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Strategic decision making for automated driving on two-lane, one way roads using model predictive control

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Abstract— **This paper presents an algorithm for strategic decision making regarding when lane change and overtake manoeuvres are desirable and feasible. By considering the task of driving on two-lane, one-way roads, as the selection of desired lane and velocity profile, the algorithm provides useful results in terms of velocity control as well as a decision variable corresponding to whether a lane change manoeuvre should be performed. The decision process is modelled through a mixed logical dynamical system which is solved through model predictive control using mixed integer program formulation. The performance of the proposed control system is explored through simulations of varying driving scenaria on a two-lane, one-way road, which shows the capability of the system to achieve appropriate longitudinal and lateral control strategies depending on the traffic situation.**

I. INTRODUCTION

The last decades have witnessed an intense evolution within the field of intelligent vehicles. Advanced driver assistance systems (ADAS) such as adaptive cruise control (ACC), lane keeping aid (LKA), and traffic jam assists (TJA) for stop and go traffic are (or are soon to be) commercially available, and many research projects e.g. the California PATH project [1], Demo 2000 in Japan [2], the US DARPA challenges [3]-[4] and subsequent projects [5]-[6], and more recently SARTRE [7], demonstrates the possibilities of increased automated functionality. This development can crudely be deduced to environmental, economic, safety, and convenience factors, since an increased level of autonomy has the potential to improve traffic flow, reduce fuel consumption, and support the driver such that the impact of human factors can be decreased.

One area where a high level of autonomy is both realizable and desirable is in two-lane, one-way roads. In this area (i.e. highways) a substantial percentage of traffic accidents and fatalities are related to lane change and overtake manoeuvres [8]. Thus, ADAS or even fully automated systems, for these types of manoeuvres are of great interest.

An abundant amount of research has been made in terms of trajectory generation and controller design for longitudinal and lateral movement for vehicle following and collision avoidance [9]-[11]. This paper will therefore focus on higher level, strategic decision making regarding when lane change and overtake manoeuvres are desirable and feasible, assuming that once a lane change decision has been made, a lower-level controller will be able to track a pre-computed reference trajectory for that manoeuvre, alternatively a human driver can follow the recommendation and perform the lane change.

Methods for strategic decision-making in fully or highly automated driving systems designed for lane change and overtake manoeuvres, can roughly be divided into either rule-based [12]-[13], or utility-based [14]-[16] approaches, where the more advanced applications also include probabilistic methods to handle uncertainties [17]-[18]. Rule-based systems have the advantage of traceability and ease of implementation for specified scenaria but can require a substantial effort in order to be extended into more complex scenaria. One the other hand, approaches based on utility functions have the advantage of allowing combined weighting of multiple criteria and can thus more easily be extended to complex scenaria. However, a large amount of different weighting parameters can result in time-consuming parameter tuning and tractability difficulties.

In this paper, the problem of deriving decisions regarding appropriate driving manoeuvres i.e. selection of desired lane and velocity profile, on two-lane, one-way roads, is considered as a mixed logical dynamical (MLD) system [19] to be solved through model predictive control (MPC) [20] using mixed integer program formulation. This approach allows for propositional logic regarding mandatory lane changes, i.e. lane changes depending on route and road properties such as lane ends and lane destinations, and collision avoidance constraints, to be incorporated with an objective function to attain the possibility of discretionary lane changes i.e. lane changes resulting from a desire to improve ones own driving conditions. Thus, the proposed algorithm combines the benefits of rule- and utility-based approaches since the MLD formulation maintains the simplicity of rule-based systems by allowing logic constraints, and by using MPC the benefits of utility functions are maintained while less parameter tuning is required, and evaluation over a prediction horizon is easily obtained. This is beneficial since in order to make decisions regarding preferred lane and consequently whether a lane change or overtake manoeuvre is desired, current and future states must be taken into consideration.

The proposed control system will at each time instance provide acceleration/deceleration request as well as a decision variable corresponding to whether a lane change manoeuvre should be initiated, all in purpose of allowing the ego vehicle to retain desired velocity (v_{des}), while avoiding collisions with other vehicles; allowing smooth and efficient transportation. The algorithm can be considered as either a compliment to ADAS or as a step towards highly automated driving where the vehicle makes intelligent decisions and

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displays behaviour similar to that of an everyday driver. In either aspect, the proposed algorithm can be regarded as a decision-unit combining the functionality of other ADAS such as ACC and LKA.

