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Comprehensive Treated Sections in a Trajectory Planner for Realizing Autonomous Driving in Braunschweig's Urban Traffic

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Abstract— **With the announcement of the "Stadtpilot"-Project the Technische Universität Braunschweig has accepted the challenge of guiding a vehicle fully autonomously in the complex environment of Braunschweig's entire ring road. Autonomous driving on this two-lane urban road includes interaction with traffic, behavior at intersections, lane change maneuvers at speeds up to 60 km/h as well as merging into moving traffic. The demanding terms of the surroundings require an extremely high precision of all modules. Compared to former research activities in autonomous driving, the width of the ring road's roadway varies significantly and has both narrow curves close to the turn radius of the vehicle and straight roads. Therefore, a new approach for trajectory planning having the comprehensive treatment of path-planned sections as its distinctive feature has been realized and the first major success within the context of the "Stadtpilot"-Project. Curvature optimized trajectories are generated over the whole roadway that are independent from the way driving decision are found, resulting in a safe, smooth and comfortable driving behavior. This paper introduces the algorithm and shows its potential on the basis of a typical driving maneuver on the ring road.**

I. INTRODUCTION

While former research activities were mainly focused on highway environments [1]-[4], selected complex maneuvers [5], [6] or off-road scenarios [7], [8], numerous autonomous vehicles interacted with each other for the first time in an urban environment in the 2007 DARPA Urban Challenge. Due to its visibility the Challenge developed into a demonstration of the technically possible.

For the Urban Challenge, the team CarOLO of the Technische Universität Braunschweig equipped a 2006 Volkswagen Passat station wagon named "Caroline" to compete in this latest contest of the DARPA. The team mastered the qualification process consisting of a video presenting the vehicle's abilities, a site visit and a National Qualification Event. Among only 11 teams out of the initially 89 competitors, the CarOLO team passed the challenges to qualify early for the final DARPA Urban Challenge event [9], [10].

With its participation in the DARPA Urban Challenge the Technische Universität Braunschweig proved that it was capable of building up a state-of-the-art autonomous vehicle. To further increase the difficulty of driving autonomously in an urban area, the Technische Universität Braunschweig has continued the effort with the follow-up project called "Stadtpilot" and the objective to drive fully autonomously on Braunschweig's entire ring road, which is known as the arterial road of its inner city traffic. Fig. 1 shows a map of the proposed track and two typical scenes of the ring road environment.

Fig. 1. Proposed track for autonomous driving around Braunschweig with an overall length of 12 km (marked in red).

II. TRANSFERRING AUTONOMOUS DRIVING INTO A REAL URBAN ENVIRONMENT

While the Urban Challenge was an artificial, strongly simplified urban environment, the goal of "Stadtpilot" is to drive fully autonomously in real urban traffic. The challenges of the project are constituted by the complex environmental conditions of the ring road: the dense traffic on this two-lane urban road imposes high demands on the modules for environmental perception, the road geometry requires a precise trajectory planning and shadowing effects due to tall houses or trees can interfere with the reception of the GPS signals resulting in a sophisticated positioning of the vehicle.

The differences between "Stadtpilot" and Urban Challenge are manifold and vary from modified basic conditions to a totally changed infrastructure. The race contest is substituted by a long-term project with the goal of achieving a high research sustainability. Instead of the disused military area of the Urban Challenge the scenario takes place in the densely populated surroundings of Braunschweig's inner ring road. With the "Stadtpilot"-Project the expertise gained by building up an autonomous vehicle for the Urban Challenge is applied to a real urban environment. Table I contrasts the major differences of both scenarios.

