Low-Complexity Explicit MPC Controller for Vehicle Lateral Motion Control

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***Abstract*— We consider the problem of controlling the vehicle lateral motion in highway scenarios while guaranteeing safety. We propose a solution consisting of a Low-Complexity Explicit Model Predictive Controller (LC-EMPC), where the lateral deviation from the desired path is hard constrained according to prescribed bounds. The robust satisfaction of such safety constraints can be achieved by imposing the terminal state to enter a Robust Invariant Set (RIS), which is known to result into a potentially high number of additional constraints, thus increasing the computational complexity of the controller. Our controller, instead, relies on recent results on the calculation of low-complexity RIS to significantly reduce the number of constraints in the MPC controller. Simulation results show that the designed controller is able to meet the desired objectives with highly reduced complexity.**

I. Introduction

The advances in sensing, perception and control engineering, are pushing automated driving technologies into our society. Although low-level (autonomous driving systems are categorized in five levels by national highway traffic safety administration (NHTSA)) automated driving systems like, to name a few, lane keeping, lane change assist are already available since a decade, as explained in [1], high-level autonomous vehicles will have to guarantee the satisfaction of recently introduced Automotive Safety Integrity Level (ASIL) requirements mentioned in ISO 26262.

One of the main element in the vehicle motion control problem is the lateral controller. Under ASIL requirements, autonomous lateral motion control is classified to meet the ASIL D type of requirement, which imply, the vehicle manufacturer has to guarantee a failure rate of not more than 10^{-8} events per hour. In conclusion, keeping the vehicle within predefined safety bounds around the desired path should be guaranteed by the control algorithm. Furthermore, the control strategy should be computationally less expensive as an onboard computational resource in the vehicle is limited and shared between many subsystems like a sensor unit, a communication unit, a controller unit etc.

One of the initial paper which discusses vehicle lateral control problem is [2]. It analyses the steering controller for a highway like scenarios and proposes criteria for the safety and performance. Due to the highly constrained nature of the problem, later many works were specifically focused on various MPC based strategies. An approach based on the nonlinear MPC scheme was proposed in [4], [7], the main challenge with such an approach resides in the real-time solution of a nonlinear programming which are known to be computationally demanding. In [5], a robust tube based MPC approach is used for lateral control problem in the semi-autonomous vehicle to guarantee state and input constraints satisfaction in the presence of disturbances and model error. Also, these approaches lack feasibility guarantee of underlying optimization problem.

An ASIL D type of requirement suggests MPC controller should guarantee persistent feasibility and stability. In MPC schemes a commonly used strategy to guarantee persistent feasibility is by using robust control invariant (RCI) set as a terminal constraint [3]. A tracking lateral controller considered in [6] guarantees the vehicle constraints satisfaction while tracking a piecewise clothoidal trajectory from a designed path planner. The approach uses a polyhedral RCI set of arbitrary complexity obtained from an algorithm similar to geometric approach but specialized for the case where disturbance set and dynamics are known. In our previous work [1], the MPC controller uses terminal constraint in the form of maximal RCI set to guarantee persistence feasibility and stability. The major drawback of such an approach is they are computationally very expensive due to a large number of constraints introduced by RCI set.

Moreover, all the above-mentioned approaches use an online optimization solver. This implies, the used solvers also need to be ASIL D compliant, which is lacked in case of all available solvers. Hence implicit MPC scheme, in general, cannot be used for a vehicle control problem unless an ASIL D compliant solver is available.

In this paper, we propose an LC-EMPC controller for vehicle lateral motion control, specifically for a highway scenario. The main advantage of using an explicit MPC over implicit is the obtained offline solution can be extensively verified and validated offline before implementation. Further, we aim at reducing the computational complexity of the EMPC scheme while guaranteeing to meet the desired safety and performance criteria in the presence of bounded side wind disturbance. The safety and performance requirements are formulated in terms of the maximum deviation from the path and constraints on the vehicle states stemming from the desired comfort envelope. These constraints are guaranteed to be persistently satisfied within a known set of vehicle states for the desired path within given boundaries.

