

# Legible Model Predictive Control for Autonomous Driving on Highways

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## Abstract:

Safety and efficiency are two defining factors for autonomous vehicles. While there is already extensive literature on how to safely cope with highway situations, approaches for higher efficiency are usually designed on an individual basis and are often accompanied by an increased risk of collision. Here, in addition to focusing on the individual behavior of the autonomous ego vehicle, we also consider how to support other traffic participants in correctly inferring the ego vehicle's future maneuvers, thus, enabling secure and efficient traffic flow. We propose a legible model predictive control method that provides a framework to improve the readability of the ego vehicle's planned maneuvers, while simultaneously optimizing factors such as comfort and energy efficiency. A simulation of a highway scenario is presented to demonstrate the effectiveness of our proposed method.

*Keywords:* Model Predictive Control, Legibility, Autonomous Vehicles

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## 1. INTRODUCTION

Autonomous driving has gained increasing interest in both research and industry over the past decade. In the future, autonomous vehicles will offer, among many benefits, increased safety and personal independence, particularly by supporting humans on long distance drives. Advances in sensor and computing technology, cognition, and control have all paved the way for highly advanced driver assistance systems as well as fully autonomous prototypes. One of the key requirements for such autonomous vehicles is ensuring safe driving behavior. However, these requirements can lead to a tradeoff between safety and efficiency. As autonomous vehicles must first and foremost be safe, the resulting maneuvers and trajectories are often chosen conservatively. Consider a highway scenario with two autonomous vehicles intending to overtake a leading vehicle. If both vehicles pursue a safe course, this can lead to a situation where both cars hesitate; each vehicle inhibits the other from overtaking due to safety constraints, resulting in an unresolved standoff, as algorithms of autonomous vehicles are typically conservative. As turn signals are, if at all, often only used when the lane change actually occurs, they are not sufficient for reasonable predictions. This raises the question of how to improve efficiency in such scenarios, while keeping risk at a similar level. It is assumed that vehicle-to-vehicle (V2V) communication is not a dependable solution in the near future as this would require a substantial number of vehicles communicating with each other to be effective. Furthermore, in other scenarios aiming at increasing legibility for pedestrians or bicycles, V2V communication is not applicable. One of the approaches to reduce conservativeness of high-

way maneuvers was suggested by Schildbach and Borrelli (2015) and Carvalho et al. (2014) using stochastic Model Predictive Control (MPC). In these works, chance constraints are used to loosen hard constraints and allow for a non-zero collision probability within a defined confidence level. However, safety is decreased in order to obtain a less conservative trajectory. Among other works, Gray et al. (2013) assume stochastic driver models and subsequently use these models to then plan trajectories. Sadigh et al. (2016a) suggest forcing other traffic participants to react to an unexpected ego vehicle (EV) maneuver, cutting into the lane of another car, for example, in order to gather information about the human's internal state, e.g. his awareness. However, this could be interpreted as a potential hazard by other drivers and possibly cause accidents. Sadigh et al. (2016b) apply Inverse Reinforcement Learning to acquire the human driver's reward function, which is then incorporated into a controller, planning to interfere with the human driver in a manner that benefits the autonomous EV. However, if the human driver's controls vary from the learned model due to overfitting, the EV's elevated aggressiveness can potentially result in collisions. The summarized prior works all focus on improving the ego vehicle performance, either by allowing riskier maneuvers or by gathering more information on other drivers.

In contrast, we suggest cooperatively including other vehicles in the EV's trajectory planning process, eventually resulting in benefits for all vehicles involved. Other traffic participants can plan their trajectories more efficiently if they can correctly assess the EV's intention. We therefore propose a novel legibility MPC method that increases the readability of the EV's future maneuvers, while optimizing

other factors such as energy efficiency and comfort at the same time. Inferring the EV's planned maneuver then enables other, either human driven or autonomous vehicles to plan trajectories efficiently and resolve situations where multiple vehicles block one another due to competing intentions. Thus, conservativeness can be reduced, while preserving safety constraints.

Due to its ability to handle constraints and nonlinear systems on a finite horizon, MPC has proved effective for trajectory planning in autonomous driving as stated by Katrakazas et al. (2015) or Levinson et al. (2011). Alami et al. (2006) and Kruse et al. (2012), among others, state that legibility results from predictability; however, we use a definition of legibility in human-robot interaction provided by Dragan and Srinivasa (2013). They declare that a motion is legible if it allows the spectator to confidently derive the robot's correct goal given an initial trajectory. Predictable motion is defined as the trajectory an observer would expect if he knew the robot's goal prior to the execution. To illustrate this, they give an example of a robot reaching out to grasp the right one of two bottles that are located next to each other. Knowing which bottle the goal is, the observer would predict the robot to follow a straight path to the bottle. However, the beginning of this trajectory would make it difficult for another spectator, who does not have knowledge of the goal, to infer which bottle the robot is aiming to grasp. Therefore, the robot should start a motion that exaggerates its movement to the right to emphasize its goal, the right bottle, which is then considered a legible motion. Dragan et al. (2013) state that the legibility of a motion depends on the required time until an observer derives the robot's actual goal. To reduce the necessary time, the goal inference probability of a spectator is modeled. The robot then uses this inference model to generate legible motion.

In summary, the contribution of this work is to develop a legible model predictive control method for an autonomous vehicle. In our case, the framework is used to increase the probability that other traffic participants will correctly infer the EV's planned maneuvers without loosening safety constraints. This is done while simultaneously optimizing for other objectives such as energy consumption and comfort. The presented method is evaluated in a highway scenario simulation exhibiting the effectiveness of the method.