This paper specially focuses on the approach taken to assure a precise trajectory planning on the ring road environment. Most of the so far applied trajectory planners are based on clothoids, sigmoid functions or splines and generate piecewise trajectories [12]-[14]. In the following a new trajectory planning algorithm is stated that is based

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TABLE I
COMPARISON OF DARPA URBAN CHALLENGE 2007 AND "STADTPILOT"-PROJECT

	DARPA Urban Challenge	Project Stadtpilot
Basic conditions	Race contest	Long-term project
	Winning a prize	High research sustainability
	Non-public area	Real urban traffic
	DARPA regulations	Road traffic regulations
Infrastructure	Disused military area	Highly populated area
	4-Way-Stops	Traffic lights
	Paved and unpaved roads	Only paved roads
	Uninterrupted GPS-receiving	GPS shadowing effects
	Fault-tolerant infrastructure	Highest precision mandatory
Environment	Stunt drivers	Real traffic conditions
	Only cars and static obstacles	Manifold road users
	Defensive driving behavior	Divergent driving styles
	Interaction between robots	Erratic behavior of road users
Vehicle	Manifold vehicle modifications	Constrained modifications
	Remote emergency stop	Surveillance driver, monitoring

on elastic band theory and so-called smoothing splines. Its distinctive feature is the comprehensive treatment of path-planned sections generating curvature optimized trajectories for the whole roadway resulting in a safe and smooth driving behavior.

III. THE HARDWARE

Two 2007 Passat station wagons named "Henry" and "Leonie" after the famous duke of Braunschweig "Henry the Lion" are currently being equipped to fulfill the requirements of an autonomous vehicle. "Leonie" serves as the first vehicle platform in the "Stadtpilot"-Project and has been equipped with a variety of actuators, sensors and computers as shown in Fig. 2. In addition, "Leonie's" brother "Henry" is currently built up to enable parallel research activities.To generate an object-based model of the environment, lidar and radar sensors are used. The combination of the advantages of each sensor modality leads to a redundant environment model. Laser scanners mounted on both sides of the vehicle provide information about the lane marking to improve the vehicle's self-positioning. A server rack mounted in the trunk of "Leonie" serves as the computational core for different software modules. A CAN (Controller Area Network) interface opens the opportunity to access "Leonie's" electronic steering, brake and throttle system.

IV. REQUIREMENTS FOR TRAJECTORY PLANNING

The requirements of the trajectory planning are deduced from the non-holonomic constraints of the test vehicle, the scenario and the special conditions of the ring road

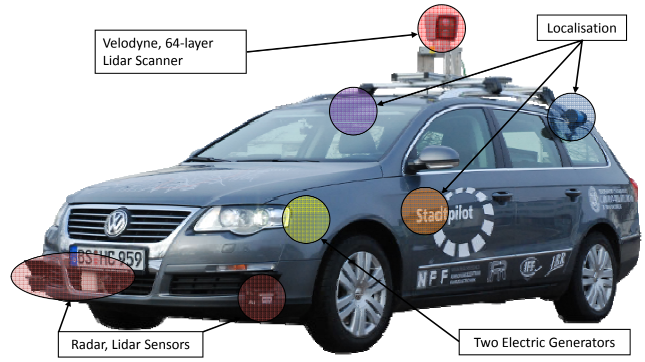

Fig. 2. Autonomous vehicle "Leonie".

environment. While for instance in the Urban Challenge an optimization of the driving comfort was subordinate, a smooth and comfortable driving behavior is postulated in the "Stadtpilot"-Project. Also, the limited dynamics of the steering actuator require optimized trajectories in terms of reduced actuating variables. While a reduced curvature of the desired track leads to lower steering angles and minimizes side acceleration, a reduced curvature derivative results in less activity of the steering actuator and therefore in a shortened lateral jerk. By reducing curvature and its derivative the remaining actuator potential and the distance to vehicle dynamics limits can be increased.

The course of the ring road has developed over time. Compared to highway scenarios or the Urban Challenge environment, the width of roadway varies significantly and has narrow curves close to the turn radius of the vehicle as well as straight roads. However, the trajectory planner is in charge to generate optimized trajectories independent from those divergent constraints. The purpose of the shown trajectory planner in this paper can be summarized as follows:

- Realizing of as much as possible remaining actuator potential
- Increased vehicle dynamics reserve
- Realizing sophisticated driving maneuvers close to the vehicle's turn radius
- Increased driving comfort through less lateral accelerations and actuator activities
- Implementation of a driving maneuver independent trajectory planner

