CSE 21 Hw 1

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1. (a) Use regular induction to show that for all n $\geq$ 1 that

$$\sum_{k=1}^{n} \frac{1}{(k)(k+1)} = \frac{n}{n+1}$$

i) Prove the base case: (n = 1)

$$\sum_{k=1}^{1} \frac{1}{k(k+1)} = \frac{1}{2} = \frac{1}{1+1}$$

ii) Assume for some n = j

$$\sum_{k=1}^{j} \frac{1}{k(k+1)} = \frac{j}{j+1}$$

and show that it holds for j + 1

$$\begin{aligned}
& \sum_{k=1}^{j+1} \frac{1}{k(k+1)} && (1)\\
= & \frac{1}{(j+1)(j+2)} + \frac{j}{j+1} && (2)\\
= & \frac{1 + j(j+2)}{(j+1)(j+2)} && (3)\\
= & \frac{(j+1)^2}{(j+1)(j+2)} && (4)\\
= & \frac{(j+1)}{(j+2)} && (5)
\end{aligned}$$

(b) Use regular induction to show that for all n $\geq$ 0 that

$$\sum_{k=0}^{n} 2^k < 2^{n+1}$$

i) Prove the base case: (n = 0)

$$\sum_{k=0} n2^k = 1 < 2$$

ii) Assume for some n = j

$$\sum_{k=0} n2^k = 2^{n+1}$$

and show that it holds for j + 1

$$\begin{aligned} & \sum_{k=0}^{k} 2^{j+1} && (6) \\ < \; & 2^{j+1} + 2^{j+1} && (7) \\ = \; & 2 * 2^{j+1} = 2^{j+2} && (8) \end{aligned}$$

2. A password is a string over the alphabet of the 26 uppercase letters, the 26 lowercase letters, and the 10 digits

(a) How many 8-char passwords start and end with a digit?

$$k = 10 \cdot (26 + 26 + 10)^6 \cdot 10 = 10^2 \cdot 62^6$$

- 10 *: number of ways to start the string with a number*
- $(26 + 26 + 10)^6$ *number of 6-char strings*
- 10 *: number of ways to end the string with a number*

(b) How many 8-character passwords have exactly one uppercase and exactly one lowercase letter?

$$k = 26^2 \cdot 10^6$$

- 26^2 *: number of combinations of 1 uppercase and 1 lowercase*
- (10^6) *: number of 6 char strings with only digits*

(c) How many 8-character passwords avoids having the word COUNT (with any combinationof upper and lowercase letters)

$$k = 26^2 \cdot 10^6$$

- 26^2 *: number of combinations of 1 uppercase and 1 lowercase*
- (10^6) *: number of 6 char strings with only digits*