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Dynamic Trajectory Planning for Vehicle Autonomous Driving

Sumin Zhang, Weiwen Deng*, Qingrong Zhao, Hao Sun, and Bakhtiar Litkouhi

Abstract—**Trajectory planning is one of the key and challenging tasks in autonomous driving. This paper proposes a novel method that dynamically plans trajectories, with the aim to achieve quick and safe reaction to the changing driving environment and optimal balance between vehicle performance and driving comfort. With the proposed method, such complex maneuvers can be decomposed into two sub-maneuvers, i.e., lane change and lane keeping, or their combinations, such that the trajectory planning is generalized and simplified, mainly based on lane change maneuvers. A two fold optimization-based method is proposed for stationary trajectory planning as well as dynamic trajectory planning in the presence of a dynamic traffic environment. Simulation is conducted to demonstrate the efficiency and effectiveness of the proposed method.**

Keywords- ***autonomous driving; dynamic trajectory planning; vehicle models***

I. INTRODUCTION

Vehicle trajectory is defined as a path, along with time stamps. It contains not only the geometric position, but also the velocity and acceleration information. Trajectory planning is one of the most important yet complex tasks in vehicle autonomous driving. A well-planned trajectory accounts for obstacles, keeps the vehicle in the intended path, and achieves smooth and efficient maneuvers for driving comfort and stability.

Overtaking, for example, is one of the most common yet challenging maneuvers in driving both manually and autonomously [1, 2]. For autonomous overtaking, one of the key issues is planning ahead an appropriate trajectory by taking into account, dynamically, the vehicle(s) in front and also other moving or stationary obstacles that may be intrusive or lead to potential collision during the course of the maneuver.

In recent years, intensive research on vehicle trajectory planning has been driven mainly by the increasing interest in autonomous driving. While various approaches have been proposed, many are still based on methods traditionally adopted in robotics. They are primarily global and stationary based with assumptions that the driving environment is given or known. However, the driving environment can be very dynamic. Lacaze and Coombs [3, 4] propose some search-based algorithms that rely on a pre-computed database stored with approximate trajectories, generated either based on geometric approaches or vehicle models [5-9]. While the geometric technique has been shown to be successful in trajectory planning, it considers little vehicle dynamics or vehicle/terrain interaction.

There are other approaches and algorithms proposed for vehicle trajectory planning, including artificial potential fields [10], the dynamic window approach [11], the velocity occupancy space (VOS) approach [12] and the elastic band approach [13]. Although some of these approaches are effective for some applications, they can be very computationally intensive, and thus may not be real-time feasible. Overtaking, for example, generally consists of two lane-changing and one car-following maneuver. The duration of a single lane change maneuver may be about 4-6 seconds [14]. Therefore the algorithms cannot be effectively applied based on a prior knowledge of the intended maneuver, to include the path from its beginning to the end, without taking into account potential collisions with dynamically emerging obstacles.

This paper proposes a method that dynamically plans the trajectory, not only to achieve quick reaction to the changing driving environment, but also to optimize the balance between vehicle performance and driving efficiency. An overall architecture is briefly introduced in Section II. The maneuver planning is presented in Section III, where maneuvers are first decomposed into two atomic sub-maneuvers, i.e., lane change and lane keeping, or their combinations. Thus the trajectory planning is generalized and simplified mainly based on lane change maneuvers. An optimization-based method for dynamic trajectory planning is proposed in Section IV, with which a steering angle command is generated by satisfying two often competing objectives of driving, i.e., comfort and efficiency. With this steering angle at a given vehicle speed, an analytical model-based approach is employed to derive, dynamically, a candidate trajectory. Similar modeling is also used to predict traffic vehicle movement. Simulations are conducted and presented in Section V followed by conclusions in Section VI.

