University of Massachusetts at Dartmouth Physics 421, Fall 2025
Advanced Undergraduate Physics Laboratory
Determination of the Hubble Constant from Type Ia Supernovae

Introduction

In the late 1990s, the physics world was rocked by the announcement that not only was the universe expanding (which had been known for some 70 years since the work of Edwin Hubble), but its expansion was also *accelerating*. This in turn implied the existence of a mysterious form of energy which became known as *dark energy*. The Nobel committee recognized the significance of this work by awarding the 2011 Nobel prize in physics to the leaders of two projects which spearheaded these discoveries – Adam Riess and Brian Schmidt of the Supernova Search Team, based primarily at the Harvard Center for Astrophysics, and Saul Perlmutter, who led the Supernova Cosmology Project at Lawrence Berkeley National Laboratory.

More recently, the same class of Type Ia supernovae utilized in the 2011 Nobel-winning research have been at the center of one of the biggest crises in modern physics. Astrophysicists have measured the rate of expansion of the universe, which is quantified by the Hubble constant H_0, using Type Ia supernovae in the local universe. They have simultaneously obtained measurements of H_0 derived from the cosmic microwave background from the early universe. The values of H_0 measured using these independent methods do not agree with one another, and the disagreement has become statistically more significant over time, as each of the independent methods have become more precise (see figure 1). The discrepancy may point to new early universe physics, such as "early dark energy," or new light particles, such as neutrinos. Alternatively, the discrepancy may be resolved by a more accurate calibration of the nearby supernovae, accounting for dust in the nearby universe, or possibly poorly-understood systematics relating to the unknown explosion mechanism of Type Ia supernovae.

Reading: Cosmologists Debate How Fast the Universe is Expanding : `http://bit.ly/31aFfFk`

In this lab, you will determine the Hubble constant for yourself using a large sample of Type Ia supernovae.

1. First Steps : Calibrating Type Ia Supernovae as Standard Candles

The problem of determining distances in astronomy is a very challenging one. Stars and galaxies do not carry with them road signs which mark off their distances, so astronomers have cleverly devised numerous methods to measure distances. These are shown schematically in figure 2. Note that each method is valid only over a certain distance range – for example, parallax is useful only up to about 5000 pc, even with the most advanced and dedicated space telescopes such as Gaia. Astronomers refer to the overlapping methods of distance determinations as rungs on the cosmic *distance ladder*. Fortunately, the various rungs on the cosmic ladder overlap, enabling us to calibrate the more distant methods against the closer methods.

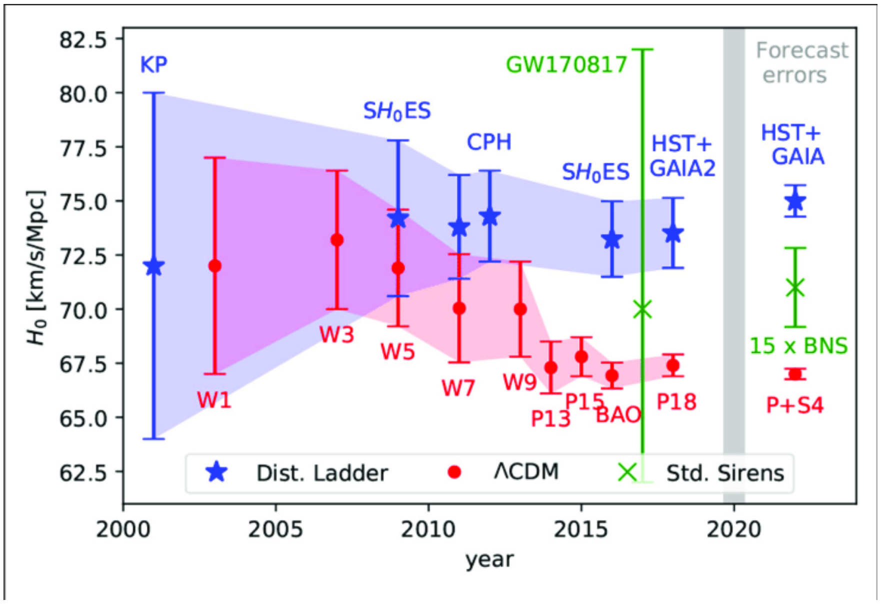

Figure 1: The Hubble constant, H_0, measured over time using both early universe physics, assuming a Λ cold dark matter (CDM) universe (red), and using Type Ia supernovae in the local universe (blue). Also shown is the one existing data point from a "standard siren" as inferred from the binary neutron star merger event GW170817 (BNS, green). The data points to the right of the grey bar are the errors forecast from future measurements using each of the three methods.

