Energy Efficient Driving in Dynamic Environment: Considering Other Traffic Participants and Overtaking Possibility

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Chapter 4
Energy efficient driving in dynamic environment: Considering other traffic participants and overtaking possibility

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Abstract: This chapter studies energy efficient driving of (semi)autonomous electric vehicles operating in a dynamic environment with other traffic participants on a unidirectional, multi-lane road. This scenario is considered to be a so called hard problem, as constraints imposed are varying in time and space. Neglecting the constraints imposed from the surrounding traffic, the generation of an energy optimal speed trajectory may lead to bad results, with the risk of low driver acceptance when applied in a real driving environment. An existing approach satisfies constraints from surrounding traffic by modifying an existing unconstrained trajectory. In contrast to this, the proposed approach incorporates a leading vehicle's motion as constraint in order to generate a new optimal speed trajectory in a global optimal sense.

First simulation results show that energy optimal driving considering other vehicle participants is important. Even in simple setups significantly (8%) less energy is consumed at only 1.3% travelling time prolongation compared to the best constant speed driving strategy. Additionally, the proposed driving strategy is using 4.5% less energy and leads to 1.6% shorter travelling time compared to the existing overtaking approach.

Using simulation studies, the proposed energy optimal driving strategy is analyzed in different scenarios.

Keywords: overtaking, car following, ecodriving, optimal speed trajectory

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1.1 Introduction

Increasing environmental awareness, strict regulations on greenhouse gas emissions and constant desire to increase the range of electric vehicles as well as the big economic benefits drive a lot of research in the field of energy efficient driving. As a result, there are many different approaches addressing this topic. Some approaches are related to the vehicle design optimization, some to using alternative propulsion systems and some to the driving behavior optimization. In [1] the authors present a study which shows that the driving behavior has a rather big influence on energy consumption. It is shown that energy consumption may vary in a range of approx. 30% depending on moderate or aggressive driving behavior. Driving behavior related approaches for improving energy efficiency can be grouped into: “eco routing”, “using road slope information”, “traffic light assist”, “platooning” and “overtaking” as shown in Fig.1.

Fig. 1: Approaches to increase driving energy efficiency.

Eco routing is based on determining the most energy-efficient route for the trip, which may differ from the shortest or fastest one. E.g. [2] shows an example using historical data to determine an eco-route.

Knowledge about the upcoming driving route, the road conditions and the ability to control the vehicle’s propulsion enables the optimization of the speed trajectory of the vehicle with respect to the energy consumption. This problem has been extensively studied. Discrete dynamic programming (DP) for energy efficient driving has been used for over a decade now e.g. in research focused on heavy duty vehicles [3] [4]. A comparison between different optimization methods (Euler-Lagrange, Pontryagin’s Maximum Principle, DP, and Direct Multiple Shooting) was presented in [5]. The reader will find there an analysis on the DP grid choice, tips on backward and forward dynamic programming, and on how to incorporate traffic lights. By using model predictive control (MPC) to drive vehicles on free roads with up and down slopes notable fuel savings are shown in [6]. MPC was also used to control a hybrid vehicle driving over a hill and performing vehicle following in [7]. In [8] an overview of the existing approaches and current state of the art can be found. Optimized speed trajectories are usually proposed to advice a human driver via an appropriate human-machine-interface (HMI). Rarely optimized speed trajectories are used to directly provide a reference value for underlying low-level controllers such as cruise control. Increasing vehicle automation is expected to change this in the near future.

The integration of time varying constraints such as traffic lights has also been studied intensively. Incomplete knowledge about upcoming traffic lights' timing was studied in [9], complete knowledge of the upcoming traffic lights' timing together with Dijkstra's algorithm was studied in [10] and an MPC based controller was developed with additional constraints imposed from a vehicle in front in [11]. MPC based controller proposed in [11] only considers vehicle following but not overtaking.

A lot of research on a topic of platooning was done within the SATRE project generating benefits for heavy duty vehicles [12].

In [13] a possible solution for a vehicle following problem is presented showing different concepts for safe vehicle following, defining helpful concepts such as the safe distance, time-inter-vehicular and time-to-collision. In [14] a possible solution for comfort oriented vehicle following with leading vehicle movement prediction treated as disturbance in MPC controller is presented. In publications dealing with the execution of optimal overtaking [15] [16] [17] [18], speed trajectory planning is done in a way, that modifying an optimal speed trajectory leads to the smallest deviation from the desired speed while the vehicle is overtaking. These approaches are treating the problem locally and partially and don't give an energy consumption based decision if a vehicle should overtake or not and where the best location is for overtaking. Up to now not enough attention is paid on considering other participants in traffic especially leading vehicles.