II. ARICHTECTURE OF TRAJECTORY PLANNING

Trajectory planning is an integral part of and a key enabler for vehicle autonomous driving. Vehicle navigation in general can be divided into three different levels to carry out different tasks including route planning (RP) at macro level, maneuver planning (MP) at meso level and trajectory planning (TP) at micro level, as shown in Figure 1. The route planning defines an optimal route or journey between two geographical locations under a given road network, either based on the

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shortest distance or minimum expected travelling time, or other criteria. Many online mapping websites or commercial software tools offer road route planning functions. The route planning provides more strategic, global and stationary path information with time duration to be in minutes or hours. The route, however, does not typically consider time stamps or take into account roads and lanes, vehicle interactions with other parts of the traffic or obstacles, and vehicle behavior itself. The sensor information required for route planning is typically a GPS with digital map, and sometimes information about traffic conditions and road constructions, etc.

Maneuver planning instead generates a series of vehicle behavioral demands dynamically, with time scale often in seconds, to carry out part of the overall driving tasks defined by a planned route. A planned maneuver determines vehicle navigation behavior more tactically and locally in responding to surrounding roads and traffic, environmental conditions and driving scenarios. Some typical maneuvers are lane change, overtaking (or double lane change), or lane keeping, etc. The required sensor information is often from on-board sensors, such as radar and camera, with data fusion to perceive potential safety hazards, in addition to road, traffic signs and lights, weather conditions, etc.

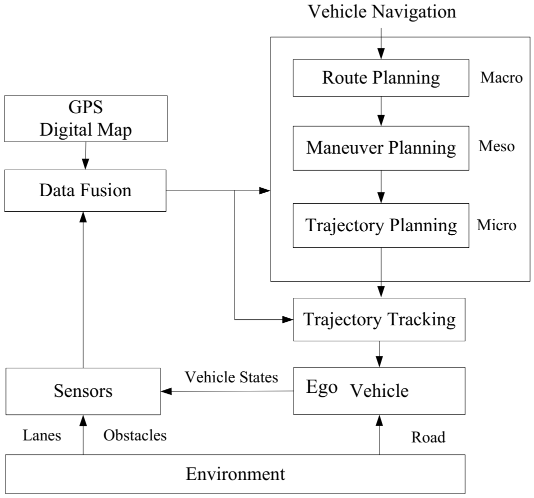

Figure 1: Proposed architecture of vehicle navigation

Finally, the trajectory planning at the micro level is to provide a dynamic path for a vehicle to follow with time scale often in milliseconds, which directly impacts vehicle dynamic performance, such as driving comfort and stability. The planned trajectory combines the geometric path information with time stamps to perform the driving tasks defined by a maneuver. It determines, dynamically, via trajectory tracking how much the vehicle is to be steered with or without braking and/or throttling. In addition to the sensor information for maneuver planning, some vehicle dynamic state measurements or estimations may be required.

This paper mainly deals with both maneuver and trajectory planning, assuming the route is defined by the Route Planner.

III. MANEUVER PLANNING

As defined earlier, a route represents a driving task from a geometric origin to a destination. A route consists of many individual or combined maneuvers, such as lane changes, intersection crossing, left or right turns, stops, lane keeping, etc. From a trajectory planning point of view, an individual maneuver can be further decomposed into one or more basic behavioral actions, such as Lane Change (left or right), or Lane Keeping. These actions can be defined to be atomic and are constituent to any maneuvers. For example, a left turn maneuver can be considered as a left lane change, while a stop maneuver can be interpreted as lane keeping. Further, an overtaking maneuver can be decomposed into a series of combined atomic actions, or sub-maneuvers, as (left) lane change, lane keeping and (right) lane change. Figure 2 illustrates an overtaking maneuver, which is composed of a (left) lane change, lane keeping and a (right) lane change, the combination of the two atomic sub-maneuvers.

