Chapter 1

Introduction: Charles Peirce, the Logician, the Philosopher... the Mystic

Charles Sanders Peirce is probably best known as initial founder of the now popular school of thought in philosophy called pragmatism. He was far more obscure in his day and likely would have been completely so if it were not for his close friend, William James, who adapted and popularized some of Peirce's ideas (Most notably his Pragmatic Maxim). As a philosopher, Peirce was insightful, ambitious, and exceptionally creative. However, his complexity and penchant for inventing new technical terms sometimes led to an underwhelming reception of his work.

Peirce led a troubled life. It is ironic that now we know him primarily as a philosopher, as in his own lifetime he made his living primarily as a working scientist. The only academic position he held was his appointment as a lecturer on logic at Johns Hopkins. He held this position from 1879

until 1884, when he was fired for reasons tied to his divorce and subsequent marriage of his second wife Juliette **?**. His isolation from the academic community partially explains the difficulty of his philosophy. There were few people he could bounce ideas off of and a little criticism to help rein him in. His student and long time correspondent and co-author, Christine Ladd-Franklin once wrote: “If Charles S. Peirce had happened to have a longer period of activity at the Johns Hopkins University—if the years had not been cut off during which he was kept upon the solid ground of intelligible reason by discussions with a constantly growing group of level-minded students,—there is no doubt that his work would have been of more certain value than it can be affirmed to be now" (Ladd-Franklin, 1916). Despite his tenuous academic career, Peirce remained keenly interested in philosophy and logic, devoting all of his time to these subjects when he was not working on the intellectual odd jobs he used to sustain himself.

There are many facets of Peirce: the logician, the philosopher, the scientist, the mathematician, etc. At times when his philosophical theses become especially grand, he also appears to take on the character of a mystic. In this thesis, we will primarily be interested in Peirce the logician. However, explaining his logical endeavours will also require us to speak to the philosopher and even (perhaps unfortunately) to the mystic.

Peirce has been somewhat pushed out of the history of logic by towering presence of Frege and Russell. Peirce’s importance in the history of logic was, for a long time, not well understood until the pioneering work on the subject by Hintikka (1997) and Dipert (1995) was published. As a logician, Peirce worked within the algebraic tradition, which finds its roots in Boole’s algebra of logic. He and his student, O.H. Mitchell, discovered quantification

independently of Frege (though about four years later). He showed that propositional logic is expressively complete under a single operator, which we now refer to as “Peirce’s Arrow." Furthermore, the notation he and his pupils used was appreciated and expounded by Schröder and is only a typographical variant of the notation logicians currently use (Putnam, 1982). Lowenheim proved his famous theorem in this notation (Ibid). Zermelo wrote his axiomatic set theory in Peirce’s notation (Ibid). Peirce also appears to be the first person to distinguish between first and second order logics (Ibid). He even attempted to work with non-classical logics to deal with issues like modality. This brings us to the topic of my thesis.

In 1909, from around January 7th to February 23, Peirce began experimenting with with three-valued logic, anticipating the pioneering work on the subject by Łukasiewicz (1920) and Post (1921) by about 10 years. Now, Peirce’s work on three-valued logic is nowhere near as sustained and complete Łukasiewicz’s or Post’s. It only spans about 6 handwritten pages in Peirce’s logic notebook. Nonetheless, it is striking, and speaks to his logical instincts, we observe Peirce wrestling with an additional truth value. It seems a natural question to ask why he saw fit to do this. What was the defect with classical logic that Peirce hoped to address with his three-valued logic (which he terms ‘triadic logic’)? This is the question I hope to address throughout the course of this thesis.

The reasons others have taken up three-valued logics are quite diverse. Some, Łukasiewicz most notably, have been motivated by worries about future contingent propositions. Some use an additional truth value in an attempt to deal with vague predicates and the Sorites Paradox. Others thought that results in quantum mechanics necessitated a third value. And others

still have wanted to accommodate undecideable statements in mathematics. It is unclear whether Peirce's motivations were akin to any of these.

The strongest indication of what is motivating Peirce to conduct his three-valued experiments comes from the last of the connected pages in his notebook. There he gives two indications that at a glance appear to take us in entirely distinct directions. Throughout, I will argue and present evidence to the contrary of this view.