If leading vehicles are neglected in the optimization, the unconstrained planning will not be fully achievable in real driving conditions, and may in some situations lead to drawbacks in energy consumption and very likely to bad driver acceptance. This work focuses on the integration of constraints imposed by leading vehicles and a global approach to optimize energy consumption.

1.2 Problem definition

This chapter shortly states the problem definition from a mathematical point of view as an optimal control problem. The aim is to determine a vehicle speed trajectory (necessary motor torque) which results in optimal energy consumption for the given transportation task while taking into account additional constraints. First, a widely used, rather generic model of a vehicles longitudinal motion is defined.

Then an appropriate cost function and relevant constraints are outlined. Finally, incorporation of time and/or space varying constraints (e.g. leading traffic) is discussed.

1.2.1 System model

Since low model complexity is crucial for efficient optimization, the vehicle is modelled as particle-mass. This approach suits most applications and is the standard choice. The vehicle model (which we shortly call the system) is represented by two states: s – the longitudinal distance travelled by the vehicle and v – the longitudinal velocity of the vehicle. The system input is the motor torque T_m. The mathematical model is described by

$$\dot{s} = v\,, \tag{1}$$

$$\dot{v} = \frac{F_m(t)}{m} - \frac{F_r}{m}\,, \tag{2}$$

$$F_r = \frac{1}{2}\rho_a c_d A_f v(t)^2 + c_r mgcos\left(\alpha\big(s(t)\big)\right) + mgsin\left(\alpha\big(s(t)\big)\right). \tag{3}$$

The upper equation (1) originates from kinematics. Equation (2) is derived from Newton's second law with F_m being the force produced by the propulsion motor, m denoting the vehicle mass and F_r being the resistive force. This resistive force is defined in (3), with air density ρ_a, aerodynamic drag coefficient c_d, the vehicle's frontal area A_f, rolling resistance coefficient c_r, gravity acceleration g and the road slope angle α. The propulsion element (which is here considered to be an electric motor with inner torque T_m) is modelled statically by:

$$F_m(t) = \frac{kT_m\eta^{sign(\boldsymbol{T}_m(t))}}{r_w}, \tag{4}$$

using an efficiency coefficient η scheduled by a map, a combined transmission ratio of the powertrain k and the radius of the wheels r_w. The ratio between rotational speed ω of the motor and vehicle speed is defined by

$$\omega(t) = \frac{kv(t)}{2r_w\pi}\;. \tag{5}$$

1.2.2 Cost function

Based on the system, in a next step a cost function is defined. In this contribution the main focus is on energy efficiency, so the cost function will be designed to represent the overall energy consumption of the vehicle.

If only energy used by the propulsion system of the vehicle is considered, the optimal solution generally speaking will most likely lead to rather slow movement. To avoid this, some authors additionally introduced a term to the cost function weighting travelling time [8] [13]. The weighting coefficient is then tuned such that the travel time is comparable to times achieved by human drivers. Although such an approach avoids slow movement, it results in a suboptimal solution from an energy efficiency point of view.

In the following, instead of introducing weighted travelling time we include power consumption from the auxiliary devices such as air conditioning system, infotainment system, thermal management system, etc. leading to straight forward energy related cost function. These auxiliary consumptions are assumed to be a constant load P_{bn}. The total power consumption of the vehicle is then the sum of boardnet power consumption and power consumption of the motor, which is calculated from the product of the rotational speed of the motor ω and motor torque T_m. Consequently, the used energy is the integral of the power over time of the trip represented by

$$E_{min} = \min_{T_m} \int_0^T (\omega(t) T_m(t) + P_{aux})\, dt\,. \tag{6}$$

Instead of using time for integration, the distance can be used. This offers some distinct advantages for solving as final time is not known, but final distance is, and road slope appears as a function of distance [5].

1.2.3 Internal constraints

Internal constraints in the optimization problem originate from constrained system dynamics, constraints on states, initial and final conditions. The following *internal* system constraints are considered:

Vehicle maximum speed and acceleration limits:

$$v_{min} < v(t) < v_{max}, \tag{7}$$

$$\dot{v}_{min} < \dot{v}(t) < \dot{v}_{max}, \tag{8}$$

Initial and final conditions for position and velocity:

$$v(0) = v_i, \qquad v(T) = v_f, \tag{9}$$

$$s(0) = 0, \qquad s(T) = S, \tag{10}$$

Note that the complete behavior of the electric motor such as maximum available torque, rotational speed and efficiency is modelled within the efficiency map. Apart from internal constraints, constraints which are imposed from environment and external factors exist.

