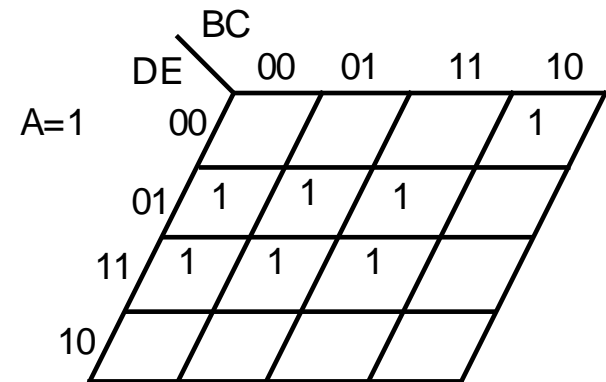
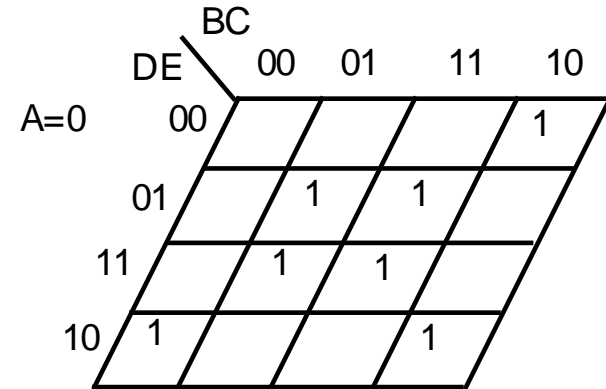
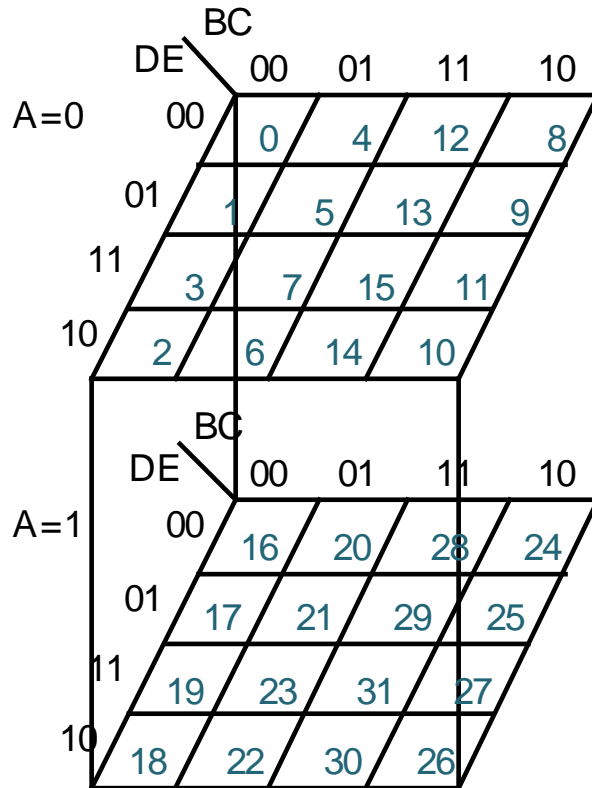


