Lecture 10: Indirect Minimum-Variance / Stochastic STR.

- Indirect Minimum Variance Adaptive Control.
- Stochastic Self Tuning Regulators.

Stochastic Linear Models

Assume the plant is represented by special ARMAX model

$$A(q)\,y(t) = B(q)\,u(t) + C(q)\,e(t)$$

where $\{e(t)\}$ – white noise, $d_0 = 1$,

$$\begin{aligned} A(q) &= q^n + a_1\,q^{n-1} + \cdots + a_n, & \deg\{A\} &= n \\ B(q) &= b_1\,q^{n-1} + \cdots + b_n, & \deg\{B\} &= n-1 \\ C(q) &= q^n + c_1\,q^{n-1} + \cdots + c_n, & \deg\{C\} &= n \end{aligned}$$

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Equivalently,

$$\begin{aligned} y(t) &= -a_1\, y(t-1) - \cdots - a_n\, y(t-n) \\ &\quad + b_1\, u(t-1) + \cdots + b_n\, u(t-n) \\ &\quad + e(t) + c_1\, e(t-1) + \cdots + c_n\, e(t-n) \end{aligned}$$

Example: approximate regression for dynamic noise

Consider the model

$$y(t) = -a\,y(t-1) + b\,u(t-1) + c\,e(t-1) + e(t)$$

where $|c| < 1$. It can not be transformed into regression form $y = \phi^T\theta$ but an approximation is possible.

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With bounded $\{y(k), u(k)\}$ given for $k < t$, we are given

$$f(k) = y(k) + a\,y(k-1) - b\,u(k-1)$$

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Then, we can try to compute $e(t-1)$ as an infinite series:

$$e(t-1) = f(t-1) - c\,e(t-2)$$

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$$e(t-1) = f(t-1) - c\,(f(t-2) - c\,e(t-3))$$

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Alternatively, we can estimate $e(t-1)$ recursively:

$$e(t) \approx \varepsilon(t): \qquad \varepsilon(t) = f(t) - c\,\varepsilon(t-1)$$

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Extended Least Square (ELS)

For the model

$$
\begin{aligned}
y(t) &= -a_1\, y(t-1) - \cdots - a_n\, y(t-n) \\
&\quad + b_1\, u(t-1) + \cdots + b_n\, u(t-n) \\
&\quad + e(t) + c_1\, e(t-1) + \cdots + c_n\, e(t-n)
\end{aligned}
$$

with $\theta^0 = \left[a_1,\ \ldots,\ a_n,\ b_1,\ \ldots,\ b_n,\ c_1,\ \ldots,\ c_n\right]^T$

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\end{aligned}
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with $\theta^0 = \Big[a_1,\ \ldots,\ a_n,\ b_1,\ \ldots,\ b_n,\ c_1,\ \ldots,\ c_n\Big]^T$

Define the prediction error

$$\varepsilon(t) = y(t) - \phi(t-1)^T \hat{\theta}(t-1)$$

with the regression vector

$$\phi(t-1)^T = \Big[-y(t-1),\ \ldots,\ u(t-n),\ \varepsilon(t-1),\ \ldots,\ \varepsilon(t-n)\Big]$$

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with $\theta^0 = \Big[a_1,\ \ldots,\ a_n,\ b_1,\ \ldots,\ b_n,\ c_1,\ \ldots,\ c_n\Big]^T$

Define the prediction error

$$\varepsilon(t) = y(t) - \phi(t-1)^T \hat{\theta}(t-1)$$

with the regression vector

$$\phi(t-1)^T = \Big[-y(t-1),\ \ldots,\ u(t-n),\ \varepsilon(t-1),\ \ldots,\ \varepsilon(t-n)\Big]$$

and apply standard RLS for the approximate regression model

$$y(t) = \phi(t-1)^T \theta + e(t)$$

ELS / RML

The ELS algorithm is given by RLS

$$
\begin{aligned}
\hat{\theta}(t) &= \hat{\theta}(t-1) + P(t)\,\phi(t-1)^T \varepsilon(t) \\
P(t)^{-1} &= P(t-1)^{-1} + \phi(t-1)\,\phi(t-1)^T \\
\varepsilon(t) &= y(t) - \phi(t-1)^T \hat{\theta}(t-1)
\end{aligned}
$$

