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MTH20D
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MTH20D
Matlab HW3

EXERCISE 3.1:
Part A
Output >>

```
x =
                 0
   0.200000000000000
   0.400000000000000
   0.600000000000000
   0.800000000000000
   1.000000000000000
   1.200000000000000
   1.400000000000000
   1.600000000000000
   1.800000000000000
   2.000000000000000


y =
   1.000000000000000
   0.800000000000000
   0.608000000000000
   0.440000000000000
   0.312000000000000
   0.240000000000000
   0.240000000000000
   0.328000000000000
   0.520000000000000
   0.832000000000000
   1.280000000000000
```

Estimates

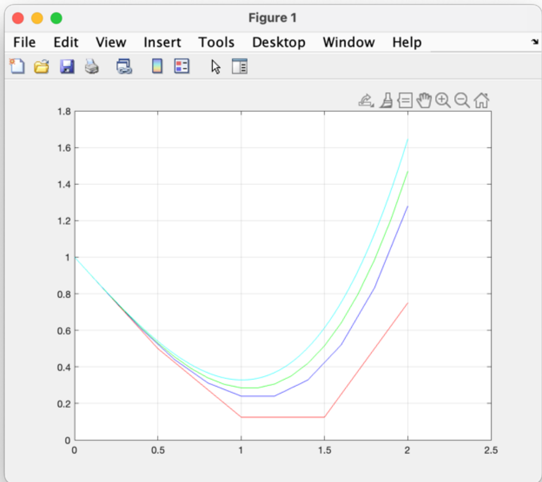
y(1) = 0.240000000000000
y(2) = 1.280000000000000

h=0.1

y(1)= 0.285000000000000
y(2)= 1.470000000000001

h=0.01

y(1)= 0.328350000000000
y(2)= 1.646700000000004



Part B

Dy/dx = x^2 – 1
y = (1/3)x^3 – x + C
C = 1

y(1) = 1/3
y(2) = 5/3

Part C

I) Euler's formula gives better solutions as h decreases. As the interval shrinks the estimation that the derivative represents comes closer to approximating the actual function y = I(t)

II) Euler's formula gives estimations with decreasing accuracy the farther it moves away from the initial value. This is because each y value is computed off the previous estimated y value, so the mistakes and inaccuracy compound to given increasingly incorrect results. There are probably classes of functions, with an oscillating y, where this effect does not occur.

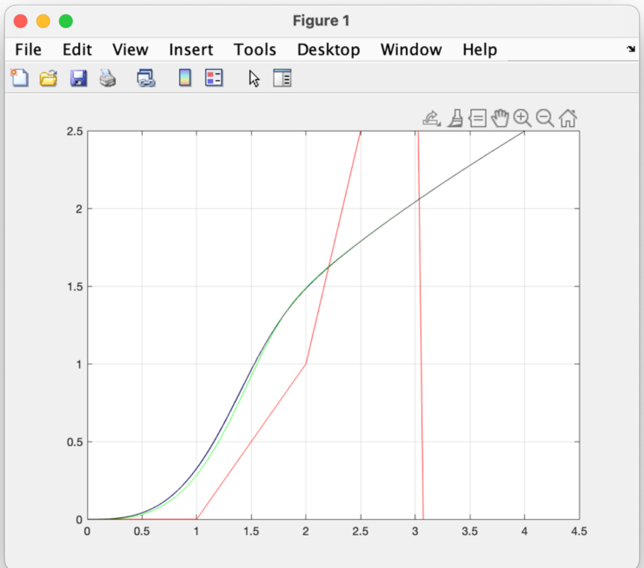
Exercise 3.2

(5 & 6) Matlab accesses the first element in the newly generated x and y row vectors and sets them to the initial values passed into the function.
(7) Matlab loops through the number of increments in the interval. It uses the value it computed at the start of the function. The loop starts at i=2 since we have already defined the initial condition at i=1.

(8 & 9) Matlab accesses the ith element in the x and y row vectors, and computes their value based on the previous values of x and y given by Euler's formula. It accesses those previous values of x and y by writing either x(i-1) or y(i-1).

