Air Pressure Gradient in a Rotating Space Station

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Abstract

A formula for computing the air pressure at various distances from the center of a rotating space station is developed.

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1 Introduction

Artificial gravity in a radially symmetric space station can be created by rotating it around its axis. From this, it is theoretically possible to construct a space station with roughly earth-like conditions on the inside surface. This idea has been explored in fiction, most notably *2001: A Space Odyssey*[KCD+68] and *Babylon 5*[Str93]. The book *Eon*[Bea85], describes a hollowed out asteroid rotating to produce artificial gravity. In this case the air pressure is less than earth normal and the rotation produces about 0.6 of earth's gravity. Larry Niven's *Ringworld*[Niv77] is perhaps the ultimate expression of a rotating cylinder space station with a radius equal to about one AU.

A formula giving the air pressure at any distance from the center of a rotating space station is desired.

2 Development

The coordinates in the space station are x, the distance along the axis, r, the distance from the axis of rotation, and θ, the angle from a fixed reference point. Note that this coordinate system is rotating. A number of parameters and constants are defined in Table 1.

Table 1: Definition of Parameters

Parameter	Description	Value	Dimensions
r	Distance from axis of rotation		m
R_1	Radius of space station		m
A_0	Acceleration at R_1	9.8	$m \times s^{-2}$
ΔV	Small volume	$r \times \Delta x \times \Delta r \times \Delta\theta$	m^3
ω_0	Angular velocity of space station	$\sqrt{\frac{A_0}{R_1}}$	$radians^2$
p	Air pressure		kPa
P_1	Air pressure at outside inner surface	101.325	kPa
V	Volume		m^3
n	Number of molecules		mol
R	Ideal gas constant	8.314472	$J \times K^{-1} \times mol^{-1}$
T	Temperature	293.15	K
M	Molar mass of the gas		$g \times mol^{-1}$
M_{air}	Approximate molar mass of air	0.029	$kg \times mol^{-1}$

The centripetal force is given by:

$$F = mr\omega^2 \tag{1}$$

With some manipulation of (1), the angular velocity can be determined to be:

$$\omega_0 = \sqrt{\frac{A_0}{R_1}} \tag{2}$$

The ideal gas law is:

$$pV = nRT \tag{3}$$

The mass of a quantity of gas is given by:

$$m = nM \tag{4}$$

Assuming that air consists of 75% Nitrogen and 25% Oxygen, we have a molar mass of air (M_{air}) approximately 29 g/mol.

Substituting (4) into (3) and solving for mass gives:

$$m = \frac{pVM_{air}}{RT} \tag{5}$$

Substituting (5) into (1) gives the force (weight) of a small volume of gas:

$$F = \frac{p\Delta VM_{air}r\omega^2}{RT} \tag{6}$$

Since pressure is force divided by area, we can obtain the pressure contribution of this small volume as:

$$\Delta p = \frac{\Delta F}{\Delta x \times r\Delta\theta} = \frac{pM_{air}\omega^2}{RT} \times \frac{r \times \Delta x \times \Delta r \times r\Delta\theta}{r \times \Delta x \times \Delta\theta} \tag{7}$$

Simplifying and canceling (7) and substituting the angular velocity from 2 gives:

$$\Delta p = \frac{pM_{air}A_0}{RTR_1}r\Delta r \tag{8}$$

Since M_{air}, A_0, R, T, and R_1 are constants and p is a function of r, (8) can be rewritten as:

$$\Delta p = kp(r)r\Delta r \tag{9}$$

Where

$$k = \frac{M_{air}A_0}{R_1RT} \tag{10}$$

Plugging in known values and canceling units in (10) gives:

$$k = \frac{0.029\frac{kg}{mol} \times 9.8\frac{m}{s^2}}{R_1 \times 8.314472\frac{J}{K \times mol} \times 293.15K} = \frac{1.662 \times 10^{-4}}{R_1} \times m^{-1} \tag{11}$$

Taking the limit of (9) as Δr approaches zero give the differential equation:

$$dp = kprdr \tag{12}$$

This has a solution of the form:

$$p(r) = Ce^{\frac{kr^2}{2}}, p(R_1) = P_1 \tag{13}$$

The value of the constant C can now be determined as:

$$C = \frac{P_1}{e^{\frac{M_{air}A_0R_1}{2RT}}} \tag{14}$$

Note that both constants, k and C are both functions of the size, R_1 of the space station, which gives a formula

$$p(R_1, r) = \frac{P_1}{e^{\frac{M_{air}A_0R_1}{2RT}}} \times e^{\frac{M_{air}A_0r^2}{2R_1RT}} \tag{15}$$

This can be rewritten as:

$$p(R_1, r) = P_1e^{\frac{M_{air}A_0(r^2-R_1^2)}{2R_1RT}} \tag{16}$$

Putting in numbers where possible gives (note that R_1 and r are in meters):

$$P(R_1, r) = 101.325kPa \times e^{\frac{5.83\times10^{-5}(r^2 - R_1^2)}{R_1}} \quad (17)$$

A contour plot of this function is shown in Figure 1. The air pressures are given in kPa, where standard atmospheric pressure is slightly greater than 100 kPa. Figure 2 shows the air pressure at the axis of the station for various sizes of the station. For comparison, elevations for some air pressures are given in Table 2.

