Growth, Interest Rates, and Deficits: Forecasting Fiscal Policy

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Abstract

ARIMA, ETS, and dynamic regression models are applied to several methodological approaches which work to predict the spread between nominal growth and an adjusted safe interest rate. This spread can be used to interpret how healthy the current debt to output ratio is within the US. I find with some confidence that the spread will remain positive but narrow over the next five years as growth slows.

1. Problem

In this paper I will use four methods to forecast the spread between US national output and the maturity adjusted safe rate of interest net of the implicit tax rate as proposed by Olivier Blanchard (Blanchard, 2019) in his recent presidential address at the AEA conference. There are many exogenous variables influencing national output and interest rates which makes forecasting them difficult. I will use four different approaches in forecasting the spread of output to Blanchard's adjusted rate. The ARIMA and ETS method will be used to forecast (1) the spread directly, (2) the adjusted safe rate and national output directly, (3) various exogenous variables which will then be applied to a several models for predicting growth and interest rates, and (4) dynamic regression techniques will be applied using the exogenous variables in (3). The predictions from (2), (3), and (4) will then be transformed into the spread which is what we are primarily interested in.

2. Significance

The US debt to GDP ratio is at it's highest level since the WWII era. There has been a great deal discussion around this lately. It's not immediately obvious what level of debt is healthy as this depends on many exogenous variables. We know that debt is the lifeblood that keeps the creative destruction of capitalism churning, while excessive debt can also have harmful effects if future returns on that debt don't compensate it's cost. There is some balance of debt to income required to achieve maximum sustainable growth. When debt is cheap, as indicated by low interest rates, we have reason to believe that a higher ratio isn't necessarily bad as this can contribute additional capital to growth.

In a hypothetical world of certainty where we know income will be higher than interest expenses in the future, we will be subject to no intertemporal budget constraint. An increase in debt today can be used in lucrative projects which will earn higher returns than interest expenses in the future – this is an arbitrage opportunity in this world of certainty. Therefore, assuming higher incomes increase utility, a rational decision maker will not be concerned about high debt levels and will in fact seek it. This paper will attempt to understand how close the present conditions in the US are to this hypothetical world with a focus on US fiscal policy.

The US government is in a unique position because it's financing is generally considered riskless to investors which keeps interest expenses minimal. If national output growth continues at some level above the adjusted riskless rate, which will be introduced later, then fiscal expenses could be covered indefinitely without increasing the tax rate. Higher incomes will contribute more to the treasury at the same rate of taxation.

3. Literature Review

At this years AEA Presidential Address the Economist Olivier Blanchard asked how worried we should be about the current debt levels. He went on to formulate some models which try to understand how the safe rate and nominal growth rate effect net utility within a country. Of central importance to his model and discussion is our ability to forecast the safe interest rate and nominal growth.

Mehra (1995) proposed a model for predicting treasury bill rates which requires a forecast of the federal reserve policy, national output, prices typically measured using the consumer price index, and historical data for treasury bill rates and prices. In this paper I will apply a simplified Mehra model to make predictions on the safe interest rate. There are other ways to predict interest rates by deriving demand and supply functions for bonds using instrumental variables and applying those functions to observed data. This approach can be useful in some contexts but I've decided to consider the Mehra model in its place.

Many techniques have been derived to forecast output and the uncertainty around output. Quantile regressions can be used to construct a probability distribution of forecasted GDP, rather than mean point forecasts (Laurent & Kozluk, 2012). Other methods can be found in (Schumacher & Breitung, 2008), (Marcellino, Stock, & Watson, 2003) and (Armesto, Engemann, & Owyang, 2010). In addition to these forecasting techniques there are production models which can be applied to make predictions. These models include the well-known Solow growth model (Solow, 1956) and endogenous growth models such as (Romer, 1990). Other methods can be used to estimate growth indirectly first by using forecasting techniques to predict a models inputs and then applying those forecasts to some framework such as the one used in (Solow, 1957) which was originally used to measure changes in total factor productivity.

The methods I will use will borrow heavily from some of these researchers – especially Blanchard, Mehra, and Solow.

