KU Leuven

Mechanica 2: Dynamica

Case studie

Team A2 - 4

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1 Kinematica

1.1 Transformatiematrices

T_1 van x'y'z' (en dus ook van x"y"z") naar xyz:

$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$

T_2 van x"'y"'z"' naar x"y"z":

$T_2 = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$

1.2 Vraag 1

Bereken de ogenblikkelijke totale rotatiesnelheidsvector $\vec{\omega}_w$ en rotatieversnellingsvector $\vec{\alpha}_w$ van het wiel.

$$\begin{aligned}
\vec{\omega}_w &= \vec{\omega}_g + \vec{\omega}_i + \vec{\omega}_w \\
&= \omega_g * \vec{e}_{z'} + \omega_i * \vec{e}_{y''} + (-\omega_w) * \vec{e}_{x'''} \\
&= \omega_g * \vec{e}_{z'} + \omega_i * \vec{e}_{y'} + (-\omega_w) * (\cos\beta * \vec{e'}_x - \sin(\beta) * \vec{e'}_z) \\
&= \begin{Bmatrix} \Big(-\omega_w * \cos(\beta)\Big) * \vec{e}_{x'} \\ \Big(\omega_i\Big) * \vec{e}_{y'} \\ \Big(\omega_g - \omega_w * \sin(\beta)\Big) * \vec{e}_{z'} \end{Bmatrix} \\
&= \begin{Bmatrix} \Big(-\omega_w * \cos(\beta)\Big) * \vec{e}_x \\ \Big(-\omega_g * \sin(\alpha) + \omega_i * \cos(\alpha) - \omega_w * \sin(\alpha) * \sin(\beta)\Big) * \vec{e}_y \\ \Big(\omega_g * \cos(\alpha) + \omega_i * \sin(\alpha) + \omega_w * \cos(\alpha) * \sin(\beta)\Big) * \vec{e}_z \end{Bmatrix}
\end{aligned} \tag{1}$$

$$\begin{aligned}
\vec{\alpha}_w &= \frac{d\vec{\omega}_g}{dt} + \frac{d\vec{\omega}_i}{dt} + \frac{d\vec{\omega}_w}{dt} \\
&= \alpha_g * \vec{e}_{z'} + \omega_g \frac{d\vec{e}_{z'}}{dt} + \alpha_i \vec{e}_{y''} + \omega_i * \frac{d\vec{e}_{y''}}{dt} + \alpha_w * \vec{e}_{x'''} + (-\omega_w) \frac{d\vec{e}_{x'''}}{dt} \\
&= \begin{Bmatrix} \Big(-\omega_g\omega_i + \alpha_w \cos(\beta) + \omega_i\omega_w \sin\beta\Big) * \vec{e}_{x'} \\ \Big(\alpha_i - \omega_g\omega_w \cos(\beta)\Big) * \vec{e}_{y'} \\ \Big(\alpha_g - \alpha_w \sin\beta + \omega_i\omega_w \cos(\beta)\Big) * \vec{e}_{z'} \end{Bmatrix} \\
&= \begin{Bmatrix} \Big(-\omega_g\omega_i + \alpha_w \cos(\beta) + \omega_i\omega_g \sin(\beta)\Big) * \vec{e}_x \\ \Big((-\alpha_g + \alpha_w \sin(\beta) - \omega_i\omega_w \cos(\beta)) \sin(\alpha) + (\alpha_i - \omega_g\omega_w \cos\beta) \cos\alpha\Big) * \vec{e}_y \\ \Big((\alpha_i - \omega_g\omega_w \cos\beta \sin\alpha + (\alpha_g - \alpha_w * \sin(\beta) + \omega_i\omega_w \cos\beta) \cos\alpha\Big) * \vec{e}_z \end{Bmatrix}
\end{aligned} \tag{2}$$

met

$\vec{e'''}_x = cos(\beta) * \vec{e'}_x - sin(\beta) * \vec{e'}_z$

$\vec{e'}_x = \vec{e}_x$

$\vec{e''}_y = \vec{e'}_y$

$\vec{e'}_y = \cos(\alpha) * \vec{e}_y + \sin(\alpha) * \vec{e}_z$

$\vec{e'}_z = -\sin\alpha * \vec{e}_y + \cos\alpha * \vec{e}_z$

$\frac{d\vec{e}_{z'}}{dt} = \vec{0}$

$\omega_i * \frac{d\vec{e}_{y''}}{dt} = \vec{\omega_g} \times \vec{\omega_i} = -\omega_i * \omega_g * \vec{e}_{x'} = -\omega_i * \omega_g * \vec{e}_x$

