# Valuation Project 3: Monte Carlo

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**Executive summary**

Product: Contingent Coupon Autocallable Yield Notes due June 21, 2024
Underlying asset: S&P 500® Index, EURO STOXX 50® Index, and Nasdaq-100 Index®
Issuer: Credit Suisse AG, London branch
Estimated value per security by Credit Suisse: $977.10
Estimated value per security for Project #3: $977.59

The product has three underlying assets, pays a contingent coupon on eight different coupon payment dates, and is auto-callable on six different observation dates. To value the product, we use Monte Carlo Method because the method is good at dealing with multiple underlying assets. Our method also includes eight time steps since there are eight observation dates, and an analysis on the correlation between different assets within one, three, and five years.

**Data collection and parameters used**

We have eight time steps for eight of the observation dates.

The USD risk-free rate is 3.3271% and the EURO risk-free rate is 1.5772%, both quoted from Bloomberg terminal's OIS rate after linear interpolation of the interest rate curve.

The SX5E dividend yield is 1.866%, 1.839% for SPX, and 1.015% for NDX, are also quoted from Bloomberg terminal.

We pick the sigma combination of 22.764% for SX5E, 24.056% for SPX, and 30.120% for NDX because the input gives us the value closet to the estimated value by the issuer.

Our sensitivity analysis shows that 3-year historical correlations yields a much closer results to the estimated value than 1-year and 5-year correlations.

**Key terms**

Initial index level and autocall level are 3438.46, 3674.84, and 11265.99 for SX5E, SPX, and NDX respectively. Barrier level and knock-in level are 2234.999, 2388.646, and 7322.8935 respectively.

Principal amount per security is $1000, which we'll call it face value, pays a coupon of $27 quarterly per security on the coupon payment dates.

On observation dates, check if **each** of the underlying were greater than its respective barrier level so that the coupon is payable and occurs each quarter. On autocall observation dates, check if **any** of the underlying were equal or greater than its respective autocall level so that the note is callable.

**Methodology and Algorithms**

We did ten trials for our Monte Carlo Method, where in each trial we did one hundred thousand simulations so that our estimated value would converge to a more accurate estimate.

In order to implement the probabilistic solution with multiple underlying assets, we need to first find the square roots of the covariance matrix using Cholesky factorization. The result gives us:

$$M = \begin{pmatrix} m_{11} & 0 & \dots & 0 \\ m_{21} & m_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ m_{N1} & m_{N2} & \dots & m_{NN} \end{pmatrix}$$

Then, create a vector of standard random normal numbers, multiplying the two gives us a vector of diffusion terms, $\phi$. With that, we can easily use our probabilistic solution to do our simulation on indices' levels. The probabilistic solution gives:

$$S_{t,j} = S_{t-1,j} * \exp\left(\left(r_{US} - d_j - 0.5 * \sigma_j^2\right) * dt_t + \sigma_j \phi_j dt_t^{1/2}\right)$$

Where *S* is the index level, *t* is the observation dates, *j* represents each of the three indices, $r_{US}$ is the USD interest rate, *d* is the dividend yield, $\sigma$ is the volatility, and *dt* is time difference between observation dates.

As for EU based index, SX5E, we need a slightly different solution for the drift term since it's a Quanto. The probabilistic solution for SX5E gives:

$$S_{t,j} = S_{t-1,j} * \exp\left(\left(r_{EU} - d_j - \rho_e \sigma_j \sigma_e - 0.5 * \sigma_j^2\right) * dt_t + \sigma_j \phi_j dt_t^{1/2}\right)$$

Where $r_{EU}$ is the EURO interest rate, $\rho_e$ is the historical correlation between $r_{EU}$ and USDEUR exchange rate, $\sigma_e$ is the historical volatility of USDEUR exchange rate.

Then, we can determine the coupon payments and the redemption event using the simulated price of the indices.

For all early redemption and coupon payments, we discount them all to valuation date to value the payments at maturity. And then, we discount them again back to time 0 to get the price on the trade date.