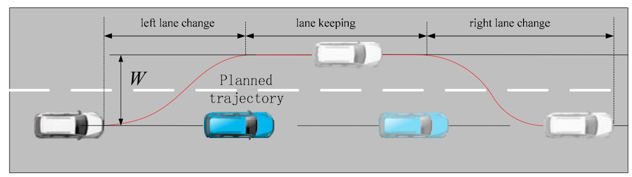

Figure 2: An overtaking maneuver consists of two atomic sub-maneuvers

. Figure 3 shows a typical and complete left lane change maneuver. Its trajectory can be mathematically expressed as a polynomial curve:

$$\begin{cases} x(t) = A_3t^3 + A_2t^2 + A_1t + A_0 \\ y(t) = B_3t^3 + B_2t^2 + B_1t + B_0 \end{cases} \tag{1}$$

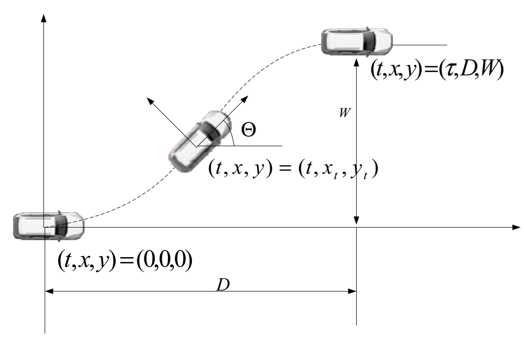

Figure 3: (Left) lane change as one of the atomic sub-maneuvers

Assume constant vehicle speed, the boundary conditions at both initial and final times can be listed below as:

$$x(0) = 0, x(\tau) = D, \dot{x}(0) = u, \dot{x}(\tau) = u$$
$$y(0) = 0, y(\tau) = W, \dot{y}(0) = 0, \dot{y}(\tau) = 0$$

where u is the vehicle speed.

Thus, the parameterized curve can be expressed as:

$$\begin{cases} x(t) = 2(u\tau - D)\left(\dfrac{t}{\tau}\right)^3 - 3(u\tau - D)\left(\dfrac{t}{\tau}\right)^2 + ut \\ y(t) = -2W\left(\dfrac{t}{\tau}\right)^3 + 3W\left(\dfrac{t}{\tau}\right)^2 \end{cases} \tag{2}$$

The planned trajectory on a maneuver, followed by trajectory tracking, greatly influences vehicle dynamic performance in terms of comfort, handling and stability, and driving efficiency. To quantify these, a cost function is introduced, which takes into account the magnitude of the vehicle acceleration and the time duration to complete the maneuver:

$$J = w_1 a^2 + w_2 \tau^2 \tag{3}$$

where w_1 and w_2 are the two weighting factors and a is the maximum total vehicle acceleration:

$$a = \max(\sqrt{\ddot{x}^2 + \ddot{y}^2})|_{t=0\sim\tau} \tag{4}$$

Based on (2)-(4), we aim to minimize the cost function:

$$\min J = \min\left[w_1 a^2 + w_2 (\frac{3W^3 a}{u^4} + 4.8\sqrt{3}\frac{3W^2}{u^2} + 2.4^2\frac{W}{a}) \right] \tag{5}$$

By solving the above, we can obtain the optimal values of both a^* and τ^*:

$$a^* = \left(\frac{5.76 w_2 W}{2 w_1}\right)^{1/3}$$
$$\tau^* = \sqrt{3}\frac{W^{3/2}}{u^2}\left(\frac{5.76 w_2 W}{2 w_1}\right)^{1/6} + 2.4\sqrt{W}\left(\frac{5.76 w_2 W}{2 w_1}\right)^{-1/6} \tag{6}$$

Further, the optimal value of the vehicle travel distance D can be obtained approximately:

$$D^* = 2.4u\frac{\sqrt{W}}{\sqrt{a^*}} \tag{7}$$

Since the lane width W is generally known, thus, the optimal trajectory under a given vehicle speed u can be derived to meet the optimal requirements on both vehicle performance and driving efficiency:

$$\begin{cases} x(t) = 2\sqrt{3}\dfrac{W^{3/2}\sqrt{a^*}}{u^2}\left(\dfrac{t}{\tau^*}\right)^3 - 3\sqrt{3}\dfrac{W^{3/2}\sqrt{a^*}}{u^2}\left(\dfrac{t}{\tau^*}\right)^2 + ut \\ y(t) = -2W\left(\dfrac{t}{\tau^*}\right)^3 + 3W\left(\dfrac{t}{\tau^*}\right)^2 \end{cases} \tag{8}$$

Figure 4 shows an example of optimal lane change maneuvers. With this method, by varying the two weighting factors w_1 and w_2, different trajectories can be generated to meet different requirements in terms of vehicle performance or turning efficiency. The trajectories generated by this method, however, do not take into account the changing driving environment, thus, are stationary in nature.

