Lecture 14: General MRAS

Plan:

1. Find a parametrization for the controller.
2. Derive the error model of the form

$$\varepsilon(t) = \underbrace{G_1(p)}_{\text{SPR}}\left[\phi^T(t)\,(\theta - \theta^0)\right]$$

   with ε constructed from the available on-line signals.

3. Use the passivity-based adaptation law

$$\dot{\theta} = -\gamma\,\phi(t)\,\varepsilon(t) \qquad \gamma > 0$$

   or a normalized version

$$\dot{\theta} = -\gamma\,\frac{\phi(t)\,\varepsilon(t)}{\alpha + \phi^T(t)\,\phi(t)}, \qquad \gamma, \alpha > 0$$

Defining Controller Structure

Plant:

$$\boxed{A(p)\, y(t) = b_0\, B(p)\, u(t)}$$

- $A(s)$ and $B(s)$ are stable monic polynomials with no common factors.
- $b_0 \neq 0$ is called high-frequency gain.
- $\deg\{A\} > \deg\{B\}$ – strictly proper transfer function.

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Model:

$$\boxed{A_m(p)\,y(t) = B_m(p)\,u_c(t)}$$

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Controller:

$$\boxed{R(p)\,u(t) = -S(p)\,y(t) + T(p)\,u_c(t)}$$

Pole Placement Design

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for $R_p(s)$ and $S(s)$.

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for $R_p(s)$ and $S(s)$.

Step 3: Take

$$R(s) = R_p(s)\, B(s), \qquad T(s) = t_0\, A_o(s)$$

assuming $B_m(s) = b_m \neq 0$ and so $t_0 = b_m/b_0$.

(Typically, $b_m = A_m(0)$ so that $B_m(0)/A_m(0) = 1$.)

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$$e = y - y_m \quad \Rightarrow \quad A_o A_m e = A_o A_m y - A_o A_m y_m$$

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Using $A\, R_p + b_0\, S = A_o\, A_m$ and $A_m\, y_m = b_0\, t_0\, u_c$:

$$A_o\, A_m\, e = (A\, R_p + b_0\, S)\, y - A_o\, b_0\, t_0\, u_c$$

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Manipulations with the error model

$$e = y - y_m \quad \Rightarrow \quad A_o A_m e = A_o A_m y - A_o A_m y_m$$

Using $T = A_o t_0$ and $R = R_p B$:

$$A_o A_m e = b_0 (R u + S y - T u_c)$$

Manipulations with the error model

$$e = y - y_m \quad \Rightarrow \quad A_o\, A_m\, e = A_o\, A_m\, y - A_o\, A_m\, y_m$$

We obtain:

$$e = \frac{b_0}{A_o\, A_m}\left(R\, u + S\, y - T\, u_c\right)$$

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$$e_f = \frac{Q}{P}\,e$$

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such that $\dfrac{b_0\,Q}{A_o\,A_m}$ is SPR and $\dfrac{Q}{P}, \dfrac{R}{P}, \dfrac{S}{P}, \dfrac{T}{P}$ are proper.

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If b_0 is known, it is left to rewrite

$$\left(\frac{R}{P}\, u + \frac{S}{P}\, y - \frac{T}{P}\, u_c\right)$$

in the form useful for parameter estimation.

Parametrization of the error model

Let

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where

$$\bar{R} = R - P_2 = (s^k + \cdots + r_k) - (s^k + \dots) = \bar{r}_1\, s^{k-1} + \cdots + \bar{r}_k$$

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The error dynamics can be now rewritten as

$$e_f = \underbrace{\frac{b_0\, Q}{A_o\, A_m}}_{\text{SPR}} \left(\frac{1}{P_1}\, u + \underbrace{\frac{\bar{R}}{P}\, u + \frac{S}{P}\, y - \frac{T}{P}\, u_c}_{\phi(t)^T\, \theta^0} \right)$$

Parametrization of the error model (cont'd)

The parametrization

$$\phi(t)^T\,\theta^0 = \frac{\bar{R}}{P}\,u + \frac{S}{P}\,y - \frac{T}{P}\,u_c$$

is defined by

$$\begin{aligned}\phi(t)^T &= \Big[\frac{p^{k-1}}{P(p)}u, \ldots, \frac{1}{P(p)}u, \frac{p^l}{P(p)}y, \ldots, \frac{1}{P(p)}y, \\ &\qquad -\frac{p^m}{P(p)}u_c, \ldots, -\frac{1}{P(p)}u_c\Big] \\ \theta^0 &= \left[\bar{r}_1,\ \ldots,\ \bar{r}_k,\ s_0,\ \ldots, s_l,\ t_0,\ \ldots,\ t_m\right]^T\end{aligned}$$