## Sensitivity analysis

Instead of doing a sensitivity analysis on sigma values, our analysis focuses on the historical correlations between indices of different year span. We choose 1-, 3-, and 5-year historical data for our correlations and simulations. (right: 1-year, mid: 3-year, left: 5-year)

| | Simulations | Monte |
|---|---|---|
| 0 | 100000 | 976.640892 |
| 1 | 100000 | 975.789782 |
| 2 | 100000 | 976.621568 |
| 3 | 100000 | 975.899005 |
| 4 | 100000 | 976.471005 |
| 5 | 100000 | 975.420972 |
| 6 | 100000 | 976.480209 |
| 7 | 100000 | 977.013983 |
| 8 | 100000 | 976.260140 |
| 9 | 100000 | 976.024787 |

| | Simulations | Monte |
|---|---|---|
| 0 | 100000 | 977.755968 |
| 1 | 100000 | 977.403001 |
| 2 | 100000 | 977.271922 |
| 3 | 100000 | 977.942355 |
| 4 | 100000 | 977.593990 |
| 5 | 100000 | 978.544490 |
| 6 | 100000 | 977.402286 |
| 7 | 100000 | 977.537811 |
| 8 | 100000 | 977.148652 |
| 9 | 100000 | 977.502357 |

| | Simulations | Monte |
|---|---|---|
| 0 | 100000 | 977.434251 |
| 1 | 100000 | 977.637798 |
| 2 | 100000 | 976.466137 |
| 3 | 100000 | 976.605641 |
| 4 | 100000 | 976.174772 |
| 5 | 100000 | 976.665491 |
| 6 | 100000 | 976.964248 |
| 7 | 100000 | 976.304823 |
| 8 | 100000 | 975.734033 |
| 9 | 100000 | 976.373351 |

As shown above, 3-year historical correlation come the closet to the estimated value by Credit Suisse, with the 5-year estimates get slightly lower values and the 1-year estimates get much lower values but the difference is still within two dollars. The average of the 3-year simulations is $977.61, therefore, we pick the closet simulated value to the average value is $977.59 as our estimated value for the product.

## Conclusion

The estimated value for this project is $977.59. Picking 3-year historical correlations as the model input came close to the estimated value by the issuer, thus we use it as our correlations for the model.

Considerations for this project might be the choice of volatility input. We should be able to develop a better way of choosing the volatility input. There are still a number of different combinations of sigma that we haven't test yet. We just use the first combination we found that came close to the estimated value.

## Supplement data

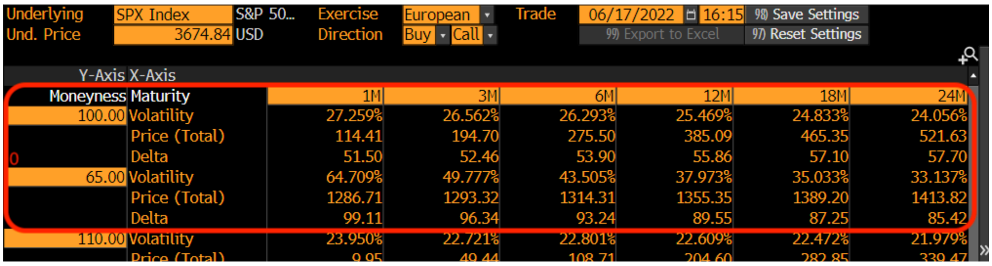
Volatility source SX5E

Underlying SX5E Index Euro St... Exercise European Trade 06/17/2022 12:00 98) Save Settings
Und. Price 3438.46 EUR Direction Buy Call 99) Export to Excel 97) Reset Settings

Y-Axis X-Axis

| Moneyness | Maturity | 1M | 3M | 6M | 12M | 18M | 24M |
|---|---|---|---|---|---|---|---|
| 100.00 | Volatility | 27.554% | 26.016% | 25.223% | 23.313% | 22.764% | 21.591% |
| | Price (Total) | 106.22 | 174.75 | 232.47 | 262.23 | 326.72 | 322.07 |
| | Delta | 50.90 | 51.74 | 51.68 | 47.41 | 49.41 | 46.01 |
| 65.00 | Volatility | 65.210% | 49.430% | 42.732% | 35.922% | 32.929% | 30.529% |
| | Price (Total) | 1201.01 | 1206.31 | 1209.53 | 1152.25 | 1179.04 | 1123.67 |
| | Delta | 99.02 | 96.49 | 93.23 | 86.57 | 84.42 | 79.71 |
| 110.00 | Volatility | 22.087% | 20.960% | 20.998% | 20.193% | 20.323% | 19.649% |