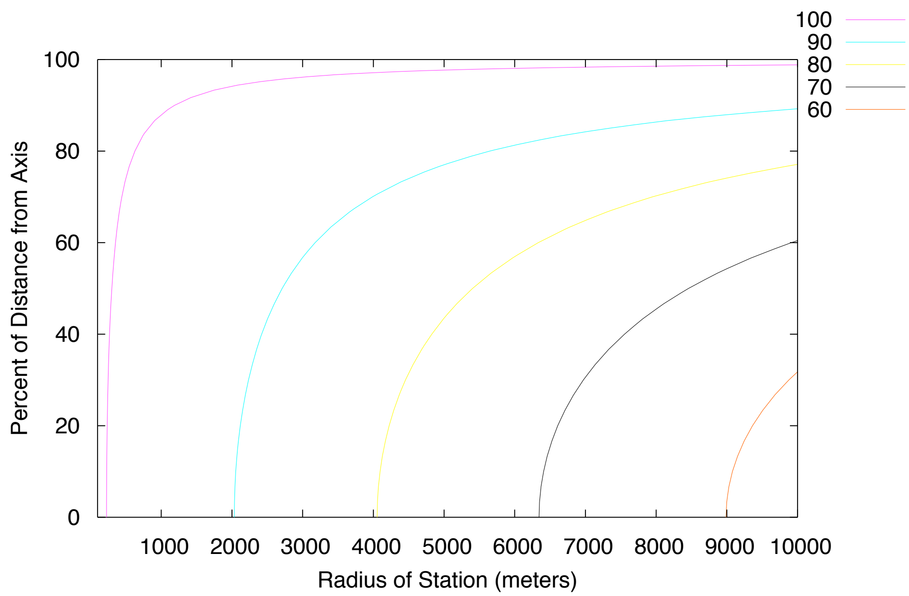

Figure 1: Air Pressure Curves for Various Space Station Sizes

Table 2: Air Pressure per Elevation on Earth

Pressure (kPa)	Altitude (ft
100	Sea Level
80	6600
60	14500
40	24750
20	41000

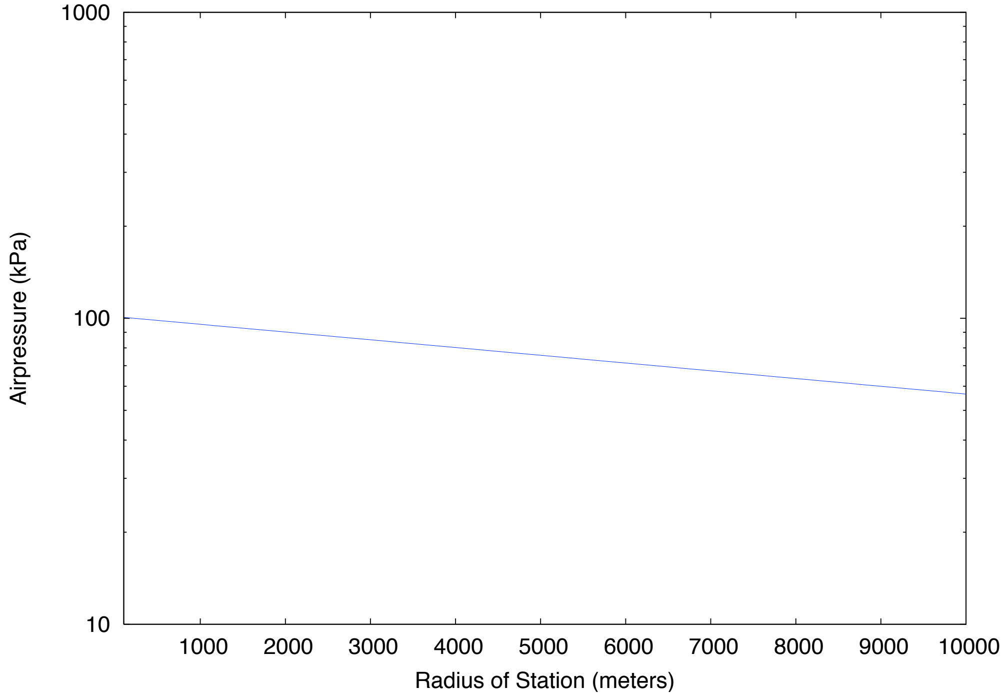

Figure 2: Air Pressure at Axis for Various Space Station Sizes

3 Conclusions

The air at the center of a rotating space station should be easily breathable for stations up to about 4000 meters in radius. People would be able to acclimatize to the air pressure at the center of a 9000 meters radius station.

For a sufficiently large station, the decrease in pressure would reduce the need for pressure sealing along the axis. Larry Niven's *Ringworld*[Niv77] is a rather extreme example of this situation.

Note that the pressures given should be considered an approximation. There will no doubt be fluctuations due to weather and similar effects. Also, temperature has been assumed to be constant. Further investigation could be done to consider the effects of temperature varying with altitude.

References

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