1.2.4 External constraints

External constraints can be classified according to the dependence on two variables relevant for the optimization problem. These are space and time. This means that external constraints can be grouped into four different groups summarized in Table 1.

Tab. 1: External constraints classification.

	Time variant	Time invariant
Space variant	*other traffic participants*	*resting time (e.g. every 2h)*
Space invariant	*traffic lights*	*traffic signs (e.g. speed limits), road curvature*

Generally speaking invariant constraints are easier to integrate into the optimization problem than variant constraints. In some cases time/space variant constraints can lead to tremendous efforts when considered.

Time-and-space-invariant constraints

This type of constraints is straight forward to integrate as it is constant in time and space. Examples are: Traffic signs, which limit maximum speed on some road segments or road curvature which also limits maximum speed because of the risk of roll-over. On curved segments a minimum longitudinal acceleration limit can also be imposed. The resulting acceleration would generate an inertial force pushing back the driver and improving the drivability feeling since it may partly compensate the uncomfortable centrifugal forces.

Time-invariant-space-variant constraints

An example of a constraint of this type could be resting time. Usually the driver has to make resting stops after continuously driving for longer periods. This generally doesn’t explicitly depend on space as there are more resting spots along the road. With some approximation, charging of the vehicle could be considered as a constraint of this type.

Time-variant-space-invariant constraints

Constraints of this type do not change in space but change with time. A typical example is a traffic light, which sets the maximum speed to zero when the red light is active. This happens in discrete time intervals at a fixed location in space.

Time-and-space-variant constraints

Constraints of this type are usually hard to consider, sometimes even impossible with reasonable effort. Typical examples for a constraint of this type are other vehicles moving in the surrounding traffic. Constraints are on the speed and position of the controlled vehicle. Time and space of such constraints are not fixed, since the velocity of the controlled vehicle itself influences the constraint. For example if the controlled vehicle is moving faster, it will reach a leading vehicle sooner in time and space. Things get even more complicated when considering the possibility of overtaking. In this case the speed of the controlled vehicle has to be significantly higher than the speed of the leading vehicle, providing a speed difference to safely overtake. Such constraints can be mathematically expressed as:

$$|s(t) - s_{lead}(t)| > d_{safe} \tag{11}$$

$$v(t) - v_{lead}(t) > v_{safe}. \tag{12}$$

Constraint (11) is active in case of vehicle following and constraint (12) in case of overtaking a leading vehicle.

From a practical point of view, as the future movement of other vehicles is usually unknown, some assumptions must be made. In this work, the most simple movement prediction is used by assuming, that other vehicles will continue to move with their actual speed. Additionally, it will be assumed that other vehicles will not overtake the controlled vehicle and may slow down behind in order to avoid collisions once they have been overtaken. This assumption leads to the possibility of neglecting vehicles following the controlled vehicle.

1.2.4 Challenges

There are a few challenges connected to the discussed optimization, which are shortly highlighted in the following:

Nonlinear problem

The problem is nonlinear, as air drag resistance is proportional to the square of the velocity. Including the motor efficiency adds additional nonlinearity, as there is a sign function in motor torque calculation.

Dynamic constraints

The incorporation of the dynamic constraints increases the problem complexity. Many methods such as backward dynamic programming with free final time cannot cover them as these constraints depend on the trajectory from the start.

Course of dimensionality

If the optimization is solved numerically, state discretization is necessary. Increasing the number of system states considered in optimization problem dramatically increases the number of possible state combinations. Each additional state multiplies the number of state combinations by the number of its discretization values. E.g. Considering multiple propulsion sources as in hybrid cars, or even considering distinctive gear ratios will highly increase the computational effort for solving.

Real-time computation vs precision

As the focus is on real-time applications within a vehicle the general tradeoff between speed of computation and precision has to be taken into account. This will have an impact on the choice of methods.

1.3 Optimal motion planner

As explained above, publications considering road slope to generate energy efficient speed trajectory do not fully consider other vehicles in traffic and the possibility to overtake them. On the other hand publications dealing with optimal overtaking tackle the problem locally and consider efficiency only as deviation from a desired speed which leads to a local optimal solution.

In contrary to existing approaches, this work aims to achieve a global optimal solution by generating a new optimal speed trajectory. A new optimal speed trajectory has to be generated since the constraints are assumed to be violated. Therefore, the unconstrained optimal speed trajectory in this case is not optimal and there is no argument in persisting to still track it.

