Modeling and Analysis of the Lane-Changing Execution in Longitudinal Direction

Da Yang, Liling Zhu, Bin Ran, Yun Pu, and Pan Hui

Abstract—**Lane changing is a common driving behavior in traffic, and the algorithm of lane-changing maneuver is an indispensable part of the design of autonomous vehicle control and adaptive cruise control. Although extensive research studies focused on modeling drivers' decision mechanism, lane-changing execution (LCE) happening after the lane-changing decision has not attracted much attention, which has a significant impact on driving safety and traffic simulation results. This paper attempts to replicate the real LCE behavior by proposing new LCE models. Depending on whether the lag vehicle on the target lane is considered in an LCE or not, two types of lane-changing execution are defined, namely, the cooperative LCE (CLCE) and the forced LCE (FLCE). The real vehicle trajectory data, i.e., the NGSIM data, are applied to train and test the proposed models. The results illustrate that the proposed models have good performance in replicating the CLCE and FLCE behavior and outperform the first LCE model proposed by Moridpour *et al*. Furthermore, when drivers decide to conduct a lane-changing execution, their considerations of the vehicles on the target lane have happened, and their considerations of the vehicles on the current lane decrease sharply during the LCE. In addition, the driver generally pays more attention to the preceding vehicle than to the lag vehicle on the target lane in an LCE.**

Index Terms—**Driver behavior, lane-changing execution, modeling, vehicle dynamics.**

I. Introduction

As ONE of the basic driving behaviors on highways, the lane-changing maneuver has a significant influence on traffic safety and characteristics of traffic flow [1]–[6] and has attracted much attention in the community of microscopic traffic flow [1]–[8]. The lane-changing model is also a key component of traffic simulation tools like VISSIM, CORSIM, AIMSUN, etc. A complete lane-changing process includes three parts, the lane-changing decision-making, lane-changing preparation and lane-changing execution. Each part is important and needs to be considered in lane-changing behavior study and traffic simulation tools. However, the majority of the existing studies related to lane-changing behavior, even the traffic simulation tools, only care about the decision part and take the lane-changing maneuver as an instantaneous event [7], [9], [10]. The lane-changing preparation and execution parts are generally neglected.

This paper focuses on the lane-changing execution (LCE) part of lane-changing behavior. The LCE refers to the process that a driver finishes the final lane-changing displacement from the current lane to the target lane. The beginning and end of a lane-changing execution are the times that the lateral movement of the subject vehicle starts and finishes [11]. The studies showed that a LCE took around 3–8 seconds [12], [13] and had a significant impact on traffic safety [11], traffic flow characteristics [11] and traffic simulation results [10]. More importantly, the lane-changing execution model is an indispensable part of control design for autonomous vehicles [14] and an important component of Adaptive Cruise Control (ACC) [15], [16].

In the existing ACC and autonomous vehicle control studies, during the entire LCE process, the lane-changing vehicle followed a predefined trajectory and adopted a constant acceleration [15], [16], and the surrounding vehicles were assumed to keep constant velocities [17]. However, in real traffic, the velocities of the surrounding vehicles change with time [10], [11], and drivers of lane-changing vehicles need to adjust their accelerations in real time according to the velocity and position variations of the surrounding vehicles. Compared to real LCE behavior, the existing ACC and autonomous vehicle algorithms are too simple and cannot react to the real-time change of traffic condition, so advanced and more human-like algorithms need to be developed. In recent years, more and more sensors are installed on autonomous vehicles, such as radar, camera, GPS and other sensors [18], and the connected vehicles technology, especially the vehicle-to-vehicle (V2V) technology, has obtained fast development [19]. That greatly improves the vehicles' capability of sensing the surrounding traffic conditions and makes it easy for the subject vehicle to gather the position, speed, distance and time gap of other vehicles. Therefore, it provides a possibility of applying the human-like algorithm to control vehicle movement.

Improving the existing control design of autonomous vehicles needs a well understanding of the real lane-changing execution behavior [14], [20]. Therefore, this paper attempts to investigate the characteristics of the real LCE and propose appropriate LCE models, which will provide a reference for algorithm design of future advanced ACC and autonomous vehicle control.

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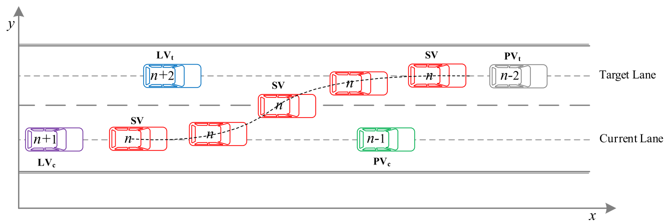

Fig. 1. Trajectory sketch of a lane-changing vehicle.

Despite the importance of LCE, few studies [10], [12], [13], [21] have focused on LCE, and only one LCE model was proposed by Moridpour *et al.* [11], which was also the first model to investigate the LCE behavior in the longitudinal direction. However, this model did not take into account the following aspects. First, the algorithm was based on the classical stimulus-response mechanism; however, the stimulus-response mechanism had been proven to be insufficient in reflecting real driver behavior [22]. Second, collision-avoid mechanism was not considered in the model [23], while safety is the most major concern of drivers in reality. Third, this model did not distinguish the difference between the two typical types of lane-changing mode, the cooperative lane-changing and forced lane-changing, which were observed widely in real traffic [24], [25]. Therefore, more sophisticated LCE models need to be developed.

