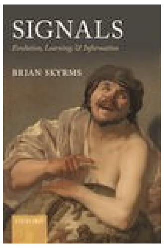

Signals: Evolution, Learning, and Information
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CHAPTER

14 14 Learning to Network

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Abstract

This chapter introduces a low-rationality *probe and adjust* dynamics to approximate higher rationality learning in the basic Bala–Goyal models. Both best response dynamics and *probe and adjust* learned networks that reinforcement learning did not. In general, *probe and adjust* learns a network structure if *best response with inertia* does.

Keywords: signals, signaling systems, signaling networks

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We now suppose that, in one of the ways investigated in preceding chapters, individuals have learned to signal. Building on this basis, how can they learn to combine these signaling interactions to form signaling networks? This is the next question for a naturalistic theory of signaling. It is too large a question for a single chapter, or even a single book. Here I will give an introduction to this growing area of research. I hope that you will find the simple examples treated here interesting and suggestive.

The spirit of the enterprise is intended to be consonant with the rest of this book: start with the simplest and most naive forms of trial-and-error learning and see what they can do. If they fail to solve a problem, climb up the ladder of cognitive sophistication to see what it takes. We start in a somewhat roundabout way by noting the importance of ring structures of symbolic exchange in primitive societies.

Rings in primitive societies

In 1920, Bronislaw Malinowski published an article entitled “Kula” in *Man*, the journal of the Royal
p. 162 Anthropological Society. In it, he described the Kula Ring, later made famous ↳

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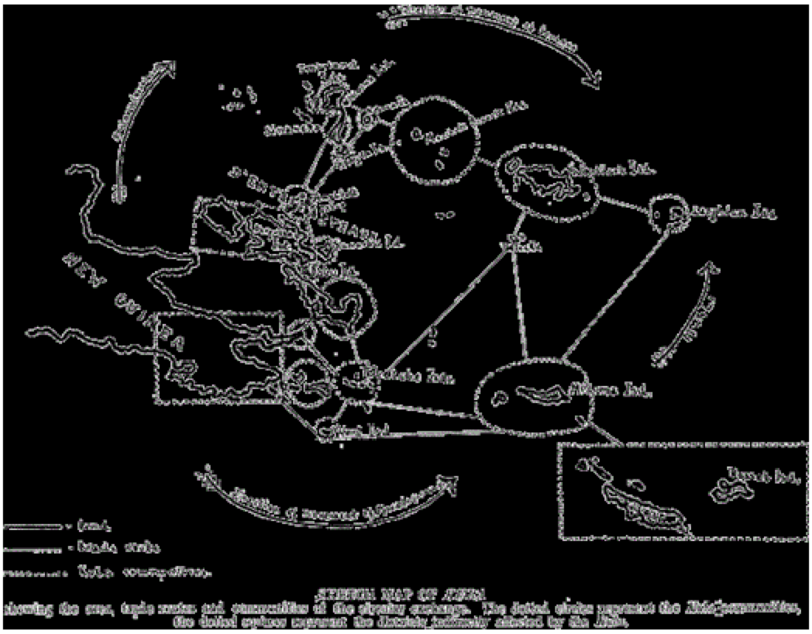
by his book *Argonauts of the Western Pacific*.[1] The ring, in Malinowski's words:

> is based primarily upon the circulation of two articles of high value, but of no real use,—these are armshells made of the *Conus milleounctatus*, and necklets of red shell-discs, both intended for ornaments, but hardly ever used, even for this purpose. These two articles travel, in a manner to be described later in detail, on a circular route which covers many miles and extends over many islands. On this circuit, the necklaces travel in the direction of the clock hands and the armshells in the opposite direction. Both articles never stop for any length of time in the hands of any owner; they constantly move, constantly meeting and being exchanged.[2]



The necklaces and armshells have social, symbolic, and even magical value. They become more valuable as they circulate. Each subsequent owner adds to the history, power and value of an item.