Gate Logic: Two-Level Simplification

5-Variable K-maps

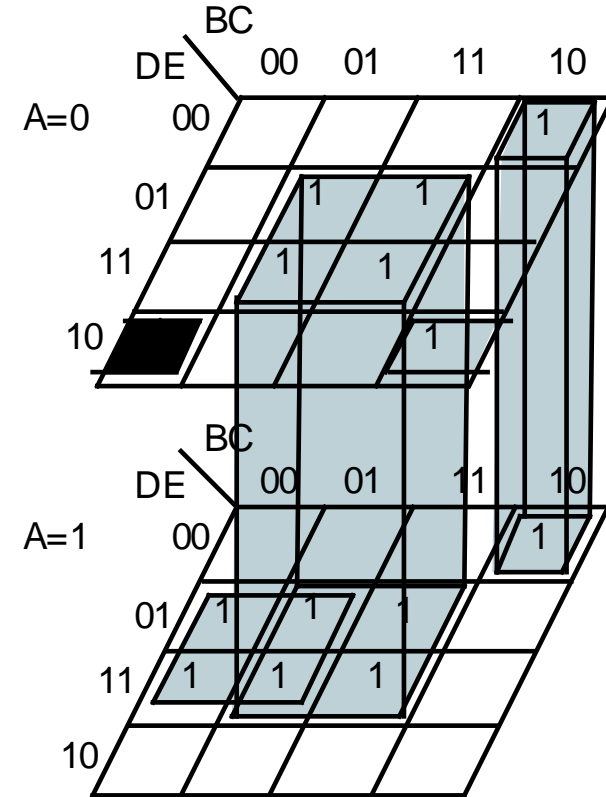
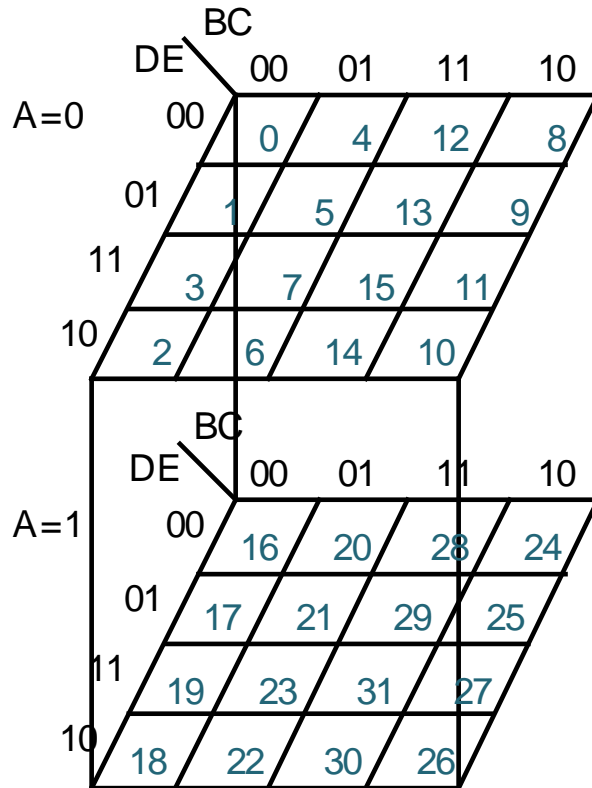


$$f(A,B,C,D,E) = \sum m(2,5,7,8,10,13,15,17,19,21,23,24,29,31)$$

=

Gate Logic: Two-Level Simplification

5-Variable K-maps



$$f(A,B,C,D,E) = \sum m(2,5,7,8,10,13,15,17,19,21,23,24,29,31)$$

$$= CE + AB'E + BC'D'E' + A'C'DE'$$

Gate Logic: Two Level Simplification

6- Variable K-Maps

		CD			
		EF	00	01	11
AB=00	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10
AB=01	00	16	20	28	24
	01	17	21	29	25
	11	19	23	31	27
	10	18	22	30	26
AB=11	00	48	52	60	56
	01	49	53	61	57
	11	51	55	63	59
	10	50	54	62	58
AB=10	00	32	36	44	40
	01	33	37	45	41
	11	35	39	47	43
	10	34	38	46	42