The remainder of this paper is structured as follows. Section 2 presents an introductory example. The legible model predictive controller is derived in Sec. 3, whereas Sec. 4 introduces the vehicle models. The implementation and simulation setup as well as the results are shown in Sec. 5. This is followed by concluding remarks and an outlook in Sec. 6.

## 2. INTRODUCTORY EXAMPLE: TWO-LANE HIGHWAY OVERTAKING SCENARIO

Consider a highway scenario consisting of three vehicles on two lanes illustrated in Fig. 1. Initial velocities of the three vehicles are assumed to fulfill

$$v_0^{\text{OV}} > v_0^{\text{EV}} > v_0^{\text{LV}}, \quad (1)$$

where  $v^{\text{EV}}$ ,  $v^{\text{OV}}$ , and  $v^{\text{LV}}$  are the velocities of the ego vehicle (EV) and the two target vehicles, i.e., the observing

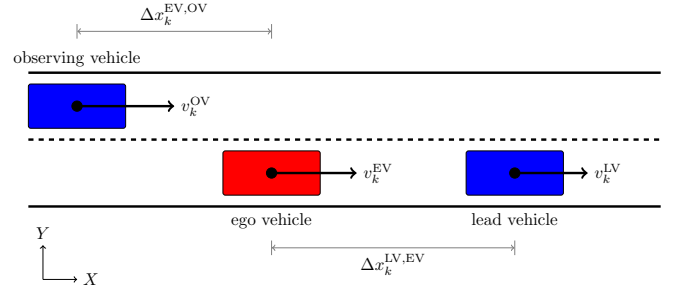


Fig. 1. Qualitative illustration of a two-lane highway scenario: the two target vehicles, LV and OV, are depicted by blue boxes, the EV is the red box. The EV is planning a legible maneuver to safely and efficiently overtake the LV, while considering the OV's presumed goal to overtake as well.

vehicle (OV) and lead vehicle (LV), respectively.

We introduce the longitudinal distances between the LV and the EV  $\Delta x_k^{\text{LV,EV}} = \|x_k^{\text{LV}} - x_k^{\text{EV}}\|$  and between the EV and the OV  $\Delta x_k^{\text{EV,OV}} = \|x_k^{\text{EV}} - x_k^{\text{OV}}\|$  with the longitudinal positions  $x_k^{\text{OV}}$ ,  $x_k^{\text{EV}}$ ,  $x_k^{\text{LV}}$  at time step  $k \in \mathbb{N}_0$ . The LV maintains a constant speed and keeps its position in the right lane throughout the scenario. As the OV's velocity is larger than the velocities of the LV and EV, it aims to overtake both vehicles. The OV can therefore adapt its velocity but stays in its left lane. The EV's target is to overtake the LV. If we assume the OV is autonomous, it will only overtake if a safe maneuver is possible, as algorithms for autonomous vehicles are generally conservative. This results in two possible EV maneuvers  $\mathcal{M} = \{M^{\text{lk}}, M^{\text{ot}}\}$ : a lane keeping maneuver  $M^{\text{lk}}$ , where the EV keeps its lane until the OV has passed, and an overtaking maneuver  $M^{\text{ot}}$ , where the EV overtakes the LV before the OV passes.

## 3. LEGIBLE MODEL PREDICTIVE CONTROL

### 3.1 Problem Statement

We assume an EV maneuver was selected, and our aim is to now generate an EV trajectory that is readable for other traffic participants, i.e., the OV, so that traffic efficiency is improved. For trajectory generation we design a legible model predictive controller with the cost function

$$J = J_{\text{gen}} + w_{\text{leg}} J_{\text{leg}}^M, \quad (2)$$

consisting of two cost function terms. The first term  $J_{\text{gen}}$  focuses on non-legibility related factors such as energy efficiency or comfort. The second term  $J_{\text{leg}}^M$  addresses the legibility of the EV's planned maneuver  $M$ , with weight  $w_{\text{leg}}$ . The objective therefore is to design the legibility cost function term  $J_{\text{leg}}^M$  such that the EV input  $\mathbf{U} = (\mathbf{u}_k, \mathbf{u}_{k+1}, \dots, \mathbf{u}_{k+N-1})^\top$  optimizes the legibility of the maneuver, i.e., the probability that the OV correctly infers the EV maneuver. This results in the MPC problem structure

$$\mathbf{U}^* = \arg \min_{\mathbf{U}} J_{\text{gen}} + w_{\text{leg}} J_{\text{leg}}^M \quad (3a)$$

$$\text{s. t. } \xi_{k+1} = f(\xi_k, \mathbf{u}_k), \quad k \in \mathbb{N}_0 \quad (3b)$$

$$\xi_{k+j} \in \Xi, \quad j = 0, \dots, N-1 \quad (3c)$$

$$\xi_{k+N} \in \Xi_f \quad (3d)$$

$$\mathbf{u}_{k+j} \in \mathcal{U}, \quad (3e)$$

with the EV input  $\mathbf{U}$  and EV state  $\xi$ , time step  $k$ , prediction horizon  $N$ , and the sets of admissible states, terminal states, and inputs  $\Xi$ ,  $\Xi_f$ , and  $\mathcal{U}$ , respectively. The function  $f(\xi_k, \mathbf{u}_k)$  represents the vehicle model introduced in Sec. 4.

