Lane Change Maneuvers for Automated Vehicles

Julia Nilsson, Mattias Brännström, Erik Coelingh, and Jonas Fredriksson

***Abstract*—By considering a lane change maneuver as primarily a longitudinal motion planning problem, this paper presents a lane change maneuver algorithm with a pragmatic approach to determine an inter-vehicle traffic gap and time instance to perform the maneuver. The proposed approach selects an appropriate inter-vehicle traffic gap and time instance to perform the lane change maneuver by simply estimating whether there might exist a longitudinal trajectory that allows the automated vehicle to safely perform the maneuver. The lane change maneuver algorithm then proceeds to solve two loosely coupled convex quadratic programs to obtain the longitudinal trajectory to position the automated vehicle in the selected inter-vehicle traffic gap at the desired time instance and the corresponding lateral trajectory. Simulation results demonstrate the capability of the proposed approach to select an appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of a lane change maneuver in various traffic scenarios. The real-time ability of the lane change maneuver algorithm to generate safe and smooth trajectories is shown by experimental results of a Volvo V60 performing automated lane change maneuvers on a test track.**

***Index Terms*—Autonomous driving, automated driving, lane change, trajectory planning, model predictive control.**

I. INTRODUCTION

DUE to congestion, accidents, and high variation in vehicles' velocity, highway driving can be both tedious and stressful. To improve the driving experience, automotive companies have developed Advanced Driver Assistance Systems (ADAS) such as Adaptive Cruise Control (ACC) and Lane Keeping Aid (LKA) which aim to make driving safer and more comfortable [1], [2]. To further increase the capability of ADAS and eventually progress to fully automated highway driving, one particularly interesting maneuver is the lane change. This maneuver is one of the riskiest maneuvers that a driver has to perform on a highway, and can be perceived as challenging since it involves changes in both the longitudinal and lateral velocity as well as movement in the presence of other moving vehicles [3]. Hence, this paper focuses on the following real-time lane change trajectory planning problem: *given that the ego vehicle, i.e., the vehicle which is controlled by an intelligent vehicle system, should perform a lane change maneuver, determine in which inter-vehicle traffic gap and at what time instance the maneuver should be performed, and calculate a feasible maneuver (if such exists) in terms of a longitudinal and a lateral trajectory, i.e., the control signals, which allow the ego vehicle to position itself in the selected inter-vehicle traffic gap at the desired time instance. Furthermore, the maneuver should be planned such that the trajectory is robust to prediction errors and that the ego vehicle*:

- *maintains safety margins to all relevant surrounding traffic participants and objects*,
- *respects traffic rules and regulations, and*
- *satisfies physical and design limitations.*

Inspired by reachability analysis [4], [5], the proposed lane change maneuver algorithm determines whether a lane change maneuver might be possible in a certain inter-vehicle traffic gap at a certain time instance. When an appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver have been selected, the trajectory planning problem of positioning the ego vehicle in the selected inter-vehicle traffic gap at the desired time instance is formulated as two loosely coupled longitudinal and lateral Model Predictive Control (MPC) [6], [7] problems. The general idea of the trajectory planning algorithm applied to lane change maneuvers was first introduced in [8] and generalized to various automated maneuvers in [9]. This paper extends the results presented in [8] and [9] by integrating the trajectory planning algorithm for lane change maneuvers with a novel method for selecting an appropriate inter-vehicle traffic gap and time instance to perform the maneuver. Furthermore, while [8] and [9] presents simulation results of the trajectory planning algorithm, this paper provides experimental results of lane change maneuvers performed at a test track using a Volvo V60.

Various research studies have previously applied MPC for trajectory planning and motion control of automated vehicles e.g., [10]–[14]. However, although the proposed approaches do provide good results in their respective applications, it can be concluded that in order to reduce the computational complexity due to e.g., vehicle dynamics and collision avoidance constraints, a particular optimal control trajectory planning algorithm often assumes a given reference trajectory or considers either the longitudinal or the lateral aspects of the trajectory planning problem. For instance, in [10] a Quadratic Program

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(QP) MPC formulation is presented with the purpose of computing the spacing control laws of an ACC system to maintain a specified inter-vehicle distance between the ego vehicle and its preceding vehicle. However, the proposed approach does not consider any aspects of lateral trajectory planning in terms of e.g., lane change maneuvers. In [11] a trajectory planning algorithm for hazard avoidance scenarios is formulated as a QP with the purpose of maintaining the ego vehicle within a constraint bounded corridor whenever dynamically feasible. However, the presented approach does only consider lateral trajectory planning and assumes constant longitudinal velocity. In [12] a method for obstacle avoidance trajectory planning using convex optimization is presented. However, although the presented approach is able to plan trajectories which accounts for both longitudinal and lateral dynamics, it assumes the existence of reference trajectories. In [13] a two-layer hierarchical control framework for the obstacle avoidance problem of automated and semi-automated vehicles is presented. However, although the proposed approach allows for real-time implementation of the trajectory planning algorithm, the approach assumes a given reference trajectory and utilizes motion primitives which entails that only a subset of all possible maneuvers is considered. In [14] a MPC lane change trajectory planning algorithm that is fairly similar to the approach which initially was introduced in [8] is presented. However, [14] provides no details regarding the proposed algorithm's computational complexity or its applicability to real-time implementation on a standard vehicle platform.

To overcome these limitations, the lane change maneuver algorithm proposed in this paper considers both the longitudinal and the lateral planning aspects of a lane change maneuver without the assumption of a reference trajectory. The proposed algorithm thereby captures the lane change maneuver properties of adjusting the longitudinal position and velocity prior to initializing the lateral motion of the maneuver. In order to generate safe and smooth trajectories in real-time, the MPC trajectory planning problem is formulated as low-complexity convex QPs which can be efficiently solved [15]. Furthermore, by selecting an appropriate inter-vehicle traffic gap and time instance to perform the lane change maneuver in a preprocessing step, the computational complexity of the proposed lane change maneuver algorithm is further reduced. The proposed algorithm is thereby considered as a building block for ADAS regarding lane change maneuvers and eventually fully automated vehicles in a mixed highway traffic environment with both human drivers and automated vehicles with or without vehicle-to-vehicle and vehicle-to-infrastructure communication.

The remainder of this paper is organized as follows: in Section II the proposed lane change maneuver algorithm is presented, while Section III and Section IV respectively provides results of simulation and experimental studies. Finally, Section V summarizes the contributions of the paper and provides suggestions for future work.