$$f(A,B,C,D,E,F) = \Sigma m(2,8,10,18,24, 26,34,37,42,45,50, 53,58,61)$$

=

		CD				
		EF	00	01	11	10
AB=00	00					1
	01					
	11					
	10	1			1	

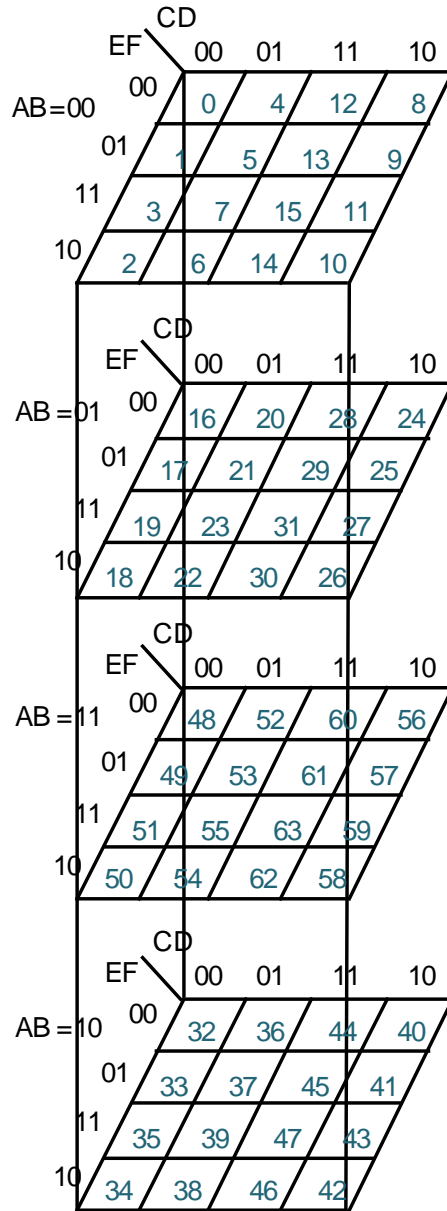
		CD				
		EF	00	01	11	10
AB=01	00					1
	01					
	11					
	10	1				1

		CD			
		EF	00	01	11
AB=11	00				
	01		1	1	
	11				
	10	1			1

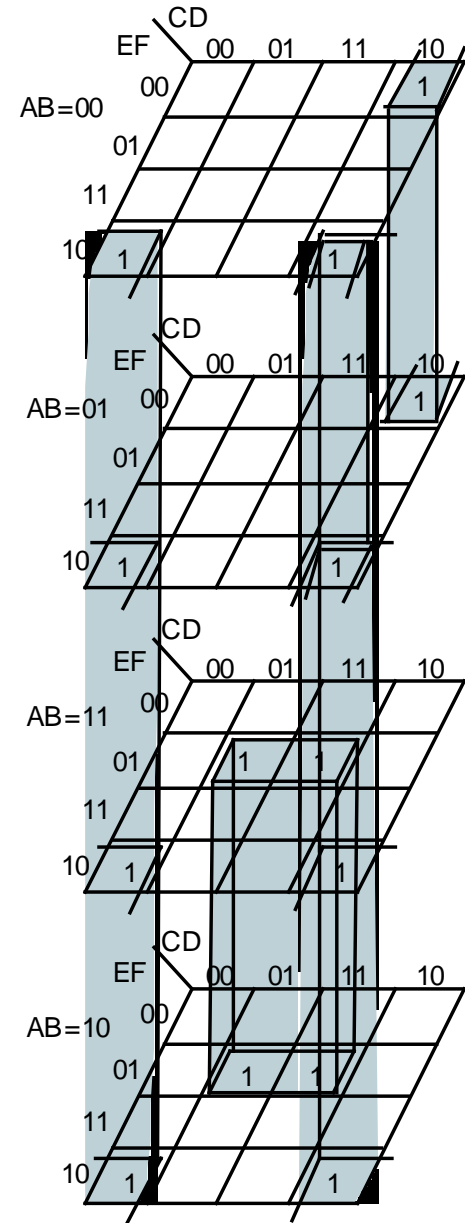
		CD				
		EF	00	01	11	10
AB=10	00					
	01		1	1		
	11					
	10	1			1	

Gate Logic: Two Level Simplification

6- Variable K-Maps



$$\begin{aligned}
 f(A,B,C,D,E,F) &= \sum m(2,8,10,18,24, \\
 &\quad 26,34,37,42,45,50, \\
 &\quad 53,58,61) \\
 &= D' E F' + A D E' F \\
 &\quad + A' C D' F'
 \end{aligned}$$



Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method

Tabular method to systematically find all prime implicants

$$f(A,B,C,D) = \Sigma m(4,5,6,8,9,10,13) + \Sigma d(0,7,15)$$

Stage 1: Find all prime implicants

Step 1: Fill Column 1 with ON-set and DC-set minterm indices. Group by number of 1's.

