Beantown Paper Mill[1]

Beantown Paper Mills Inc., located in Cohoes, New York, produces up to 200 tons of paper per day to fill orders from 250 customers. The mill's products include:

- fine papers (used in printing applications such as magazines and annual reports)
- bleached paperboard (used in packaging such as milk cartons)
- kraft paper (used for corrugated boxes and decorative laminates such as Formica).

The mill is served by six carriers, and twenty shipments per day are not unusual. The carriers will generally accept shipments to any destination that they serve, subject to daily volume commitments and equipment availability. Each carrier has a different and somewhat complex rate structure.

Rob Wood is the mill's load planner. Each morning, Rob reviews the previous day's production to divide large orders and consolidate less-than-truckload (LTL) orders into truckload quantities. Orders typically weigh between 1000 and 15,000 pounds, and a truck can generally pull a trailer with 46,000 to 48,000 pounds of paper. Rob plans the consolidated loads so that each truck makes no more than four stops.

After loads are planned, they are turned over to Ed Wong, the transportation planner. Ed's job is to assign carriers to loads so as to minimize the total shipping cost. He has a contract for each carrier that gives the rates to each destination served. The rates include a mileage charge, a stop-off charge, and a minimum charge per truckload.

Tables 1 – 3 show a typical problem that Ed must solve. In this day's scenario, there are 32 loads that must be sent on 12 different routes. Some of the routes contain interim stops before the final destination. When multiple loads must be sent to the same destination, the load assignment may be split among different carriers. The destinations and load requirements are listed in Table 1.

Table 1: Routes and Loads

Destination	State	# Loads	Interim	Miles
Atlanta	GA	4	0	612
Everett	MA	1	3	612
Ephrata	PA	3	0	190
Riverview	MI	5	0	383
Carson	CA	1	2	3063
Chamblee	GA	1	0	429
Roseville	MN	1	3	600
Hanover	PA	1	0	136
Sparks	NV	2	0	2439
Parsippany	NJ	1	1	355
Effingham	IL	5	0	570
Kearny	NJ	7	0	324

[1] This problem is adapted from the book *Winston & Albright – Practical Management Science, 4th edition*

In the shipping area, there are 33 drivers from the six carriers waiting for their trucks to be loaded; one truck will not be needed today. Some carriers have minimum daily load commitments that must be met. For example, at least seven loads must be assigned to IRST. Minimum charges per truckload, interim stop-off charges, numbers of available trucks, and the minimum load commitments for each carrier are displayed in Table 2.

Table 2: Carrier Information

	Carrier					
	ABCT	**IRST**	**LAST**	**MRST**	**NEST**	**PSST**
Minimum Charge per Truckload	$350	$400	$350	$300	$350	$300
Interim Stop-off Charge	$50	$75	$50	$35	$50	$50
Available Pulls (Trucks)	4	8	7	7	3	4
Minimum Load Commitment	1	7	6	0	0	4

The per-mile transportation charges to each destination for each of the six carriers are displayed in Table 3. Not every carrier services every destination.

Table 3: Per-Mile Carrier Rates

		Per-Mile Rates for Carriers					
Destination	**State**	**ABCT**	**IRST**	**LAST**	**MRST**	**NEST**	**PSST**
Atlanta	GA	**	$0.88	$1.15	$0.87	$0.95	$1.05
Everett	MA	**	$1.18	$1.27	$1.39	$1.35	$1.28
Ephrata	PA	**	$3.42	$1.73	$1.71	$1.82	$2.00
Riverview	MI	$0.79	$1.01	$1.25	$0.96	$0.95	$1.11
Carson	CA	**	$0.80	$0.87	**	$1.00	**
Chamblee	GA	**	$1.23	$1.61	$1.22	$1.33	$1.47
Roseville	MN	$1.24	$1.13	$1.89	$1.32	$1.41	$1.41
Hanover	PA	**	$4.78	$2.23	$2.39	$2.26	$2.57
Sparks	NV	**	$1.45	**	$1.20	**	**
Parsippany	NJ	**	$1.62	$1.36	$1.39	$1.03	$1.76
Effingham	IL	$0.87	$0.87	$1.25	$0.87	$0.90	$1.31
Kearny	NJ	**	$2.01	$1.54	$1.53	$1.28	$1.95

(HINT: when building your model, if a carrier does not service a location, you can put in that the cost [for instance for ABCT to carry a load to Atlanta] is very high, like 100. This might make some of your formulae more consistent).