We will follow work done by Allan Sandage and others in the late 90s, who noted that the Type Ia supernova 1990N went off in the nearby galaxy NGC 4639. This galaxy is fortuitously close enough that Cepheid variables have been imaged in it using the Hubble Space Telescope. You will measure the distance to NGC 4639 using a catalog of Cepheids, and from this distance, as well as the peak apparent magnitude of supernova 1990N, you will infer the absolute magnitude of SN 1990N. Type Ia supernovae are excellent standard candles, as we will see later, so this calibration of their absolute magnitudes establishes a critical rung on the distance ladder.

A set of sample Type Ia light curves is shown in figure 6. Distances to each of these events were determined using an independent method, from which the absolute magnitude could be obtained. What is being plotted on the top portion of the figure is essentially the absolute peak V-band magnitude of each event (higher is more negative on this plot and therefore brighter). It can be seen that all "normal" Type Ia supernovae explode with nearly the same intrinsic luminosity, differing by only about one magnitude, or about a factor of three in terms of peak luminosity.

Cepheid variables undergo a regular variation in their brightness—see figure 4. Crucially, Cepheid variables have a well-defined correlation between their luminosity and period, which makes them excellent standardizable candles. Recent Gaia astrometric measurements have enabled a recalibration of this relation using precise parallaxes for Galactic Cepheids. Trahin

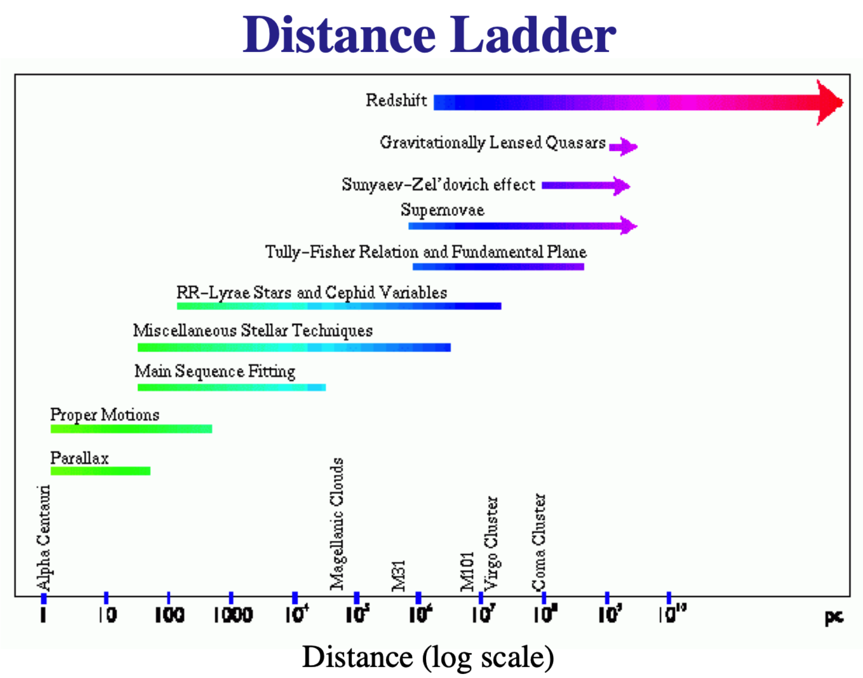

Figure 2: A schematic diagram (courtesy of George Djorgovski) of the cosmic distance ladder from several years ago. The reach of the parallax method has expanded dramatically with the Gaia space telescope, which has measured accurate parallaxes for stars out to several kiloparsecs (up to about 5 kpc for bright sources) in its published data releases.

et al. (2021) analyzed 63 Milky Way Classical Cepheids with Gaia EDR3 parallaxes and determined the following updated period–luminosity relation in the V band:

$$\langle M_V \rangle = -2.775(\pm 0.103) \log_{10} P - 1.53(\pm 0.09), \tag{1}$$

where $\langle M_V \rangle$ is the mean absolute magnitude and P is the pulsation period in days. This calibration, based directly on Gaia parallaxes, refines the older Madore & Freedman (1991) relation and provides a more accurate zero point tied to the Galactic distance scale. Thanks to Gaia's astrometry, the relative uncertainty in the luminosity scale has been reduced to about 1%, compared to several percent in pre-Gaia datasets, representing a major improvement in the calibration of Cepheid standard candles.