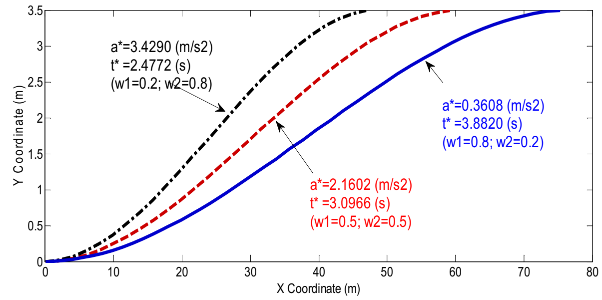

Figure 4: Optimal lane change maneuvers with different criteria

IV. DYNAMIC TRAJECTORY PLANNING

In every day driving and through driver actions, a vehicle trajectory is largely affected by the complex and dynamic driving environment, including the road, traffic and weather conditions. Similarly in automated vehicles, the trajectory has to be planned dynamically to cope with the instant changes of the driving scenarios.

During a lane change maneuver under a given vehicle speed, a rather large steering input would cause a large vehicle lateral acceleration, or in general more discomfort and instability. On the other hand, a smaller steering input may lead to longer time to complete the lane change, which is generally less desirable. This paper proposes a novel method for dynamic trajectory planning. First, an optimal steering angle is determined to satisfy the often competing objectives of vehicle performance and driving comfort.

This optimal steering angle is then used to generate a candidate trajectory, based on prediction via vehicle kinematic model. This trajectory is further checked to confirm that, by following it, the vehicle does not collide with any potential traffic obstacles. A rough estimation on the possible movement of the traffic obstacles (typically traffic vehicles) is conducted in both the space and the time.

A. Optimal Steering Angle

At any given time t during a lane change maneuver, the vehicle's position and orientation can be assumed known by taking information from GPS, digital map, or some other measurements, as shown in Figure 5. We set both the origins of the inertial frame and the vehicle coordinates on the vehicle's CG position, where the X is parallel to the target lane center line, and x points to its heading direction with angle θ.

As shown in Figure 5, the duration from the current position at any time t to the end of the maneuver is denoted by τ, the distance in x-direction is denoted by $D(t)$ and in y-direction by $W(t)$.

Given vehicle current states, i.e., speed and yaw rate, its movement (x_{pre}, y_{pre}, θ_{pre}) can be approximately predicted via a model-based estimation within a short time interval τ_1:

$$x_{pre} = u\tau_1 \cos\theta - R_s(1 - \cos(r_s\tau_1))\sin\theta \tag{9}$$

$$y_{pre} = u\tau_1 \sin\theta + \text{sgn}(\delta) R_s(1 - \cos(r_s\tau_1))\cos\theta \tag{10}$$

$$\theta_{pre} = \theta + r_s \tau_1 \quad (11)$$

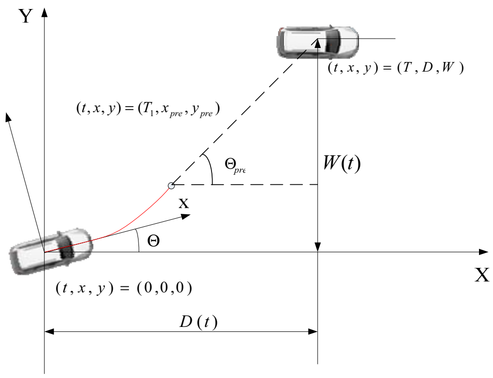

Figure 5: Dynamic trajectory planning for lane change maneuver

where θ is the current orientation of the vehicle and r_s is the steady-state yaw rate. We can then compute the vehicle traveling distance $D(t)$:

$$D(t) = \frac{W(t) - y_{pre}}{\tan(\theta + r_s \tau_1)} + x_{pre}$$
$$\approx \frac{W(t) - u\tau_1 \sin\theta - \dfrac{u\tau_1^2 \delta}{2L(1+Ku^2)}\cos\theta}{\theta + \dfrac{u\tau_1\delta}{L(1+Ku^2)} + \dfrac{\left(\theta + \dfrac{u\tau_1\delta}{L(1+Ku^2)}\right)^3}{3}} \quad (12)$$