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So that in the case of known θ^0:

$$u = -P_1(p)\Big(\phi(t)^T\,\theta^0\Big) \quad \Leftrightarrow \quad u = \frac{T(p)}{R(p)}\,u_c - \frac{S(p)}{R(p)}\,y$$

Augmentation error-based design

In the case when θ^0 is unknown, we still have

$$e_f = \frac{b_0\, Q(p)}{A_o(p)\, A_m(p)} \left(\frac{1}{P_1(p)}\, u + \phi(t)^T\, \theta^0 \right)$$

and so if one could take

$$u = -P_1(p) \Big(\phi(t)^T\, \theta(t) \Big)$$

the result would be

$$e_f = -\frac{b_0\, Q}{A_o\, A_m} \Big(\phi(t)^T\, (\theta(t) - \theta^0) \Big)$$

in the form siutable for passivity-based adaptation.

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Since the closed-loop system with differentiations of θ in u for $\deg\{P_1\} \neq 0$ is impossible to realize, we take:

$$\boxed{u = -\theta(t)^T\, P_1(p) \Big(\phi(t) \Big)}$$

Augmentation-based adaptation

The error dynamics with the chosen controller are

$$e_f = \frac{b_0\, Q}{A_o\, A_m} \left(-\frac{1}{P_1}\, \theta^T P_1\, \phi + (\theta^0)^T \phi \right)$$

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The error dynamics with the chosen controller are

$$e_f = \frac{b_0\, Q}{A_o\, A_m} \Big[\underbrace{\left(\theta^0 - \theta\right)^T \phi}_{\text{zero when } \theta = \theta^0} + \underbrace{\theta^T \phi - \frac{1}{P_1} \left(\theta^T P_1\, \phi\right)}_{\text{zero when } \theta(t) = \text{const}} \Big]$$

Let us introduce the augmented error

$$\varepsilon(t) = \frac{b_0\, Q(p)}{A_o(p)\, A_m(p)}\, \phi(t)^T \left(\theta^0 - \theta(t)\right)$$

and the error augmentation

$$\eta(t) = -\theta(t)^T \phi(t) + \frac{1}{P_1(p)} \underbrace{\left(\theta(t)^T P_1(p)\, \phi(t)\right)}_{-u(t)}$$

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So that

$$\varepsilon(t) = \frac{Q(p)}{P(p)} \Big(y(t) - y_m(t) \Big) + \frac{b_0\, Q(p)}{A_o(p)\, A_m(p)}\, \eta(t)$$

General MRAS: Summary

Model to follow

$$y_m(t) = \frac{b_m}{A_m(p)}\, u_c(t)$$

Filtered error

$$e_f(t) = \frac{Q(p)}{P(p)}\Big(y(t) - y_m(t)\Big)$$

Error augmentation

$$\eta(t) = -\left(\phi(t)^T\theta(t) + \frac{1}{P(p)}\, u(t)\right)$$

Augmented error

$$\varepsilon(t) = e_f(t) + \frac{b_0\, Q(p)}{A_o(p)\, A_m(p)}\, \eta(t)$$

Adaptation law and control law

$$\theta(t) = -\frac{\gamma}{p}\Big(\phi(t)\,\varepsilon(t)\Big), \qquad u(t) = -\theta(t)^T\Big(P_1(p)\,\phi(t)\Big)$$