Sandage and colleagues imaged 18 distinct Cepheid variables in NGC 4639 and fit their data to obtain both the period of oscillation of each Cepheid as well as its apparent mean V band magnitude. These results are shown in the table in figure 5.

The absolute and apparent magnitudes M_V and m_V are related to each other by

$$m_V = M_V + \mu \tag{2}$$

where μ is the distance modulus:

$$\mu = 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right) \tag{3}$$

From the period-luminosity (PL) relation for Cepheids:

$$\langle M_V \rangle = A \log_{10} P + B \tag{4}$$

where a and b are the slope and y-intercept of this calibrated fit from Equation 1.

Substituting $M_V = m_v - \mu$, we obtain

$$\langle m_V \rangle = (A \log_{10} P + B) + \mu \tag{5}$$

So, on a linear fit of $\langle m_V \rangle$ versus $\log_{10} P$,

$$\langle m_V \rangle = a \log_{10} P + b, \tag{6}$$

where A and B are the slope and y-intercept, respectively, of the fit that you will obtain. Comparing Equations 5 and 6, the y measured y-intercept will be $b = B + \mu$.

Thus, we obtain the key result

$$\boxed{\mu = b - B} \tag{7}$$

Given a calibrated PL relation between absolute magnitude and period for a population of galactic Cepheids, and a more distant population with measured apparent magnitudes and periods, we can infer the distance modulus and hence the distance to the more distant set of Cepheids.

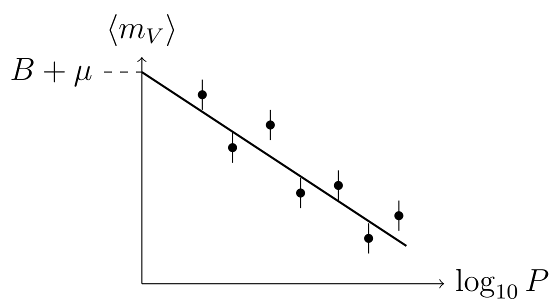

Figure 3: Schematic showing the relationship between apparent magnitude m_V and period for Cepheids. The apparent magnitude slope a should be identical to the calibrated M_V slope A.

Plot the data $\langle m_V \rangle$ versus log period from figure 5, and do a linear fit. Use the y-intercept of your fit to determine the distance modulus to NGC 4369. What is your measured distance to NGC 4369 (in Mpc)?

SN 1990N was followed from 14 days before maximum to well afterward peak brightness. Its apparent V-band magnitude at peak brightness was measured to be $m_V = 12.61 \pm 0.05$ Leibundgut et al. 1991.

Use the distance modulus you inferred for NGC 4369 to determine the absolute magnitude of SN 1990N.

