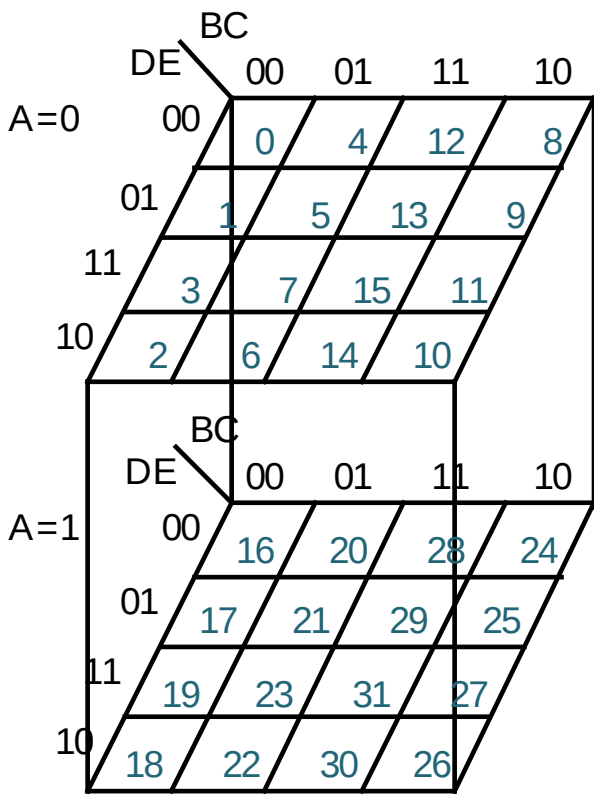
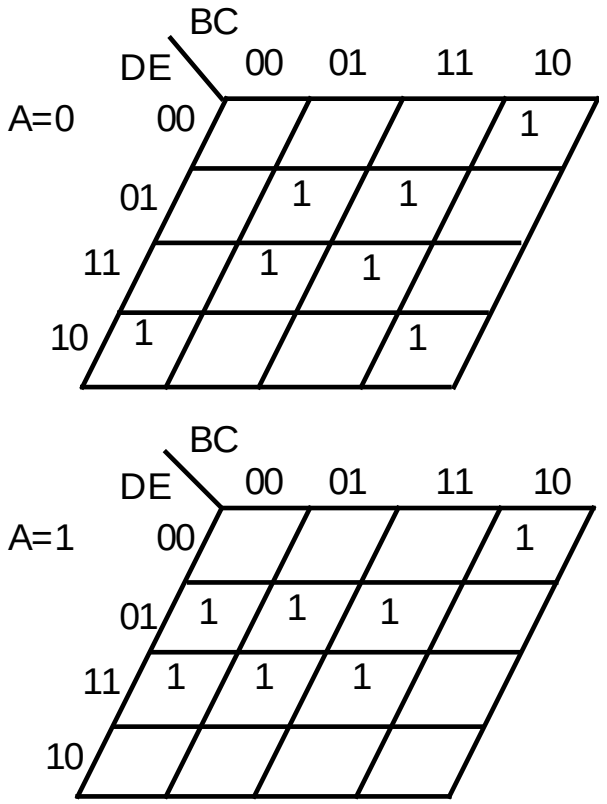
Gate Logic: Two-Level Simplification

