The compromise for achieving this efficiency is the dependency of the programs length on the length of the transform, i.e., for longer transforms longer programs are generated. But the complexity order of the program length is N loglog N, and the proportionality constant is small, resulting in the program lengths shown in Table IV. These results are of real practical significance and can be immediately applied to create digital signal processing software of very high time efficiency. Since the cost of memory chips is going down rapidly, the concept of variable length of programs will become important for efficiency even if they occupy more space than programs of fixed size do.

A program generation technique for the radix-2 FFT that generates a program of length $O(N)$ with no array references is reported in Takaoka [8].

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A Note on "Realization of First-Order Two-Dimensional All-Pass Digital Filters"

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In the above correspondence,[1] the stability constraints (15), used to satisfy the condition (14), assume Z_i, $i = 1, 2$, as the delay operators [1]. With the commonly used symbol Z_i^{-1} for the delay operator, the stability constraints (15) of the paper become

$$a_0 - a_1 - a_2 + 1 > 0 \qquad (15a)'$$

$$a_0 - a_1 + a_2 - 1 < 0 \qquad (15b)'$$

$$a_0 + a_1 - a_2 - 1 < 0 \qquad (15c)'$$

$$a_0 + a_1 + a_2 + 1 > 0. \qquad (15d)'$$

Taking the products of (15a)′ and (15b)′, and (15c)′ and (15d)′, we get

$$(a_0^2 + a_1^2 - a_2^2 - 1) - 2(a_0 a_1 - a_2) < 0 \qquad (16)'$$

and

$$(a_0^2 + a_1^2 - a_2^2 - 1) + 2(a_0 a_1 - a_2) < 0, \qquad (17)'$$

respectively. The condition (14) of the correspondence follows if we multiply (16)′ and (17)′. Therefore, the multipliers of the proposed structure are also real when Z_i^{-1} is taken as the delay operator.

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Passive Source Localization Employing Intersecting Spherical Surfaces from Time-of-Arrival Differences

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Abstract—**Problems associated with the use of intersecting hyperboloids for passive source localization from time-of-arrival difference signals are discussed. A closed-form solution for source location is presented given time-of-arrival difference measurements when the distance from the source to any arbitrary reference is unknown.**

I. INTRODUCTION

The problem of passive source localization using time-of-arrival difference signals from multiple sensors is an active research area in acoustics and radar. Application techniques include the use of sensors along a line or surface array where beamforming and wavefront curvature ranging may be employed [1]-[4], and the use of single sensors where interference effects such as Lloyd's mirror may be employed [5]. In many applications, the problem may be made more tractable by reduction to two dimensions, e.g., azimuth and range or range and depth [1]-[5].

The most common approach for passive source localization is to use time delays between pairs of sensors to define curves of constant time difference which are hyperboloids of revolution (of two sheets). Since it is known that a minimum of four sensors are usually required to locate a source, the intersection of three sets of hyperboloids defines the source location. This solution is typically found numerically since closed-formed solutions to simultaneous hyperbolic algebraic equations are difficult to find. In many applications the problem is constrained to a plane where the intersection of two hyperboloids in two variables is sufficient [3], [5]. In some applications, the source can be assumed to lie at a distance which is great compared to the sensor spacing so that the direction of the source is given by the hyperboloid asymptote [1], [2]. In two dimensions, this reduces to the intersection of two lines; whereas in three dimensions, the source is found by the intersection of three sets of cones (asymptote lines rotated about the axis of each pair of sensors). The linearized case is not computationally difficult,

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thereby reducing the numerical complexity of the solution considerably.

This correspondence is concerned with the general problem of passive localization of an acoustic source where the application is three dimensional by nature; i.e., two-dimensional simplifications may not be appropriate.

One of the basic problems with numerically solving the simultaneous hyperbolic equations is the hyperboloid itself. The point of intersection of two hyperboloids can move considerably for a relatively small change in eccentricity of one of the hyperboloids. This is not true for intersecting spheres when the radius of one of the spheres is changed. This is due to the fact that hyperboloids define surfaces of constant distance difference between pairs of sensors, while spheres define surfaces of constant distance from a single sensor. Measurement errors can, in some cases, make the existence of a solution using intersecting hyperboloids difficult. With this in mind, and because of the numerical difficulties associated with intersecting hyperboloids in a general three-dimensional geometry, we attempt to recast the problem into one which employs spheres.

