

Value Sensitive Design for Autonomous Vehicle Motion Planning

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Abstract—Human drivers navigate the roadways by balancing values such as safety, legality, and mobility. The public will likely judge an autonomous vehicle by similar values. The iterative methodology of value sensitive design formalizes the connection of human values to engineering specifications. We apply a modified value sensitive design methodology to the development of an autonomous vehicle speed control algorithm to safely navigate an occluded pedestrian crosswalk. The first iteration presented here models the problem as a partially observable Markov decision process and uses dynamic programming to compute an optimal policy to control the longitudinal acceleration of the vehicle based on the belief of a pedestrian crossing. The speed control algorithm is then tested in real-time on an experimental vehicle on a closed road course.

I. INTRODUCTION

The roadways are populated by many stakeholders, such as pedestrians, bicyclists and vehicle occupants, and these stakeholders have values that predicate their expectations. Public expectations regarding autonomous vehicle driving behavior will likely be based on similar human values. Safety, legality, and mobility are some reasonable values to consider [1]. Designers of autonomous vehicle technologies have the challenge of connecting these human values to engineering specifications. One way to address this challenge is to integrate the stakeholders and their values into the design process of autonomous vehicle motion planning algorithms.

Value sensitive design (VSD) is a tripartite methodology that can be applied to any generic design task by iterating over conceptual, technical, and empirical investigations [2], [3]. At every stage of the design process, human values are connected to the designed technology. VSD is most applicable to a design task in which value conflicts exist for high-stakes ethical issues. As Friedman, Kahn, and Borning indicate, VSD has been found to be widely useful in the human-computer interaction community to help balance privacy concerns with usability for end-users [3], [4]. In the design of an office space with a virtual window viewing a public plaza, they also demonstrate that recognizing indirect stakeholders (a component of the conceptualization phase) uncovered privacy concerns for passersby. Denning et al. use VSD to construct a list of specifications to guide future designs of a security system in implantable medical devices [5]. Furthermore, the ubiquity of VSD allows for modification to be in-line with certain design tasks. Van Wynsberghe appends to VSD the moral theory of care ethics in the design

of health care robots to ensure the robot reflects stakeholder values that are in line with the morality of care [6].

In designing autonomous vehicle motion planning algorithms, engineers already account for some human values. Most often, the algorithm design focuses on the values of safety and efficiency as demonstrated by Chen, Zhao, and Peng's evaluation framework of an autonomous vehicle approaching an unsignalized pedestrian crosswalk [7]. Bandyopadhyay et al. [8] and Bandyopadhyay et al. [9] also focus on safety and efficiency in the construction of the reward function of a partially observable Markov decision process (POMDP) for speed control in pedestrian environments. Brechtel, Gindele, and Dillmann also design for safety and efficiency in the reward function of a speed control POMDP for entering occluded intersections [10]. These examples demonstrate the good intentions of engineers to connect human values to an engineered technology. The focus on safety and efficiency in the evaluation framework and motion planning policies highlights the difficulty of designing for two conflicting values. Brechtel et al. additionally consider occupant comfort in the reward function, and suggest traffic rules can likewise be included in future iterations of the POMDP design. Their discussion indicates a desire to account for the various human values at stake in the design of a motion planning policy. To account for these values, it would be useful to have a methodology that can help with determining which values to include and that can be used for resolving conflicts among them.

We propose that VSD can help fill the gaps in the design process of motion planning algorithms for autonomous vehicles. We use VSD to formalize the connection of human values to engineering specifications by identifying a more complete list of human values that are at stake in the design problem and by resolving value conflicts through justification of design choices. We demonstrate a modified application of VSD for autonomous vehicle motion planning with the design task of a speed controller for the scenario of an occluded pedestrian crosswalk. The speed along the path is only upper bounded by the speed limit. In order for the autonomous vehicle to navigate the scenario safely, it likely needs to reduce its speed. The speed is controlled by acceleration commands from a POMDP policy designed using VSD. This VSD speed controller is the first iteration from the design process and is not the final product.

