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Why to use an Articulated Vehicle in Underground Mining Operations?

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Abstract

Engineers in the underground mining industry have known for a long time that an articulated vehicle is preferable to a normal truck for the navigation in the narrow environments of an underground mine because of its higher maneuverability. The two advantages of the articulated configuration can be explained in terms of degrees of freedom in the selection of the directions needed to span the whole tangent space and of the reduced gap between the trajectories followed by the wheels of the truck when steering. Both characteristics can be used in a constructive way for path planning and navigation purposes.

1 Introduction

The action of transporting the material from the stope to the dumping point of an underground mine is performed by a truck called LHD (Load-Haul-Dump). The LHD is an articulated vehicle composed of two bodies connected by a kingpin hitch. Each body has a single axle and the wheels are all non-steerable. The steering action is performed on the joint, changing the angle between the front and rear part by means of hydraulic actuators. Both the shape and the steering mechanism are intended to improve the maneuverability of the vehicle. The effect of the actuated articulation is twofold: the truck can steer on place i.e. the orientation of the vehicle changes varying the steering angle alone and the width spanned by the vehicle when turning is reduced with respect to, for example, a car-like vehicle. Both properties descend directly from the geometry of the articulation and can be explained considering the kinematic model of the vehicle.

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The articulated truck is an underactuated drift-free nonlinear system with two inputs, which was proven to be controllable in [6]. The *steering on place* can be justified looking at the vector fields associated with the two inputs. In particular, when comparing with a car-like vehicle, it turns out that the articulated configuration has as consequence a richer range of possible maneuvers that can be explained in terms of independence of the higher order Lie brackets of the vector fields i.e. with the presence of an extra degree of freedom in the selection of the directions needed to complete the tangent space of the configuration space. This degree of freedom can be used to optimize a cost function in a path planning problem. Here we provide a simple example where the cost to minimize is the number of elementary maneuvers required to reach a given point in the configuration space.

Also the second characteristic can be explained considering the geometry of the vehicle. In general, in a multiaxis wheeled vehicle, a nonnull steering angle implies that the wheels (or better the midpoints of the axles) follow different trajectories. This fact becomes a relevant problem when the free space in which the vehicle is allowed to move is limited, like in a narrow road or in an underground tunnel. It is intuitively easy to understand that a vehicle without articulation, say a car-like vehicle, would be more cumbersome i.e. would span a larger area than the LHD when turning. For the articulated vehicle case, the *off-tracking* between the trajectories can be easily calculated in some situations (see also [4]). The main problem for the autonomous navigation of the mining truck is to be able to follow the tunnel keeping a safety margin from both walls. Navigation of the truck requires proper interaction with the environment: in this case the environment (the tunnel) can be modeled as a path to follow and the proper criterion (keeping the middle of the tunnel) can be reformulated as reducing the off-tracking of the *whole vehicle* from the path. This idea,

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formulated in [3], finds here its most suitable application because of the limited width of the workspace of the mining truck. All the several approaches proposed in the literature to solve the path following problem for wheeled vehicles are essentially based on the selection of a single point of the vehicle and on the definition of a tracking criterion for this guidepoint. The task of the controller, then, is to have the corresponding tracking error converging to zero. Here the proposed solution consists in redefining the tracking error of the path following problem not based only on one single distance but on the *sum* of the signed distances of the midpoints of both axles of the vehicle from their orthogonal projections on the path. Stability can be proven locally for paths of constant curvature.

Finally, these being the two advantages of having an articulated configuration, it is also easy to see a drawback: as consequence of the central joint there is no direction of motion in which the open-loop system has a stable equilibrium point: in a sense that will be clarified below the system always behaves as a car moving backwards. Therefore such a configuration is useful only for applications in which the speed range is quite low, like mining, earth moving, forest industry or similar.