### 3.2 Legibility Background

The legibility cost function in (2) is designed using the legibility definition of Dragan et al. (2013). They define legible motion using an inference function

$$\mathcal{I}_L : \mathcal{Z} \rightarrow \mathcal{G}. \quad (4)$$

This function infers a goal  $G \in \mathcal{G}$  from the goal set  $\mathcal{G}$  based on an initial part of a trajectory  $\zeta \in \mathcal{Z}$ . A motion is considered legible if the observer can conclude the correct goal

$$\mathcal{I}_L(\zeta_{S \rightarrow Q}) = G \quad (5)$$

based on an incomplete initial trajectory  $\zeta_{S \rightarrow Q}$ , which starts at  $\zeta(t_0)$  and is evaluated at  $\zeta(t_Q)$ . Given the initial trajectory  $\zeta_{S \rightarrow Q}$ , there are multiple possible goals with individual probabilities. The observer's inferred goal is modeled to be the most likely goal, which is determined by

$$\mathcal{I}_L(\zeta_{S \rightarrow Q}) = \arg \max_{G \in \mathcal{G}} P(G \mid \zeta_{S \rightarrow Q}). \quad (6)$$

In other words, observing the initial path  $\zeta_{S \rightarrow Q}$ , the spectator would then assume that the planned goal at the end of the completed trajectory is  $G$ . This inference model can subsequently be used to plan legible motion.

### 3.3 Design of Legibility Cost Function

In order to generate legible maneuvers, the OV's inference function must first be chosen. Using the probability  $P(M \mid \xi_k)$  that the EV will perform maneuver  $M \in \mathcal{M}$  given the current vehicle state  $\xi_k$  at time step  $k$ , we can formulate the EV's assumption of the OV inference model

$$\mathcal{I}_L^{\text{OV}}(\xi_k) = \arg \max_{M \in \mathcal{M}} P(M \mid \xi_k). \quad (7)$$

It is then the EV's aim to maximize  $P(M^{\text{plan}} \mid \xi_k)$ , i.e., increasing the OV's probability of correctly inferring the EV's planned maneuver  $M^{\text{plan}}$ . This results in the requirements that  $J_{\text{leg}}^M$  is minimal for  $P(M^{\text{plan}} \mid \xi_k) = 1$  and maximal for  $P(M^{\text{plan}} \mid \xi_k) = 0$ . Thus, multiple cost function designs to generate legible trajectories are possible. We propose the legibility cost function term

$$J_{\text{leg}}^M = \sum_{j=0}^N \frac{1}{c + P(M \mid \xi_{k+j})}, \quad (8)$$

where  $c > 0$  is a parameter and the prediction horizon is  $N$ . The constant  $c$  is necessary to guarantee a non-zero denominator. However, the choice of  $c$  affects the weight  $w_{\text{leg}}$  associated with the cost function term  $J_{\text{leg}}^M$ , especially if the planned maneuver is not perceptible, i.e.,

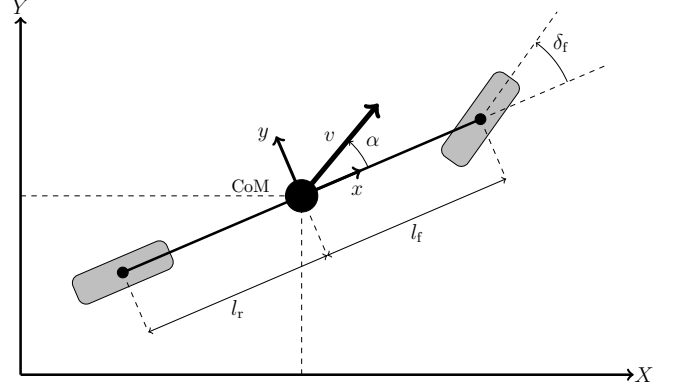


Fig. 2. Qualitative illustration of the dynamic bicycle model: the vehicle velocity, body slip angle, and steering angle are represented by  $v$ ,  $\alpha$ , and  $\delta_f$ , respectively. The center of mass is denoted by CoM and the distance of the front and rear axles to the CoM is given by  $l_f$  and  $l_r$ , respectively.

$P(M \mid \xi_k) \rightarrow 0$ .

The general cost function term  $J_{\text{gen}}$  has the form

$$J_{\text{gen}} = J_{\text{gen},f}(\xi_{k+N}) + \sum_{j=0}^{N-1} l(\xi_{k+j}, \mathbf{u}_{k+j}), \quad (9)$$

where the function  $l(\xi_{k+j}, \mathbf{u}_{k+j})$  is the stage cost and the terminal cost is  $J_{\text{gen},f}(\xi_{k+N})$  with the final state  $\xi_{k+N}$ .

## 4. VEHICLE MODELING

### 4.1 Ego Vehicle Dynamics

The EV is represented by a nonlinear dynamic bicycle model based on the approach of Rajamani (2005), as seen in Fig. 2. Kong et al. (2015) show that this model is more appropriate for high velocities than the kinematic bicycle model. The vehicle motion is described by the differential equations

$$\ddot{x} = a_x, \quad (10a)$$

$$\ddot{y} = -\dot{x}\dot{\psi} + \frac{2}{m}(F_{c_f} \cos \delta_f + F_{c_r}), \quad (10b)$$

$$\ddot{\psi} = \frac{2}{I_z}(l_f F_{c_f} \cos \delta_f - l_r F_{c_r}), \quad (10c)$$

where  $x$  and  $y$  represent the vehicle longitudinal and lateral position in the vehicle frame, respectively, and  $\psi$  is the vehicle orientation. The EV state vector is  $\xi^{\text{EV}} = [x, \dot{x}, y, \dot{y}, \psi, \dot{\psi}]^T$ . The input vector  $\mathbf{u}^{\text{EV}} = [a_x, \delta_f]^T$  consists of the longitudinal acceleration  $a_x$  and the steering angle  $\delta_f$ . In the following we omit the index EV, as only the EV state and input are considered. The vehicle mass is given by  $m$ , while  $I_z$  denotes the yaw inertia. The lateral forces on the front and rear tires are given by  $F_{c_f}$  and  $F_{c_r}$ , respectively. We use a linear tire model according to Rajamani (2005) resulting in