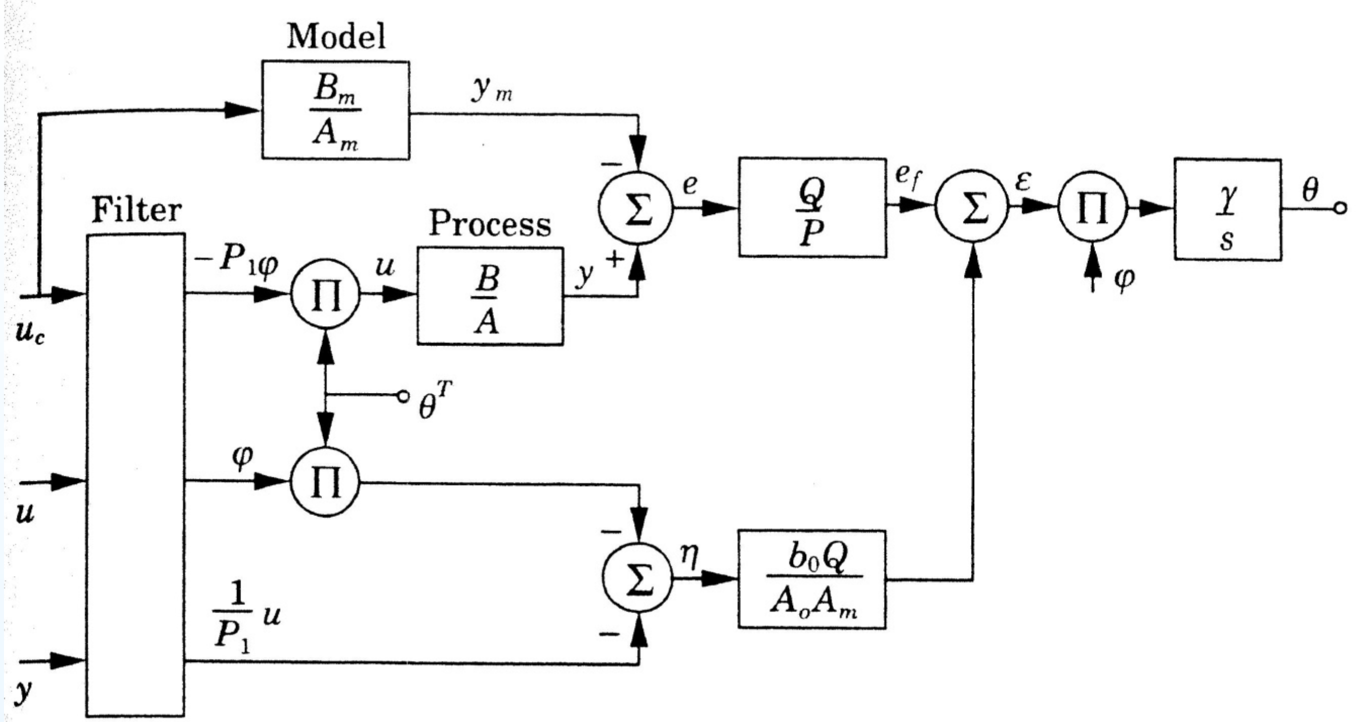

Figure 5.21 Block diagram of a model-reference adaptive system for a SISO system.

