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Part A: Defining concepts

1. Provide a brief description of each of the following biological concepts.
    a. **Eukaryotic cell** – A domain of organisms whose cells contains a defined nucleus, in additions to a variety of cellular organelles to aid in its functions. Humans, plants, animals are contained in this domain.
    b. **Maternal effect gene** – expressed by nurse cells, these genes are released into the embryos to direct the development of the organism. For fruit flies, these cells attach at the front and the back of the fruit fly oocytes.
    c. **Somitogenesis** – development of the embryonic structures that makes up a segment of an animal's vertebrae. These cells are regulated by a clock-wavefront system regulated by a negative-feedback system.
2. Provide a brief description of some of the sources of spatial heterogeneity in bacterial cells.
    Cells maintain spatial heterogeneity through the separation of important cellular functions into organelles. For example, the genetic material is stored in the cell's nucleoid.
3. Provide a detailed description of the genetic circuit that controls skin patterning in *Danio rerio.*
    The skin patterning is controlled by a self-activating activator protein and a repressor protein. The activator protein contributes to the darker spots found on the fish in addition to activating the repressor gene. The repressor gene prevents the darker spots from forming on the fish.
4. Provide a brief description of the clock-wavefront model.
    The clock-wavefront model is responsible for the spinal segmentation found in vertebrates. This simulates the interactions between the activator-inhibitor pair that controls how the spinal column is formed.
5. Provide a one sentence definition for each of the following mathematical and physical concepts:
    a. **Fick's law of diffusion** – the relationship between the change in concentration over time and space.
    b. **Compartment Model** – the division of space into discrete segments to decrease computational costs
    c. Brownian Motion – movement of particles in
6. Provide a detailed description of how a Reaction-Diffusion may be simulated using forward Euler and finite difference.
    Forward Euler and finite difference can be used to describe the diffusion of particles in a finite space. Finite difference is used to separate a bound-continuous space into discreet chunks. First the variables and constants are set. An array is used containing the position of the chunk and the point in time. The value stored here is the concentration of the substrate in question. Next for each point in time, we iterate through all the substrates in each position and calculate the rate of change in the substrate. In addition to the rate of change in the substrate due to interactions between each substrate an additional term is used to represent the diffusion of substrates from neighboring cells.
7. Provide a brief description of a particle-based stochastic reaction diffusion model.
    In this model, an array is used containing the position of the chunk and the point in time of the chunk. In each iteration of time, we iterate through each of the substrates. For each substrates we calculate the probability of a reaction happening and then randomly select the next reaction. Repeat for each step.

Part B: Applying concepts

1. Consider Somitogenesis in an developing vertebrate as modeled with a Clock-Wavefront model where we describe the clock as a sine-wave (in time) and the wavefront as a step function (in distance) that moves at

a constant rate. The expression of the clock gene oscillates while within the "on" region of the wavefront; when the wavefront is reached, the expression of the clock gene is "locked" in its current state. This can be described by the following system of equations:

$$\text{Clock}: \quad \frac{dX(t,d)}{dt} = \frac{A\pi\Omega(d)}{\omega}\sin\left(\frac{2\pi t}{\omega}\right), \quad X(0,0) = 0$$

$$\text{SwitchFunction}: \quad \Omega(d) = \begin{cases} 1 & \text{if} \quad d \geq d_0 \\ 0 & \text{if} \quad d < d_0 \end{cases}$$

$$\text{Wavefront}: \quad d_0(t) = \alpha t$$

where *t* is time, *d* is the distance of from the posterior (tail), *A* and ω are the amplitude and period of the clock oscillation respectively, and α is a parameter that describes the rate of progression of the wavefront (or growth of the embryo).

a. Find an expression for X(d) in the limit of large time; that is, the state of the clock gene as a function of distance from the posterior assuming that the wavefront has passed through the entire organism.