A Simple and Efficient Estimator for Hyperbolic Location

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Introduction

- TDOA
  Determine the difference in the time-of-arrival (TD) between the master signal and the signals from each of the secondary stations of a chain so that we will get the position.
- Arbitrary and Linear Array of the sensors
  In a 2-D plane, the sensors will be set to the two kinds
- Near and distant sources
  noise
  MSE
- Solution
  Taylor-series, proposed, CRLB, SI

Contents

Solution

- Taylor-series
  We first get the Gt

$$G_t = \begin{bmatrix} (x_1 - x)/r_1 - (x_2 - x)/r_2 & (y_1 - y)/r_1 - (y_2 - y)/r_2 \\ (x_1 - x)/r_1 - (x_3 - x)/r_3 & (y_1 - y)/r_1 - (y_3 - y)/r_3 \\ \cdots & \\ (x_1 - x)/r1 - (x_M - x)/r_M & (y_1 - y)/r1 - (y_M - y)/r_M \end{bmatrix}$$

  while the ht

$$ht = \begin{bmatrix} r_{2,1} - (r_2 - r_1) \\ r_{3,1} - (r_3 - r_1) \\ \cdots \\ r_{M,1} - (r_M - r_1) \end{bmatrix}$$

Solution

- Taylor-series
  We first get the Gt

$$G_t = \begin{bmatrix} (x_1 - x)/r_1 - (x_2 - x)/r_2 & (y_1 - y)/r_1 - (y_2 - y)/r_2 \\ (x_1 - x)/r_1 - (x_3 - x)/r_3 & (y_1 - y)/r_1 - (y_3 - y)/r_3 \\ \dots & \\ (x_1 - x)/r1 - (x_M - x)/r_M & (y_1 - y)/r1 - (y_M - y)/r_M \end{bmatrix}$$

  while the ht

$$ht = \begin{bmatrix} r_{2,1} - (r_2 - r_1) \\ r_{3,1} - (r_3 - r_1) \\ \dots \\ r_{M,1} - (r_M - r_1) \end{bmatrix}$$

- So we can get the x and y deviation

$$\begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = (G_t^T Q^{-1} G_t)^{-1} G_t^T Q^{-1} h_t$$

Solution

- Proposed 2
  First we get the z_a

$$z_a = (G_a^T \psi^{-1} G_a)_{-1} G_a^T \psi^{-1} h \tag{1}$$

  Then is the z_a' :

$$z_a' = (G_a'^T \psi'^{-1} G_a')^{-1} G_a'^T \psi'^{-1} h' \tag{2}$$

  So we get the Zp as

$$z_p = \begin{bmatrix} x \\ y \end{bmatrix} = \sqrt{z_a'} + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \tag{3}$$

- Proposed 1

$$z_a \approx (G_a^T Q^{-1} G_a)^{-1} G_a^T Q^{-1} h \tag{4}$$

$$z_a' \approx (G_a'^T B'^{-1} G_a Q^{-1} G_a B'^{-1} G_a')^{-1} (G_a'^T B'^{-1} G_a Q^{-1} G_a B'^{-1}) h' \tag{5}$$

Solution

- CRLB
  First we get the B"

$$B'' = \begin{bmatrix} (x^0 - x_1) & 0 \\ 0 & (y^0 - y_1) \end{bmatrix}$$

  So

$$\Phi = c^2 (B'' G_a'^T B'^{-1} G_a^{0\,T} B^{-1} Q^{-1} B{-}1 G_a^0 B'^{-1} G_a' B'')^{-1} \quad (6)$$

- When the CRLB linear

$$\Phi = c^2 (T^T G_l^{0\,T} B^{-1} Q^{-1} B^{-1} G_l^0 T)^{-1} \quad (7)$$

Solution

- Three sensors situation
  There is a special formula to solve it:

$$\begin{bmatrix} x \\ y \end{bmatrix} = - \begin{bmatrix} x_{2,1} & y_{2,1} \\ x_{3,1} & y_{3,1} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} r_{2,1} \\ r_{3,1} \end{bmatrix} r_1 + \frac{1}{2} \begin{bmatrix} r_{2,1}^2 - K_2 + K_1 \\ r_{3,1}^2 - K_3 + K_1 \end{bmatrix} \right\}$$

