COMP319-004 Algorithm 1

Assignment #1

마감: 4/1 (금) 23:59

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1. $\frac{n(n-1)}{2}$ = $O(n^2)$ 임을 증명하시오. (20pt)

$\Rightarrow \frac{n(n-1)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n \le \frac{1}{2}n^2$ at $n \ge 0$

$\Rightarrow$ So, we can take $c = \frac{1}{2}$ and $n_0 = 0$

$\Rightarrow$ Then $\frac{n(n-1)}{2} \in O(n^2)$

2. T(n) = 2 T(n/2) +1 의 복잡도는 O(n) 임을 증명하시오. (20pt)

Using master method for solving recurrence of above T(n)

$a=2$, $b=2$ $f(n) = n^{\log_b a} = n^{\log_2 2} = n^1 = n$ $f(n) = 1 \le O(n^{1-\epsilon})$, $0 < \epsilon \le 1$

$\Rightarrow$ we can apply case 1 of the M.T ($\because \epsilon = 1$, $f(n) = O(n^{1-\epsilon})$

$\Rightarrow \therefore T(n) = \Theta(n)$

3. 다음 알고리즘을 이해하고 아래 문제에 답하시오. (20pt)

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for(i = 1; i <= 1.5n; i++)
    cout << i;
for (i = n; i >= 1; i--)
    cout << i;
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$\Rightarrow \frac{3}{2}n$.

$\Rightarrow n$

1) n=2, n=4, n=6 일 때 결과를 각각 쓰시오.

① n=2 $\Rightarrow$ 1 2 3 2 1

② n=4 $\Rightarrow$ 1 2 3 4 5 6 4 3 2 1

③ n=6 $\Rightarrow$ 1 2 3 4 5 6 7 8 9 6 5 4 3 2 1

2) 복잡도 T(n)을 구하시오. 입력 n은 2의 배수로 가정.

$\Rightarrow$ ① $T(n) = \frac{3}{2}n + n = \frac{5}{2}n$

$\Rightarrow 0 \le 2n \le T(n) \le 3n$ at $n \ge 0$

$\Rightarrow$ ② $T(n) = \Theta(n)$ $(c_1 = 2, c_2 = 3, n_0 = 0)$

4. 스트라센 알고리즘을 이용해 다음 행렬 곱을 계산하는 과정을 보여라 (20pt)

$$\begin{bmatrix}1 & 3\\ 7 & 5\end{bmatrix}\begin{bmatrix}6 & 8\\ 4 & 2\end{bmatrix}$$

$M_1 = (1+5)(6+2) = 48$

$M_2 = (7+5)\cdot 6 = 72$

$M_3 = 1\cdot(8-2) = 6$

$M_4 = 5\cdot(4-6) = -10$

$M_5 = (1+3)\cdot 2 = 8$

$M_6 = (7-1)\cdot(6+8) = 84$

$M_7 = (3-5)\cdot(4+2) = -12$

$C_{11} = M_1 + M_4 - M_5 + M_7$
$= 48 - 10 - 8 - 12$
$= 18$

$C_{12} = M_3 + M_5$
$= 14$

$C_{21} = M_2 + M_4$
$= 62$

$C_{22} = M_1 - M_2 + M_3 + M_6 = 48 - 72 + 6 + 84 = 66$

$\Rightarrow C = \begin{bmatrix}18 & 14\\ 62 & 66\end{bmatrix}$

5. $T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$. 에서, T(n)을 구하시오 (20pt)

(Hint: Master method를 사용하세요)

$a = 4 \quad b = 2 \quad f(n) = n^2\sqrt{n}$
$= n^{5/2}$

$\Rightarrow h(n) = n^{\log_b a} = n^2$

$\Rightarrow$ ① $f(n) = \Omega(n^{2+\epsilon}) \rightarrow f(n) \geq n^{2+\epsilon}$ for $\epsilon < 1/2$

② $af(n/b) \leq cf(n)$

$\Rightarrow 4f(n/2) = 4\cdot(n/2)^{5/2} = 2^2/2^{5/2}\cdot n^{5/2} = \frac{1}{2}n^{5/2} \leq \frac{1}{2}f(n)$

$\rightarrow c = \frac{1}{2} < 1$

$\therefore$ case 3

$T(n) = \Theta(n^{5/2})$
$= \Theta(n^2\cdot\sqrt{n})$

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