Exercise 3.3
Part A

Looking at the graph, the h=0.1 case is significantly more accurate, and does not blow up to extreme +y or -y values as the h=0.25, h=0.5 cases do



Part B

The smaller changes to the increment size have increasingly less significant effects on the accuracy of the plot. The jump from h=1 to h=0.1 has the most considerable effect ('r' to 'g'), as the plot no longer sporadically jumps between extreme values of y, however each 1/10 change to the increment size maintains the same shape of the graph.

The initial plot in the assignment and all plots for h<=0.1 are almost entirely consistent on the selected interval.

Exercise 3.4
Part A

X	y	h(x)	error
0	1.000000000000000	1.000000000000000	0
0.250000000000000	1.031119947345269	1.031087578289355	0.000032369055914
0.500000000000000	1.122417709585642	1.122417438109627	0.000000271476015
0.750000000000000	1.268272990286521	1.268311131126179	0.000038140839658
1.000000000000000	1.459695924024716	1.459697694131860	0.000001770107144
1.250000000000000	1.684755462492111	1.684677637604731	0.000077824887380

1.500000000000000	1.929274542715971	1.929262798332297	0.000011744383674
1.750000000000000	2.178179042739787	2.178246055649492	0.000067012909705
2.000000000000000	2.416144096880461	2.416146836547143	0.000002739666682
2.250000000000000	2.628224524718298	2.628173622722739	0.000050901995559
2.500000000000000	2.801155208225341	2.801143615546934	0.000011592678407
2.750000000000000	2.924267278132111	2.924302378632464	0.000035100500353
3.000000000000000	2.989990479330401	2.989992496600445	0.000002017270044
3.250000000000000	2.994106029243575	2.994129676080546	0.000023646836971
3.500000000000000	2.936456643128134	2.936456687290796	0.000000044162663
3.750000000000000	2.820587613605972	2.820559357339561	0.000028256266411
4.000000000000000	2.653643353778242	2.653643620863612	0.000000267085369
4.250000000000000	2.446010208156413	2.446087489913793	0.000077281757380
4.500000000000000	2.210783332149258	2.210795799430780	0.000012467281522
4.750000000000000	1.962462654583433	1.962397847112023	0.000064807471409
5.000000000000000	1.716338716312398	1.716337814536774	0.000000901775624
5.250000000000000	1.487853831690964	1.487914522758159	0.000060691067195
5.500000000000000	1.291315970788756	1.291330225708740	0.000014254919984
5.750000000000000	1.138848530943809	1.138807582838479	0.000040948105330
6.000000000000000	1.039830128017163	1.039829713349634	0.000000414667529
6.250000000000000	1.000561453605040	1.000550581775501	0.000010871829539
6.500000000000000	1.023408610740486	1.023412374271977	0.000003763531491
6.750000000000000	1.106972269670238	1.106993655310923	0.000021385640686
7.000000000000000	1.246096465091977	1.246097745656695	0.000001280564718
7.250000000000000	1.432147439046900	1.432075826711305	0.000071612335595
7.500000000000000	1.653374043314734	1.653364682164974	0.000009361149760
7.750000000000000	1.896140758372729	1.896205642780747	0.000064884408018
8.000000000000000	2.145497408476206	2.145500033808613	0.000002625332407
8.250000000000000	2.385813623362045	2.385747937452222	0.000065685909823
8.500000000000000	2.602024955037178	2.602011902684824	0.000013052352354
8.750000000000000	2.780796127975161	2.780845683605749	0.000049555630588
9.000000000000000	2.911127878622349	2.911130261884677	0.000002383262328
9.250000000000000	2.984763714748071	2.984765173467324	0.000001458719253
9.500000000000000	2.997176072598027	2.997172156196378	0.000003916401649
9.750000000000000	2.947590311462322	2.947579803977993	0.000010507484328
10.000000000000000	2.839070752163858	2.839071529076453	0.000000776912595