$$F_{c_i} = -C_{\alpha_i} \alpha_i, \quad i \in \{f, r\}, \quad (11a)$$

$$\alpha_i = \arctan \frac{v_{c_i}}{v_{l_i}}. \quad (11b)$$

The tire slip angle is denoted by  $\alpha_i$ , the tire cornering stiffness by  $C_{\alpha_i}$ , and

$$v_{c_i} = v_{y_i} \cos \delta_i - v_{x_i} \sin \delta_i, \quad i \in \{f, r\}, \quad (12a)$$

$$v_{l_i} = v_{y_i} \sin \delta_i + v_{x_i} \cos \delta_i, \quad (12b)$$

$$v_{y_f} = \dot{y} + l_f \dot{\psi}, \quad (12c)$$

$$v_{y_r} = \dot{y} - l_r \dot{\psi}, \quad (12d)$$

$$v_{x_i} = \dot{x} - \frac{l_w}{2} \dot{\psi}, \quad (12e)$$

where the rear tire angle is  $\delta_r = 0$  and the lateral and longitudinal tire velocities are  $v_c$  and  $v_l$ , respectively. The longitudinal distance of the tires to the vehicle center of mass (CoM) is denoted by  $l_f$  and  $l_r$ . The vehicle's track width is  $l_w$ .

The vehicle dynamics in the inertial frame is given by

$$\dot{X} = \dot{x} \cos \psi - \dot{y} \sin \psi, \quad (13a)$$

$$\dot{Y} = \dot{x} \sin \psi + \dot{y} \cos \psi, \quad (13b)$$

with the position vector  $[X, Y]^\top$ . The vehicle model is discretized using the Runge-Kutta-Fehlberg method.

#### 4.2 Target Vehicles

The OV and LV are represented by a double integrator model, similar to (10a), as only longitudinal acceleration is considered. The LV's velocity remains constant throughout the scenario, whereas the OV's velocity depends on which EV maneuver it expects. Based on the EV's observed current state  $\xi_k$  and the possible maneuvers  $\mathcal{M} = \{M^{\text{ot}}, M^{\text{lk}}\}$  the OV determines the probabilities

$$P(M | \xi_k), \quad M \in \mathcal{M}, \quad (14)$$

of the EV either performing a future overtaking maneuver or keeping its lane until the OV passes. Figure 3 exemplifies that once the EV moves towards the right side of its lane, the OV expects an increased probability  $P(M^{\text{lk}} | \xi_k)$  of the EV staying in its lane. In contrast, if the EV approaches the left lane, the OV assesses it more likely that the EV will perform a future lane changing maneuver. We design a simple routine for the OV depending on the inferred probabilities. If the belief  $P(M^{\text{lk}} | \xi_k)$  exceeds the threshold  $\bar{P}^{\text{lk}}$ , the OV expects the EV to keep its lane. Thus, the OV accelerates until it reaches its maximal allowed velocity and overtakes both other vehicles. If both probabilities,  $P(M^{\text{lk}} | \xi_k)$  and  $P(M^{\text{ot}} | \xi_k)$ , lie underneath their respective thresholds  $\bar{P}^{\text{lk}}$  and  $\bar{P}^{\text{ot}}$ , the OV is unable to infer the EV's intent with certainty. Thus, the OV decelerates until  $\Delta x_k^{\text{EV,OV}} > x_{\text{safe}}$  to keep a safe distance  $x_{\text{safe}}$  to the EV and oscillates about this safety distance until it is able to reliably infer the EV's maneuver. If  $P(M^{\text{ot}} | \xi_k) > \bar{P}^{\text{ot}}$ , the OV expects the EV to overtake the LV without waiting for the OV to pass. The OV decelerates until  $\Delta x_k^{\text{EV,OV}} > \Delta x_{\text{large}}$  to provide the EV with increased space to change lanes.

### 5. EXEMPLARY SIMULATION STUDY

#### 5.1 Implementation and Simulation Setup

A simulation was carried out in MATLAB<sup>®</sup> as a proof of concept to validate the legible model predictive controller of (3). We used the MPC routine utilizing *fmincon* developed by Grüne and Pannek (2017) as the base for

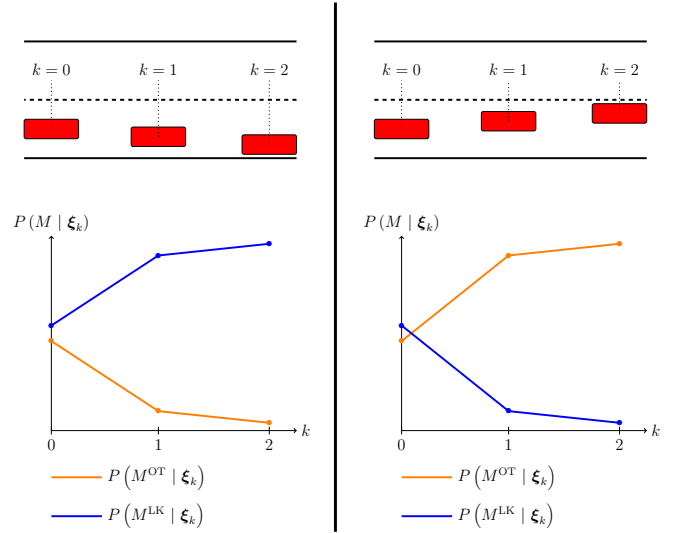


Fig. 3. Qualitative development of the probability for two maneuvers given three steps; Left side: EV lane keeping maneuver; Right side: EV overtaking maneuver; The OV's belief  $P(M | \xi_k)$  of an EV maneuver (lane keeping or overtaking) given the steps  $k \in \{1, 2, 3\}$  is displayed in the bottom graphs.

implementing our method. It is to note, however, that the implementation is not yet optimized for online trajectory planning. All quantities in this section are given in SI units unless stated otherwise.