Implication Table		
Column I	Column II	Column III
0000		
0100		
1000		
0101		
0110		
1001		
1010		
0111		
1101		
1111		

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method

Tabular method to systematically find all prime implicants

$$f(A,B,C,D) = \sum m(4,5,6,8,9,10,13) + \sum d(0,7,15)$$

Stage 1: Find all prime implicants

Step 1: Fill Column 1 with ON-set and DC-set minterm indices. Group by number of 1's.

Step 2: Apply Uniting Theorem—
Compare elements of group w/
N 1's against those with N+1 1's.
Differ by one bit implies adjacent.
Eliminate variable and place in
next column.

E.g., 0000 vs. 0100 yields 0-00
0000 vs. 1000 yields -000

When used in a combination,
mark with a check. If cannot be
combined, mark with a star. These
are the prime implicants.

Repeat until no further combinations can be made.

Implication Table		
Column I	Column II	Column III
0000 ✓	0- 00 - 000	
0100 ✓		
1000 ✓	010- 01- 0	
0101 ✓	100- 10- 0	
0110 ✓		
1001 ✓		
1010 ✓	01-1 -101	
0111 ✓	011- 1-01	
1101 ✓		
1111 ✓	-111 11-1	

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method

Tabular method to systematically find all prime implicants

$$f(A,B,C,D) = \sum m(4,5,6,8,9,10,13) + \sum d(0,7,15)$$

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mark with a check. If cannot be
combined, mark with a star. These
are the prime implicants.

Repeat until no further combinations can be made.

Implication Table		
Column I	Column II	Column III
0000 ✓	0- 00 * - 000 *	01-- *
0100 ✓ 1000 ✓	010- ✓ 01- 0 ✓	-1-1 *
0101 ✓ 0110 ✓ 1001 ✓ 1010 ✓	100- * 10-0 * 01-1 ✓ ✓ -101 ✓	
0111 ✓ 1101 ✓	011- ✓ 1-01 *	
1111 ✓	-111 ✓ 11-1 ✓	

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method Continued

AB \ CD		A			
		00	01	11	10
C	00	X	1	0	1
	01	0	1	1	1
	11	0	X	X	0
	10	0	1	0	1

Diagram showing the Karnaugh map with groupings for variables A, B, C, and D. The map is a 4x4 grid with columns labeled 00, 01, 11, 10 and rows labeled 00, 01, 11, 10. The variables A, B, C, and D are indicated by brackets around the grid.

Prime Implicants:

