

KU LEUVEN

MECHANICA 2: DYNAMICA

CASE STUDIE

Team A2 - 4

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1 Kinematica

1.1 Transformatiematrices

T_1 van $x'y'z'$ (en dus ook van $x''y''z''$) naar xyz :

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

T_2 van $x''y''z''$ naar $x''y''z''$:

$$T_2 = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

1.2 Vraag 1

Bereken de ogenblikkelijke totale rotatiesnelheidsvector $\vec{\omega}_w$ en rotatieversnellingsvector $\vec{\alpha}_w$ van het wiel.

$$\begin{aligned} \vec{\omega}_w &= \vec{\omega}_g + \vec{\omega}_i + \vec{\omega}_w \\ &= \omega_g * \vec{e}_{z'} + \omega_i * \vec{e}_{y''} + (-\omega_w) * \vec{e}_{x'''} \\ &= \omega_g * \vec{e}_{z'} + \omega_i * \vec{e}_{y'} + (-\omega_w) * (\cos(\beta) * \vec{e}_x - \sin(\beta) * \vec{e}_z) \\ &= \begin{Bmatrix} (-\omega_w * \cos(\beta)) * \vec{e}_{x'} \\ (\omega_i) * \vec{e}_{y'} \\ (\omega_g - \omega_w * \sin(\beta)) * \vec{e}_{z'} \end{Bmatrix} \\ &= \begin{Bmatrix} (-\omega_w * \cos(\beta)) * \vec{e}_x \\ (-\omega_g * \sin(\alpha) + \omega_i * \cos(\alpha) - \omega_w * \sin(\alpha) * \sin(\beta)) * \vec{e}_y \\ (\omega_g * \cos(\alpha) + \omega_i * \sin(\alpha) + \omega_w * \cos(\alpha) * \sin(\beta)) * \vec{e}_z \end{Bmatrix} \end{aligned} \quad (1)$$

$$\begin{aligned} \vec{\alpha}_w &= \frac{d\vec{\omega}_g}{dt} + \frac{d\vec{\omega}_i}{dt} + \frac{d\vec{\omega}_w}{dt} \\ &= \alpha_g * \vec{e}_{z'} + \omega_g \frac{d\vec{e}_{z'}}{dt} + \alpha_i \vec{e}_{y''} + \omega_i * \frac{d\vec{e}_{y''}}{dt} + \alpha_w * \vec{e}_{x'''} + (-\omega_w) \frac{d\vec{e}_{x'''}}{dt} \\ &= \begin{Bmatrix} (-\omega_g \omega_i + \alpha_w \cos(\beta) + \omega_i \omega_w \sin(\beta)) * \vec{e}_{x'} \\ (\alpha_i - \omega_g \omega_w \cos(\beta)) * \vec{e}_{y'} \\ (\alpha_g - \alpha_w \sin(\beta) + \omega_i \omega_w \cos(\beta)) * \vec{e}_{z'} \end{Bmatrix} \\ &= \begin{Bmatrix} (-\omega_g \omega_i + \alpha_w \cos(\beta) + \omega_i \omega_w \sin(\beta)) * \vec{e}_x \\ ((-\alpha_g + \alpha_w \sin(\beta) - \omega_i \omega_w \cos(\beta)) \sin(\alpha) + (\alpha_i - \omega_g \omega_w \cos(\beta)) \cos(\alpha)) * \vec{e}_y \\ ((\alpha_i - \omega_g \omega_w \cos(\beta) \sin(\alpha) + (\alpha_g - \alpha_w * \sin(\beta) + \omega_i \omega_w \cos(\beta)) \cos(\alpha)) * \vec{e}_z \end{Bmatrix} \end{aligned} \quad (2)$$