The remaining of this paper is organized as follows: Section II presents the decision model, while Section III describes the decision and control algorithm. Simulation results of the proposed algorithm for varying traffic scenaria on a two-lane, one-way road are given in Section IV, and conclusions are stated in Section V.

II. Decision model

The problem of deriving decisions regarding appropriate driving manoeuvres on a two-lane, one-way road is hereby considered as two coupled questions:

1) selection of most appropriate lane and
2) selection of velocity profile.

For the purpose of designing a decision algorithm regarding preferred lane and desired acceleration/deceleration, it is sufficient to represent the vehicle as a point-mass. This simplification is adequate since the focus of the algorithm is on decision making regarding appropriate driving manoeuvres while keeping safe distances to surrounding vehicles, and not on the actual control of the vehicle. Further, since the algorithm is intended to provide a decision regarding which lane is preferable at each time instance, assuming a lane change reference trajectory exists, the lateral control i.e. steering of the vehicle, can be excluded from the model. Hence, the longitudinal motion of the vehicle and the force acting upon it can be modelled as a simple double integrator system. However, to represent the preferred lane, i.e. the lane change decision variable, an additional binary control input, u_b, is introduced, and the control signal becomes; $u = [u_c, u_b]^T, u_c \in R, u_b \in \{0, 1\}$, where u_c is the acceleration/deceleration control signal and $u_b = 0$ represents the right lane, whereas $u_b = 1$ corresponds to the left lane.

In order to describe the interaction between the continuous dynamics and logic rules based on the binary control input u_b, in a format suitable for MPC formulation the system model should be expressed as a MLD system. The main idea of MLD systems is to link logic and dynamics of a system through mixed integer linear inequalities i.e. linear inequalities involving both real and binary variables. This is achieved by associating each statement e.g. $f_j(z) \leq 0$ with a binary logic variable δ_j such that $f_j(z) \leq 0 \in \{false, true\} \Leftrightarrow \delta_j \in \{0, 1\}$. The association is accomplished by

$$[f_j(z) \leq 0] \leftrightarrow [\delta_j = 1] \Leftrightarrow \begin{cases} f_j(z) \leq M(1-\delta_j) \\ f_j(z) > m\delta_j \end{cases} \tag{1}$$

where $M \triangleq \max f(z)$ and $m \triangleq \min f(z)$. For details on the general MLD system formulation the reader is referred to [19].

By considering the presented problem as a MLD system, collision avoidance constraints and conditions on mandatory lane changes due to e.g. lane ends, can be expressed as logic constraints which can be transformed into linear inequalities through propositional calculus. As an example, a logic constraint regarding which lane the vehicle must be positioned in after a certain longitudinal position p, can be transformed into mixed integer liner inequalities by first assigning

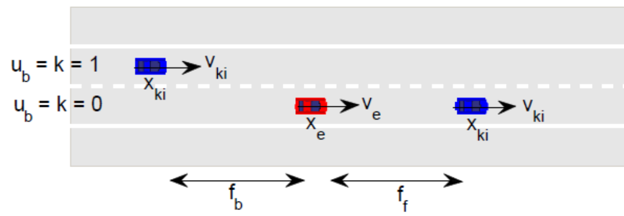

Fig. 1: Problem setup for a two-lane, one way road.

$$\begin{aligned} x_e \geq p \leftrightarrow \delta_1 = 1 \\ u_b = k \leftrightarrow \delta_2 = 1 \end{aligned} \tag{2}$$

making

$$x_e \geq p \rightarrow u_b = k \Leftrightarrow \delta_1 - \delta_2 \leq 0. \tag{3}$$

Likewise, logic safety constraints to avoid collision with surrounding vehicles travelling in the same lane, can be transformed into mixed integer liner inequalities from the form

$$\begin{aligned} u_b = k \rightarrow x_e - x_{k_i} \geq f_b(v_{k_i}) = c + 1.5 v_{k_i} \\ or \quad x_{k_i} - x_e \geq f_f(v_e, v_{k_i}) = c + 3v_e - v_{k_i} \end{aligned} \tag{4}$$

where x_e and v_e is the longitudinal position and velocity of the ego vehicle, x_{k_i} and v_{k_i} is the respective longitudinal position and velocity of the *ith* surrounding vehicle in the right or left lane ($k = \{0, 1\}$), and c is a constant representing an additional safety marginal, (here c is set to 2). See Fig 1 for an illustrative picture.