5-Variable K-maps



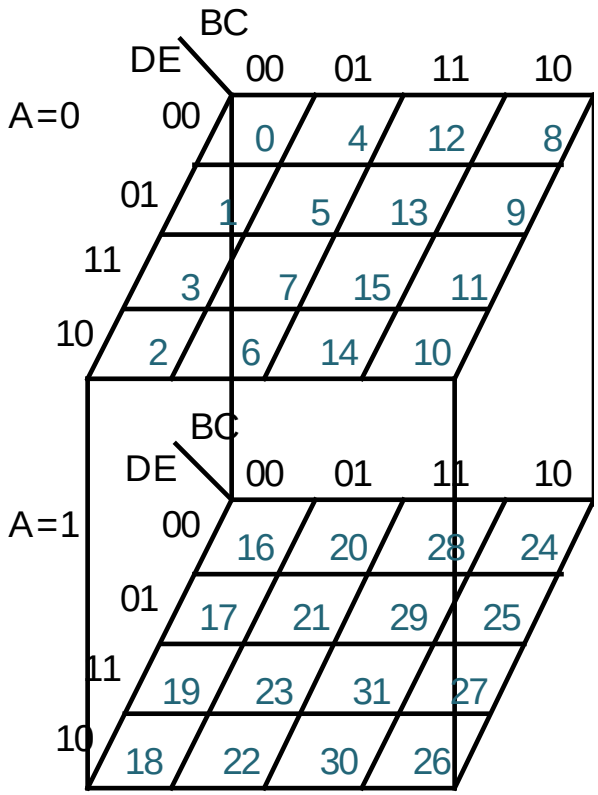


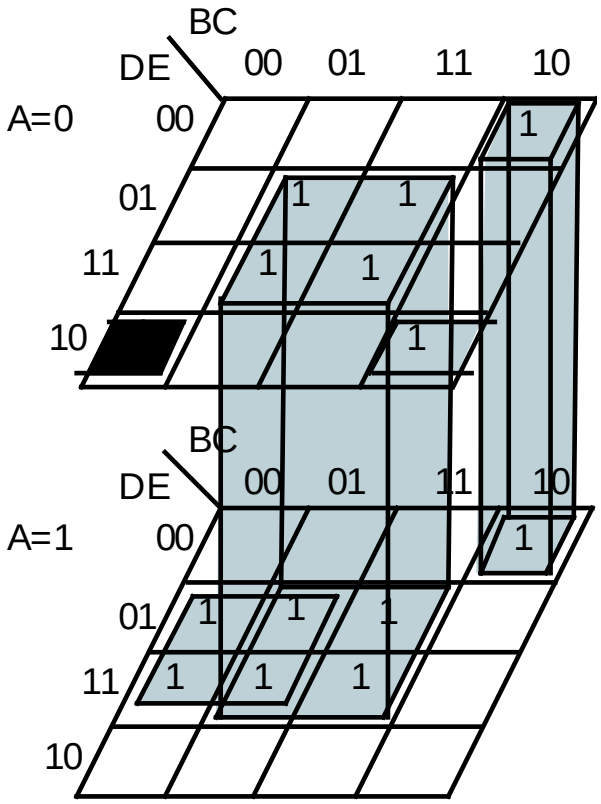
$f(A,B,C,D,E) = \Sigma m(2,5,7,8,10,$
$13,15,17,19,21,23,24,29\ 31)$

