Topology Final

Name: ______________________________

Problem 1 [20 points]

(a) Give the definition of a topological space.

(b) Give the definition of connected.

(c) Give the definition of compact.

(d) Give the definition of simply connected.

Problem 2 [20 points]

For this problem, just write true or false. **You do NOT need to justify your answers.**

(a) True or False: If X is compact and $q : X \to Y$ is a quotient map, then Y is compact.

(b) True or False: Let A be a connected subspace of X. Then the closure of A is connected.

(c) True or False: Let A be a subspace of X. If the closure of A is connected, then A is connected.

(d) True or False: The symbols 0 and ∞ (subspaces of $\mathbb{R}^2$) are homeomorphic.

Problem 3 [20 points]

This question is asking you to prove statements that we have proven in class, do not just cite them.

(a) Let $f : X \to Y$ be a continuous function. Prove that if X is connected, then $f(X)$ is connected.

(b) Let Z be a compact subspace of a Hausdorff space X. Prove that Z is closed.

Problem 4 [40 points]

Pick 2 of the following problems to solve. If you submit more than two, mark which two you would like graded.

(a) Let $q : X \to \mathbb{S}^1$ be a continuous map where X is simply connected and let $i : \mathbb{S}^1 \to \mathbb{S}^1$ be the identity map. Prove that there does not exist a continuous map $\widetilde{i} : \mathbb{S}^1 \to X$ such that $q \circ \widetilde{i} = i$.

(b) Let X be a topological space and suppose there is a subspace $Z \subseteq X$ that is infinite, closed and has the discrete topology (in the subspace topology). Prove that X is not compact.

(c) Let A and B be compact subspaces of a Hausdorff space X. Prove that $A \cup B$ and $A \cap B$ are compact.