V. MULTI STAGE VEHICLE GUIDANCE SYSTEM

Three different kinds of input data are available to guide the vehicle through the surroundings of the ring road: a-priori-knowledge in form of a digital map, grid data containing static obstacles, and object data representing the dynamic part of the environment like other vehicles and so on. Fig. 3 emphasizes how available environmental data are used in the vehicle guidance system in the "Stadtpilot"-Project. It consists of multiple stages resulting in a configuration space which takes dynamic and static environmental data into account. First, the configuration space is spanned by a digital map limited with a left and a right border according to

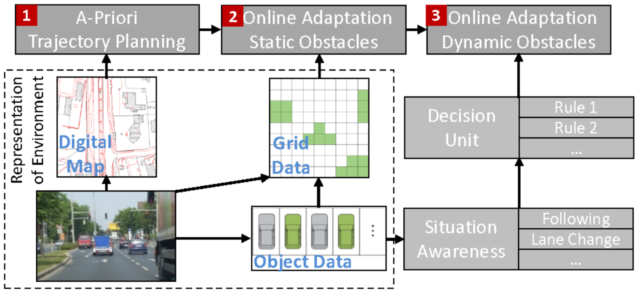

Fig. 3. Vehicle guidance consisting of multiple stages.

lane markings. Based on these limitations an a-priori planner determines an optimized trajectory for the whole course. This approach opens a new degree of optimization especially on twisty sections. Using the knowledge of the whole curve progression it becomes feasible to approach curves optimized, resulting in an improved directional stability and less controller activity.

In the absence of any obstacles the vehicle can follow a trajectory that is calculated based on these a-priori data. During the drive the a-priori trajectory is adapted to the actual driving situation through avoiding static and dynamic obstacles. Static obstacles are considered by adapting the borders of the configuration space according to the grid-based representation of the environment; dynamic object data serve the situation awareness as well as the decision unit to realize an online adaptation of the configuration space based on the current driving situation. The realization unit is now in charge to determine trajectories in between these limitations taking the non-holonomic constraints of the vehicle through a partly online adaptation of the pre-calculated trajectory into account. In addition, maneuvers like lane changes cannot be planned in advance. Therefore, it is calculated online how the trajectory is conveyed from one precalculated trajectory to the other using the same algorithm (Fig 4).

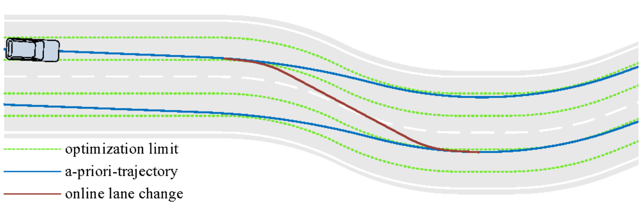

Fig. 4. A-priori trajectories and online lane change maneuver.

Instead of the described a-priori planning, also a solely partial planning seems to be feasible. However, data collected in several manual test runs have shown that an online adaptation due to static and dynamic obstacles happens very rarely. Even though the environment consists of a huge number of dynamic obstacles, it usually leads only to an adaptation of the speed profile while the width of the configuration space can remain unmodified. Therefore, an a-priori planning in combination with certain online adaptation leads to less trajectory sections compared to an approach that calculates trajectories only partially.

The described process is mapped into a special data structure as shown in (Fig. 5). A drivable corridor in front of "Leonie" is determined that takes data from the a-priori digital map as well as the grid- and object-based representation of the surroundings into account. Pearls on each side bound this area representing the configuration space for the trajectory planning; thus, connecting these pearls results in a representation of the road edge. Both the distance of the pearls and the width of the path can be adjusted to current conditions.

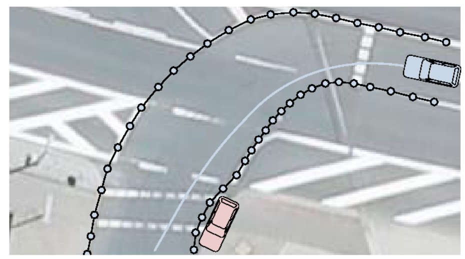

Fig. 5. Representation of driving decisions.