The approach proposed in this work gives valid conclusion if it is more beneficial (in terms of overall energy consumption) to overtake a leading vehicle or to slow down based on predicted energy usage for the trip and actual overtaking maneuver and on which segment of the road to overtake. The planning is based on a motion prediction (assumption) of the leading vehicle moving in front and information about the upcoming road.

In this work dynamic programming is used as a method to derive the optimal control (here the speed trajectory) because of its broad range of applicability and the flexibility to integrate different constraints.

1.3.1 Dynamic programming

Dynamic programming is a very common method used for solving the optimal control problem discussed in this work. The main advantages are its flexibility and possibility to incorporate different kinds of models and constraints. It is based on the Principle of Optimality, and it was introduced by R. Bellman [19]. The interested reader is referred to [20].

Principle of Optimality

An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions. [19]

This principle enables iterative search for optimal solution starting from the end point and building an optimal trajectory to the start. In each step transitions from all possible current states to the all previous states with accumulated previous values are calculated and minimum costs and respective transitions are selected for each possible current state. In this way going backwards in time, trajectories are growing by using calculation results from the "previous" step.

For this work a tailored and computationally optimized solution for optimal speed trajectory planning based on dynamic programming was developed in MATLAB making intensive use of matrix calculus. For a trip of ***4200 m*** with discretization steps ***ds = 5m*** and ***dv = 0,1 m/s*** solving on a standard PC (intel i5) it takes ***8.83 seconds*** to plan an optimal speed trajectory using energy efficiency maps for the electric motor and ***4.81 seconds*** using a constant efficiency.

To validate the implementation both, forward and backward dynamic programming schemes were implemented. The achieved results are identical, as it was expected. The advantage of the backward calculation is that the calculated result can be reused during the trip as it only depends on the final state. This is not the case with the forward calculation, where results are related to the specific initial state. On the other hand the advantage of the forward calculation is that other states such as the position of other vehicles can be calculated as the initial time is always known.

The implementation of dynamic programming can use time or distance as a basis for discretization. Using distance is useful in finding the energy optimal trajectory as the final time is not fixed but it has several disadvantages such as, when the speed is zero it is impossible to calculate time. An additional disadvantage is that on high speeds time shortens and with fixed speed discretization steps the number of possible transitions which satisfy the maximum acceleration constraints decreases. Nevertheless, an appropriate selection of the discretization steps leads to a valid solution. Therefore, distance as a base is used in this implementation.

In order to deeply understand the optimal control discussed in this work we introduce two helpful concepts: the optimal trajectory tree and the cost-to-go map.

1.3.2 Optimal speed trajectory tree

We define the so called optimal speed trajectory tree as a tree like structure formed by connecting all optimal transitions by a line. This results in a nice representation of the optimal control problem and together with a cost-to-go map it gives insight into an optimal behavior of the case without leading vehicles. This will be useful in understanding the advantage of the proposed approach compared to existing approaches, which are discussed in section 4.2.

What can be noted is that generally if two different trajectories have a common node they will continue on same trajectory towards the goal.

An optimal speed trajectory tree for a problem considered in this work with discretization steps of 10 m for distance and 0.3 m/s for speed is shown on Fig. 3. This map is generated from end to front in backwards DP. Additionally to the optimal solution for the given initial condition, we can see optimal solutions for different initial conditions and the impact of different initial conditions.

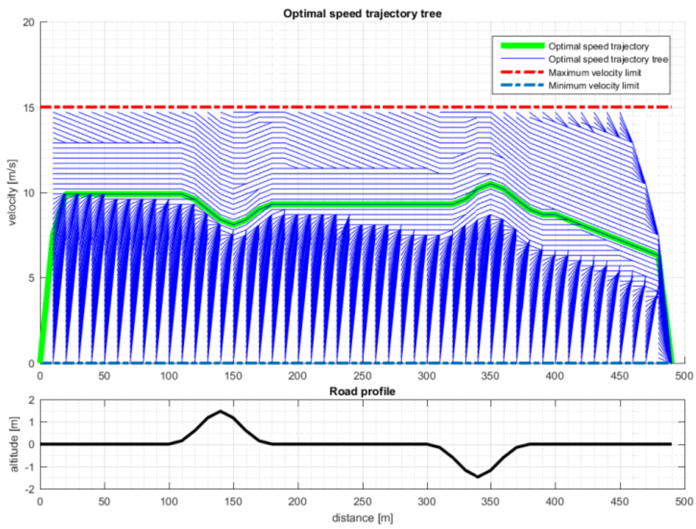

Fig. 3: Optimal speed trajectory tree in backward planning dynamic programming.