4. Data

This paper uses annual time series data on the safe interest rate and nominal growth starting in 1962 to the present year. I choose to leave out earlier dates due to their diminished significance to our analysis of present and future events. Additionally, while training the models discussed below I bifurcate the data into a training set of 52 observations and a test set of 5 observations which will be used at the end to test model accuracy and select for optimality.

Historically, nominal growth is 37 basis points above the ten year safe rate and 140 points above the one year safe rate. I mentioned earlier that for our purposes we're more interested in Blanchard's adjusted rate. This rate adjusts for debt maturity as this varies overtime. For example, average debt maturity was as high as 9 years in 1946 and declined gradually to about 2.5 years in the mid-1970's. Today, the average maturity is just over 5 years on treasury bonds. Additionally, most government bond holders pay taxes on their holdings, excluding municipal bonds. The adjusted rate subtracts the implicit tax rate. Refraining from doing this would result in an upward bias in rates for our purposes. The adjusted rate is historically 2.1 percent below nominal growth and is observed to be higher than nominal growth 14 percent of the time for our time series.

We can observe that nominal growth rates have been declining over the past several decades. Some economists believe in a phase transition to a secular stagnation across developed economies. Our forecasts will consider this possibility due to the implications it would have on our model. Is this mainly due to lower rates of inflation or are these lower growth rates derived from real effects in output? We'll work to answer this question in method (3) below.

We can also observe trends in adjusted inflation which is confirmed by an observed gradual decrease in autocorrelation from the present to the past while there are no obvious signs of seasonality

The data used in methods (1) – (4) exhibit limited to no seasonality, largely because I am using annual data. There are also large trends in most of the variables such as growth, capital stock and labor force statistics used in method (3).

5. Models

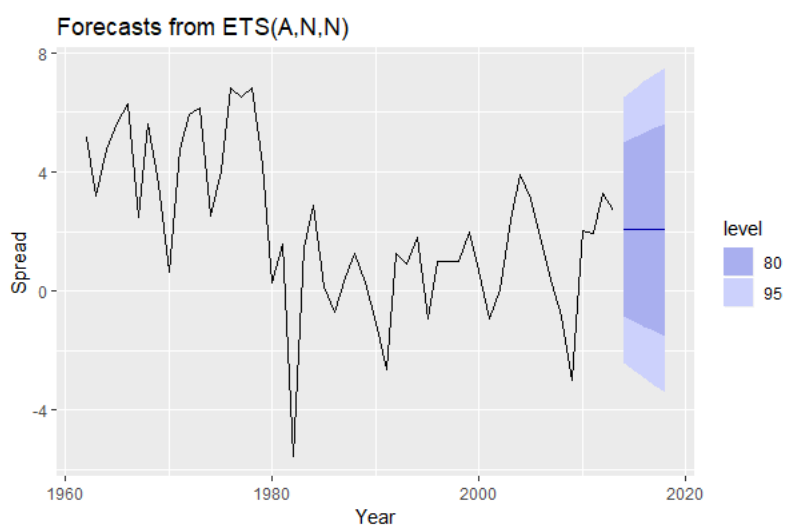
Four modeling approaches are used for forecasting the data. The first is strictly inductive in that is uses historical observations for our variable of interest, named *g-r spread* from here, for nominal growth minus the adjusted interest rate appropriate for analysis of fiscal policy. The second is also inductive, but first bifurcates the spread into the two relevant time series – nominal growth and the adjusted rate. The forecasts from these variables are then used to make a prediction on the future *g-r spread*. The third uses a theoretical framework for predicting the *g-r spread.* A Cobb-Douglas production function with returns to capital being (1/3) and returns to labor of (2/3) is applied to historical observations for the capital stock, total factor productivity, and the labor supply. Forecasts are done on these variables and applied to a growth accounting formula: $\%\Delta Yt = \%\Delta At + \alpha\%\Delta Kt + (1 - \alpha)\%\Delta Lt$ where $\alpha = (1/3)$. This model is then used to predict future growth in real output. Forecasts for inflation are then derived and applied to the real output predictions to arrive at predictions for nominal growth. I also need to estimate future adjusted interest rates. This is done by first forecasting three variables which influence our dependent variable: inflation expectations, real growth, and the federal funds rate. I then regress these historical variables onto the ten year and one year safe rate. Now that my model is trained I apply it to my forecasted data to make forecasts on the dependent variables: the ten and one year safe rates. Lastly, I convert these rates into the relevant adjusted rate by forecasting maturity expectations as well as the adjusted tax rates as given by Blanchard. (Blanchard, 2019) From here I derive forecasted values for *g-r spread*. The fourth method utilizes a dynamic regression model on the adjusted rate as defined by Blanchard and on nominal growth which is then used to forecast the future spread five years forward

