KU Leuven

Mechanica 2: Dynamica

Case studie

Team A2 - 4

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1 Kinematica

1.1 Transformatiematrices

T_1 van x'y'z' (en dus ook van x"y"z") naar xyz:

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

T_2 van x"'y"'z"' naar x"y"z":

$$T_2 = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

1.2 Vraag 1

Bereken de ogenblikkelijke totale rotatiesnelheidsvector $\vec{\omega}_w$ en rotatieversnellingsvector $\vec{\alpha}_w$ van het wiel.

$$\begin{aligned}
\vec{\omega}_w &= \vec{\omega}_g + \vec{\omega}_i + \vec{\omega}_w \\
&= \omega_g * \vec{e}_{z'} + \omega_i * \vec{e}_{y''} + (-\omega_w) * \vec{e}_{x'''} \\
&= \omega_g * \vec{e}_{z'} + \omega_i * \vec{e}_{y'} + (-\omega_w) * (\cos(\beta) * \vec{e'}_x - \sin(\beta) * \vec{e'}_z) \\
&= \left\{ \begin{array}{c} \Big(-\omega_w * \cos(\beta)\Big) * \vec{e}_{x'} \\ \Big(\omega_i\Big) * \vec{e}_{y'} \\ \Big(\omega_g - \omega_w * \sin(\beta)\Big) * \vec{e}_{z'} \end{array} \right\} \\
&= \left\{ \begin{array}{c} \Big(-\omega_w * \cos(\beta)\Big) * \vec{e}_x \\ \Big(-\omega_g * \sin(\alpha) + \omega_i * \cos(\alpha) - \omega_w * \sin(\alpha) * \sin(\beta)\Big) * \vec{e}_y \\ \Big(\omega_g * \cos(\alpha) + \omega_i * \sin(\alpha) + \omega_w * \cos(\alpha) * \sin(\beta)\Big) * \vec{e}_z \end{array} \right\}
\end{aligned} \tag{1}$$

$$\begin{aligned}
\vec{\alpha}_w &= \frac{d\vec{\omega}_g}{dt} + \frac{d\vec{\omega}_i}{dt} + \frac{d\vec{\omega}_w}{dt} \\
&= \alpha_g * \vec{e}_{z'} + \omega_g * \frac{d\vec{e}_{z'}}{dt} + \alpha_i * \vec{e}_{y''} + \omega_i * \frac{d\vec{e}_{y''}}{dt} + \alpha_w * \vec{e}_{x'''} + (-\omega_w) * \frac{d\vec{e}_{x'''}}{dt} \\
&= \left\{ \begin{array}{c} [-\omega_g * \omega_i + \alpha_w * \cos(\beta) + \omega_i * \omega_w * \sin(\beta)]\vec{e}_{x'} \\ [\alpha_i - \omega_g * \omega_w * \cos(\beta)]\vec{e}_{y'} \\ [\alpha_g - \alpha_w * \sin(\beta) + \omega_i * \omega_w * \cos(\beta)]\vec{e}_{z'} \end{array} \right\} \\
&= \left\{ \begin{array}{c} [-\omega_g * \omega_i + \alpha_w * \cos(\beta) + \omega_i * \omega_g * \sin(\beta)] * \vec{e}_x \\ \Big((-\alpha_g + \alpha_w * \sin(\beta) - \omega_i * \omega_w * \cos(\beta))\sin(\alpha) + (\alpha_i - \omega_g\omega_w\cos\beta)\cos\alpha\Big) * \vec{e}_y \\ \Big((\alpha_i - \omega_g\omega_w\cos\beta\sin\alpha + (\alpha_g - \alpha_w * \sin(\beta) + \omega_i\omega_w\cos\beta)\cos\alpha\Big) * \vec{e}_z \end{array} \right\}
\end{aligned} \tag{2}$$