1.3.3 Cost-to-go map

The so called cost-to-go map represents the energy needed to finish a trip on an optimal trajectory from each point in distance-velocity plane. In Fig. 4 the cost-to-go map for the same problem as in Fig.3 is shown for the backward DP approach.

The characteristics of the cost to go map are closely related to the cost function and it can be noted that costs increase as distance is closer to the start. Additionally, costs are smaller on higher speeds because of the bigger kinetic energy stored in a moving vehicle. Costs are also smaller on a hill and bigger in the valley because of the potential energy.

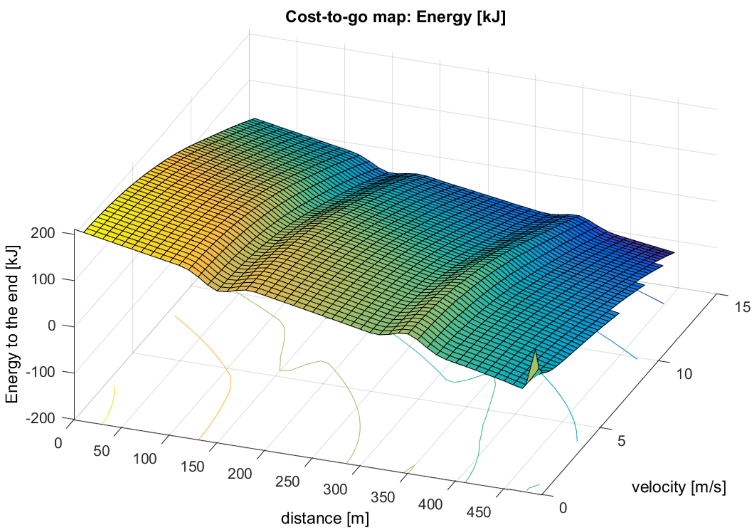

Fig. 4: Cost-to-go map of cumulative costs to travel to the end (backward planning).

1.3.4 Considering external constraints

Time-and-space-invariant constraints

As mentioned in Section 2 examples of constraints of this type are: traffic signs, which limit the maximum speed on some road segment or due to road curvature which also limits the maximum speed because of the risk of roll-over. These constraints can be easily incorporated into the optimization and can be handled by in both, forward and backward DP, if distance is used as the integration variable. This type of constraint imposes that transitions to the speed values which are conflicting with a traffic sign are not allowed on the distance segments where the traffic sign is active. Impossible transitions are implemented by assigning an infinite cost value to that transition.

Time-and-space-variant constraints

Constraints of this type are a main focus of this work, since it deals with other vehicles moving in the leading traffic. These constraints are imposed on speed and position of the controlled vehicle. As mentioned previously, both time and space of such constraints are not fixed, because the velocity of the controlled vehicle itself influences this constraint.

Due to this, forward DP is used here, enabling to calculate the distance between the controlled vehicle and the leading vehicle for each trajectory, which is possible since the initial distance is known. The movement of the other vehicle is calculated using a simple prediction model that assumes that the leading vehicle is moving with constant speed and that it will slow down if it reaches the controlled vehicle (after being overtaken). Note that a more sophisticated model of the leading vehicle's choice of velocity which may depend on space, time and the controlled vehicle can be easily included.

Finally, the transitions which do not satisfy the constraints (11) or (12) explained in Section 2 are defined to be impossible. Disabling a transition will disable all trajectories leading to a collision. As a consequence only trajectories which do not lead to a collision will be checked and the best out of these will be selected.

Time-variant-space-invariant constraints & time-invariant-space-variant constraints

Constraints of this type are not considered now, they will be tackled in future works. These constraints can be considered as a special case of time-and-space-variant constraints, therefore incorporating them should be straight forward once time-and-space-variant constraints are incorporated.

1.4 Simulation results

In this section, analyses and comparison to other approaches will be presented in order to highlight fundamental attributes and advantages of the proposed approach. Discussion of the results will be based on the vehicle trajectories in distance-velocity and time-distance plots, appropriate tables will show the differences in consumed energy as well as travelled time.

In the first subsection, driving with constant speed is compared to optimal driving. In the second subsection, differences between the proposed and existing overtaking approach will be investigated. In the third subsection, a short analysis of the influence of the road gradient on the overtaking event will be made. After that, the influence of the speed difference on overtaking event will be analyzed.

For this purpose, a road segment of 500m with a hill and a valley will be used. The hill is 1.6 m high with maximum slope of 6 %; the valley has the same shape but negative. Discretization step size for distance is 1 m and for speed 0.03 m/s have been used. If not stated differently, this discretization applies to all simulation experiments.