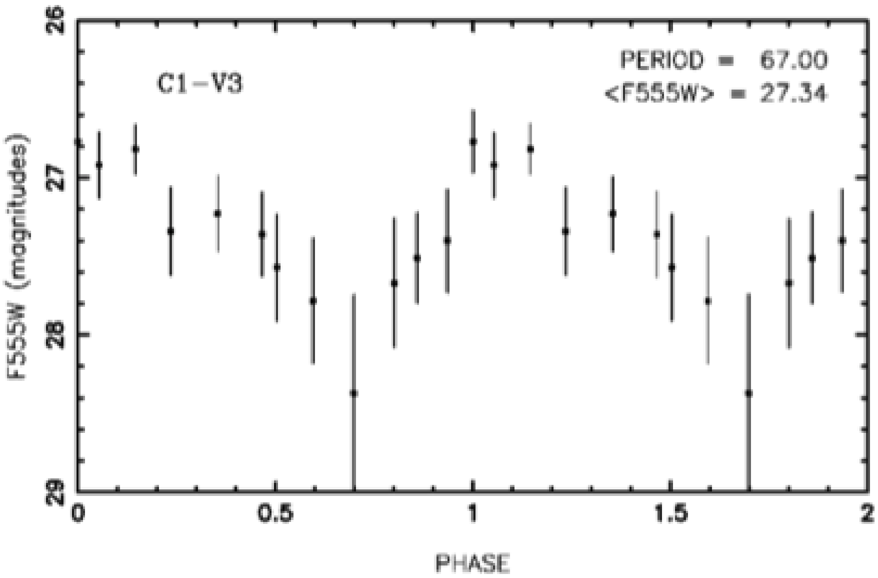

Figure 4: A Hubble Space Telescope light curve from a Cepheid variable (called CV-V3 in the Sandage paper) in NGC 4639. The light is gathered through a filter (F555W) on Hubble's Wide Field Planetary Camera 3 (WFC3), so what is being plotted is the apparent F555W magnitude versus the phase of the cycle. The best-fit period (67.00 days) and mean F555W magnitude (27.34) are shown. These magnitudes were converted to V band magnitudes in the table shown in figure 5 by Sandage and colleagues.

2. The Hubble Constant from Standard Candle Type Ia Supernovae

A set of sample Type Ia light curves is shown in figure 6. Distances to each of these events were determined using an independent method, from which the absolute magnitude could be obtained. What is being plotted on the top portion of the figure is essentially the absolute peak V-band magnitude of each event (higher is more negative on this plot and therefore brighter). It can be seen that all "normal" Type Ia supernovae explode with nearly the same intrinsic luminosity, differing by only about one magnitude, or about a factor of three in terms of peak luminosity.

We will begin by making the simplifying assumption that all Type Ia explode with the same intrinsic peak luminosity, regardless of when or where they exploded in the universe. In making this approximation, we will say that Type Ia supernovae are *standard candles.* In doing so we are following in the footsteps of the those in the field, who themselves took much the same path up to the mid-to-late 1990s.

For this lab, we will utilize automated measurements made with the Apache Point Telescope during the Sloan Digital Sky Survey, which measured light curves for 146 spectroscopically-confirmed Type Ia in the redshift range $z = 0.05 - 0.4$. You will access this dataset through a containerized Docker environment hosted by Github. Go to

`https://github.com/rtfisher/supernova-sdss-lab`

Object	Period (days)	$\langle V \rangle$	$\sigma\langle V \rangle$
C1-V1	29.7	27.09	0.28
C1-V2	21.7	26.84	0.26
C1-V3	67.0	27.33	0.35
C1-V5	32.0	26.20	0.13
C2-V1	41.0	25.79	0.18
C2-V3	48.0	25.24	0.26
C2-V4	26.0	26.67	0.34
C2-V5	17.0	26.86	0.29
C2-V6	30.0	26.18	0.25
C2-V7	21.0	26.96	0.30
C3-V1	34.5	26.72	0.17
C3-V6	34.0	25.87	0.13
C3-V7	32.0	26.26	0.17
C3-V8	21.0	26.28	0.25
C3-V9	17.0	27.22	0.29
C3-V10......	40.0	25.65	0.11
C3-V11......	58.0	26.39	0.19
C4-V1	52.0	25.98	0.10

Figure 5: A table showing the period, mean V band magnitude, and error bar on the V-band magnitude for the Cepheid sample in NGC 4369, from Saha et al, 1997.

First, click on the "Fork" button on the top right. This will create a copy of the repository in your own Github account. Click on the "Create fork" green button on the next page. Next, navigate to this new repository in your own account.

In the web browser, click on the green "Code" button, and then select the Codespces tab. From there, you can create a free Codespace by clicking on the green "Create codespace on main" button. A new tab will open with your codespace environment. IN the lower right, you will see a progress bar, "Setting up remote connection: Building codespace..." which will take a few minutes to configure. Once the codespace has been configured, you will have access to a Linux terminal and a Python 3.x environment. The free accounts have a limit of 120 hours of compute, which should be more than sufficient for this lab.

The data comes packed in a tarball, and you will need to untar it to access the contents. This can be accomplished with the command `tar -xf data.tar` in the Linux terminal at the bottom of the codespace window.

The working environment is similar to JupyterHub if you have used it previously, with a full Linux VM + Docker container running in the cloud, tied to the repo.

