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MTH20D
May 29 2024

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Matlab HW4

Exercise 4.1

Part A

```
B = [1.2, 2.5; 4, 0.7];

B =

    1.2000    2.5000
    4.0000    7.0000
```

Part B

```
[eigVectors, eigValues] = eig( B )

eigValues =

   0.650087273644288  -0.589893864978993
   0.759859550605068   0.807480791140040


eigVectors =

   4.122144385112380                   0
                   0  -2.222144385112380
```

Exercise 4.2

Part A

```
A = [ 3, 4; -1, -2 ]

A =

     3     4
    -1    -2
```

Part B

```
[eigVectors, eigValues] = eig( A )

eigVectors =

   0.970142500145332  -0.707106781186547
  -0.242535625036333   0.707106781186547
```

```
eigValues =

     2     0
     0    -1
```

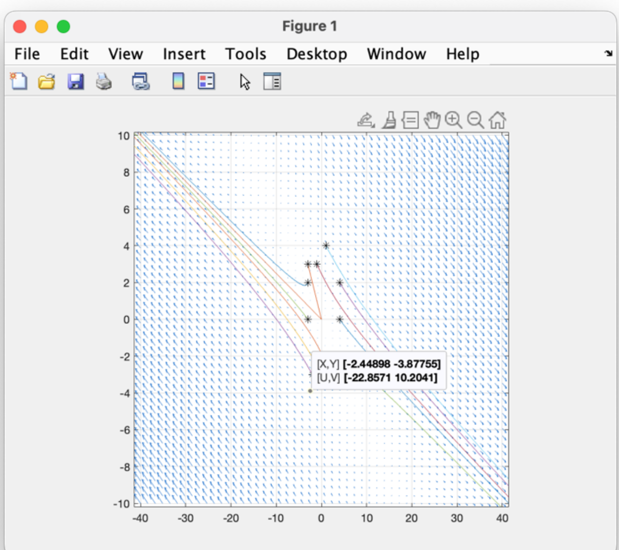
Part C

```
y1 = c1 * exp(1) ^ ( eigValues(1, 1) * t ) * eigVectors(: , 1);
y2 = c2 * exp(1) ^ ( eigValues(2, 2) * t ) * eigVectors(: , 2);
y = y1 + y2;
```

$$y(t) = c_1 e^{2t} \begin{bmatrix} 0.9701 \\ -0.2425 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} -0.7071 \\ 0.7071 \end{bmatrix}$$

Depending on the starting condition (the value of c1), the value of x either approaches negative infinity while the value of y approaches infinity, or the value of x approaches infinity while the value of y approaches negative infinity.

Part D



Yes, this answer exemplifies the behavior I identified in part c

Exercise 4.3

Part A

```
A = [2.7, -1; 4.1, 3.7]

[eigVectores, eigValues] = eig( A )
```

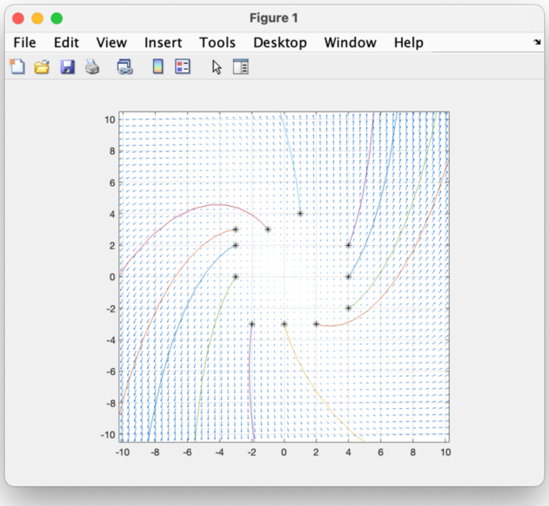
```
A =

   2.700000000000000  -1.000000000000000
   4.100000000000000   3.700000000000000


eigVectores =

  -0.109343504210174 + 0.429094895634507i -0.109343504210174 -
0.429094895634507i
  0.896616734523426 + 0.000000000000000i  0.896616734523426 +
0.000000000000000i


eigValues =

  3.200000000000000 + 1.962141687034858i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i  3.200000000000000 -
1.962141687034858i
```

Part B

```
y1 = c1 * exp(1) ^ ( eigValues(1, 1) * t ) * eigVectors(: , 1);
y2 = c2 * exp(1) ^ ( eigValues(2, 2) * t ) * eigVectors(: , 2);
y = y1 + y2;
```

$$y(t) = c_1 e^{(3.2 + 1.962i)t} \begin{bmatrix} -0.10934 + 0.4291i \\ 0.89662 \end{bmatrix} + c_2 e^{(3.2 - 1.962i)t} \begin{bmatrix} -0.10934 - 0.4291i \\ 0.89662 \end{bmatrix}$$

Part C



The eigen values of this system of equations were 3.2 +- Bi. Since a=3.2 > 0, the e ^ (at) expression grows to infinity as t increases.