The *or* condition allows for different safety margins to be activate depending on if the surrounding vehicle is in front or behind the ego vehicle. This is advantageous since it allows for a more appropriate traffic modelling than would be possible with symmetric safety conditions.

III. Decision and control algorithm

The MLD formulation allows the manoeuvre decision making problem to be converted to a mixed integer program suitable to be solved as a decision and control algorithm within the MPC framework. In MPC, an optimal control input sequence is computed at each time instance, by solving a finite time optimal control problem. By utilizing receding horizon control principle, the determined control input is only applied to the system during the next consecutive sampling interval. At the next time instance, the finite optimal control problem is resolved using the latest measurement information.

The optimization problem to be solved at each time instance to achieve a decision and control algorithm for lane selection on a two-lane, one way road, while abiding constraints (Eqs. (2) - (4)) and retaining a desired velocity, can thus be formulated as

$$
\begin{aligned}
&\min_{U_t} \sum_{j=0}^{H_c-1} Q \mid u_{t+j,t} \mid + w \mid v_{e_{t+j,t}} - v_{des} \mid \\
&s.t. \;\; x_{t+j+1,t} = A x_{t+j,t} + B u_{t+j,t}, \\
&\qquad\qquad j = 0, \ldots, H_p - 1 \\
&\qquad E_1 x_{t+j,t} + E_2 u_{t+j,t} + E_3 \delta_{t+j,t} \leq E_4, \\
&\qquad\qquad j = 0, \ldots, H_p \\
&\qquad u_{\min} \leq u_{t+j,t} \leq u_{\max}, \\
&\qquad\qquad j = 0, \ldots, H_p \\
&\qquad \triangle u_{\min} \leq \triangle u_{t+j,t} \leq \triangle u_{\max}, \\
&\qquad\qquad t = 0, \ldots, H_c - 1 \\
&\qquad \triangle u_{t+j,t} = 0, \\
&\qquad\qquad j = H_c, \ldots, H_p.
\end{aligned} \tag{5}
$$

where t denotes the current time instance, $x_{t+j,t} \in R$ is the predicted state of the MLD system (i.e. longitudinal position and velocity) at time instance $t + j$ obtained by applying the control signal $U_t = [u_{t,t}, \ldots, u_{t+j,t}]$ to the system, $u \in R \times \{0, 1\}$ are the continuous and binary inputs, and $\delta \in \{0, 1\}$ represent auxiliary binary variables generated from the conditions in Eqs. (2) - (4). A, and B are the standard matrices for the double integrator system, and E_1, E_2, E_3, and E_4 are matrices enforcing the constraints given in Eq. (2) - (4). Q and w are weighting matrices, H_c denotes the control horizon, while H_p denotes the prediction horizon ($H_c \leq H_p$). If H_p is chosen larger than H_c the control signal is kept constant during the prediction time beyond H_p. This is beneficial for reducing the required computational time which is strongly influenced by the number of introduced binary variables.

The MPC formulation allows the decisions to be evaluated over a prediction horizon, thus allowing consequences of each decision to be included in the current decision criteria. By re-evaluating and resolving the problem at each time instance, changes in the environment can also be accounted for in the decision process.

From the problem formulation another benefit of the binary decision variable becomes apparent since by using u_b to represent the lanes, the right lane is implicitly implemented as the preferred lane of travel. This behaviour which correlates to driving convention on highways with right-hand traffic is obtained by the inclusion of u_b in the cost function in Eq. (5).

IV. Simulation Results

Simulation tests are implemented in Matlab using the Multi-Parametric Toolbox (MPT) [21] to obtain the implicit controller for the system. For implementation purposes the system is discretize with sampling time, $t_s = 0.1s$. In the simulation a traffic situation on a two-lane, one-way road, as shown in Fig. 2, is considered. The simulated scenario includes four vehicles: the ego vehicle initially positioned in the right lane (Veh_e), a vehicle in front of the Veh_e moving in the same initial lane (Veh_{0f}), a vehicle in front of the Veh_e moving in the left lane (Veh_{1f}), and a vehicle behind the Veh_e also moving in the left lane (Veh_{1b}). All vehicles except the Veh_e are assumed to move with constant longitudinal velocity and not to perform any lane change manoeuvres.