Method 2 and 3 avoid some ambiguity of method 1 that might arise from overlooking important patterns in the spreads exogenous influencers. Interpretability diminishes with the second and third method however as we are forced to abandon reliable prediction intervals. Furthermore, method 3 assumes a Cobb-Douglas production function and a liner-relationship in interest rates to the applied exogenous variables. These assumptions may not hold which would corrupt the models predictive capacity. Method 4 also has difficulties as it relies heavily on naïve forecasts of the included variables.

I apply two forecasting methods to the data. The first is an exponential smoothing algorithm (ETS) and the second is an autoregressive integrated moving average (ARIMA). I apply various versions of ETS and ARIMA models such as a simple exponential smoothing with additive/multiplicative errors, multiplicative Holt-Winters with additive errors, and versions of ARIMA to the data relevant for the three methods and select the model with the best performance. This is done by comparing model performance and selecting the model with the lowest AIC score. These selected forecasts are then applied through the various methods described above.

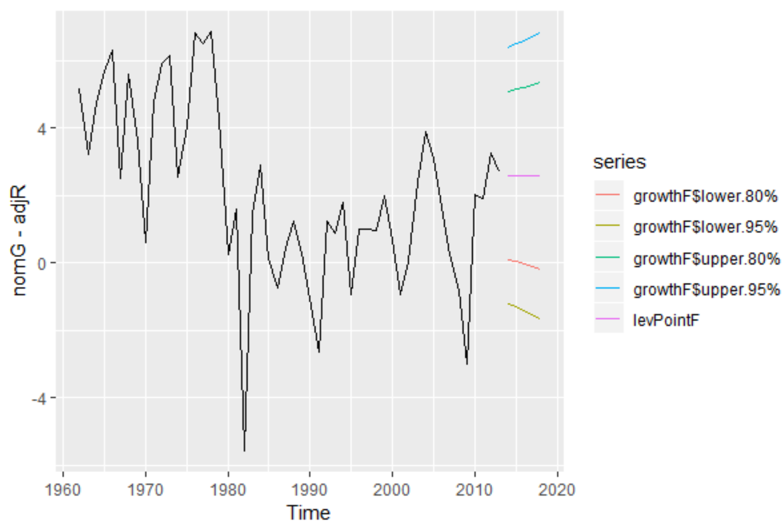
6. Formulation and Results

I find the optimal forecasting model for method 1 is a simple exponential smoothing model with additive errors. The model satisfies the assumption that the residuals are uncorrelated and normally distributed. Cross validating the predictions shows a mean squared error of 5.97 (MAE of 1.85) compared to the optimal ARIMA (1,0,1) which had an MSE of 6.33 (MAE of 2).



We can see the model is relatively naïve in that it makes a constant prediction. It also predicts positive future interest rates, but we should notice how the 80 percent prediction intervals are well below zero indicating it is possible that the *g-r spread* will become negative in the future based off this model, suggesting fiscal policy makers should be cautious of rising debt-income ratios.

In method two I conclude in forecasting nominal growth that a simple exponential smoothing method with additive errors is the optimal model while the best model for forecasting the adjusted rate uses a simple exponential smoothing method with multiplicative errors. The MSE's for nominal growth and the adjusted rate derived from cross validation are 5.02/1.03 for the ETS models and 5.81/1.12 for the ARIMA models. It should be stated that the basic model assumptions hold, the residuals are uncorrelated with mean zero and represent a normal distribution, it can be assumed this is true from here unless I note otherwise.