II. Lane Change Maneuver Algorithm

The general idea of the lane change maneuver algorithm is to divide the trajectory planning problem into one longitudinal and one lateral trajectory planning module. The algorithm thereby consists of five main steps:

I. Determine the inter-vehicle traffic gap and time instance to perform the maneuver.
II. Determine the longitudinal safety corridor.
III. Determine the longitudinal trajectory.
IV. Determine the lateral safety corridor.
V. Determine the lateral trajectory.

The lane change maneuver algorithm is executed in receding horizon, i.e., at every time instance the trajectory planning problem is formulated and solved over a shifted time horizon based on newly available sensor information. Further details regarding Step I, Step II, and Step III of the algorithm are respectively given in Section II-A.3, II-A.1, and II-A.2. Note that this paper is related to longitudinal trajectory planning i.e., Step I–III of the lane change maneuver algorithm while further details regarding the lateral trajectory planning i.e., Step IV and V are provided in [8] and [9]. Furthermore, although not included in this paper it is possible to perform a feasibility check after Step I, Step III, and Step V of the lane change maneuver algorithm in order to determine whether the algorithm should continue planning the lane change trajectory or if a backup trajectory should be planned and executed.

The lane change maneuver algorithm is formulated based on the following set of assumptions:

- A1 The ego vehicle, E, is equipped with sensor systems which measure its position on the road as well as e.g., the relative positions and velocities of surrounding traffic participants and objects.
- A2 E is equipped with prediction systems which estimate the motion trajectories of surrounding traffic participants and objects over a time horizon.
- A3 E is equipped with low-level control systems capable of following the planned trajectory.
- A4 E is equipped with a decision-making system which provides a desired maneuver request e.g., a left lane change maneuver.

Examples of the assumed decision-making system, low-level control system, and the necessary sensor technology are given in [16]–[18] respectively, while the assumed prediction systems can utilize any prediction model e.g., [19], [20] to estimate the motion trajectories of surrounding traffic participants and objects over a time horizon. Furthermore, uncertainties resulting from the sensor and prediction systems can be taken into account by e.g., increasing the safety margins which E must maintain to the surrounding traffic participants and objects over the prediction horizon in relation to the confidence level of the assumed systems. In addition, the re-planning nature of receding horizon MPC allows changes in the perceived environment to be accounted for at each time instance, which makes the lane change maneuver algorithm robust to prediction errors and uncertainty. A simplified schematic architecture of the intelligent vehicle system for lane change maneuvers is illustrated in Fig. 1.

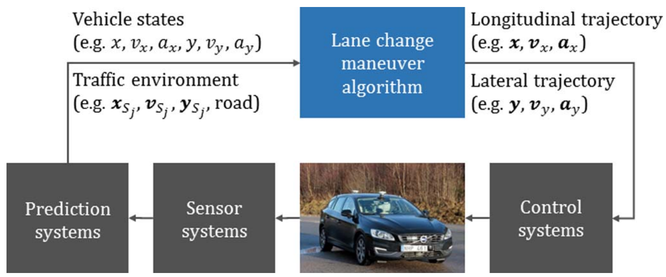

Fig. 1. Schematic architecture of an intelligent vehicle system for lane change maneuvers.

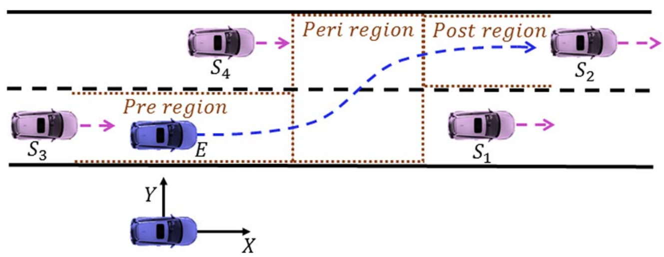

Fig. 2. Vehicles driving on a one-way road with two lanes. The ego vehicle (E) is shown in blue, and the surrounding vehicles (S_1, S_2, S_3, and S_4) are displayed in purple. The dashed arrows represent the predicted paths of S_j, $j = 1, \dots, 4$, and the planned route of E. The space which E should traverse to perform a left lane change maneuver is schematically illustrated by the Pre, Peri, and Post regions.

A. Longitudinal Trajectory Planning

For E to avoid collisions, its trajectory must be planned so that it maintains safety margins to all relevant traffic participants and objects e.g., vehicles, pedestrians, and road barriers in the surrounding traffic environment. In terms of longitudinal motion, this implies that E must be positioned within a safety corridor which defines upper and lower boundaries on its longitudinal position throughout the execution of a certain maneuver e.g., lane change. As illustrated in Fig. 2, the space which E should traverse during a lane change maneuver can be divided into three regions: Pre, Peri, and Post. Hence, for a lane change trajectory to be feasible, E must be able to traverse the Pre, Peri, and Post regions while maintaining safety margins to all relevant surrounding traffic participants and objects in each region. The following two subsections clarify how the Pre, Peri, and Post regions are defined and utilized when determining the longitudinal safety corridor and corresponding trajectory for the lane change traffic situation illustrated in Fig. 2, where E should perform a left lane change maneuver in the inter-vehicle traffic gap between the surrounding vehicles S_2 and S_4.

1) Longitudinal Safety Corridor: Prior to initializing the lateral motion of the lane change maneuver, i.e., while in the Pre region, E must be able to maintain safety margins to the preceding vehicle S_1 and trailing vehicle S_3. Once the lateral motion of the lane change maneuver is initiated i.e., while in the Peri region, E must be able to maintain safety margins to all relevant surrounding vehicles i.e., S_1, S_2, S_3, and S_4. Finally, when E has completed the lateral motion of the lane change maneuver, i.e., while in the Post region, it must be able to maintain safety margins to the preceding vehicle S_2 and trailing vehicle S_4. The upper and lower bounds on E's longitudinal position in each region can thus be determined as

$$x_{\max_k} = x_{S_{1_k}} - s_m\left(S_{1_k}\right) \tag{1a}$$

$$x_{\min_k} = x_{S_{3_k}} + s_m\left(S_{3_k}\right) \quad \forall\, k = 0, \dots, N_{\text{peri}} \tag{1b}$$