met

$\vec{e'''}_x = cos(\beta) * \vec{e'}_x - sin(\beta) * \vec{e'}_z$

$\vec{e'}_x = \vec{e}_x$

$\vec{e''}_y = \vec{e'}_y$

$\vec{e'}_y = \cos(\alpha) * \vec{e}_y + \sin(\alpha) * \vec{e}_z$

$\vec{e'}_z = -\sin(\alpha) * \vec{e}_y + \cos(\alpha) * \vec{e}_z$

$\frac{d\vec{e}_{z'}}{dt} = \vec{0}$

$\omega_i * \frac{d\vec{e}_{y''}}{dt} = \vec{\omega}_g \times \vec{\omega}_i = -\omega_i * \omega_g * \vec{e}_{x'} = -\omega_i * \omega_g * \vec{e}_x$

$$-\omega_w * \frac{d\vec{e}_{x'''}}{dt} = (\vec{\omega_i} + \vec{\omega_g}) \times \vec{\omega_w} = \begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & \omega_i & \omega_g \\ -\omega_w * \cos(\beta) & 0 & \omega_w * \sin(\beta) \end{vmatrix} = \begin{Bmatrix} \omega_i * \omega_w * \sin(\beta) * \vec{e}_{x'} \\ \omega_g * \omega_w * \cos(\beta) * \vec{e}_{y'} \\ \omega_i * \omega_w * \cos(\beta) * \vec{e}_{z'} \end{Bmatrix}$$

1.3 Vraag 2

1.4 Vraag 3

1.5 Vraag 4

Bereken de ogenblikkelijke snelheid $\vec{v}_d$ en de ogenblikkelijke versnelling $\vec{a}_d$ van het punt D.

Positie van D tov B uitgedrukt in het x"y"z"- assenstel

$$\vec{r}_{d|b} \mapsto \begin{Bmatrix} \left(-\frac{1}{4} * l_4 \cos(\beta) - \frac{3}{4} * l_3 * \sin(\beta)\right) * \vec{e}_{x''} \\ 0 * \vec{e}_{y''} \\ \left(\frac{1}{4} * l_4 * \sin(\beta) - \frac{3}{4} * l_3 * \cos(\beta)\right) * \vec{e}_{z''} \end{Bmatrix} \tag{3}$$

We berekennen $\vec{v}_d$ met mebehulp van samengestelde beweging.

$$\vec{v}_d = \vec{v}_b + \vec{\omega}_g \times \vec{r}_{d|b} + \vec{v}_{rel} \tag{4}$$

$$\begin{aligned} \vec{v}_b &= \vec{v}_a + \vec{\omega} \times \vec{r}_{b|a} + \vec{v}_{rel} \\ &= v_v * \vec{e}_{y''} + \vec{0} + \vec{\omega}_g \times *(l) * \vec{e}_{x'} \\ &= (v_v + \omega_g * l_1) * \vec{e}_{y''} \end{aligned} \tag{5}$$

$$\begin{aligned} \vec{\omega}_g \times \vec{r}_{d|b} &= \begin{vmatrix} \vec{e}_{x''} & \vec{e}_{y''} & \vec{e}_{z''} \\ 0 & 0 & \omega_g \\ \frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta & 0 & \frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta \end{vmatrix} \\ &= \begin{Bmatrix} 0 * \vec{e}_{x'} \\ \left(\omega_g * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{6}$$

$$\begin{aligned} \vec{v}_{rel} &= \vec{\omega}_i \times \vec{r}_{d|b} \\ &= \begin{vmatrix} \vec{e}_{x''} & \vec{e}_{y''} & \vec{e}_{z''} \\ 0 & \omega_i & 0 \\ \frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta & 0 & \frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta \end{vmatrix} \\ &= \begin{Bmatrix} \left(\omega_i * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\right) * \vec{e}_{x''} \\ 0 * \vec{e}_{y''} \\ \left(-\omega_i * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) * \vec{e}_{z''} \end{Bmatrix} \end{aligned} \tag{7}$$