The method used here prevents me from presenting nice prediction intervals such as those shown in method 1. Still I'm able to apply predictions and present them as shown. We can see that our prediction is still relatively naïve but has a narrower prediction interval than what is shown for method 1, presumably because some information was lost from the general approach taken in method 1. This approach makes us slightly more confident that our *g-r spread* will be positive five years out, but we see there is a 10 percent probability that the spread will turn negative within the next 5 years.

Method three is the most involved as it attempts to constrain our predictions within theoretical models. To begin, logarithmic transformations are applied to real output and the consumer price index. I then apply the data to various ETS and ARIMA models and cross validate the results. My findings suggest

```
Call:
lm(formula = y1 ~ cpi + ffr + rgdp, data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-1.5526 -0.7701 -0.1771  0.6197  2.1450

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.9573726  0.7381351   6.716 2.00e-08 ***
cpi          0.0617096  0.0127283   4.848 1.35e-05 ***
ffr          0.6600952  0.0456879  14.448  < 2e-16 ***
rgdp        -0.0010291  0.0002173  -4.735 1.97e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9621 on 48 degrees of freedom
Multiple R-squared:  0.8821,	Adjusted R-squared:  0.8747
F-statistic: 119.7 on 3 and 48 DF,  p-value: < 2.2e-16
```

```
Call:
lm(formula = y2 ~ cpi + ffr + rgdp, data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-1.57539 -0.27996 -0.07376  0.38203  1.02877

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.8035837  0.3780820   4.770 1.75e-05 ***
cpi          0.0191893  0.0065196   2.943  0.00499 **
ffr          0.8648932  0.0234019  36.958  < 2e-16 ***
rgdp        -0.0003632  0.0001113  -3.262  0.00204 **
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4928 on 48 degrees of freedom
Multiple R-squared:  0.978,	Adjusted R-squared:  0.9766
F-statistic: 711.8 on 3 and 48 DF,  p-value: < 2.2e-16
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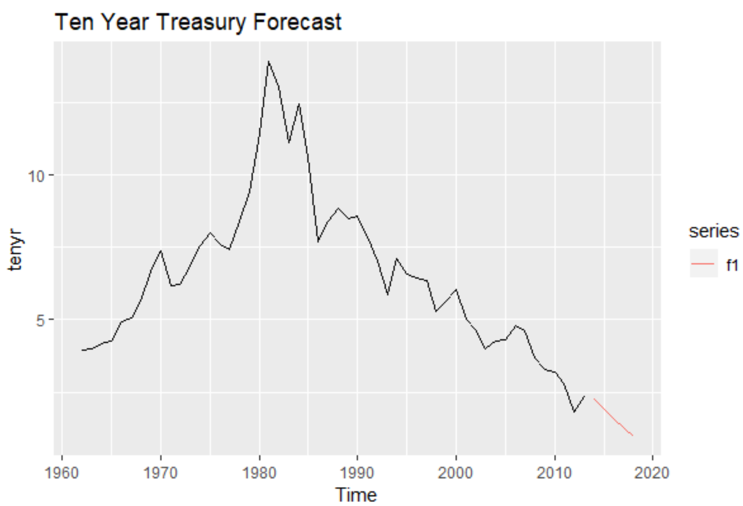
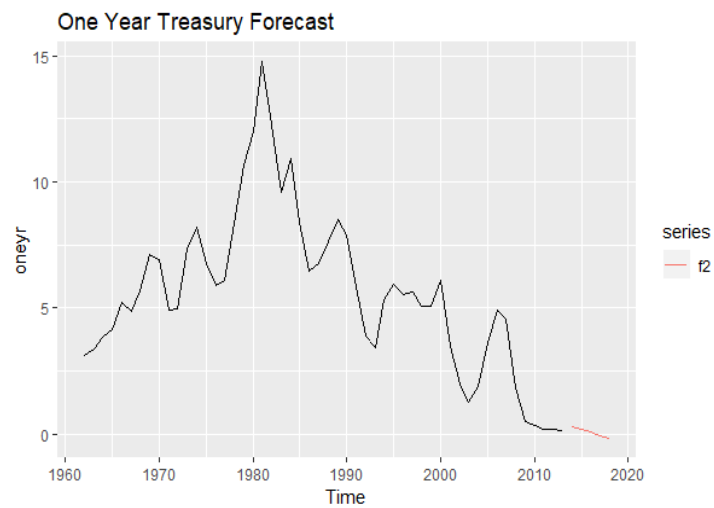
that the ARIMA model is superior to the ETS for all three predictors as the MSE is consistently lower for the ARIMA models.

Regressing this shows all three variables are significant and the federal funds rate is the main explanatory variable for both ten year and one year safe interest rates. We can see that our results fit traditional theories. The Fisher equation suggests that nominal interest rates are a combination of real interest rates and inflation expectations. Therefore, we'd expect interest rates to be positively correlated with inflation expectations. Additionally, monetary theory suggests that output and interest rates are



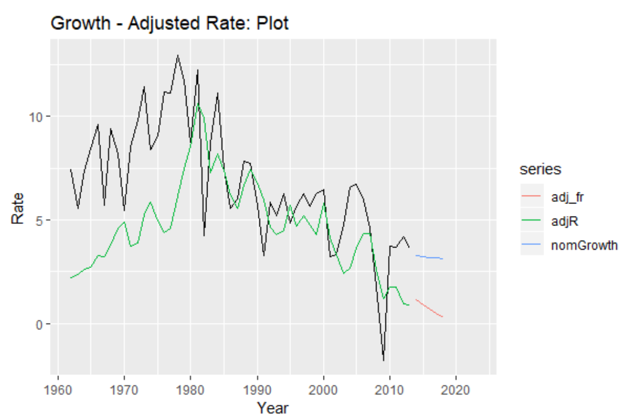


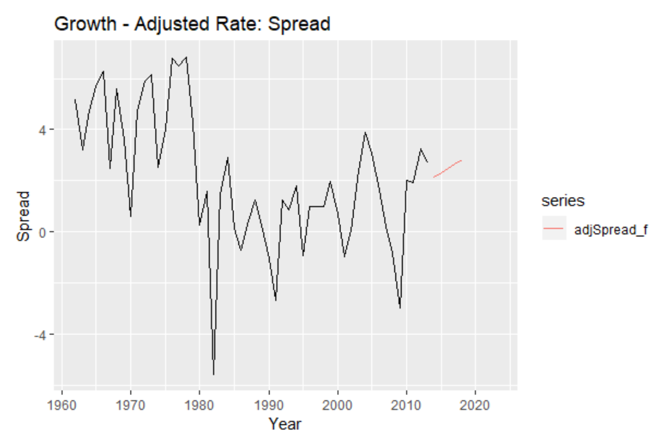
inversely correlated which this model can confirm, though the relationship is shown to be very weak.