=

Gate Logic: Two-Level Simplification

5-Variable K-maps



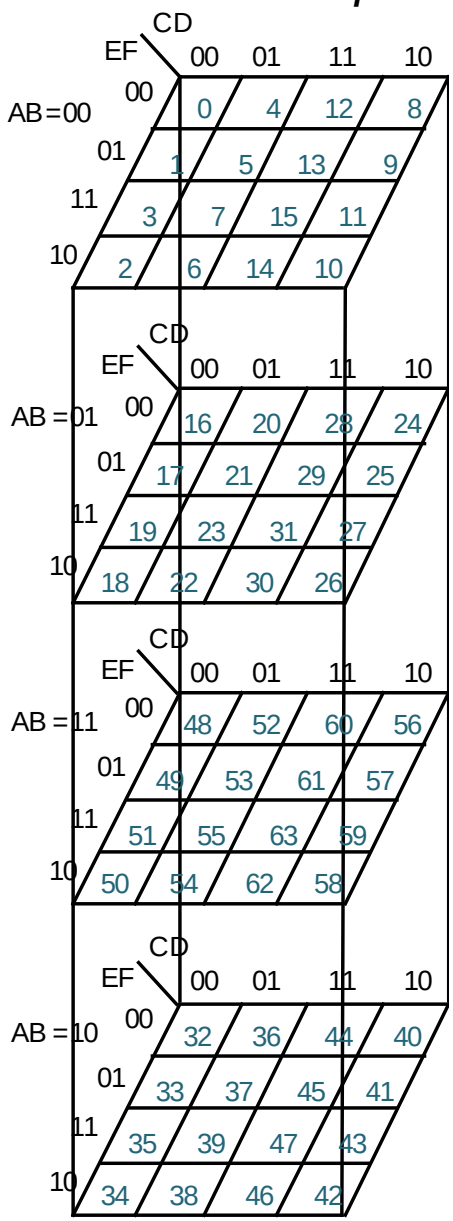


$f(A,B,C,D,E) = \Sigma m(2,5,7,8,10, 13,15,17,19,21,23,24,29\ 31)$

$= C\ E\ +\ A\ B'\ E\ +\ B\ C'\ D'\ E' + A'\ C'\ D\ E'$

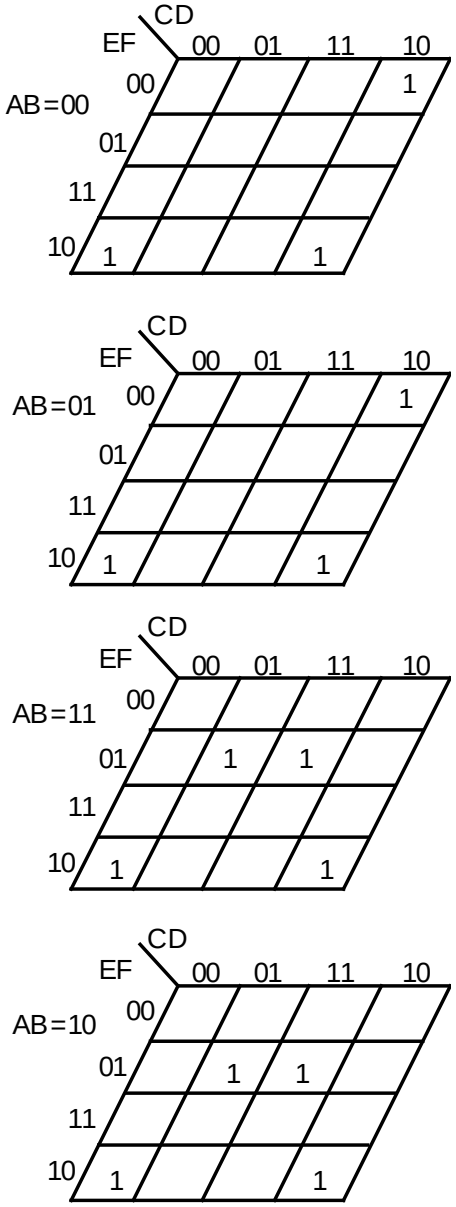
Gate Logic: Two Level Simplification

6- Variable K-Maps



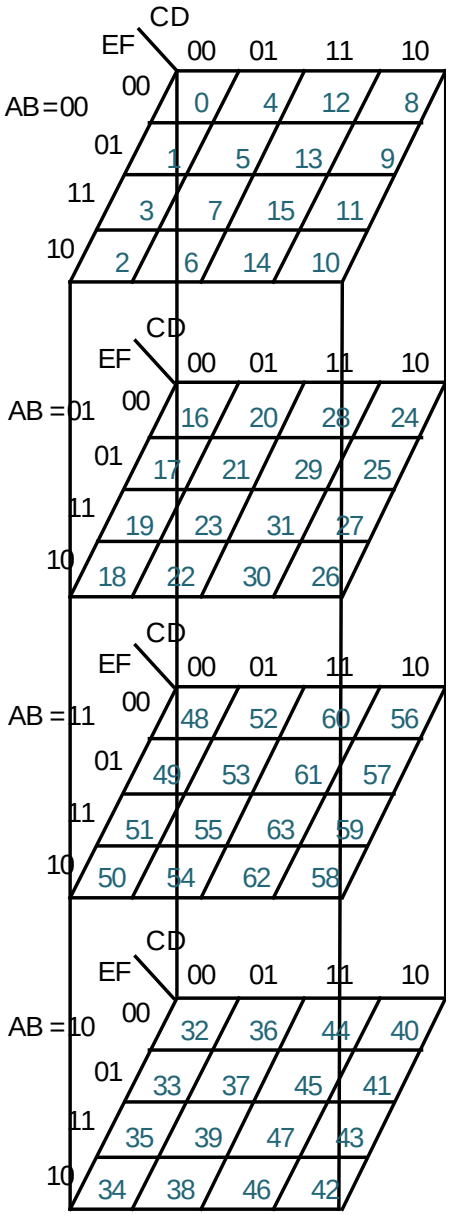
$f(A,B,C,D,E,F) = \Sigma m(2,8,10,18,24,26,34,37,42,45,50,53,58,61)$