Computer LAN rings

Before computer networking power was almost free, computers were sometimes organized in local area networks (LAN) with the structure of a ring. Each node passes information to an immediate neighbor in a specified direction—say clockwise—along the ring. Information then flows to all nodes around the ring. One disadvantage of a ring network is that it is not robust. If one node is disabled, information flow is disrupted. As insurance, sometimes ring networks also add counter-rotation, passing information to neighboring nodes in both clockwise and counter-clockwise directions, just like the Kula ring.

p. 164 Some of the resemblance is misleading. Rotation combined with counter-rotation in the Kula has more to do with reciprocity than with robustness. But some resemblances may be significant. In particular, note that the good being passed on does not degrade as it passes along. The information is passed along reliably without appreciable decay in the computer LAN. The articles in the Kula ring also have a value that does not decay. It actually increases as they are passed along. This will prove to be an important consideration in game theoretic analysis of network formation.

The Bala–Goyal ring game

Venkatesh Bala and Sanjeev Goyal[3] introduce an informational network game in which a ring structure[4] has a special equilibrium status. Individuals get private information by observing the world. Each gets a different piece of information. Information is valuable. An individual can pay to connect to another and get her information. The individual who pays does not give any information; it only goes from payee to payer. The payer gets not only the information from private observations of those whom she pays, but also that which they have gotten from subscribing to others for their information. Information flows freely in this community, and without degradation, along the links so established. It flows in one direction, from payee to payer. We assume that information flow is fast, relative to any adjustment of the network structure.

If the cost of subscribing to someone's information is too high, then it won't pay for anyone to do it. But let's suppose that the cost of establishing a connection is less than the value of each piece of information. Then connections certainly make sense. We assume that any individual can make as many connections as she wishes. This model can be viewed as a game, with an individual's strategy p. 165 ↳ being a decision of what connections to make. It could be none, all, or some. The game has multiple equilibria, but one is special. This is the ring (or circle). There is an example in figure 14.1.

The ring structure in this game is special in two ways. The first is that it is *strict*, the second that it is *efficient*. It is a strict equilibrium in that someone who unilaterally deviates from such a structure finds herself worse off. It is *Pareto efficient* in that there is no way to change it to make someone better off without making someone worse off. It is efficient in an even stronger sense. There is simply no way at all to make anyone better off. Everyone has the highest possible payoff that they could get in any network structure. The key to both these properties is that information flows freely around the ring, so that for the price of one connection a player gets all the information that there is.

Consider a player in such a ring who changes her strategy. She could establish additional links, in which case she pays more and p. 166 ↳ gets no more information. She could break her link, in which case she would forego the cost but get no information. She could break the link and establish one or more new ones, but every way to do that would deliver less than total information. Every deviation leaves her worst off. That is to say that the ring is a *strict* Nash equilibrium of the game.

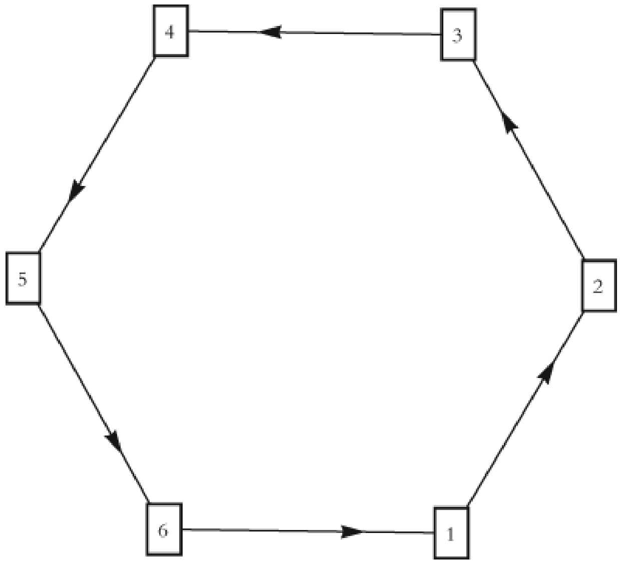


Figure 14.1: An information ring.

Now let us ask a different question. Suppose that, starting from the ring, there is some lucky guy that everyone else would like to make better off, even if they have to sacrifice something to do it. There is nothing they can do! He is already getting all the information at the cost of one link. They cannot alter their links so as to give him more information, since he is already getting it all. Only he can avoid the cost of the link by breaking it—that is, not visiting anyone—but then he gets no information at all. The ring is strongly *efficient*.