Exercise 4.4
Part A

```
A = [1.25, -0.97, 4.6; -2.6, -5.2, -0.31; 1.18, -10.3, 1.12]
[eigVectors, eigValues] = eig(A)

eigVectors =

  Columns 1 through 2

  0.735103536321409 + 0.000000000000000i  0.449031928453424 +
0.259118786157820i
 -0.196145936873525 + 0.000000000000000i -0.337494431036665 +
0.224201610414725i
  0.648960370389260 + 0.000000000000000i -0.753033152521308 +
0.000000000000000i

  Column 3

  0.449031928453424 - 0.259118786157820i
 -0.337494431036665 - 0.224201610414725i
 -0.753033152521308 + 0.000000000000000i


eigValues =
```

```
  Columns 1 through 2

  5.569771441242941 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i -4.199885720621474 +
2.660595237935089i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i

  Column 3

  0.000000000000000 + 0.000000000000000i
  0.000000000000000 + 0.000000000000000i
 -4.199885720621474 - 2.660595237935089i
```

Part B
The system is unstable: the first eigen value has a real component great than 0. This means that at least on of the exponential terms with approach infinity as t increases, meaning the whole system approaches infinity, and thus is unstable

---

Exercise 4.5
Part A

```
A = [-0.0558 -0.9968 0.0802 0.0415; 0.598 -0.115 -0.0318 0; -3.05
0.388 -0.465 0; 0 0.0805 1 0];
B = [0.01; -0.175; 0.153; 0];

[eigVectors, eigValues] = eig(A)

eigVectors =

  Columns 1 through 2

  0.199374640137239 - 0.106271229547151i  0.199374640137239 +
0.106271229547151i
 -0.077977305236739 - 0.133260428240184i -0.077977305236739 +
0.133260428240184i
 -0.016547449083871 + 0.666768207597794i -0.016547449083871 -
0.666768207597794i
  0.693010617551841 + 0.000000000000000i  0.693010617551841 +
0.000000000000000i

  Columns 3 through 4

 -0.017177857798322 + 0.000000000000000i  0.006721774358807 +
0.000000000000000i
```

```
 -0.011827816177923 + 0.000000000000000i  0.040421900660712 +
0.000000000000000i
 -0.489531260037862 + 0.000000000000000i -0.010525414999835 +
0.000000000000000i
  0.871736295798027 + 0.000000000000000i  0.999104650842706 +
0.000000000000000i


eigValues =

  Columns 1 through 2

 -0.032935458097396 + 0.946653235187099i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i -0.032935458097396 -
0.946653235187099i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i

  Columns 3 through 4

  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
 -0.562651115485759 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i -0.007277968319449 +
0.000000000000000i
```

Part B

This system is asymptotically stable, since all the real components of the eigen values are negative, meaning all the exponential terms in the solution go to 0 as t increases.

Part C

the fourth eigen value is very close to 0. In the associated eigen vector the fourth component (x4 = pitch rate) is nearly 1. This suggests that the eigenvector / value are most closely related to the pitch of the plane.

Exercise 4.6
part A
```
ans =

0   0.070000000000000                  0  -0.010000000000000
0  -1.225000000000000                  0   0.175000000000000
0   1.071000000000000                  0  -0.153000000000000
0                   0                  0                   0
```

Part B
```
ans =

0   0.050000000000000                  0  -0.001000000000000
0  -0.875000000000000                  0   0.017500000000000
0   0.765000000000000                  0  -0.015300000000000
0                   0                  0                   0
```

Part C
```
A + B*F
ans =

  -0.055800000000000  -0.946800000000000   0.080200000000000
0.040500000000000
   0.598000000000000  -0.990000000000000  -0.031800000000000
0.017500000000000
  -3.050000000000000   1.153000000000000  -0.465000000000000  -
0.015300000000000
                   0   0.080500000000000   1.000000000000000
0

eigVectors =

  Columns 1 through 2

 -0.190613894122166 + 0.000071205250351i -0.190613894122166 -
0.000071205250351i
 -0.066835246548876 + 0.112601603593222i -0.066835246548876 -
0.112601603593222i
  0.252053236780036 - 0.597092719268404i  0.252053236780036 +
0.597092719268404i
 -0.725582329863229 + 0.000000000000000i -0.725582329863229 +
0.000000000000000i

  Columns 3 through 4

 -0.019657016636972 + 0.000000000000000i -0.087341952341295 +
0.000000000000000i
```

```
 -0.035550723966758 + 0.000000000000000i -0.078915297371967 +
0.000000000000000i
  0.090767200904648 + 0.000000000000000i -0.587905087259993 +
0.000000000000000i
 -0.995043246779912 + 0.000000000000000i  0.800319540918329 +
0.000000000000000i


eigValues =

  Columns 1 through 2

 -0.339965554948602 + 0.810422561268811i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i -0.339965554948602 -
0.810422561268811i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i

  Columns 3 through 4

  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
 -0.088343263380559 + 0.000000000000000i  0.000000000000000 +
0.000000000000000i
  0.000000000000000 + 0.000000000000000i -0.742525626722238 +
0.000000000000000i
```

The eigen values of A + BF are different than A.

Part D

the vector F = [0, 2.6, 0, -0.09] satisfies the given constraints