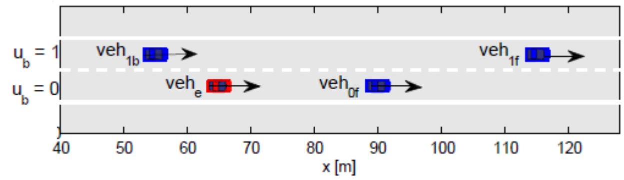

Fig. 2: Highway scenario including four vehicles: the ego vehicle initially positioned in the right lane (Veh_e), a vehicle in front of the Veh_e moving in the same initial lane (Veh_{0f}), a vehicle in front of the (Veh_e) moving in the left lane (Veh_{1f}), and a vehicle behind the (Veh_e) also moving in the left lane (Veh_{1b}).

In order to demonstrate the ability of the proposed decision and control algorithm to select preferred lane while keeping the distance to surrounding vehicles, the following three scenaria of the described traffic situation are considered:

1) Veh_e is approaching the slower moving Veh_{0f}, while Veh_{1b} and Veh_{1f} are at sufficient large distances to allow for an immediate overtake manoeuvre.
2) Veh_e is approaching the slower moving Veh_{0f}, while Veh_{1f} is at a sufficient large distance to allow for lane change but Veh_{1b} is approaching at high speed, hence Veh_e initially slows down to allow Veh_{1b} to pass before overtaking Veh_{0f}.
3) Veh_e is approaching the slower moving Veh_{0f}, while Veh_{1b} and Veh_{1f} are at sufficient large distances to allow for an immediate overtake manoeuvre, however Veh_e is approaching its designated exit lane and consequently does not performed the overtake manoeuvre.

In the above described scenaria it is assumed that the velocities and positions of all the surrounding vehicles are known and deterministic. To demonstrate the algorithms ability to handle unexpected events or sensor measurement errors, via the replanning step of MPC, a fourth scenario is considered as follows:

4) Veh_e is approaching the slower moving Veh_{0f}, while Veh_{1b} and Veh_{1f} are at sufficient large distances to allow for an immediate overtake manoeuvre, however after the lane change to the left has been performed it is detected that Veh_{1f} has a lower velocity than expected (in this example -5 m/s), whereas Veh_e adjusts its velocity accordingly and the overtake manoeuvre is aborted.

For each of the describe scenaria the respective initial conditions are given in Table I whereas the design parameters of the predictive controller is given in Table II. Note that the initial conditions for scenario 2 and scenario 3 are identical since the purpose of scenario 3 is to demonstrate the influence of the condition described by Eq. (3). Note also that in Table II only conditions for u_c is given since u_b is a binary integer variable.

As a mean to evaluate whether the proposed decision and

TABLE I: Initial conditions for the four considered scenaria, x denotes the longitudinal position [m] and v is the velocity [m/s].

	x_e	x_{0f}	x_{1f}	x_{1b}	v_e	v_{0f}	v_{1f}	v_{1b}
scenario 1	65	120	130	30	20	15	20	20
scenario 2	65	120	130	50	20	15	22	22
scenario 3	65	120	130	50	20	15	22	22
scenario 4	65	120	130	0	20	15	20	0

TABLE II: Design parameters for the decision and control algorithm (5).

$H_c = 20$	$H_p = 50$	$Q = diag(1, 1)$
$w = 1$	$v_{des} = 20\ m/s$	$u_{min} = -1\ m/s^2$
$u_{max} = 1\ m/s^2$	$\triangle u_{min} = -0.2\ m/s^3$	$\triangle u_{max} = 0.2\ m/s^3$

control algorithm satisfy safe distance keeping, the time to collision (TTC) defined as

$$TTC = \frac{|x_e - x_{k_i}|}{|v_e - v_{k_i}|} \tag{6}$$

and the intervehicular time (TIV) defined as

$$TIV = \frac{|x_e - x_{k_i}|}{v_e} \tag{7}$$

are introduced as performance criteria [22].