Volatility source SPX

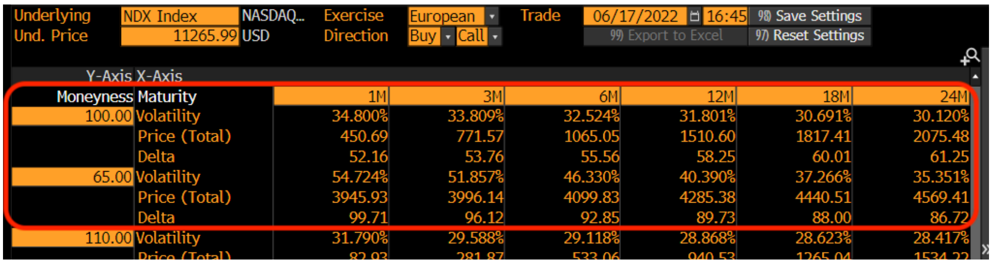


| Underlying | SPX Index | S&P 50... | Exercise | European | Trade | 06/17/2022 | 16:15 | 98) Save Settings |
|---|---|---|---|---|---|---|---|---|
| Und. Price | 3674.84 | USD | Direction | Buy Call | | 99) Export to Excel | | 97) Reset Settings |

Y-Axis X-Axis

| Moneyness | Maturity | 1M | 3M | 6M | 12M | 18M | 24M |
|---|---|---|---|---|---|---|---|
| 100.00 | Volatility | 27.259% | 26.562% | 26.293% | 25.469% | 24.833% | 24.056% |
| | Price (Total) | 114.41 | 194.70 | 275.50 | 385.09 | 465.35 | 521.63 |
| | Delta | 51.50 | 52.46 | 53.90 | 55.86 | 57.10 | 57.70 |
| 65.00 | Volatility | 64.709% | 49.777% | 43.505% | 37.973% | 35.033% | 33.137% |
| | Price (Total) | 1286.71 | 1293.32 | 1314.31 | 1355.35 | 1389.20 | 1413.82 |
| | Delta | 99.11 | 96.34 | 93.24 | 89.55 | 87.25 | 85.42 |
| 110.00 | Volatility | 23.950% | 22.721% | 22.801% | 22.609% | 22.472% | 21.979% |
| | Price (Total) | 9.95 | 49.44 | 108.71 | 204.60 | 282.85 | 339.47 |

