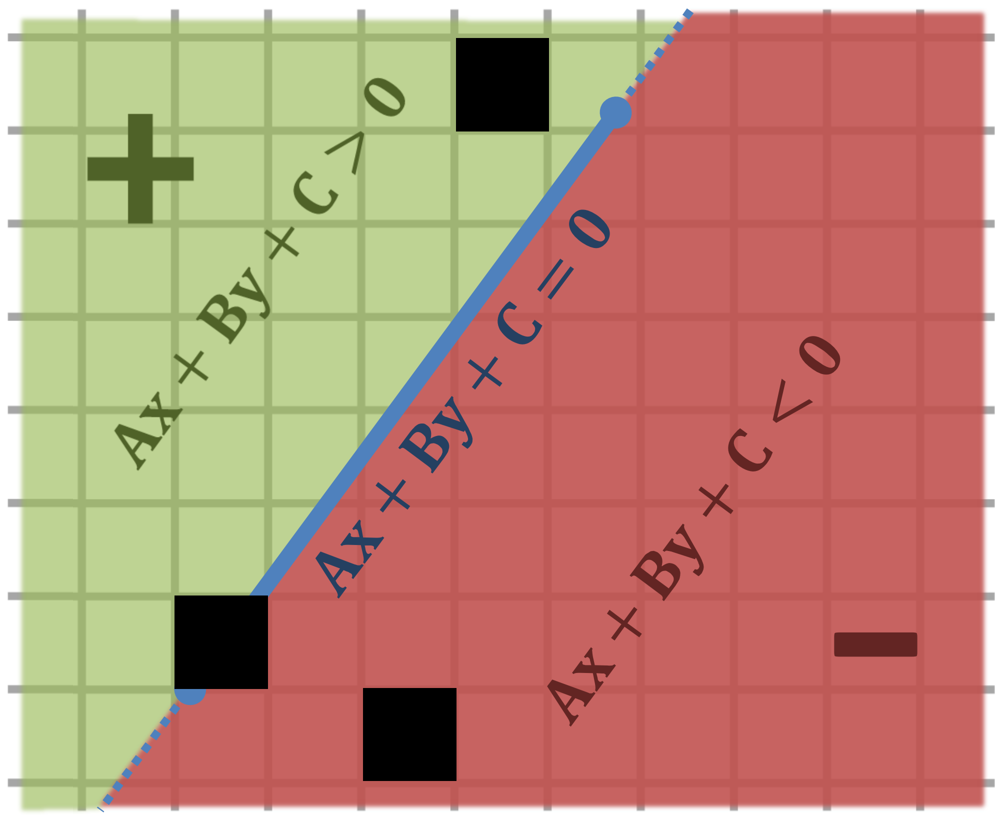


- 2 highest/lowest points



Edge Equations

- Defines positive/negative half-spaces
- Reverse spaces by multiplication by -1
- $E(x, y) = Ax + By + C$
- Value for pixels?
  - $E(P_x, P_y)$



Given 2 points $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, compute A,B,C