The subsequent sections proceed as follows: Section II details the three phases of the VSD methodology: conceptualization, technical implementation, and empirical analysis. For the scenario of an occluded pedestrian crosswalk, Section III presents the first iteration of the conceptualization

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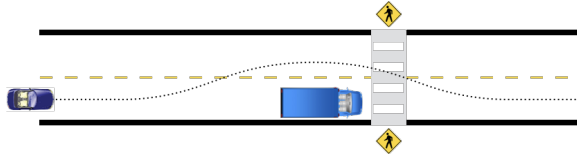


Fig. 1. Experimental scenario of occluded pedestrian crosswalk.

of the speed control design task, and Section IV describes a technology implementation through a POMDP formulation. Section V provides empirical analyses to determine how well the conceptualization and technical implementation were realized as demonstrated in real-time on an experimental vehicle. Conclusions and next steps for the second VSD iteration are in Section VI.

II. VALUE SENSITIVE DESIGN

The methodology of value sensitive design (VSD) consists of three phases: conceptual, technical, and empirical [2], [3]. During the conceptual phase, the methodology involves identifying the values encompassed by the designed technology. Additionally, the conceptualization phase determines the direct and indirect stakeholders of the technology. For the technical phase, a feature of VSD holds that some technological implementations are better suited to uphold certain values over other implementations. During this phase, the technical solutions most in line with the values identified in the conceptual phase are used to develop the technology being designed. Finally, the empirical phase allows for quantitative and qualitative analyses of the developed design, such as data analysis or observations from human-user studies. This period allows for inspection of how successful the designed technology meets the conceptualization. Throughout the design development, one iterates over the various phases until all three align. Engineers implicitly iterate over conceptual, technical and empirical phases throughout the design of a new technology, and VSD provides a tool to help formalize this process to explicitly account for values embedded in the technology.

III. CONCEPTUALIZATION

To start the VSD process, we specify the scenario and design task, and then identify the stakeholders involved. We additionally determine the human values consistent with the scenario and design task. The scenario in Fig. 1 depicts a two-lane roadway with a single, dashed yellow line. The roadway also comprises a marked pedestrian crosswalk. In front of the crosswalk is a large, illegally parked van. From the perspective of the autonomous vehicle approaching the crosswalk, the crosswalk is partially occluded because of the obstructing van. The design task is to develop a speed control algorithm such that the autonomous vehicle safely navigates through the scenario.

Identifying both the direct and indirect stakeholders forces engineers and programmers to think more deeply about the consequences and who is affected by the designed technology. For this scenario, the direct stakeholders are the au-

tonomous vehicle, occupants in the autonomous vehicle, the pedestrian potentially crossing the street, and the authority of traffic laws. An indirect stakeholder is the obstructing vehicle parked on the road because we assume the autonomous vehicle can follow an obstacle-free path around the occlusion. We choose to focus on these stakeholders for this first iteration, but there can be many more stakeholders, such as bicyclists or bystanders.

Determining the human values involved in the scenario and design task is critical to VSD and the engineering process. Traffic scenarios, in general, relate to balancing the human values of safety, legality, and mobility. By considering the stakeholders, we uncover more values at stake, such as care and respect for others, fairness and reciprocity, respect for authority, trust and transparency, and individual autonomy. How we define these moral values could lead to different technology designs, so we more clearly define what is meant by each value for this particular scenario:

- *Care and respect for others* manifests by our desire to not harm other persons.
- *Fairness and reciprocity* affect both the vehicle occupants and pedestrian stakeholders in that the autonomous vehicle should not take biased or discriminatory actions based on information about the stakeholders. The autonomous vehicle should treat all individuals involved as equal agents.
- *Respect for authority* engages the relationship between the autonomous vehicle and adherence to traffic laws.
- *Trust* emerges when the pedestrian assumes an oncoming vehicle yields to his or her right-of-way while crossing within the crosswalk. *Transparency* occurs when the autonomous vehicle's actions facilitate this trust.
- *Individual autonomy* of the vehicle occupants acknowledges the desire to get from one destination to another with little impedance.

