

# MAT 425 - Stochastic Processes

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# 1 Week of August 14th, 2016

**Stochastic Process** a collection of random variables

## 1.1 Basic Probability Review

### 1.1.1 Example 1.12

Compute the probability that event E occurs before event F if we repeat the experiment.

$$\begin{aligned}
 p &= \mathbb{P}(E) + \mathbb{P}((E \cup F)^C) \cdot p \\
 &= \mathbb{P}(E) + 1 - \mathbb{P}(E \cup F) \cdot p \\
 &= \mathbb{P}(E) + (1 - \mathbb{P}(E) - \mathbb{P}(F)) \cdot p \\
 &= \frac{\mathbb{P}(E)}{\mathbb{P}(E) + \mathbb{P}(F)}
 \end{aligned}$$

### 1.1.2 Partitions

If  $\{B_1, B_2, \dots, B_n\}$  is a partition of  $\Omega$  and  $A \subset \Omega$  then,

$$\begin{aligned}
 \mathbb{P}(A) &= \mathbb{P}\left(\bigcup_{i=1}^n A \cap B_i\right) \\
 &= \sum_{i=1}^n (\mathbb{P}(A \cap B_i)) \\
 &= \sum_{i=1}^n \mathbb{P}(A \mid B_i) \mathbb{P}(B_i)
 \end{aligned}$$

### 1.1.3 Example 1.13 - Craps

Roll two dice, then if

1. the sum is 7 or 11  $\longrightarrow$  you win!
2. the sum is 2, 3, or 12  $\longrightarrow$  you lose!
3. the sum,  $i$ , is such that  $i \in \{4, 5, 6, 8, 9, 10\}$ , keep rolling until
  - (a) the sum is 7  $\longrightarrow$  you lose!
  - (b) the sum is  $i$   $\longrightarrow$  you win!

Let  $W$  be the event that you win and  $D$  the sum of the two dice.

$$\begin{aligned}
 \mathbb{P}(W) &= \sum_{i=2}^{12} (\mathbb{P}(W \mid D = i) \mathbb{P}(D = i)) \\
 &= \mathbb{P}(D = 7) + \mathbb{P}(D = 11) + \sum_{i \in \{4, 5, 6, 8, 9, 10\}} (\mathbb{P}(W \mid D = i) \mathbb{P}(D = i)) \\
 &= \frac{6}{36} + \frac{2}{36} + \sum_{i \in \{4, 5, 6, 8, 9, 10\}} \left( \frac{\mathbb{P}(D = i)^2}{\mathbb{P}(D = i) + \mathbb{P}(D = 7)} \right) \\
 &= \frac{6}{36} + \frac{2}{36} + 2 \left( \frac{1}{36} + \frac{4}{90} + \frac{25}{396} \right) \\
 &= \frac{4880}{9900} \approx 0.4929
 \end{aligned}$$

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