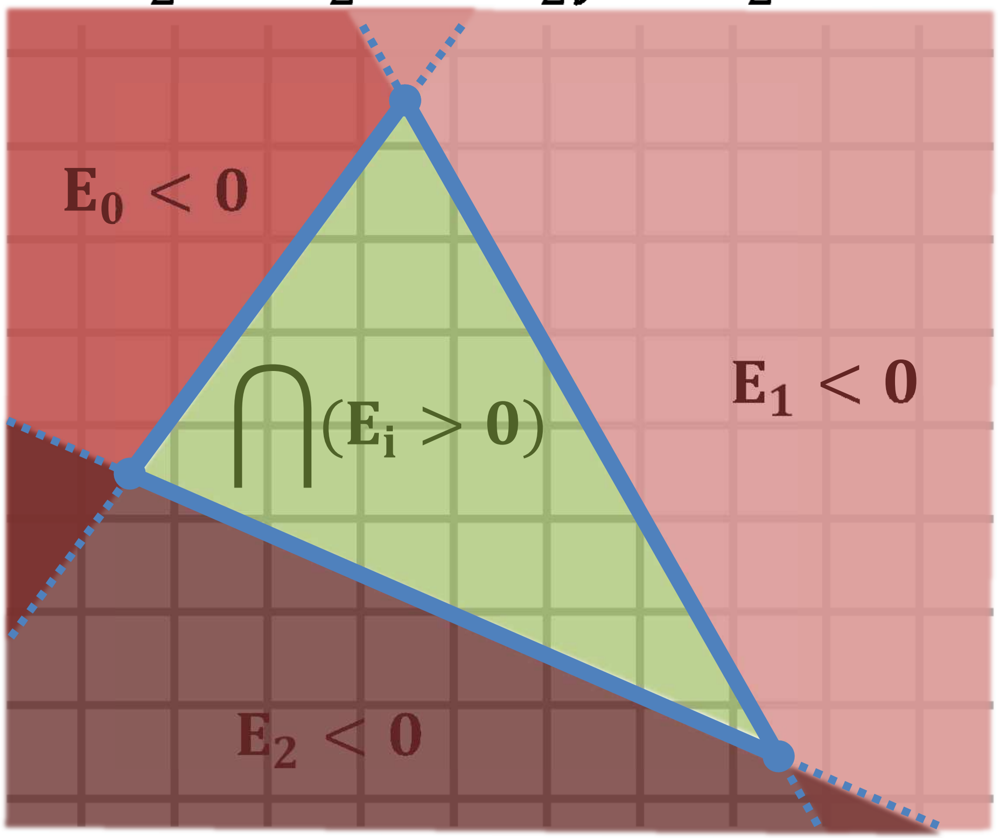
1. Setup equation system
$$Ax_0 + By_0 + C = 0 \quad Ax_1 + By_1 + C = 0$$
2. Matrix representation
$$\begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} + \begin{bmatrix} C \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \leftrightarrow \begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = -C \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
3. Solve
$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{-C}{\begin{vmatrix} x_0 & y_0 \\ x_1 & y_1 \end{vmatrix}} \begin{bmatrix} \begin{vmatrix} 1 & y_0 \\ 1 & y_1 \end{vmatrix} \\ \begin{vmatrix} x_0 & 1 \\ x_1 & 1 \end{vmatrix} \end{bmatrix} = \frac{-C}{x_0 y_1 - y_0 x_1} \begin{bmatrix} y_1 - y_0 \\ x_0 - x_1 \end{bmatrix}$$
4. Choose C

