

# MAT 415 - Introduction to Combinatorics

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# 1 Week of August 14th, 2016

## 1.1 Principle Definitions

### 1.1.1 Product Principle

Suppose a task can be broken into  $k$  subtasks,  $t_1, t_2, \dots, t_k$ , and further suppose there are  $c_i$  ways to perform subtask  $t_i$  and each way leads to a unique result. Then the number of ways to perform the task is  $c_1 \cdot c_2 \cdot \dots \cdot c_k$ .

### 1.1.2 Sum Principle

Suppose the objects in a counting problem can be divided into  $k$  disjoint and exhaustive cases. If there are  $n_i$  objects in the  $i^{th}$  case for  $i = 1, 2, \dots, k$  then there are  $n_1 + n_2 + \dots + n_k$  objects.

### 1.1.3 Bijection Principle

Two finite sets have the same cardinality if and only if there exists a bijection between them.

**Example** How many subsets does  $\{k_1, k_2, k_3, k_4\}$  have?

Find a bijection between the binary string  $b_1b_2b_3b_4$  and  $\{k_1, k_2, k_3, k_4\}$ .

$$S \subseteq \{k_1, k_2, k_3, k_4\} \longleftrightarrow b_1b_2b_3b_4 \quad \text{where} \quad b_i = \begin{cases} 0 & \text{if } k_i \notin S \\ 1 & \text{if } k_i \in S \end{cases}$$

There are  $2^4 = 16$  possibilities for the binary string so the set has 16 subsets.

### 1.1.4 Quotient Principle

A *partition* of a set,  $S$ , is a division of a set into disjoint subsets whose union is  $S$ . The subsets in a set of partitions are often called blocks of the partition.

Suppose a set  $S$  has  $p$  elements. If we partition  $S$  into  $q$  blocks of size  $r$ , then  $q = p/r$  and  $r = p/q$ .

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