- When it comes to linear

$$-2x_{i,1}(x + ay) - 2r_{i,1}r_1 = r_{i,1}^2 - K_i + K_1$$

Contents

Simulation

Near Source

TABLE I

COMPARISON OF MSE FOR THE SI, TAYLOR-SERIES AND PROPOSED METHODS; ARBITRARY ARRAY AND NEAR SOURCE

MSE	M = 3	M = 4	M = 5	M = 6	M = 7	M = 8	M = 9	M = 10
A	no sol.	1.5768	0.1597	0.1480	0.1229	0.1164	0.1148	0.1103
B	2.1726	0.7061	0.1460	0.1341	0.1144	0.1057	0.1034	0.09462
C	2.1726	0.7282	0.1456	0.1359	0.1159	0.1077	0.1055	0.09697
D	2.1726	0.6986	0.1451	0.1337	0.1141	0.1055	0.1034	0.09480
E	1.9794	0.6884	0.1451	0.1334	0.1143	0.1054	0.1032	0.09432

A: SI method. B: Taylor series method. C: proposed method, { (14*b*), (22*b*), (24) }. D: proposed method, { (14*b*), (14*a*), (22*a*), (24) }. E: theoretical MSE of the new method = CRLB.

Figure: paper result

Table 1

MSE	M=3	M=4	M=5	M=6	M=7	M=8	M=9	M=10
B	2.51247	0.67408	0.12023	0.10776	0.09558	0.09580	0.09507	0.08809
C	2.55841	0.74229	0.11825	0.10341	0.09768	0.09671	0.09882	0.08813
D	2.53063	0.71126	0.11648	0.10457	0.09740	0.09848	0.09625	0.08798
E	1.97937	0.68835	0.14508	0.13343	0.11427	0.10538	0.10322	0.09432

Figure: My result

Simulation

Near Linear Source

TABLE II
COMPARISON OF MSE FOR THE PROPOSED AND TAYLOR-SERIES METHODS; LINEAR ARRAY AND NEAR SOURCE

MSE	M = 3	M = 4	M = 5	M = 6	M = 7	M = 8	M = 9	M = 10
B	8.2574	1.2836	0.3566	0.1222	0.06155	0.02854	0.01746	0.009540
C	8.2574	1.1131	0.3556	0.1225	0.06199	0.02884	0.01772	0.009726
D	8.2574	1.1117	0.3545	0.1219	0.06148	0.02852	0.01746	0.009541
E	7.2819	1.1000	0.3548	0.1219	0.06123	0.02840	0.01750	0.009599

B: Taylor series method. C: proposed method, {(28) with $\Psi = Q$, (29)}. D: proposed method, {(28) with $\Psi = Q$, (28), (29)}. E: theoretical MSE of the new method = CRLB.

Figure: paper result

Table 2

MSE	M=3	M=4	M=5	M=6	M=7	M=8	M=9	M=10
B	2.49104	0.86361	0.24959	0.10094	0.04916	0.02498	0.01488	0.00865
C	2.50821	0.82232	0.25265	0.09891	0.04864	0.02409	0.01470	0.00835
D	2.52143	0.83038	0.24806	0.09678	0.04843	0.02405	0.01484	0.00829
E	7.28191	1.10001	0.35481	0.12192	0.06123	0.02840	0.01750	0.00960

Figure: My result

Simulation

Distant Source

TABLE III
COMPARISON OF MSE FOR THE SI, TAYLOR-SERIES AND PROPOSED METHODS; ARBITRARY ARRAY AND DISTANT SOURCE

MSE	M = 4	M = 5	M = 6	M = 7	M = 8	M = 9	M = 10
A	101881	213.93	48.12	40.40	41.87	40.11	37.23
B	346.86	147.57	44.38	38.64	38.63	36.55	33.80
C	348.74	144.84	44.06	38.41	38.47	36.50	33.87
E	328.82	143.94	44.06	38.54	38.53	36.47	33.73

A: SI method. B: Taylor series method. C: proposed method, {(14*b*), (22*b*), (24)}. E: theoretical MSE of the new method = CRLB.