The first indication is this statement characterizing his triadic logic: "Triadic Logic is that logic which, though not rejecting entirely the Principle of Excluded Middle, nevertheless recognizes that every proposition, S is P, is either true or false, *or else S has a lower mode of being such that it can neither be determinately P, nor determinately not P, but is at the limit between P and not P*" (My emphasis). Here Peirce is claiming that his reasons for deviating from classical logic are to accommodate propositions in which the subjects have a lower mode of being than the predicate. Because these subjects have a lower mode of being, they are at the limit between the predicate and its denial. These propositions receive the third truth value, L, which is to be interpreted as limit, or not *determinately* true nor false.

The second indication comes from a rather odd example on the same page: "Thus, a blot is made on a sheet. Then every point of the sheet is unblackened or blackened. But there are points on the boundary line; and these points are incapable of being unblackened or of being blackened, since these predicates refer to the area about S and a line has no area about any point of it." Reserving discussion of the oddities of this example for later, it will become clear in subsequent chapters that it is connected to Peirce's views on continuity, continua, and breaches of continuity.

So far there have been two attempts to explain the philosophical motivations behind Peirce's triadic logic. The first is due to Max Fisch and Atwell Turquette (1964). The second is due to Robert Lane (1999). Each of these accounts takes a different one of the just mentioned clues as a starting point and arrive at seemingly distinct conclusions. Fisch and Turquette link Peirce's comment about modes of being to his special theory of modality, called 'triadic modality.' Lane, on the other hand, denies that triadic modality factored into Peirce's motivations, and instead tries to locate the project entirely within his views on continuity. I will argue that these views are not actually incompatible and present a synthesis of these views that brings us much closer to Peirce's true motivations.

Before I give a brief explanation of Peirce's modes of being and continuity, it will be helpful to bring up a running theme in his thought. This is his notion of categories. Peirce had a concept of three categories that he used in his analysis of all philosophical ideas. He sometimes calls them categories of being, sometimes of ideas, of thoughts, and of nature. Depending on the subject matter, these categories will have different names, but in the most abstract sense they are always the same. In the abstract, the categories are firsts (things that are what they are without reference to anything else), seconds (things that exist only in reference or connection something else), and thirds (which exist in bringing together a second and a third). An example of the sort of thing that would be a first for Peirce is a quality, like a color. There is some sense in which colors exist without reference to anything else. We all have a concept of 'redness' that we can think of independently of red objects. So the color red is a first. Seconds are like particular objects, like a red blanket. A red blanket is what it is in reference to two things: being red

and being a blanket. Thirds do not lend themselves to simple examples as easily, and are more recognizable within the various contexts Peirce applies his categories. He sometimes calls them laws, sometimes reasons, and other times powers.

Having some idea of Peirce's categories, we can now examine his three modes of being. Each of the objects that falls into his three categories has an associated mode of being: "My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future" (CP 1.23).[1] So, firsts, being merely qualities, have the mode of being of a possibility. This is because they do not exist on their own except for in the possibility that they are instantiated by some object. Seconds, have the mode of being of actuality. These are ordinary objects that occur in the world around us. All the objects that we normally see and interact with are seconds. Thirds have the mode of being of necessity. Again, this notion is much more difficult to understand as precisely as the first two. The things that are thirds for Peirce can loosely be understood as laws of nature. They are basically general facts about seconds. He says "This mode of being which *consists*, mind my word if you please, the mode of

[1] Throught this document I will often refer to passages written by Peirce in published and unpublished collections. It will be convenient to use the typical abbreviated citations for these. Passages from *The Collected Papers of Charles S. Peirce* will be referred to in the text by CP v.p, where v is the volume and p is the paragraph number. Citations from *New Elements of Mathematics* will be abbreviated to NEM v:p, where p will be the page number. *The Essential Peirce* by EP x:y where x is the volume and y is the entry number. Unpublished manuscripts will be referred to as MS followed by the manuscript number. Most references to unpublished manuscripts will be to MS 339, Peirce's logic notebook. I refer to these pages by seq.xyz, according to the order in which they appear in the Harvard Mirador reproduction of this manuscript.

being which *consists* in the fact that future facts of Secondness will take on a determinate general character, I call a Thirdness" (CP 1.26). Part of the difficulty with understanding and stating precisely what thirds are is thinking of them as objects. We do not normally think of laws as objects. Nonetheless, suppose that every time a diamond is dragged across a pane of glass, from now into the indefinite future, a scratch is produced. Then the third in this case is the law or whatever determines that all diamonds scratch all panes of glass.

Having cleared up Peirce's categories and modes of being, it is already clear that modality was built in to these notions. When Peirce discusses triadic modality, as well as when Fisch and Turquette, Lane, and myself do so, it is the possibility, actuality, and necessity built into these modes of being that we are referring to. So when Peirce refers to "modes of being" in his logic notebook, there is clearly some sense in which modality is involved.

