MAT 425 - Stochastic Processes

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Stochastic Process a collection of random variables

1.1 Basic Probability Review

1.1.1 Example 1.12

Compute the probability that event E occurs before event F if we repeat the experiment.

$$\begin{aligned} p &= \mathbb{P}(E) + \mathbb{P}\left((E \cup F)^C\right) \cdot p \\ &= \mathbb{P}(E) + 1 - \mathbb{P}(E \cup F) \cdot p \\ &= \mathbb{P}(E) + (1 - \mathbb{P}(E) - \mathbb{P}(F)) \cdot p \\ &= \frac{\mathbb{P}(E)}{\mathbb{P}(E) + \mathbb{P}(F)} \end{aligned}$$

1.1.2 Partitions

If $\{B_1, B_2, \ldots, B_n\}$ is a partition of Ω and $A \subset \Omega$ then,

$$\begin{aligned} \mathbb{P}(A) &= \mathbb{P}\left(\bigcup_{i=1}^{n} A \cap B_i\right) \\ &= \sum_{i=1}^{n} \left(\mathbb{P}(A \cap B_i)\right) \\ &= \sum_{i=1}^{n} \mathbb{P}(A \mid B_i)\,\mathbb{P}(B_i) \end{aligned}$$

1.1.3 Example 1.13 - Craps

Roll two dice, then if

1. the sum is 7 or 11 $\longrightarrow$ you win!
2. the sum is 2, 3, or 12 $\longrightarrow$ you lose!
3. the sum, i, is such that $i \in \{4, 5, 6, 8, 9, 10\}$, keep rolling until
   (a) the sum is 7 $\longrightarrow$ you lose!
   (b) the sum is i $\longrightarrow$ you win!

Let W be the event that you win and D the sum of the two dice.

$$\begin{aligned} \mathbb{P}(W) &= \sum_{i=2}^{12} \left(\mathbb{P}(W \mid D = i)\,\mathbb{P}(D = i)\right) \\ &= \mathbb{P}(D = 7) + \mathbb{P}(D = 11) + \sum_{i \in \{4,5,6,8,9,10\}} \left(\mathbb{P}(W \mid D = i)\,\mathbb{P}(D = i)\right) \\ &= \frac{6}{36} + \frac{2}{36} + \sum_{i \in \{4,5,6,8,9,10\}} \left(\frac{\mathbb{P}(D = i)^2}{\mathbb{P}(D = i) + \mathbb{P}(D = 7)}\right) \\ &= \frac{6}{36} + \frac{2}{36} + 2\left(\frac{1}{36} + \frac{4}{90} + \frac{25}{396}\right) \\ &= \frac{4880}{9900} \approx 0.4929 \end{aligned}$$