=



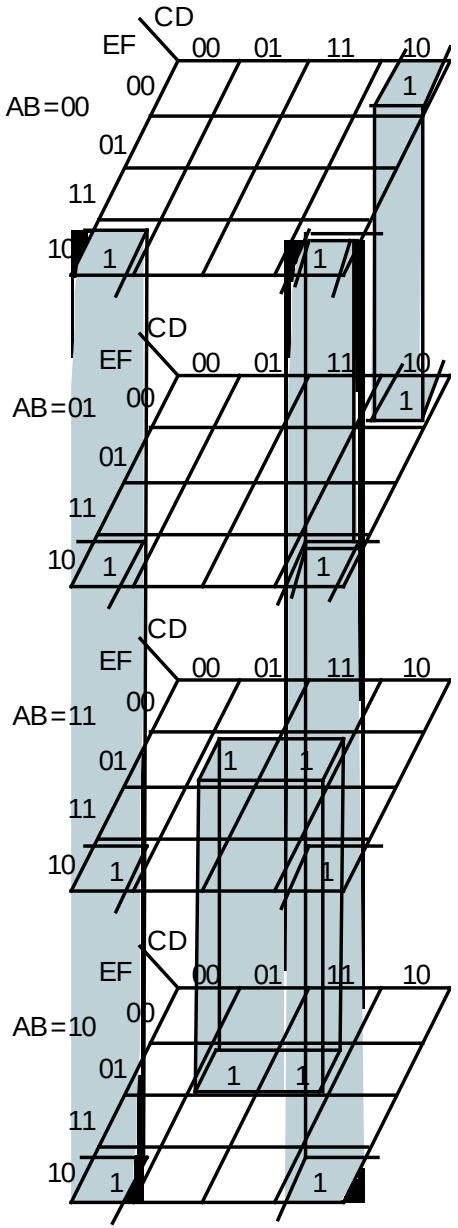
Gate Logic: Two Level Simplification

6- Variable K-Maps



$f(A,B,C,D,E,F) = \Sigma m(2,8,10,18,24,26,34,37,42,45,50,53,58,61)$

$= D'\,E\,F' + A\,D\,E'\,F + A'\,C\,D'\,F'$



Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method

Tabular method to systematically find all prime implicants

$f(A,B,C,D) = \Sigma m(4,5,6,8,9,10,13) + \Sigma d(0,7,15)$

Stage 1: Find all prime implicants

Step 1: Fill Column 1 with ON-set and DC-set minterm indices. Group by number of 1's.

Implication Table		
Column I	Column II	Column III
0000		
0100 1000		
0101 0110 1001 1010		
0111 1101		
1111		

Gate Logic: CAD Tools for Simplification

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Step 2: Apply Uniting Theorem—
Compare elements of group w/ N 1's against those with N+1 1's. Differ by one bit implies adjacent. Eliminate variable and place in next column.

E.g., 0000 vs. 0100 yields 0-00
0000 vs. 1000 yields -000

When used in a combination, mark with a check. If cannot be combined, mark with a star. These are the prime implicants.

Implication Table

Column I	Column II	Column III
0000 ✓	0- 00	
	- 000	
0100 ✓		
1000 ✓	010-	
	01- 0	
0101 ✓	100-	
0110 ✓	10- 0	
1001 ✓		
1010 ✓	01-1	
	-101	
0111 ✓	011-	
1101 ✓	1-01	
1111 ✓	-111	
	11-1	

Repeat until no further combinations can be made.