Having dealt with those preliminaries, we can now turn to the other piece of the puzzle: Peirce's views on continuity. Continuity was an obsession for Peirce throughout his entire life. In the last 25 years of his life especially, he repeatedly revised his definition of continuity and continua. He started out with a definition he attributes to Kant, which "confounds [continuity] with infinite divisibility" (CP 6.120). He refined this definition, possibly after studying Cantor whom he had immense respect for, after noticing that the rational numbers were infinitely divisible but not continuous. The development of his view will be detailed in later sections but it is clear that the nature of continuity always remained an open question to Peirce. The view he expresses closest to the time he wrote about triadic logic is this passage written in 1908:

> "A perfect continuum belongs to the genus, of a whole all whose parts without any exception whatsoever conform to one general law to which same law conform likewise all the parts of each single part. *Continuity* is thus a special kind of *generality*, or conformity to one Idea. More specifically, it is a *homogeneity*, or generality among all of a certain kind of parts of one whole. Still more specifically, the characters which are the same in all the parts are a certain kind of relationship of each part to all the coordinate parts; that is, it is a *regularity*. The step of specification which seems called for next, as appropriate to our purpose of defining, or logically analyzing the Idea of continuity, is that of asking ourselves what kind [of] relationship between parts it is that constitutes the regularity a continuity; and the first, and therefore doubtless the best answer for our purpose, not as the ultimate answer, but as the proximate one, is that it is the relation or relations of *contiguity*; for continuity is unbrokenness (whatever that may be,) and this seems to imply a *passage* from one part to a contiguous part. What is this 'passage'? This passage seems to be an act of turning the attention from one part to another part; in short an actual event in the mind. This seems decidedly unfortunate, since an event can only take place in Time, and Time is a continuum; so that the prospect is that we shall rise from our analysis with a definition of continuity in general in terms of a special continuity. However, it is possible that this objection will disappear as we proceed" (CP 7.535).

It should be clear from this passage that Peirce found himself on shaky ground here. He does attribute some properties to continua though. Every part of them has parts, and the regularity between these parts, and parts of parts, is that of having common borders (though it is by no means obvious what these borders are in this general context). He also claims continua are unbroken but admits that he is uncertain of what that means. Peirce's remarks about a 'passage' here are somewhat opaque. He probably means that, since a continuum is unbroken, it should be possible to pass from part to part seamlessly. He also refers to this passage as an "event in the mind" and relates this to time. When Peirce deals with continuity, he is not just thinking of the continuum of the reals. He meant his theory to apply to lots of things he thought were continuous, not only time and space, but thought as well. He never successfully worked this theory out to his satisfaction, so it is only natural that some of the details are a bit fuzzy at the edges.

One further detail to note is that throughout the manuscript where the last passage came from, Peirce frequently relates his explanation of continuity to three special universes, of "arbitrary possibilities, physical things, and minds" (MS 204). These universes come up again in the notebook pages where he works out his triadic logic and are explained in an earlier entry (which will be transcribed in chapter 1). They are obviously related to his three categories and modes of being. The nature of this relationship, be it identity or correspondence, is unclear.

So why was continuity so important for Peirce? This is where the somewhat mystical side of Peirce comes in (as he himself reluctantly alludes to in CP 6.102). The answer has to do with another doctrine that Peirce remained committed to until the end of his life: his synechism. Roughly put,

synechism is the doctrine that regards everything as continuous. Peirce contrasts it with materialism, idealism, and dualism: “Thus, materialism is the doctrine that matter is everything, idealism the doctrine that ideas are everything, dualism the philosophy which splits everything in two. In like manner, I have proposed to make synechism mean the tendency to regard everything as continuous" (EP 2.1). He goes on to say that “that continuity governs the whole domain of experience in every element of it" (Ibid). The comparison with these other doctrines seems to imply contrary to them that mind and matter is continuous in Peirce’s view.