Edge Equations for the Triangle

$$E_0 = A_0x + B_0y + C_0 = 0$$

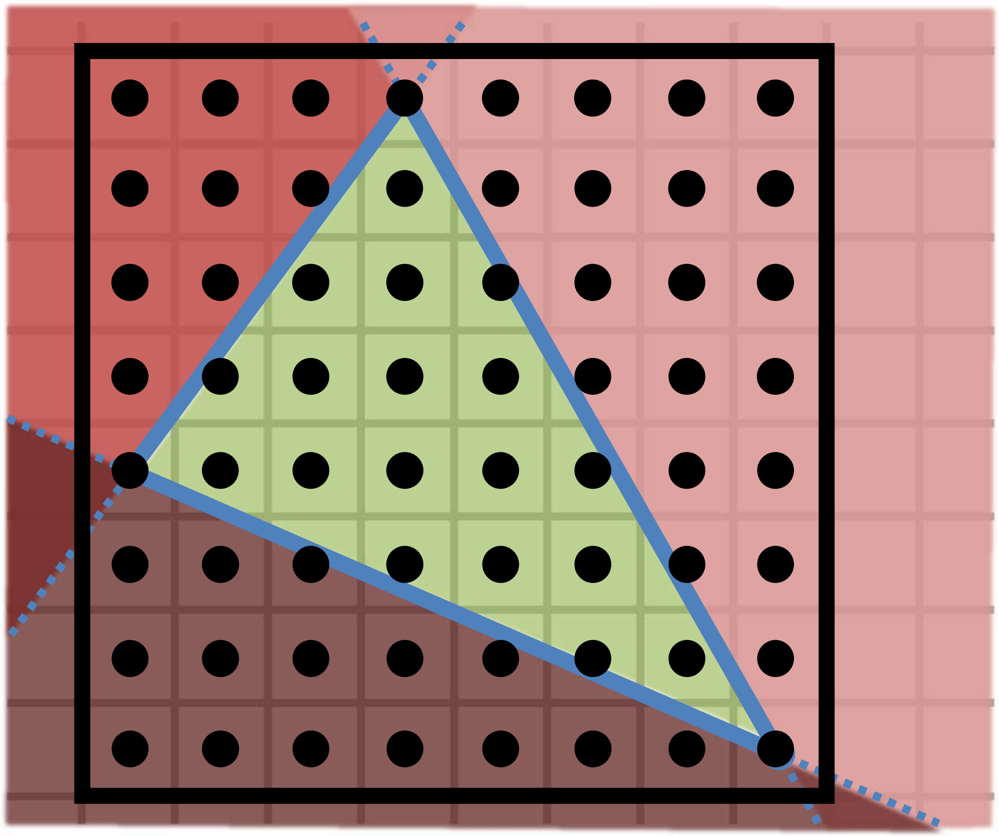
$$E_1 = A_1x + B_1y + C_1 = 0$$

$$E_2 = A_2x + B_2y + C_2 = 0$$



Testing Pixels

- Find bounding box
- Test $\bigcap(\mathbf{E_i} > \mathbf{0})$ for each pixel
- Happy?



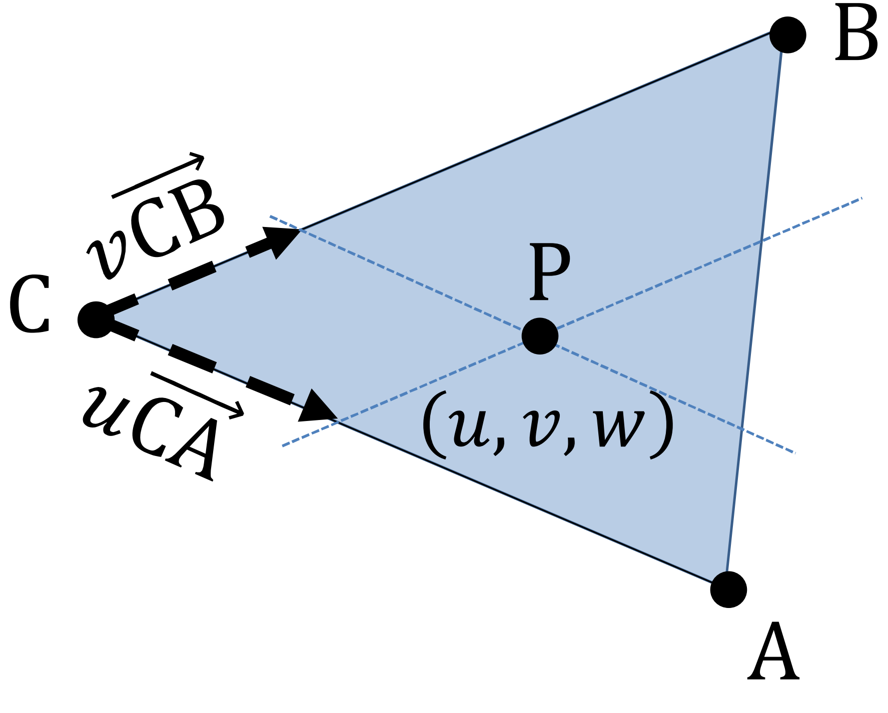
Barycentric Coordinates of P

- Define $\mathrm{P} = \mathrm{C} + u\overrightarrow{\mathrm{CA}} + v\overrightarrow{\mathrm{CB}}$