For the implementation a probability function  $P(M | \xi_k)$  needs to be chosen with  $0 \leq P(M | \xi_k) \leq 1$ . This function represents the OV's estimated probabilities for the two possible EV maneuvers, overtaking  $M^{\text{ot}}$  and lane keeping  $M^{\text{lk}}$ , and is then used in the legibility cost function term in (8). In this work, the probability function of an expected overtaking maneuver is chosen to be

$$P(M^{\text{ot}} | \xi_k) = q_1 \exp \left( y_k^{\text{EV}} - \left( \Delta y_{\text{lane}} - \frac{w^{\text{EV}}}{2} \right) \right) + q_2 \exp \left( q_3 \left( \Delta x_{\text{safe}}^{\text{EV,LV}} - \Delta x_k^{\text{EV,LV}} \right) \right), \quad (15)$$

with time step  $k$  and weights  $q_1 = 0.2$ ,  $q_2 = 0.8$ , and  $q_3 = 0.2$ . The EV's lateral position is denoted by  $y_k^{\text{EV}}$ , the lane width by  $\Delta y_{\text{lane}}$ , and the vehicle width by  $w^{\text{EV}}$ . The right boundary of the right lane is set to  $y = 0$ . The first term of (15) is maximal if the EV is on the left boundary of its lane and decreases exponentially the further it moves to the right side, whereas the second term analyzes the EV's longitudinal distance to the LV. We assume that the EV always keeps at least a minimal distance to the LV, i.e.,  $\Delta x_k^{\text{LV,EV}} \geq \Delta x_{\text{safe}}$ , as part of (3c), which therefore results in a maximal value for the second term. The further the EV then falls back, the smaller the second term gets, decreasing the expected probability of an EV overtaking maneuver. While it is possible to increase the longitudinal distance indefinitely to lower the second term of (15) to zero, this is not possible for the first term, as the lane boundary limits the lateral motion to the right. It is necessary that  $q_1 + q_2 = 1$ , so that the probability is bounded to  $P(M^{\text{ot}} | \xi_k) \leq 1$ . Analogously, the probability of a lane keeping maneuver is defined by  $P(M^{\text{lk}} | \xi_k) = 1 - P(M^{\text{ot}} | \xi_k)$ . It is to note that the

