Math Question

December 11, 2015

Brian Mascitello

The problem posed is ...

$$1 + \sqrt{3}^x = 2^x \text{ , solve for x?}$$

1. The first way I attempted solving this was the laziest. Try $x = 0$ ...

$$1 + \sqrt{3}^x = 2^x$$

$$1 + \sqrt{3}^0 = 2^0$$

$$1 + 1 = 1$$

$$2 = 1$$

This is of course false, thus $x \neq 0$. However it tells us that at $x = 0$ the left side of the equation is greater than the right, but in analyzing the equation we note the right will grow faster than the left side. This is because 1 is constant, thus not dependent on x. $\sqrt{3} \approx 1.732$ and $1.732 < 2$, thus 2^x increases faster than $1 + \sqrt{3}^x$ as x increases. Try $x = 1$ ...

$$1 + \sqrt{3}^x = 2^x$$

$$1 + \sqrt{3}^1 = 2^1$$

$$1 + 1.732 = 2$$

$$2.732 = 2$$

Also false, since left side is still larger, try $x = 2$ ...

$$1 + \sqrt{3}^x = 2^x$$

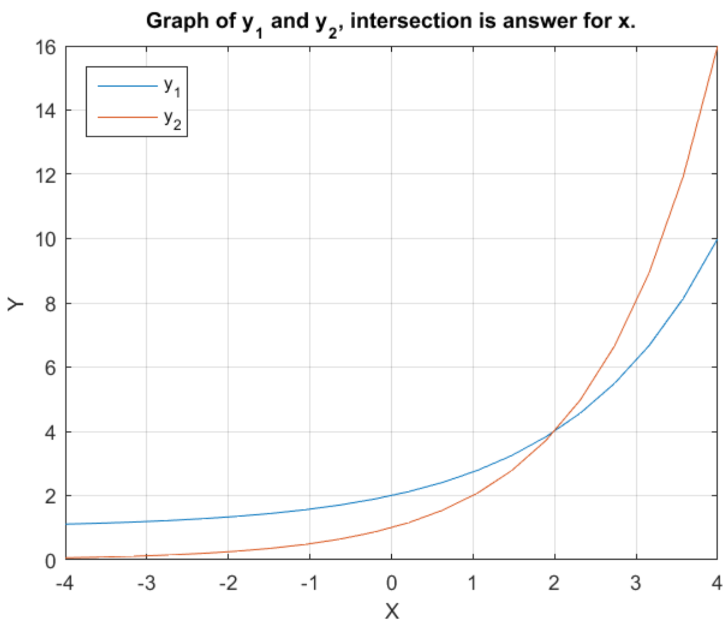
$$1 + \sqrt{3}^2 = 2^2$$

$$1 + 3 = 4$$

$$4 = 4$$

Since four is indeed equivalent to four, the answer to $1 + \sqrt{3}^x = 2^x$ is $x = 2$.

2. The second way I attempted to solve it, as well as verify the first method, is by graphing it. Let $y_1 = 1 + \sqrt{3}^x$ and $y_2 = 2^x$, then



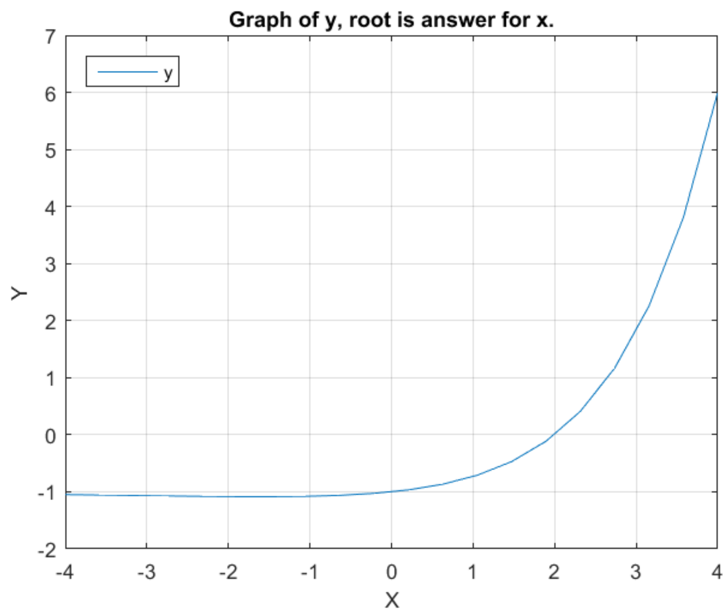
x	y_1	y_2
4	10	16
-4.000	1.111	0.063
-3.579	1.140	0.084
-3.158	1.176	0.112
-2.737	1.222	0.150
-2.316	1.280	0.201
-1.895	1.353	0.269
-1.474	1.445	0.360
-1.053	1.561	0.482
-0.632	1.707	0.645
-0.211	1.891	0.864
0.211	2.123	1.157
0.632	2.415	1.549
1.053	2.783	2.074
1.474	3.247	2.777
1.895	3.831	3.719
2.316	4.568	4.979
2.737	5.497	6.666
3.158	6.667	8.925
3.579	8.142	11.950
4.000	10.000	16.000