$$x_{\max_k} = \min\left(x_{S_{1_k}} - s_m\left(S_{1_k}\right), x_{S_{2_k}} - s_m\left(S_{2_k}\right)\right) \tag{1c}$$

$$x_{\min_k} = \max\left(x_{S_{3_k}} + s_m\left(S_{3_k}\right), x_{S_{4_k}} + s_m\left(S_{4_k}\right)\right) \tag{1d}$$

$$\forall\, k = N_{\text{peri}}, \dots, N_{\text{post}}$$

$$x_{\max_k} = x_{S_{2_k}} - s_m\left(S_{2_k}\right) \tag{1e}$$

$$x_{\min_k} = x_{S_{4_k}} + s_m\left(S_{4_k}\right) \quad \forall\, k = N_{\text{post}}, \dots, N \tag{1f}$$

where x_{S_j} denotes the longitudinal position of the jth surrounding vehicle, S_j. The time instants $N_{\text{peri}} \leq N_{\text{post}} \leq N$ respectively denotes when E enters the Peri and Post regions, and the prediction horizon. N_{post} is defined as

$$N_{\text{post}} = N_{\text{peri}} + n_{\min} \tag{2}$$

where $n_{\min}$ is the discrete time version of the minimum time it takes for E to laterally traverse the Peri region i.e., move from its current into its target lane, and $N_{\text{peri}} \in \{0, \dots, N - n_{\min}\}$. The safety margin which E must maintain to each S_j is denoted by s_m, which may be defined as

$$s_m\left(S_{j_k}\right) = \max\left(\epsilon_{S_{j_k}}, \tau_{S_{j_k}} v_{S_{j_k}}\right) \quad \forall\, k = 0, \dots, N \tag{3}$$

where v_{S_j} denotes the longitudinal velocity of S_j, and ϵ_{S_j} and τ_{S_j} respectively denotes the minimum distance and time gap which E must maintain to S_j. Furthermore, by allowing τ_{S_j} to be selected over a range from the minimum safe value to the maximum driver preferred value, it can be set to accommodate different driver styles without compromising the safety of the intelligent vehicle system.

2) Longitudinal Trajectory: Since the purpose of the lane change maneuver algorithm is to determine an appropriate trajectory for the traffic situation rather than to capture the full vehicle dynamics, the motion of E is modeled by a simple double integrator [21]. The longitudinal dynamics of E can thus be linearly expressed as

$$x_{k+1} = x_k + v_{x_k} h + a_{x_k} \frac{h^2}{2} \quad \forall\, k = 0, \dots, N-1 \tag{4a}$$

$$v_{x_{k+1}} = v_{x_k} + a_{x_k} h \quad \forall\, k = 0, \dots, N-1 \tag{4b}$$

where x, v_x, and a_x denote E's longitudinal position, velocity, and acceleration respectively, and h denotes the discrete sampling time. Furthermore, the system described by (4) is subjected to the following set of constraints:

$$x_{\min_k} \leq x_k \leq x_{\max_k} \quad \forall\, k = 1, \dots, N \tag{5a}$$

$$v_{x_{\min_k}} \leq v_{x_k} \leq v_{x_{\max_k}} \quad \forall\, k = 1, \dots, N \tag{5b}$$

$$a_{x_{\min_k}} \leq a_{x_k} \leq a_{x_{\max_k}} \quad \forall\, k = 1, \dots, N \tag{5c}$$

$$\Delta a_{x_{\min_k}} \leq \Delta a_{x_k} \leq \Delta a_{x_{\max_k}} \quad \forall\, k = 1, \dots, N \tag{5d}$$

where $\Delta a_{x_k} = a_{x_k} - a_{x_{k-1}}$. Constraint (5a) ensures that E remains within the longitudinal safety corridor while (5b) limits the longitudinal velocity and thereby constrains the velocity of E to the allowed velocity limits. Conditions (5c) and (5d) respectively limits the longitudinal acceleration and jerk in order to allow for smooth and comfortable maneuvers. Furthermore, (5c) and (5d) ensure that the planned trajectory is within the capability of the assumed low-level control systems i.e., Assumption A3.

Since the sets of states i.e., x and v_x, and control inputs i.e., a_x are convex, the longitudinal trajectory planning problem can be written as a standard QP problem for which the cost function is defined as

$$\sum_{k=1}^{N} \vartheta_k \left(v_{x_k} - v_{x_{\mathrm{des}_k}}\right)^2 + \kappa_k a_{x_k}^2 + \varrho_k \Delta a_{x_k}^2 \tag{6}$$

where ϑ, κ, and ϱ are positive scalar weights and the $\vartheta(v_x - v_{x_{\mathrm{des}}})^2$ term penalizes deviations from E's desired velocity, $v_{x_{\mathrm{des}}}$, e.g., the velocity limit, while ride comfort is favored by the κa_x^2 and $\varrho \Delta a_x^2$ terms. In addition, deviations from a desired longitudinal position of E, x_{des}, can be included in (6) by the term $\varsigma(x - x_{\mathrm{des}})^2$ where ς is a positive scalar weight. The QP optimization problem has N optimization variables i.e., control input a_x, and $10N$ linear constraints corresponding to system dynamics (4) and the physical and design constraints (5).

Remark 1: For a lane change trajectory to be feasible, it must allow E to respect the constraints imposed by each surrounding traffic participant and object e.g., vehicles, pedestrians, and road barriers in the surrounding traffic environment. Naturally, these constraints vary depending on the inter-vehicle traffic gap and time instance for which the lane change maneuver is planned to be performed. Hence, the lane change maneuver algorithm can iterate Step II and Step III in order to find the most appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver, i.e., the optimization problem which has the lowest cost function (6). However, since the required computational resources of the lane change maneuver algorithm increases with each optimization problem which has to be solved, it can become intractable to iterate the longitudinal trajectory optimization over each inter-vehicle traffic gap and time instance. Hence, in order to limit the computational complexity of the lane change maneuver algorithm, the inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver can be selected in a preprocessing step i.e., Step I. Thus allowing the longitudinal optimization problem to only be solved once i.e., for the selected inter-vehicle traffic gap and time instance.