$$\vec{v}_d = \begin{Bmatrix} \left(\omega_i * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\right) * \vec{e}_{x''} \\ \left(v_v + \omega_g * l_1 + \omega_g * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) * \vec{e}_{y''} \\ \left(-\omega_i * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) * \vec{e}_{z''} \end{Bmatrix} \tag{8}$$

$$\vec{v}_d = \begin{Bmatrix} \left(\omega_i * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\right) * \vec{e}_x \\ \left(\left(v_v + \omega_g * l_1 + \omega_g * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) \cos\alpha + \sin\alpha * \omega_i * \left(\frac{-1}{4} l_4 * \cos\beta - \frac{3}{4} l_3 \sin\beta\right)\right) * \vec{e}_y \\ \left(\left(v_v + \omega_g * l_1 + \omega_g * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) \sin\alpha + \cos\alpha * \omega_i * \left(\frac{-1}{4} l_4 * \cos\beta - \frac{3}{4} l_3 \sin\beta\right)\right) * \vec{e}_z \end{Bmatrix} \tag{9}$$

$$\vec{a}_d = \vec{a}_b + \vec{\alpha} \times \vec{r}_{d|b} + \omega_g \times (\omega_g \times \vec{r}_{d|b}) + \vec{a}_{rel} + 2 * (\omega_g \times \vec{v}_r) \tag{10}$$

met

$$\begin{aligned}\vec{a}_b &= \vec{a}_a + \vec{\alpha}_g \times \vec{r}_{b|a} + \vec{\omega}_g \times (\vec{\omega}_g \times \vec{r}_{b|a}) + \vec{a}_{rel} \\ &= a_v * \vec{e}_{y'} + (\alpha_g * \vec{e}_{z'} + \omega_g * \frac{d\vec{e}_{z'}}{dt}) \times l_1 \vec{e}_{x'} + \omega_g * \vec{e}_{z'} \times (\omega_g * \vec{e}_{z'} \times l_1 \vec{e}_{x'}) \\ &= \left\{ \begin{matrix} -\omega_g^2 l_1 * \vec{e}_{x'} \\ (a_v + \alpha_g * l_1) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{matrix} \right\}\end{aligned} \tag{11}$$

$$\begin{aligned}\vec{\alpha}_g \times \vec{r}_{d|b} &= (\alpha_g * \vec{e}_{z'}) \times \left\{ \begin{matrix} \left(-\frac{1}{4} * l_4 \cos(\beta) - \frac{3}{4} * l_3 * \sin(\beta)\right) * \vec{e}_{x'} \\ 0 * \vec{e}_{y'} \\ \left(\frac{1}{4} * l_4 * \sin(\beta) - \frac{3}{4} * l_3 * \cos(\beta)\right) * \vec{e}_{z'} \end{matrix} \right\} \\ &= \left\{ \begin{matrix} 0 * \vec{e}_{x'} \\ \alpha * (-\frac{1}{4} * l_4 \cos(\beta) - \frac{3}{4} * l_3 * \sin(\beta)) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{matrix} \right\}\end{aligned} \tag{12}$$

$$\begin{aligned}\omega_g \times (\omega_g \times \vec{r}_{d|b}) &= \omega_g \vec{e}_{z'} \times (\omega_g * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)) \\ &= \left\{ \begin{matrix} \left(\omega_g^2 * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\right) * \vec{e}_{x'} \\ 0 * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{matrix} \right\}\end{aligned} \tag{13}$$

$$\begin{aligned}\vec{a}_{rel} &= \vec{\alpha}_i \times \vec{r}_{d|b} + \vec{\omega}_i \times (\vec{\omega}_i \times \vec{r}_{d|b}) \\ &= \begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ -\omega_g \omega_i & \alpha_i & 0 \\ \frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta & 0 & \frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta \end{vmatrix} + \\ &\begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & \omega_i & 0 \\ \omega_i * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta) & 0 & \frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta \end{vmatrix}\end{aligned} \tag{14}$$

2 Kinematica