To address each of these values, we relate them to the specifications safety and legality, mobility and efficiency, and smoothness, which can be captured by engineering terms.

A. Safety and legality

When navigating an occluded pedestrian crosswalk, the relationship between legality and safety are closely tied. According to the California Driver's Handbook [11], pedestrians have the right-of-way in a crosswalk. When a pedestrian may be present, a driver is encouraged to reduce the vehicle speed when approaching a crosswalk and be prepared to stop as specified by California Vehicle Code §21950:

- (a) The driver of a vehicle shall yield the right-of-way to a pedestrian crossing the roadway within any marked crosswalk or within any unmarked crosswalk at an intersection, except as otherwise provided in this chapter.
- (b) This section does not relieve a pedestrian from the duty of using due care for his or her safety. No pedestrian may suddenly leave a curb or other place of safety and walk or run

into the path of a vehicle that is so close as to constitute an immediate hazard. No pedestrian may unnecessarily stop or delay traffic while in a marked or unmarked crosswalk.

- (c) The driver of a vehicle approaching a pedestrian within any marked or unmarked crosswalk shall exercise all due care and shall reduce the speed of the vehicle or take any other action relating to the operation of the vehicle as necessary to safeguard the safety of the pedestrian.
- (d) Subdivision (b) does not relieve a driver of a vehicle from the duty of exercising due care for the safety of any pedestrian within any marked crosswalk or within any unmarked crosswalk at an intersection.

As the vehicle code alludes, following the law and driving safely converge to the same objective in this scenario: if the autonomous vehicle adheres to the legal requirement, then it also takes safe actions. The key pieces of information necessary for safe and legal decision-making are vehicle speed (v_t), distance to crosswalk (d_t), and whether a pedestrian is crossing the street or not (c_t). Again, this connects to the ethical values of care and respect for others, respect for authority, and fairness and reciprocity.

B. Mobility and efficiency

The metric of time efficiency captures the human value of mobility. Time efficiency is directly related to the speed of the vehicle (v_t) for the given path. This objective relates to the moral value of individual autonomy.

C. Smoothness

Smooth driving affects occupant comfort and interjects trust and transparency between the stakeholders. For longitudinal control, smoothness can be captured through the change in vehicle speed, which is equivalent to knowing the acceleration command (a_t) and change in time (Δt).

IV. TECHNICAL IMPLEMENTATION

Many motion planning approaches can be tailored to solve the pedestrian occlusion scenario. Rather than just choosing an arbitrary approach, VSD maintains that the choice of technology or algorithm implicates certain values. We choose to formulate the technical realization of the above conceptualization through a stochastic optimization problem. A stochastic optimization problem can account for modeled uncertainty present in the driving scenario while balancing the identified values through the objective function. Furthermore, we can formulate the problem as an open-loop or a closed-loop planning problem. To obtain an offline policy that we can inspect and verify before putting on an autonomous vehicle, we will use a closed-loop planning approach and construct the problem as a partially observable Markov decision process (POMDP) [12]. Throughout the design of the POMDP, every design choice is connected back to a value from the conceptualization phase in order to justify

the engineering and explicitly record the embedding of said values.