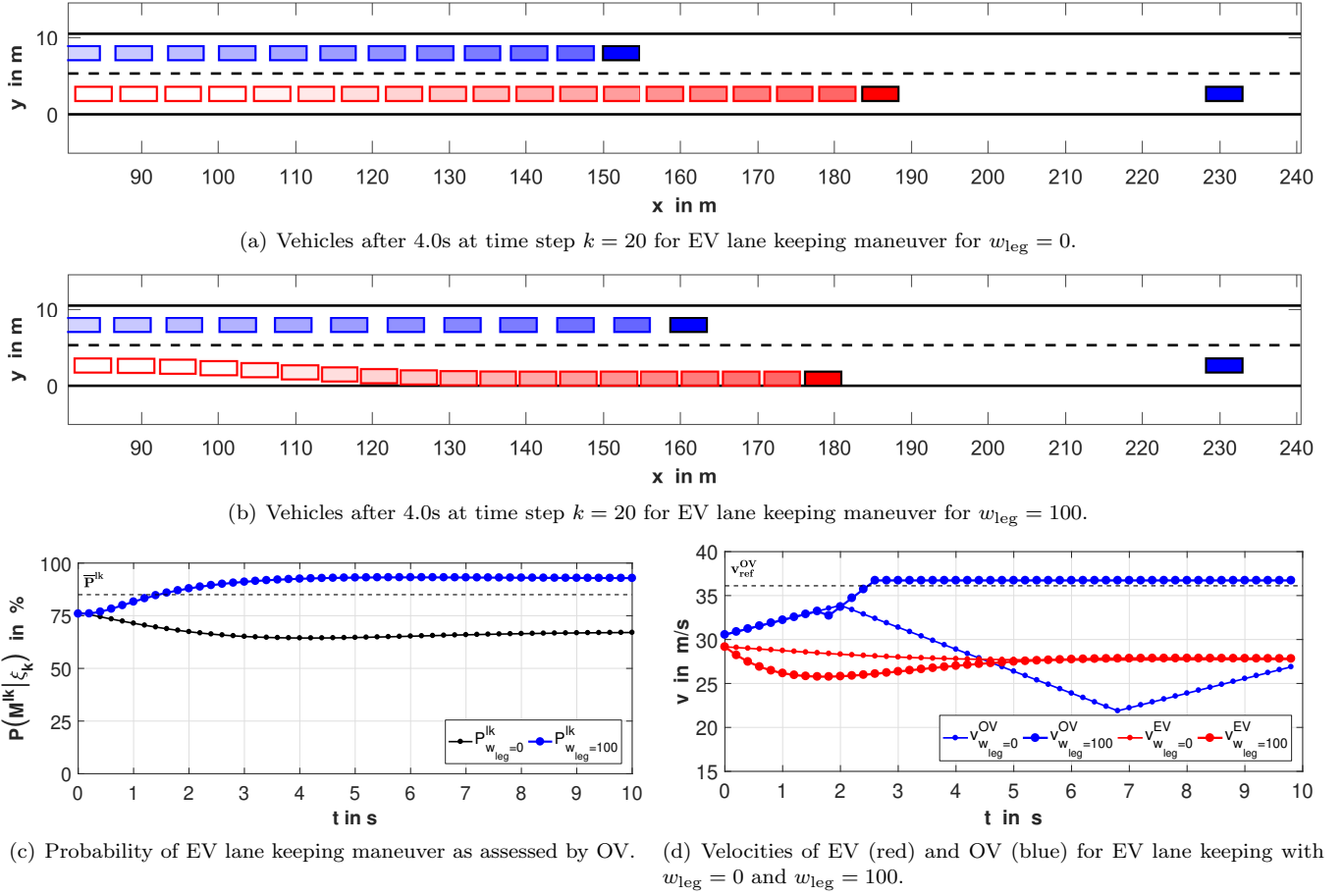


Fig. 4. (a): The highway scenario without a legibility cost function term after four seconds is illustrated. The red box represents the EV, the blue boxes display the target vehicles. Fading boxes show past states. Past states of the LV are omitted since its velocity remains constant. Increasing space between past boxes represents acceleration. The OV is unable to identify the EV's planned lane keeping maneuver and stays behind. (b): It is shown that the EV supports the OV in inferring the EV's correct maneuver, which leads to the OV overtaking. (c): The OV's expected probability of an EV lane keeping maneuver increases. (d): The EV decelerates greater than in the non-legible case. The OV needs to decelerate if it cannot infer the EV maneuver but accelerates and overtakes if the OV's motion is legible.

probability function of (15) is a design choice and can be adapted depending on the scenario and the respective OV inference model.

The general cost function  $J_{gen}$  in (9) is designed according to suggested costs in Althoff et al. (2017), which yields the stage and terminal cost

$$l_{gen} = q_a (a_{x,k+j})^2 + q_{\Delta\delta} (\Delta\delta_{f,k+j})^2 + q_{\Delta x} (\Delta x_{k+j}^{LV, EV} - \Delta x_{ref})^2 + q_{\psi} (\psi_{k+j})^2, \quad (16a)$$

$$J_{f,gen} = q_{\Delta x} (\Delta x_{k+N}^{LV, EV} - \Delta x_{ref})^2 + q_{\psi} (\psi_{k+N})^2, \quad (16b)$$

where  $\Delta\delta_{f,k} = \delta_{f,k} - \delta_{f,k-1}$  represents the steering rate of the EV at time step  $k$ . This cost function minimizes acceleration and steering changes for comfort, tracks the reference distance to the LV  $\Delta x_{ref} = 45$ , and aims to align the EV with the road. The weights are set to  $[q_a, q_{\Delta\delta}, q_{\Delta x}, q_{\psi}] = [1, 100, 0.1, 50]$ , whereas the weight used for (3a) is  $w_{leg} = 100$  and the constant of the legibility cost function term in (8) is  $c = 0.001$ . The probability thresholds are set to  $\bar{P}^{lk} = \bar{P}^{ot} = 0.85$ . The lane width is  $\Delta y_{lane} = 5.25$  and for the EV we assume  $[l_f^{EV}, l_r^{EV}, l_w^{EV}] =$

$[2.25, 2.25, 1.5]$  and  $[w^{EV}, m^{EV}, I_z^{EV}, l_{f,r}^{EV}, C_{\alpha f,r}^{EV}] = [1.83, 2000, 3344, 2.25, 34377]$ . The state and input constraints according to  $\Xi$ ,  $\Xi_f$ , and  $\mathcal{U}$  in (3) are

$$\Delta x_k^{LV, EV} \geq \Delta x_{safe}, \quad (17a)$$

$$Y_k^{EV} \in \left[ \frac{w^{EV}}{2}, \Delta y_{lane} - \frac{w^{EV}}{2} \right], \quad (17b)$$

$$a_{x,k} \in [-9, 6], \quad (17c)$$

$$\delta_{f,k} \in [-0.245, 0.245], \quad (17d)$$

$$\delta_{f,k} - \delta_{f,k-1} = \Delta\delta_{f,k} \in [-0.5, 0.5]. \quad (17e)$$

We also choose  $\Delta x_{safe} = 40$  and  $\Delta x_{large} = 50$ . At the initial time, all three vehicles are centered in their respective lanes and heading straight with starting positions and velocities  $[x_0^{OV}, v_0^{OV}] = [31, 30.6]$ ,  $[x_0^{EV}, v_0^{EV}] = [78, 29.2]$ , and  $[x_0^{LV}, v_0^{LV}] = [125, 27.8]$ . The time step is  $\Delta t = 0.2$  with the horizon length  $N = 20$ .

## 5.2 Results

It is first assumed that the EV plans to let the OV pass, and then the EV will change lanes and overtake the LV.