Given these rather strong optimality properties of the ring, it is of interest to see if individuals playing this game can *learn* to form a signaling network with the structure of the ring. Experimental evidence is that in small group interactions with this game structure, individuals *do* spontaneously learn to form rings.[5] Do we have a plausible model of network dynamics that can account for this?[6]

A dynamic model of network formation

Robin Pemantle and I advanced a low rationality, trial-and-error model of network formation in 2000. The idea was the individuals start out by interacting at random and then learn *with whom to interact* by the kind of reinforcement learning with which we are familiar. Here is a simple model of the process. Each individual starts out with an urn with one ball of one color for each possible p. 167 ↳ choice. Each day each individual chooses a ball from his urn, visits the indicated individual and has an interaction. Visits are always received. Visitor and visitee take numbers of balls of the partner's color proportional to the payoff received and add them to their respective urns. This is just learning whom to visit by the kind of reinforcement learning we have already studied in connection with learning to signal.

The Bala–Goyal game fits within this framework. We substitute altering connections for visiting, assume that information transfer is fast between changes in connections, and keep the reinforcement learning. Then we can justify the payoff function used by Bala and Goyal, and the equilibrium analysis is unchanged. However, network formation by reinforcement does not learn the ring. Rather extensive simulations show individuals maintaining probabilistic links with a variety of contacts. The structure is different each time. We just don't see the ring crystallizing out.[7]

There is no reason at all to believe that reinforcement learning should lead to an optimal solution to *every* problem. This is a situation where a little more sophistication in learning might be useful.

Simple inductive learning[8]

Suppose we move up to simple inductive learning. Individuals observe others′ acts, form predictive probabilities by taking the average of their acts in the past, and choose a best response to those acts. If there are ties for best response, the players flip a coin. Instead of keeping track of rewards, individuals see how others′ acts affect their payoffs, attempt to predict those acts in a simple way, and choose strategically. Now individuals often learn ↳ the ring, but not always. It is still possible (but not likely) to get stuck in a sub-optimal state.

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Slight modifications to this process, however, lead to uniform success. If players treat approximate ties as ties—for instance by computing expected payoff just to two decimal places—they always learn the ring network. The little bit of noise generated by approximate ties gets them out of the sub-optimal states. Here a little decrease in rationality helps. Can we reduce it more and still succeed?

Best response with inertia

Suppose we decrease our agent′s sophistication a little more. Let′s get rid of the inductive logic and just keep the best response. Most of the time players just keep on doing what they did last time, but once in a while someone wakes up and chooses a best response to what others did last time. She remembers the whole network structure as it was, assumes that no one else will change, and alters her network connections in the optimal way given that assumption. This is called *best response with inertia*. Bala and Goyal prove that this dynamics always learns the ring.

Low information—low rationality

The level of rationality has been lowered to rather modest levels. But our players still have to know some things in order to best-respond. They need to know the structure of the game: that is, how the actions of others affect their payoffs. And they need to know the existing network structure—what everyone did last time. There are circumstances in which these requirements are not plausible. Consider Malinowski′s own observation about the Kula:

> Yet it must be remembered that what appears to us an extensive, complicated, and yet well ordered institution is the outcome of so many doings and pursuits, carried on by savages, who have no laws or aims or charters definitively laid down. They have no knowledge of the *total outline* of any ↳ of their social structure. They know their own motives, know the purpose of individual actions and the rules which apply to them, but how, out of these, the whole collective institution shapes, this is beyond their mental range. Not even the most intelligent native has any clear idea of the Kula as a big, organized social construction…[9]

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So we are led to ask whether there is a plausible low-information, low-rationality learning that succeeds here where reinforcement learning fails.

Probe and adjust

Suppose our individuals don't know the payoff structure of the game, and don't know what others have done. They only know what actions are open to them, what they have done, and what payoffs they got. We are almost back where we started. Again we suppose that individuals usually just keep doing what they are used to, but occasionally an individual chooses to explore. These choices are infrequent and independent, just like in the Bala–Goyal best-response dynamics. Call them *probes*. When an individual probes, she notes her payoff and compares it with what she is used to. If it is better she sticks with the probe strategy. If it is worse, she goes back to her old strategy. Ties are broken by a coin flip.