1.4.1 Constant speed vs. optimized variable speed

At first driving with constant speed is compared to driving at optimal variable speed. The example simulation is setup as follows: Three different speed set points (7 m/s, 9 m/s, 11 m/s) are used. The vehicle starts from standstill and accelerates to reach the set speed. Then it continues to travel with this speed until it has to decelerate to finally stop at the end of the trip.

In contrast to this, optimal driving, as described in Section 2 and 3, takes road slope, air resistance, roll coefficient, board net power consumption, and power-train efficiency into consideration. An (in the sense of energy usage) optimal speed trajectory is generated for the given travel.

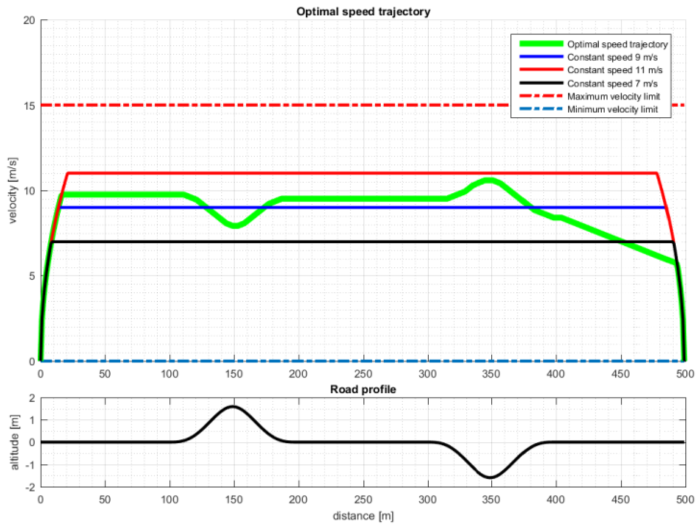

Fig. 5: Constant speed vs. optimized variable speed.

As can be seen in Fig. 5, this results in moving on a constant speed of round about 10 m/s (for this vehicle and air drag resistance) on a flat road, slowing down uphill and speeding up downhill.

Tab. 2: Constant speed vs. optimized variable speed.

	Energy used [kJ]	*difference [%]*	*Time travelled [s]*	*time difference [%]*
Optimal traj.	213.87	0	59.25	0
speed 9 m/s	231.68	+8.3	58.49	-1.28
speed 11 m/s	237.98	+11.3	49.06	-17.20
speed 7 m/s	237.97	+11.3	73.75	+24.47

Table 2 summarizes the results from Fig. 5 with respect to energy consumption and traveling time. Concluding this example, an improvement in energy consumption above 8% can be observed. Prolongation of the trip is only about 1.3%.

1.4.2 Proposed vs. existing overtaking approach

In this subsection, the proposed approach is compared to existing overtaking approach. As mentioned in the introduction in literature optimal overtaking is usually incorporated as a least quadratic (LQ) deviation from the desired vehicle trajectory. This means that an unconstrained optimal speed trajectory is generated and then modified to satisfy safety requirements and avoid collisions.

In the following simulation example, it is assumed that a leading vehicle will move with a constant speed of 8 m/s located initially 20m ahead of the controlled vehicle. In Fig. 6 the green speed trajectory represents the optimal speed trajectory for the unconstrained case. This trajectory is used as reference trajectory for generating a collision free trajectory (red color) as proposed in literature. In contrast to this, the blue trajectory is the optimal speed trajectory resulting from directly incorporating the constraint into the optimization problem.

Tab. 3: Proposed vs. existing overtaking approach. Summary on energy consumption and traveling time

	Energy used [kJ]	*difference [%]*	*Time travelled [s]*	*difference [%]*
Unconstrained	213.87	0	59.25	0
Constrained	215.45	+ 0.74	57.34	-3.23
LQ deviation	225.15	+ 5.27	58.27	-1.66

As expected from Fig. 6, Table 3 reveals that regarding the energy consumption as well as traveling time considering the constraint in the optimization leads to better results. The constrained optimization leads to only slightly larger energy consumption (+0.74%) compared to the unconstrained case. The standard approach leads to a bigger difference (+5.27%). This implies that integration of the leading vehicle as a constraint into the optimization problem may play a considerable role in reducing energy consumption. Even there is the potential to additionally save traveling time.

Note that Fig. 7 shows that the location and point in time of the overtaking event for both approaches differ. In the case of constrained planning, the overtaking event is at 224 meters. In the case of LQ deviation planning overtaking event is at 202 meters.

Computation time for calculating LQ deviation trajectory can be approximated to be almost twice as big as necessary, because first the unconstrained planning has to be generated and then LQ deviation trajectory has to be calculated.