met

$$\begin{aligned} \vec{e}_{x'''} &= \cos(\beta) * \vec{e}_x - \sin(\beta) * \vec{e}_z \\ \vec{e}_{x'} &= \vec{e}_x \\ \vec{e}_{y'} &= \vec{e}_y \\ \vec{e}_{y''} &= \cos(\alpha) * \vec{e}_y + \sin(\alpha) * \vec{e}_z \\ \vec{e}_{z''} &= -\sin(\alpha) * \vec{e}_y + \cos(\alpha) * \vec{e}_z \\ \frac{d\vec{e}_{z'}}{dt} &= \vec{0} \end{aligned}$$

$$\begin{aligned}\omega_i * \frac{d\vec{e}_{y''}}{dt} &= \vec{\omega}_g \times \vec{\omega}_i = -\omega_i * \omega_g * \vec{e}_{x'} = -\omega_i * \omega_g * \vec{e}_x \\ -\omega_w * \frac{d\vec{e}_{x''}}{dt} &= (\vec{\omega}_i + \vec{\omega}_g) \times \vec{\omega}_w = \begin{vmatrix} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & \omega_i & \omega_g \\ -\omega_w * \cos(\beta) & 0 & \omega_w * \sin(\beta) \end{vmatrix} = \begin{cases} \omega_i * \omega_w * \sin(\beta) * \vec{e}_{x'} \\ \omega_g * \omega_w * \cos(\beta) * \vec{e}_{y'} \\ \omega_i * \omega_w * \cos(\beta) * \vec{e}_{z'} \end{cases}\end{aligned}$$

1.3 Vraag 2

1.4 Vraag 3

1.5 Vraag 4

Bereken de ogenblikkelijke snelheid \vec{v}_d en de ogenblikkelijke versnelling \vec{a}_d van het punt D.

Positie van D tov B uitgedrukt in het x''y''z''-assenstel

$$\vec{r}_{d|b} \mapsto \begin{cases} \left(-\frac{1}{4} * l_4 \cos(\beta) - \frac{3}{4} * l_3 * \sin(\beta) \right) * \vec{e}_{x''} \\ 0 * \vec{e}_{y''} \\ \left(\frac{1}{4} * l_4 * \sin(\beta) - \frac{3}{4} * l_3 * \cos(\beta) \right) * \vec{e}_{z''} \end{cases} \quad (3)$$

We berekenen \vec{v}_d met mebehulp van samengestelde beweging.

$$\vec{v}_d = \vec{v}_b + \vec{\omega}_g \times \vec{r}_{d|b} + \vec{v}_{rel} \quad (4)$$

$$\begin{aligned}\vec{v}_b &= \vec{v}_a + \vec{\omega} \times \vec{r}_{b|a} + \vec{v}_{rel} \\ &= v_v * \vec{e}_{y''} + \vec{0} + \vec{\omega}_g \times (l) * \vec{e}_{x'} \\ &= (v_v + \omega_g * l_1) * \vec{e}_{y''}\end{aligned} \quad (5)$$

$$\begin{aligned}\vec{\omega}_g \times \vec{r}_{d|b} &= \begin{vmatrix} \vec{e}_{x''} & \vec{e}_{y''} & \vec{e}_{z''} \\ 0 & 0 & \omega_g \\ \frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta & 0 & \frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \end{vmatrix} \\ &= \begin{cases} 0 * \vec{e}_{x'} \\ \left(\omega_g * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) \right) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{cases}\end{aligned} \quad (6)$$

$$\begin{aligned}\vec{v}_{rel} &= \vec{\omega}_i \times \vec{r}_{d|b} \\ &= \begin{vmatrix} \vec{e}_{x''} & \vec{e}_{y''} & \vec{e}_{z''} \\ 0 & \omega_i & 0 \\ \frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta & 0 & \frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \end{vmatrix} \\ &= \begin{cases} \left(\omega_i * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{x''} \\ 0 * \vec{e}_{y''} \\ \left(-\omega_i * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) \right) * \vec{e}_{z''} \end{cases}\end{aligned} \quad (7)$$