$-\omega_w * \frac{d\vec{e}_{x'''}}{dt} = (\vec{\omega_i} + \vec{\omega_g}) \times \vec{\omega_w} = \begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & \omega_i & \omega_g \\ -\omega_w \cos\beta & 0 & \omega_w \sin\beta \end{vmatrix} = \begin{Bmatrix} \omega_i\omega_w \sin(\beta) * \vec{e}_{x'} \\ \omega_g\omega_w \cos(\beta) * \vec{e}_{y'} \\ \omega_i\omega_w \cos(\beta) * \vec{e}_{z'} \end{Bmatrix}$

1.3 Vraag 2

1.4 Vraag 3

1.5 Vraag 4

Bereken de ogenblikkelijke snelheid $\vec{v}_d$ en de ogenblikkelijke versnelling $\vec{a}_d$ van het punt D.

Positie van D tov B uitgedrukt in het x"y"z"- assenstel

$$\vec{r}_{d|b} \mapsto \begin{Bmatrix} \Big(-\frac{1}{4}l_4\cos(\beta) - \frac{3}{4}l_3\sin(\beta)\Big) * \vec{e}_{x''} \\ 0 * \vec{e}_{y''} \\ \Big(\frac{1}{4}l_4\sin(\beta) - \frac{3}{4}l_3\cos(\beta)\Big) * \vec{e}_{z''} \end{Bmatrix} \tag{3}$$

We berekennen $\vec{v}_d$ met mebehulp van samengestelde beweging.

$$\vec{v}_d = \vec{v}_b + \vec{\omega}_g \times \vec{r}_{d|b} + \vec{v}_{rel} \tag{4}$$

$$\begin{aligned} \vec{v}_b &= \vec{v}_a + \vec{\omega} \times \vec{r}_{b|a} + \vec{v}_{rel} \\ &= v_v\vec{e}_{y''} + \vec{0} + \vec{\omega}_g \times *(l) * \vec{e}_{x'} \\ &= (v_v + \omega_g * l_1) * \vec{e}_{y''} \end{aligned} \tag{5}$$

$$\begin{aligned} \vec{\omega}_g \times \vec{r}_{d|b} &= \begin{vmatrix} \vec{e}_{x''} & \vec{e}_{y''} & \vec{e}_{z''} \\ 0 & 0 & \omega_g \\ \frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta & 0 & \frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta \end{vmatrix} \\ &= \begin{Bmatrix} 0 * \vec{e}_{x'} \\ \Big(\omega_g(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)\Big) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{6}$$

$$\begin{aligned} \vec{v}_{rel} &= \vec{\omega}_i \times \vec{r}_{d|b} \\ &= \begin{vmatrix} \vec{e}_{x''} & \vec{e}_{y''} & \vec{e}_{z''} \\ 0 & \omega_i & 0 \\ \frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta & 0 & \frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta \end{vmatrix} \\ &= \begin{Bmatrix} \Big(\omega_i * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\Big) * \vec{e}_{x''} \\ 0 * \vec{e}_{y''} \\ \Big(-\omega_i * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta)\Big) * \vec{e}_{z''} \end{Bmatrix} \end{aligned} \tag{7}$$

$$\vec{v}_d = \begin{Bmatrix} \Big(\omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta)\Big) * \vec{e}_{x''} \\ \Big(v_v + \omega_g l_1 + \omega_g(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)\Big) * \vec{e}_{y''} \\ \Big(-\omega_i(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)\Big) * \vec{e}_{z''} \end{Bmatrix} \tag{8}$$