A. POMDP

A POMDP makes decisions based on the history of state estimates or observations o_1, \dots, o_t . To reduce the information stored, it is summarized in a belief state b , which is a distribution over the states. The optimal policy is represented as a set of alpha vectors, which convert the belief state to a control input or action. Given the values of safety and legality, efficiency and mobility, and smoothness, the information necessary to address each value in the objective function is captured by the state vector

$$x_t = \begin{bmatrix} v_t \\ d_t \\ c_t \end{bmatrix} \quad (1)$$

and the control input

$$u_t = a_t, \quad (2)$$

where v_t is the vehicle speed, d_t is the vehicle distance to the crosswalk, c_t is the pedestrian detection and a_t is the longitudinal acceleration. The top speed of the roadway is assumed to be 10 m/s, so the vehicle speed is upper bounded by the speed limit to coincide with the safety and legality objective. The pedestrian detection is a Boolean value because the pedestrian is either crossing or not, and, in order to uphold the values of fairness and reciprocity, the detection does not rely on other information about the pedestrian that may be discriminatory. The control input, or action, is limited to ± 3 m/s² to provide comfortable acceleration and deceleration values, which further addresses the objective of smoothness for occupant comfort.

For the dynamics (or state transitions), a point mass model of the vehicle is used to calculate the distance to the crosswalk and vehicle speed. The detection of a pedestrian crossing maintains some uncertainty over time. When the pedestrian is detected, there is a 90% probability the pedestrian will continue to be detected at the next time step. When the pedestrian is not detected, then there is a 50% chance he or she will continue to not be detected, which captures the uncertainty due to the occlusion. The control loop assumes perfect information for the distance to the crosswalk and vehicle speed. However, there is observation uncertainty for the pedestrian crossing with a false positive of 5% for detecting and a false positive of 5% for not detecting the pedestrian, which captures sensor uncertainty.

The reward function defines the stage cost $g(x_t, u_t)$ for every state and action, which allows us to further connect the conceptualization values to the technical implementation. The reward for a state-action pair involves adding stage costs (3), (4), and (5) for that state and action. The stage cost for safety and legality is

$$g_{\text{safe}}(x_t, u_t) = - \left(\zeta \frac{v_t^2}{d_t + \epsilon} + \eta \mathbf{1}(d_t = 0) \right) \mathbf{1}(c_t), \quad (3)$$

where $\epsilon > 0$ is a buffer in the denominator to soften the constraint, $\zeta > 0$ is a weight on the penalty incurred by

TABLE I
WEIGHTS IN THE REWARD FUNCTION

Variable	Weight
Safety and legality (ζ)	0.2 s ² /m
Safety and legality (η)	0.2
Buffer (ϵ)	8 m
Efficiency (λ)	0.25 s/m
Smoothness (ξ)	1 s ² /m ²

driving quickly as the vehicle gets closer to the crosswalk, and $\eta > 0$ is a terminal penalty independent of velocity to encourage the vehicle to stop when the pedestrian is crossing. This term comes from the constant acceleration point mass equations relating the constant deceleration needed to come to a complete stop given the distance to the crosswalk and vehicle speed. For efficiency and mobility, the stage cost takes the form of

$$g_{\text{efficient}}(x_t, u_t) = \lambda v_t \mathbf{1}(-c_t), \quad (4)$$

where $\lambda > 0$ is a reward weight to encourage higher speed when the pedestrian is not crossing. To achieve smoothness for occupant comfort, the objective is realized through a penalty stage cost on the change in velocity

$$g_{\text{smooth}}(x_t, u_t) = -\xi(v_t - v_{t+1})^2 = -\xi(a_t \Delta t)^2, \quad (5)$$

which only depends on the current input and the time step. The goal is for the autonomous vehicle to smoothly drive safely and efficiently through the occluded crosswalk while adhering to the relevant traffic laws.

To solve the POMDP, we use the method QMDP to approximate an optimal solution [12]. Although QMDP assumes that at the next time step the state will be fully observable, it is well suited for this problem because the actions are not information gathering, meaning the actions do not directly reduce the uncertainty of the scenario. To solve the POMDP with this method, the state and action spaces are discretized but continuousness is maintained using multilinear grid interpolations [13] for the state transitions. Vehicle speed increments in steps of 0.5 m/s, vehicle distance to crosswalk increments by 1 m, and accelerations are quantized by 0.1 m/s² intervals. The choice of weights in the reward function are summarized in Table I.