3) Inter-Vehicle Traffic Gap and Initiation Time: To determine whether there exists a longitudinal trajectory, T, which allows E to safely position itself in a certain inter-vehicle traffic gap at a certain time instance, the reachable set of E can crudely be approximated by a set of trajectories, $\mathbb{T}$, which e.g., is generated by constant acceleration profiles ranging over a discrete interval from the maximum to the minimum feasible acceleration which satisfies E's physical and design limitations. As such, each trajectory, $T \in \mathbb{T}$, can be expressed as a control

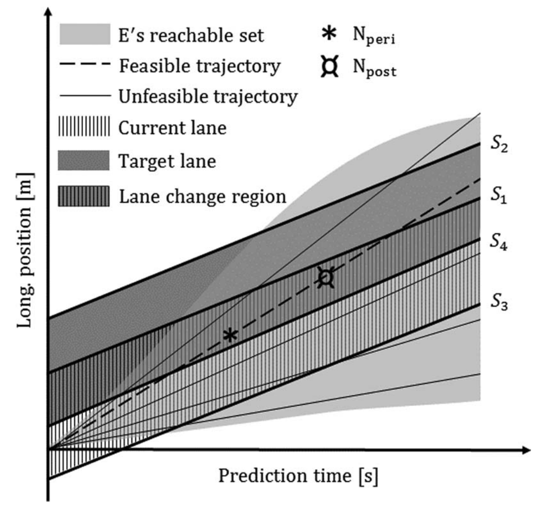

Fig. 3. Schematic illustration of the inter-vehicle traffic gap and initiation time selection for the traffic situation illustrated in Fig. 2.

sequence in terms of e.g., E's longitudinal position, T_x, velocity, T_{v_x}, and acceleration, T_{a_x}. For each inter-vehicle traffic gap and discrete time instance to initialize the lateral motion of the maneuver, it can thereby easily be determined if a trajectory, $T \in \mathbb{T}$, allows E to be safely positioned in its current lane, x_{cl}, enter the corresponding lane change region, x_{lcr}, of the inter-vehicle traffic gap at the specified time instance, remain in that region for $n_{\min}$, and thereafter safely be positioned in the target lane, x_{tl}, until the end of the prediction horizon. A subset of feasible lane change trajectories can thus be created as

$$\begin{aligned}\mathbb{FT} = \big\{\mathbb{FT} \subset \mathbb{T}:\ & T_{x_k} \in x_{\mathrm{cl}_{k,q}} \quad \forall\, k = 0,\ldots,N_{\mathrm{peri}}\\ & T_{x_k} \in x_{\mathrm{lcr}_{k,q}} \quad \forall\, k = N_{\mathrm{peri}},\ldots,N_{\mathrm{post}}\\ & T_{x_k} \in x_{\mathrm{tl}_{k,q}} \quad \forall\, k = N_{\mathrm{post}},\ldots,N\\ & N_{\mathrm{peri}} = [0,\ldots,N-n_{\min}]\\ & q = [1,\ldots,Q]\big\}\end{aligned} \tag{7}$$

where q denotes a certain inter-vehicle traffic gap and Q is the number of inter-vehicle traffic gaps within sensor range. To allow for smooth and comfortable maneuvers, the most appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver are selected as the gap with corresponding time instance for which $\mathrm{argmin}_{T\in\mathbb{FT}}\ |T_{a_x}|$. If $\mathbb{FT} = \emptyset$, the algorithm cannot plan a lane change maneuver for E at the current time instance and should rather wait for an appropriate inter-vehicle traffic gap to approach.

Fig. 3 provides a simple illustration of the inter-vehicle traffic gap and initiation time selection for the traffic situation depicted in Fig. 2. As seen in the figure, there exist a trajectory, $T \in \mathbb{T}$, which allows E to be safely positioned in its current lane, enter the corresponding lane change region of the inter-vehicle traffic gap between S_2 and S_4, remain in that region for $n_{\min}$, and thereafter safely be positioned in the target lane until the end of the prediction horizon. Hence, it might be possible for E to perform the desired lane change maneuver.

Remark 2: In order to avoid collisions with vehicles in the surrounding traffic environment E should maintain a safe distance to all S_j at each time instance as described by (3). To further account for uncertainty in the surrounding traffic environment, a safety critical event which could be taken into account is the possibility of any relevant preceding vehicle unexpectedly performing a substantial brake maneuver. To account for this possibility, E should not perform a lane change maneuver for which it does not have the ability to avoid a collision if such an event occurs. Thus, when selecting the inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver, in addition to $T \in \mathbb{FT}$ allowing E to enter a lane change region, remain in that region for $n_{\min}$, and thereafter safely be positioned in the target lane over the prediction horizon, it should also be possible for E to apply a deceleration trajectory which allows it to not get closer than ϵ_{S_j} to any relevant preceding vehicle which may perform a substantial brake maneuver at the next time instance.

To further account for the possibility of any relevant preceding vehicle unexpectedly performing a substantial brake maneuver, whilst in the Peri region the maximum bound on the velocity of E, i.e., (5b), could be updated as

$$v_{x_{\max_k}} = \min_{j \in \mathbb{J}} \left(v_{x_{\max_k}}, \sqrt{v^2_{S_{j_k}} - 2 a_{x_{\min}} d_b\left(S_{j_k}\right)} \right) \quad \forall\, k = N_{\text{peri}}, \ldots, N_{\text{post}} \tag{8}$$

where $\mathbb{J}$ is the set of all relevant preceding vehicles, $a_{x_{\min}}$ denotes the minimum acceleration of E i.e., maximum deceleration, and d_b denotes the available braking distance which may be defined as

$$d_b\left(S_{j_k}\right) = \tau_{S_{j_k}} v_{S_{j_k}} - \epsilon_{S_{j_k}} \quad \forall\, k = N_{\text{peri}}, \ldots, N_{\text{post}}. \tag{9}$$

As such, when determining an appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the lane change maneuver, the possibility of a relevant preceding vehicle performing a substantial brake maneuver is accounted for, and thereby it should be possible for E to avoid a collision in such a situation. By accounting for the possibility of any relevant preceding vehicle unexpectedly performing a substantial brake maneuver, E can further be considered to somewhat account for the possibility of a preceding vehicle performing an unexpected lane change maneuver into its current or target lane.

III. Simulation Results

To study the performance of the proposed lane change maneuver algorithm, this section presents simulation results regarding three aspects of the algorithm. Firstly, in Section III-A the impact of Step I i.e., the preprocessing step, is evaluated by comparing the result of the algorithm with Step I with the result of the algorithm without Step I i.e., when Step II and Step III are iterated in order to find the most appropriate solution as mentioned in Remark 1. Secondly, in Section III-B the consistency of the algorithm's solution i.e., the lane change trajectory, is evaluated in receding horizon. Thirdly, in Section III-C the impact on the algorithm's solution of including the additional safety considerations as mentioned in Remark 2 is evaluated.

