MAT 415 - Introduction to Combinatorics

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Contents

1 Week of August 14th, 2016

1.1 Principle Definitions

1.1.1 Product Principle

Suppose a task can be broken into k subtasks, $t_1, t_2, \ldots, t_k$, and further suppose there are c_i ways to perform subtask t_i and each way leads to an unique result. Then the number of ways to perform the task is $c_1 \cdot c_2 \ldots c_k$.

1.1.2 Sum Principle

Suppose the objects in a counting problem can be divided into k disjoint and exhaustive cases. If there are n_i objects in the i^{th} case for $i = 1, 2, \ldots, k$ then there are $n_1 + n_2 + \ldots + n_k$ objects.

1.1.3 Bijection Principle

Two finite sets have the same cardinality if and only if there exists a bijection between them.

Example How many subsets does $\{k_1, k_2, k_3, k_4\}$ have?
Find a bijection between the binary string $b_1b_2b_3b_4$ and $\{k_1, k_2, k_3, k_4\}$.

$$S \subseteq \{k_1, k_2, k_3, k_4\} \longleftrightarrow b_1b_2b_3b_4 \quad \text{where} \quad b_i = \begin{cases} 0 & \text{if} \quad k_i \notin S \\ 1 & \text{if} \quad k_i \in S \end{cases}$$

There are $2^4 = 16$ possibilites for the binary string so the set has 16 subsets.

1.1.4 Quotient Principle

A *partition* of a set, S, is a division of a set into disjoint subsets whose union is S. The subsets in a set of partitions are often called blocks of the partition.
Suppose a set S has p elements. If we partition S into q blocks of size r, then $q = p/r$ and $r = p/q$.