Fig. 1 illustrates a trajectory sketch of a LCE vehicle. Different from the car-following trajectory, the trajectory of the LCE vehicle stretches to both the longitudinal direction and lateral direction. In this paper, we only study the longitudinal movement of the LCE, and the lateral movement problem is solved in another paper. From Fig. 1 we can observe that the four vehicles may be involved in a LCE process. They are the lane-changing vehicle n (the subject vehicle, SV), the preceding vehicle $n-1$ of SV on the current lane $(\mathrm{PV_c})$, the preceding vehicle $n-2$ on the target lane $(\mathrm{PV_t})$ and the lag vehicle $n+2$ on the target lane $(\mathrm{LV_t})$. The existing lane-changing studies [14], [26] do not consider the lag vehicle $n+1$ behind SV on the current lane $(\mathrm{LV_c})$, so in the subsequent analysis and modeling we only concern the three vehicles, $\mathrm{PV_c}$, $\mathrm{PV_t}$, and $\mathrm{LV_t}$.

In the rest of this paper, we first review the literature related to this study in Section II, including the lane-changing models, cooperative and forced lane-changing and lane-changing execution. Section III introduces the used data, the methods of data processing, and the empirical analysis of the two types of lane-changing execution, the cooperative lane-changing execution (CLCE) and forced lane-changing execution (FLCE). Section IV is the methodology part in which the models for the CLCE and FLCE are proposed based on a crash-avoiding algorithm, and the methods for model training and testing are presented. Section V is the results analysis, and the paper is concluded in Section VI.

II. Related Work

In this section, we will review the literature related to this study, on three aspects, lane-changing models, cooperative and forced lane-changing and lane-changing execution. The merits and shortcomings of the existing studies will be discussed.

A. Lane-Changing Models

According to Rahman *et al.* [14], the existing lane-changing models can be categorized into four groups: rule-based models, discrete-choice-based models, artificial intelligence models and incentive-based models.

The rule-based models assumed that drivers obeyed several rules to make their lane-changing decisions, such as the preferred target lanes, the necessity of changing lanes, the feasibility of changing lanes, etc. The representative model in this category was Gipps' lane-changing model [1]. Discrete-choice-based models adopted the classical Discrete Choice Model framework to capture the decision mechanism of drivers in lane-changing. In this category, three decisions need to be made, whether or not to make a lane change, target lane choice, and acceptance of a gap that is sufficient to execute the lane-changing. The advantage of this category of models was that they can reflect the driver heterogeneity. The first discrete-choice-based lane-changing model was proposed by Ahmed *et al.* [7] and further improved by Toledo *et al.* [4]. Artificial intelligence models applied some artificial intelligence algorithms, such as Fuzzy Logic [27] and Artificial neural network [28] to mine the potential factors influencing the drivers' lane-changing decision. In incentive-based models drivers chose to change or not to maximize their benefits. The representative model of this category was the one proposed by Kesting *et al.* [29].

However, all the above four categories of model only concerned drivers' lane-changing decision mechanism [1], [5], [14], while neglecting to investigate the lane-changing execution characteristics [26].

B. Cooperative and Forced Lane-Changing

Cooperative and forced lane-changing behavior had been widely observed both on freeways [24], [30] and arterials [9], [25], [31], which were found to have a significant impact on macroscopic characteristics of highway traffic flow [32]. Due to their importance, they had been embedded into several traffic simulation tools [5], [33].

The difference between cooperative and forced lane-changing depends on how the lag vehicle reacts to the lane-changing request of the SV [2]. If the $\mathrm{LV_t}$ chooses to yield the SV in courtesy and is followed by a deceleration maneuver, and then it is recognized as a cooperative lane-changing. In a cooperative lane-changing, the $\mathrm{LV_t}$ will provide an acceptable gap of sufficient length to allow the SV to conduct lane-changing execution [25], [31], [32]. In contrast, if the $\mathrm{LV_t}$ chooses not to slow down or even accelerate to prevent the lane-changing execution, it will be a forced lane-changing. In a forced lane-changing, the acceptable gap for further lane-changing execution is not provided by the lag vehicle voluntarily but created by the SV itself forcedly [25].

Sun and Elefteriadou [25], [32] presented a method to discriminate the two types of lane-changing according to the variation characteristic of the spacing gap between the SV and $\mathrm{LV_t}$. If the gap was increasing before the execution point, it would be a cooperative lane-changing. Otherwise, if the gap was either keeping constant or narrowing before the execution point and started to widen after execution, it revealed that the SV had forced the LV_t to slow down.

Similarly to the lane-changing models, the existing cooperative and forced lane-changing studies have only focused on the lane-changing decision but ignored the lane-changing execution part.

C. Lane-Changing Execution Models

So far, only a few studies concerning LCE have been conducted. The early studies mainly focused on statistical analysis and modeling the LCE duration [10], [12], [13], [21], and the results displayed that the LCE duration dropped in the range of [3 s, 8 s] [12], [13].

