HW3
Theory of Computation

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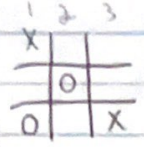
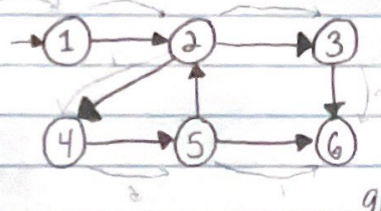
1) 

Optimizing your moves to block what the opponent is trying to do while furthering your own victory chance is important. The best move from the current position is X on A3 to block the other player and secure a victory as no matter where O is placed they can no longer win. As far as a general winning strategy, trying to get the board to look the way it currently is would be a valid strategy. At minimum you want 2 of your placements of X or O to be diagonal so you can maximize winning potential.

2) 

Player 1 always choses 1, player 2 goes to 2, player 1 has a choice now, but if they want to win they must choose 4 from 2. Doing so guarantees that player 2 chooses 5 because no other options are available and then player 1 secures the victory with 6.

3) $A_{DFA} \in L$ To prove this we can construct a TM M to decide A_{DFA} in log space.

$$A_{DFA} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts } w\}$$

and M = "on input $\langle B, w\rangle$ where B is a DFA and w is a string:
1) Simulate B on input w by keeping track of its head location.
2) If it ends in an accept state, accept.
3) If it ends in a reject state, reject.

This decides A_{DFA} in $O(\log(n))$ space by trading time for space because M only computes individual symbols at a time before restarting. (credit to theorem 8.23 in Sipser)

4) A. If $A \leq_L B$ and $B \in L$ then $A \in L$. We build TM M such that for every positive integer x the inverse is computed. The "inverse" is always going to add with the regular computed value to 1 meaning that the computation $A_2 = \{(x, y) \mid R^+(x) = y\}$ is equivalent to $R^+(x) = x + x^R$ since there will be no leading 0s in x and the rest are cancelled out by binary addition $0 + (0)^R = 0 + 1 = 1$

B) Since the only change to A_3 is another round of computation and the inverse of an inverse is just the original value, we can safely say that A_2 is functionally equivalent to $R^+(x) = x + x^R$, which we said to be in L. Using a transitive property of equality we can then say $A_3 \in L$ because A_3 is poly-space reducible to $R^+(x) = x + x^R$.

5) Show UCYCLE $\in$ L. Let M be a deterministic TM that decides UCYCLE.

M = "On input (G)
1) Select a vertex x
2) Select an edge (x, y)
3) Start traversing the graph G through (x, y), if we come back to x through an edge that isn't (x, y) then accept otherwise reject."

If we are able to get back to x without encountering (x, y) then we can say the graph has a cycle, this is done in log space so UCYCLE $\in$ L.

6) Given $G = (V, E)$ the n nodes, the reduction produces $H = (V', E')$ Where $V' = \{(i, v, b) \mid 1 \leq i \leq n, v \in V, \text{ and } b \in \{0, 1\}\} \cup \{s, t\}$ and $E' = \{((i, v_1, b_1), (i, v_2, b_2)) \mid (v_1, v_2) \in E \text{ and } b_1 = 1 - b_2\} \cup \{(s, (i, v_1, 0)), (t, (i, v_2, 1)) \mid i < n\}$

(Credit to the Sipser text for this. I'm not 100% on it but did my best following along with the reduction examples online & in the Chapter)

7) Show that $BOTH_{NFA}$ is NL-complete. In order to non-deterministically compute M_1 and M_2 (NFAs) and to have $L(M_1) \cap L(M_2) = \emptyset$ we simulate both M_1 and M_2 in parallel for each guessed symbol and accept when they both end in accept. In order to prove completeness we reduce $BOTH_{NFA}$ to PATH. PATH is NL-complete so if we reduce $BOTH_{NFA}$ to PATH then $BOTH_{NFA}$ is NL-complete too.

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7 continued) Given a generic instance of PATH $\langle G, s, t \rangle$ we map it to $BOTH_{NFA}$.

M_1 = "M_1 is an NFA with $\Sigma = \{0\}$ and a state q_i for each node i in G. Then, for every edge (i,j) in G there's a δ from state q_i to q_j for input 0. $\delta_{M_1}(q_i, 0) = \{q_j \mid (i,j)$ is an edge in $G\}$. Next we use M_2 = "M_2 is an NFA and $\Sigma = \Sigma^*$". Since $L(M_1) \cup L(M_2) \neq \emptyset$ we know that neither language can include $\emptyset$ which is functionally equivalent to there being a path from s to t in G. Since we can map $BOTH_{NFA}$ to PATH like this, we can safely say $BOTH_{NFA}$ is NL-complete.

8) 2SAT is NL-complete. Since NL = coNL and that $2SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable boolean formula in 2CNF$\}$ we can show 2SAT is NL-complete by showing $\overline{2SAT}$ is NL-complete. The first condition for NL-completeness is proving $\overline{2SAT} \in NL$. For NTM M

M = "On input ϕ

1) Construct graph G such that ϕ is not satisfiable if for vertice (u,v) there are the paths uv and vu in G

2) for every variable x create a node x and $\bar{x}$

3) nondeterministically choose a vertex in G

4) check if G has both $x\bar{x}$ and $\bar{x}x$ paths via the NL-algorithm

5) Accept if both paths exist reject otherwise"

This decides $\overline{2SAT}$ in log space, so the first requirement is met: $\overline{2SAT} \in NL$.

Next we prove $\overline{2SAT}$ is NL Hard by reducing PATH to $\overline{2SAT}$ by mapping G to a 2cnf expression with a variable x_v for every v in G. $\phi = (\bar{x}_i \vee x_j) \wedge (\bar{x}_t \vee \bar{x}_s) \wedge (x_s \vee x_s)$ If we want ϕ to be true then x_s can't be false (i.e. G has a path from s to t) but we can reach a satisfiable assignment with x_i = False, x_j = False, x_T = False and x_s = True. This mapping of nodes in G to clauses in ϕ, and the subsequent assignment, shows 2SAT is NL-complete because $\overline{2SAT}$ is and NL = coNL