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method

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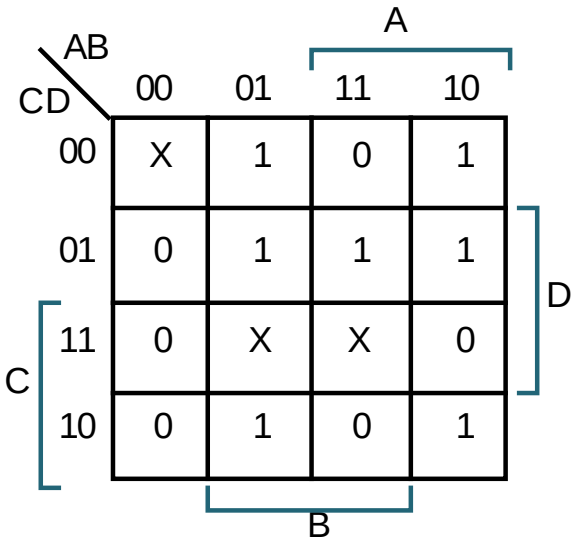
Implication Table

Column I	Column II	Column III
0000 ✓	0- 00 *	01-- *
	- 000 *	
0100 ✓		-1-1 *
1000 ✓	010- ✓	
	01- 0 ✓	
0101 ✓	100- *	
0110 ✓	10-0 *	
1001 ✓		
1010 ✓	01-1 ✓ ✓	
	-101 ✓	
0111 ✓	011- ✓	
1101 ✓	1-01 *	
1111 ✓	-111 ✓	
	11-1 ✓	

Repeat until no further combinations can be made.

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method Continued

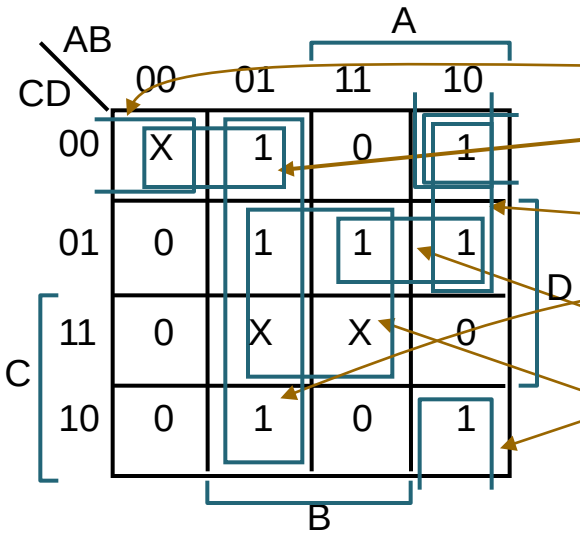


Prime Implicants:

```
0-00 = A' C' D'     -000 = B' C' D'

100- = A B' C'      10-0 = A B' D'

1-01 = A C' D       01-- = A' B

-1-1 = B D
```

Gate Logic: CAD Tools for Simplification

Quine-McCluskey Method Continued



Prime Implicants:

```
0-00 = A' C' D'      -000 = B' C' D'
100- = A B' C'       10-0 = A B' D'
1-01 = A C' D        01-- = A' B
-1-1 = B D
```