with $e(\cdot)$ approximated by $\varepsilon(\cdot)$ in $\phi(t-1)$

ELS / RML

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\end{aligned}
$$

with $e(\cdot)$ approximated by $\varepsilon(\cdot)$ in $\phi(t-1)$

Another possible choice is Recursive Maximum Likelihood method (RML):

$$
\begin{aligned}
\hat{\theta}(t) &= \hat{\theta}(t-1) + P(t)\,\phi_f(t-1)^T\varepsilon(t) \\
P(t)^{-1} &= P(t-1)^{-1} + \phi_f(t-1)\,\phi_f(t-1)^T \\
\varepsilon(t) &= y(t) - \phi_f(t-1)^T\hat{\theta}(t-1) \\
\hat{C}(q)\,\phi_f(t) &= \phi(t)
\end{aligned}
$$

Example 4.4 (MV+ELS)

Consider the plant from previous lecture described by

$$y(t) = -a\, y(t-1) + b\, u(t-1) + c\, e(t-1) + e(t)$$

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$$y(t) = -a\,y(t-1) + b\,u(t-1) + c\,e(t-1) + e(t)$$

Suppose the real parameters $a = 0.9$, $b = 3$, $c = -0.3$ are not known and so the minimum variance controller

$$u(t) = \frac{a-c}{b}\,y(t) = s_0\,y(t), \qquad s_0 = (-0.3 - (-0.9))/3 = 0.2$$

cannot be applied directly.

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cannot be applied directly.

Let us combine MV controller

$$u(t) = \hat{s}_0(t)\,y(t), \qquad \hat{s}_0(t) = \frac{\hat{a}(t) - \hat{c}(t)}{\hat{b}(t)}$$

with ELS estimator for $\quad \hat{\theta}(t) = \left[\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t)\right]$ using

$$\phi(t-1)^T = \left[-y(t-1),\ u(t-1),\ \varepsilon(t-1)\right]$$

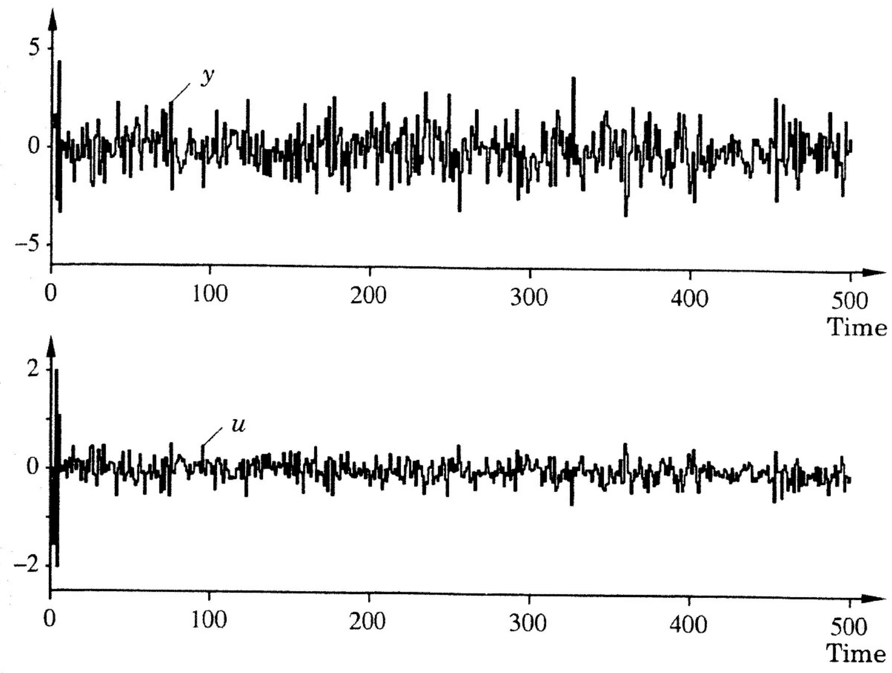

Figure 4.2 Output and input when an indirect self-tuning regulator based on minimum-variance control is used to control the system in Example 4.4.