In Figs. 3, 4, 5, and 6 the intervehicle distances, TTC, and TIV with respect to Veh_e, as well as the preferred lane decision variable for the four scenaria are shown. In scenario 1 the TTC and TIV for Veh_{0f} drops to zero as Veh_e passes in the adjacent lane. While the TTC and TIV for Veh_{1f} and Veh_{1b} are constant since their velocity profiles are the same as for the Veh_e, making the TTC approach infinity and thus not displayed in Fig. 3. For scenario 2 the TTC and TIV for Veh_{1b} drops to zero as it passes Veh_e. When the TTC and TIV no longer are at critical levels Veh_e initialize the overtake manoeuvre allowing the TTC and TIV for Veh_{0f} to approach zero as Veh_e passes. In scenario 3, the TTC and TIV for Veh_{1b} drops to zero as it passes Veh_e. Since Veh_e is approaching its exit lane, it does not perform a lane change but rather follows Veh_{1f} at a safe distance. In scenario 4 the Veh_e performs a lane change after which it must reduce its velocity due to new information. Once, the TTC and TTV for Veh_{1f} are at a safe levels the overtake manoeuvre is aborted and Veh_e returns to the right lane. Why the Veh_e returns to the right lane instead of following Veh_{1f} in the left lane is a consequence of including u_b in the cost function (Eq. (5)) and thereby making the right lane the preferred lane of travel. Note, if the TTC is not displayed in Figs. 3, 4, 5, and 6 it means that the TTC > 10 s, and thus not relevant to take into consideration in the decision making.

From the figures it can be seen that the collision avoidance constraints (Eq. (4)) ensures that Veh_e maintains safe distances to the relevant surrounding vehicles by never allowing TTC or TIV to go below a certain threshold value ϑ. The unsymmetrical safety constraint allows ϑ to be kept as approximately 2 s when Veh_e is approaching or following a vehicle, and 1 s when Veh_e is changing to a lane with a vehicle behind.

Figure 7 shows the velocity profiles of the Veh_e for the four scenaria. In the figure it can be seen that the algorithm generates a control signal for the lateral movement that allows the Veh_e to maintain its desired velocity if possible, or else adjusts it to the velocity of an appropriate surrounding vehicle.

In the described algorithm, limits on acceleration and jerk have been set to obtain comfortable driving. Thus, if sudden changes, which require sever braking, occur in the environment the described MPC problem can become infeasible. To handle these types of emergency situations are beyond the scope of this paper. However, by incorporating the proposed algorithm with other collision avoidance and mitigation systems these serious situations can be assumed to be handled.

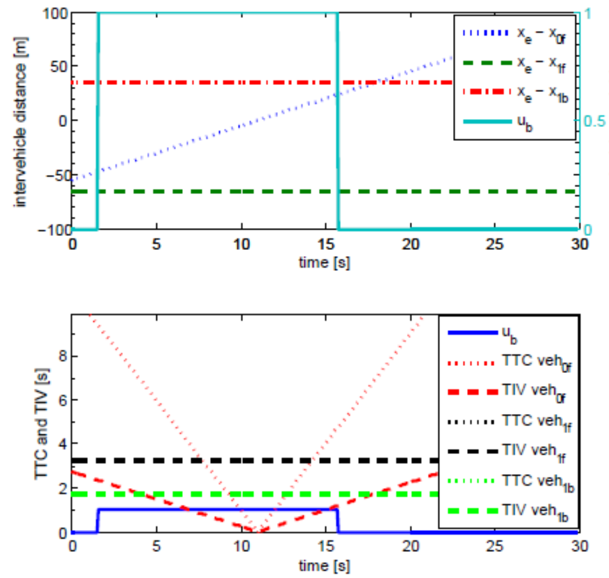

Fig. 3: Intervehicle distances, TTC, TIV, and preferred lane decision variable for scenario 1.

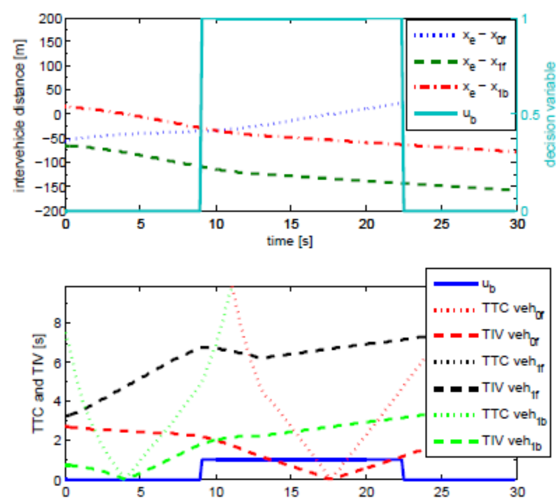

Fig. 4: Intervehicle distances, TTC, TIV, and preferred lane decision variable for scenario 2.

