Appendix

The Replicator Dynamics

There are a few forms of the replicator dynamics. One that I use in many of the simulations here is the *discrete time* replicator dynamics. Under these dynamics, a population playing a game changes at time steps. At each time step, the expected payoff for each strategy is calculated given the current population. These expected payoffs are then compared to the average expected payoff for the population. Strategies that beat the average expand and those that do not contract. This dynamics is formulated as

$$x_i' = x_i \left(\frac{f_i(x)}{\sum_{j=1}^{n} f_j(x) x_j} \right) \tag{A.1}$$

where x_i is the proportion of a population playing strategy i, $f_i(x)$ is the fitness of type i in the population state x and $\sum_{j=1}^{n} f_j(x) x_j$ is the average population fitness in this state. It says that $x_i\prime$, the proportion of actors playing x_i in the next time step, is a function of the current proportion x_i multiplied by the ratio of the payoff of those playing i to the average group payoff.

The *continuous time* replicator dynamics can be thought of as the result of taking the limit of the change process modeled by the discrete time version as the length of each time step goes to 0. These dynamics consist of differential equations that model the smooth change of a population undergoing the sort of process already described—increasing relatively good strategies and decreasing poor ones. They are formulated as

$$\dot{x}_i = x_i \left(f_i(x) - \sum_{j=1}^{n} f_j(x) x_j \right) \tag{A.2}$$

I will appeal to both versions of these dynamics throughout the book. While the continuous and discrete time versions of the replicator dynamics vary, they are generally taken as representationally similar, with the continuous time version used for analytic solutions and the discrete time version for simulation solutions.

I will use slightly different versions of these dynamics to represent two-type populations. In perfectly divided populations, actors have two types and each interacts only with the other type. This corresponds well to the standard two-population replicator dynamics where actors exist in two populations and each interacts only with the other population. They then compare strategies with those in their own group and the strategies that are doing better proliferate in each population. The two-population continuous time replicator dynamics are formulated as

$$\dot{x}_i = x_i \left(f_i(y) - \sum_{j=1}^{n} f_j(y) x_j \right) \tag{A.3}$$

$$\dot{y}_i = y_i \left(f_i(x) - \sum_{j=1}^{n} f_j(x) y_j \right) \tag{A.4}$$

where x represents strategies in one population and y represents strategies in the other. Note that the payoff terms in each equation—$f_i(y)$ and $f_i(x)$—depend on the other population state because actors interact with the other population and receive payoffs that way. But strategies expand and contract in comparison to the success of those only in their own population. (The discrete time version involves two update functions like those in (1), but where, again, the payoffs depend on the other population state.)

For two-type mixing populations, actors have types, but also interact with their own type. The evolutionary process for this is best modeled by a version of the replicator dynamics employed by Neary (2012); Bruner and O'Connor (2015); Bruner (2017); O'Connor and Bruner (2017) where actors have two types, but their payoffs are derived from interactions with everyone in the population. These dynamics are formulated as

$$\dot{x}_i = x_i \left(f_i(x, y) - \sum_{j=1}^{n} f_j(x, y) x_j \right) \tag{A.5}$$

$$\dot{y}_i = y_i \left(f_i(x, y) - \sum_{j=1}^{n} f_j(x, y) y_j \right) \tag{A.6}$$

for the continuous time version. The only difference from the standard two-population replicator dynamics is that the fitnesses of the strategies from each population now depend on everybody, $f_i(x, y)$, rather than only the other population, $f_i(y)$ or $f_i(x)$.[1]

[1] Models with two types could involve any level of correlation or anti-correlation of interactions between types. Here I have laid out dynamics for complete anti-correlation (perfectly divided) and no correlation at all (two-type mixing). Similar dynamics can be introduced for any other levels of correlation.

In the book, I also look at populations where one type is in the minority. These models employ the two-type mixing replicator dynamics. It is assumed that actors interact with minority population members proportional to their prevalence in the entire population. So the behaviors of minority actors are less significant in determining the payoffs of strategies in either population than the behaviors of the majority members are. The only change, then is to the $f_i(x, y)$ terms of the last equation, where the proportions determine how much fitness is influenced by each group.