Thus, the relationship between the traveling time period τ and the steering angle δ can be approximately derived as:

$$\tau(\delta) \approx \frac{\dfrac{W(t)}{u} - \tau_1 \sin\theta - \dfrac{u\tau_1^2 \delta}{2L(1+Ku^2)}\cos\theta}{\theta + \dfrac{u\tau_1\delta}{L(1+Ku^2)} + \dfrac{\left(\theta + \dfrac{u\tau_1\delta}{L(1+Ku^2)}\right)^3}{3}} \quad (13)$$

Also, the relationship between the steering angle δ and the lateral acceleration a_y is well known based on vehicle steady-state turn:

$$a_y(\delta) = \frac{u^2}{L}\delta \quad (14)$$

For driving comfort with good handling and stability, a smaller $a_y(\delta)$ is more desirable, which however may lead to undesirable longer time to complete the maneuver, and vice versa. Similarly from (3)-(5) in Section III, this becomes an optimization challenge. With proper weighting factors w_1 and w_2, the cost function to be minimized can be defined in a quadratic form as:

$$\min(J) = \min\left[w_1 a_y^2(\delta) + w_2 \tau^2(\delta)\right] \quad (15)$$

Taking the derivatives with respect to both a_y and τ, respectively, the optimal steering angle at any time t that satisfies both vehicle performance and driving efficiency can be obtained numerically.

B. Vehicle Trajectory Prediction

With this optimized steering angle input, an optimal vehicle trajectory can be obtained or predicted at a given vehicle speed. A kinematic vehicle model that has reasonable fidelity and computational efficiency within a short time interval is used. The trajectory generated can be a candidate of the planned trajectory, dynamically.

As shown in Figure 6, the vehicle steady state yaw rate is:

$$r_s = \frac{u}{L(1+Ku^2)} \cdot \delta \quad (16)$$

where $K = \frac{m}{L^2}\left(\frac{a}{K_{yr}} - \frac{b}{K_{yf}}\right)$, r_s is the steady-state yaw rate, L is the vehicle wheelbase, $L = a+b$, m is the vehicle mass, a and b are the distances from the center of gravity to the front and rear wheel axles, respectively; and K_{yf} and K_{yr} are the cornering stiffness coefficients of the front and rear wheel axles, respectively.

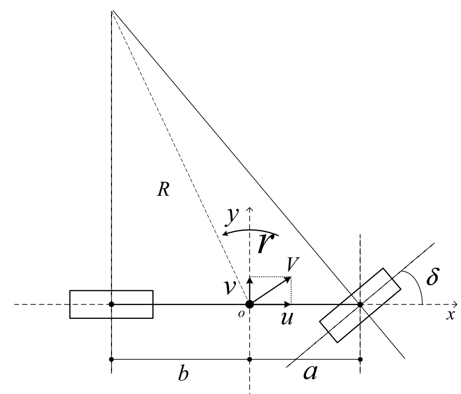

Figure 6: The steady-state vehicle model to predict vehicle trajectories

The maximum vehicle lateral acceleration $a_{y\max}$ is limited by the physic law of the tire-road contact,

$$\alpha_{y\max} = \mu g \quad (17)$$

where g is the gravity. Therefore, the maximum vehicle steady-state yaw rate is limited by:

$$\left|r_{s\max}\right| = \frac{\mu g}{u} \quad (18)$$

Under a steady-state turn, the cornering radius R_s is roughly,

$$R_s = \left|\frac{u}{r_s}\right| = \left|\frac{L(1+Ku^2)}{\delta}\right| \quad (19)$$

The new vehicle coordinates can be obtained by,

$$x(t) = \frac{L(1+Ku^2)}{\delta}\sin(\frac{u}{L(1+Ku^2)}\cdot\delta t) \quad (20)$$

$$y(t) = \frac{L(1+Ku^2)}{\delta}(1-\cos(\frac{u}{L(1+Ku^2)}\cdot\delta t)) \quad (21)$$

$$\theta(t) = r_s t \quad (22)$$

where $x(t)$ and $y(t)$ are the coordinates of vehicle center of gravity at time t. $\theta(t)$ is the vehicle heading orientation.