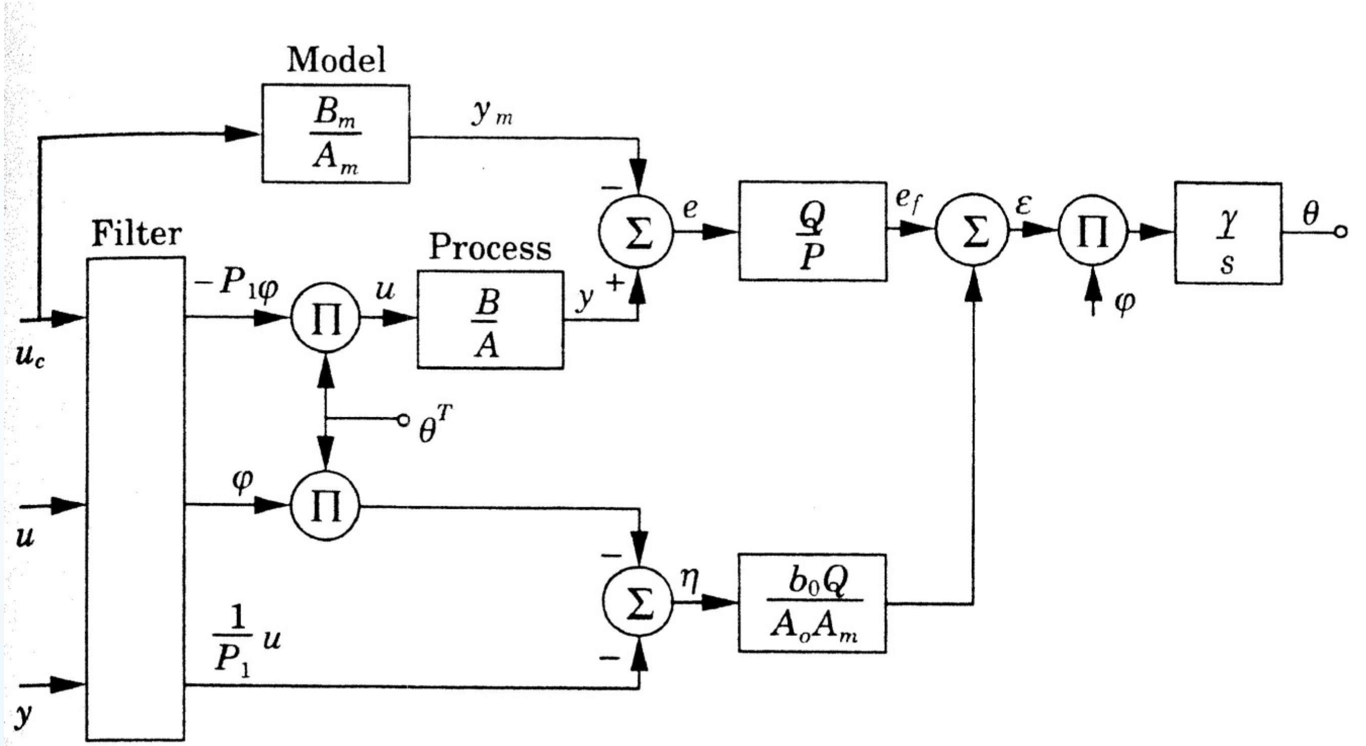

Figure 5.21 Block diagram of a model-reference adaptive system for a SISO system.

The filter block has three outputs:

$$-P_1(p)\,\phi(t), \qquad \phi(t), \qquad \frac{1}{P_1(p)}\,u(t)$$

$$\phi^T = \left[\frac{p^{k-1}}{P(p)}u, \ldots, \frac{1}{P(p)}u, \frac{p^l}{P(p)}y, \ldots, \frac{1}{P(p)}y, -\frac{p^m}{P(p)}u_c, \ldots, -\frac{1}{P(p)}u_c\right]$$

Realization of the filter

The filter should be realized carefully for ($n \geq k = \deg\{R\}$)

$$P_1(s) = s^n + \alpha_1 p^{n-1} + \cdots + \alpha_n, \qquad P_2(s) = s^k + \beta_1 p^{k-1} + \cdots + \beta_k$$

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One should implement

$$\dot{x} = \begin{bmatrix} -\beta_1 & -\beta_2 & \ldots & -\beta_{k-1} & -\beta_k \\ 1 & 0 & & 0 & 0 \\ & \ddots & & & \\ 0 & 0 & & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$\dot{z} = \begin{bmatrix} -\alpha_1 & -\alpha_2 & \ldots & -\alpha_{k-1} & -\alpha_k \\ 1 & 0 & & 0 & 0 \\ & \ddots & & & \\ 0 & 0 & & 1 & 0 \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} x_k$$

$$x = \begin{bmatrix} x_1 & \ldots & x_k \end{bmatrix}^T = \begin{bmatrix} \frac{p^{k-1}}{P(p)} u & \ldots & \frac{1}{P(p)} u \end{bmatrix}^T$$

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- Relative degree $\deg\{A\} - \deg\{B\} > 0$ should be known:

$$
\begin{aligned}
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&= \deg\{A_o\} + \deg\{A_m\} - (\deg\{A\} - \deg\{B\})
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- For minimal-degree pole-placement:

$$\deg\{A_m\} = \deg\{A\}, \quad \deg\{B_m\} = \deg\{B\}$$

$$\Rightarrow \quad \deg\{R\} = \deg\{A_m\} - 1$$

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$$\deg\{A_m\} = \deg\{A\}, \quad \deg\{B_m\} = \deg\{B\}$$

$$\Rightarrow \quad \deg\{R\} = \deg\{A_m\} - 1$$

Remark: The following choice is often possible:

$$Q(s) = A_o(s)\, A_m(s), \quad P_1(s) = A_m(s), \quad P_2(s) = A_o(s)$$

Example: 2nd-order MRAS

Consider the process and the model

$$G(s) = \frac{b_0}{s(s+a)}, \qquad G_m(s) = \frac{\omega^2}{s^2 + 2\zeta\omega\, s + \omega^2}$$

assuming that $b_0 = 2$ is known and $a = 1$ is unknown.