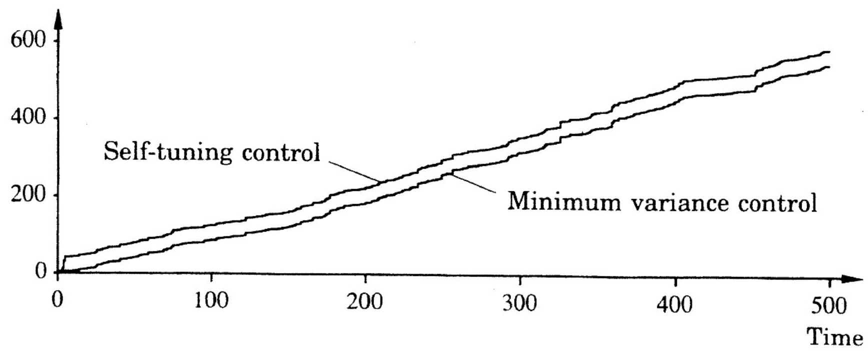

Figure 4.3 The accumulated loss when a self-tuning regulator and the optimal minimum-variance controller are used on the system in Example 4.4.

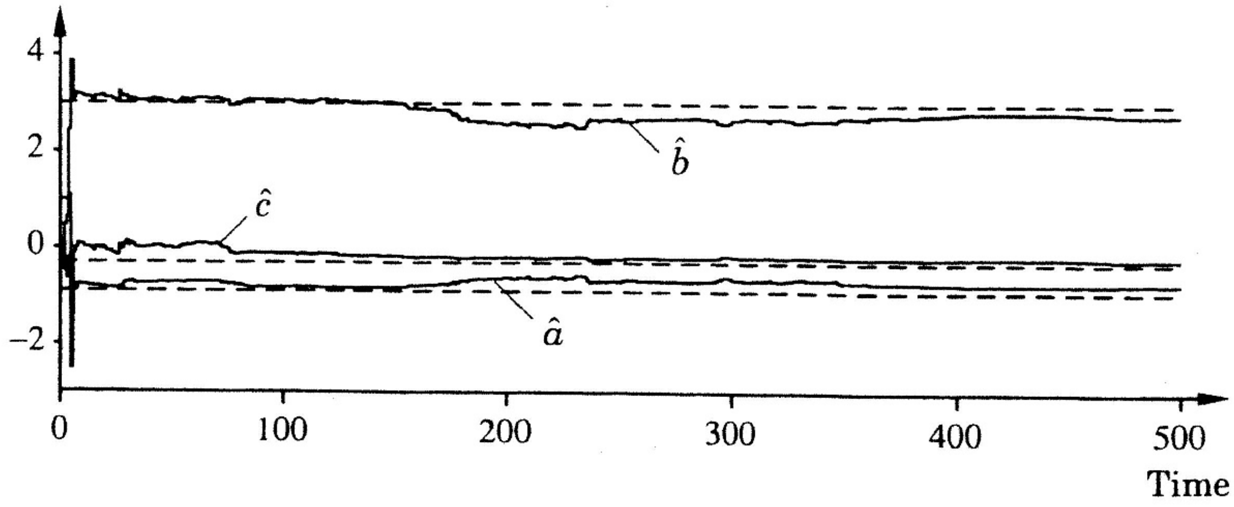

Figure 4.4 The estimated parameters $\hat{a}(t)$, $\hat{b}(t)$, and $\hat{c}(t)$ when the system in Example 4.4 is controlled. The dashed lines correspond to the true parameter values.