If probes are infrequent and independent, then it is very unlikely that multiple individuals probe at the same time, or subsequent times. Most individual probes occur in a context where everyone has been doing the same thing, a single individual tries an alternative while others stay the same, and the individual adjusts to the new strategy while others remain the same if it gives an increased payoff. If probes are very infrequent, then almost all probes will have this character. We can analyze a simplified process where they all do.

In the simplified process, nothing happens unless there is a probe. We can then focus on the states before and p. 170 after a probe, with the ↳ transition probabilities being governed by the probabilities of probes (uniform) and their results. This gives us an embedded Markov chain. The rings are absorbing states, and the only ones. From any other state we can get to a ring with positive probability.[10] It follows that the embedded Markov process *always learns the ring*. That means that the original probe and adjust learning, learns the ring and stays close to it except for little fluctuations caused by ongoing probes. We have shown that it is possible for decentralized agents without global knowledge or strategic sophistication to learn the ring structure.[11]

Breakdown of the ring

Many primitive societies have ring exchanges of one sort or another. Susan McKinnon finds "male" and "female" gifts flowing in opposite directions in a ring structure in the Tanimbar islands.[12] Ceremonial exchange cycles are often accompanied with real economic exchange—with trade. They often interact with kinship. Exchange rings have been studied among aboriginals in Australia and Bantu in Africa. Some take place on land, so one cannot simply assume that the ring follows from the geography of a set of islands.

But ring structures are not to be found in every society. As societies become more complex, rings give way to other topologies. Claude Lévi-Strauss and others associate rings with egalitarian exchange, and the breakdown of the ring with the development of inegalitarian arrangements. Whether this generalization holds good or not, it is of interest to see why rings may not persist. In order to understand the breakdown of the ring, we p. 171 might begin by ↳ investigating what happens when the assumptions behind the Bala–Goyal ring model are relaxed.

Relaxing assumptions

Information may decay (e.g. be corrupted by noise) as it passes form one node to another. This would be increasingly important when the ring grows large enough so that information passing through many links becomes highly degraded. Information flow may be two-way, rather than one-way. If so, costs of connection might be shared in various ways—perhaps with individuals bargaining over how the costs are shared. The items of information originating with different individuals might not be independent, or may not be equally valuable. Each of these variations, taken either separately or in combination, can make a difference. They are being actively explored in a growing literature.[13]

To get a little feeling for these factors, suppose we have a ring as in the Bala–Goyal game, and information flows without decay in two directions. Then any member of the ring would be happy to drop her connection, saving the connection cost and still getting all the information. The resultant topology is a line:

> •→•→•→•→•→•

The individual at the foot of the line now free-rides on the others. He is not cut off because his information is worth more than the cost his neighbor pays to connect to him.

This arrangement is an equilibrium, in that no one can now do better by changing. But there are changes in which others do no worse. For instance, the individual at the head of the line might just as well break his connection and connect to someone else. He still pays the cost of one connection and gets all of the
p. 172 information. ↳ This is also an equilibrium. There are now lots of possible equilibria, and the population could drift among them.

But now suppose that one player, the center, pays to connect to each of the other players, and that is all. They are now in a star topology, as shown in figure 14.2.

Since the center is paying for all connections, it is called a *center-sponsored star*. Suppose that value of a unit of information is 1 and cost

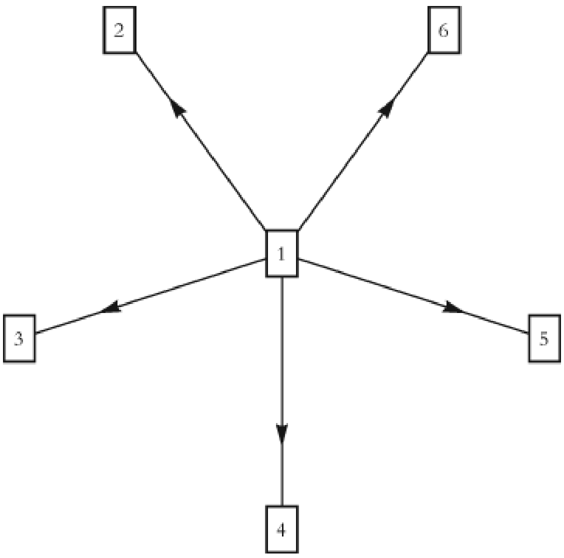

Figure 14.2: A star network with six players.

of establishing a link is 0.1. Then the center gets all the information, 6.0, less the cost of maintaining 5 links, for a net payoff of 5.5. Each of the other players free-rides on the links for a net payoff of 6.0.