2 Steering on place

A typical configuration for a mining truck is the one shown in Fig. 1, where (x_i, y_i), $i = 0, 1$ are the

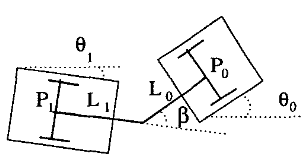


Figure 1: Two-unit articulated vehicle.

cartesian coordinates and θ_i the orientation angles of the midpoints P_0 and P_1 of the axles of the vehicle. At kinematic level, the inputs of the system can be taken to be the speed vector v of the point P_1 and the steering speed $u = \dot{\beta}$ where $\beta \triangleq \theta_0 - \theta_1$.

A set of variables that describes the configuration space of the truck is given by $\mathbf{q} \triangleq [x_1, y_1, \theta_1, \beta]$ with the equations:

$$\begin{aligned} \dot{\mathbf{q}} &= \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \\ \frac{\sin\beta}{L_0+L_1\cos\beta} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ -\frac{L_0}{L_0+L_1\cos\beta} \\ 1 \end{bmatrix} u \\ &= g_1(\mathbf{q})v + g_2(\mathbf{q})u \end{aligned} \tag{1}$$

The system (1) is well-defined in the domain

$$D = \left\{ \mathbb{R} \times \mathbb{R} \times \mathbb{S}^1 \times \left] -\arccos\left(-\frac{L_0}{L_1}\right), \arccos\left(-\frac{L_0}{L_1}\right) \right[\right\}$$

If $L_0 > L_1$ the system presents no singularity.

The system (1) was proven to be controllable in [6] using tools from differential geometry, like the rank of the Control Lie Algebra generated by the vector fields associated with the inputs. For the definition of nonlinear controllability, as well as Lie bracket, filtration, distribution etc. refer to a standard textbook on nonlinear control systems like [5].

When $L_0 = 0$, we obtain a car-like vehicle with the well-known system:

$$\begin{aligned} \dot{\mathbf{q}} &= \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \\ \frac{\tan\beta}{L_1} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_c \\ &= g_{1_c}(\mathbf{q})v + g_{2_c}(\mathbf{q})u_c \end{aligned} \tag{2}$$

where u_c stands for the steering input of the car. The domain of definition is:

$$D_c = \left\{ \mathbb{R} \times \mathbb{R} \times \mathbb{S}^1 \times \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\right\}$$

The difference between the two models (1) and (2) is that in the articulated truck the steering input is entering also into the equation for the orientation angle θ_1. This allows to change θ_1 by means of the steering actuator alone, whereas, in the car, θ_1 can be varied only through a sequence of the two inputs. It is equivalent to say that θ_1 is locally accessible by u without need of Lie bracketing the two inputs. The steerability on place can be checked using linearization of the equations around the origin: for the system (1) the two vector fields are $g_1(0) = [1\ 0\ 0\ 0]^T$, $g_2(0) = \left[0\ 0\ -\frac{L_0}{L_0+L_1}\ 1\right]^T$, whereas for the car-like (2) $g_{1_c}(0) = [1\ 0\ 0\ 0]^T$ and $g_{2_c}(0) = [0\ 0\ 0\ 1]^T$.

We can use the original nonlinear system to analyze more in depth this difference. As in all nonholonomic systems, g_1 and g_2 (or for the car g_{1_c} and g_{2_c}) are non-commuting vectors and the distribution they generate *span* $\{g_1, g_2\}$ is not involutive: the combination of the two vector fields gives a new direction (here the "wriggle"). For the articulated vehicle, the wriggle has the following expression:

$$[g_1,\, g_2] = \begin{bmatrix} \frac{L_0 \sin\theta_1}{L_0+L_1\cos\beta} \\ \frac{-L_0\cos\theta_1}{L_0+L_1\cos\beta} \\ \frac{L_0\cos\beta+L_1}{(L_0+L_1\cos\beta)^2} \\ 0 \end{bmatrix}$$