II. Theory—Alternate Derivation Employing Spherical Surfaces

For four sensors, one can define the difference in distance from source to sensor i and source to sensor j, d_{ij} in terms of the (unknown) absolute distances D_i from the source to each of the sensors as follows:

$$\begin{aligned} d_{ij} &= D_i - D_j \quad i, j = 1, 4 \\ d_{ij} &= -d_{ji} \\ d_{ij} &= c(t_i - t_j) = c\Delta t, \end{aligned} \tag{1}$$

where data must have the self-consistency shown below, c is the speed of sound, and Δt is the time-of-arrival (TOA) difference.

$$\begin{aligned} d_{12} &= d_{13} - d_{23} = d_{14} - d_{24} \\ d_{13} &= d_{12} + d_{23} = d_{14} - d_{34} \\ d_{23} &= d_{13} - d_{12} = d_{24} - d_{34} \\ d_{14} &= d_{12} + d_{24} = d_{13} + d_{34} \\ d_{24} &= d_{14} - d_{12} = d_{23} + d_{34} \\ d_{34} &= d_{14} - d_{13} = d_{24} - d_{23}. \end{aligned} \tag{2}$$

Consider the definition of the distance from source to sensor; this yields four equations

$$(x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2 = D_i^2, \quad i = 1, 4 \tag{3}$$

where (x_1, y_1, z_1) is the position of sensor 1 (similarly for sensors 2 and 3), (x_s, y_s, z_s) is the source position, D_1 is the distance of sensor 1 from the source (similarly for sensors 2 and 3). Expanding (3) and substituting the values for D_1, D_2, D_3 given by (1) yields

$$\begin{aligned} x_i^2 - 2x_i x_s + x_s^2 + y_i^2 - 2y_i y_s + y_s^2 + z_i^2 - 2z_i z_s + z_s^2 &= (d_{i4} + D_4)^2, \quad i = 1, 3 \\ x_4^2 - 2x_4 x_s + x_s^2 + y_4^2 - 2y_4 y_s + y_s^2 + z_4^2 - 2z_4 z_s + z_s^2 &= D_4^2. \end{aligned} \tag{4}$$

In the above set, the direct solution to equation set (1) has been employed, i.e., $D_1 = d_{14} + D_4$, etc.

In some cases, an alternative least-squares solution to (1) may be employed more effectively (depending on the specific application)

$$\begin{aligned} D_1 &= (d_{12} + d_{13} + 2d_{14} + d_{24} + d_{34})/4 + D_4, \\ D_2 &= (-d_{12} + d_{23} + d_{14} + 2d_{24} + d_{34})/4 + D_4, \\ D_3 &= (-d_{13} - d_{23} + d_{14} + d_{24} + 2d_{34})/4 + D_4. \end{aligned} \tag{5}$$

Solving the fourth equation in (4) for the distance from origin to source squared, R_s^2, yields

$$\begin{aligned} R_s^2 &\equiv x_s^2 + y_s^2 + z_s^2 \\ &= -(x_4^2 + y_4^2 + z_4^2) + D_4^2 + 2x_4x_s + 2y_4y_s + 2z_4z_s \\ &= -R_4^2 + D_4^2 + 2x_4x_s + 2y_4y_s + 2z_4z_s. \end{aligned} \tag{6}$$

Substituting into the first three equations in (4) yields

$$\begin{aligned} R_i^2 - (d_{i4} + D_4)^2 + D_4^2 - R_4^2 = 2x_ix_s + 2y_iy_s + 2z_iz_s - 2x_4x_s \\ - 2y_4y_s - 2z_4z_s, \quad i = 1, 3. \end{aligned} \tag{7}$$