Part B

ODE45 Results	Euler Results	h(x)	error1
1.000000000000000	1.000000000000000	1.000000000000000	0
1.031119947345269	1.000000000000000	1.031087578289355	0.000032369055914
1.122417709585642	1.061850989813631	1.122417438109627	0.000000271476015
1.268272990286521	1.181707374464682	1.268311131126179	0.000038140839658
1.459695924024716	1.352117064470515	1.459697694131860	0.000001770107144
1.684755462492111	1.562484810672489	1.684677637604731	0.000077824887380

1.929274542715971	1.799730965511386	1.929262798332297	0.000011744383674
2.178179042739787	2.049104712162400	2.178246055649492	0.000067012909705
2.416144096880461	2.295101198880884	2.416146836547143	0.000002739666682
2.628224524718298	2.522425555587304	2.628173622722739	0.000050901995559
2.801155208225341	2.716943854809284	2.801143615546934	0.000011592678407
2.924267278132111	2.866561890835273	2.924302378632464	0.000035100500353
2.989990479330401	2.961977138848356	2.989992496600445	0.000002017270044
2.994106029243575	2.997257140863323	2.994129676080546	0.000023646836971
2.936456643128134	2.970208357230796	2.936456687290796	0.000000044162663
2.820587613605972	2.882512550308391	2.820559357339561	0.000028256266411
2.653643353778242	2.739622220622805	2.653643620863612	0.000000267085369
2.446010208156413	2.550421596795823	2.446087489913793	0.000077281757380
2.210783332149258	2.326674257238677	2.210795799430780	0.000012467281522
1.962462654583433	2.082291727822403	1.962397847112023	0.000064807471409
1.716338716312398	1.832468530578558	1.716337814536774	0.000000901775624
1.487853831690964	1.592737461912774	1.487914522758159	0.000060691067195
1.291315970788756	1.378003838556126	1.291330225708740	0.000014254919984
1.138848530943809	1.201618757163528	1.138807582838479	0.000040948105330
1.039830128017163	1.074548987788713	1.039829713349634	0.000000414667529
1.000561453605040	1.004695113238982	1.000550581775501	0.000010871829539
1.023408610740486	0.996400309102093	1.023412374271977	0.000003763531491
1.106972269670238	1.050180306124046	1.106993655310923	0.000021385640686
1.246096465091977	1.162691324569201	1.246097745656695	0.000001280564718
1.432147439046900	1.326937974248898	1.432075826711305	0.000071612335595
1.653374043314734	1.532708194001774	1.653364682164974	0.000009361149760
1.896140758372729	1.767208188195459	1.896205642780747	0.000064884408018
2.145497408476206	2.015857882973253	2.145500033808613	0.000002625332407
2.385813623362045	2.263197444629099	2.385747937452222	0.000065685909823
2.602024955037178	2.493848497188934	2.602011902684824	0.000013052352354
2.780796127975161	2.693470275344807	2.780845683605749	0.000049555630588
2.911127878622349	2.849651263783355	2.911130261884677	0.000002383262328
2.984763714748071	2.952680885093794	2.984765173467324	0.000001458719253
2.997176072598027	2.996153256438903	2.997172156196378	0.000003916401649
2.947590311462322	2.977365476323450	2.947579803977993	0.000010507484328
2.839070752163858	2.897485677917882	2.839071529076453	0.000000776912595

ErrorEuler
0
0.031087578289355
0.060566448295996
0.086603756661497
0.107580629661345
0.122192826932242
0.129531832820911

0.129141343487093
0.121045637666259
0.105748067135435
0.084199760737649
0.057740487797190
0.028015357752089
0.003127464782777
0.033751669940000
0.061953192968830
0.085978599759193
0.104334106882030
0.115878457807897
0.119893880710380
0.116130716041785
0.104822939154614
0.086673612847386
0.062811174325049
0.034719274439079
0.004144531463481
0.027012065169884
0.056813349186877
0.083406421087495
0.105137852462407
0.120656488163200
0.128997454585288
0.129642150835360
0.122550492823123
0.108163405495890
0.087375408260943
0.061478998101322
0.032084288373529
0.001018899757476
0.029785672345457
0.058414148841429

The answers generated by the ODE45 call gave far more accurate answers