You can verify that this is a strict Nash equilibrium. Any players would be worse off by deviating. If the center broke a link he would save 0.1 and lose 1.0. If a peripheral player made a link he would have to pay for it, whereas he now gets all the information without paying. If he broke a link he would simply lose information.

Bala and Goyal establish that in the case of two-way flow of information without decay, the center sponsored
p. 173 star is the unique ↳ strict Nash equilibrium. (Providing the value of a unit of information is greater than the cost of making a link.) They also show that the process of best response with inertia learns this star configuration with probability one. For a low rationality, payoff-based approach, we find that *probe and adjust* dynamics here approximates the star in the same way that it approximated the ring in the earlier model.

Laboratory experiments, however, show that human subjects perform quite differently in the two Bala–Goyal games. In experiments by Armin Falk and Michael Kosfeld[14] subjects learned the ring quite well, but never learned the center-sponsored star. Explanations that have been floated focus on the difficulty of deciding who would be in the center, and distaste for being the one player who pays for all the connections.

Everything is changed if there is *information decay*. Suppose that 10% of the information is lost as it passes through a link. The value of a bit of information is taken as 1 and the cost of connection as .1.

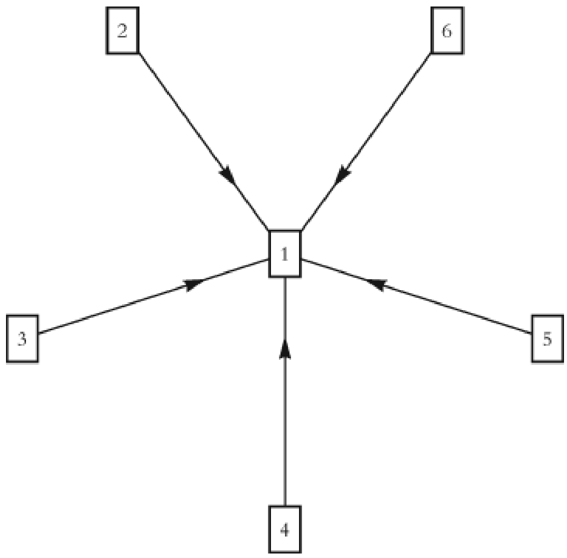

Figure 14.3: A star network with six players.

p. 174 Now consider a *periphery-sponsored star*, where five players on the periphery pay to connect to the one in the center.

The lucky center pays no costs and is one link away from everyone else, for a payoff of 5.5. The players on the periphery, after cost, have a payoff of 5.04. This is a strict equilibrium. It isn't worth it for players on the periphery to pay .1 for a direct connection to one another to replace second-hand information that passed through the center (value .81) with first-hand information (value .9). They certainly don't want to break the connection to the center, who provides lots of second-hand information. His centrality is an asset. But note that with these values for cost of link formation and information decay, the center-sponsored star that we considered earlier is also a strict Nash equilibrium. If we raise the cost of connection up to 1.1, the center-sponsored star is not viable. The center would have to pay more for each connection than it is worth. But the periphery-sponsored star is viable if the players can get to it. Each peripheral player pays 1.1, which is well worth it given all the information she gets from the center.

Daniel Hojman and Adam Szeidl[15] have taken information decay further. They have constructed a class of models in which the *periphery-sponsored star* plays a special role. There is two-way flow of information, with strong decay in the value of information received, and with some "cut-off" distance such that information flowing from more distant sources is worthless. In the basic model, the initiator of a link bears its full costs, and all individuals are the same, although modifications of these assumptions are also explored.

In one realization of the basic model, each individual observes the state of the world with some error, errors being independent across individuals and error probabilities being the same for each individual. Increasing information exhibits decreasing returns—when you know with high probability the state, further
p. 175 confirmation is not worth so much. When observations are communicated ↳ from one to another, further

errors are introduced. For this basic model, Hojman and Szeidl prove that the unique Nash equilibrium of the network formation game is a *periphery-sponsored star*.

