Goal:

prove

$$\int (\vec{\nabla} \times \vec{v})\, d\tau = -\oint \vec{v} \times d\vec{a}$$

where $\vec{v} = \vec{v} \times \vec{c}$

$$\int \vec{\nabla} \times (\vec{v} \times \vec{c}) = -\oint (\vec{v} \times c) \times da$$

Left side 1st

$$\int \vec{\nabla} \times (\vec{v} \times \vec{c})$$

use product rule 5

$$\int (\vec{c} \cdot \vec{\nabla})\vec{v} - (\vec{v} \cdot \nabla)\vec{c} + \cancel{\vec{v}(\nabla \cdot \vec{c})} - \vec{c}(\nabla \cdot \vec{v})$$

divergence of a constant vector

$$\int (\cancel{c_x \vec{\nabla}_x} + c_y \vec{\nabla}_y + c_z \vec{\nabla}) \vec{v} - (\cancel{\vec{v}_x \vec{\nabla}_x +} \vec{v}_y \vec{\nabla}_y + v_z \vec{\nabla}_z)\vec{c} - \vec{c}(\vec{\nabla} \cdot \vec{v})$$

c integral of this would be 0.

taking derivatives of constants.

so we are left with

$-\int \vec{c}(\vec{\nabla} \cdot \vec{v})$ on the left side right side.

$\oint v \cdot da$

$\oint (\vec{v} \times \vec{c}) \cdot da$, dot product doesn't matter the order.

$\oint d\vec{a} \cdot (\vec{v} \times \vec{c})$, $\vec{v} \times \vec{c} = -\vec{c} \times \vec{v}$

$\oint -d\vec{a} \cdot (\vec{c} \times \vec{v})$

$\oint -\vec{c} \cdot (\vec{v} \times d\vec{a})$

$\int \cancel{\vec{c}} (\vec{\nabla} \cdot \vec{v}) = \oint -\cancel{\vec{c}} (\vec{v} \times d\vec{a})$

$\int (\vec{\nabla} \cdot \vec{v}) d\tau = -\oint \vec{v} \times d\vec{a}$