The redshifts z to each supernova is measured by the survey, and are known. Therefore, for nearby supernovae, we can determine the rate of expansion of the Universe as parameterized by the value of Hubble's constant H_0 :

$$v = H_0 d_L \tag{8}$$

Noting that $v = cz$ for nearby supernovae, we have

$$cz = H_0 d_L \tag{9}$$

Therefore, the value of H_0 can be determined by a straight-line fit to a plot of cz versus d_L for a sample of nearby supernovae.

Note that the units of H_0 are velocity per distance, or equivalently, inverse time. For the standard Λ CDM cosmology, it can be shown that the age of the universe, written as t_0, is very close to the the inverse Hubble constant :

$$t_0 \simeq \frac{1}{H_0} \tag{10}$$

However, instead of writing the Hubble constant in units of inverse seconds, astronomers have long measured it in more astronomically-friendly units of km/s per megaparsec (Mpc). While this may seem like an odd choice of units at first, these units are easier to use with astronomical data. In particular, the numerical value of the Hubble constant as measured in km/s/Mpc is simply the velocity of recession of a galaxy at a distance of 1 Mpc. For reference, the nearest major galaxy to the Milky Way, the Andromeda galaxy, is about 0.8 Mpc from the Milky Way.

Fit the SDSS light curve data to obtain the apparent peak magnitude for each event. Using these apparent peak magnitudes, as well as the absolute magnitude you determined for SN 1990N, generate a Hubble diagram for your dat, assuming all SNe Ia events are standard candles. Fit your Hubble diagram to obtain the Hubble constant H_0 in units of km/s/Mpc.

3. Next Steps : The Phillips Relation and the Standardizable Candle

The key discovery which made both the discovery of dark energy as well as high-precision measurements of the Hubble constant possible, was another discovery published in 1993 by Mark Phillips. Phillips studied a sample of 9 relatively nearby, low-redshift supernovae, for which distances, and therefore also their intrinsic brightnesses, as measured by their absolute magnitudes, were determined. When plotting the light curves, he realized that, the greater the intrinsic brightness of a source, the longer the decay time of the light curve – see figure 6. He quantified this decay time in terms of a parameter Δm_{15}, which is the number of magnitudes with which the light curve of the supernova would decline over the 15 days since maximum brightness.

Reading : Phillips, M., The Absolute Magnitudes of Type Ia Supenovae, 1993 – see refs.

Fit your light curves to obtain the decline rate Δm_{15} for each event. Using the Phillips relation and the apparent peak magnitude, you will obtain a second, more accurate Hubble diagram. Fit to obtain the standardizable SNe Ia candle H_0 value in units of km/s/Mpc.