$$\vec{v}_d = \begin{Bmatrix} \left(\omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta)\right) * \vec{e}_x \\ \left(\left(v_v + \omega_g l_1 + \omega_g(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)\right)\cos\alpha + \sin\alpha\omega_i\left(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta\right)\right) * \vec{e}_y \\ \left(\left(v_v + \omega_g l_1 + \omega_g(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)\right)\sin\alpha + \cos\alpha\omega_i\left(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta\right)\right) * \vec{e}_z \end{Bmatrix} \tag{9}$$

$$\vec{a}_d = \vec{a}_b + \vec{\alpha} \times \vec{r}_{d|b} + \omega_g \times (\omega_g \times \vec{r}_{d|b}) + \vec{a}_{rel} + 2 * (\omega_g \times \vec{v}_r) \tag{10}$$

met

$$\begin{aligned} \vec{a}_b &= \vec{a}_a + \vec{\alpha}_g \times \vec{r}_{b|a} + \vec{\omega}_g \times (\vec{\omega}_g \times \vec{r}_{b|a}) + \vec{a}_{rel} \\ &= a_v * \vec{e}_{y'} + (\alpha_g * \vec{e}_{z'} + \omega_g * \frac{d\vec{e}_{z'}}{dt}) \times l_1\vec{e}_{x'} + \omega_g * \vec{e}_{z'} \times (\omega_g * \vec{e}_{z'} \times l_1\vec{e}_{x'}) \\ &= \begin{Bmatrix} -\omega_g^2 l_1 * \vec{e}_{x'} \\ (a_v + \alpha_g * l_1) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{11}$$

$$\begin{aligned} \vec{\alpha}_g \times \vec{r}_{d|b} &= (\alpha_g * \vec{e}_{z'}) \times \begin{Bmatrix} \left(-\frac{1}{4}l_4\cos(\beta) - \frac{3}{4}l_3\sin(\beta)\right) * \vec{e}_{x'} \\ 0 * \vec{e}_{y'} \\ \left(\frac{1}{4}l_4\sin(\beta) - \frac{3}{4}l_3\cos(\beta)\right) * \vec{e}_{z'} \end{Bmatrix} \\ &= \begin{Bmatrix} 0 * \vec{e}_{x'} \\ \alpha_g(-\frac{1}{4}l_4\cos(\beta) - \frac{3}{4}l_3\sin(\beta)) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{12}$$

$$\begin{aligned} \omega_g \times (\omega_g \times \vec{r}_{d|b}) &= \omega_g\vec{e}_{z'} \times (\omega_g(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)) \\ &= \begin{Bmatrix} \left(\omega_g^2(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta)\right) * \vec{e}_{x'} \\ 0 * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{13}$$

$$\begin{aligned} \vec{a}_{rel} &= \vec{\alpha}_i \times \vec{r}_{d|b} + \vec{\omega}_i \times (\vec{\omega}_i \times \vec{r}_{d|b}) \\ &= \begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ -\omega_g\omega_i & \alpha_i & 0 \\ \frac{-1}{4}l_4\cos\beta - \frac{3}{4} * l_3 * \sin\beta & 0 & \frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3 * \cos\beta \end{vmatrix} + \\ &\begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & \omega_i & 0 \\ \omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta) & 0 & \omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta) \end{vmatrix} \\ &= \begin{Bmatrix} \left(\alpha_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta) + \omega_i^2(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta)\right) * \vec{e}_{x'} \\ \left(\omega_g\omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta)\right) * \vec{e}_{y'} \\ \left(\alpha_i(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta) - \omega_i^2(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta)\right) * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{14}$$

En tot slot de corrioliskracht:

$$\begin{aligned} 2(\vec{\omega}_g \times \vec{v}_{rel}) &= 2 * \begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & 0 & \omega_g \\ \omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta) & 0 & -\omega_i(\frac{-1}{4}l_4\cos\beta - \frac{3}{4}l_3\sin\beta) \end{vmatrix} \\ &= \begin{Bmatrix} 0 * \vec{e}_{x'} \\ \left(2\omega_g\omega_i(\frac{1}{4}l_4\sin\beta - \frac{3}{4}l_3\cos\beta)\right) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{Bmatrix} \end{aligned} \tag{15}$$