Based on steering structure, vehicle's steering angle is also limited by its geometry:

$$\delta_{\min} \le \delta \le \delta_{\max}, \dot{\delta}_{\min} \le \dot{\delta} \le \dot{\delta}_{\max} \quad (23)$$

where δ_{min} is the minimum steering angle and δ_{max} is the maximum steering angle.

Figure 7 shows that the proposed kinematic vehicle model is sufficiently accurate in comparison with CarSim® full dynamic vehicle model within a short time interval, thus, is well suitable to this application.

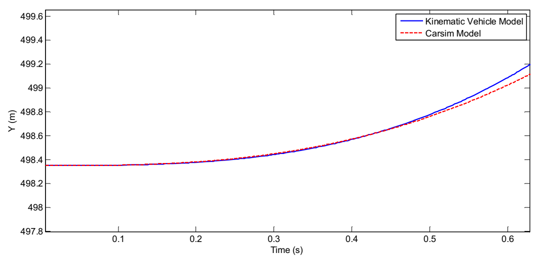

Figure 7: Comparison of the kinematic model and CarSim® model

C. Traffic Vehicle Prediction

Traffic conditions are one of the most influencing factors in vehicle trajectory planning. However, prediction of traffic movement is quite challenging, not only because it requires sufficient and accurate sensing information on the surrounding traffic vehicles, but also because their future movement is in general unknown or hard to predict.

Although it is difficult or even impossible to predict a traffic vehicle's exact future movement, one can estimate an envelope of its future movement in a set of possible trajectories within a short time (few seconds). Assuming a traffic vehicle's current speed and heading angle are known from the host vehicle's on-board sensors, such as radar and camera, its movement likely will form a cluster of trajectories, mainly depending on the possible changes of its future speed and steering angle. These changes, however, are subject to physics laws of tire-road contact and vehicle steering limitations.

The traffic vehicle's movement, for any vehicle, is governed by the following differential equation:

$$\dot{x} = f(x(t), u(t)) \quad (24)$$

where x is the state variables and u is the input, such as steering, braking and/or throttling, constrained by their maximum boundary set of U. Therefore, the possible vehicle movement within a short time interval τ can be expressed as:

$$R^e(r) = \left\{ \begin{array}{l} x(r) \mid x(r) = x(0) + \int_0^\tau f(x(\tau), u(\tau)d\tau, \\ x(0) \in X_0, u(\tau) \in U \end{array} \right\} \quad (25)$$

Figure 8 is an illustration of an envelope of a traffic vehicle motion with a cluster of trajectories. Although each of the possible trajectories carries a probability, generally speaking, keeping its current status is more likely than making a change.

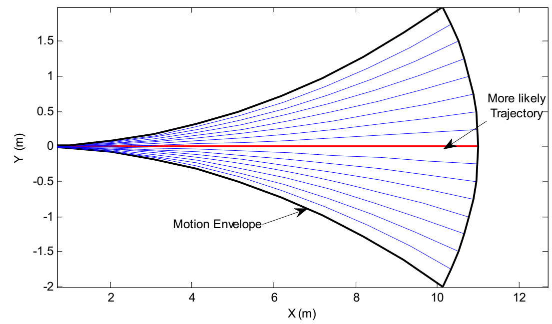

Figure 8: Illustration of traffic vehicle's possible movement

D. Dynamic Trajectory Planning

As discussed above, to ensure real-time computational feasibility while maintaining sufficient accuracy, an analytical model based approach, or more specifically, a closed-form kinematic vehicle model is adopted to generate candidate vehicle trajectories and an envelope of the traffic vehicle movement. Then the dynamic trajectory planning becomes a selection process in which the planned trajectory does not have overlap with the envelope of the traffic obstacles, or one or more trajectories within the envelope both in space and time.