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The minimal-degree pole-placement controller is defined by

$$A_o(s) = s + a_0 = s + 2, \quad R(s) = s + r_1, \quad S(s) = s_0\, s = s_1,$$

$$t_0 = \frac{b_m}{b_0} \qquad \Rightarrow \quad T(s) = t_0\, s + t_1 = t_0\, A_0 = \frac{\omega^2}{b_0}(s+2)$$

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Let us take

$$Q(s) = A_o(s)\, A_m(s), \quad P_1(s) = A_m(s), \quad P_2(s) = A_o(s)$$

so that $\frac{b_0\, Q(p)}{A_o(p)\, A_m(p)} = b_0$ is SPR.

MRAS can be designed to estimate all five parameters

$$\theta = \left[r_1 - 2,\ s_0,\ s_1,\ t_0,\ t_1 \right]^T$$

$$\phi^T = \left[\frac{1}{P(p)} u, \frac{p}{P(p)} y, \frac{1}{P(p)} y, -\frac{p}{P(p)} u_c, -\frac{1}{P(p)} u_c \right]$$

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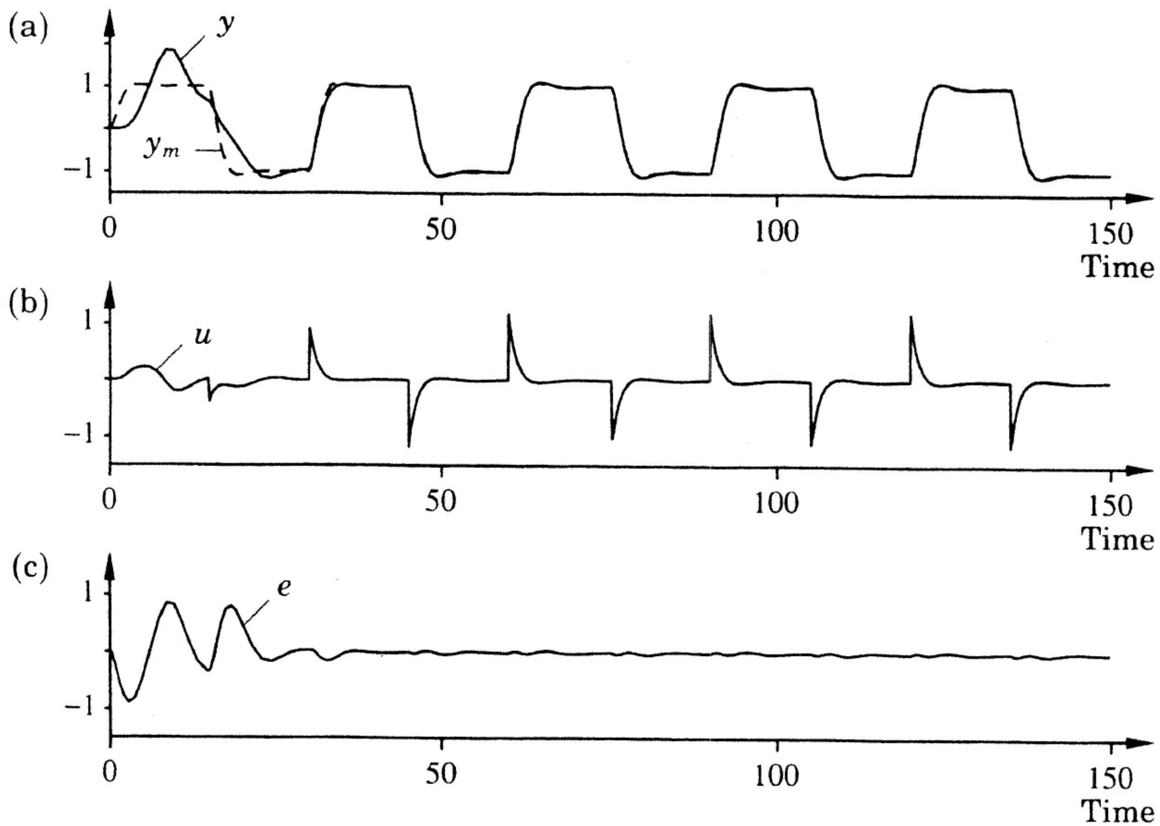

Figure 5.22 Simulation of the system in Example 5.14. (a) The process output (solid line) and the model output (dashed line). (b) The control signal. (c) The error $e = y - y_m$.