$$= \begin{Bmatrix} \Big(-\omega_g^2 l_1 + \omega_g^2 * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta) + \alpha_i(\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta) \\ +\omega_i^2 * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\Big) * \vec{e}_{x'} \\ \Big(a_v + \alpha_g l_1 + \alpha_g * (-\frac{1}{4} l_4 \cos(\beta) - \frac{3}{4} l_3 * \sin(\beta)) + 3\omega_g\omega_i(\frac{1}{4} l_4 \sin\beta - \frac{3}{4} l_3 \cos\beta)\Big) * \vec{e}_{y'} \\ \Big(\alpha_i * (\frac{1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta) - \omega_i^2(\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\Big) * \vec{e}_{z'} \end{Bmatrix}$$

$$= \begin{Bmatrix} \Big(-\omega_g^2 l_1 + \omega_g^2 * (\frac{-1}{4} * l_4 * \cos\beta - \frac{3}{4} * l_3 * \sin\beta) + \alpha_i(\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta) \\ +\omega_i^2 * (\frac{1}{4} * l_4 * \sin\beta - \frac{3}{4} * l_3 * \cos\beta)\Big) * \vec{e}_x \\ ((\Big(a_v + \alpha_g l_1 + \alpha_g * (-\frac{1}{4} l_4 \cos(\beta) - \frac{3}{4} l_3 * \sin(\beta)) + 3\omega_g\omega_i(\frac{1}{4} l_4 \sin\beta - \frac{3}{4} l_3 \cos\beta)\Big) \cos\alpha \\ +\Big(\alpha_i * (\frac{1}{4} l_4 \cos\beta - \frac{3}{4} l_3 \sin\beta) - \omega_i^2(\frac{1}{4} l_4 \sin\beta - \frac{3}{4} l_3 \cos\beta)\Big) * (-\sin\alpha) * \vec{e}_y \\ ((\Big(a_v + \alpha_g l_1 + \alpha_g * (-\frac{1}{4} l_4 \cos(\beta) - \frac{3}{4} l_3 \sin(\beta)) + 3\omega_g\omega_i(\frac{1}{4} l_4 \sin\beta - \frac{3}{4} l_3 \cos\beta)\Big) \sin\alpha \\ +\Big(\alpha_i(\frac{1}{4} l_4 \cos\beta - \frac{3}{4} l_3 \sin\beta) - \omega_i^2(\frac{1}{4} l_4 \sin\beta - \frac{3}{4} l_3 \cos\beta)\Big)(\cos\alpha) * \vec{e}_z \end{Bmatrix} \tag{16}$$

2 Kinematica

2.1 Vraag 1

Bereken de ogenblikkelijke impulsvector en de verandering van de impulsvector van het landingsgestel en die van het wiel.

$$\vec{p}_d = m * \vec{v}_d$$

$$= m * \begin{Bmatrix} \Big(\omega_i(\frac{1}{4} l_4 \sin\beta - \frac{3}{4} l_3 \cos\beta)\Big) * \vec{e}_x \\ \Big(\big(v_v + \omega_g l_1 + \omega_g(\frac{-1}{4} l_4 \cos\beta - \frac{3}{4} l_3 \sin\beta)\big) \cos\alpha + \sin\alpha\omega_i\big(\frac{-1}{4} l_4 \cos\beta - \frac{3}{4} l_3 \sin\beta\big)\Big) * \vec{e}_y \\ \Big(\big(v_v + \omega_g l_1 + \omega_g(\frac{-1}{4} l_4 \cos\beta - \frac{3}{4} l_3 \sin\beta)\big) \sin\alpha + \cos\alpha\omega_i\big(\frac{-1}{4} l_4 \cos\beta - \frac{3}{4} l_3 \sin\beta\big)\Big) * \vec{e}_z \end{Bmatrix} \tag{17}$$

2.2 Vraag 2

Bereken de ogenblikkelijke impulsmomentvector en de verandering van de impulsmometvector van het landingsgestel en die van het wiel rond hun respectievelijke massacentra.