Throughout the technical implementation, every design choice in the POMDP is connected back to a value from the conceptualization phase as a way to document and justify the engineering of this technology. The next section shows how well this implementation realizes the conceptualization.

V. EMPIRICAL ANALYSIS

The third phase of the VSD methodology entails qualitative and quantitative analyses. In order to discuss how the VSD process impacts the design of a speed control algorithm, we compare it to a deterministic proportional speed control as a baseline. We will qualitatively compare the resulting policies of the baseline and POMDP. Quantitative analysis comes from real-time in-vehicle experiments on an automated Ford Fusion.

A. Baseline

The baseline for comparison is a deterministic proportional speed control. Once the pedestrian is detected, a constant deceleration is commanded based on the current vehicle velocity and distance to the crosswalk as in Eq. (6a), which also assumes a point mass model with constant acceleration. When no pedestrian is detected, then the vehicle resumes a proportional cruise control with gain k_p and known desired velocity v_{des} as in Eq. (6b). Admittedly, the baseline is simple, but it is appropriate because it uses the same information as the POMDP. In future iterations, other baselines can be considered.

$$\begin{aligned} &\text{if } c_t \\ &\quad a_t = -\frac{v_t^2}{2d_t} \end{aligned} \quad (6a)$$

$$\begin{aligned} &\text{else} \\ &\quad a_t = k_p(v_{\text{des}} - v_t) \end{aligned} \quad (6b)$$

B. Policy Comparison

The baseline controller (6) is a closed-loop policy, mapping every state to an action, and is graphically represented in Fig. 2. The values along the horizontal-axis are the vehicle speed, and the values along the vertical-axis are the distance to the crosswalk. The colors indicate the action. The baseline policy seems to be safe when the pedestrian is detected, but it lacks efficiency. In contrast, when the pedestrian is not detected, it is efficient but not safe. This dichotomous behavior highlights the need to anticipate transitioning from one set of logic to the other because of the uncertainty about a pedestrian crossing. The baseline policy lacks resolution of the value conflict between safety and efficiency for the scenario.

The closed-loop policy of the POMDP designed in the technical implementation phase is represented as a set of alpha vectors, where each alpha vector corresponds to an action. Using the optimal expected utility for each state, the corresponding action is shown in Fig. 3. When the pedestrian is crossing, the policy indicates a balance between efficiency further away from the crosswalk and safety as the vehicle approaches. There is significant improvement in terms of safety while the pedestrian is not crossing while there is still some efficiency. The policy also indicates smooth actions across the state space. The policy comparison highlights that the human values identified in the conceptualization phase are better realized with the POMDP.

C. Experimental Results

To further illustrate how well the VSD speed controller works, we conducted an in-vehicle experiment using a fully automated hybrid Ford Fusion known as Trudi. Trudi is equipped with an Oxford Technical Solutions RT4000 differential GPS/INS unit, which obtains pose information within 2 cm of accuracy using corrections from a Novatel base station. Vehicle state information is calculated using the pose information as well as velocities and accelerations from the INS. Additionally, Trudi has four Velodyne HDL-32E

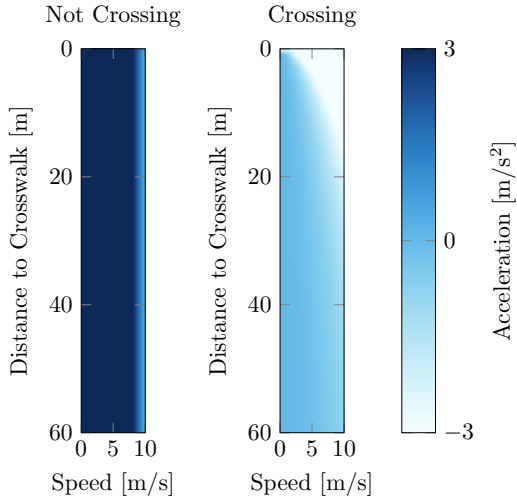


Fig. 2. Baseline closed-loop policy mapping each state to an action.

