MAT 472 - Intermediate Real Analysis I

Instructor: Dr. Steven Kaliszewski
Notes written by Brett Hansen

Contents

1 Week of August 21st, 2016

1.1 The Completeness Property of Real Numbers

$\mathbb{R}$ and $\mathbb{Q}$ are ordered fields.

- $|\,\mathbb{Q}\,| < |\,\mathbb{R}\,|$
- $\mathbb{Q}$ is countable, $\mathbb{R}$ is uncountable (cardinality)
- $x^2 - 2 = 0$ has solutions in $\mathbb{R}$ but none in $\mathbb{Q}$
- $\mathbb{R}$ has no gaps (order)

1.1.1 Axiom of Completeness

Every non-empty subset of $\mathbb{R}$ that has an upper bound has a least upper bound. Along with the greatest lower bound, these bounds can be classified using the following definitions:

1 Defn: Upper Bound. *Let the set S be a non-empty subset of $\mathbb{R}$ and suppose $b \in \mathbb{R}$. b is an upper bound of the set S if $\forall x \in S,\ x \leq b$.*

2 Defn: Lower Bound. *Let the set S be a non-empty subset of $\mathbb{R}$ and suppose $b \in \mathbb{R}$. b is a lower bound of the set S if $\forall x \in S,\ x \geq b$.*

3 Defn: Supremum. *Let B be the set of upper bounds of the non-empty subset, S, of $\mathbb{R}$ and suppose $b \in B$. If $\forall x \in B,\ b \leq x$, then b is the least upper bound of S, or supremum of S. This can be denoted as sup $S = b$.*

4 Defn: Infinum. *Let B be the set of lower bounds of the non-empty subset, S, of $\mathbb{R}$ and suppose $b \in B$. If $\forall x \in B,\ b \geq x$, then b is the greatest lower bound of S, or infinum of S. This can be denoted as inf $S = b$.*

Example. $A = [0,\ 2] \quad \longrightarrow \quad \mathit{sup}\ A = 2$
$A = [0,\ 2) \quad \longrightarrow \quad \mathit{sup}\ A = 2$

5 Defn: Maximum. *If the supremum of a set is also a member of the set, then the supremum is also the maximum.*

2 Week of August 28th, 2016

3 Week of September 4th, 2016

4 Week of September 11th, 2016

5 Week of September 18th, 2016

6 Week of September 25th, 2016

7 Week of October 2nd, 2016

8 Week of October 9th, 2016

9 Week of October 16th, 2016

10 Week of October 23rd, 2016

11 Week of October 30th, 2016

12 Week of November 6th, 2016

13 Week of November 13th, 2016

14 Week of November 20th, 2016

15 Week of November 27th, 2016