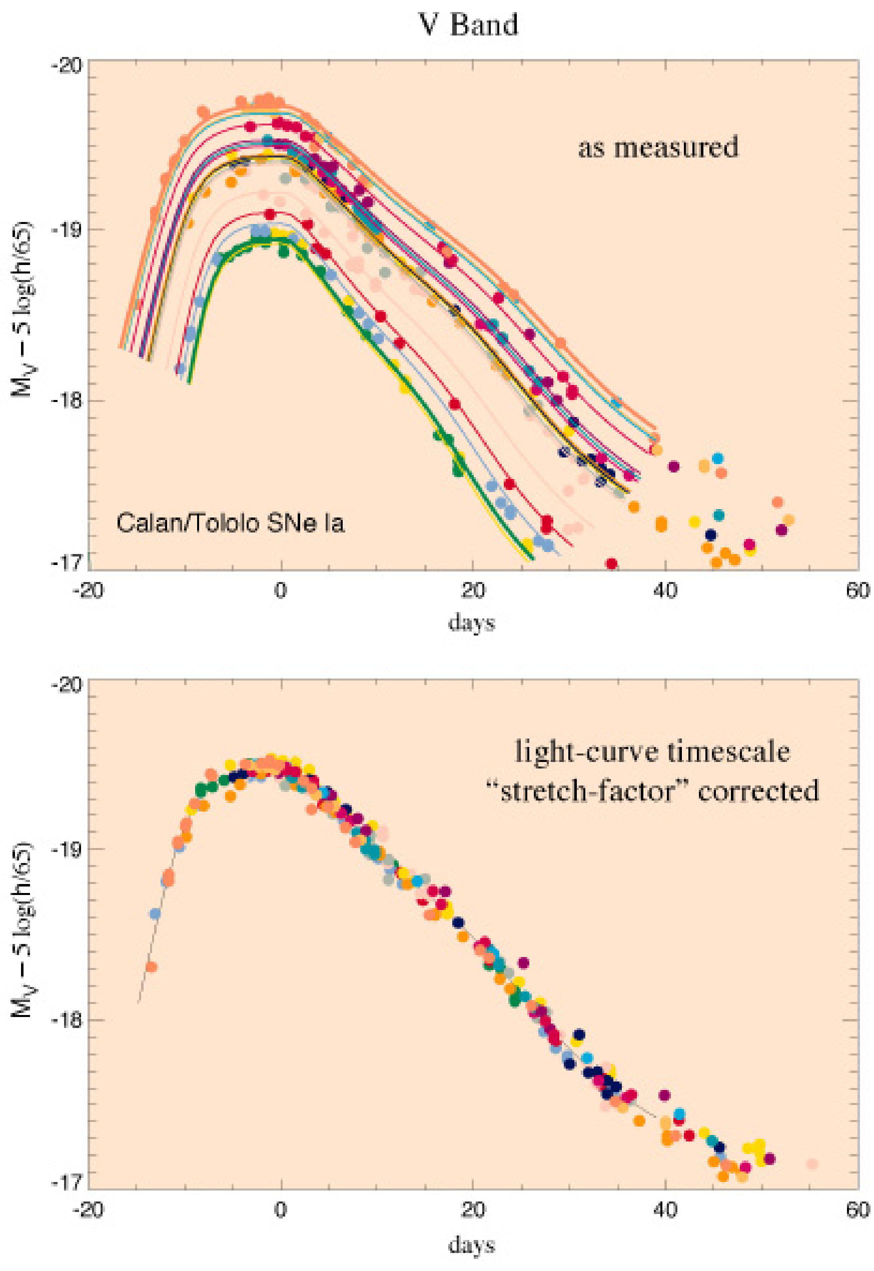

Figure 6: Light curves for a nearby sample of supernovae, plotted in terms of absolute magnitude versus time since peak brightness. The top plot clearly shows that, the greater the intrinsic brightness of a supernova, the greater its decay time. The bottom plot demonstrates the resulting dispersion in the light curves, once their decay time is calibrated to account for this correlation between peak brightness and decay time.

Appendix A : Observational Astronomy Background

In terms of basic physical quantities, the electromagnetic energy radiated by a body, per unit time, is referred to by astronomers as its *luminosity*, L. The total electromagnetic luminosity radiated across the entire spectrum is known as the *bolometric luminosity.* It is important to bear in mind that although the bolometric luminosity is a very useful physics concept, most detectors are sensitive only to a portion of the electromagnetic spectrum – e.g. the optical, IR, UV, X-ray, etc. Thus the bolometric luminosity is rarely directly measured; instead the energy which is detected in certain "broadband" filters is more relevant. We will return to the concepts of filters below.

Furthermore, the luminosity would only be directly measured if one could encircle the entire source with a sphere capable of observing all of the emergent radiation from the source. Instead, detectors measure only the radiation hitting their detectors per unit time, per unit area : the *flux.* The measured flux of the source is simply

$$F = \frac{L}{4\pi d^2}, \tag{11}$$

where d is the distance to the source, assuming that the radiation is emitted equally in all directions. In fact, we can use this equation to *define* the *luminosity distance* to the source by

$$d_L = \sqrt{\frac{L}{4\pi F}} \tag{12}$$

where L is the luminosity of the source and F is the flux.

The vast distances encountered in cosmology make the standard SI meter unit unwieldy. In the popular press, astronomical distances are frequently referred to in light-years. However, you will never hear an astronomer speak in terms of light-years in a professional context. The reason why is because light-years cannot be measured directly, and there is a much more direct measure of distance in astronomy, as inferred from the *stellar parallax.* One arcsecond of parallax across the 1 AU baseline of the Earth-sun distance is defined to be one *parsec*, or pc. It is straightforward to show from basic geometry that 1 pc = 206265 AU = 3.08578×10^{16} m.