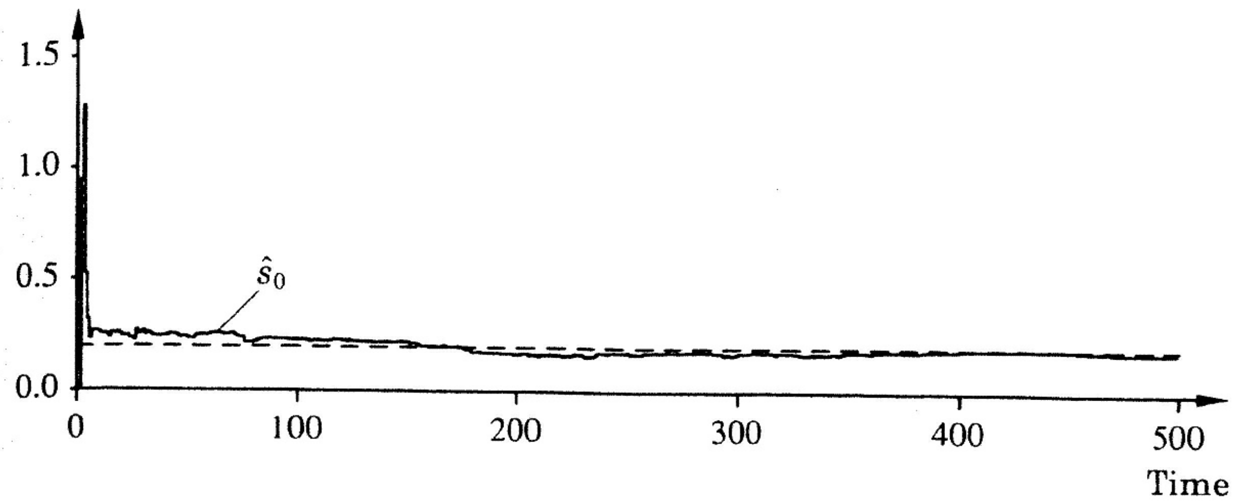

Figure 4.5 The controller parameter $\hat{s}_0(t)$ when the system in Example 4.4 is controlled. The dashed line is the optimal parameter for the minimum-variance controller.

Direct Minimum Variance Controller

The closed-loop system under MV controller

$$u = -\frac{S(q)}{R(q)}\, y(t), \qquad R(q) = B(q)\, F(q), \quad S(q) = G(q)$$

can be written in the form

$$y(t+d_0) = \frac{q\, G(q)}{C(q)}\, y(t) + \frac{q\, B(q)\, F(q)}{C(q)}\, u(t) + q^{1-d_0}\, F(q)\, e(t+d_0)$$

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Let us introduce the polynomials with backward shifts as

$$\frac{q\,R}{C} = \frac{r_0 q^n + \cdots + r_{n-1} q}{q^n + c_1 q^{n-1} + \cdots + c_n} = \frac{r_0 + \cdots + r_{n-1} q^{-n+1}}{1 + c_1 q^{-1} + \cdots + c_n q^{-n}} = \frac{R^*}{C^*}$$

to rewrite the system in the *direct form* for adaptation

$$y(t+d_0) = \frac{R^*(q^{-1})}{C^*(q^{-1})}\,u(t) + \frac{S^*(q^{-1})}{C^*(q^{-1})}\,y(t) + R_p^*(q^{-1})\,e(t+d_0)$$

Moving Average Self-Tuning Controller

The MA controller is defined by

$$u = -\frac{S(q)}{R(q)}\,y(t), \qquad R = R_p\,B^{+}, \quad q^{d-1}\,C = A\,R_p + B^{-}\,S$$

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The closed-loop system can be rewritten as follows

$$q^{d-1}\,C\,y(t) = A\,R_p\,y(t) + B^{-}\,S\,y(t)$$

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$$q^{d-1}\, C\, y(t) = R_p\, \boxed{\left(B\, u(t) + C\, e(t)\right)} + B^{-}\, S\, y(t)$$

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The closed-loop system can be rewritten as follows

$$C\,y(t+d) = q\,R_p\,B^{-}\,B^{+}\,u(t) + q\,R_p\,C\,e(t) + q\,B^{-}\,S\,y(t)$$

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$$y(t+d) = \frac{q\, B^{-}\, R}{C}\, u(t) + \frac{q\, B^{-}\, S}{C}\, y(t) + q\, R_p\, e(t)$$

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Using the polynomials with backward shifts

$$C^* = 1 + \cdots + c_n q^{-n}, \;\; R^* = r_0 + \cdots + r_{n-1} q^{-n+1}, \;\; S^* = \ldots$$

we obtain the system in the *direct form* for adaptation

$$y(t+d_0) = \frac{B^{-*}}{C^*}\Big(R^*(q^{-1})u(t) + S^*(q^{-1})y(t)\Big) + R_p^*(q^{-1})e(t+d_0)$$

Adding Parameter Estimation

The closed-loop system under MV and MA controllers is

$$y(t+d) = \frac{Q^*}{P^*}\Big(R^*(q^{-1})u(t)+S^*(q^{-1})y(t)\Big)+R_p^*(q^{-1})e(t+d)$$

with $d = d_0$ and $Q^*/P^* = 1/C^*$ for MV and

with $d \geq d_0$ and $Q^*/P^* = B^{-*}/C^*$ for MA.