$$\vec{v}_d = \begin{cases} \left(\omega_i * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{x''} \\ \left(v_v + \omega_g * l_1 + \omega_g * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) \right) * \vec{e}_{y''} \\ \left(-\omega_i * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) \right) * \vec{e}_{z''} \end{cases} \quad (8)$$

$$\vec{v}_d = \left\{ \begin{array}{c} \left(\omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) * \vec{e}_x \\ \left((v_v + \omega_g l_1 + \omega_g \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right)) \cos \alpha + \sin \alpha \omega_i \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \right) * \vec{e}_y \\ \left((v_v + \omega_g l_1 + \omega_g \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right)) \sin \alpha + \cos \alpha \omega_i \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \right) * \vec{e}_z \end{array} \right\} \quad (9)$$

$$\vec{a}_d = \vec{a}_b + \vec{\alpha} \times \vec{r}_{d|b} + \omega_g \times (\omega_g \times \vec{r}_{d|b}) + \vec{a}_{rel} + 2 * (\omega_g \times \vec{v}_r) \quad (10)$$

met

$$\begin{aligned} \vec{a}_b &= \vec{a}_a + \vec{\alpha}_g \times \vec{r}_{b|a} + \vec{\omega}_g \times (\vec{\omega}_g \times \vec{r}_{b|a}) + \vec{a}_{rel} \\ &= a_v * \vec{e}_{y'} + (\alpha_g * \vec{e}_{z'} + \omega_g * \frac{d\vec{e}_{z'}}{dt}) \times l_1 \vec{e}_{x'} + \omega_g * \vec{e}_{z'} \times (\omega_g * \vec{e}_{z'} \times l_1 \vec{e}_{x'}) \end{aligned} \quad (11)$$

$$\begin{aligned} &= \left\{ \begin{array}{c} -\omega_g^2 l_1 * \vec{e}_{x'} \\ (a_v + \alpha_g * l_1) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{array} \right\} \\ \vec{\alpha}_g \times \vec{r}_{d|b} &= (\alpha_g * \vec{e}_{z'}) \times \left\{ \begin{array}{c} \left(-\frac{1}{4} * l_4 \cos(\beta) - \frac{3}{4} * l_3 * \sin(\beta) \right) * \vec{e}_{x'} \\ 0 * \vec{e}_{y'} \\ \left(\frac{1}{4} * l_4 * \sin(\beta) - \frac{3}{4} * l_3 * \cos(\beta) \right) * \vec{e}_{z'} \end{array} \right\} \end{aligned} \quad (12)$$

$$\begin{aligned} &= \left\{ \begin{array}{c} 0 * \vec{e}_{x'} \\ \alpha_g * \left(-\frac{1}{4} * l_4 \cos(\beta) - \frac{3}{4} * l_3 * \sin(\beta) \right) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{array} \right\} \\ \omega_g \times (\omega_g \times \vec{r}_{d|b}) &= \omega_g \vec{e}_{z'} \times \left(\omega_g * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) \right) \\ &= \left\{ \begin{array}{c} \left(\omega_g^2 * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) \right) * \vec{e}_{x'} \\ 0 * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{array} \right\} \end{aligned} \quad (13)$$

$$\begin{aligned} \vec{a}_{rel} &= \vec{\alpha}_i \times \vec{r}_{d|b} + \vec{\omega}_i \times (\vec{\omega}_i \times \vec{r}_{d|b}) \\ &= \left| \begin{array}{ccc} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ -\omega_g \omega_i & \alpha_i & 0 \\ \frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta & 0 & \frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \end{array} \right| + \\ &\quad \left| \begin{array}{ccc} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & \omega_i & 0 \\ \omega_i * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) & 0 & \omega_i * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \end{array} \right| \\ &= \left\{ \begin{array}{c} \left(\alpha_i \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) + \omega_i^2 * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{x'} \\ \left(\omega_g \omega_i \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{y'} \\ \left(\alpha_i * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) - \omega_i^2 \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{z'} \end{array} \right\} \end{aligned} \quad (14)$$