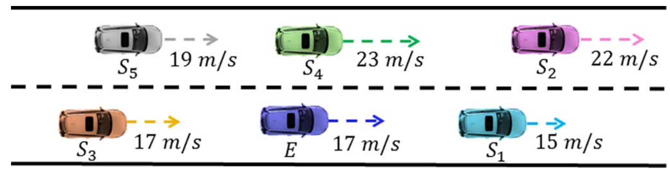

Fig. 4. Example of Scenario VI of the considered traffic situation where the vehicles are driving on a one-way road with two lanes. The ego vehicle (E) is shown in blue, and the surrounding vehicles (S_1, S_2, S_3, S_4, and S_5) are displayed in cyan, magenta, orange, green, and gray, respectively.

TABLE I
General Design Parameters for the Longitudinal Trajectory Planning Problem

$v_x \in \{0, 30\}$ [m/s]	$\tau = 0.5$ [s]	$\vartheta = 1$
$a_x \in \{-4, 2\}$ [m/s^2]	$N = 10$	$\kappa = 1$
$\Delta a_x \in \{-3h, 1.5h\}$ [m/s^2]	$h = 1$ [s]	$\varrho = 1$
$\epsilon = 1$ [m]		

In the entire section the proposed lane change maneuver algorithm is evaluated in simulated lane change traffic situations on a one-way, two-lane highway as illustrated in Fig. 4. Six scenarios of the traffic situation are considered where each scenario includes E and the following surrounding vehicles: I) S_1 and S_2, II) S_1, S_2, and S_4, III) S_1, S_2, S_4, and S_5, IV) S_1, S_2, and S_3, V) S_1, S_2, S_3, and S_4, VI) S_1, S_2, S_3, S_4, and S_5. In each subsection the proposed lane change maneuver algorithm is evaluated on 100 versions of each of the six scenarios where each version is initialized so that the velocity of each vehicle is randomly selected over an interval of $[5:25]$ m/s and the time gap between each vehicle driving in a lane is randomly selected over an interval of $[1:4]$ s. In each scenario, it is assumed that $v_{x_{\text{des}}} = 20$ m/s, and that S_j drives at a constant velocity without performing any lane change maneuvers over the prediction horizon. The assumption regarding the behavior of S_j is a simple assumption purely in order to illustrate the lane change maneuver algorithm. However, any predicted behavior, i.e., trajectory of S_j can be incorporated into the algorithm when selecting the inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver, as well as determining the longitudinal safety corridor and corresponding trajectory.

The aim of the lane change maneuver algorithm in the described traffic situation is to determine in which inter-vehicle traffic gap and at what time instance the lane change maneuver should be performed, and calculate the corresponding trajectory in terms of the control signals which allow E to position itself in the selected inter-vehicle traffic gap at the desired time instance. The algorithm has been implemented in Matlab where the longitudinal QP optimization problem for trajectory planning is solved using CVXGEN [15]. The general design parameters for the longitudinal trajectory planning problem are given in Table I. Note that the purpose of the simulation study is to indicate and evaluate different aspects of the algorithm's performance in the considered traffic situation, rather than to provide statistically conclusive results.

TABLE II
MEAN, μ, AND STANDARD DEVIATION, σ, OF THE REQUIRED COMPUTATIONAL TIME IN [s] FOR THE LANE CHANGE MANEUVER ALGORITHM WITH AND WITHOUT STEP I

Scenario	I	II	III	IV	V	VI
With Step I						
μ	0.0487	0.0486	0.0479	0.0455	0.0467	0.0430
σ	0.0013	0.0041	0.0068	0.0128	0.0102	0.0165
Without Step I						
μ	0.7445	1.1218	1.4837	0.7480	1.1209	1.4968
σ	0.0068	0.0081	0.0090	0.0090	0.0082	0.0113

TABLE III
COMPARISON OF THE LANE CHANGE MANEUVER ALGORITHM WITH AND WITHOUT STEP I FOR 100 RANDOM VERSIONS OF EACH OF THE SIX SCENARIOS OF THE CONSIDERED TRAFFIC SITUATION AS WELL AS THE MEAN VALUE, μ, OF SCENARIOS I–VI IN [%]

Scenario	I	II	III	IV	V	VI	μ
Same gap	85	87	81	90	88	87	86
Same time	30	46	37	33	43	52	40
Same gap and time	30	46	35	32	43	50	39
Both feasible	92	97	94	87	90	91	92
Both unfeasible	7	3	4	13	10	8	8

A. Inter-Vehicle Traffic Gap and Initiation Time Selection

The mean and standard deviation of the required computational time for the lane change maneuver algorithm with and without Step I, on a Dell Optiplex 990 stationary computer, is shown in Table II. From the table it can be seen that the required computational time is significantly reduced when Step I is included in the algorithm. In the table it can further be seen that when Step II and Step III are iterated in order to find the most appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver, the required computational time increases with the number of inter-vehicle traffic gaps to evaluate while it remains fairly constant when Step I is included.

Table III provides the result of the lane change maneuver algorithm with Step I compared with the result of the algorithm without Step I i.e., when Step II and Step III are iterated over each inter-vehicle traffic gap within sensor range and time instance discretized at intervals of one second over a time horizon of ten seconds in order to find the most appropriate solution i.e., the optimization problem which has the lowest cost function (6). From the table it can be seen that both with and without Step I the algorithm selects the same inter-vehicle traffic gap in approximately 86% of the cases while the time instance for initializing the lateral motion of the maneuver coincides in approximately 40% of the cases. The difference in selected solutions is most likely a result of the reachable set of E being approximated by constant acceleration profiles and that the inter-vehicle traffic gap and time instance to perform the lane change maneuver are selected without accounting for $v_{x_{\mathrm{des}}}$ and jerk constraints in Step I.

From Table III it can further be seen that both with and without Step I the algorithm finds a feasible trajectory in approximately 92% of the cases, and that approximately 8% of the cases are unfeasible. As such, although the algorithm with Step I offers a rough approximation in comparison with iterating Step II and Step III to find the most appropriate inter-vehicle traffic gap and time instance to perform the maneuver, it only fails to find a feasible trajectory if such exist in less than 1% of the cases. The failure in computing a feasible trajectory is most likely a consequence of the solution trajectory requiring a large shift in deceleration and acceleration and as such Step I fails to find the possible solution since it assumes constant acceleration profiles when selecting an appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the lane change maneuver.

