Name:____

3.2 Standard Notations and Common Functions

Monotonicity

1. Complete the **Monotonicity** definitions:
   (a) A function $f(n)$ is *monotonically increasing* if...
   (b) A function $f(n)$ is *monotonically decreasing* if...
   (c) A function $f(n)$ is *strictly increasing* if...
   (d) A function $f(n)$ is *strictly decreasing* if...

Exponentials

1. Complete the useful **Exponential** identities:
   (a) $a^{-1} =$
   (b) $(a^m)^n =$
   (c) $a^m a^n =$

2. We can relate rates of growth of polynomials and exponentials:
   (a) For all real constants a and b such that $a > 1$, $\lim_{n\to\infty} \frac{n^b}{a^n} =$ (Why?)
   (b) which implies that $n^b =$ _____ (a^n) (write/circle O, Ω, Θ, o, or ω).
   (c) Thus any exponential function with a base strictly greater than 1 grows _______________ than any polynomial function (write *faster* or *slower*).

3. e^x
   (a) Write the Taylor series for $e^x =$
   (b) This gives a surprisingly useful inequality: for all real x, e^x_____$1 + x$ (write $\geq$ or $\leq$).
   (c) And as x gets closer to 0, e^x gets closer to __________.

Logarithms

1. Complete the useful **Logarithm** identities for $a > 0, b > 0, c > 0$:
   (a) $b^{\log_b a} =$
   (b) $\log_c(ab) =$
   (c) $\log_b(a^n) =$
   (d) $\frac{\log_c a}{\log_c b} =$ (Change of base formula)
   (e) $\log_b(1/a) =$
   (f) Relate $\log_b a$ and $\log_a b$:
   (g) $a^{\log_b c} = c^?$ where ? = __________

2. Changing the base of a logarithm from one constant to another only changes the value by a constant factor, so we usually don't worry about logarithm bases in asymptotic notation. Convention is to use __________ within asymptotic notation, unless the base actually matters.

3. Just as polynomials grow more ________________ than exponentials, logarithms grow more ________________ than polynomials (write *slowly* or *quickly* in each case).

4. Justify the limit:

   (a) $\lim_{n\to\infty} \frac{\lg^b n}{(2^a)^{\lg n}} = \lim_{n\to\infty} \frac{\lg^b n}{n^a} = 0$

   (b) which implies that $\lg^b n =$ _____ (n^a) (write/circle O, Ω, Θ, o, or ω).

Factorials

1. $n!$

   (a) $n! =$

   (b) $0! =$ (Why?)

   (c) Write down *Stirling's approximation*:

   (d) $n! = o($___________$)$

   (e) $n! = \omega($___________$)$

   (f) $\lg(n!) = \Theta($___________$)$

Test your understanding

1. Let $k \geq 1$ and $c > 0$ Consider $\lg^k n$ and n^c. Justify your responses.

   (a) Is $\lg^k n = O(n^c)$?

   (b) Is $\lg^k n = o(n^c)$?

   (c) Is $\lg^k n = \Omega(n^c)$?

   (d) Is $\lg^k n = \omega(n^c)$?

   (e) Is $\lg^k n = \Theta(n^c)$?

2. Let $k \geq 1$ and $c > 0$ Consider n^k and c^n. Justify your responses.

   (a) Is $n^k = O(c^n)$?

   (b) Is $n^k = o(c^n)$?

   (c) Is $n^k = \Omega(c^n)$?

   (d) Is $n^k = \omega(c^n)$?

   (e) Is $n^k = \Theta(c^n)$?

3. Consider $\lg(n!)$ and $\lg(n^n)$. Justify your responses.

   (a) Is $\lg(n!) = O(\lg(n^n))$?

   (b) Is $\lg(n!) = o(\lg(n^n))$?

   (c) Is $\lg(n!) = \Omega(\lg(n^n))$?

   (d) Is $\lg(n!) = \omega(\lg(n^n))$?

   (e) Is $\lg(n!) = \Theta(\lg(n^n))$?