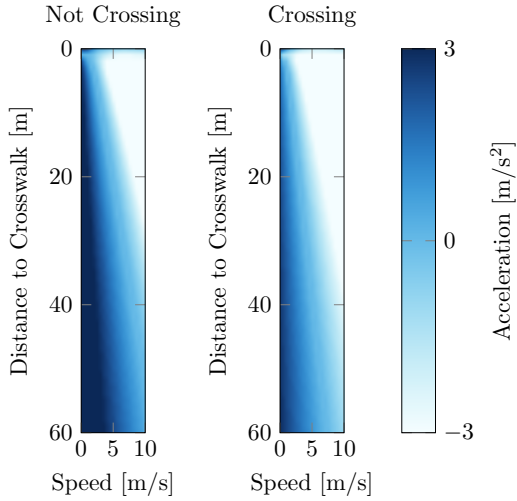


Fig. 3. Closed-loop policy depicting optimal action at that state assuming perfect state information.

lidars, which return a dense 3D point cloud with intensity values. The intensity values are used as a simple classifier for the pedestrian, which is a large person-shaped piece of cardboard (Fig. 4) covered in retro-reflective material. The vehicle is tasked with following an obstacle-free path around the occluding vehicle using a deterministic model predictive steering control as in Brown, Funke, Erlien and Gerdes [14].

For the POMDP policy execution, an observation of the vehicle speed, vehicle distance to crosswalk and detection of the pedestrian are used to update the belief with a Bayesian filter. The approximate optimal action taken is then

$$\arg \max_a \alpha_a^\top \mathbf{b}, \quad (7)$$

where α_a is an alpha vector for action a and \mathbf{b} is the belief state as a vector. Both the policy solver and policy execution are implemented with the POMDPs.jl library [15].

The experimental scenario involves an occluded pedestrian crosswalk on a two-lane roadway. The vehicle starts from a



Fig. 4. Experimental setup of occluded pedestrian crosswalk using an inflatable van for the occluding vehicle and a retro-reflective cardboard cutout for the pedestrian that moves along a track.

stopped position at the beginning of the road. As the vehicle approaches the crosswalk, the pedestrian suddenly appears in the crosswalk from behind the occluding vehicle. The control algorithms have no prior knowledge as to when the pedestrian will appear. Figures 5 and 6 depict the overhead driven trajectory, acceleration commands, and speed profile for the baseline and POMDP policies, respectively. The circles in Fig. 5 and 6 indicate when the pedestrian was detected by the intensity filter. For both approaches, the immediate time step after detection commands a large deceleration. Because the baseline control is at full speed when the pedestrian appears, it is unable to legally yield to the pedestrian. In contrast, the POMDP policy has the vehicle decelerate much earlier and has it reach a lower maximum speed. Consequently, Trudi successfully stops for the suddenly appearing pedestrian.

The in-vehicle experiments give further insight into how well the speed control incorporates the values defined in the conceptualization phase. For example, although we intentionally designed for occupant comfort in the POMDP formulation, we only optimized for smoothness in velocity and did not account for the jerk vehicle occupants experience due to choppy acceleration commands. When just looking at the closed-loop policies, the value of smoothness seems to be achieved, but the in-vehicle experiments indicate it may not have been properly accounted for by this first iteration.

