Probability Problems   Brian Ko

1\. 1. A professor has 15 students and during lecture will (uniformly) at random choose a student to answer a question. The professor asks 8 questions during the lecture. What is the probability no student will have to answer more than one question?

$\left(\frac{15}{15}\right) \cdot \frac{14}{15} \cdots \frac{8}{15}$ possible choices $= 15^8$

P(student has to answer more than one)

$$= \frac{\frac{15!}{7!}}{15^8} = \boxed{0.101}$$

2\. 2. An integer from the range 00000 - 99999 is generated uniformly at random. We are interested only in even integers that start with 2 odd digits where all digits are unique. If we randomly generate 8 of these numbers in succession, what is the probability we get exactly 5 numbers that meet our criteria?

total space $= 10^5$

3 digits $= \binom{5}{1} \cdot \binom{4}{1} \binom{5}{1} = 100$

4 digits $= \binom{5}{1} \binom{4}{1} \binom{7}{1} \binom{5}{1} = 700$

5 digits $= \binom{5}{1}\binom{4}{1}\binom{7}{1}\binom{6}{1}\binom{5}{1} = 4200$

P(randomly generated number is formatted): 0.042

Binomial dist.

$$P(5) = \binom{8}{5}(0.042)^5(0.958)^3$$

$$= (56)(0.042)^5(0.958)^3$$

$$\boxed{= 0.000006} ✓$$

3. 3. You roll 3 six-sided, fair dice. Let A be the event that at least 2 dice show 4 or above. Let B be the event that all 3 dice show the same value. Are A and B independent?

Independence : $P(A) \cdot P(B) = P(A \cap B)$

$P(A) = P($ at least 2 dice $\geq 4)$ + $P($ all 3 dice show same value$)$ (A_1, A_2)

B.) Binomial distribution:

$$P(A_1) = \binom{3}{2}\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right) = \tfrac{3}{8}$$

$$P(A_2) = \binom{3}{3}\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^0 = \tfrac{1}{8}$$

$$P(A) = \tfrac{3}{8} + \tfrac{1}{8} = \tfrac{4}{8} = \tfrac{1}{2}$$

$$P(B) = \binom{6}{6}\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$$

$$P(A \cap B) = (4,4,4), (5,5,5), (6,6,6)$$

$$= \frac{3}{6\times6\times6} = \frac{1}{72}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{36} = \frac{1}{72}$$

$$P(A)\cdot P(B) = P(A \cap B)$$

$\therefore$ A and B are independent ✓

4. 1. In poker, a flush is any 5-card hand where all the cards of the same suit. For this problem we will not distinguish between an ordinary flush and special flushes (like straight and royal flushes), meaning we will call any hand that has all 5 cards from the same suit a flush. Poker-player Paul loves a flush. What is the expected number of hands of poker he has to play to get a flush. (We assume each hand is dealt from a new deck containing of randomly ordered cards).

P(flush given 5 drawn cards)

$$= \binom{4}{1}\left(\frac{\binom{13}{5}}{\binom{52}{5}}\right)$$

$$E[X] = \frac{1}{p} = \frac{1}{\binom{4}{1}\left(\frac{\binom{13}{5}}{\binom{52}{5}}\right)} = \boxed{504.8}$$

5. A basketball team has a superstar. When their superstar plays, they win 70% of the time. When their superstar does not play they win 50% of the time. Entering a 5 game stretch, the superstar had been recovering from an injury and said the chance they would play the next 5 games was 75%. You go on a trip to the jungle (no internet access). When you return you find out the team won 4 of the 5 games. What is the probability the superstar played those 5 games? You may assume the superstar doesn't get injured during those games (either they play all or none of the 5).

Conditional probability:

P(event team wins $\frac{4}{5}$ games) = 0.8 = P(F)

P(event that superstar plays) = 0.75 P(S)

P(event that star doesn't play) = 1 − P(F) = 0.25 = $P(S^c)$

P(star played | team won $\frac{4}{5}$)

= P(S | F)

Baye's Rule: $P(S|F) = \dfrac{P(F|S)\,P(S)}{P(S|F)\,P(S) + P(F|S^c)}$

By Binomial dist.

P(team wins 4/5 w/ star) = $P(F|S) = p(4) = \binom{5}{4}(0.7)^4(0.3)^{5-4}$

$= 0.360$

P(team wins 4/5 w/o star) = $P(F|S^c) = \binom{5}{4}(0.5)^4 \cdot (0.5)^1$

$= (5)(0.5)^5 = 0.156$

$P(S|F) = \dfrac{(0.36)(0.75)}{(0.36)(0.75) + (0.156)(0.25)} = \boxed{0.874}$