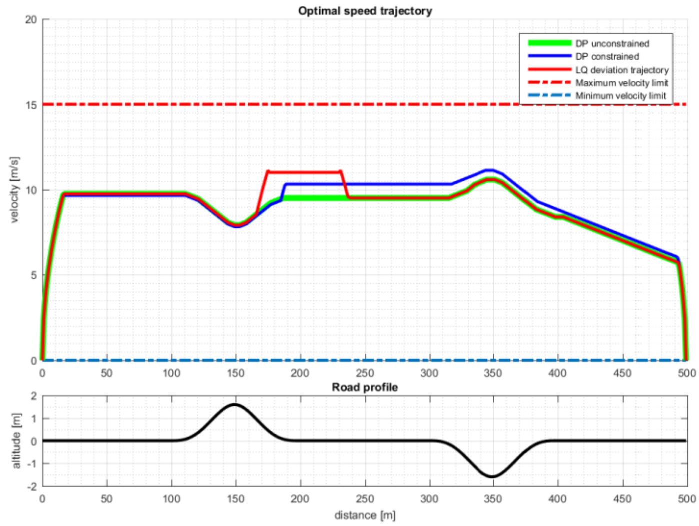

Fig. 6: Proposed vs. existing overtaking approach.

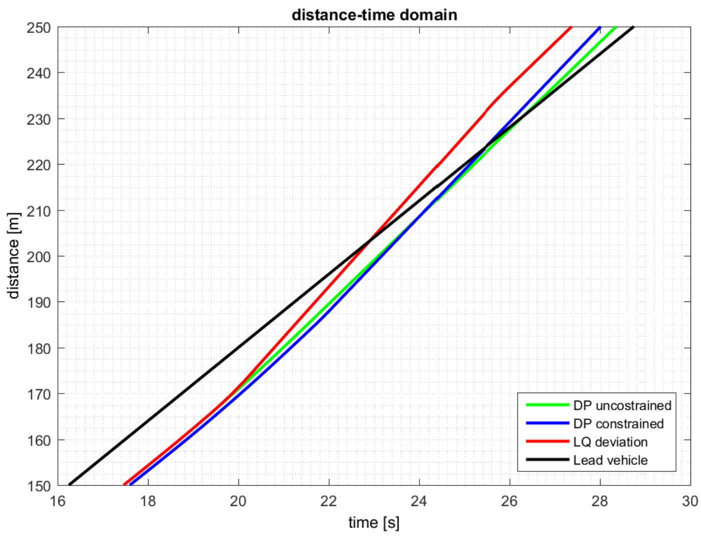

Fig. 7: Different locations of overtaking events in proposed and existing overtaking approach.

1.4.3 Influence of the road gradient

Within another simulation study the influence of the road gradient on a constrained trajectory will be analyzed. The leading vehicle's initial position will be varied so that the leading vehicle trajectory crosses the unconstrained trajectory at different locations (possible collision at different locations). The leading vehicle will move with a constant speed of 8 m/s and its initial position will be set to 5m, 10m, 15m, and 20m.

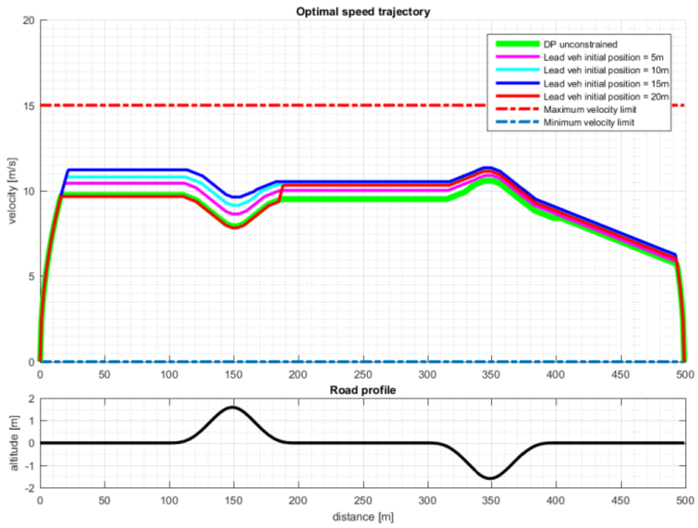

Fig. 8: Influence of the road gradient.

As shown in Fig. 8 the algorithm doesn't give uniform results for all cases and it seems that there are some preferred segments for overtaking. Either the vehicle speeds up and overtakes the leading vehicle before the hill or it slows down and waits until it passed the hill and overtakes on a downhill section (red trajectory). This complies with usual human driving behavior (at least when considering energy consumption).