Simulation results show that the proposed approach is able to meet the desired safety and performance objective while being computationally less expensive at the cost of slightly reduced feasibility set of the controller.

The paper is structured as follows. We present vehicle lateral dynamics, constraints and problem statement in section II. The section III presents an approach to find a low-complexity RCI (LC-RCI) set followed by LC-EMPC controller design. In section IV, we show the simulation result of the considered vehicle control problem. The paper is concluded in section V with final remarks about the presented results and future research directions.

II. Modeling and Problem Formulation

A. Vehicle Dynamics and Constraints

We consider a standard bicycle model for vehicle lateral dynamics with side wind disturbance and constant longitudinal velocity V_x [8], [9]. The continuous-time model is described by

$$\begin{bmatrix}\dot{e}_y\\ \ddot{y}\\ \dot{e}_\psi\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}0 & 1 & V_x & 0\\ 0 & \frac{-(C_f+C_r)}{mV_x} & 0 & -V_x-\frac{(l_fC_f-l_rC_r)}{mV_x}\\ 0 & 0 & 0 & 1\\ 0 & \frac{-(l_fC_f-l_rC_r)}{I_zV_x} & 0 & \frac{-(l_f^2C_f+l_r^2C_r)}{I_zV_x}\end{bmatrix}\begin{bmatrix}e_y\\ \dot{y}\\ e_\psi\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{C_f}{m}\\ 0\\ \frac{l_fC_f}{I_z}\end{bmatrix}\delta+\begin{bmatrix}0 & 0\\ \frac{1}{m} & 0\\ 0 & 0\\ 0 & \frac{1}{I_z}\end{bmatrix}\begin{bmatrix}\mathrm{F}_w\\ \mathrm{M}_w\end{bmatrix}, \quad (1)$$

where m, I_z are the vehicle mass, and inertia. C_f, C_r are the front and rear cornering stiffness coefficients and l_f, l_r are the distance of the front and rear wheel axle from the vehicle center of gravity, respectively. e_y, e_ψ are the vehicle lateral and orientation error, w.r.t. a predefined path, $\dot{y}$ is the lateral velocity in the vehicle body frame, $\dot{\psi}$ is the yaw rate, δ is the steering angle and $\mathrm{F}_w, \mathrm{M}_w$ represent the force and moment exerted by a side wind impacting the vehicle. These quantities are assumed to be modeled according to

$$\mathrm{F}_w=\frac{2.5\pi}{2}V_w^2, \quad (2)$$

$$\mathrm{M}_w=\left(2.5\frac{\pi}{2}-3.3(\frac{\pi}{2})^3\right)V_w^2+\frac{l_f-l_r}{2}\mathrm{F}_w, \quad (3)$$

where V_w is wind velocity. Considering $x = \begin{bmatrix}e_y & \dot{y} & e_\psi & \dot{\psi}\end{bmatrix}^T$ as state vector, $u=\delta$ as a control input and $w=V_w^2$ as a disturbance input, a discrete time system (1)-(3) can be compactly written as

$$x(k+1)=Ax(k)+Bu(k)+Ew(k). \quad (4)$$

The state and input vector in (4) are subject to a set of physical and design constraints to meet the desired safety and performance requirements. For example, as part of ASIL D-type safety requirements, the lateral deviation of the vehicle from the desired path could be requested to be bounded as follows

$$e_{ymin}\le e_y\le e_{ymax}. \quad (5)$$

The lateral vehicle speed could be bounded as well, to preserve the ride comfort for the passengers

$$\dot{y}_{min}\le \dot{y}\le \dot{y}_{max}. \quad (6)$$

Finally, the steering angle can be limited as well, reflecting additional safety requirements aiming at limiting the vehicle lateral acceleration

$$\delta_{min}\le\delta\le\delta_{max}. \quad (7)$$

Constraints (5)-(7) can be compactly written as

$$\mathcal{X}\triangleq\{x|\ Hx(k)\le\mathbf{1}\},\ \mathcal{U}\triangleq\{u|\ Gu(k)\le\mathbf{1}\}, \quad (8)$$

where $\mathbf{1}$ is a vector of ones of a compatible dimension. The bounds on the input disturbance in (4) can be similarly written as

$$\mathcal{W}\triangleq\{w|\ |Dw(k)|\le\mathbf{1}\}. \quad (9)$$

Note that the constraints related to performance sometimes also include the bounded lateral acceleration is translated into bounds on $\dot{\psi}$, see [1].