The results in Table III thus indicate that the algorithm with Step I normally finds a feasible trajectory if such exists, and that the inclusion of Step I mainly influences the time instance to initialize the lane change maneuver, while the selected inter-vehicle traffic gap is often the same as when Step II and Step III are iterated. Since an important feature of the lane change maneuver algorithm is to determine whether a lane change maneuver is safe and possible rather than to find the most appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver, Step I can thereby be considered to offer a crude yet fair estimation regarding which inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver while significantly reducing the algorithm's required computational time as seen in Table II.

TABLE IV
LANE CHANGE MANEUVER ALGORITHM WITH STEP I IN RECEDING HORIZON FOR 100 RANDOM VERSIONS OF EACH OF THE SIX SCENARIOS OF THE CONSIDERED TRAFFIC SITUATION AS WELL AS THE MEAN VALUE, μ, OF SCENARIOS I–VI IN [%]

Scenario	I	II	III	IV	V	VI	μ
Change gap	3	4	5	4	7	7	5
Unfeasible initially	8	1	2	4	4	3	4
Feasibility lost	1	2	2	0	1	3	2
Change gap and feasibility lost	1	2	1	0	0	3	1
Acc. to find solution	9	2	3	4	2	1	4
Wait to find solution	0	1	1	0	3	5	2

B. Receding Horizon Solutions

To study the performance of the proposed lane change maneuver algorithm in terms of the consistency of the algorithm's solution, the algorithm with Step I is applied in receding horizon until the maneuver is completed i.e., until E has positioned itself in the selected inter-vehicle traffic gap. Table IV provides the result of the algorithm with Step I in receding horizon, and from the table it can be seen that the algorithm modifies the selected inter-vehicle traffic gap in approximately 5% of the cases which coincides with lost feasibly in approximately 1% of the cases.

In Table IV, it can further be seen that although the algorithm sometimes fails to find a feasible trajectory, the acceleration profile resulting from Step I can be applied as lane change maneuver preparation until a feasible lane change trajectory is found. Similarly, if Step I fails to find a possible inter-vehicle traffic gap and initiation time, the lane change maneuver algorithm does nothing and waits until a lane change opportunity arise.

TABLE V
COMPARISON OF THE LANE CHANGE MANEUVER ALGORITHM WITH STEP I WITH AND WITHOUT ADDITIONAL SAFETY CONSIDERATIONS FOR 100 RANDOM VERSIONS OF EACH OF THE SIX SCENARIOS OF THE CONSIDERED TRAFFIC SITUATION AS WELL AS THE MEAN VALUE, μ, OF SCENARIOS I–VI IN [%]

Scenario	I	II	III	IV	V	VI	μ
Same gap	99	97	97	96	95	94	96
Same time	92	88	91	92	91	88	90
Same gap and time	92	88	91	92	91	88	90
Both feasible	75	80	84	63	75	67	74
Both unfeasible	8	3	6	13	10	9	8

TABLE VI
LANE CHANGE MANEUVER ALGORITHM WITH STEP I WITH ADDITIONAL SAFETY CONSIDERATIONS IN RECEDING HORIZON FOR 100 RANDOM VERSIONS OF EACH OF THE SIX SCENARIOS OF THE CONSIDERED TRAFFIC SITUATION AS WELL AS THE MEAN VALUE, μ, OF SCENARIOS I–VI IN [%]

Scenario	I	II	III	IV	V	VI	μ
Change gap	13	21	26	7	11	15	16
Unfeasible initially	9	9	7	7	10	6	8
Feasibility lost	8	10	10	4	5	7	7
Change gap and feasibility lost	7	9	10	4	4	7	7
Acc. to find solution	15	14	11	9	9	5	11
Wait to find solution	2	5	6	2	6	8	5

The results in Table IV thus indicate that the proposed lane change maneuver algorithm offers fairly consistent solutions in receding horizon. Furthermore, the results show that the acceleration trajectory resulting from Step I can be utilized as a backup trajectory if the trajectory planning optimization fails to produce a feasible solution, and if no inter-vehicle traffic gap has been selected at the current time instance, E should wait for a reachable gap to approach.

C. Safety Considerations

Table V provides the result of the lane change maneuver algorithm with Step I without additional safety considerations compared with the result of the algorithm with Step I including additional safety considerations as mentioned in Remark 2. From the table it can be seen that both with and without additional safety considerations the lane change maneuver algorithm selects the same inter-vehicle traffic gap in approximately 96% of the cases. Similarly, the time instance for initializing the lateral motion of the maneuver coincides in approximately 90% of the cases, and thereby it can be seen that the algorithm selects the same inter-vehicle traffic gap and time instance in approximately 90% of the cases. Furthermore, the table shows that both with and without additional safety considerations the algorithm finds a feasible trajectory in approximately 74% of the cases, and that approximately 8% of the cases are unfeasible. As such it can be seen that when additional safety considerations are taken into account the algorithm fails to find a feasible trajectory if such exist in approximately 18% of the cases compared to the algorithm with Step I without the additional safety considerations taken into account. The failure to compute feasible trajectories although the algorithm selects the same inter-vehicle traffic gap and time instance in Step I is most likely a result of the optimization problem accounting for both jerk constraints and the safety bound on the maximum velocity of E, while the inter-vehicle traffic gap and time instance to perform the lane change maneuver are selected in Step I without accounting for the jerk constraints.

To study the impact of the additional safety considerations on the performance of the lane change maneuver algorithm in terms of the consistency of the algorithm's solution, the algorithm is applied in receding horizon until the maneuver is completed i.e., until E has positioned itself in the selected inter-vehicle traffic gap. Table VI provides the result of the algorithm in receding horizon with Step I including additional safety considerations and from the table it can be seen that the algorithm modifies the selected inter-vehicle traffic gap in approximately 16% of the cases which coincides with lost feasibly in approximately 7% of the cases.