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The fact that the denominator $C^*(q^{-1})$ is unknown is an obstacle to obtaining the standard regression form and using ELS or RML with over parametrization might be heavy.

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The fact that the denominator $C^*(q^{-1})$ is unknown is an obstacle to obtaining the standard regression form and using ELS or RML with over parametrization might be heavy.

However, standard RLS can be used for the approximate model

$$\varepsilon(t) = y(t)-R^*\,u_f(t-d) - S^*\,y_f(t-d) = y(t)-\phi(t-d)^T\hat{\theta}(t-1)$$

with a stable filter Q^*/P^* for the signals:

$$y_f(t) = \frac{Q^*(q^{-1})}{P^*(q^{-1})}\,y(t), \qquad u_f(t) = \frac{Q^*(q^{-1})}{P^*(q^{-1})}\,u(t)$$

Stochastic Self-Tuning Regulators

Surprisingly, adaptive controllers with sufficiently high degree of the polynomials with incorrect filters P^*/Q^*,

$$u(t) = -\frac{\hat{S}^*(q^{-1})}{\hat{R}^*(q^{-1})}\, y(t)$$

recover MV (no zero cancellations) for $d = d_0$ and

MA (with correct number of canceled zeros) for $d > d_0$.

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MA (with correct number of canceled zeros) for $d > d_0$.

The modification to include feedforward term is often useful:

$$y(t+d) = R^*(q^{-1})\, u(t) + S^*(q^{-1})\, y(t) - T^*(q^{-1})\, v(t) + \varepsilon(t+d)$$

where $v(t)$ can be a measurable disturbance and

$$u(t) = -\frac{\hat{S}^*(q^{-1})}{\hat{R}^*(q^{-1})}\, y(t) + \frac{\hat{T}^*(q^{-1})}{\hat{R}^*(q^{-1})}\, v(t)$$

Example 4.5 (Direct MV STR)

Consider the plant from Example 4.4

$$y(t) = -a\, y(t-1) + b\, u(t-1) + c\, e(t-1) + e(t)$$

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$$y(t) = -a\, y(t-1) + b\, u(t-1) + c\, e(t-1) + e(t)$$

The closed-loop system with MV controller can be approximated by the process

$$y(t+1) = r_0\, u(t) + s_0\, y(t) + \varepsilon(t+1), \qquad \frac{s_0}{r_0} = \frac{a-c}{b} = 0.2$$

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Let us use STR with $\hat{r}_0 = 1$ and

$$u(t) = -\frac{\hat{s}_0(t)}{\hat{r}_0}\, y(t),$$

with RLS estimator for $\quad \hat{\theta}(t) = \hat{s}_0(t)$

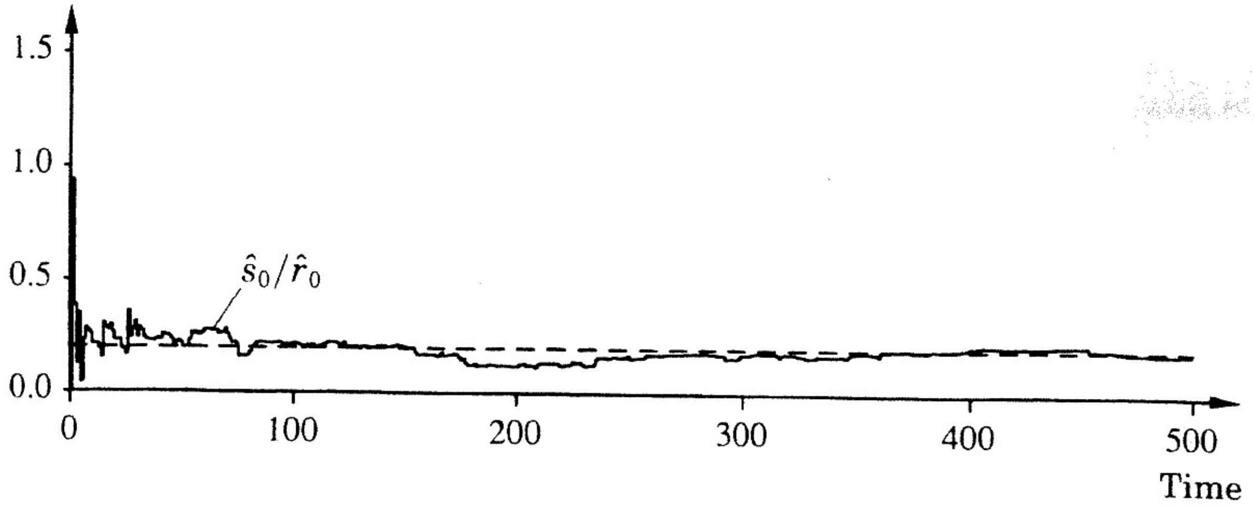

Figure 4.6 The parameter $\hat{s}_0/\hat{r}_0$ in the controller, when the process in Example 4.5 is controlled by using the direct minimum-variance self-tuning controller.