If $\cos\beta = -\frac{L_0}{L_1}$, then $[g_1,\, g_2]$ is aligned with g_1 and further bracketing does not generate any independent vector. Therefore, defining $\mathcal{L} \triangleq min\left\{\frac{L_0}{L_1},\, \frac{L_1}{L_0}\right\}$, we limit our analysis to the subset of D given by:

$$D_a = \{\mathbb{R}\times\mathbb{R}\times\mathbb{S}^1\times\,]-\arccos(\mathcal{L})\,,\, \arccos(\mathcal{L})[\}$$

which, $\forall\mathcal{L}$, contains an open neighborhood of the origin. In practice, this is not a critical limitation because in a real mining truck the two semichassis have lengths L_0 and L_1 that are similar, when not equal. Both systems (1) and (2) are completely nonholonomic with a filtration that grows regularly in their respective domains D_a and D_c and both distributions that contain vector fields and the Lie brackets of vector fields become involutive (i.e. a Lie Algebra) already at the second level of Lie bracketing (the degree of nonholonomicity of the system is 2 in both cases) implying therefore controllability. The difference between the two cases lies in the arbitrariness of the selection of the fourth vector that complete the distribution to the whole tangent space. If we consider the possible combinations of the vector fields for the two generator case, stopped at the degree 2, we have the five vectors:

$$g_1,\, g_2,\, [g_1,\, g_2]\,, [g_1, [g_1,\, g_2]]\,, [g_2, [g_1,\, g_2]]$$

and among those we have to choose the basis of the Control Lie Algebra.

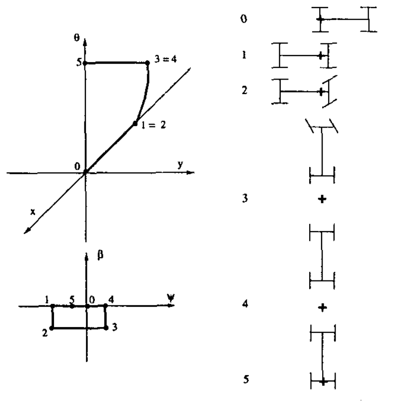
For the car-like system, we obtain that there is an unique way to complete the basis. In fact, $[g_2, [g_1,\, g_2]]$ results always aligned with the wriggle $[g_1,\, g_2]$ $\forall \mathbf{q} \in D_c$. In the articulated vehicle instead, both $[g_1, [g_1,\, g_2]]$ and $[g_2, [g_1,\, g_2]]$ can be used to complete the basis which means that both inputs can be used to move along the new direction.

The richer behavior of the steering actuator on the articulation joint can be interpreted substituting the articulation with a virtual steering wheel (see [1]). In fact, at kinematic level, the articulated truck is equivalent to a tricycle model or to a couple of car-like models having the steering wheel in common. The steering action on the articulation is equivalent to a combination of steering plus translation on the corresponding car-like models.

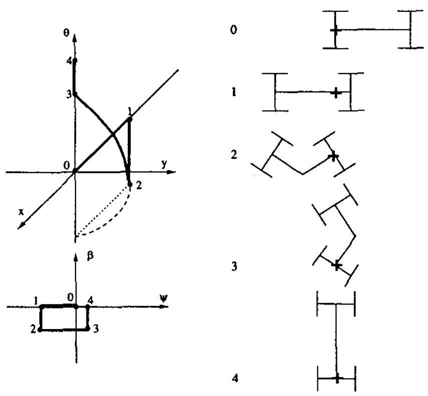
We want to exploit now this peculiarity of the articulated truck comparing the two systems in a given maneuver. For an autonomous system it is reasonable to generate motion using the simplest possible sequence of inputs functions, for example piecewise constant inputs, and to give them in a decoupled way, i.e. our input functions have to be:

- piecewise constant
- $u(t)\cdot v(t) = 0\ \ \forall\, t \in [0,\, T]$

We take as maneuver a rotation of 90° around the midpoint of the rear axle $(x_1,\, y_1)$. This movement can correspond to a docking maneuver, a typical maneuver that has to be performed to place the truck in the correct load/unload position. In the more realistic case of a limitation in the steering angle, say $|\beta| \leq \pi/3$, this has to be accomplished with a sequence of input commands since both vehicles have also to satisfy to the nonholonomic constraints. If we call ψ the phase angle of the rear (actuated) wheels, then $v = \dot{\psi}\rho$ with ρ the radius of the wheels. In order to have a clear picture of how is the evolution of the integral curves of the system, we consider ψ as an extra state and split the state vector into *base* variables $(\beta,\, \psi)$ which are directly controlled by the inputs and *shape* variables $(x,\, y,\, \theta)$ which evolve in $SE(2)$, the special planar Euclidean group. In both cases, simple geometric considerations allow to compute a path between the two shape points $(0,\, 0,\, 0)$ and $(0,\, 0,\, \frac{\pi}{2})$. Controllability on $SE(2)$ as a subspace of D_a and D_c assures that such a path exists. The corresponding path on the base space, with the requirement that $\beta = 0$ at the end point (again the existence is guaranteed by the nonlinear controllability), and therefore the corresponding piecewise constant time-varying input functions can be calculated using the inverse kinematic of the system. In Fig. 2 the paths that realize the desired motion with a *minimal* sequence of input commands are shown in the two cases. The minimal sequence is not unique in the sense that other combinations of base movements can lead to the same endpoint. On the real system, the controllability of ψ is of no practical relevance, therefore we are not interested in achieving closed paths on the base space but only in the final value of β. Comparing (a) and (b) of Fig. 2, we notice that the minimal sequence for the articulated vehicle is shorter than the other one: this is again a consequence of the direct accessibility of θ from the steering input. Loosely speaking, we can say that for the car-like case the impossibility of generating the whole tangent space only by repeated Lie bracketing g_2 with g_1 is reflected here in the need to apply g_1 three times (instead of the two of the articulated configuration) in order to reach the



(a) Car-like vehicle



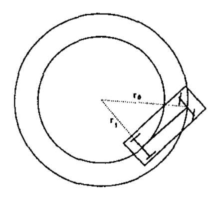
(b) Articulated vehicle

Figure 2: Change of orientation.

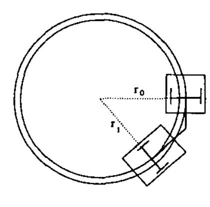
desired endpoint, i.e. the arbitrariness in the choice of the fourth vector needed to span the whole tangent space reflects in the larger range of combinations of input commands and implies here the possibility of finding simpler combinations that generate a path between two given points.

3 Off-tracking

The typical work environment for a mining truck is an underground tunnel of limited width. Navigating in such an environment implies an high risk of crashing against the walls of the gallery. It is common characteristic of multiaxis vehicles that during a bend the midpoints of the axles tend to follow different trajectories. The difference between these trajectories can be taken as a measure of how much cumbersome a vehicle is. Comparing again the mining truck with the car-like vehicle, it is intuitively clear that the articulation helps reducing the gap between the trajectories of the midpoints of the two axles. Detailed calculations for the articulated vehicle are reported in [4]. In synthesis, in the two cases, the distance between the two trajectories can be easily computed for a motion with constant steering angle $\beta \neq 0$. In fact, for $v \neq 0$, the two midpoints follow concentric circles whose radius r_0 and r_1 (respectively for the front axle and the rear axle) can be calculated using the geometry of the vehicle. For the car-like vehicle, the off-tracking is (see Fig. 3):



(a) Car-like vehicle



(b) Articulated vehicle

Figure 3: Off-tracking margins.

$$r_0 - r_1 = L\frac{1-\cos\beta}{\sin\beta}$$

whereas, for the articulated vehicle:

$$r_0 - r_1 = L_1\frac{\cos\left(\frac{L_0}{L_0+L_1}\beta\right) - \cos\left(\frac{L_1}{L_0+L_1}\beta\right)}{\sin\left(\frac{L_1}{L_0+L_1}\beta\right)}$$

When the two units are symmetric, then $L_0 = L_1$ and the off-tracking is zero i.e. the two trajectories overlay.

3.1 A path tracking criterion

The rest of the paper is dedicated to the formulation of an algorithm for the navigation of the articulated vehicle, aiming at reducing the off-tracking of the whole vehicle from a given path. The idea is adapted from [3].