The research mostly related to this study was the one conducted by Moridpour *et al.* [11]. It was the first model to investigate the LCE behavior in the longitudinal direction, which was based on the stimulus-response mechanism. They fit an acceleration model and a deceleration model to real data and use the two models to examine the acceleration and deceleration characteristics of LCE behavior. However, they only used these two models to analyze the characteristics of LCE behavior but did not give an explanation how to use the models to simulate the LCE behavior. The formulations of the Moridpour model for cars are presented as follows

$$a_n(t+\tau) = \lambda_0 v_n(t+\tau)^{\lambda_1} \frac{(v_{n+1}(t) - v_n(t))^{\lambda_2}}{(x_{n-1}(t) - x_n(t))^{\lambda_3}} \tag{1}$$

$$d_n(t+\tau) = \lambda_0 v_n(t+\tau)^{\lambda_1} \frac{(v_{n+2}(t) - v_n(t))^{\lambda_2}}{(x_{n-1}(t) - x_n(t))^{\lambda_3}} \tag{2}$$

where, $v_n(t)$ and $x_n(t)$ are the velocity and position of the vehicle n at time t respectively, $v_{n-1}(t)$ and $x_{n-1}(t)$ are the velocity and position of the vehicle $n-1$ at time t respectively, $a_n(t+\tau)$, $d_n(t+\tau)$ and $v_n(t+\tau)$ are the acceleration, deceleration and velocity of the vehicle n at time $t+\tau$ respectively, and $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ are parameters need to be calibrated.

From Equations (1) and (2), we can find the Moridpour model did not reflect the difference between CLCE and FLCE. In addition, if the velocity difference in the numerator of the models is 0, the output of the models, acceleration or deceleration, will be 0 as well, no matter what the denominator (the distance gap) is, which is not consistent with reality.

III. Data Description and Analysis

In this section, the used real traffic data, NGSIM (Next Generation Simulation) data, will be introduced. The methods adopted to extract and process the original data is then presented. Based on the extracted data, the two types of typical LCE, the CLCE and FLCE, will be explored empirically.

A. Data Source

The well-known real vehicle trajectory data, NGSIM trajectory data [34], is adopted to explore the characteristics of lane-changing execution in the longitudinal direction and further calibrate and validate the proposed models. NGSIM project used videos to capture real traffic information, such as location, speed, vehicle type, lane, etc. The vehicle trajectory data has a temporal resolution of 0.1 s and includes both the longitudinal and lateral position information of vehicles, which makes it feasible to study LCE. More description of the data see the reference [35].

B. Data Extraction and Process

In this study, we only focus on the behavior of passenger cars, so heavy trucks and motorcycles are excluded from the data. The lane-changing execution data supporting this study should include the starting time and end time of a LCE, position, and velocity of vehicles involved in a LCE process. The data extracting and processing steps are as follows,

1) Recognize the IDs of the lane-changing vehicles.
2) Search the lane-changing time t_{lc} from NGSIM data, and extract the vehicle position information in $T \in [t_{\mathrm{lc}} - 5\text{ s}, t_{\mathrm{lc}} + 5\text{ s}]$. After several trials, we find the interval 5 s before and after t_{lc} is an appropriate period to ensure the entire lane-changing process is covered.
3) Find the starting point and end point of each LCE. By observing the trajectories of lane-changing vehicles in the lateral direction, we can find the trajectories in the LCE are obviously different from the trajectories before and after LCE. The slopes of trajectories before and after LCE are close to 0, while in the LCE they are less than 0 changing from a left lane to a right lane and more than 0 changing from a right lane to a left lane. This characteristic is shown in Fig. 2. We use this feature to determine the starting point and end point of the LCE.
4) Extract positions, velocities and accelerations of all the vehicles involved in the LCE process.

C. Empirical Analysis

The classification of the two types of lane-changing, the cooperative and forced lane-changing, is also applicable to lane-changing execution, that is, CLCE and FLCE. Thus, similarly, we give the definitions of the CLCE and FLCE respectively. The cooperative lane-changing execution refers to the case that the LV_t chooses to yield the SV in courtesy and actively provides an acceptable gap for SV to conduct LCE. In contrast, the forced lane-changing execution refers to the case that the LV_t accelerates to prevent the SV's lane-changing execution, and the acceptable gap is created by the SV forcedly in the lane-changing execution process.

The difference between CLCE and FLCE will be reflected in modeling. In a CLCE, the driver of SV does not need to consider the LV_t due to the cooperation behavior of the driver of LV_t. However, in a FLCE, LCE drivers have to take into account the LV_t due to the prevention behavior of LV_t.

The criterion of discriminating the CLCE and FLCE is slightly different from Sun and Elefteriadou's method [25], [32] used to discriminate cooperative and forced lane-changing. We use time gap as the criterion. If the time gap between the SV and LV_t keeps increasing until some point in the LCE or the end of