Stage 2: find smallest set of prime implicants that cover the ON-set

recall that essential prime implicants must be in all covers

another tabular method– the prime implicant chart

Gate Logic: CAD Tools for Simplification

Prime Implicant Chart

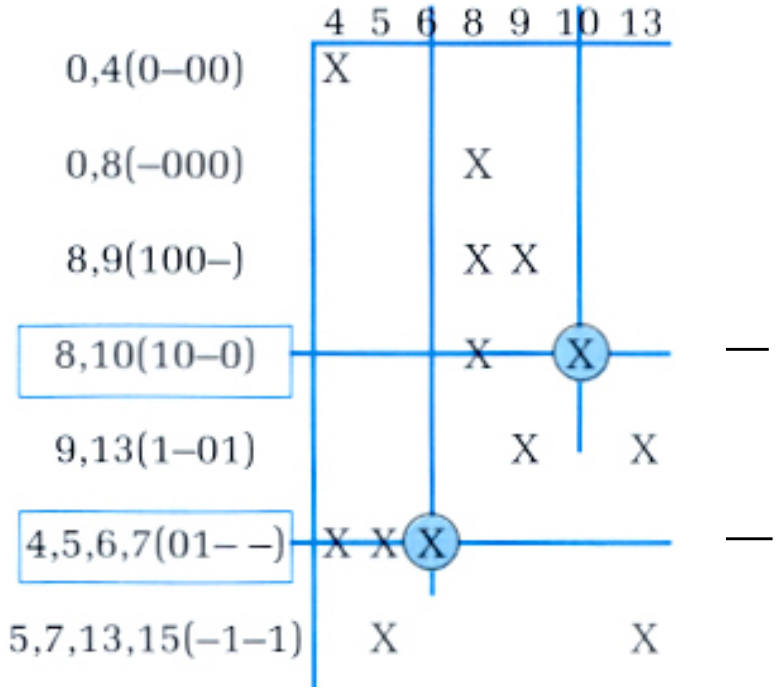
	4	5	6	8	9	10	13
0,4(0–00)	X						
0,8(–000)				X			
8,9(100–)				X	X		
8,10(10–0)				X		X	
9,13(1–01)					X		X
4,5,6,7(01– –)	X	X	X				
5,7,13,15(–1–1)		X					X

rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is
covered by the prime implicant

Gate Logic: CAD Tools for Simplification

Prime Implicant Chart

	4	5	6	8	9	10	13
0,4(0–00)	X						
0,8(–000)				X			
8,9(100–)				X	X		
8,10(10–0)				X		X	
9,13(1–01)					X		X
4,5,6,7(01– –)	X	X	X				
5,7,13,15(–1–1)		X					X

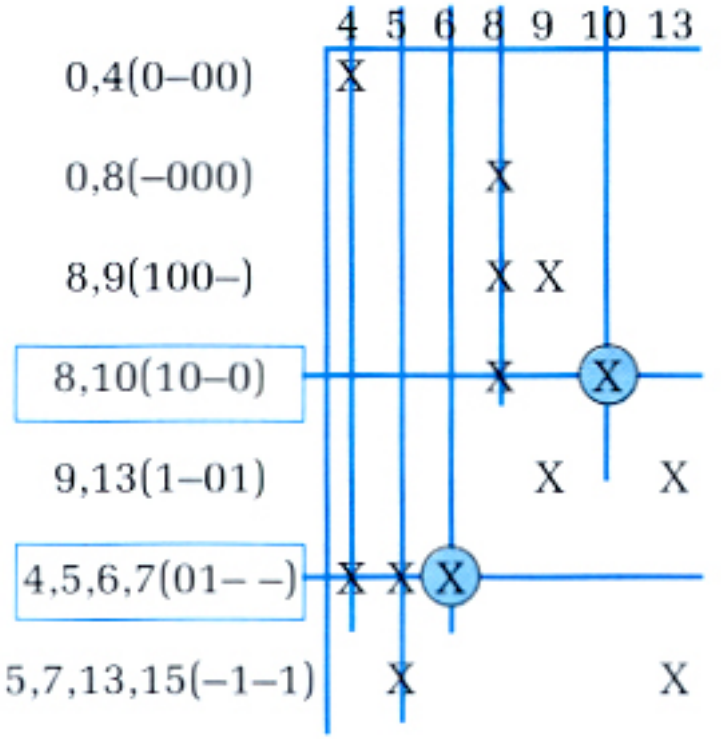


rows = prime implicants
columns = ON-set elements
place an "X" if ON-set element is covered by the prime implicant

If column has a single X, than the implicant associated with the row is essential. It must appear in minimum cover