The underground tunnel in which the truck is navigating is usually represented in terms of a curvature function associated with the length of a trajectory representing for example the middle of the tunnel. Translating this into the cartesian coordinates of an inertial frame is not possible analytically because of the absence, except for trivial cases, of a closed form in the line integral expressing the length of the path covered. Therefore, a particularly convenient local representation is given by a frame moving on the path to follow (see [7, 8]). Under the assumption that the path is at least C^1 and that the curvature has an upper bound (see [9] for the details), a Frenet frame can be used to locally describe the motion of the point with respect to the reference path γ of known curvature κ_γ. The continuity of the curvature function is not required and so also simple paths, composed of straight lines and arcs of circle, can be considered. The set in which the local coordinates are well defined is essentially a "tube" around the path.

In our case, we consider two Frenet frames moving on the curve to follow, corresponding to the projections on γ of the two points P_0 and P_1 of our vehicle. In our frames, we assume to have chosen a base

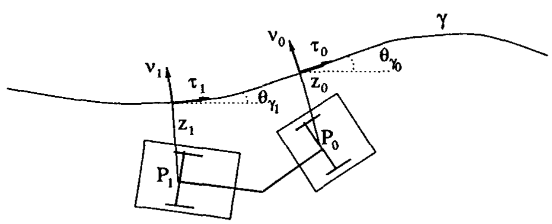


Figure 4: Frenet frames associated with P_0 and P_1.

with the conventions of Fig. 4. Each of the curvilinear frames is represented by two coordinates $(s_{\gamma_i}, \theta_{\gamma_i})$ where s_{γ_i} is the curvilinear abscissa and θ_{γ_i} is the orientation of the frame with respect to the inertial frame. In the Frenet frame, the point P_i is represented by the signed distance z_i between the point itself and its projection and by the relative orientation angle $\tilde{\theta}_i \triangleq \theta_i - \theta_{\gamma_i}$. The equations describing the dynamics of the point P_i in the local frame can be found for example in [7].

For this system, the main issue of the autonomous navigation is by far to *keep the middle* of the tunnel, i.e. to keep control of the lateral dynamics. The main advantage of the Frenet frame is that it provides a natural way to describe the lateral displacement of a point from the path, since z_i represents the signed distance of P_i from its orthogonal projection on γ. This property has been used by several authors as tracking criterion for the path following problem. We will also use it, but redefining the tracking error as the sum of the two signed distances corresponding to the two points P_0 and P_1. In other words, we replace the tracking criterion normally used $z_1 \to 0$ (or an equivalent one) with

$$z_0 + z_1 \to 0 \tag{3}$$

Using the Frenet frame, the control of the longitudinal dynamics, i.e. how fast the path is covered, becomes an independent problem and relatively secondary with respect to the lateral control. Therefore, it will be neglected in this study. This is equivalent to drop one of the degrees of freedom of the control design that is to say, in our case, to consider the input speed v as a given (open-loop) function. Furthermore, also the dynamic equations for the length covered along the path can be neglected: in fact s_{γ_0} and s_{γ_1} enter into the model only when the curvature κ_γ is varying. Since the stability analysis of our path tracking algorithm will be carried out along an arc of circle, the model is completely independent from s_{γ_0} and s_{γ_1}. In order to keep track of both the distances z_0 and z_1 simultaneously, we must increase the dimension of the state. The new state vector is $\mathbf{p} = \left[z_0\, z_1\, \tilde{\theta}_0\, \tilde{\theta}_1\, \beta_1\right]^T$ with the dynamic equations (see [2] for details):