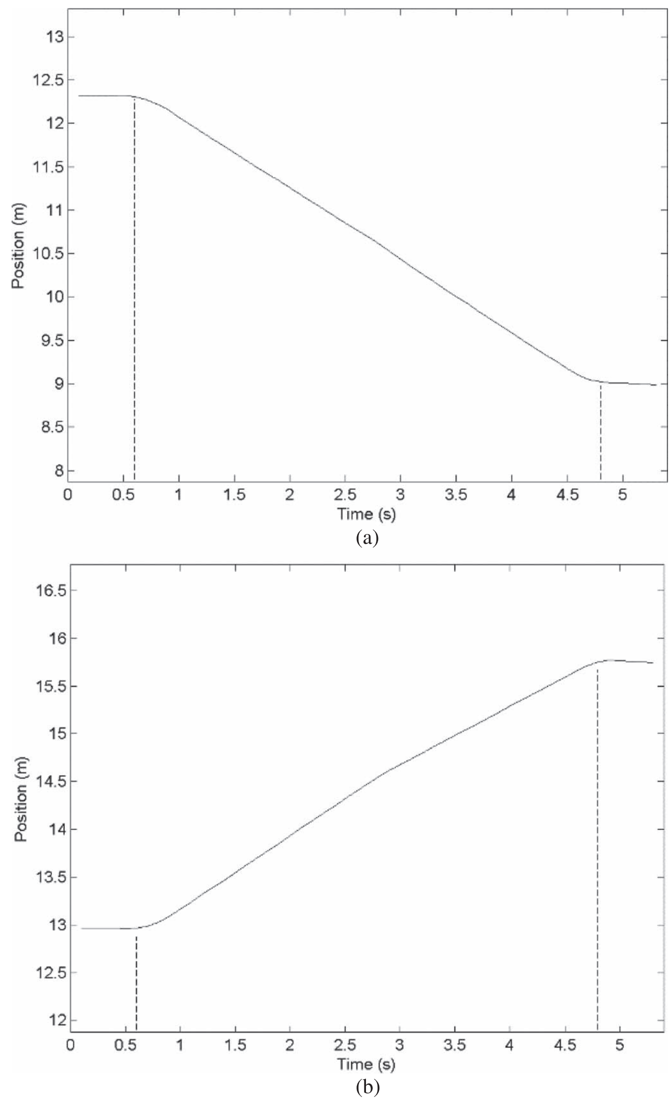

Fig. 2. Trajectories of vehicles in LCE in the lateral direction. (a) From a left lane to a right lane. (b) From a right lane to a left lane.

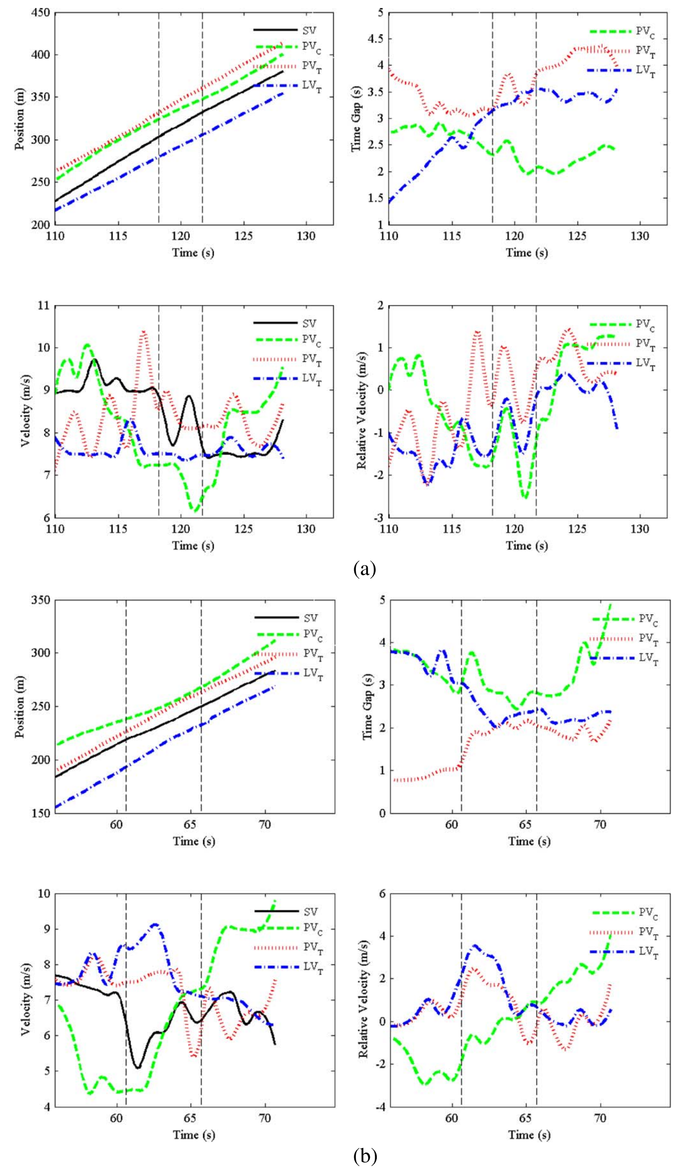

Fig. 3. Examples of the two types of LCE. (a) A case of the cooperative LCE. (b) A case of the forced LCE.

the LCE, it is recognized as a CLCE. Otherwise, if the time gap keeps constant or narrows before some point in the LCE and widens after that point, it is a FLCE. It should be noted that time gap is just one indication of CLCE and FLCE, as the vehicle speed or acceleration are the results of many factors, such as the initial conditions (e.g. gap size), less predictable psychological factors, etc.

According to the above criterion, 452 CLCEs and 321 FLCEs are obtained respectively from the NGSIM data, and 13 lane-changings are neither CLCE nor FLCE, which may need further investigation in future.

