

Name: _____

7.1 Description of quicksort

Listing 1: quicksort

```
QUICKSORT(A, p, r)
  if p < r
    q = PARTITION(A, p, r)
    QUICKSORT(A, p, q-1)
    QUICKSORT(A, q+1, r)
```

Initial call is $\text{QUICKSORT}(A, 1, n)$.

Partitioning

Partition subarray $A[p..r]$ by the following procedure:

Listing 2: partitioning

```
PARTITION(A, p, r)
  x = A[r]
  i = p - 1
  for j = p to r - 1
    if A[j] <= x
      i = i + 1
      exchange A[i] with A[j]
  exchange A[i + 1] with A[r]
  return i + 1
```

- PARTITION always selects the last element $A[r]$ in the subarray $A[p..r]$ as the *pivot*—the element around which to partition.
- As the procedure executes, the array is partitioned into four regions, some of which may be empty:

Loop invariant:

1. All entries in $A[p..i]$ are \leq pivot.
2. All entries in $A[i + 1..j - 1]$ are $>$ pivot.
3. $A[r] =$ pivot.

It's not needed as part of the loop invariant, but the fourth region is $A[j..r - 1]$, whose entries have not yet been examined, and so we don't know how they compare to the pivot.

1. Using Figure 7.1 pg. 172 as a model, illustrate the operation of PARTITION on the array $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11 \rangle$.
2. Give a brief explanation of why the running time of PARTITION on a subarray of size n is $\Theta(n)$.
3. Use the loop invariant to prove correctness of PARTITION. Hint: The answer is in the book! Try to do each step without the book first, then check and correct if necessary.