En tot slot de corioliskracht:

$$\begin{aligned} 2(\vec{\omega}_g \times \vec{v}_{rel}) &= 2 * \left| \begin{array}{ccc} \vec{e}_{x'} & \vec{e}_{y'} & \vec{e}_{z'} \\ 0 & 0 & \omega_g \\ \omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) & 0 & -\omega_i \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \end{array} \right| \\ &= \left\{ \begin{array}{c} 0 * \vec{e}_{x'} \\ \left(2\omega_g \omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) * \vec{e}_{y'} \\ 0 * \vec{e}_{z'} \end{array} \right\} \end{aligned} \quad (15)$$

$$\begin{aligned}
&= \left\{ \begin{aligned} &\left(-\omega_g^2 l_1 + \omega_g^2 * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) + \alpha_i \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right. \\ &\quad \left. + \omega_i^2 * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{x'} \\ &\left(a_v + \alpha_g l_1 + \alpha_g * \left(-\frac{1}{4} l_4 \cos(\beta) - \frac{3}{4} l_3 \sin(\beta) \right) + 3\omega_g \omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) * \vec{e}_{y'} \\ &\left(\alpha_i * \left(\frac{1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) - \omega_i^2 \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_{z'} \end{aligned} \right\} \\
&= \left\{ \begin{aligned} &\left(-\omega_g^2 l_1 + \omega_g^2 * \left(\frac{-1}{4} * l_4 * \cos \beta - \frac{3}{4} * l_3 * \sin \beta \right) + \alpha_i \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right. \\ &\quad \left. + \omega_i^2 * \left(\frac{1}{4} * l_4 * \sin \beta - \frac{3}{4} * l_3 * \cos \beta \right) \right) * \vec{e}_x \\ &\left(\left(a_v + \alpha_g l_1 + \alpha_g * \left(-\frac{1}{4} l_4 \cos(\beta) - \frac{3}{4} l_3 \sin(\beta) \right) + 3\omega_g \omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) \cos \alpha \right. \\ &\quad \left. + \left(\alpha_i * \left(\frac{1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) - \omega_i^2 \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) * (-\sin \alpha) * \vec{e}_y \right. \\ &\left(\left(a_v + \alpha_g l_1 + \alpha_g * \left(-\frac{1}{4} l_4 \cos(\beta) - \frac{3}{4} l_3 \sin(\beta) \right) + 3\omega_g \omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) \sin \alpha \right. \\ &\quad \left. + \left(\alpha_i \left(\frac{1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) - \omega_i^2 \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) (\cos \alpha) * \vec{e}_z \right. \end{aligned} \right\} \quad (16)
\end{aligned}$$

2 Kinematica

2.1 Vraag 1

Bereken de ogenblikkelijke impulsvector en de verandering van de impulsvector van het landingsgestel en die van het wiel.

$$\begin{aligned}
\vec{p}_d &= m * \vec{v}_d \\
&= m * \left\{ \begin{aligned} &\left(\omega_i \left(\frac{1}{4} l_4 \sin \beta - \frac{3}{4} l_3 \cos \beta \right) \right) * \vec{e}_x \\ &\left(\left(v_v + \omega_g l_1 + \omega_g \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \right) \cos \alpha + \sin \alpha \omega_i \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \right) * \vec{e}_y \\ &\left(\left(v_v + \omega_g l_1 + \omega_g \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \right) \sin \alpha + \cos \alpha \omega_i \left(\frac{-1}{4} l_4 \cos \beta - \frac{3}{4} l_3 \sin \beta \right) \right) * \vec{e}_z \end{aligned} \right\} \quad (17)
\end{aligned}$$