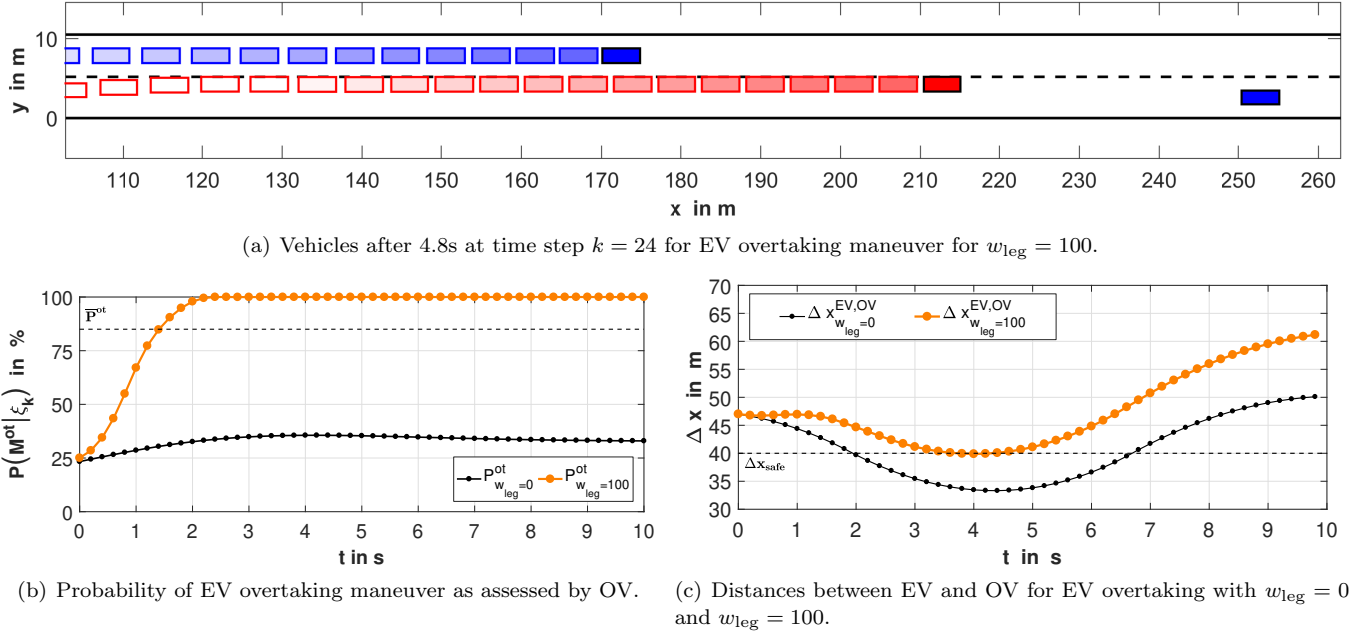


Fig. 5. (a): The EV helps the OV in correctly inferring the EV’s planned overtaking maneuver. (b): The probability of a future EV overtaking maneuver as estimated by the OV increases. (c): If an EV overtaking maneuver can be inferred, the OV extends its distance to the EV to enable an EV overtaking maneuver. In the case  $w_{\text{leg}} = 0$  the OV also starts increasing its distance at 4.6s but only to regain the safety distance  $\Delta x_{\text{safe}}$ .

Figure 4(a) shows the scenario after time  $t = 4.0$ s if no legibility term is included in the EV’s cost function, i.e.,  $w_{\text{leg}} = 0$ , and it keeps following the LV regularly. Despite aiming at overtaking both other vehicles, the OV decides to decelerate in order to satisfy the safety requirements. The situation remains unsolved until the OV infers the EV’s maneuver with enough certainty and can react accordingly. Adding the legibility cost function term, i.e.,  $w_{\text{leg}} = 100$ , the EV is increasing its distance to the LV as well as moving to the right side of its current lane, as depicted in Fig 4(b). The OV then infers that the probability that the EV will keep its lane has increased, as shown in Fig. 4(c). Once the threshold is reached, i.e., the OV is confident enough that the EV will keep its lane, the OV accelerates, depicted in 4(d), and securely overtakes both vehicles. This now provides the EV with the possibility of safely changing its lane to overtake the LV as well.

Figure 5(a) displays the case of the EV planning to overtake prior to the OV. Planning an overtaking maneuver including a legibility cost function term, i.e.,  $w_{\text{leg}} = 100$ , the EV will move towards the left side of its lane, clearly signaling to the OV that it plans to overtake. This time the probability, assessed by the OV, of an expected EV overtaking maneuver increases, as seen in Fig. 5(b). This causes the OV to increase its distance to the EV, shown in Fig. 5(c), and gives the EV the opportunity to safely change lanes and perform its overtaking maneuver prior to the OV.

### 5.3 Discussion

As previously mentioned, safety is not affected by considering legibility in the optimization problem as the same safety constraints hold as before. Even if an EV trajectory intended to increase legibility of a planned EV overtaking

maneuver causes a human driver to act more aggressively, i.e., aiming to pass first, the EV simply aborts its intended maneuver. Thus, efficiency is positively influenced without increasing the risk of collision.

The advantage of adding legibility to the cost function, instead of expressing it as a constraint, is that legibility can be neglected if the focus is directed more to optimizing other objectives or if constraints need to be met, safety requirements, for example. Legibility is only optimized to an extent that still allows the vehicle to reasonably consider other superior objectives.

The actual OV inference model described in Sec. 4.2 uses thresholds. If the probabilities for overtaking and lane keeping both remain below their respective thresholds, no maneuver is inferred. This varies from the assumed OV inference model in (7) used in the EV algorithm, which always selects the most probable maneuver as the inferred maneuver. Thus, the inference model maximized by the legibility cost function term in (8) is not required to perfectly match the OV’s actual inference model but still is effective.