All necessary information can be represented in this data structure: position of traffic lights, areas with banned lane changes, number of lanes and so on. Also the desired vehicle speed at each pearl position is stored in the same data structure and serves a velocity planner to determine a dynamically optimized speed profile. Low level controllers are in charge of generating the required command to meet the desired speed. If necessary, it is feasible to extend the data structure with comfort levels to make the decision unit capable of influencing the driving comfort of the car. Comfort levels are represented by a set of parameters that influence the maximal and minimal accelerations.

Since the drivable corridor contains all necessary information about the edge of the path and the speed limits, a trajectory planner can transfer the driving decision into a suitable trajectory for the low level control module according to the non-holonomic constraints of the vehicle. A trajectory planner with the comprehensive treatment of path-planned sections is based on a unique approach that determines a curvature optimized trajectory inside this corridor and results in less controller activity compared to approaches that treat maneuver sections separately. The goal is not only to realize autonomous driving in general, but also to convert abstract decisions into a smooth driving experience. The approach has already been implemented and tested successfully and can be considered as the first success of the "Stadtpilot"-Project. The basic idea is outlined in the next section.

VI. TRAJECTORY PLANNING WITH COMPREHENSIVE TREATMENT OF PATH-PLANNED SECTIONS

Both the a-priori planning and the online adaptation are based on the same algorithm. Since the decision unit maps its decisions into a drivable corridor, the trajectory planning module first needs to perform a dynamic placement of nodes inside this area in real time. A unique optimization algorithm including a mechanical model describing the path and application of smoothing splines generates curvature optimized trajectories in the drivable corridor. As shown in Fig. 6 the algorithm consists of three major steps:

1) Analysis and typification of route
2) Reducing maxima of curvature by a mechanical model
3) Smoothing through optimizing in κ-s-plane

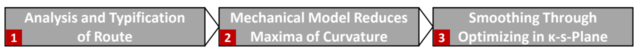

Fig. 6. Optimized trajectory planning with a three-step algorithm.

The introduction of a certain margin inside the configuration space opens the opportunity to directly influence the level of optimization. Initially, this margin is the difference of lane width and vehicle size. Also, it takes any uncertainties into consideration for instance in environment recognition or the vehicle's positioning. This offers the potential that improvements in those modules can be transformed directly into a smoother driving behavior since a widening of the margin establishes a bigger optimization space for the trajectory planner. In the following the three step algorithm is discussed in detail.

A. Analysis and Typification of Route

The overall optimization starts with a typification of the path's interpolation nodes. Those nodes are initialized centered in the configuration space; thus, a typification algorithm can determine its node class as the basis for the adjustment of its position which is essential for applying a mechanical model based on elastic band theory. Three classes are used:

1) Endpoints: Beginning and end of the route.
2) Smoothingpoints: Nodes on a left or right curve.
3) Ripplepoints: Remaining nodes.

To determine the type of node number i the perpendicular distances d_i are needed and can be determined with the equation $d_i = \frac{a}{\sqrt{a^2+b^2}} \cdot x_i + \frac{b}{\sqrt{a^2+b^2}} \cdot y_i + \frac{c}{\sqrt{a^2+b^2}}$ where $a = y_{i-1} - y_i$, $b = x_i - x_{i-1}$, and $c = x_{i-1} \cdot y_i - x_i \cdot y_{i-1}$ are the coefficients of the linear equation given by the points $i-1$ and i and its x and y position. First, distances d_{i-2} and d_{i+1} to their connecting line $g_{i-1,i}$ are calculated according to Fig. 7. Next, d_{i-1} and d_{i+2} are determined based on $g_{i,i+1}$. A comparison of the different signs of d_i allows the desired

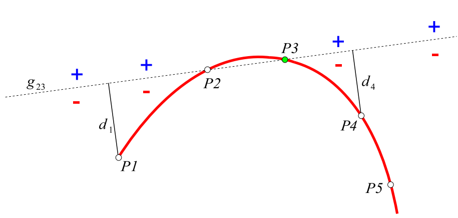

Fig. 7. Typification of a node based on its neighbors.

typification and results in a set of nodes separated into three different classes. A direction vector is determined for each node based on its class to achieve the desired adjustment. It is the goal on the one hand that the curve is expanded and on the other hand the vertex is moved in the direction of the center of curvature of the curve.