There are laboratory experiments on a game related to Hojman and Szeidel's model, but not quite the same. Seigfreid Berninghaus, Karl-Martin Ehrhart, Marion Ott, and Bodo Vogt.[16] consider a game where you get all your connection's information and if you paid for the link, you get all his connection's information as well, and that's all. There is no information decay per se, but the sharp cutoffs put a similar premium on short path length. Where the strict Nash equilibrium is a periphery-sponsored star, they find experimental subjects learning this star structure. However, when in such a star subjects sometimes fluctuate out and end up back in a star with a different individual as center.

It is remarkable how we now see breakdown of the ring and evolution of hierarchy, even though the population is perfectly homogeneous. Real populations are not homogeneous, of course, and some individuals have more valuable information than others. This may be because they are deeper thinkers, or better situated observers, or some combination of such factors. Hojman and Szeidl consider the effect of modifying their basic model to take account of such heterogeneity.

The equilibrium result is a population structure of interlinked stars, where the central figure of one star establishes a link to the central figure of another, either directly or through an intermediary. If there is a little noise operating, the central figures are the ones with the high-value information.

Conclusion

What are we to make of the parallels between dynamics of information networks and the networks of symbolic
p. 176 exchange ↳ discussed by anthropologists? I don't know the answer, but the question merits examination. The transition from rings to stars, linked stars, and more complicated structures is clearly of fundamental importance. The models discussed here already tell us very interesting things, but there is clearly a lot more to learn.

In this chapter, we introduced a low-rationality *probe and adjust* dynamics to approximate higher rationality learning in the basic Bala–Goyal models. Both best response dynamics and *probe and adjust* learned networks that reinforcement learning did not. Most of the dynamic models of network formation in the literature are either based on simple reinforcement learning,[17] or on some kind of best-response dynamics. The kind of best response with inertia used by Bala and Goyal is also used by other game theorists.[18] We would therefore like to know how far our results generalize.

It should be evident that, in general, *probe and adjust* learns a network structure if *best response with inertia* does. In the literature on *best response with inertia*, it is always assumed that responses are rare enough that we can assume that players don't respond simultaneously. This simpler idealized process is analyzed, just as we did with *probe and adjust*. If there is a best response move, then with positive probability *probe and adjust* will make it. Then suppose that, for every state, there is a positive probability best response path to an absorbing network state. It follows that, for every state, there is also a positive probability *probe and adjust* path to that state. If high rationality best response learns the network, the low rationality *probe and adjust* does so as well.[19]

Notes

1 Malinowski 1922.
2 Malinowski 1920.
3 Bala and Goyal 2000.

4 They call it a “wheel.”

5 Callander and Plott 2005.

6 The interested reader should consult modeling specifically directed at the Kula ring by Rolf Ziegler, which takes a somewhat different approach. See Ziegler 2007, 2008.

7 One might think of trying alternative forms of reinforcement learning, but—if anything—they tend to do worse.

8 See Huttegger and Skyrms 2008.

9 Malinowski 1922.

10 This follows from the analysis given by Bala and Goyal for best response dynamics. [Theorem 3.1].

11 This “Probe and Adjust” dynamics is similar to, but not identical with, the dynamics studied by Marden et al. 2009. For more on low rationality, payoff-based dynamics see Young 2009.

12 McKinnon 1991.

13 Bloch and Jackson 2007; Galeotti et al. 2006; Jackson 2008; Goyal 2007.

14 Falk and Kosfeld 2003.

15 Hojman and Szeidl 2008.

16 Berninghaus et al. 2007.

17 Skyrms and Pemantle 2000; Liggett and Rolles 2004; Pemantle and Skyrms 2004a, 2004b.

18 For instance, Watts 2001; Jackson and Watts 2002, apply this to network formation.

19 There are network games in which these dynamics do not learn the optimal network, such as the case of the Bala–Goyal peripheral sponsored star with high cost of connection discussed above. Starting without connections, any connection costs more than it is worth. It is only when the structure is in place that it pays everyone to maintain it.