$$\vec{L}_0 = I(t)\vec{\omega} \tag{18}$$

$$\vec{\omega} = \vec{\omega}_w + \vec{\omega}_g + \vec{\omega}_i$$
$$= \begin{Bmatrix} (-\vec{\omega}_w - \vec{\omega}_g sin(\beta))\vec{e}_{x''''} \\ \vec{\omega}_i * \vec{e}_{y''''} \\ \vec{\omega}_g cos\beta(\beta)\vec{e}_{z''''} \end{Bmatrix} \tag{19}$$
$$= \begin{Bmatrix} I_{\omega,x'''',x''''}(-\vec{\omega}_w - \vec{\omega}_g sin(\beta))\vec{e}_{x''''} \\ I_{\omega,y'''',y''''}\vec{\omega}_i * \vec{e}_{y''''} \\ I_{\omega,z'''',z''''}\vec{\omega}_g cos\beta(\beta)\vec{e}_{z''''} \end{Bmatrix}$$

$$\left(\frac{d\vec{L}_c}{dt}\right) = \left(\frac{d\vec{L}_C}{dt}\right)_{rel} + \vec{\Omega} \times \vec{L}_c \tag{20}$$

$$\vec{\Omega} \times \vec{L}_c = \begin{vmatrix} \vec{e}_{x''''} & \vec{e}_{y''''} & \vec{e}_{z''''} \\ -\omega_w - \omega_g \sin\beta & \omega_i & \omega_g \cos\beta \\ L_{o,x''''} & L_{o,y''''} & L_{o,z''''} \end{vmatrix}$$
$$= \begin{Bmatrix} \omega_i L_{o,z''''} - \omega_g \cos\beta L_{o,y''''} \\ \omega_g \cos\beta L_{o,x''''} + (\omega_w + \omega_g)L_{o,z''''} \\ -(\omega_w + \omega_g \sin(\beta))L_{o,y''''} + \omega_i L_{o,x''''} \end{Bmatrix} \tag{21}$$

$$\left(\frac{\vec{L}_c}{dt}\right)_{rel} = I\frac{d\vec{\omega}}{dt} = I\begin{Bmatrix} \Big(\alpha_w - \alpha_g \sin\beta + (-\omega_i\omega_g)\cos\beta\Big)\vec{e}_{x''''} \\ \Big(\alpha_i + \omega_g\omega_w\cos\beta\Big)\vec{e}_{y''''} \\ \Big(\alpha_g\cos\beta - \omega_i\omega_g\sin\beta - \omega_i\omega_w\Big)\vec{e}_{z''''} \end{Bmatrix} \tag{22}$$

want

$$\begin{aligned} \frac{d\vec{\omega}}{dt} &= \alpha_g\vec{e}_{z'} + \alpha_i\vec{e}_{y''} + (-\omega_i\omega_g)\vec{e}_{x''} + \alpha_w\vec{e}_{x'''} + \begin{Bmatrix} -\sin\beta\vec{e}_{x''''}\vec{e}_{x''''} \\ \cos\beta\omega_g\omega_w\vec{e}_{y''''} \\ -\cos\beta\omega_i\omega_w\vec{e}_{z''''} \end{Bmatrix} \\ &= \begin{Bmatrix} \alpha_w - \alpha_g + (-\omega_i\omega_g)\cos\beta\vec{e}_{x''''} \\ \alpha_i + \cos\beta\omega_g\omega_w\vec{e}_{y''''} \\ \alpha_g\cos\beta + (-\omega_i\omega_g)\sin\beta - (\omega_i\omega_g)\vec{e}_{z''''} \end{Bmatrix} \end{aligned} \tag{23}$$

$$\left(\frac{d\vec{L}_c}{dt}\right) = \begin{Bmatrix} \omega_i\omega_g\cos\beta(I_{z'''',z''''} - I_{y'''',y''''}) \\ \omega_g\cos\beta(\omega_w + \omega_g\sin\beta)I_{z'''',z''''} - I_{x'''',x''''} \\ (\omega_i + \omega_g\sin\beta)\omega_i I_{x'''',x''''} - I_{y'''',y''''} \end{Bmatrix} + \begin{Bmatrix} I_{x'''',x''''}(\alpha_w - \alpha_g\sin\beta + (-\omega_i\omega_g))\cos\beta \\ I_{y'''',y''''}(\alpha_i + \omega_g\omega_w\cos\beta) \\ I_{z'''',z''''}(\alpha_g\cos\beta - \omega_i\omega_g\sin\beta - \omega_i\omega_w) \end{Bmatrix} \tag{24}$$