Applying the model to the forecasted data yields the results shown above. These forecasts are rather contrarian as most experts expect interest rates to rise gradually as the Federal Reserve begins to tighten it's monetary policy approach. Still, there is ambiguity about how the Fed will act in the coming years and recent signals suggest the rate hikes will be slow or non-existent in the coming future. Because of this ambiguity I will trust the models predictions which are largely based off a strong direct relationship to the federal funds rate prediction. It's valid to argue these models cannot be used to predict something like the federal funds rate because it is not exactly governed by market forces, but by decisions made by economists at the Fed. This makes forecasting the federal funds rate difficult and complex and I will say the ARIMA model should still be used for the reasons described earlier.

Next, I turn to real output predictions by applying our data to a Cobb-Douglas production model defined above. There are three variables which we will forecast to apply to our model. The first is total factor productivity which is shown to grow at a rather stochastic rate with a mean of 0.85 percent. The second is the capital stock which has a negative trend overtime from around 4 percent to near 1 percent at the end of the time series. The third is the labor force which interestingly shows a negative trend currently with around a 0.5 percent growth rate. I find that after cross validation, applying a simple exponential smoothing model with additive errors to all three of these variables offers superior results with consistently lower MSE's when compared to various ARIMA models. I also apply an ETS(M,A,N) model due to it's low AIC score to predict inflation so that I can convert real growth into nominal growth.

Earlier we predicted the ten year and one year safe rate, but we need the adjusted rate to make any relevant predictions. To do this I forecast the maturity and implied tax rate of these bonds and then make the conversion to our adjusted rate. I find the optimal forecasting method the same way as four the previous variables and decide to use an ARIMA model for these variables. These predictions are then applied to the conversion formula (Blanchard, 2019) and I derive the *r-g spread.*





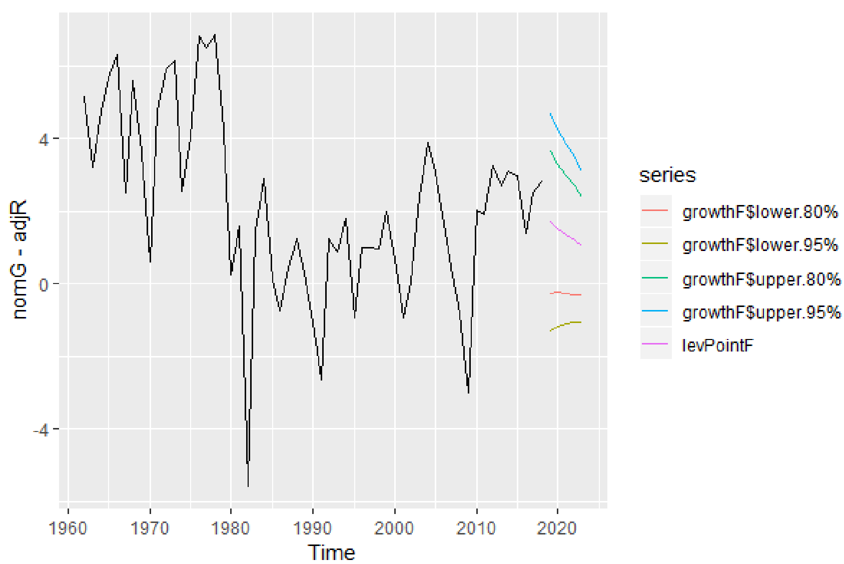
We can see this method is weak because it lacks prediction intervals while also strong because is it less naïve than our method 1 and 2. We can see it predicts the spread will increase over time but it is misleading due to the discontinuity shown from our observed data compared to the predictions from the model.

Lastly, method 4 shows the regressed independent variables have low significance on the model with small coefficients and higher standard errors. Additionally, the order for the ARIMA model used in the adjusted interest rate component was (0,1,1) and (0,1,0) for the nominal growth component. Now that we have developed these four methods we can test our models to see which performed better.

