

CSE 101 Homework 1

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1. Following the Algorithm Defined in HW1. Consider the case when $A[i] + A[j] = V$ for some $1 \leq i < j \leq n$. Prove the invariant: If the while loop has not terminated, then $I \leq i < j \leq J$

i) Prove the base case: ($I = 1, J = n$, assuming $n > 1$.)

Given $1 \leq i < j \leq n \implies I \leq i < j \leq J$

ii) Prove the general case:

Assume that after $x > 1$ iterations, $I \leq i < j \leq J$. Show the loop invariant after the $x + 1$ iteration.

If $i = I$ and $j = J \implies$

$A[i] + A[j] = V \implies$ returns True

If $j = J$

$$\implies I < i \tag{1}$$

$$\implies (\text{By Sorted Array}) A[I] + A[J] = A[i] + A[j] < V \tag{2}$$

$$\implies I++ \tag{3}$$

$$\implies I \leq i < j \leq J \tag{4}$$

If $i = I$

$$\implies J > j \tag{5}$$

$$\implies (\text{By Sorted Array}) A[I] + A[J] = A[i] + A[j] > V \tag{6}$$

$$\implies J-- \tag{7}$$

$$\implies I \leq i < j \leq J \tag{8}$$

Else:

$$\implies J > j \ \& \ i < I \tag{9}$$

$$\implies \text{Either } I++ \text{ or } J-- \text{ will uphold the loop invariant.} \tag{10}$$