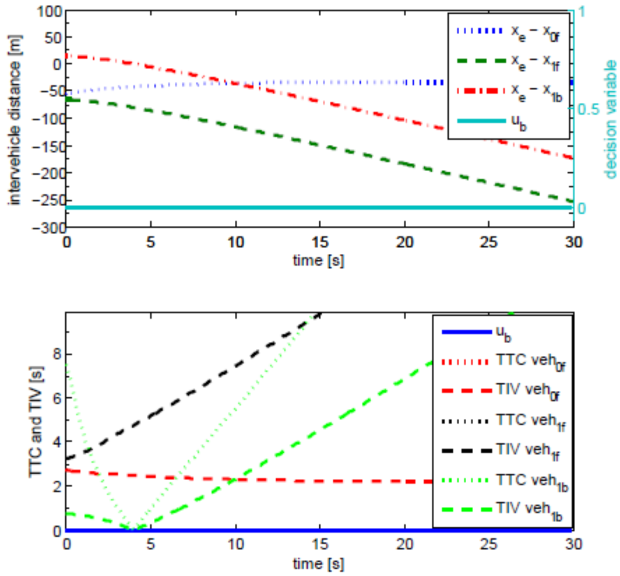

Fig. 5: Intervehicle distances, TTC, TIV, and preferred lane decision variable for scenario 3.

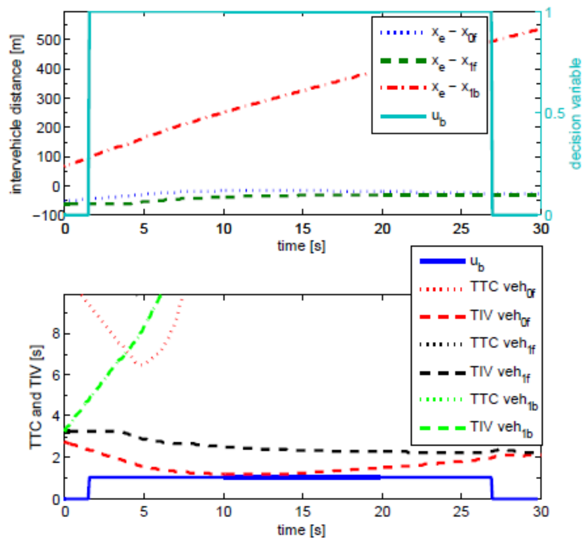

Fig. 6: Intervehicle distances, TTC, TIV, and preferred lane decision variable for scenario 4.

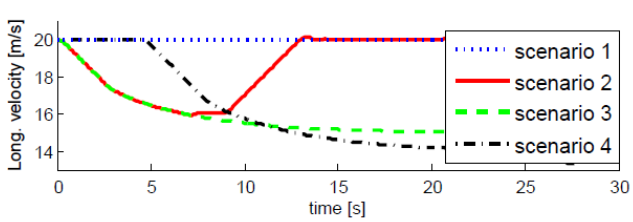

Fig. 7: Velocity profile for Veh_e.

V. CONCLUSIONS

This paper present a novel decision and control algorithm for lane change and overtake manoeuvres. By reducing the complexity of the system model and introducing a binary decision variable, a model predictive controller is efficiently computed. The predictive controller allows full control of acceleration/deceleration as well as providing a decision variable regarding preferred lane at each time instance. Simulation results demonstrates and confirms the flexibility and capability of the algorithm to make decisions and control actions similar to natural driving behaviour i.e. maintaining a desired velocity while preventing intervehicle distances to become unsafe, for two-lane, one-way road traffic scenaria.

These results motivates further work in refining the algorithm to incorporate a prediction model for the dynamic behaviour of surrounding vehicles i.e. no longer assume that surrounding vehicles move at constant velocity without lane change, and also include uncertainties and noise in the measured sensor information.

However, a drawback of the proposed algorithm is that mixed integer programming suffers from combinatorial complexity and the required computational time is strongly influenced by the number of binary variables included in the problem formulation. Thus there is a need to further investigate the computational time required for the optimization process in embedded systems for vehicle control.

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