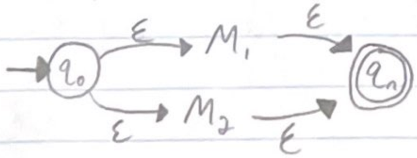
Foundations HW2

Brian DeFlaminio

1) Prove that the set of regular languages is closed under complements.
L is regular, prove $\Sigma^* \setminus L$ is regular

If L is regular then there exists a DFA $M=(Q,\Sigma,\delta,q_0,F)$ that accepts it. Now let's define another DFA $M'=(Q,\Sigma,\delta,q_0,Q\setminus F)$ that flips every accepting state to a non-accepting one and vice-versa. For any input $i \in L$, i drives our first DFA M to an accept state, and that same input results in a non-accepting state in M'. Moving further with this idea we find that $i \notin L$ (which is equivalent to $\Sigma^* \setminus L$) will drive M to a non-accepting state, but will drive M' to an accepting state. Now that there exists a DFA M' that accepts $\Sigma^* \setminus L$ we can conclude that the set of regular languages is closed under complements.

Prove that if L_1 and L_2 are regular, then so is $L_1 \cap L_2$
By De Morgans law we can say $L_1 \cap L_2 = \overline{(\bar{L_1} \cup \bar{L_2})}$.
As proven above the complement of a regular language L_1 is also regular, so $\bar{L_1}$ and $\bar{L_2}$ are regular. We also know that the union of two regular languages is regular by the following:
Say M_1 is the DFA that represents $\bar{L_1}$ and M_2 is the DFA that represents $\bar{L_2}$. Then the following NFA accepts both M_1 and M_2



A language is regular if and only if there exists a finite automata that accepts it, and we've made one that does.

We've proven that $(\bar{L_1} \cup \bar{L_2})$ is regular and using the first proof one last time we can say the complement $\overline{(\bar{L_1} \cup \bar{L_2})}$ is also regular and $L_1 \cap L_2 = \overline{(\bar{L_1} \cup \bar{L_2})}$ so $L_1 \cap L_2$ is regular

2. Prove that if L_1 and L_2 are CFLs, then so is $L_1 \circ L_2$

Lets say L_1 is generated by G_1 and L_2 is generated by G_2 where $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$

In order to prove closure under concatenation we create a new G that has all of the same rules that G_1 and G_2 had, with one extra: $S \to S_1 S_2$ where S is the new start symbol.

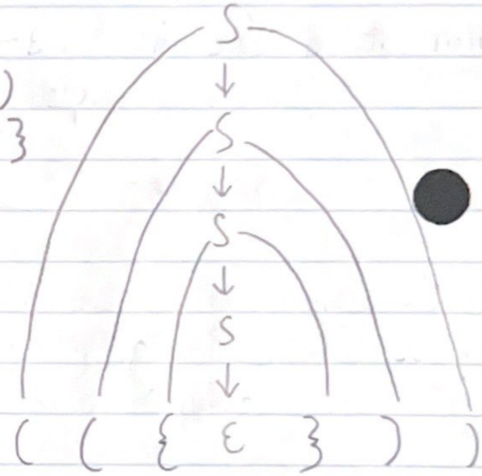
We can now define G as $(V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \to S_1 S_2\}, S)$

We can use the newly defined G to prove that the concatenation ↑ of $L_1 \circ L_2$ is a CFL because now every word generated Our new rule by G is a word found in L_1 followed by a word in L_2.

3. $S \to S_1 S_2$
$S_1 \to Aa$
$A \to \varepsilon$
$S_2 \to Bb$
$B \to \varepsilon$

Adding $S \to SS$ doesn't close this grammar under Kleene Star, it also doesn't include ε.

4. $S \to \varepsilon$
$S \to (S)$
$S \to \{S\}$
$S \to \varepsilon$
$S \to SS$



5. A Language is context free if it can be generated by a CFL

a) $\emptyset^* 1^*$ can be generated by $S \to \emptyset S$, $S \to S1$, $S \to \varepsilon$

b) $1^n \emptyset^n$ or $\emptyset^n 1^n$, $n \in \mathbb{N}$ can be generated by $S \to \emptyset S$, $S \to 1S$, $S \to S\emptyset$, $S \to S1$, $S \to \varepsilon$

c) ww^R where w^R is w in reverse.

$S \to S1$, $S \to S\emptyset$, $S \to \emptyset S$, $S \to 1S$, $S \to \varepsilon$