Without loss of generality, we may take sensor 4 to be the origin. This makes $(x_4, y_4, z_4) = (0, 0, 0)$, $R_4 = 0$. Substituting into (7) yields

$$\begin{bmatrix} R_1^2 - d_{14}^2 \\ R_2^2 - d_{24}^2 \\ R_3^2 - d_{34}^2 \end{bmatrix} - 2(x_s^2 + y_s^2 + z_s^2)^{1/2} \begin{bmatrix} d_{14} \\ d_{24} \\ d_{34} \end{bmatrix} = 2 \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} \tag{8}$$

after recalling that now $R_s^2 = D_4^2$ from (6). Symbolically, (8) can be written

$$\mathbf{\Delta} - 2R_s \boldsymbol{d} = 2\mathfrak{M}\boldsymbol{x} \tag{9}$$

where

$$\mathbf{\Delta} = \begin{bmatrix} R_1^2 - d_{14}^2 \\ R_2^2 - d_{24}^2 \\ R_3^2 - d_{34}^2 \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} d_{14} \\ d_{24} \\ d_{34} \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix}, \quad \mathfrak{M} = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}.$$

In (9) the unknown appears in both sides of the equation, the source position $\boldsymbol{x}$ on the right-hand side, and source radius R_s on the left-hand side. If one assumes for the moment that the source radius R_s were known, (9) has the solution

$$\boldsymbol{x} = \tfrac{1}{2}\mathfrak{M}^{-1}(\mathbf{\Delta} - 2R_s\boldsymbol{d}) \tag{10}$$

provided the matrix of sensor positions $\mathfrak{M}$ is not singular (sensors do not provide redundant information). Solution (10) still involves the as-yet unknown source radius R_s. To solve this problem, recall

$$R_s = (\boldsymbol{x}^T\boldsymbol{x})^{1/2}. \tag{11}$$

Substituting (10) into (11) yields after expansion

$$\begin{aligned} R_s^2[4 - 4\boldsymbol{d}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\boldsymbol{d}] + R_s[2\boldsymbol{d}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\mathbf{\Delta} \\ + 2\mathbf{\Delta}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\boldsymbol{d}] - [\mathbf{\Delta}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\mathbf{\Delta}] = 0. \end{aligned} \tag{12}$$

This equation involves only the unknown source radius R_s, all other quantities are known from the sensor locations ($\mathfrak{M}$), measured time delay differences ($\boldsymbol{d}$) or both, ($\mathbf{\Delta}$). The quantities in brackets are seen to represent scalar results of inner products of the individual vectors and matrices. Equation (12) has solution

$$R_s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad R_s \geq 0, \tag{13}$$

where

$$\begin{aligned} a &= [4 - 4\boldsymbol{d}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\boldsymbol{d}], \\ b &= [2\boldsymbol{d}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\mathbf{\Delta} + 2\mathbf{\Delta}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\boldsymbol{d}], \\ c &= -[\mathbf{\Delta}^T(\mathfrak{M}^{-1})^T\mathfrak{M}^{-1}\mathbf{\Delta}]. \end{aligned}$$

Thus, (10) and (13) represent a two-step solution to the passive source localization problem. Equation (13) is first solved to find the source radius, this radius is substituted into (10) to find source position. In some instances, (13) will permit two physical solutions ($R_s \geq 0$). In these cases, two source locations will be found which will reproduce the measured data. In these situations, the two locations are usually far enough apart that the correct solution can be discerned by other physical reasoning such as one solution lying outside the domain of interest.

III. Conclusion

The difficulties of passive localization of sound sources through intersecting hyperboloids has been discussed. To circumvent these difficulties, an alternate method which employs intersecting spheres has been presented. This formulation of the problem permits a simple closed-form solution for source location from the time delay estimates and sensor locations.

The specific solution presented herein is for the case of four receiving sensors. Implicit in the development is the assumption that a mechanism for measuring TOA differences exists. More general aspects of this problem, such as methodology for measuring TOA differences, multiple sources, and situations where more than four receiving sensors are employed, are capable of development from the discussion presented. Each of the subjects is application dependent, particularly the N sensor case ($N > 4$) which requires a pseudoinverse in (10). Since detailed analysis for this case can only be carried forth for a specific sensor number and position, its development is more appropriately considered for each application separately.

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