Fig. 3(a) displays an example of CLCE. The time between the two black dashed lines is the period of the LCE process, and the duration for this example is 5.9 s. From Fig. 3(a), we can observe that the time gap between the LV_t and SV keeps increasing before the end of LCE, and the LV_t always keep a lower velocity than the SV in the entire LCE, although the SV is decelerating. This indicates the LV_t is avoiding the SV, which gives the SV a relaxed situation to finish the LCE.

Fig. 3(b) is an example of FLCE. In Fig. 3(b), we can observe before the SV begins to conduct LCE, the time gap between LV_t and SV reduces until time $= 63$ s, and after that begins to increase. The velocity of LV_t is always more than the velocity of SV, and under the pressure from LV_t the SV chooses to accelerate. The actions of SV and LV_t reveal that the LV_t chooses to accelerate rather than to yield the SV at first, and when the LV_t finds it is impossible to prevent this LCE, the LV_t chooses to decelerate to keep a safe distance.

IV. Methodology

In this section, we first present the Gipps' general crash-avoiding algorithm and then apply it in the situations of avoiding crashes with the preceding and lag vehicles on the target lane in LCE. The models for the two types of LCE, the CLCE and FLCE, are then proposed. Following that, the methods of model training and testing are introduced, which will be used to find the optimal reaction time and parameters.

A. Crash-Avoiding Algorithm

We adopt the safe driving algorithm introduced by Gipps [36] and apply it to model the LCE behavior. Gipps' model assumes drivers tend to choose a safe velocity to avoid a rear crash. It has been applied widely in microscopic simulation tools, such as AIMSUN, DRACULA and SITRAS [37]. In the paper, we assume the drivers of the lane-changing vehicles tend to pursue a safe driving state in the LCE process, namely, the drivers would like to keep safe distances with the surrounding vehicles. If the preceding vehicle brakes emergently, the following vehicle will brake emergently too after a reaction time and choose a new velocity that can make the two vehicles stop without rear-crash.

According to Gipps [36], to avoid a crash with $\mathrm{PV_c}$, the velocity of the SV obeys the following equation:

$$v_n^{\mathrm{PV_c}}(t+\tau) = b_n\tau + \sqrt{b_n^2\tau^2 - b_n\begin{bmatrix}2\left(x_{n-1}(t) - l_{n-1} - x_n(t)\right)\\ -v_n(t)\tau - v_{n-1}(t)^2/b_{n-1}\end{bmatrix}} \quad (3)$$

where, $v_n^{\mathrm{PV_c}}(t+\tau)$ denotes the velocity of SV to avoid a crash with the vehicle $\mathrm{PV_c}$, l_{n-1} denotes the length of vehicle $n-1$, and b_n and b_{n-1} denote the maximum decelerations of the vehicle n and $n-1$ respectively.

Similarly, to avoid a crash with $\mathrm{PV_t}$, the velocity of the SV has to obey the following equation:

$$v_n^{\mathrm{PV_t}}(t+\tau) = b_n\tau + \sqrt{b_n^2\tau^2 - b_n\begin{bmatrix}2\left(x_{n-2}(t) - x_n(t) - l_{n-2}\right)\\ -v_n(t)\tau - v_{n-1}(t)^2/b_{n-2}\end{bmatrix}} \quad (4)$$

where, $v_n^{\mathrm{PV_t}}(t+\tau)$ denotes the velocity of the SV to avoid a crash with the vehicle $\mathrm{PV_t}$, $x_{n-2}(t)$ denotes the position of the vehicle $n-2$ at time t, and l_{n-2} and b_{n-2} denote the length and maximum deceleration of vehicle $n-2$ respectively.

Meanwhile, the SV may consider avoiding a crash with the lag vehicle on the target lane in the FLCE. To avoid a crash with $\mathrm{LV_t}$, Equation (3) cannot be applied directly as the cases of the vehicle $\mathrm{PV_c}$ and $\mathrm{PV_t}$. In this case, the stopping position of vehicle n in avoiding a crash with $\mathrm{LV_t}$ has the following function:

$$x_n^{\mathrm{stop}} = x_n(t) + 0.5\tau\left[v_n(t) + v_n(t+\tau)\right] - v_n(t+\tau)^2/2b_n \quad (5)$$

where, the superscript stop denotes the stopping state of the vehicle.

Similarly, the stopping position of vehicle $n+2$ has the following function:

$$x_{n+2}^{\mathrm{stop}} = x_{n+2}(t) + v_{n+2}(t)\tau - v_{n+2}(t)^2/2b_{n+2} \quad (6)$$

where, the superscript stop denotes the stopping state of the vehicle.

Therefore, we can obtain the safe velocity function for SV to avoid a crash with the vehicle $\mathrm{LV_t}$ as follows:

$$v_n^{\mathrm{LV_t}}(t+\tau) = 0.5\tau b_n + \sqrt{(0.5\tau b_n)^2 + b_n\begin{bmatrix}2\left(x_n(t) - x_{n+2}(t) - l_n\right) + \tau v_n(t)\\ -2v_{n+2}(t)\tau + v_{n+2}(t)^2/b_{n+2}\end{bmatrix}} \quad (7)$$

where, $v_n^{\mathrm{LV_t}}(t+\tau)$ denotes the velocity of the SV to avoid a crash with the vehicle $\mathrm{LV_t}$, $x_{n+2}(t)$ denotes the position of the vehicle $n+2$ at time t, and l_{n+2} and b_{n+2} denote the length and maximum deceleration of vehicle $n+2$ respectively.