$$= u\mathrm{A} + v\mathrm{B} + (1 - u - v)\mathrm{C}$$

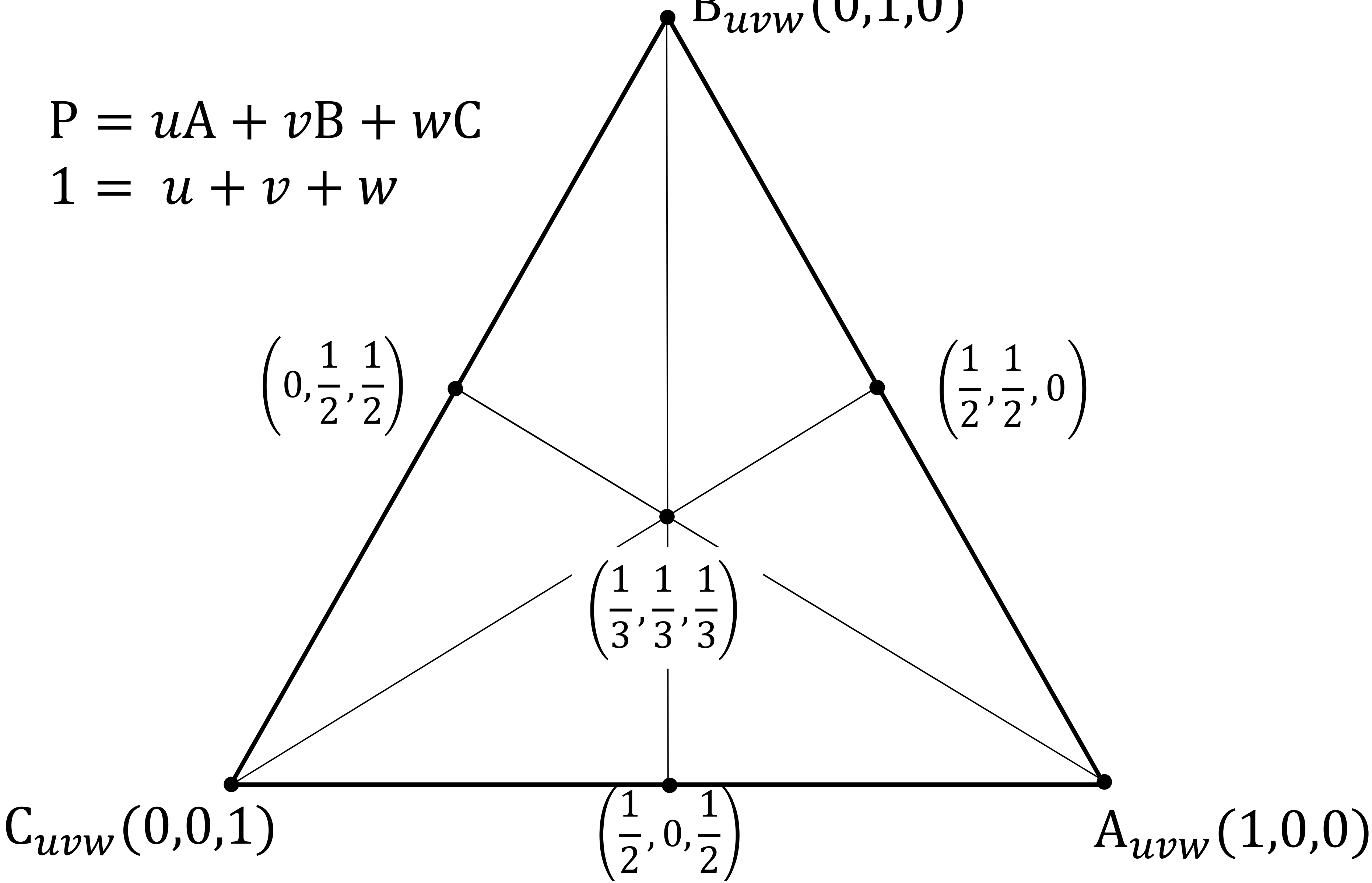
$$= u\mathrm{A} + v\mathrm{B} + w\mathrm{C} \quad \text{with} \quad 1 = u + v + w$$