The results in Tables V and VI thus indicate that when the additional safety considerations in Remark 2 are taken into account, the algorithm becomes more conservative and as such it fails to produce feasible trajectories in comparison to the case when the additional safety considerations are not taken into account. For instance, when the additional safety considerations are taken into account the lane change maneuver algorithm can fail to produce lane change trajectories which require large acceleration in order to perform a lane change maneuver into a faster moving lane since the relative velocity to relevant preceding vehicles is limited throughout the maneuver.

IV. EXPERIMENTAL RESULTS

To further study the ability of the lane change maneuver algorithm to plan appropriate trajectories, an experimental lane change traffic situation corresponding to Scenario I in Section III is considered, where E initially drives in the right lane with a preceding vehicle S_1 driving in the same lane and a surrounding vehicle S_2 driving in the adjacent left lane. As in the simulated traffic situation, it is assumed that S_j does not perform any lane change maneuvers, drives at approximately constant velocity, and that $v_{x_{\mathrm{des}}} = 20$ m/s.

The purpose of the experimental lane change traffic situation is to test the ability of the lane change maneuver algorithm to plan safe and smooth trajectories which allow E to perform a left lane change maneuver. Hence, upon a manually given lane change request, the lane change maneuver algorithm determines an appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver. When the decision whether to change lane ahead or behind S_2 is made, the lane change maneuver algorithm proceeds to plan the corresponding lane change trajectory. In the vehicle implementation the longitudinal trajectory optimization problem is implemented as a C-coded s-Function using CVXGEN running at 4 Hz, while the lateral planning problem is performed by a standard LQ-controller tracking a spline reference [22]. When not performing a lane change maneuver, E is longitudinally controlled by an ACC system and laterally controlled by a LKA system.

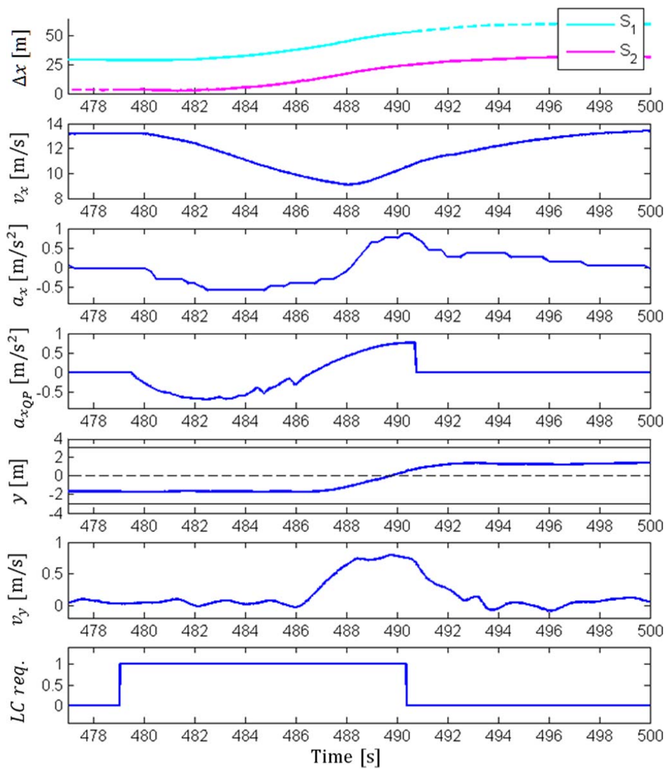

Fig. 5. Automated lane change maneuver for scenario Version I of the experimental traffic situation. From top to bottom, the plots illustrate: 1) the longitudinal position trajectory of the surrounding vehicles S_1 and S_2 relative to the ego vehicle (E) for which a solid line represents that E accounts for the surrounding vehicle, 2) E's longitudinal velocity trajectory, 3) E's longitudinal acceleration trajectory, 4) the longitudinal acceleration trajectory resulting from the QP optimization, 5) E's lateral position trajectory in a road-aligned coordinate frame, 6) E's lateral velocity trajectory towards the road, and 7) the lane change request signal. When the lane change request signal is nonactive, i.e., equal to zero, E is longitudinally controlled by an ACC system.

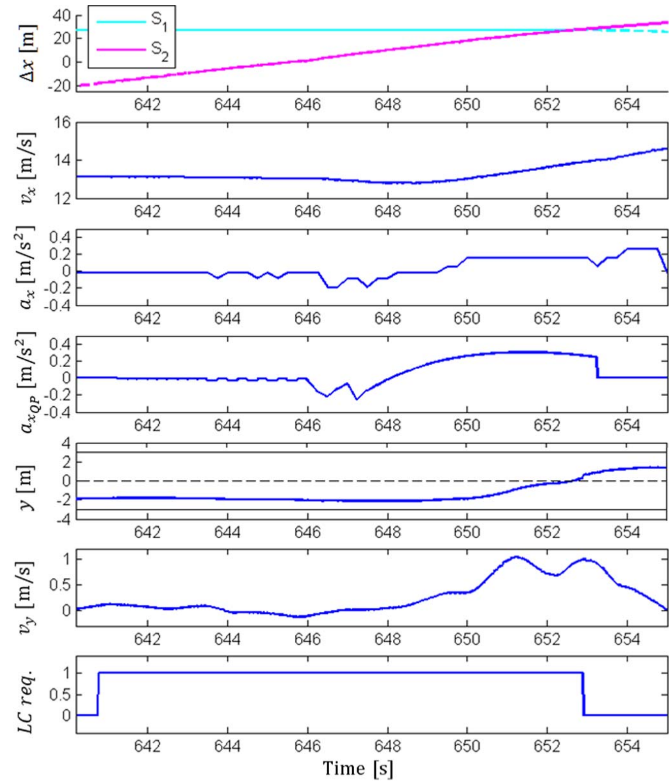

Fig. 6. Automated lane change maneuver for scenario Version II of the experimental traffic situation. From top to bottom, the plots illustrate: 1) the longitudinal position trajectory of the surrounding vehicles S_1 and S_2 relative to the ego vehicle (E) for which a solid line represents that E accounts for the surrounding vehicle, 2) E's longitudinal velocity trajectory, 3) E's longitudinal acceleration trajectory, 4) the longitudinal acceleration trajectory resulting from the QP optimization, 5) E's lateral position trajectory in a road-aligned coordinate frame, 6) E's lateral velocity trajectory towards the road, and 7) the lane change request signal. When the lane change request signal is nonactive, i.e., equal to zero, E is longitudinally controlled by an ACC system.