The mileages in Table 1 represent the total number of miles for the trip from the Cohoes mill to the final destination, including any intermediate stops. The total charge for a specific carrier-to-destination assignment is calculated as follows:

(Miles) * (Per-Mile Rate) + (# Interim Stops) * (Interim Stop-off Charge)

For example, suppose that the Roseville, MN trip is assigned to carrier IRST. The transportation cost would be 600(1.13)+3(75)=$903. Stop-off charges only apply to intermediate stops and not the final destination. If the cost calculated this way had been less than IRST's minimum truckload charge of $400, the cost would have been $400.

(Some) Steps towards a Solution

Ed can formulate his assignment task as a network flow problem, which can then be translated to a linear program. The graph for this problem is shown in Figure 1. The six nodes on the left side of the graph represent the carriers; the twelve nodes on the right side of the graph represent the destinations.

The flow on an arc from a carrier node to a destination node represents how many loads that carrier will carry to the destination city. The minimum and maximum flow on this arc is the number of loads that must be sent to the destination. (We assume every load must be delivered.)

The per-unit flow cost on the arc is the cost of sending a load to the destination using the carrier, computed from the information in Tables 1-3.

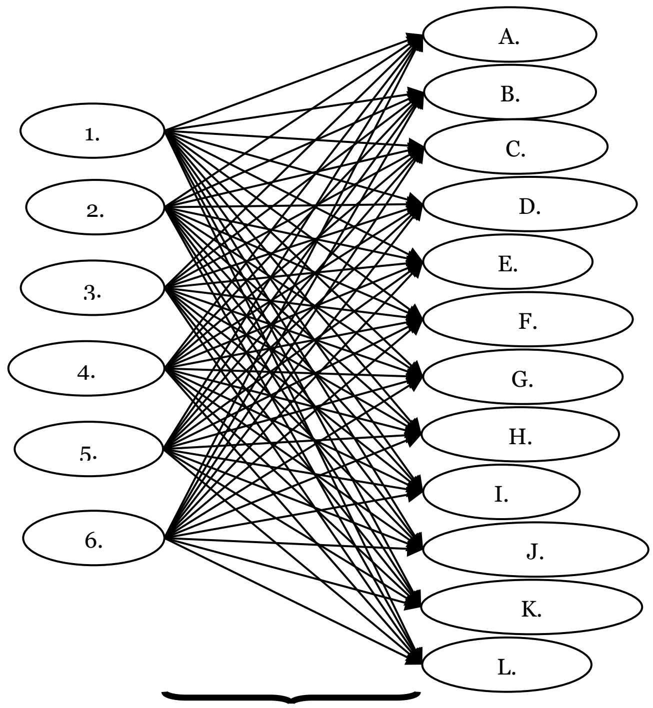

Figure 1: Beantown's Assignment Problem and Decision Variables

To translate the graph shown in Figure 1 to a linear program, we first assign a decision variable to every arc. The decision variables are shown in Figure 1. The decision variable X_{3F} for instance would correspond to how many loads Carrier 3 (which is the LAST carrier) brings from the mill to location F (the 6th demand location which is Chamblee, GA). The decision variable X_{5B} denotes how many loads carrier 5 (NEST) brings from the mill to location B(the second location, or Everett, MA).

The **objective is to minimize the sum of the decision variables** shown in Figure 1 **multiplied by the corresponding per-unit flow costs** that you must calculate from tables 1, 2, and 3. There are three constraints. First, the **overall quantity shipped using each carrier must be greater than or equal to the minimum commitment**. For example, this constraint for the 3.LAST carrier node is:

$$X_{3A} + X_{3B} + \ldots + X_{3L} .>= \text{Min commitment}$$

Second, the **overall quantity shipped using each carrier must be less than or equal to the available capacity**. For example, this constraint for the 3.LAST carrier node is:

$$X_{3A} + X_{3B} + \ldots + X_{3L} .<= \text{Available Trucks}$$

Third, **the total supply to each destination node must be equal to the demand**. The balance constraint for the D.Riverview destination node is:

$$X_{1D} + X_{2D} + \ldots + X_{6D} = X_D$$

HINT:

1) To validate your model:

   If:

   - ABCT brings 3 loads to riverview
   - IRST brings 1 load to Atlanta
   - LAST brings 1 load to Atlanta, 1 load to Everett
   - MRST brings no loads
   - NEST brings 1 load to riverview
   - PSST brings 1 load to Ephrata and and 1 load to Chamblee, 1 load to Kearney, and 1 load to Parsippany

   Then

   - you're total cost will be $5900.68
   - Your solution will be infeasible (for instance, IRST doesn't have its minimum loads and riverview isn't getting it's required delivery)

2) Do not use binary constraints here!
3) The optimal cost is between $22,200 and $22,500, and the last three digits including the decimal are "XXX4.38". (Make sure you accounted for the min charge per truckload)