- Triangle can also be 3d



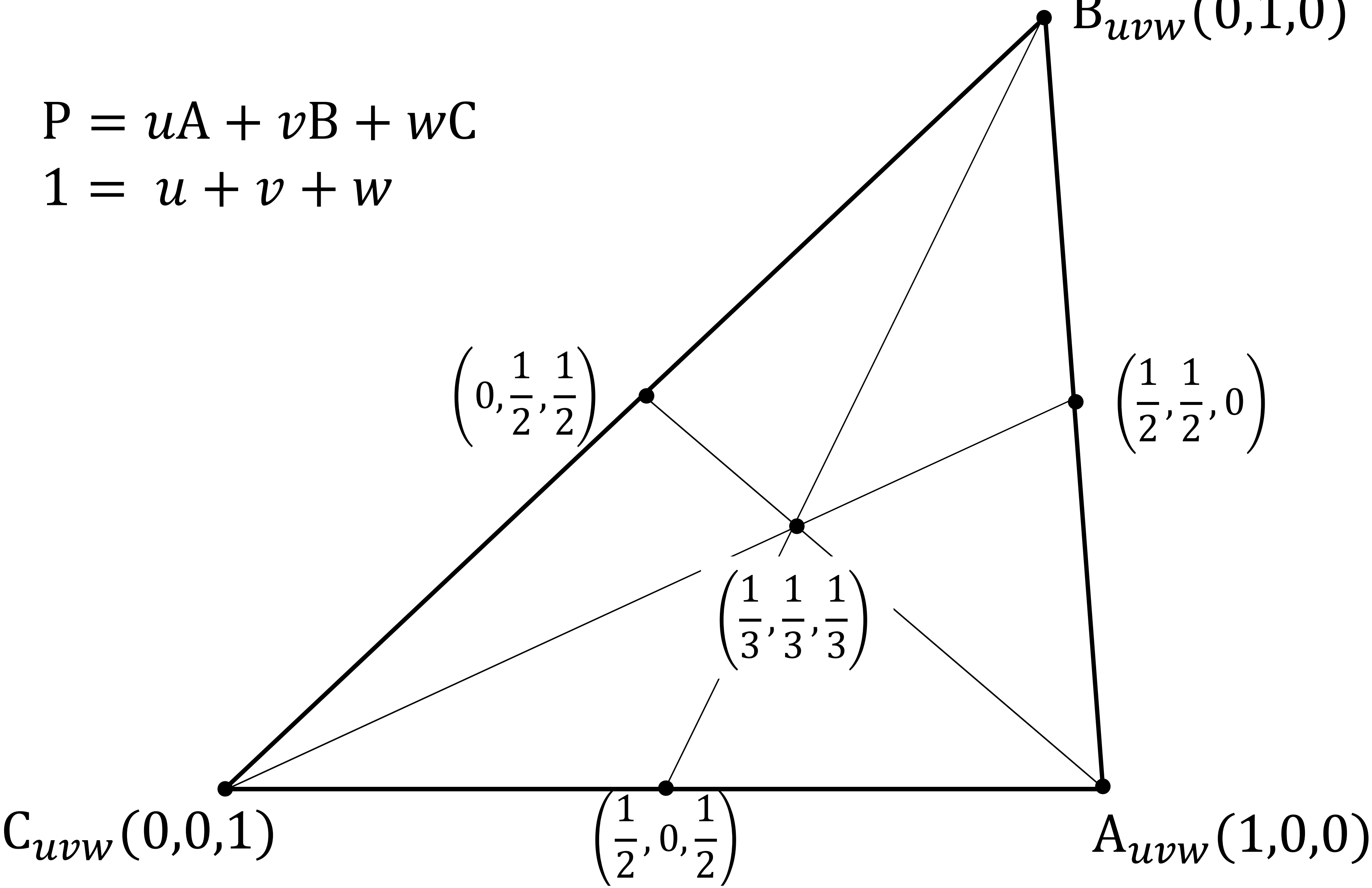
BC – Special Points

$$P = uA + vB + wC$$
$$1 = u + v + w$$



Barycentric Coordinates – Invariance

$$P = uA + vB + wC$$
$$1 = u + v + w$$



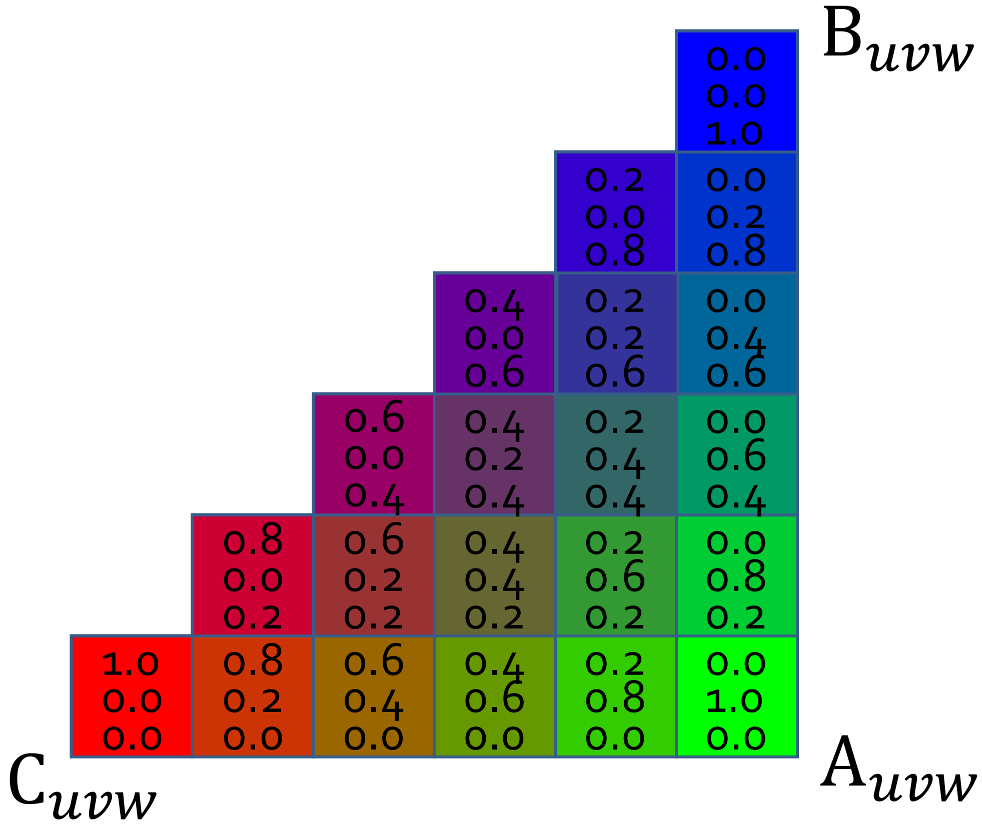
BC – Inside Triangle Test

- Also outside triangle
- In triangle if (u, v, w) all same sign
  - For CCW $(u, v, w) \geq 0$

1.2 -1.4 1.2	1.0 -1.2 1.2	0.8 -1.0 1.2	0.6 -0.8 1.2	0.4 -0.6 1.2	0.2 -0.4 1.2	0.0 -0.2 1.2	-0.2 0.0 1.2
1.2 -1.2 1.0	1.0 -1.0 1.0	0.8 -0.8 1.0	0.6 -0.6 1.0	0.4 -0.4 1.0	0.2 -0.2 1.0	0.0 0.0 1.0	-0.2 0.2 1.0