B. Model for the Cooperative Lane-Changing Execution

In a LCE, the driver of a SV needs to consider the vehicles on the current lane and the target lane at the same time. Thus, the final velocity of the SV includes two parts; the first part is determined by the vehicles on the current lane, and the second part is determined by the vehicles on the target lane. We combine the two parts together using a linear form, and derive the final velocity of vehicle n after a reaction time τ as follows:

$$v_n(t+\tau) = \alpha v_c(1-\alpha)v_t \quad (8)$$

where, v_c denotes the safe velocity part determined by the vehicles on current lane, v_t denotes the safe velocity part determined by the vehicles on target lane, and α denotes the contribution of the vehicles on the current lane in deciding the final velocity and $\alpha \in [0, 1]$.

In the CLCE, only the two vehicles, $\mathrm{PV_c}$ and $\mathrm{PV_t}$, have an impact on the behavior of the SV. Thus, there is

$$v_c = v_n^{\mathrm{PV_c}}(t+\tau), \text{ and } v_t = v_n^{\mathrm{PV_t}}(t+\tau). \quad (9)$$

Hence, the velocity of the SV in CLCE can be formulated as the linear combination of $v_n^{\mathrm{PV_c}}(t+\tau)$ and $v_n^{\mathrm{PV_t}}(t+\tau)$, that is

$$v_n(t+\tau) = \alpha v_n^{\mathrm{PV_c}}(t+\tau) + (1-\alpha)v_n^{\mathrm{PV_t}}(t+\tau). \quad (10)$$

When the SV moves from the current lane to target lane gradually, the driver's consideration of the vehicles on the current lane decreases and the consideration of the vehicles on the target lane increases. That is to say, in a LCE, the contribution α of $\mathrm{PV_c}$ in deciding the final velocity of SV decreases and the contribution of $\mathrm{PV_t}$ increases. Therefore, in Equation (10) α is not a constant but a monotonically decreasing function of the lateral distance from the current to the end position of the LCE. To find a better function for α, we propose the following three commonly used formulations, and the one with the best performance in fitting real data will be adopted

$$\alpha = k \cdot p_n(t) \quad (11)$$

$$\alpha = \tan\left(p_n(t)\right)/k \quad (12)$$

$$\alpha = \tanh\left(p_n(t)\right)/k \quad (13)$$

in which, k is a parameter that needs to be calibrated together with other model parameters, and $p_n(t)$ has the following expression:

$$p_n(t) = y_n(t)/Y \tag{14}$$

where, $y_n(t)$ denotes the lateral position of the vehicle n at time t, namely, the distance from current lateral position to the end position of the LCE where $y = 0$, and Y is the lateral distance between the starting and end position of LCE, which is close to the width of a lane.

C. Model for the Forced Lane-Changing Execution

In contrast, in the forced LCE, the lag vehicle $n+2$ on the target lane also impacts the LCE behavior of the vehicle n, so in modeling FLCE the three vehicles, $\mathrm{PV_c}$, $\mathrm{PV_t}$, and $\mathrm{LV_t}$, should be taken into account. In this case, the v_t in Equation (8) are determined by the two vehicles, the $\mathrm{PV_t}$ and $\mathrm{LV_t}$, that is

$$v_c = \beta v_n^{\mathrm{PV_t}}(t+\tau) + (1-\beta) v_n^{\mathrm{LV_t}}(t+\tau) \tag{15}$$

in which, β denotes the contribution of $\mathrm{PV_t}$ in deciding the velocity v_c, and it is a constant.

Therefore, the final velocity of the SV, $v_n(t+\tau)$, is the combination of the three velocities, $v_n^{\mathrm{PV_c}}(t+\tau)$, $v_n^{\mathrm{PV_t}}(t+\tau)$ and $v_n^{\mathrm{LV_t}}(t+\tau)$, that is,

$$\begin{aligned} v_n(t+\tau) &= \alpha v_n^{\mathrm{PV_c}}(t+\tau) \\ &\quad + (1-\alpha)\left[\beta v_n^{\mathrm{PV_t}}(t+\tau) + (1-\beta) v_n^{\mathrm{LV_t}}(t+\tau)\right] \end{aligned} \tag{16}$$

where, α has the same meaning with the one in the CLCE model.

D. Model Training and Testing

To ensure that the model calibration and validation can be performed effectively, we adopt the 10-fold cross-validation method [38]. The obtained dataset is randomly split into 10 folders, and then the model is tested using 9 folders of them and trained using the remaining 1 folder. This process is repeated 10 times, in which all the 10 folders are used once as the validation data. Finally, the results of the 10 times are averaged to represent the final model performance. In this method all observations are used for both training and validation.

The model training procedure can be summarized as a numerical optimization problem. The objective function is a function of the estimation error with respect to field data. The constraints are physical boundaries of the parameters. Table I illustrates the feasible ranges of the parameters. Since the value of p_n varies in [0, 1], to ensure α is in [0, 1] too, k should not be a big number. Therefore, to save the training and testing time, the feasible range of k is narrowed down to 0$\sim$10 using trail-error method. The crash-avoiding algorithm in the proposed models has no restriction on the values of acceleration, which may produce unphysical acceleration values, so in model training and testing, we give the acceleration a range of $-3.5 \sim 3.5\ \mathrm{m/s^2}$ [39].