B. Problem Formulation

Next, we focus on an Explicit MPC (EMPC) controller for regulating the state variables of the vehicle dynamics (4) to origin while satisfying constraints (8) in the presence of disturbance (9). To guarantee persistent feasibility and stability we opt for an approach similar to [1] where the considered MPC controller was as follows:

Problem 1.

$$J(x(k))=\min_{U_k}\|x_{k+N|k}\|_P^2+\sum_{i=0}^{N-1}\|x_{k+i|k}\|_Q^2+\|u_{k+i|k}\|_R^2$$

$$\begin{aligned}\textit{St:}\ \ x_{k+i+1|k}&=Ax_{k+i|k}+Bu_{k+i|k}+Ew_{k+i|k},\\ x_{k+i|k}&\in\mathcal{X},\ w_{k+i|k}\in\mathcal{W}\\ u_{k+i|k}&\in\mathcal{U},\ \forall i=0,\cdots,N-1\\ x_{k+N|k}&\in\mathcal{C},\\ x_{k|k}&=x(k).\end{aligned}$$

where $U_k=\left[u_{k|k},\ldots,u_{k+N-1|k}\right]$, N is prediction horizon, $P,\ Q\succeq 0$ and $R\succ 0$ are weighting matrices of appropriate dimensions. The set $\mathcal{C}$ is a maximal RCI set for some stabilizing feedback $u=Kx$. It is common to select K as an LQR controller for some tuning matrices Q and R used in MPC and P as the corresponding cost-to-go matrix for the LQR controller. The above scheme is referred as dual-mode MPC controller in [3]. In practice, only the first control move from the obtained sequence U_k is applied to the system (1) i.e.

$$u^*(x(k))=u_{k|k},$$

after which the **Problem 1** is solved again for time $k+1$.

Remark 1. *The choice of K defines $\mathcal{C}$ which in turn affects the size of the feasibility set (the set of initial states $x(0)$ such that Problem 1 is feasible) of a MPC controller. Hence, choosing $\mathcal{C}$ as a maximal RCI set for the LQR gain K corresponding to Q and R in Problem 1 can potentially* *lead to a small feasibility set. It would be then convenient to decouple the choice of $\mathcal{C}$ from the choice of the matrices Q and R (i.e., the tuning of the controller) [11].*

As mentioned in the introduction, to make offline verification of the controller possible, an EMPC controller can be designed. The idea is to pre-compute the explicit solutions of **Problem 1** for all the feasible values x by using multi-parametric programming [15] in the form

$$u^*(x(k)) = \begin{cases} F_1 x(k) + h_1, \text{if } x(k) \in \mathcal{R}_1 \\ \vdots \\ F_L x(k) + h_L, \text{if } x(k) \in \mathcal{R}_L, \end{cases} \tag{10}$$