2.2 Vraag 2

Bereken de ogenblikkelijke impulsmomentvector en de verandering van de impulsmomentvector van het landingsgestel en die van het wiel rond hun respectievelijke massacentra.

$$\vec{L}_0 = I(t) \vec{\omega} \quad (18)$$

$$\begin{aligned}
\vec{\omega} &= \vec{\omega}_w + \vec{\omega}_g + \vec{\omega}_i \\
&= \left\{ \begin{aligned} &(-\vec{\omega}_w - \vec{\omega}_g \sin(\beta)) \vec{e}_{x''''} \\ &\vec{\omega}_i * \vec{e}_{y''''} \\ &\vec{\omega}_g \cos \beta(\beta) \vec{e}_{z''''} \end{aligned} \right\} \quad (19)
\end{aligned}$$

$$= \left\{ \begin{aligned} &I_{\omega, x'''' , x''''} (-\vec{\omega}_w - \vec{\omega}_g \sin(\beta)) \vec{e}_{x''''} \\ &I_{\omega, y'''' , y''''} \vec{\omega}_i * \vec{e}_{y''''} \\ &I_{\omega, z'''' , z''''} \vec{\omega}_g \cos \beta(\beta) \vec{e}_{z''''} \end{aligned} \right\}$$

$$\left(\frac{d\vec{L}_c}{dt} \right) = \left(\frac{d\vec{L}_C}{dt} \right)_{rel} + \vec{\Omega} \times \vec{L}_c \quad (20)$$

$$\begin{aligned}
\vec{\Omega} \times \vec{L}_c &= \begin{vmatrix} \vec{e}_{x''''} & \vec{e}_{y''''} & \vec{e}_{z''''} \\ -\omega_w - \omega_g \sin \beta & \omega_i & \omega_g \cos \beta \\ L_{o, x''''} & L_{o, y''''} & L_{o, z''''} \end{vmatrix} \\
&= \left\{ \begin{aligned} &\omega_i L_{o, z''''} - \omega_g \cos \beta L_{o, y''''} \\ &\omega_g \cos \beta L_{o, x''''} + (\omega_w + \omega_g) L_{o, z''''} \\ &-(\omega_w + \omega_g \sin(\beta)) L_{o, y''''} + \omega_i L_{o, x''''} \end{aligned} \right\} \quad (21)
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\vec{L}_c}{dt}\right)_{rel} &= I \frac{d\vec{\omega}}{dt} \\
&= I \begin{pmatrix} \left(\alpha_w - \alpha_g \sin \beta + (-\omega_i \omega_g) \cos \beta\right) \vec{e}_{x''''} \\ \left(\alpha_i + \omega_g \omega_w \cos \beta\right) \vec{e}_{y''''} \\ \left(\alpha_g \cos \beta - \omega_i \omega_g \sin \beta - \omega_i \omega_w\right) \vec{e}_{z''''} \end{pmatrix}
\end{aligned} \tag{22}$$

want

$$\begin{aligned}
\frac{d\vec{\omega}}{dt} &= \alpha_g \vec{e}_{z'} + \alpha_i \vec{e}_{y''} + (-\omega_i \omega_g) \vec{e}_{x''} + \alpha_w \vec{e}_{x'''} + \begin{pmatrix} -\sin \beta \vec{e}_{x''''} \vec{e}_{x''''} \\ \cos \beta \omega_g \omega_w \vec{e}_{y''''} \\ -\cos \beta \omega_i \omega_w \vec{e}_{z''''} \end{pmatrix} \\
&= \begin{pmatrix} \alpha_w - \alpha_g + (-\omega_i \omega_g) \cos \beta \vec{e}_{x''''} \\ \alpha_i + \cos \beta \omega_g \omega_w \vec{e}_{y''''} \\ \alpha_g \cos \beta + (-\omega_i \omega_g) \sin \beta - (\omega_i \omega_w) \vec{e}_{z''''} \end{pmatrix}
\end{aligned} \tag{23}$$