Gate Logic: CAD Tools for Simplification

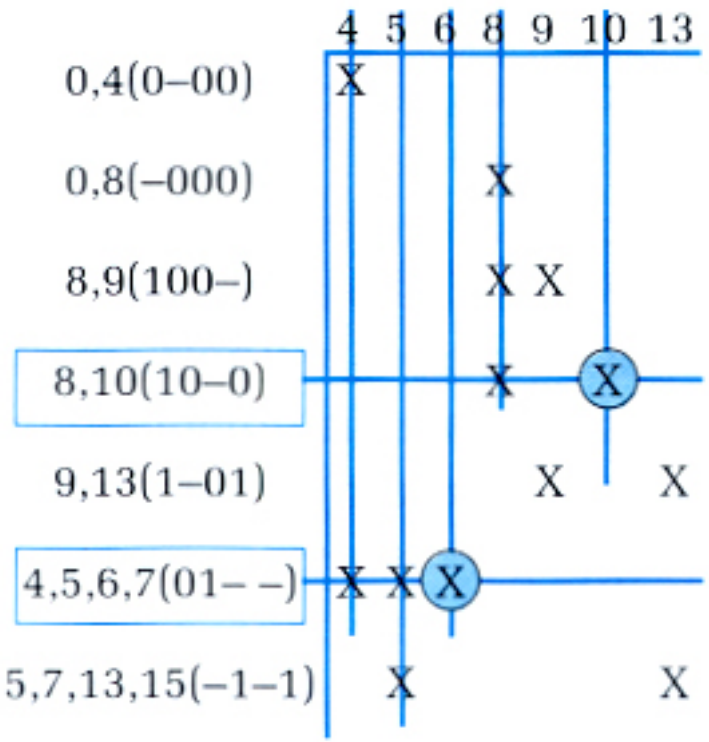
Prime Implicant Chart (Continued)

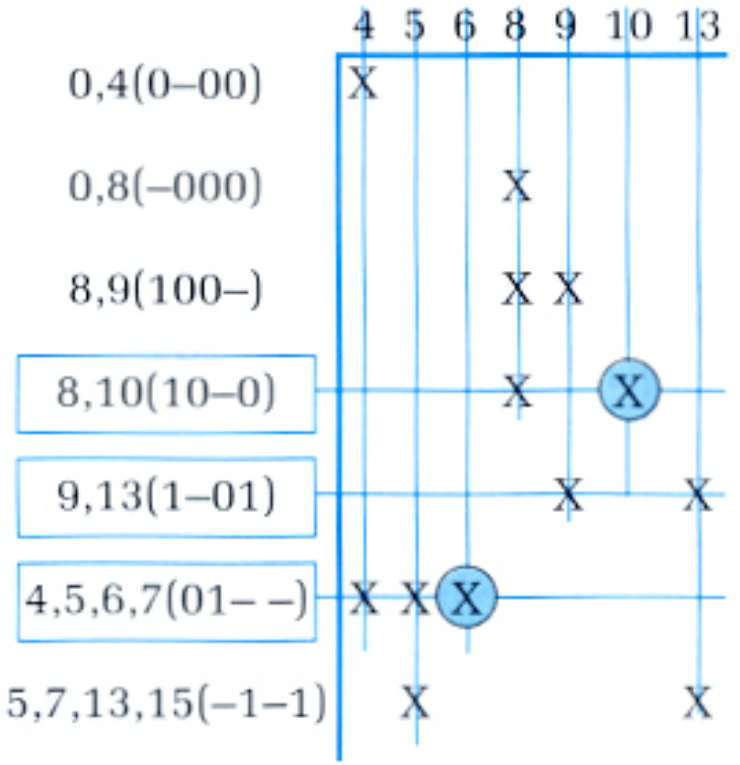


Eliminate all columns covered by essential primes

Gate Logic: CAD Tools for Simplification

Prime Implicant Chart (Continued)





Eliminate all columns covered by essential primes

Find minimum set of rows that cover the remaining columns

$f = A\,B'\,D' + A\,C'\,D + A'\,B$