Number systems

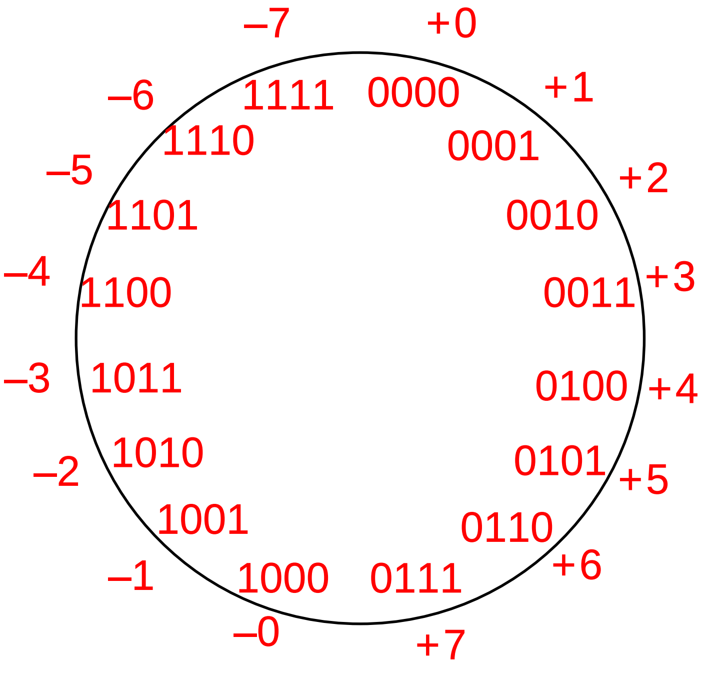
- Representation of positive numbers is the same in most systems
- Major differences are in how negative numbers are represented
- Representation of negative numbers come in three major schemes
  - sign and magnitude
  - 1s complement
  - 2s complement
- Assumptions
  - we'll assume a 4 bit machine word
  - 16 different values can be represented
  - roughly half are positive, half are negative

Sign and magnitude

- One bit dedicate to sign (positive or negative)
  - sign: 0 = positive (or zero), 1 = negative
- Rest represent the absolute value or magnitude
  - three low order bits: 0 (000) thru 7 (111)
- Range for n bits
  - +/– 2^{n-1} –1 (two representations for 0)
- Cumbersome addition/subtraction
  - must compare magnitudes
    to determine sign of result

0 100 = + 4

1 100 = – 4



1s complement

- If N is a positive number, then the negative of N (its 1s complement or N') is N' = $(2^n – 1) – N$
  - example: 1s complement of 7

$$
\begin{array}{lcr}
2^4 & = & 10000 \\
1 & = & \underline{00001} \\
2^4 - 1 & = & 1111 \\
7 & = & \underline{0111} \\
 & & 1000 \quad = \ -7 \text{ in 1s complement form}
\end{array}
$$

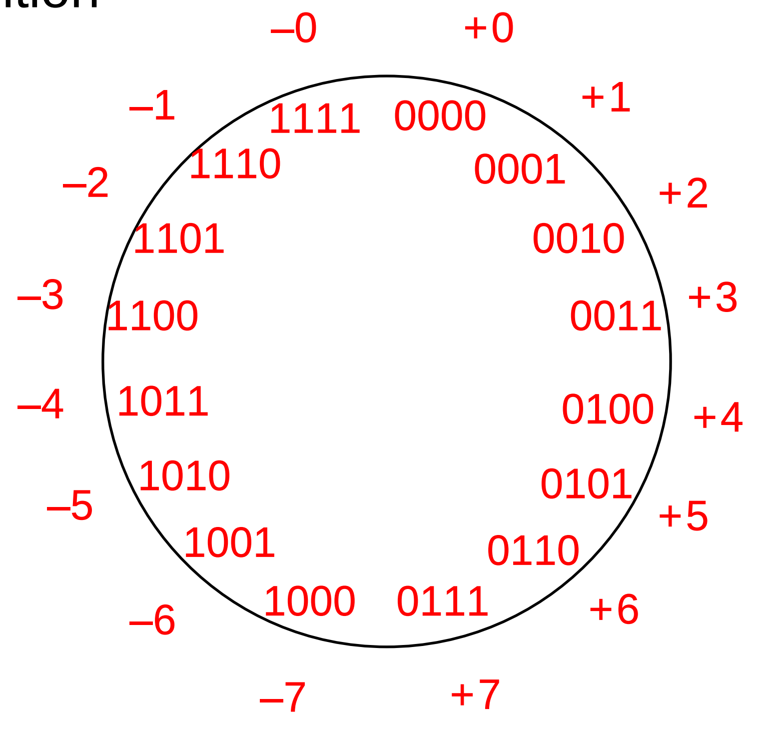
  - shortcut: simply compute bit-wise complement (0111 -> 1000)