Synechism is one of Peirce’s most ambitious ideas. He carried it so far that eventually he came up with a cosmological theory based on it that incorporates the three triads discussed above, his three categories, modes of being, and special universes. This will be detailed further in the final chapter but simply put, Peirce believed (or at least entertained as a hypothesis) that the universe begins in a vacuous state of mere possibility, a universe of firsts. From there it evolves to the existing universe, where some of the possibilities in the initial state have become determined. Peirce tells us that this evolutionary process is one where a world of Platonic forms is incrementally becoming determined:

> “From this point of view we must suppose that the existing universe, with all its arbitrary secondness, is an offshoot from, or an arbitrary determination of, a world of ideas, a Platonic world; not that our superior logic has enabled us to reach up to a world of forms to which the real universe, with its feebler logic, was inadequate... The evolutionary process is, therefore, not a

> mere evolution of the existing universe, but rather a process by which the very Platonic forms themselves have become or are becoming developed" (CP 6.192-194).

Here is where our two clues, triadic modality/modes of being and continuity meet. In chapter 4 I will argue that these were the ideas Peirce had hoped to capture with his triadic logic. He thought that classical logic was only able to represent the middle portion of this evolutionary process, the existent universe, and he wanted a logic general enough to capture the whole of it. The reason he needed L to do this has to do with the boundaries where indeterminate possibilities become determined. This will be explained in detail later.

In the next two chapters I will survey and critically review nearly[2] all of what has so far been written about Peirce's triadic logic. I will begin with what Peirce himself wrote in connection with this logic and then turn to more recent work on the topic by Atwell Turquette, Max Fisch, and Robert Lane.

The second chapter is dedicated to the pages from Peirce's logic notebook that contain his experiments with three-valued logic. The first three pages I discuss have been taken up by the commentators just mentioned. The first of these is usually thought to mostly be a failed experiment, although it did yield some useful one-place operators. The second more successfully defines a set of three-valued connectives, all versions of conjunctions and

[2]Some of Atwell Turquette's papers have been cited but not discussed. Many of these papers are dedicated to purely formal results that can be generated from the page I refer to as seq.640. While these results are not uninteresting, it is highly unlikely that Peirce would have been aware of them and so are unlikely to help advance our understanding of what triadic logic was intended for in the first place. However, I have included discussion of Turquette's findings which Peirce might feasibly have noticed or known about.

disjunctions. The final page contains Peirce's most explicit statement of what his triadic logic was intended for and seems to reference some of his comments on both triadic modality, as well as continuity. Out of these three pages, the first two are *versos* and only the third is a *recto*.

In the second half of chapter two, I discuss pages that are connected with triadic logic, but that the above commentators have left out of there discussion. The first of these contains examples of Peirce's existential graphs, his preferred representation of logic, but with an additional operator that does not appear to be used elsewhere. Peirce seems pretty quickly to have decided his graphical representation was not up to the job, as he reverted back to his symbolic notation on all subsequent pages. The second of these unmentioned pages makes reference to "special universes" as well as the "fundamental quadratic" of Boolean logic. The third of these seems to be mostly scratchwork with a concluding remark about functions on values being "known" and "generally known." All three of these pages are *rectos*, the first two of which are on the front side of the *versos* in the first half of the section.

In chapter three I discuss Fisch and Turquette's paper, "Peirce's Triadic Logic," and Lane's follow up, "Peirce's Triadic Logic Revisited." These are the most well known accounts that attempt to describe the philosophical issues that may have motivated Peirce to experiment with three-valued logic. Fisch and Turquette's discussion, as well as the historical evidence on the notebook pages, make a compelling case that triadic logic is connected with notions of modality and Peirce's tychism. After reviewing this, I give Lane's version of Peirce's motivations. Lane claims that Peirce was motivated not by modality, but by his views on continuity instead. While I agree with

Lane that continuity is an important piece of the puzzle, I argue against his claim that modality has nothing to do with it. Removing modality from our consideration entirely would involve ignoring an important part of the little historical evidence we have for why Peirce invented triadic logic. I conclude the section by providing evidence that demonstrates that the claim that Peirce was motivated by modality and the claim that he was motivated by continuity are in fact compatible. This view coincides well with much of Peirce's other discussion of continuity and continua.

In the final chapter, in light of my demonstration that we cannot properly understand the purpose of Peirce's triadic logic exclusively through modality or continuity alone, I give an account that synthesizes these considerations. In the first section, I explain why Peirce thought his subject matter required him to deviate from classical logic. In the second, I answer the questions as to why he needed his additional truth value, L. In the third, I give examples of the kinds of propositions that would be evaluated as L. I admit that this unification is by no means seamless. However, this is not necessarily a deficiency for this account but rather a natural result of its subject matter. Peirce's triadic logic was based on ideas that he himself never fully worked out to his own satisfaction. He continued writing about the ideas expressed here until he died in 1914 and he was oft to admit his own uncertainty.

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