Volatility source NDX

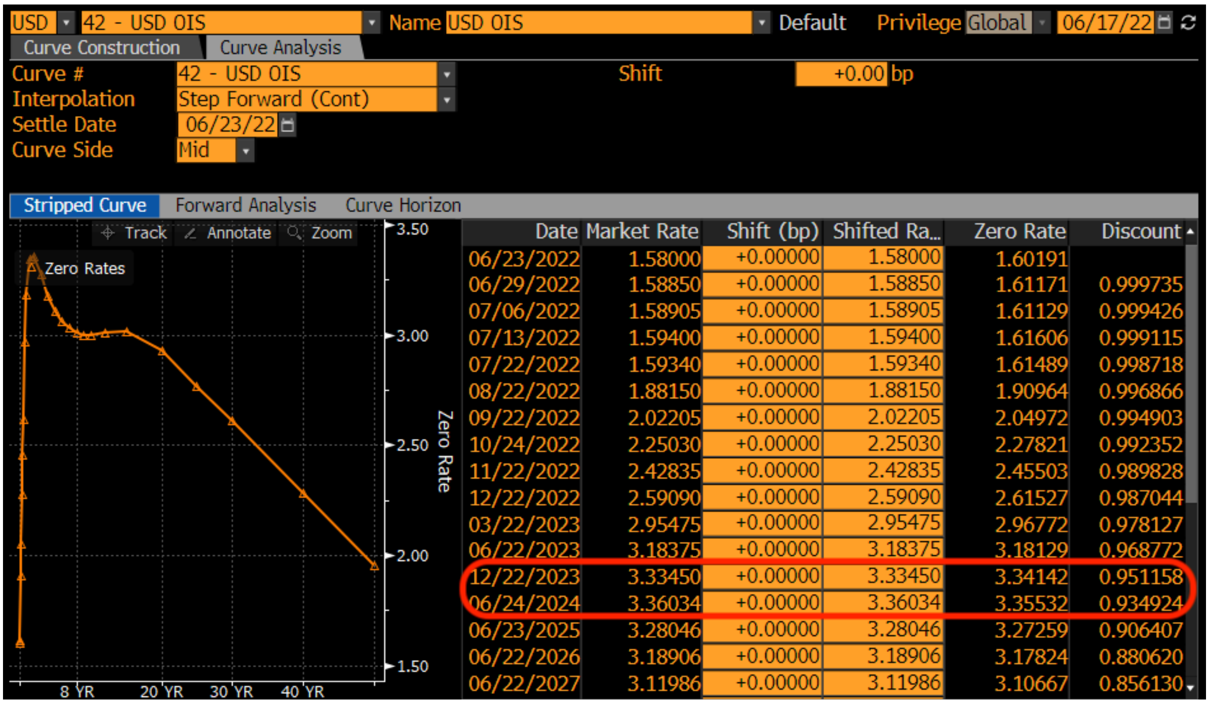


| Underlying | NDX Index | NASDAQ... | Exercise | European | Trade | 06/17/2022 | 16:45 | 98) Save Settings |
|---|---|---|---|---|---|---|---|---|
| Und. Price | 11265.99 | USD | Direction | Buy Call | | 99) Export to Excel | | 97) Reset Settings |

Y-Axis X-Axis

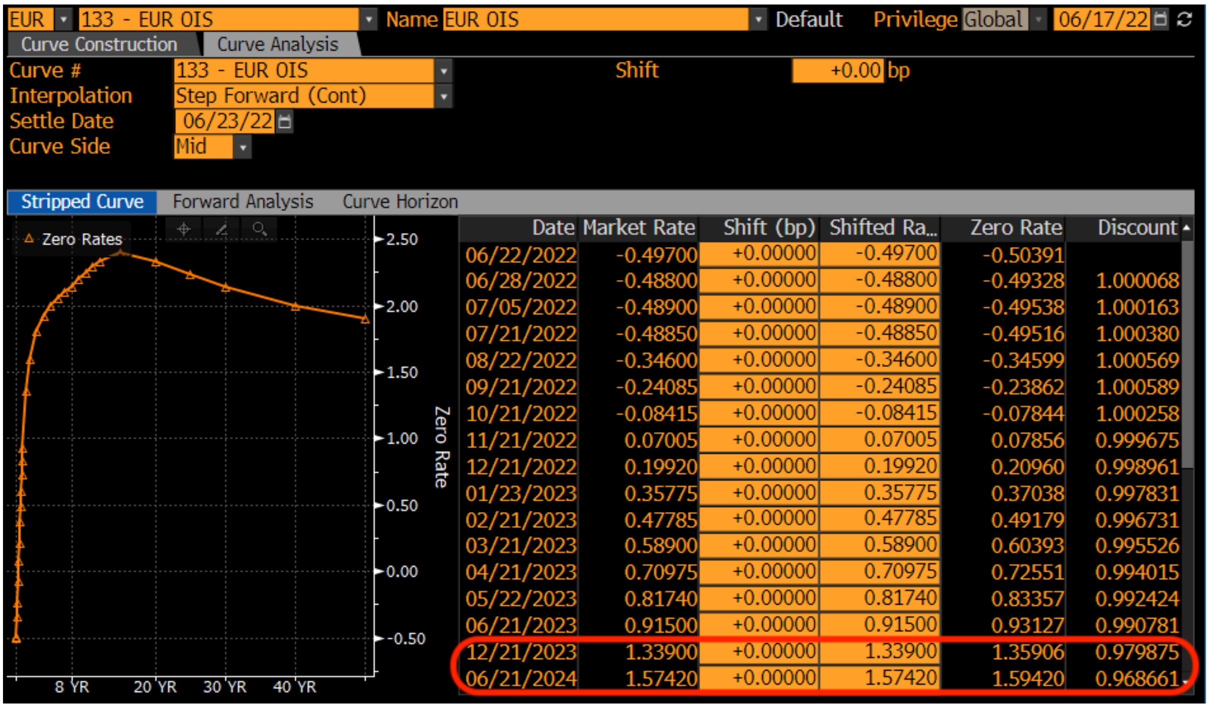
| Moneyness | Maturity | 1M | 3M | 6M | 12M | 18M | 24M |
|---|---|---|---|---|---|---|---|
| 100.00 | Volatility | 34.800% | 33.809% | 32.524% | 31.801% | 30.691% | 30.120% |
| | Price (Total) | 450.69 | 771.57 | 1065.05 | 1510.60 | 1817.41 | 2075.48 |
| | Delta | 52.16 | 53.76 | 55.56 | 58.25 | 60.01 | 61.25 |
| 65.00 | Volatility | 54.724% | 51.857% | 46.330% | 40.390% | 37.266% | 35.351% |
| | Price (Total) | 3945.93 | 3996.14 | 4099.83 | 4285.38 | 4440.51 | 4569.41 |
| | Delta | 99.71 | 96.12 | 92.85 | 89.73 | 88.00 | 86.72 |
| 110.00 | Volatility | 31.790% | 29.588% | 29.118% | 28.868% | 28.623% | 28.417% |
| | Price (Total) | 82.93 | 281.87 | 533.06 | 940.53 | 1265.04 | 1534.22 |