7. Performance

After applying methods (1), (2), (3), and (4) and making 5 year forecasts I apply the forecasts to the test data to calculate the MSE for the predictions. The results show that method 1 and 4 were the least predictive method with an MSE of 0.64 and 43.5 respectively. which is still low for method 1 considering the complexity of *g-r spread,* whereas method 4 performs the worst because of the way in which it was developed. The interest rate spread we've been considering shows little trend over our time frame while the underlying variables included in method 4 do show trends which are biasing the results enough to increase our test error significantly, more details on this can be found in this papers associated codes for replication. Additionally, method 3 has an MSE of 0.52 and method 2 has the lowest MSE of 0.38.

Method 3 is attractive because it relies less on purely inductive methods. This is true because predicting variables like the capital stock, labor force, and inflation is typically easier than forecasting something as complex as interest rates directly. Still, using theoretical models is usually not the best option when we simply want to make accurate forecasts, largely due to invalid assumptions which bias theoretical models. For this reason I will avoid the theoretical models and adopt method 2 as the optimal forecasting method for the data for our purposes.



The five year predictions are shown above. We can see the prediction intervals are strange which is a result of combining the prediction intervals from two forecasts which differ in variance. For this reason we cannot derive very much information from the intervals. The forecast tells us to expect a narrowing of the *g-r spread* which will remain positive although we cannot confidently say that it will remain positive in the coming years.

This view tends to agree with the general zeitgeist of many economists who predict slowing growth, fear rising inflation, and expect rising interest rates will occur as the Fed tightens monetary policy.

8. Conclusion

The preceding analysis suggests that 'deficit hawks' may be justified in their analysis. If the *g-r spread* turns negative we can expect a runaway effect where growth can no longer compensate government interest payments which will heavily burden the tax payer or consumer. The government will be forced to either raise taxes or print more dollars. Both forms of tax could have substantial adverse effects on economic activity – potentially leading to negative feedbacks which will be difficult to escape. This paper cannot lead us to conclude decisively in any direction. The forecasts predict a positive *g-r spread*, but it is narrow and only growing narrower.

