KU Leuven

Mechanica 2: Dynamica

Case studie

Team A2 - 4

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1 Kinematica

1.1 Transformatiematrices

T_1 van x'y'z' (en dus ook van xij"z") naar xyz:

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

T_2 van x"y"z" naar xij"z":

$$T_2 = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

1.2 Vraag 1

Bereken de ogenblikkelijke totale rotatiesnelheidsvector $\vec{\omega}_w$ en rotatieversnellingsvector $\vec{\alpha}_w$ van het wiel.

$$\begin{aligned} \vec{\omega}_w &= \vec{\omega}_g + \vec{\omega}_i + \vec{\omega}_w \\ &= \omega_g * \vec{e'}_z + \omega_i * \vec{e''}_y + (-\omega) * \vec{e'''}_x \\ &= \omega_g * \vec{e'}_z + \omega_i * \vec{e'}_y + (-\omega) * (\cos(\beta) * \vec{e'}_x - \sin(\beta) * \vec{e'}_z) \\ &= \left\{ \begin{array}{c} -\omega_w * \cos(\beta) \\ -\omega_g * \sin(\alpha) + \omega_i * \cos(\alpha) - \omega_w * \sin(\alpha) * \sin(\beta) \\ \omega_g * \cos(\alpha) + \omega_i * \sin(\alpha) + \omega_w * \cos(\alpha) * \sin(\beta) \end{array} \right\} \end{aligned} \tag{1}$$

$$\begin{aligned} \vec{\alpha}_w &= \frac{d\vec{\omega}_g}{dt} + \frac{d\vec{\omega}_i}{dt} + \frac{d\vec{\omega}_w}{dt} \\ &= \alpha_g * \vec{e'}_z + \omega_g * \frac{d\vec{e'}_z}{dt} + \alpha_i * \vec{e''}_y + \omega_i * \frac{d\vec{e''}_y}{dt} + \alpha_w * \vec{e'''}_x + (-\omega_w) * \frac{d\vec{e'''}_x}{dt} \\ &= \left\{ \begin{array}{c} -\omega_g * \omega_i + \alpha_w * \cos(beta) + \omega_i * \omega_g * \sin(\beta) \\ -\alpha_g * \sin(\alpha) + \alpha_i * \cos(\alpha) + \alpha_w * \sin(\alpha) * \sin(\beta) - \omega_g * \omega_w \cos(\alpha) * \cos(\beta) - \omega_g * \omega_w \sin(\alpha) * \cos(\beta) \\ \alpha_g * \cos(\alpha) + \alpha_i * \sin(\alpha) - \alpha_w * \cos(\alpha) * \sin(\beta) - \omega_g * \omega_w \sin(\alpha) * \cos(\beta) + \omega_g * \omega_w \cos(\alpha) * \cos(\beta) \end{array} \right\} \end{aligned} \tag{2}$$