Further, astronomers will rarely speak directly in terms of flux. Instead, they typically refer to what is known as the *magnitude scale.* The key concept underlying the magnitude scale is that it quantifies flux *relative to a standard reference.* In the past, the star Vega was defined to be magnitude zero. While modern calibration methods have replaced these older ones, Vega is still quite close to zero magnitude.

$$m - m_{\rm ref} = -2.5 \log \left(F/F_{\rm ref}\right) \tag{13}$$

Defined in this way, *each decrement of 5 on the magnitude scale is exactly an increase in flux of a factor of 100.* Thus, a star with magnitude 5 is 100 times fainter, or produces 100 times less flux, than a star of magnitude zero. Note that *lower is brighter* on the magnitude scale, and that magnitudes can be negative as well as positive. The sun, for example, has a magnitude of -27, making it 6×10^{10} brighter than Vega!

Our sun is quite apparently bright, even though it is a relative modest main sequence G type star. Other stars are apparently very faint, even though they may be intrinsically quite bright. In order to compare the intrinsic luminosities of stars directly against one another, astronomers define the *absolute magnitude* M of a star. The absolute magnitude is simply defined as the magnitude the star would have at a standard reference distance, defined to be 10 pc. An alternative way of capturing these same concepts uses the *distance modulus*. The astronomical distance modulus is defined as $m - M$, and can be related to the distance from the equation :

$$m - M = 5 \log_{10}(d_L/10) \tag{14}$$

Here m is the apparent magnitude of the star, M its absolute magnitude, and d_L is assumed to be in parsecs.

The peak apparent magnitude m of a supernova can be measured directly from its light curve. If one can also determine the absolute magnitude of the supernova, M, the distance can be inferred from this equation.

As we mentioned earlier, filters are used by astronomers to allow some portion of the electromagnetic spectrum to reach their detectors. The filter itself is a piece of colored glass. The "Johnson" UBV filters employed by Mark Phillips in his 1993 paper differ from the ugriz filters utilized by the SDSS – see figure 7. This is an issue you will confront almost immediately when you attempt to convert between your Δm_{15} as determined by SDSS and Phillips' original data.

Fortunately, this is a common issue, and there are a number of approaches on how to deal with it. You can read about some of these at `http://www.sdss.org/dr5/algorithms/sdssUBVRITransform.html`.

One of the more nuanced issues which supernova astronomers deal with is that of *dust*. Each line of sight to a distant supernova must pass through our own galaxy, as well as the parent galaxy of the supernovae, both of which are filled with interstellar gas. The dust absorbs some of the radiation, which can make supernovae appear more distant than they actually are. The dust map of our own galaxy was measured in a classic survey by Shlegel, Finkbeiner, and Davis. This dust extinction, in each of the ugriz SDSS filter bands, is listed for the line of sight to each supernova in the SDSS data files.

One further subtle effect deals with the impact of cosmological redshift on the measured luminosity, or flux. Let's consider two galaxies, which radiate primarily in the optical through sunlight. One galaxy is nearby at $z = 0$, and the other is very distant – let's say at $z = 10$. Let's say the nearby galaxy has a measured Because the redshift z is related to the shift in the rest-wavelength λ by :