Table 4 shows that energy consumption in all of the cases is not much bigger than 1 % and time is smaller as a result of speeding up, compared to the unconstrained problem.

Tab. 4: Influence of the road gradient.

	Energy used [kJ]	*difference [%]*	*Time travelled [s]*	*difference [%]*
Unconstrained	213.87	0	59.25	0
Init. pos.=0m	214.26	0.18	56.78	-4.17
Init. pos.=10m	214.99	0.52	55.12	-6.98
Init. pos.=15 m	216.07	1.03	53.85	-9.12
Init. pos.=20m	215.45	0.74	57.34	-3.23

1.4.4 Influence of the leading vehicle speed

In a last simulation study the influence of the leading vehicle speed on a constrained trajectory will be analyzed. For this reason, the leading vehicle's speed will be varied. The leading vehicle's position is also adjusted so that the leading vehicle's trajectory crosses the unconstrained trajectory always at 250 meters. The leading vehicle will have constant speeds with 7.5 m/s, 8 m/s, 8.5 m/s, and 9 m/s.

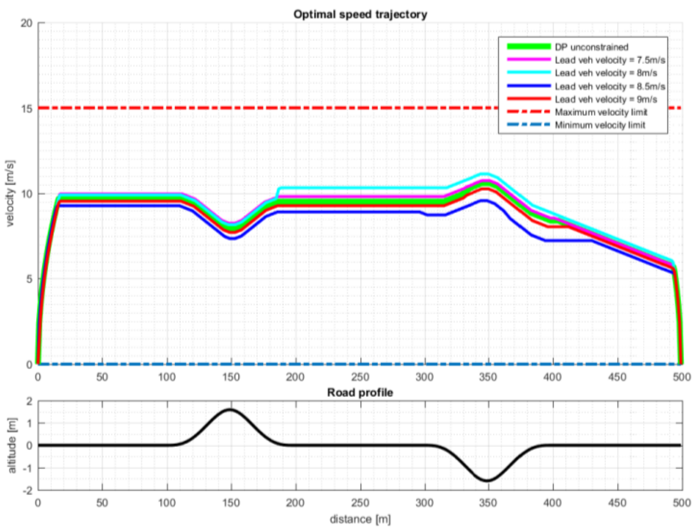

Fig. 9: Influence of the leading vehicle speed.

Fig. 9 shows the simulation results of the above described setup. It can be seen that the optimization again doesn't give uniform results for all cases and that sometimes it is better to slow down and follow the leading vehicle (blue and red) and sometimes to speed up and overtake (magenta and cyan). This also complies with usual human driving behavior. Table 5 shows that differences in energy consumption in all of the cases are rather small and differences in travel time depend on following or overtaking.

Tab. 5: Influence of the leading vehicle speed.

	Energy used [kJ]	*difference [%]*	*Time travelled [s]*	*difference [%]*
Unconstrained	213.87	0	59.25	0
LV velocity 7.5	213.94	0.03	58.07	-1.99
LV velocity 8	215.02	0.53	56.80	-4.13
LV velocity 8.5	215.61	0.81	63.49	7.16
LV velocity 9	215.42	0.72	61.56	3.89

Conclusion

Forward dynamic programming, as a flexible approach, allows the consideration of available movement prediction of leading vehicles as constraints within travel speed optimization. As a result, a global optimal solution in energy consumption can be obtained. Sometimes additional travel time reduction compared to the existing overtaking approach reported in literature has been observed.

The constrained problem showed up to require much smaller discretization steps to produce valid results compared to the unconstrained problem. A reason for this is that the optimal trajectory is usually very close to the constraints during an overtaking maneuver so a coarse grid can cause unwanted oscillations. As the minimum clearance between vehicles is only a few meters, it is also important that the space discretization step is at least one order smaller. Small space discretization steps automatically imply small velocity discretization steps to provide enough possible transitions in each step.

For the sake of simplicity, the presented work investigated one leading vehicle moving in the same direction on a two lane road. However, extending the optimization to handle more vehicles or vehicles moving in different directions is straight forward, since additional vehicles are represented by constraints of the same type. The additional constraints just increase the computational effort, but the calculation schema stays the same. The consideration of available lanes at certain segments can be accomplished by adding the lane as an additional state.

Although considering other traffic participants within the optimization increases the effort for solving compared to the original unconstrained problem, there is a big potential for improvements in both, energy consumption and travelling time.

First simulation studies also show that driver acceptance will not be a problem since the behavior is intuitive.

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