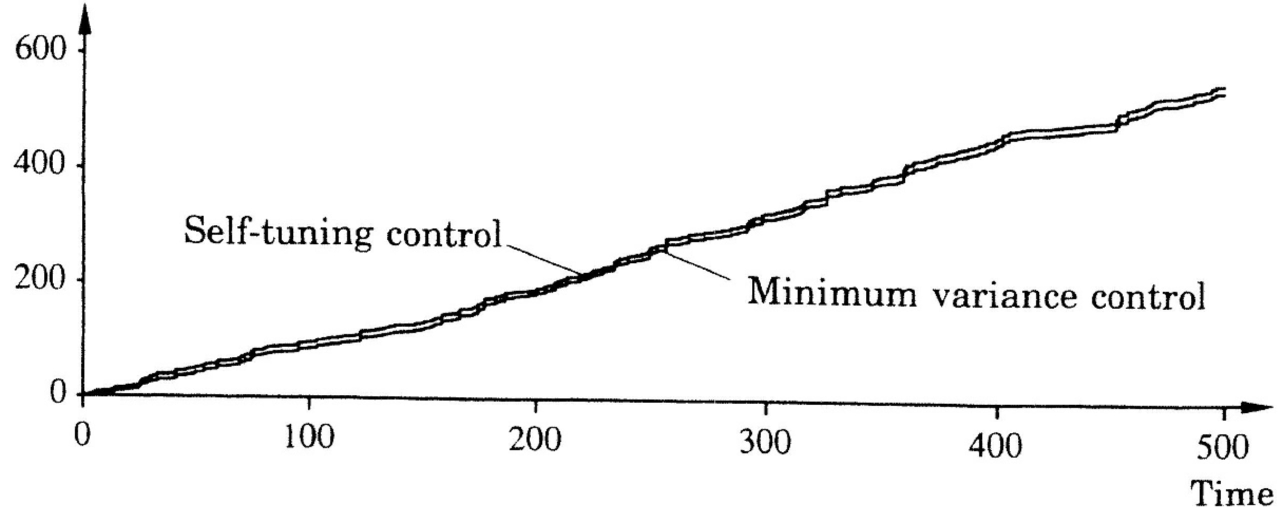

Figure 4.7 The loss function when the direct self-tuning regulator and the optimal minimum-variance controller are used on the system in Example 4.5.

Example 4.6 (Direct MV/MA STR)

Consider a sampled model of a delayed integrator

$$(q^2 - q)\, y(t) = (h - \tau)(q + b)\, u(t) + (q^2 + c\, q)\, e(t), \qquad b = \frac{\tau}{h - \tau}$$

with $c = 0.8$ and $h = 1$.

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Let us use STR with $d = 1$ or $d = 2$

$$u(t) = -\hat{s}_0(t)\, y(t) \quad \text{or} \quad u(t) = -\hat{s}_0(t)\, y(t) - \hat{r}_1(t)\, u(t-1)$$

with RLS estimator for $\hat{\theta}(t) = \hat{s}_0(t)$ or $\hat{\theta}(t) = [\hat{s}_0(t),\ \hat{r}_1(t)]^T$.

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$$u(t) = -\hat{s}_0(t)\, y(t) \quad \text{or} \quad u(t) = -\hat{s}_0(t)\, y(t) - \hat{r}_1(t)\, u(t-1)$$

with RLS estimator for $\hat{\theta}(t) = \hat{s}_0(t)$ or $\hat{\theta}(t) = [\hat{s}_0(t),\ \hat{r}_1(t)]^T$.

Note that since $|b| < 1 \quad \Leftrightarrow \quad \tau < h/2$, $d = d_0 = 1$ (MV) should not be used for $\tau > h/2 = 0.5$.