The steering angle input to this model starts with the angle derived from the optimization between vehicle performance and driving efficiency. The generated trajectory is predicted and checked against the traffic vehicle trajectories to confirm a collision free path both in space and time. If this trajectory is predicted to have potential to collide with the traffic vehicles or obstacles, the steering angle is adjusted and used to generate another trajectory, which is further checked against the traffic vehicle trajectories. The process continues until a feasible trajectory is found or the maneuver is completed.

V. Simulation

As illustrated in Figure 9, the simulated vehicle (V1) travels at speed of 50km/h and conducts a left lane change maneuver. The simulated traffic vehicle (V2) moves straight in the target lane at the constant speed of 36km/h.

Figure 10 shows three trajectories, with the red one to be from the traffic vehicle, the blue one from the stationary trajectory planning (or static TP), and the green one generated from the dynamic trajectory planning (or dynamic TP).

Figure 11 shows the above trajectories with both the space and time grid information. From the simulation, the trajectory planned stationary will likely collide with the traffic obstacle at around time t= 11s, while the dynamically planned trajectory is able to avoid the potential collision with the traffic vehicle (V2).

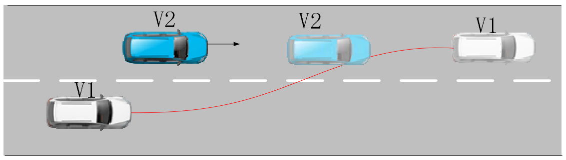

Figure 9: Illustration of trajectory planning with a traffic obstacle

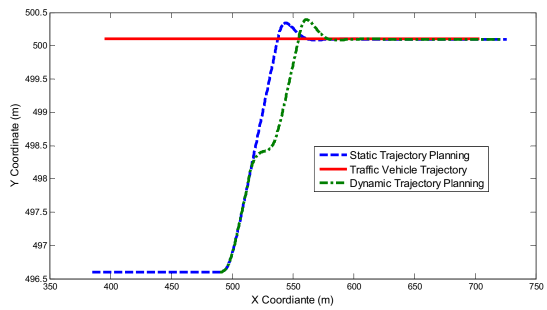

Figure 10: Comparison between dynamic and stationary trajectory planning

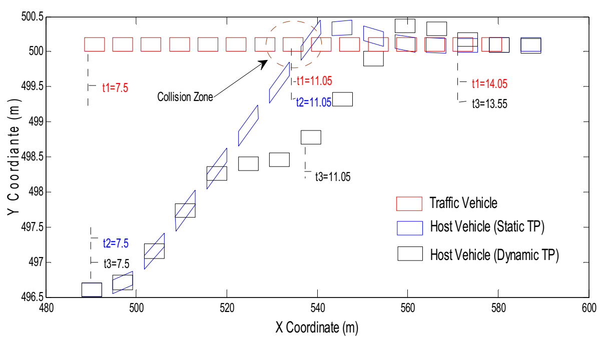

Figure 11: Trajectories with the space and the time grid information

VI. Conclusions

This paper presents a novel dynamic trajectory planning approach for vehicle autonomous driving, which is designed to accommodate complicated traffic scenarios and driving behaviors with the aim to achieve driving safety and comfort. By decomposing any complex maneuver into two sub-maneuvers, i.e., lane change and lane keeping, or their combinations, the proposed approach generalizes and simplifies a trajectory planning problem mainly based on lane change maneuvers. First, an optimal steering angle was determined to satisfy the competing objectives of vehicle performance and driving comfort. This optimal steering angle was then used to generate a candidate trajectory, based on prediction via vehicle kinematic model. This trajectory was followed while confirming that the vehicle would not collide with any potential traffic obstacles. A rough prediction of the possible movement of the traffic obstacles (typically traffic vehicles) was conducted in both the space and the time. Simulation results show the effectiveness of the approach in dynamically planning trajectories for vehicle autonomous driving. For future work, the proposed approach is to be further validated through a larger number of dynamic scenarios.

Acknowledgment

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