1.2 -1.0 0.8	1.0 -0.8 0.8	0.8 -0.6 0.8	0.6 -0.4 0.8	0.4 -0.2 0.8	0.2 0.0 0.8	0.0 0.2 0.8	-0.2 0.4 0.8
1.2 -0.8 0.6	1.0 -0.6 0.6	0.8 -0.4 0.6	0.6 -0.2 0.6	0.4 0.0 0.6	0.2 0.2 0.6	0.0 0.4 0.6	-0.2 0.6 0.6
1.2 -0.6 0.4	1.0 -0.4 0.4	0.8 -0.2 0.4	0.6 0.0 0.4	0.4 0.2 0.4	0.2 0.4 0.4	0.0 0.6 0.4	-0.2 0.8 0.4
1.2 -0.4 0.2	1.0 -0.2 0.2	0.8 0.0 0.2	0.6 0.2 0.2	0.4 0.4 0.2	0.2 0.6 0.2	0.0 0.8 0.2	-0.2 1.0 0.2
1.2 -0.2 0.0	1.0 0.0 0.0	0.8 0.2 0.0	0.6 0.4 0.0	0.4 0.6 0.0	0.2 0.8 0.0	0.0 1.0 0.0	-0.2 1.2 0.0
1.2 0.0 -0.2	1.0 0.2 -0.2	0.8 0.4 -0.2	0.6 0.6 -0.2	0.4 0.8 -0.2	0.2 1.0 -0.2	0.0 1.2 -0.2	-0.2 1.4 -0.2

