Topology Midterm

Name: ________________________________

Problem 1 [20 points]

For this problem, just write true or false. **You do NOT need to justify your answers.**

a) True or False: In a topological space, arbitrary intersections of closed sets are closed.

b) True or False: If $\mathcal{T}_1$ and $\mathcal{T}_2$ are topologies on a set X, then $\mathcal{T}_1 \cup \mathcal{T}_2$ is a topology on X.

c) True or False: If $\mathcal{T}_1$ and $\mathcal{T}_2$ are topologies on a set X, then $\mathcal{T}_1 \cap \mathcal{T}_2$ is a topology on X.

d) True or False: If Y is a subspace of X, then a subset $U \subseteq Y$ is open in Y if and only if U is open in X.

Problem 2 [20 points]

A map $f : X \to Y$ is called **open** if the image on any open subset of X is an open subset of Y.

a) Prove that an open, surjective, continuous map is a quotient map.

b) Let $\mathbb{R}_{cof}$ be $\mathbb{R}$ with the cofinite topology. Consider the function $f : \mathbb{R}_{cof} \to \mathbb{R}_{cof}$ defined by

$$f(x) = \sin(x)$$

Is f continuous? Is it open? Prove your answer.

Problem 3 [20 points]

Consider the set of real numbers $\mathbb{R}$ with the following topology:

$$\mathcal{T} = \{U \subseteq \mathbb{R} : \forall x \in U, -x \in U\}$$

You do not need to prove that $\mathcal{T}$ is a topology.

a) Let $A = [-1, 3)$. Is A open? Is A closed? Prove your answer.

b) Determine the interior and closure of A. Prove your answer.

c) What are the limit points of A. **You do NOT need to prove your answer.**

d) Is A Hausdorff with the subspace topology? Prove your answer.

Problem 4 [20 points]

Let X be a topological space, we define a subset $D \subseteq X \times X$ by

$$D = \{(x, x) : x \in X\}$$

Prove that X is Hausdorff if and only if D is closed in $X \times X$.

Bonus Problem [2 points]

What is one thing you like and one thing you would like to see changed about the class?