To obtain an EV motion similar to the simulation, a simple implementation would be to directly move the OV left, right, or adapt its distance to the LV depending on the planned maneuver, e.g. move right and decelerate for lane keeping. However, this would require new tuning and evaluation if the scenario or the OV inference model change. Using the proposed legibility cost function term allows for the easy adaptation for altered OV inference models or different maneuvers and scenarios. While two maneuvers, overtaking and lane keeping, are considered here, the presented method can be applied to other traffic maneuvers and scenarios by utilizing the suggested legibility cost function term and solely including the adapted inference model chosen for the new scenario. It is not necessary



to include all maneuvers that could possibly occur while driving, as legibility is not required for safety purposes here. Each additional maneuver considered simply has a positive effect on traffic flow if the autonomous vehicle finds itself in one of the modeled scenarios. A different scenario worth considering is that of an urban environment: the EV plans to turn right, but there are two possible streets to turn into which are close to one another. It is therefore difficult for traffic participants to predict into which of the two streets the EV plans to turn. By altering the trajectory in a legible way, the EV's intention would be clarified.

For completeness, an algorithm would be necessary that applies the legibility cost function term only if useful in a specific situation, as there will be scenarios when legible behavior is not necessary, for example an empty highway. Furthermore, the current method only accounts for one OV. In dense traffic or on roads with more than two lanes, it will nevertheless be necessary to consider multiple OVs. A subsequent aim is therefore to extend the presented method to interact legibly with multiple OVs.

## 6. CONCLUSION

In this paper we presented a legible model predictive control method and showed its effectiveness in a simulation of a highway overtaking scenario. Based on a traffic participant inference model, estimating the probability of a future ego vehicle maneuver, a cost function term was designed to enhance the ability of other traffic participants to correctly and confidently infer the planned ego vehicle maneuver. This approach improves both safety and performance as maneuver perception is increased.

The presented legible MPC method can serve as a framework to generally improve legibility in autonomous driving if applied to a wider range of scenarios, such as urban driving, for example. However, generating legible motion is not only an important step towards improving autonomous vehicles, but the legible MPC structure can be applied in other fields as well.

Future research will focus on applying the presented approach to online trajectory planning and on an increased number of maneuvers and scenarios. User studies that could improve the inference model and evaluate the exact influence of our method based on the perception of real humans are also of interest.

## REFERENCES

- Alami, R., Clodic, A., Montreuil, V., Sisbot, E., and Chatila, R. (2006). Toward Human-Aware Robot Task Planning. In *AAAI Spring Symposium: To Boldly Go Where No Human-Robot Team Has Gone Before*, 39–46. Beijing, China.
- Althoff, M., Koschi, M., and Manzing, S. (2017). Commonroad: Composible benchmarks for motion planning on roads. In *2017 IEEE Intelligent Vehicles Symposium (IV)*, 719–726. Los Angeles, USA.
- Carvalho, A., Gao, Y., Lefevre, S., and Borrelli, F. (2014). Stochastic predictive control of autonomous vehicles in uncertain environments. In *12th International Symposium on Advanced Vehicle Control*. Tokyo, Japan.
- Dragan, A.D., Lee, K.C.T., and Srinivasa, S.S. (2013). Legibility and predictability of robot motion. In *2013 8th ACM/IEEE International Conference on Human-Robot Interaction (HRI)*, 301–308. Pittsburg, USA.
- Dragan, A. and Srinivasa, S. (2013). Generating legible motion. In *Proceedings of the Robotics: Science and Systems IX*. Berlin, Germany.
- Gray, A., Gao, Y., Lin, T., Hedrick, J.K., and Borrelli, F. (2013). Stochastic predictive control for semi-autonomous vehicles with an uncertain driver model. In *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*, 2329–2334. The Hague, Netherlands.
- Grüne, L. and Pannek, J. (2017). *Nonlinear Model Predictive Control*. Springer-Verlag, London.
- Katrakazas, C., Quddus, M., Chen, W.H., and Deka, L. (2015). Real-time motion planning methods for autonomous on-road driving: State-of-the-art and future research directions. *Transportation Research Part C: Emerging Technologies*, 60, 416 – 442.
- Kong, J., Pfeiffer, M., Schildbach, G., and Borrelli, F. (2015). Kinematic and dynamic vehicle models for autonomous driving control design. In *2015 IEEE Intelligent Vehicles Symposium (IV)*, 1094–1099. Seoul, South Korea.
- Kruse, T., Basili, P., Glasauer, S., and Kirsch, A. (2012). Legible robot navigation in the proximity of moving humans. In *2012 IEEE Workshop on Advanced Robotics and its Social Impacts (ARSO)*, 83–88. Munich, Germany.
- Levinson, J., Askeland, J., Becker, J., Dolson, J., Held, D., Kammel, S., Kolter, J.Z., Langer, D., Pink, O., Pratt, V., Sokolsky, M., Stanek, G., Stavens, D., Teichman, A., Werling, M., and Thrun, S. (2011). Towards fully autonomous driving: Systems and algorithms. In *2011 IEEE Intelligent Vehicles Symposium (IV)*, 163–168. Baden-Baden, Germany.
- Rajamani, R. (2005). *Vehicle Dynamics and Control*. Mechanical Engineering Series. Springer US.
- Sadigh, D., Sastry, S.S., Seshia, S.A., and Dragan, A. (2016a). Information gathering actions over human internal state. In *2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 66–73. Daejeon, South Korea.
- Sadigh, D., Sastry, S., Seshia, S.A., and Dragan, A.D. (2016b). Planning for autonomous cars that leverage effects on human actions. In *Proceedings of the Robotics: Science and Systems*, 66–73. Ann Arbor, MI, USA.
- Schildbach, G. and Borrelli, F. (2015). Scenario model predictive control for lane change assistance on highways. In *2015 IEEE Intelligent Vehicles Symposium (IV)*, 611–616. Seoul, South Korea.