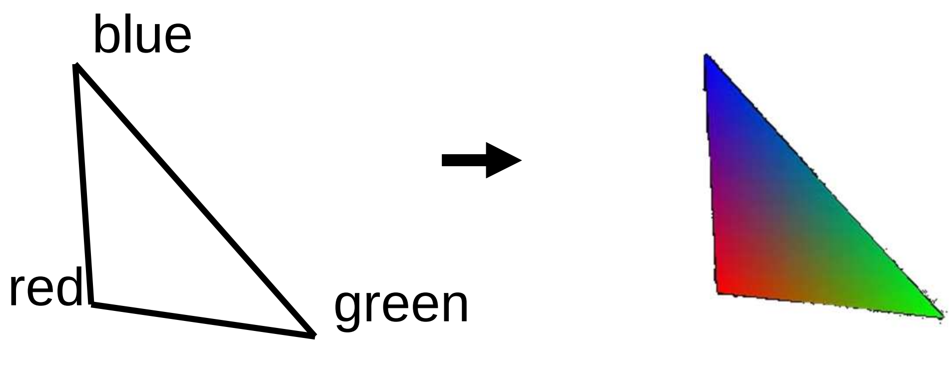
BC – Color Interpolation

- $\mathrm{P} = u\mathrm{A} + v\mathrm{B} + w\mathrm{C}$
- $\mathrm{P} = u\langle Green\rangle + v\langle Blue\rangle + w\langle Red\rangle$
- A.k.a. Gouraud interpolation



Interpolation

- Interpolate per point (a.k.a vertex) attributes (ex.: colors, z-value) over the triangle
- Attribute value for a point P
  - Easy with barycentric coordinates
  - $\mathrm{P} = u\mathrm{A} + v\mathrm{B} + w\mathrm{C}$
  - $P_{attrib.} = uA_{attrib.} + vB_{attrib.} + wC_{attrib.}$

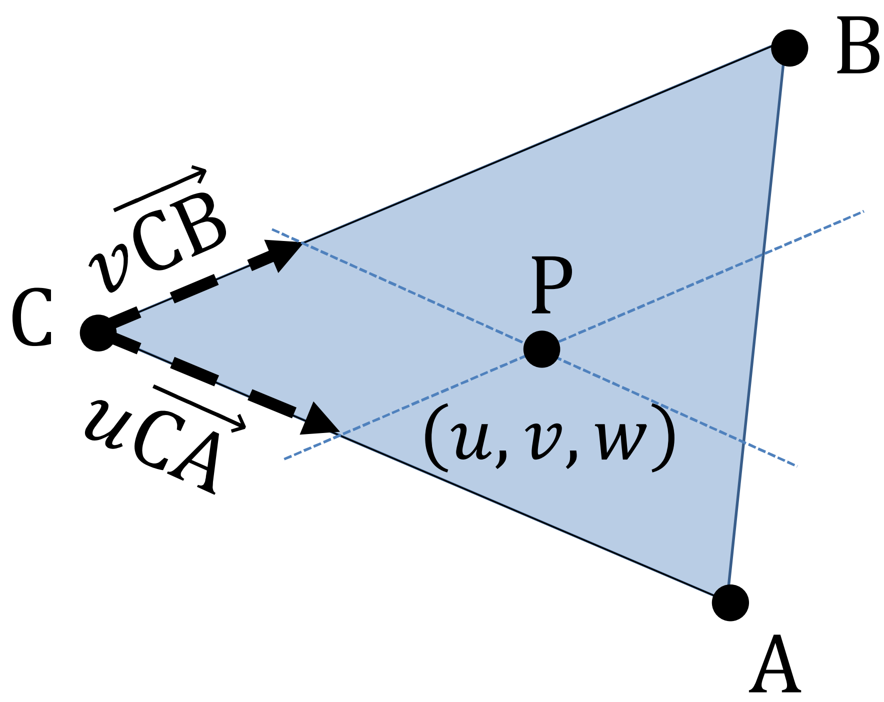


Barycentric Coordinates of P (2D)

$$\mathrm{P} = \mathrm{C} + u\overrightarrow{\mathrm{CA}} + v\overrightarrow{\mathrm{CB}}$$

$$\begin{pmatrix}\overrightarrow{\mathrm{CA}} & \overrightarrow{\mathrm{CB}}\end{pmatrix}\binom{u}{v} = \mathrm{P} - \mathrm{C}$$

$$\begin{pmatrix}\mathrm{A} - \mathrm{C} & \mathrm{B} - \mathrm{C}\end{pmatrix}\binom{u}{v} = \mathrm{P} - \mathrm{C}$$



Barycentric Coordinates of P (2D)

- Cramer's Rule

$$\binom{u}{v} = \frac{1}{|\mathrm{A-C} \quad \mathrm{B-C}|} \begin{pmatrix} |\mathrm{P-C} \quad \mathrm{B-C}| \\ |\mathrm{A-C} \quad \mathrm{P-C}| \end{pmatrix}$$

- Point is inside triangle iff (means if and only if)

$$u \geq 0 \; \cap v \geq 0 \cap (u + v) \leq 1$$