$$\dot{\mathbf{p}} = \begin{bmatrix} \frac{L_1+L_0\cos\beta}{L_0+L_1\cos\beta}\sin\tilde{\theta}_0 \\ \sin\tilde{\theta}_1 \\ \frac{\sin\beta}{L_0+L_1\cos\beta} - \frac{(L_1+L_0\cos\beta)\kappa_\gamma(s_{\gamma_0})\cos\tilde{\theta}_0}{(L_0+L_1\cos\beta)\left(1-\kappa_\gamma(s_{\gamma_0})z_0\right)} \\ \frac{\sin\beta}{L_0+L_1\cos\beta} - \frac{\kappa_\gamma(s_{\gamma_1})\cos\tilde{\theta}_1}{1-\kappa_\gamma(s_{\gamma_1})z_1} \\ 0 \end{bmatrix} v + \begin{bmatrix} -\frac{L_0L_1\sin\beta\sin\tilde{\theta}_0}{L_0+L_1\cos\beta} \\ 0 \\ \frac{L_1\cos\beta}{L_0+L_1\cos\beta} + \frac{L_0L_1\sin\beta\,\kappa_\gamma(s_{\gamma_0})\cos\tilde{\theta}_0}{(L_0+L_1\cos\beta)\left(1-\kappa_\gamma(s_{\gamma_0})z_0\right)} \\ -\frac{L_0\cos\beta}{L_0+L_1\cos\beta} \\ 1 \end{bmatrix} u$$

or, in more compact form:

$$\dot{\mathbf{p}} = \mathcal{A}(\mathbf{p}) + \mathcal{B}(\mathbf{p})u \tag{4}$$

Clearly, considering both reference systems in P_0 and in P_1 gives a redundant description of the system. In order to complete this overparameterized state representation, one has to introduce three constraints expressing the fact that the vehicle is a rigid body. These three constraints are given by line integrals that depend on the geometry of the truck and on the curvature of the path between the two projections of P_0 and P_1 on the path. They are obviously holonomic i.e. they reduce the configuration space of the system down to the original number of variables. In what follows we will simply drop them and continue working with the overparameterized model.

Stability analysis For a car-like vehicle, the path following with positive speed implies that the open loop equilibrium point is "naturally" stable whereas backward motion implies that the same equilibrium is open loop unstable. For a mining truck, the path following problem has always an unstable equilibrium point due to the steering action performed on the articulation joint.

We use Lyapunov linearization method to show that the system can be locally asymptotically stabilized to a path of constant curvature. The fact that linearization does not provide global results is not a limitation in our case since the mining truck has to navigate into a tunnel of reduced width and also the local frames are isomorphically defined only in a region around the path.

For a path of constant curvature κ_γ, the equilibrium point $\mathbf{p}_e$ can be calculated from the geometry of the problem. The Jacobian matrix calculated at $\mathbf{p}_e$ $A = \left.\frac{\partial \mathcal{A}}{\partial \mathbf{p}}\right|_{\mathbf{p}=\mathbf{p}_e}$ is a Hurwitz matrix i.e. the closed loop state matrix $A - vKB$ with $B = \mathcal{B}(\mathbf{p}_e)$ can be made stable by choosing an appropriate gain K. Therefore, the system (4) can be locally asymptotically stabilized to a circular path by means of a linear state feedback.

Simulations In Fig. 5, the path to track is an arc of circle. It can be seen that convergence is achieved from a generic admissible initial condition. A similar behavior is obtained with a negative speed.

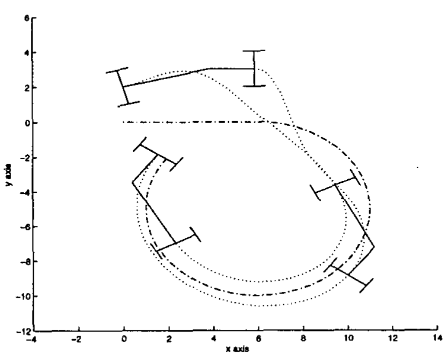


Figure 5: Following an arc of circle of curvature $\kappa_\gamma = -0.2$ (dash-dotted line) from a wrong initial posture.

4 Conclusion

In this paper we have given some mathematical insight into the "higher maneuverability" concept that makes an articulated vehicle more suitable than a car-like vehicle in an environment characterized by a limited free space like it can be an underground mine. How to explain and exploit this difference is the subject of this paper.

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