Example: reduced design

It is not hard to verify that θ^0 is

$$r_1 = 2\zeta\omega + a_0 - a, \qquad s_0 = (2\zeta\omega a_0 + \omega^2 - a r_1)/b_0,$$

$$s_1 = a_0\omega^2/b_0, \qquad t_0 = \omega^2/b_0, \qquad t_1 = a_0\omega^2/b_0$$

And so, the only unknown parameters are r_1 and s_0!

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And so, the only unknown parameters are r_1 and s_0!

Let us design a reduced MRAS.

Define

$$\theta = \Big[\underbrace{r_1 - 2,\ s_0,}_{\theta_a}\ \underbrace{s_1,\ t_0,\ t_1}_{\theta_b^0}\Big]^T,$$

$$\phi_a(t)^T = \left[\frac{1}{P(p)}u, \frac{p}{P(p)}y\right], \quad \phi_b(t)^T = \left[\frac{1}{P(p)}y, -\frac{p}{P(p)}u_c, -\frac{1}{P(p)}u_c\right]$$

Example: reduced design

It is not hard to verify that θ^0 is

$$r_1 = 2\zeta\omega + a_0 - a, \qquad s_0 = (2\zeta\omega a_0 + \omega^2 - a r_1)/b_0,$$

$$s_1 = a_0\omega^2/b_0, \qquad t_0 = \omega^2/b_0, \qquad t_1 = a_0\omega^2/b_0$$

And so, the only unknown parameters are r_1 and s_0!

Let us design a reduced MRAS.

Define

$$\theta = \Big[\underbrace{r_1 - 2,\ s_0,}_{\theta_a}\ \underbrace{s_1,\ t_0,\ t_1}_{\theta_b^0}\Big]^T,$$

$$\phi_a(t)^T = \left[\frac{1}{P(p)}u, \frac{p}{P(p)}y\right], \quad \phi_b(t)^T = \left[\frac{1}{P(p)}y, -\frac{p}{P(p)}u_c, -\frac{1}{P(p)}u_c\right]$$

So that

$$e_f(t) = \frac{b_0\,Q}{A_o\,A_m}\left[\phi^T\theta^0 + \frac{1}{P}u\right] = \frac{b_0\,Q}{A_o\,A_m}\left[\phi_a^T\theta_a^0 + \phi_b^T\theta_b^0 + \frac{1}{P}u\right]$$

Example: reduced design (cont'd)

Using the error dynamics

$$e_f(t) = \frac{b_0\, Q}{A_o\, A_m}\left[\phi_a^T \theta_a^0 + \phi_b^T \theta_b^0 + \frac{1}{P} u\right]$$

let us take

$$u(t) = -\theta_a(t)^T\, P_1(p)\, \phi_a(t) - P_1(p)\, \theta_b^0(t)^T\, \phi_b(t)$$

Example: reduced design (cont'd)

Using the error dynamics

$$e_f(t) = \frac{b_0\, Q}{A_o\, A_m}\left[\phi_a^T\theta_a^0 + \phi_b^T\theta_b^0 + \frac{1}{P}u\right]$$

let us take

$$u(t) = -\theta_a(t)^T\, P_1(p)\, \phi_a(t) - P_1(p)\, \theta_b^0(t)^T\, \phi_b(t)$$

Then, we will have

$$e_f(t) = \frac{b_0\, Q}{A_o\, A_m}\left[-\frac{1}{P_1}\left(\theta_a^T\, P_1\, \phi_a\right) + \phi_a^T\theta_a^0\right]$$

Example: reduced design (cont'd)

Using the error dynamics

$$e_f(t) = \frac{b_0\, Q}{A_o\, A_m}\left[\phi_a^T \theta_a^0 + \phi_b^T \theta_b^0 + \frac{1}{P} u\right]$$

let us take

$$u(t) = -\theta_a(t)^T\, P_1(p)\, \phi_a(t) - P_1(p)\, \theta_b^0(t)^T\, \phi_b(t)$$

Then, we will have

$$e_f(t) = \frac{b_0\, Q}{A_o\, A_m}\left[-\frac{1}{P_1}\left(\theta_a^T\, P_1\, \phi_a\right) + \phi_a^T \theta_a^0\right]$$

Hence

$$\varepsilon_a(t) = \frac{Q}{P} e + \frac{b_0\, Q}{A_o\, A_m} \eta_a, \qquad \eta_a = \frac{1}{P_1}\left(\theta_a^T\, P_1\, \phi_a\right) - \phi_a^T \theta_a$$

and we have reduced adaptation law

$$\dot{\theta}_a = -\gamma\, \phi_a(t)\, \varepsilon_a(t)$$

The case of unknown gain

Suppose only signum of b_0 is known.