B. Reducing Maxima of Curvature by a Mechanical Model

The application of a modified elastic band algorithm repositions the interpolation nodes and leads to reduced maxima of curvature. It is based on the idea of modeling the road by an elastic deformable material. The band consists of n massless nodes, which a coupled through external in internal springs with the stiffness k to its neighbors and the edge of the roadway as shown in Fig. 8.

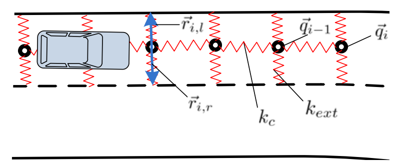

Fig. 8. Assembling a the mechanical model inside the path.

Based on Lagrangian mechanics, the effective force depending on each specific node type is composed of an internal force $\vec{f}_{int}$ and an external force on the left $\vec{f}_{ext,l}$ and right $\vec{f}_{ext,r}$ of the vehicle:

$$\vec{f}_{sum}(i) = \vec{f}_{int}(i) + \vec{f}_{ext,l}(i) + \vec{f}_{ext,r}(i) \qquad (1)$$

For $1 < i < n$ the internal forces $\vec{f}_{int}(i)$ of a node are derived from the according internal potential V_{int}, which is calculated from the potential $\vec{q}$ of the adjacent ones (Fig. 8). It is independent from the distance between two nodes:

$$\begin{aligned} \vec{f}_{int}(i) &= -\nabla_{qi} V_{int} & (2) \\ &= k_c \left(\frac{\vec{q}_{i+1} - \vec{q}_i}{\|\vec{q}_{i+1} - \vec{q}_i\|} + \frac{\vec{q}_{i-1} - \vec{q}_i}{\|\vec{q}_i - \vec{q}_{i-1}\|} \right) & (3) \end{aligned}$$

The external forces $\vec{f}_{ext}(i)$ are considered based on linear springs with a stiffness of k_{ext}. With $\vec{r}$ being a position vector to the edge of the roadway, and l being the length of the non-deflected spring, the external forces can be described as:

$$\begin{aligned} \vec{f}_{ext,l}(i) &= k_{ext} \cdot (\|\vec{r}_{i,l}\| - l_0) \cdot \frac{\vec{r}_{i,l}}{\|\vec{r}_{i,l}\|} & (4) \\ \vec{f}_{ext,r}(i) &= k_{ext} \cdot (\|\vec{r}_{i,r}\| - l_0) \cdot \frac{\vec{r}_{i,r}}{\|\vec{r}_{i,r}\|} & (5) \end{aligned}$$

The nodes' position is improved by calculating the equilibrium of forces at each node with the Euler-Cauchy-Method according to elastic band theory. The application of the balancing impact of the elastic band enables a strainless position of the path, resulting a curvature minimized trajectory. An extensive description of the method is given in [15].

C. Smoothing Through Optimizing in κ-s-Plane

At this point, the maxima of curvature are reduced; however, since the generated curve in terms of curvature and its derivative does not satisfy defined quality criteria, the curvature is directly optimized as a function of the arc length of the trajectory $\kappa(s)$ (κ-s-plane). First, B-splines are used to approximate the displaced nodes initially. Second, so-called Smoothing Splines by Schoenberg and Reimsch [16] are applied to meet the desired requirements. The basic idea of those splines is to minimize a functional J that consists of a term for approximation and a term for interpolation. The value p is chosen on the interval $[0, 1]$ to control the degree of approximation.

$$J = p \sum_{i=1}^{N} \left(\frac{\kappa_i - f(s_i)}{\delta \kappa_i} \right) + (1-p) \int_{s_0}^{s_N} \left(f''(s)\right)^2 \mathrm{d}s \quad (6)$$

The result is a smooth and minimized curvature according to the quality criteria. The inverse transformation back into x-y-plane results finally in the trajectory based on comprehensively treated sections. Both, a-priori planning and online adaptation are based on the approach described above.