TABLE I
FEASIBLE RANGES OF MODEL PARAMETERS

Model Parameters	b_n	b_{n-1}	b_{n-2}	b_{n+2}	k	β
Feasible Range	-3.5~0	-3.5~0	-3.5~0	-3.5~0	0~10	0~1

In this study, Genetic Algorithm (GA) is adopted to solve the nonlinear programming problem. The GA toolbox in Matlab is applied, which has been widely used in the community of driver behavior model calibration [40]. In searching for the optimal parameters, to guarantee α is in the range of [0, 1], the generated k making α be out of [0, 1] will be excluded from the genetic set. For each LCE model, the algorithm runs ten times with different random seeds, so that optimal parameter settings can be achieved. The objective function adopts Theil' U function as follows:

$$g = \frac{\sqrt{\frac{1}{M}\sum_{i=1}^{M}(v_i - \widehat{v}_i)^2}}{\left(\sqrt{\frac{1}{M}\sum_{i=1}^{M}(v_i)^2} + \sqrt{\frac{1}{M}\sum_{i=1}^{M}(\widehat{v}_i)^2}\right)} \tag{17}$$

where, g denotes the objective function, $i = 1, \ldots, M$ denotes the ith LCE sample, M is the total number of the samples, and $\widehat{v}_i$ and v_i respectively denote the simulated velocity based on the proposed model and the actual velocity from the field data.

To test the performance of the proposed models, we adopt the following indexes, Mean Absolute Error (MAE), Mean Absolute Relative Error (MARE) and Coefficient of Determination (R^2). Their formulations are as follows:

$$\mathrm{MAE} = \frac{\sum_{i=1}^{M}|e_i - \hat{e}_i|}{M} \tag{18}$$

$$\mathrm{MARE} = \frac{\sum_{i=1}^{M}\left(\frac{|e_i - \hat{e}_i|}{e_i}\right)}{M} \tag{19}$$

$$R^2 = \frac{\sum_{i=1}^{M}(\hat{e}_i - \bar{e})^2}{\sum_{i=1}^{M}(e_i - \bar{e})^2} \tag{20}$$

where, e_i and $\hat{e}_i$ denote the real and simulation values of the LCE sample i respectively, and $\bar{e}$ denotes the average of all the real LCE values. e has three choices, the acceleration, velocity and position, respectively.

V. RESULTS ANALYSIS

In this section, drivers' optimal reaction time in LCE is found first by examining all the feasible values, and then the training and testing results corresponding to the optimal reaction time are analyzed. In the end, the best function for the influence parameter α is analyzed.

A. Reaction Time Analysis

Before searching for the optimal model parameters, the optimal reaction time should be determined first. We attempt to find the optimal reaction times for both the CLCE and FLCE by evaluating the two models for different reaction time values ranged in 0.5$\sim$2.0 s. The evaluation index adopts the MAE values of the position. Fig. 4 illustrates the relationship curves

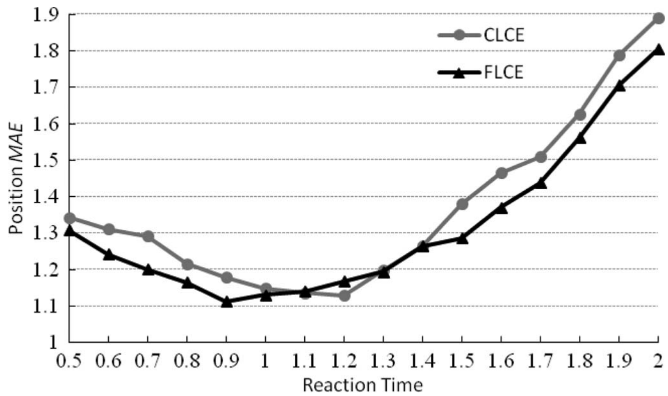

Fig. 4. Position MAE of the CLCE and FLCE for different reaction times.

TABLE II
MODEL TRAINING RESULTS

Model Parameters	b_n	b_{n-1}	b_{n-2}	b_{n+2}	k	β
CLCE	- 0.98	- 0.87	- 0.93	-	1.56	-
FLCE	- 0.73	- 0.68	- 0.71	- 1.23	2.43	0.85

of position MAE and reaction time for both the CLCE and FLCE, from which we can find that the smallest MAE values for CLCE and FLCE are corresponding to the two reaction time values, 1.2 s and 0.9 s. The difference of the optimal reaction times for CLCE and FLCE agrees with the real condition. In real traffic, drivers in CLCE and FLCE have different attention levels. The drivers in FLCE have to pay more attention to the behavior change of the surrounding vehicles, especially the behavior of the LV_t, due to the pressure from it. In contrast, the drivers in CLCE are more relaxed to perform a LCE.