Future work can be done on this topic by developing better models. Method 3 is flawed as it fails to safely predict the federal funds rate and could utilize better prediction models such as a k-nearest neighbors, random forests, or neural networks to provide higher accuracy forecasts. In addition, more methods could be devised which work to dig deeper into the data that drives the *g-r spread*, these methods could then be applied to make a better dynamic regression model which would more reliably explain the variance observed in the *g-r spread.* I only weakly considered the population decline (through the labor force) occurring in all developed countries, the ageing population, and growing inequality in the US. These are all important factors that should not be ignored when considering the wisdom of high fiscal deficits. These factors among others could be considered to improve the models proposed above.

9. Code

A more detailed analysis can be found in the code where there are many more graphs and tests to show the validity of the ARIMA and ETS models used.

The codes can be found here: https://github.com/BriVandenAkker/Forecasting-Project

10. References

Armesto, M. T., Engemann, K. M., & Owyang, M. T. (2010, December). *Federal Reserve Bank of St. Louis Review*. Retrieved from http://pds23.egloos.com/pds/201211/15/36/Forecasting_with_Mixed_Frequencies.pdf

Blanchard, O. (2019, January). *Public Debt and Low Interest Rates*. Retrieved from AEA: https://www.aeaweb.org/aea/2019conference/program/pdf/14020_paper_etZgfbDr.pdf

Laurent, T., & Koźluk, T. (2012, July). *Measuring GDP Forecast Uncertainty Using Quantile Regressions*. Retrieved from OECD Economics Department: https://www.oecd-ilibrary.org/economics/measuring-gdp-forecast-uncertainty-using-quantile-regressions_5k95xd76jvvg-en

Louis, F. R. (n.d.). *https://fred.stlouisfed.org/*. Retrieved from FRED: https://fred.stlouisfed.org/

Marcellino, M., Stockbe, J. H., & Watsonce, M. W. (2003, February). *Macroeconomic forecasting in the Euro area: Country specific versus area-wide information*. Retrieved from European Economic Review: https://www.sciencedirect.com/science/article/pii/S0014292102002064

Mehra, Y. P. (1996). *Monetary Policy and Long-Term Interest Rates*. Retrieved from Federal Reserve of Richmond: https://www.richmondfed.org/~/media/richmondfedorg/publications/research/economic_quarterly/1996/summer/pdf/mehra.pdf

Romer, P. (1990). *Endogenous Technological Change*. Retrieved from http://pages.stern.nyu.edu/~promer/Endogenous.pdf

Rudebusch, G. D. (2010, January). *Macro-Finance Models of Interest Rates and the Economy*. Retrieved from Federal Reserve Bank of San Francisco: https://www.frbsf.org/economic-research/files/wp10-01bk.pdf

Schumacher, C., & Breitung, J. (2008). *Real-time forecasting of German GDP based on a large factor model with monthly and quarterly data*. Retrieved from Science Direct: https://www.sciencedirect.com/science/article/pii/S0169207008000393

Solow, R. M. (1956, February). *A Contribution to the Theory of Economic Growth*. Retrieved from The Quarterly Journal of Economics: https://www.econ.nyu.edu/user/debraj/Courses/Readings/Solow.pdf

Solow, R. M. (1957, August). *TECHNICAL CHANGE AND THE AGGREGATE PRODUCTION FUNCTION*. Retrieved from The Review of Economics and Statistics: https://faculty.georgetown.edu/mh5/class/econ489/Solow-Growth-Accounting.pdf