Fig. 9 shows the effect of the described approach in terms of the curvature. First, the maxima of the curvature are reduced resulting in a flattened but uneven curve. Second, smoothing results in a desirable trajectory.

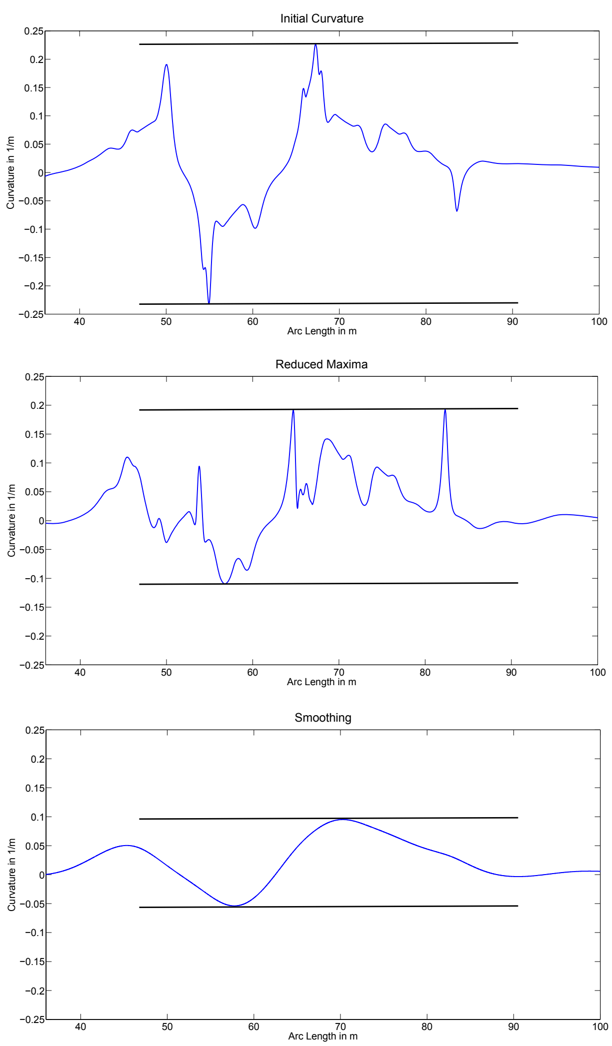

Fig. 9. Effect of the algorithm in terms of the curvature.

VII. ANALYSIS AND RESULTS

The shown approach gives the decision unit a higher flexibility compared to approaches that treat maneuvers separately. Fig. 10 shows the effect of its application to the entire ring road of Braunschweig. The enlarged part of the trajectory shows a maneuver where the vehicle starts in the upper left corner of the path, performs a lane change maneuver to the left, followed by an immediate left turn at an intersection. Fig. 11 shows the associated effect of

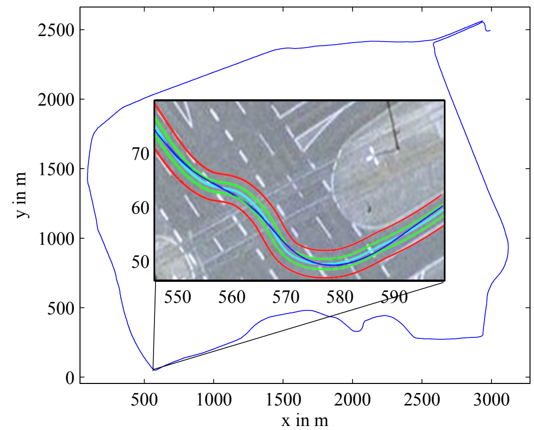

Fig. 10. Result of the comprehensive treatment of sections in the trajectory planner.

the optimization approach in terms of the curvature characteristics. The blue curve represents a smooth and flattened curvature in contrast to the non-optimized trajectory shown in red. Without the described comprehensive treatment of

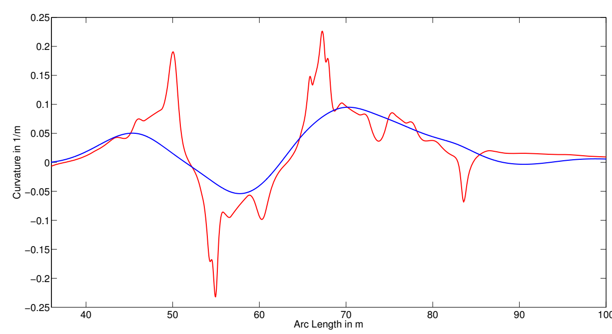

Fig. 11. Curvature of the optimized (blue) and original trajectory (red).

path-planned sections, this driving decision could not have been realized. The consecutive execution of lane change and turn maneuver in this particular example would have resulted in an unreachable steering angle due to a too high curvature.