$$0-00 = A' C' D' \quad -000 = B' C' D'$$

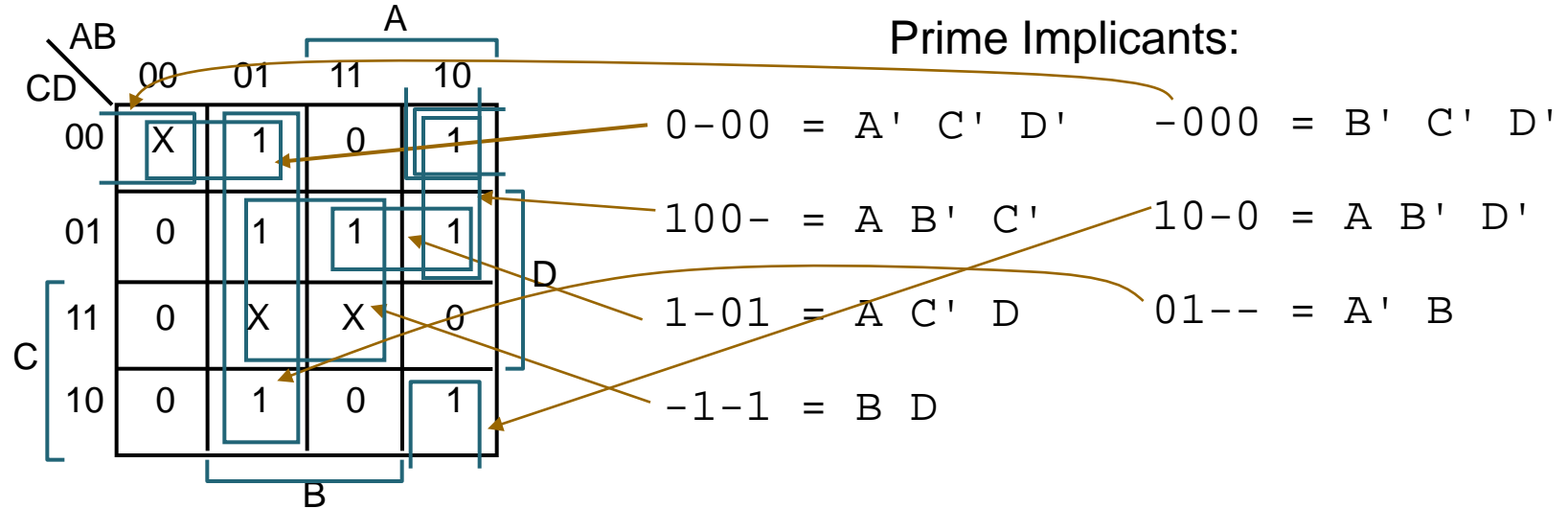
$$100- = A B' C' \quad 10-0 = A B' D'$$

$$1-01 = A C' D \quad 01-- = A' B$$

$$-1-1 = B D$$

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method Continued



Stage 2: find smallest set of prime implicants that cover the ON-set
 recall that essential prime implicants must be in all covers
 another tabular method– the prime implicant chart

Gate Logic: CAD Tools for Simplification

Prime Implicant Chart

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

Gate Logic: CAD Tools for Simplification

Prime Implicant Chart

	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

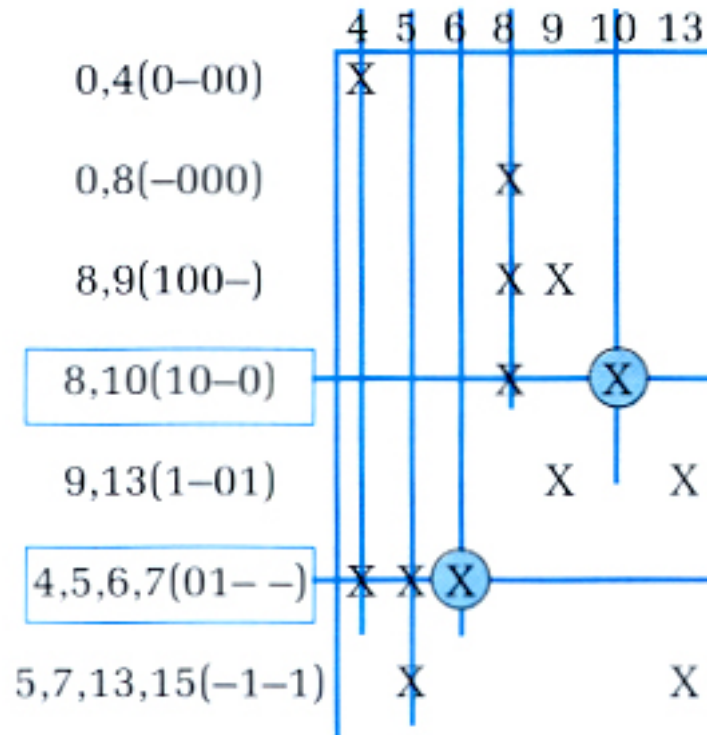
	4	5	6	8	9	10	13
0,4(0-00)	X						
0,8(-000)				X			
8,9(100-)				X	X		
8,10(10-0)				X		X	
9,13(1-01)					X		X
4,5,6,7(01--)	X	X	X				
5,7,13,15(-1-1)		X					X

rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

If column has a single X, then the
implicant associated with the row
is essential. It must appear in
minimum cover