$$z = \frac{\Delta\lambda}{\lambda} \tag{15}$$

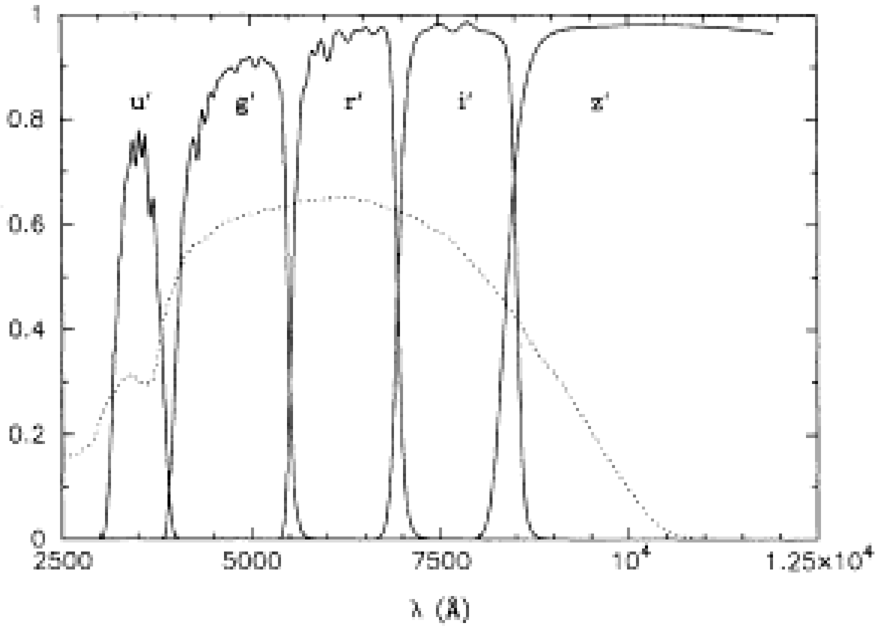

Figure 7: A plot of the transmittance of the Sloan filters as a function of wavelength, from Fukugita et al (1996). Here 0 means total absorption and 1 total transmission at a given wavelength.

the distant galaxy's luminosity is shifted into the red by the cosmological expansion by a factor of 10 ($\Delta\lambda = 10\lambda$), and therefore radiates primarily in the infrared, as opposed to the optical. Consequently, the measured optical flux will be nearly zero, and its measured optical magnitude will be very large, $m_V >> 1$.

Now, consider the same galaxy at some intermediate redshift. Its measured optical magnitude $m_V(z = 10)$ will be somewhere between the rest measurement $m_V(z = 0)$ and the measurement $m_V(z = 10)$. The cosmological redshift will cause the spectrum to shift to the red, pushing some of the radiation out of the observed bands, though not nearly as much as the high redshift case. In fact, we could invent a parameter (let's call it K) which relates the two measured magnitudes in the rest-frame ($z = 0$) and at redshift z. This is in fact, precisely what Hubble did in 1936, when he discovered that the numbers counts of galaxies differed from a hypothetical uniform distribution as a function of redshift :

$$m_V(z,t) = K(z,t) + m_V(0,t) \tag{16}$$

(Remember that the magnitude scale is a logarithmic one for the flux; adding the K correction to the magnitude is equivalent to a multiplicative correction to the overall flux.) Astronomers call this effect the "K-correction" – the name evidently traces back to the German word "Konstante," or constant.

Note that we have written $K(z,t)$ to express that for a homogeneous sample of sources, the K-parameter depends on the redshift z as well as the time of observation t. The time-dependence originates from the fact that the color of the supernova event, which fundamentally is set by how much radiation emerges in the red versus the blue part of the spectrum, varies over time in a relatively complicated way due to the physics of radiative transfer through the expanding supernova shell.

The K correction is relatively complex to implement accurately, and it is best to sidestep this issue entirely. Consequently you should attempt to minimize its effects by focusing on the lowest redshift supernovae in your sample to measure H_0. Measuring the cosmic acceleration and the acceleration parameter does require supernovae in the redshift range $z \simeq 1$, and so this K correction becomes more significant when confronting the higher-redshift supernovae.

For more on the K-correction, see Nugent et al, 2002 : `http://adsabs.harvard.edu/cgi-bin/bib_query?arXiv:astro-ph/0205351`.

Appendix B. Software Notes

1) Python is also very easy to install on your own machine. It is possible to download the basic Python language installer Windows, Mac OS X, and Linux from `http://www.python.org/download`. However, for our class, you'll also need some associated libraries, including `SciPy` and `NumPy`. Therefore if you plan to use Python in the future, I recommend installing Python on your machine using the Anaconda distribution from `http://www.anaconda.com/distribution`. Anaconda installs these and over 100 other very useful libraries, as well as the Jupyter notebook system and an integrated development environment (IDE) called Spyder – none of which come with the core Python language. Further, Anaconda also includes a package manager called `conda`, which allows you to easily keep all of these libraries updated very simply. Frequently you may find yourself working on different projects, each with distinct library requirements. The Anaconda distribution allows you to create separate *environments* for each project, each with its own set of libraries and versions.

2) As you wrap up your project, please print out your Python scripts and attach them to your lab writeup, *which should also include your key plots.*

References

M. Phillips, 1993
`http://adsabs.harvard.edu/abs/1993ApJ...413L.105P`

Riess et al, 1998
`http://iopscience.iop.org/1538-3881/116/3/1009/`