The lane change tests have been performed in Sweden at Hällered proving ground on an oval four lane road. The vehicle platform is a Volvo V60 model year 2013 equipped with a forward looking camera system, forward looking long range radar, and one medium range radar with a wide field of view in each corner of the vehicle rendering the vehicle a forward, rearward, and sideward view of approximately 200 m, 70 m, and 40 m respectively.

In Figs. 5–7 the results from automated lane change maneuvers are shown for three versions of the considered traffic situation for which the initial conditions are given in Table VII. From Fig. 5 which illustrates scenario Version I, it can be seen that since S_2 is positioned slightly ahead of E upon the lane change request, E reduces its velocity to fall back in relation to S_2. When E has obtained a safe distance to S_2 it performs the lateral motion into the left lane, increases its velocity, and safely follows S_2. Fig. 6 shows scenario Version II in which S_2 is driving at 17 m/s and is positioned behind E upon the lane change request as indicated in Table VII. Hence, in order to safely and comfortably change lane E waits until S_2 has passed before performing the lane change maneuver and accelerate to follow S_2 at a safe distance. Finally, Fig. 7 illustrates scenario Version III which is similar to scenario Version II but with the difference of S_2 being positioned further behind E upon the lane change request whereas E accelerates and change lane ahead of S_2.

In Fig. 7 it can further be seen that at time instance 884, E does not continue to accelerate to reach $v_{x_{\mathrm{des}}}$ but rather reduces its acceleration. This is due to a false detection of a ghost vehicle, G_{S_1}, i.e., a shadow of S_1 appearing in the left lane while E is performing the lane change maneuver. Hence, in order to maintain a safe distance to G_{S_1}, E reduces its acceleration until G_{S_1} is no longer detected at time instance 885.5. It can thereby be seen that since the lane change maneuver algorithm accounts for both S_1 and S_2 throughout the maneuver, the appearance of G_{S_1} does not cause the need to abort the maneuver. Rather, the algorithm adjusts the trajectory in a smooth and natural manner.

V. Conclusion

This paper presents a lane change maneuver algorithm with a novel approach to select an appropriate inter-vehicle traffic gap and time instance to initialize the lateral motion of a lane change maneuver which:

- Performs a crude yet reasonable estimation regarding which inter-vehicle traffic gap and time instance to initialize the lateral motion of the maneuver.

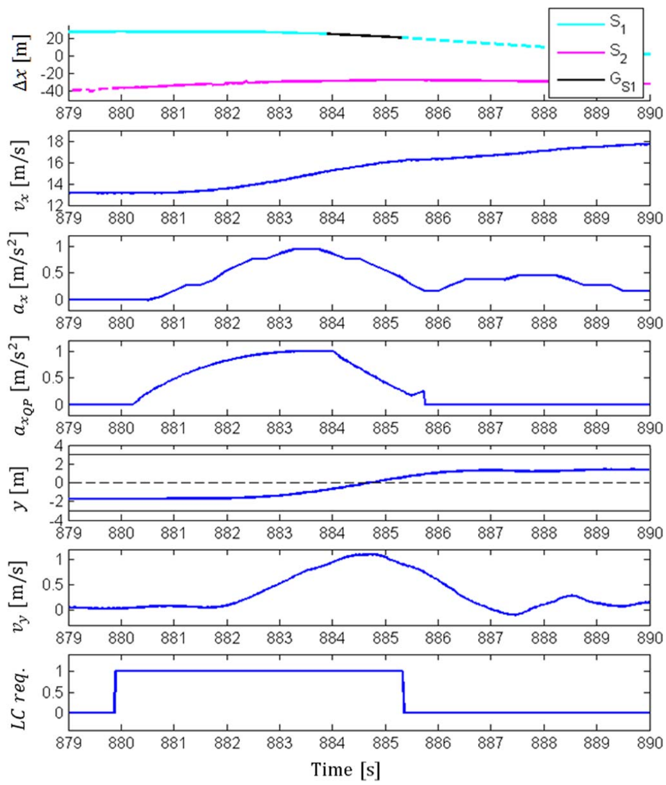

Fig. 7. Automated lane change maneuver for scenario Version III of the experimental traffic situation. From top to bottom, the plots illustrate: 1) the longitudinal position trajectory of the surrounding vehicles S_1 and S_2 and ghost vehicle G_{S_1} relative to the ego vehicle (E) for which a solid line represents that E accounts for the surrounding vehicle, 2) E's longitudinal velocity trajectory, 3) E's longitudinal acceleration trajectory, 4) the longitudinal acceleration trajectory resulting from the QP optimization, 5) E's lateral position trajectory in a road-aligned coordinate frame, 6) E's lateral velocity trajectory towards the road, and 7) the lane change request signal. When the lane change request signal is nonactive, i.e., equal to zero, E is longitudinally controlled by an ACC system.

TABLE VII
INITIAL CONDITIONS FOR THE THREE SCENARIO VERSIONS OF THE EXPERIMENTAL LANE CHANGE TRAFFIC SITUATION, Δx_0 DENOTES THE RELATIVE POSITION OF THE EGO VEHICLE (E) AND THE SURROUNDING VEHICLES (S_1 AND S_2)

Vehicle	Δx_0 [m]	v_{x_0} [m/s]	a_{x_0} [m/s^2]
E	$\{0,\ 0,\ 0\}$	$\{14,\ 14,\ 14\}$	$\{0,\ 0,\ 0\}$
S_1	$\{29.5,\ 27.5,\ 27.5\}$	$\{14,\ 14,\ 14\}$	$\{0,\ 0,\ 0\}$
S_2	$\{3.5,\ -21.5,\ -42\}$	$\{14,\ 17,\ 17\}$	$\{0,\ 0,\ 0\}$

- Generates safe and smooth lane change trajectories.
- Is real-time executable on a standard passenger vehicle platform.

The proposed algorithm is thereby considered to be a building block for ADAS regarding lane change maneuvers and eventually fully automated vehicles in highway traffic environments.

To further extend the capability of the lane change maneuver algorithm, future work include extending the inter-vehicle traffic gap and initiation time selection to include non-constant acceleration profiles, as well as the development of a method that guarantees that the automated vehicle will always be able to execute a safe maneuver without excessively conservative constraints under which the trajectory planning algorithm should operate. Furthermore, a dynamic prediction model of the traffic environment which includes prediction uncertainty and sensor noise should be incorporated in order to test the proposed algorithm in real-world traffic situations.

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