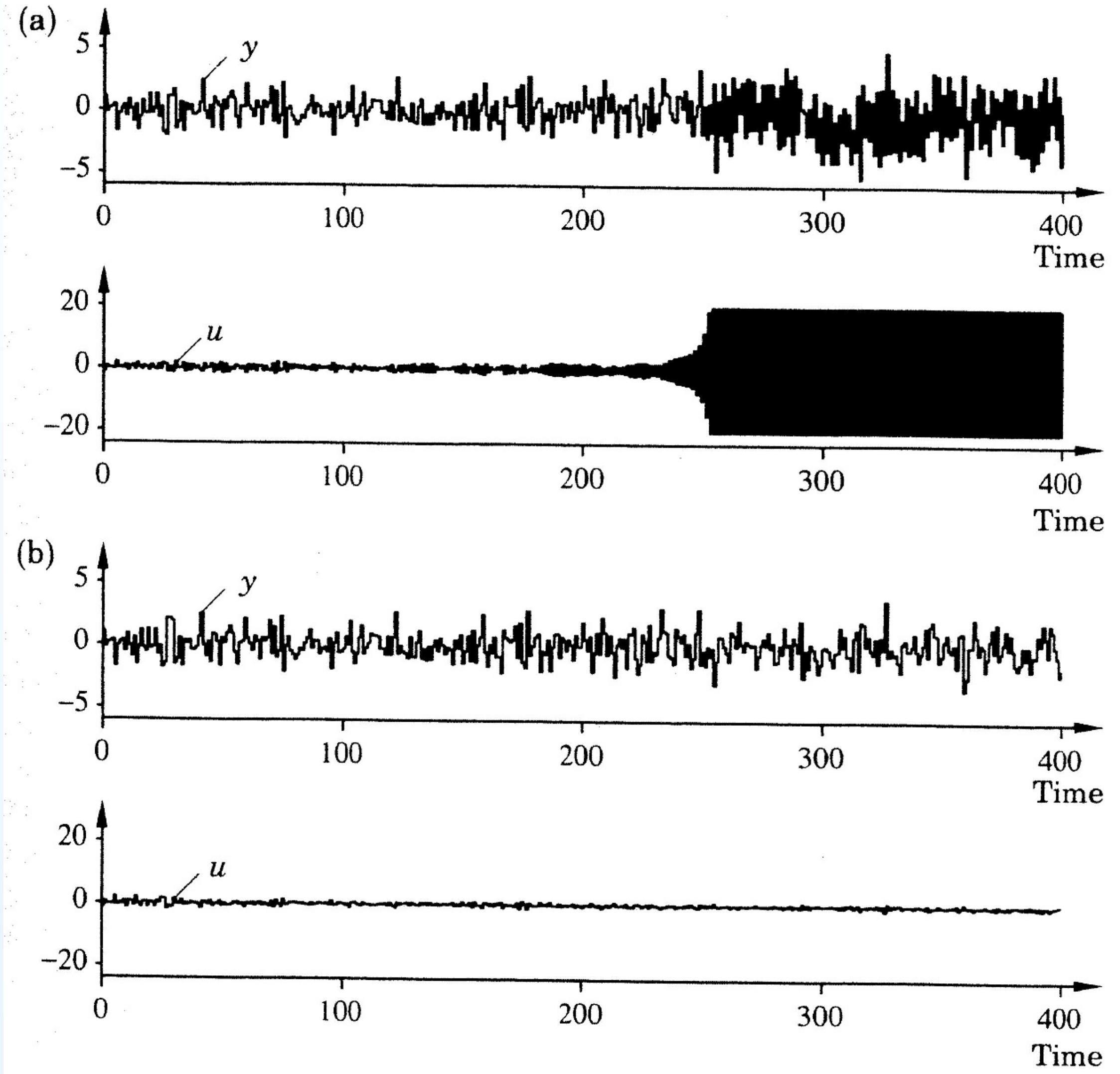

Figure 4.8 Simulation of the self-tuning algorithm on the integrator with time delay in Example 4.6. At t = 100 the delay is changed from 0.4 to 0.6. (a) d = 1; (b) d = 2.

Next Lecture / Assignments:

Next meeting (May 14, 10:00-12:00, in A206Tekn):
Model-Reference Adaptive Systems.

Homework problem: Reproduce the simulation results given on Figure 4.8.