VI. CONCLUSIONS

In this first VSD iteration of an autonomous vehicle speed control algorithm, we demonstrate the formal connection of human values to a POMDP design through the conceptualization and technical implementation phases. The empirical analysis phase helps identify areas of improvement for the next iteration. In particular, this iteration limits braking authority for occupant comfort, which limits the ability of the vehicle to be safe, so the next iteration will increase the braking authority to the maximum capability of the vehicle. Additionally, occupant comfort was not successfully realized and the next iteration will keep track of the previous

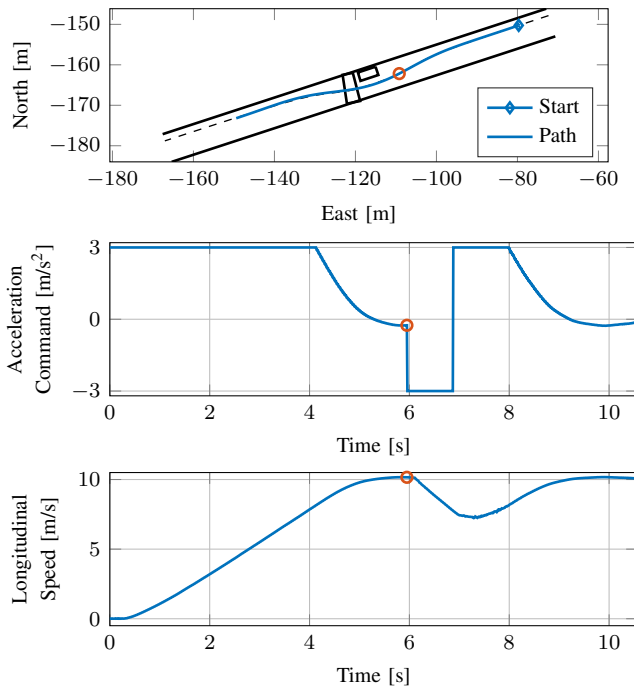


Fig. 5. Baseline trajectory overhead, acceleration command, and speed profile using deterministic speed control (circle indicates when the pedestrian was detected).

commanded acceleration to reduce longitudinal jerk. Furthermore, upon closer inspection of the California Vehicle Code, safety and legality are not strictly the same requirement. The traffic code says to exercise due care when approaching a crosswalk whereas safety may focus more on harm reduction or collision avoidance. This will be explored further in the next iteration. Overall, we found that using VSD changed the way we approach an engineering problem, and made us think deeply about each decision in the design process.

The focus here has been on engineers and programmers as designers, but VSD allows for other stakeholders to be involved in the design process, such as policymakers and civil servants. VSD is not only a valuable tool in the engineering room, but can widely encompass other contributors at a company or from third party groups to help ensure autonomous vehicles behave in socially acceptable ways.

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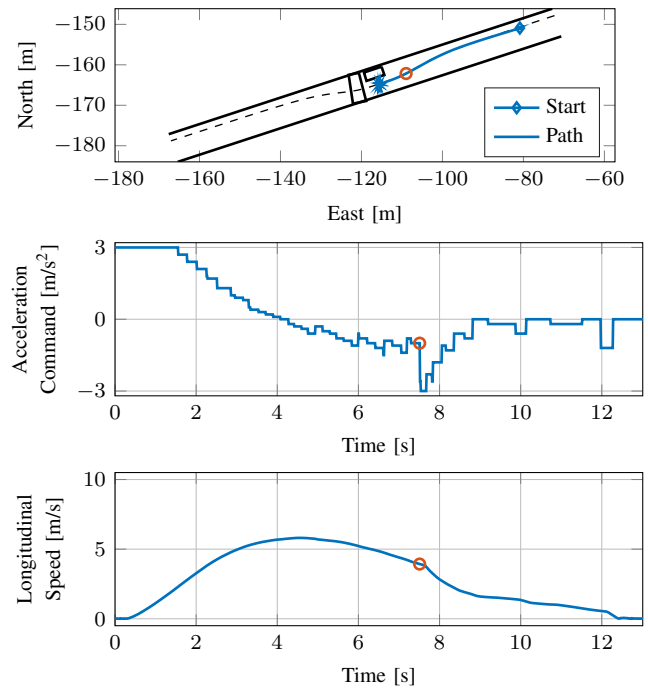


Fig. 6. POMDP trajectory overhead, acceleration command, and speed profile using belief about pedestrian detection (circle indicates when the pedestrian was detected).

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