Risk-free rate USD



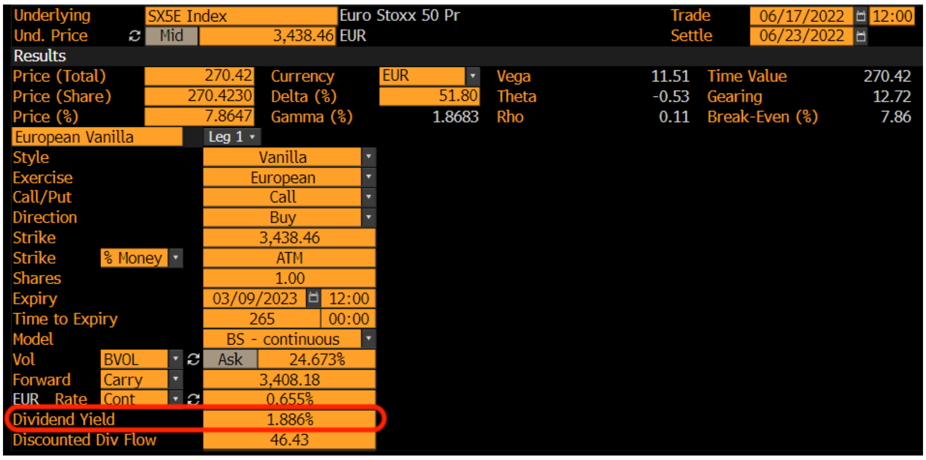

| Date | Market Rate | Shift (bp) | Shifted Ra... | Zero Rate | Discount |
|---|---|---|---|---|---|
| 06/23/2022 | 1.58000 | +0.00000 | 1.58000 | 1.60191 | |
| 06/29/2022 | 1.58850 | +0.00000 | 1.58850 | 1.61171 | 0.999735 |
| 07/06/2022 | 1.58905 | +0.00000 | 1.58905 | 1.61129 | 0.999426 |
| 07/13/2022 | 1.59400 | +0.00000 | 1.59400 | 1.61606 | 0.999115 |
| 07/22/2022 | 1.59340 | +0.00000 | 1.59340 | 1.61489 | 0.998718 |
| 08/22/2022 | 1.88150 | +0.00000 | 1.88150 | 1.90964 | 0.996866 |
| 09/22/2022 | 2.02205 | +0.00000 | 2.02205 | 2.04972 | 0.994903 |
| 10/24/2022 | 2.25030 | +0.00000 | 2.25030 | 2.27821 | 0.992352 |
| 11/22/2022 | 2.42835 | +0.00000 | 2.42835 | 2.45503 | 0.989828 |
| 12/22/2022 | 2.59090 | +0.00000 | 2.59090 | 2.61527 | 0.987044 |
| 03/22/2023 | 2.95475 | +0.00000 | 2.95475 | 2.96772 | 0.978127 |
| 06/22/2023 | 3.18375 | +0.00000 | 3.18375 | 3.18129 | 0.968772 |
| 12/22/2023 | 3.33450 | +0.00000 | 3.33450 | 3.34142 | 0.951158 |
| 06/24/2024 | 3.36034 | +0.00000 | 3.36034 | 3.35532 | 0.934924 |
| 06/23/2025 | 3.28046 | +0.00000 | 3.28046 | 3.27259 | 0.906407 |
| 06/22/2026 | 3.18906 | +0.00000 | 3.18906 | 3.17824 | 0.880620 |
| 06/22/2027 | 3.11986 | +0.00000 | 3.11986 | 3.10667 | 0.856130 |