Rewrite the error dynamics for $u = -P_1(\phi^T \theta)$

$$e_f(t) = \frac{b_0\, Q(p)}{A_o(p)\, A_m(p)} \left[\phi(t)^T \theta^0 + \frac{1}{P(p)}\, u(t)\right]$$

in the form

$$e_f(t) = b_0 \left[\phi_f(t)^T \theta^0 + u_f(t)\right]$$

using filtered signals

$$u_f(t) = \frac{Q}{A_o\, A_m(p)\, P}\, u(t), \quad \phi_f(t) = \frac{Q}{A_o\, A_m}\, \phi(t)$$

The case of unknown gain

Suppose only signum of b_0 is known.

Rewrite the error dynamics for $u = -P_1(\phi^T\theta)$

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using filtered signals

$$u_f(t) = \frac{Q}{A_o\, A_m(p)\, P}\, u(t), \quad \phi_f(t) = \frac{Q}{A_o\, A_m}\, \phi(t)$$

Let $\hat{b}_0(t)$ and $\theta(t)$ be the estimates for b_0 and θ^0 respectively.
Compute the predicted value

$$\hat{e}_f(t) = \hat{b}_0 \left[\phi_f(t)^T\theta + u_f(t)\right]$$

The case of unknown gain (cont'd)

The prediction error is

$$\varepsilon_p(t) = e_f(t) - \hat{e}_f(t) = e_f(t) - \hat{b}_0 \left[\phi_f(t)^T\theta + u_f(t)\right]$$

The case of unknown gain (cont'd)

The prediction error is

$$\varepsilon_p(t) = e_f(t) - \hat{e}_f(t) = e_f(t) - \hat{b}_0 \left[\phi_f(t)^T \theta + u_f(t)\right]$$

Using the MIT-rule:

$$\dot{\theta} = -\gamma_1 \frac{\partial \varepsilon_p}{\partial \theta} \varepsilon_p = -\gamma_1 \hat{b}_0 \phi_f \varepsilon_p = -\bar{\gamma}_1 \, \mathbf{sign}(b_0) \, \phi_f \, \varepsilon_p$$

and

$$\dot{\hat{b}}_0 = -\gamma_2 \frac{\partial \varepsilon_p}{\partial \hat{b}_0} \varepsilon_p = -\gamma_2 \left(u_f + \phi_f^T \theta\right) \varepsilon_p$$

The case of unknown gain (cont'd)

The prediction error is

$$\varepsilon_p(t) = e_f(t) - \hat{e}_f(t) = e_f(t) - \hat{b}_0 \left[\phi_f(t)^T \theta + u_f(t)\right]$$

Using the MIT-rule:

$$\dot{\theta} = -\gamma_1 \frac{\partial \varepsilon_p}{\partial \theta} \varepsilon_p = -\gamma_1 \hat{b}_0 \phi_f \varepsilon_p = -\bar{\gamma}_1 \operatorname{sign}(b_0) \phi_f \varepsilon_p$$

and

$$\dot{\hat{b}}_0 = -\gamma_2 \frac{\partial \varepsilon_p}{\partial \hat{b}_0} \varepsilon_p = -\gamma_2 \left(u_f + \phi_f^T \theta\right) \varepsilon_p$$

For the case of normalized MIT rule:

$$\dot{\theta} = -\bar{\gamma}_1 \operatorname{sign}(b_0) \frac{\phi_f \varepsilon_p}{\alpha + \phi_f^T \phi_f}, \qquad \dot{\hat{b}}_0 = -\gamma_2 \frac{\left(u_f + \phi_f^T \theta\right) \varepsilon_p}{\alpha + \phi_f^T \phi_f}$$

Next Lecture / Assignments:

Next meeting (May 21, 10:00-12:00, in A206Tekn): Adaptation in nonlinear systems.

Homework problem: Implement one of the algorithms discussed in this lecture for the second-order example.