1s complement (cont'd)

- Subtraction implemented by 1s complement and then addition
- Two representations of 0
  - causes some complexities in addition
- High-order bit can act as sign bit

0 100 = + 4

1 011 = – 4

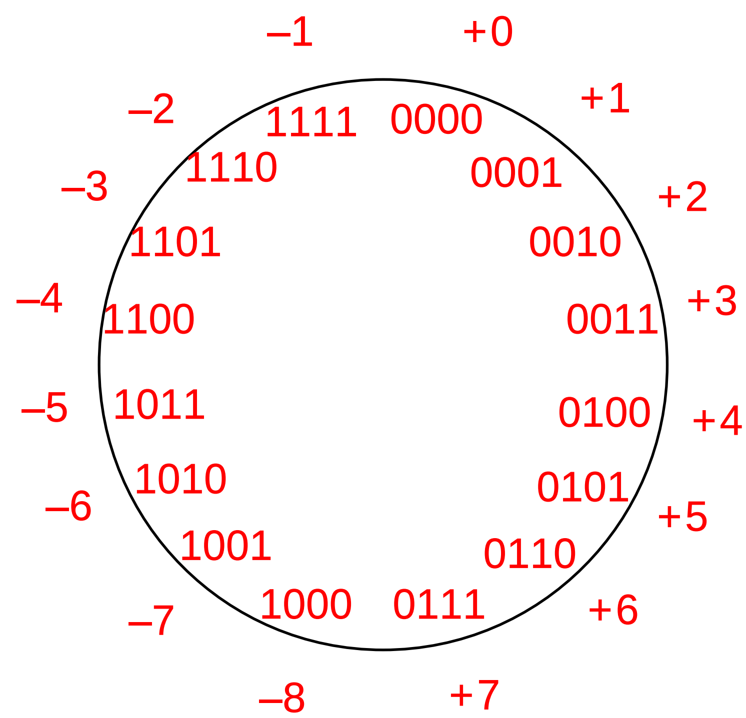


2s complement

- 1s complement with negative numbers shifted one position clockwise
  - only one representation for 0
  - one more negative number than positive numbers
  - high-order bit can act as sign bit

0 100 = + 4

1 100 = – 4



2s complement (cont'd)

- If N is a positive number, then the negative of N (its 2s complement or N*) is $N^* = 2^n - N$
  - example: 2s complement of 7

$$
\begin{array}{rrcl}
 & 2^4 & = & 10000 \\
\text{subtract} & 7 & = & \underline{\ 0111} \\
 & & & 1001 \quad = \text{repr. of } -7
\end{array}
$$

  - example: 2s complement of –7

$$
\begin{array}{rrcl}
 & 2^4 & = & 10000 \\
\text{subtract} & -7 & = & \underline{\ 1001} \\
 & & & 0111 \quad = \text{repr. of } 7
\end{array}
$$

  - shortcut: 2s complement = bit-wise complement + 1
    - 0111 -> 1000 + 1 -> 1001 (representation of -7)
    - 1001 -> 0110 + 1 -> 0111 (representation of 7)

2s complement addition and subtraction

- Simple addition and subtraction
  - simple scheme makes 2s complement the virtually unanimous choice for integer number systems in computers

4	0100	– 4	1100
+ 3	0011	+ (– 3)	1101
7	0111	– 7	11001

4	0100	– 4	1100
– 3	1101	+ 3	0011
1	10001	– 1	1111