Matlab raw code for graph:

```
>> x = linspace(-4,4,20);
>> y1 = 1 + sqrt(3).^x;
>> y2 = 2.^x;
>> plot(x, y_1, x, y_2)
>> title('Graph of y_{1} and y_{2}, intersection is answer for x.')
>> xlabel('X')
>> ylabel('Y')
>> legend('y_{1}','y_{2}','Location','northwest')
```

3. Another graph, except using a root of the function to find the value of x.

Let $y = 2^x - \sqrt{3}^x - 1$, then



x	y
-4.000	-1.049
-3.579	-1.056
-3.158	-1.064
-2.737	-1.072
-2.316	-1.079
-1.895	-1.084
-1.474	-1.085
-1.053	-1.079
-0.632	-1.061
-0.211	-1.027
0.211	-0.965
0.632	-0.865
1.053	-0.709
1.474	-0.469
1.895	-0.113
2.316	0.411
2.737	1.169
3.158	2.258
3.579	3.808
4.000	6.000

Matlab raw code for graph:

```
>> x = linspace(-4,4,20);
>> y = 2.^x - sqrt(3).^x - 1;
>> plot(x, y)
>> title('Graph of y, root is answer for x.')
>> grid on
>> xlabel('X')
>> ylabel('Y')
>> legend('y','Location','northwest')
```

4. This is the toughest way to solve for x that I can think of in this case, and it does not require graphs.

Let $y = 2^x - \sqrt{3}^x - 1$, then find the derivative of y.

$$y = 2^x - 3^{x/2} - 1$$

$$\frac{d}{dx}y = \frac{d}{dx}(2^x - 3^{x/2} - 1)$$

$$y' = \frac{d}{dx}(2^x)\frac{d}{dx}(-3^{x/2})\frac{d}{dx}(-1)$$

$$y' = \frac{d}{dx}(2^x) - \frac{d}{dx}(3^{x/2})$$

$$y' = 2^x ln(2) - \frac{d}{dx}(3^{x/2})$$

$$\boxed{y' = 2^x ln(2) - \frac{3^{x/2} ln(3)}{2}}$$

$ln(2) \approx 0.693147$ and $ln(3) \approx 1.098612$

Newton's Method If x_n is an approximation a solution of and if the next approximation is given by,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Math Problem.py raw code:

```
# Created by Brian Mascitello to calculate a specific polynomial's root.

def mathproblem(x0):
    """ function mathproblem(x0) finds a root of the nonlinear
        function specified by f and fprime. y = 2 ** x - 3 ** (x / 2) - 1;
        yprime = 0.693147 * (2 ** x) - (1.098612 * 3 ** (x / 2)) / 2; Result
        x is the root. """

    epsilon = 2.2204*10**-16
    """ governs precision of convergence
        where 2.2204*10**-16 = machine epsilion in python """

    x = x0
    xprevious = 0
    k = 0

    while abs(float(2 ** x - 3 ** (x / 2) - 1)) > epsilon*abs(float(2
    ** x0 - 3 ** (x0 / 2) - 1)) and k < 20:
        k = k+1
        xprevious = x
        x = float(x) - (float(2 ** x - 3 ** (x / 2) - 1)/float(0.693147
        * (2 ** x) - (1.098612 * 3 ** (x / 2)) / 2))
        change = abs(float(x - xprevious))
        residual = 2 ** x - 3 ** (x / 2) - 1
        print("Iteration: %d, Root: %f, Change: %f, Residual: %f" %
        (k,x,change,residual))

    print("Root at",x,"\n")

    return float(x)

print("y = 2 ** x - 3 ** (x / 2) - 1")
print("yprime = 0.693147 * (2 ** x) - (1.098612 * 3 ** (x / 2)) / 2")
print("Machine epsilon set as: 2.2204*10^-16")
x0 = float(input("Please enter your guess of the root: "))
mathproblem(x0)
```

Math Problem.py output:

```
y = 2 ** x - 3 ** (x / 2) - 1
yprime = 0.693147 * (2 ** x) - (1.098612 * 3 ** (x / 2)) / 2
Machine epsilon set as: 2.2204*10^-16
Please enter your guess of the root: 0
Iteration: 1, Root: 6.952121, Change: 6.952121, Residual: 77.270274
Iteration: 2, Root: 5.681332, Change: 1.270789, Residual: 27.651539
Iteration: 3, Root: 4.485320, Change: 1.196012, Residual: 9.648804
Iteration: 4, Root: 3.421652, Change: 1.063668, Residual: 3.165227
Iteration: 5, Root: 2.595079, Change: 0.826573, Residual: 0.882313
Iteration: 6, Root: 2.131456, Change: 0.463623, Residual: 0.156953
Iteration: 7, Root: 2.007459, Change: 0.123998, Residual: 0.008417
Iteration: 8, Root: 2.000025, Change: 0.007433, Residual: 0.000028
Iteration: 9, Root: 2.000000, Change: 0.000025, Residual: 0.000000
Iteration: 10, Root: 2.000000, Change: 0.000000, Residual: -0.000000
Iteration: 11, Root: 2.000000, Change: 0.000000, Residual: 0.000000
Root at 2.0
```

This program applies Netwon's Method to solve for x. It requires taking the derivative of the original equation and carefully implementing it to get a precise answer. You could calculate this by hand but everyone loved python so much I thought I would re-purpose newtoncube.py for this occasion. $root = x = 2$