Gate Logic: CAD Tools for Simplification

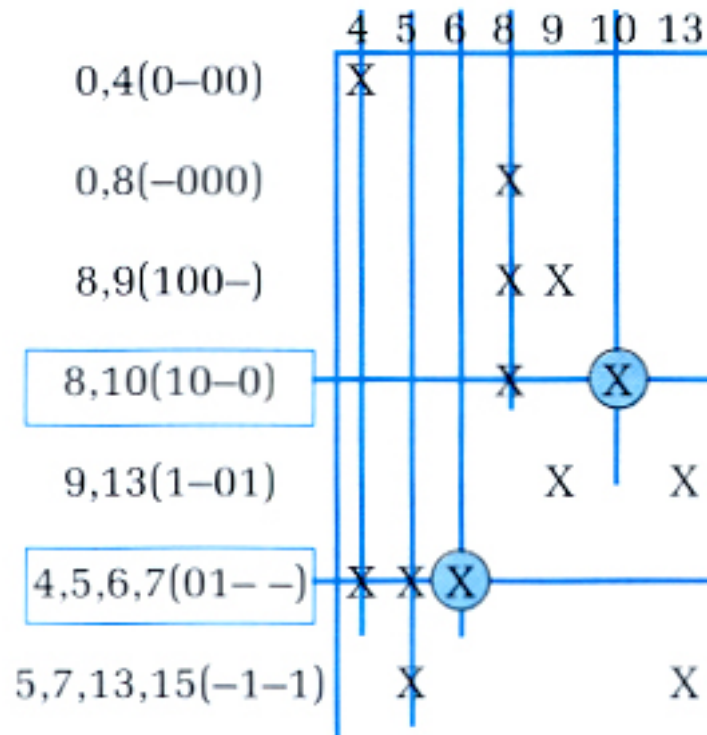
Prime Implicant Chart (Continued)



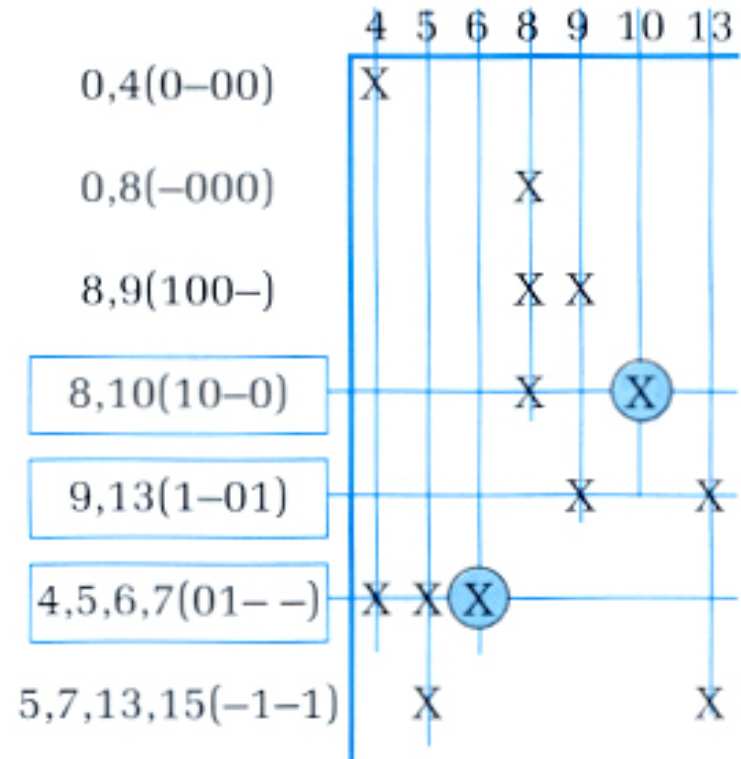
Eliminate all columns covered by essential primes

Gate Logic: CAD Tools for Simplification

Prime Implicant Chart (Continued)



Eliminate all columns covered by essential primes



Find minimum set of rows that cover the remaining columns

$$f = A B' D' + A C' D + A' B$$