UNIVERSITY OF CAMBRIDGE

MPhil in Scientific Computing

Compressive Sensing in Video Reconstruction

Outline

For the MPhil project, a complete Compressive Sensing system was developed that achieves near-perfect reconstruction of video signals from as few as $N = 0.3M$ compressed samples, where M is the original signal length.

Background

In the conventional signal processing pipeline signals are sampled at or above the Nyquist Rate and the acquired samples are then compressed for efficient storage and transmission. For many applications, this is highly inefficient since a lot of data is collected at the acquisition stage only to be - in large part - thrown away during compression.
Compressive Sensing (CS) [1, 2] is a novel technique that is able to acquire signals *directly in a compressed format*.

Formulation

Let $\mathbf{v} \in \mathbb{R}^{\mathbb{M}}$ be the underlying signal. In CS. a compressed data set $\mathbf{y} \in \mathbb{R}^N$, with $N << M$, is acquired via the linear sensing mechanism $\mathbf{\Theta v} = \mathbf{y}$, where the sensing matrix $\mathbf{\Theta}$ is independent of $\mathbf{v}$.
To reconstruct $\mathbf{v}$ from the measurements $\mathbf{y}$, CS uses the fact that many common classes of signals are *sparse* when represented in a certain basis $\mathbf{\Psi}$: $\mathbf{v} = \mathbf{\Psi w}$ and $\mathbf{w} \in \mathbb{R}^M$ has few non-zero entries. Digital image and video signals are well approximated by sparse signals when expressed in wavelet bases.
The signal $\mathbf{v}$ can be recovered by finding the sparsest solution $\mathbf{w}$ to the under-determined system

$$\mathbf{y}\ (= \mathbf{\Theta v} = \mathbf{\Theta \Psi w}) = \mathbf{\Phi w}$$

This is an NP-hard problem and alternative approaches are required.

Methods

A machine learning approach is applied and the Compressive Sensing problem is formulated as a Bayesian regression: $\mathbf{y} = \mathbf{\Phi w} + \boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon}$ is a zero-mean Gaussian error. The *Relevance Vector Machine* (RVM) [3, 4] is used to find a solution $\mathbf{w}$ with a very sparse posterior mean.
For sensing matrices such as the one used in Figure 1, a cascade of RVMs [5] is build to further imrove the reconstruction quality.

Results

Figure 1: Two consucutive frames of a sample video signal. Top row: CS measurements $\mathbf{y}$. Bottom row: Recovered video $\hat{\mathbf{v}}$

In Figure 1, the sensing mechanism $\mathbf{\Theta}$ has measured 30% of the original samples in $\mathbf{v}$. The video is recovered using a cascade of RVMs.

References and Acknowledgements

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