Figure: paper result

Table 3

MSE	M=4	M=5	M=6	M=7	M=8	M=9	M=10
B	219.12	108.47	34.72	32.48	32.71	32.01	29.75
C	374.77	109.06	33.68	32.29	32.13	32.25	29.50
E	328.82	143.94	44.06	38.54	38.53	36.47	33.73

Figure: My result

Simulation

Distant Linear Source

TABLE IV
COMPARISON OF MSE FOR THE PROPOSED AND TAYLOR-SERIES METHODS; LINEAR ARRAY AND DISTANT SOURCE

MSE	M = 4	M = 5	M = 6	M = 7	M = 8	M = 9	M = 10
B	1797.13	432.68	159.50	69.16	34.56	18.56	10.85
C	1588.31	409.66	153.92	68.41	34.44	18.56	10.86
E	1437.25	408.17	154.05	68.06	34.25	18.57	10.90

B: Taylor series method. C: proposed method, {(28) with $\Psi = Q$, (29)}. E: theoretical MSE of the new method = CRLB.

Figure: paper result

Table 4

MSE	M=4	M=5	M=6	M=7	M=8	M=9	M=10
B	230163.58	299.12	127.21	55.68	28.55	15.64	9.33
C	1066.91	285.03	122.29	56.07	28.98	15.90	9.65
E	1437.25	408.17	154.05	68.06	34.25	18.57	10.90

Figure: My result

Simulation

FIGURE

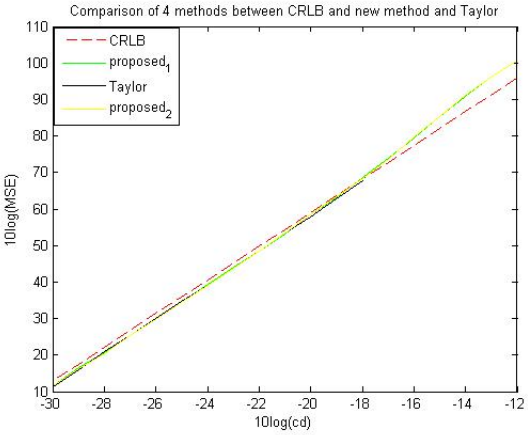

Figure: Whole mode

Simulation

FIGURE

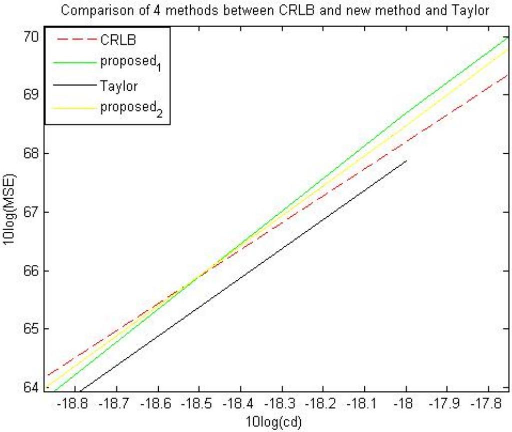

Figure: Amplify mode

Conclusion

- The MSE decreases with the increasing of the sensors or the distance.
- The linear array has a less MSE than the arbitrary.
- Proposed perform better than Taylor and SI. And the CRLB acts as the theoretical standard.
- M=4 or 5
  - Tayor will have a large error.
  - It is caused by the linear error.

END I

Thanks for watching!

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Reference

"A simple and efficient estimator for hyperbolic location."