A non real time numeric approach based on the so-called Simplex-Algorithm by Nelder und Mead [17] is used for a validation of the quality of the optimized trajectory. This reference trajectory is almost congruent to the optimized one, calculated with the approach shown above. The validation with a quality criterion in the form of the sum of the squared curvature variation $Q = \sum_{i=1}^{N} (\kappa'(s_j))^2 (s_j - s_{j-1})$ leads to an improvement of 95% of the optimized trajectory compared to a classical trajectory in the center of the roadway.

The computing time demanding reference algorithm could just achieve an improvement of 91%, whereas the attained quality depends significantly on the execution time when the numerical algorithm is stopped.

For the demonstration of effectiveness of the shown approach the impact on the autonomous driving is important. Applying the same low level control unit leads to an objective comparison of the different trajectories in between the drivable corridor. The advantages are shown in contrast to the spline approximated trajectory in the center of the road (cyan curve in Fig. 10). The resulting lateral deviation of the vehicle's position regarding the trajectory as well as the actuator activity shows the effectiveness of the optimization (Fig. 12). The steering angle with respect to the non-optimized curve is limited several times by the turn radius of the vehicle.

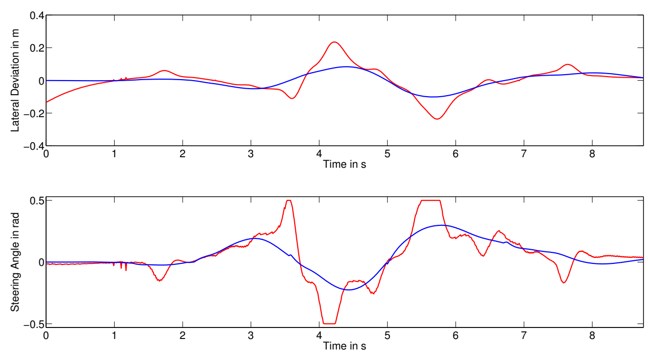

Fig. 12. Lateral deviation and steering angle for the optimized (blue) and original trajectory (red).

While the mean absolute value of the track deviation could be reduced by 42% from $5.7cm$ to $3.3cm$, the maximum of the track deviation could be decreased by 57.3% from $23.7cm$ to $10.1cm$. The achieved improvement could be mainly attributed to the release of the steering wheel actuator, which cannot realize arbitrary maneuvers. Due to the optimization the mean steering angle could be reduced by 29.2% and the maximal steering angle by 40.2%.

VIII. CONCLUSIONS

We have introduced a unique approach for generating optimized trajectories for autonomous vehicle guidance in an urban environment, which determines trajectories in between a given drivable corridor that are optimized in terms of curvature and its deviation. This approach for trajectory planning has the comprehensive treatment of path-planned sections as its distinctive feature and realizes autonomous driving with less controller deviation and less activity of the steering actuator in the "Stadtpilot"-Project.

The introduction of the tolerance margin into the algorithm opens the feasibility to map improved accuracy in the modules for environmental recognition and vehicle positioning into an augmented driving behavior. An enlarged margin results in an increased range for the optimization; therefore, a further improved trajectory characteristics can be achieved.

Optimized trajectories can be determined for an arbitrary road geometry and driving maneuver independent for instance in the environment of Braunschweig's inner ring road with narrow curves and different widths of roadway. The result is a smooth driving behavior with improved directional stability and less controller activity. The trajectory planning is done in real time on the vehicle's systems which distinguishes the shown approach from other known optimization algorithms.

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