B. Model Training and Testing Results

After training using the extracted data, we obtain the best parameter values for CLCE and FLCE in Table II. These results are corresponding to the best reaction time and best function for α. The value of parameter β in FLCE is 0.85, which indicates that the driver in forced lane-changing execution considers the PV_t more than the LV_t. The training and testing errors for the CLCE and FLCE models are shown in Table III. All the error indexes for the training and testing results are in reasonable ranges, which means the proposed models can replicate the real CLCE and FLCE behaviors on an acceptable level. Furthermore, we compare the performance of the proposed models with Moridpour model [11] that did not discriminate CLCE and FLCE. From Table III, we can observe that the proposed models outperform Moridpour model too, which proves the necessity of splitting the LCE behavior into CLCE and FLCE in studying lane-changing execution.

C. Influencing Parameter Analysis

In Section IV-B, we give three options for the formulation of α that is a function of the lateral position of the SV. All of the three Equations (11)–(13) are calibrated together with the proposed models. The testing results show that Equation (12) has the best performance in fitting the real data. That is to say,

TABLE III
TRAINING AND TESTING ERROR

			MAE	*MARE*	R^2
CLCE Model	Training	*v*	0.634	0.031	0.932
		a	0.521	0.043	0.878
		p	1.002	0.004	0.931
	Testing	*v*	0.694	0.035	0.927
		a	0.580	0.049	0.875
		p	1.129	0.004	0.923
FLCE Model	Training	*v*	0.613	0.031	0.910
		a	0.618	0.049	0.897
		p	1.032	0.005	0.942
	Testing	v	0.679	0.036	0.898
		a	0.695	0.059	0.894
		p	1.131	0.005	0.937
Moridpour Model	Training	*v*	0.812	0.061	0.804
		a	0.792	0.068	0.731
		p	1.374	0.006	0.895
	Testing	*v*	0.825	0.064	0.782
		a	0.813	0.071	0.689
		p	1.485	0.006	0.864

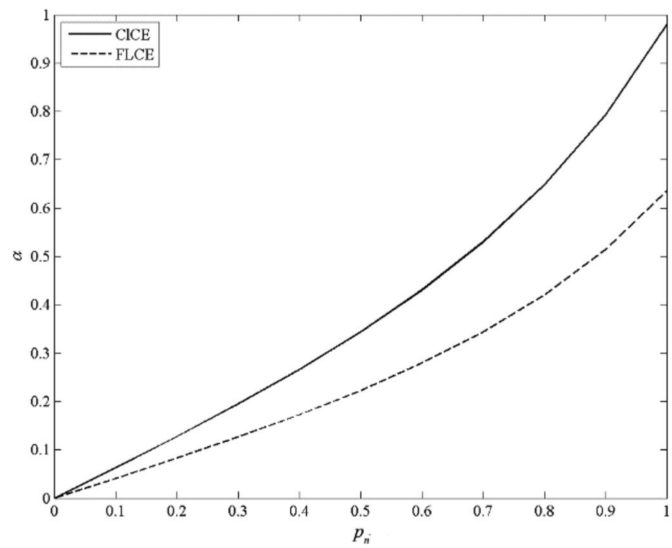

Fig. 5. Relationship of α and p_n for CLCE and FLCE.

the relationship accords to a "*Tangent*" curve, as shown in Fig. 5. From Fig. 5, we can observe that, at the beginning of a LCE, $p_n = 1$, the values of α for both CLCE and FLCE do not start from 1 but from 0.97 and 0.64, which indicates that when drivers decide to conduct a LCE, their considerations of the vehicles on the target lane has already happened. When the SV moves from the current lane to the target lane gradually, drivers' considerations of the vehicle on the current lane decrease sharply, and meanwhile their considerations of the vehicles on the target lane increase sharply. When the SV is on the lane line between the current and target lanes, that is $p_n = 0.5$, the consideration of PV_c has decreased to 0.3 for CLCE and under 0.3 for FLCE due to the pressure from the lag vehicle on the target lane. These results are also consistent with reality.

VI. CONCLUSION AND FUTURE WORK

The lane-changing execution has not attracted so much attention, although it has a significant impact on traffic safety and traffic simulation results and is also an indispensable part of the design of driving mechanism for autonomous driving and ACC vehicles. This paper attempts to replicate the real longitudinal movement of the vehicles in the LCE, which will provide a reference for future advanced LCE control algorithm on the autonomous driving and ACC vehicle. The following conclusions are drawn:

1) Two types of LCE are observed in real traffic, the cooperative LCE (CLCE) and forced LCE (FLCE). In a CLCE, the lag vehicle on the target lane provides an acceptable gap for the SV in courtesy before the LCE starts. In contrast, if the lag vehicle attempts to prevent the LCE of SV or does not choose to avoid, it is a FLCE.
2) The training and testing results of the proposed models show that they have good performance in replicating the LCE behavior and better performance than the first LCE model proposed by Moridpour *et al.*
3) When the lane-changing driver decides to conduct a lane-changing execution, their considerations of the vehicles on the target lane have happened.
4) When the SV moves from the current lane to the target lane gradually, the driver's consideration of the vehicle on the current lane decreases sharply, in contrast to the huge increase in the consideration of the vehicles on the target lane.
5) In FLCE, drivers pay more attention to the $\mathrm{PV_t}$ than $\mathrm{LV_t}$.

In future, we will further refine the proposed models to be applicable on autonomous and ACC vehicles. Meanwhile, we will test the proposed models on real vehicles in fields and evaluate them using traffic simulation tools, such as DRACSIM or AIMSUN.

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