Risk-free rate EURO

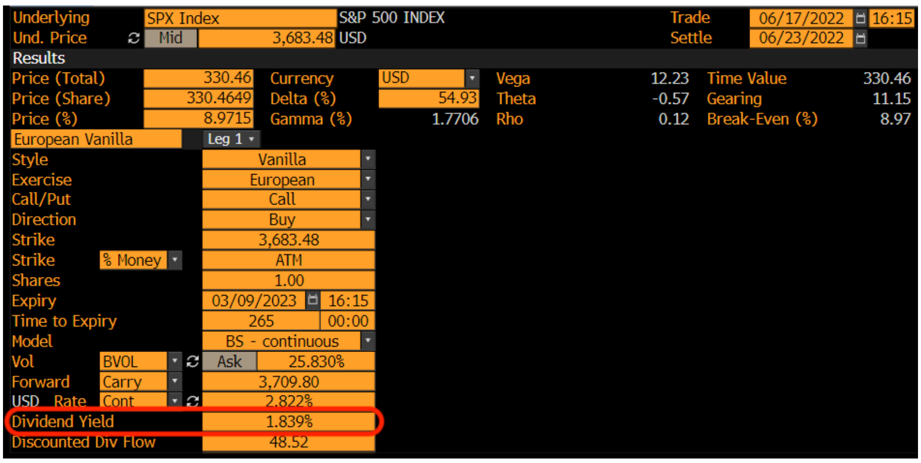



| Date | Market Rate | Shift (bp) | Shifted Ra... | Zero Rate | Discount |
|---|---|---|---|---|---|
| 06/22/2022 | -0.49700 | +0.00000 | -0.49700 | -0.50391 | |
| 06/28/2022 | -0.48800 | +0.00000 | -0.48800 | -0.49328 | 1.000068 |
| 07/05/2022 | -0.48900 | +0.00000 | -0.48900 | -0.49538 | 1.000163 |
| 07/21/2022 | -0.48850 | +0.00000 | -0.48850 | -0.49516 | 1.000380 |
| 08/22/2022 | -0.34600 | +0.00000 | -0.34600 | -0.34599 | 1.000569 |
| 09/21/2022 | -0.24085 | +0.00000 | -0.24085 | -0.23862 | 1.000589 |
| 10/21/2022 | -0.08415 | +0.00000 | -0.08415 | -0.07844 | 1.000258 |
| 11/21/2022 | 0.07005 | +0.00000 | 0.07005 | 0.07856 | 0.999675 |
| 12/21/2022 | 0.19920 | +0.00000 | 0.19920 | 0.20960 | 0.998961 |
| 01/23/2023 | 0.35775 | +0.00000 | 0.35775 | 0.37038 | 0.997831 |
| 02/21/2023 | 0.47785 | +0.00000 | 0.47785 | 0.49179 | 0.996731 |
| 03/21/2023 | 0.58900 | +0.00000 | 0.58900 | 0.60393 | 0.995526 |
| 04/21/2023 | 0.70975 | +0.00000 | 0.70975 | 0.72551 | 0.994015 |
| 05/22/2023 | 0.81740 | +0.00000 | 0.81740 | 0.83357 | 0.992424 |
| 06/21/2023 | 0.91500 | +0.00000 | 0.91500 | 0.93127 | 0.990781 |
| 12/21/2023 | 1.33900 | +0.00000 | 1.33900 | 1.35906 | 0.979875 |
| 06/21/2024 | 1.57420 | +0.00000 | 1.57420 | 1.59420 | 0.968661 |

Dividend yields SX5E




Dividend yields SPX



Dividend yields NDX