where,

$$\mathcal{R}_i = \{x | S_i x \leq z_i\}, \ i = 1, \ldots, L, \tag{11}$$

are the polytopic critical regions and L denotes the number of critical regions, also known as the complexity of the EMPC controller. By doing so an explicit affine relation (10) between current measurement $x(k)$ and optimal control input $u^*(x(k))$ is obtained. In general, computing $u^*(x(k))$ from (10) can be performed much faster compared to solving QP in **Problem 1** online. Moreover, to further reduce the computation time, it is desirable to have EMPC controller complexity L as small as possible. However, from [15], it is known that the L depends (exponentially, in the worst case) on the number of constraints and hence on the complexity of the set $\mathcal{C}$ in **Problem 1**. Thus to obtain a low complexity EMPC controller, in Section III we use an approach proposed in [10] to find a low-complexity RCI (LC-RCI) set, along with a control gain K. The obtained K is independent of the EMPC tuning matrices while the corresponding RCI set is sought with the largest volume and low complexity (described by a number of inequalities that is twice the system dimension). Nevertheless, using such LC-RCI set as a terminal set may significantly reduce the volume of the feasibility set for an EMPC controller. While this may be unacceptable in some applications, it was found to be very useful in vehicle lateral control problem to reduce the complexity of the controller, while moderately shrinking the feasibility set.

For the convenience of our presentation we recall, an EMPC controller based on **Problem 1** is referred as full-complexity EMPC (FC-EMPC) controller in further parts.

III. Lateral Controller With Guaranteed Performance

Before starting with the controller design, we first briefly recall the results proposed in [10] to find a LC-RCI set $\mathcal{C}_{low}$ and K. Next, in this section, we propose a LC-EMPC controller using the set $\mathcal{C}_{low}$ as a terminal constraint.

A. Low Complexity RCI Set

The candidate set $\mathcal{C}_{low}$ is assumed to be described as

$$\mathcal{C}_{low} = \left\{x \in \mathbb{R}^4 : -\mathbf{1} \leq W^{-1}x \leq \mathbf{1}\right\}, \tag{12}$$

where $W \in \mathbb{R}^{4\times4}$ is a square matrix to be found. The set $\mathcal{C}_{low}$ is symmetric around the origin and described by the same number of affine inequalities as twice the dimension of the state vector. The invariance is induced in the set by some static feedback law $u = Kx$. A feasible pair (W, K) can be found by using the following Theorem in [10].

Theorem 1. *A feasible pair of (W, K) can be found if there exist $N \in \mathbb{R}^{1\times4}$, $X_i = X_i^T \in \mathbb{R}^{4\times4}$, $V_i \in \mathbb{R}^{4\times4}$, $\phi_i \geq 0 \in \mathbb{R}$, a diagonal matrix $\Lambda_i \succcurlyeq 0 \in \mathbb{R}^{4\times4}, \Gamma_i \geq 0 \in \mathbb{R}$ that satisfy the following LMIs for $i = 1, \ldots, 4$ & for $j = 1, \ldots, 8$:*

$$\begin{bmatrix} X_i & 0 & 0 & V_i \\ * & \Lambda_i & 0 & (AW+BN)^T \\ * & * & D^T\Gamma_i D & E^T \\ * & * & * & V_i + V_i^T \end{bmatrix} \succcurlyeq 0, \tag{13}$$

$$\begin{bmatrix} 2\phi_i - \mathbf{1}^T\Lambda_i\mathbf{1} - \mathbf{1}^T\Gamma_i\mathbf{1} & \phi_i e_i^T \\ * & \mathcal{M}_i^k \end{bmatrix} \succcurlyeq 0, \tag{14}$$

$$HW\theta^j \leq \mathbf{1}, \tag{15}$$

$$GN\theta^j \leq \mathbf{1}, \tag{16}$$

$$\mathcal{M}_i^k = (Y_i^k)^T W + W^T Y_i^k - (Y_i^k)^T X_i Y_i^k, \tag{17}$$

$$Y_i^k = \left(X_i^k\right)^{-1} W^k. \tag{18}$$

A LC-RCI set can then be obtained as in (12) with the state feedback gain $K = NW^{-1}$.

where $*$'s represent entries that are uniquely identifiable by symmetry. e_i is the i'th column of an identity matrix of compatible dimension. W^k and X_i^k are the values of the matrix W and X respectively, obtained from the previous iteration. To get an initial solution we select $Y_i^0 = I$. The vector θ^j's are the non-symmetric vertices of a compact set Θ defined as

$$\Theta = \{\theta \in \mathbb{R}^4 : -\mathbf{1} \leq \theta \leq \mathbf{1}\}. \tag{19}$$

Conditions (13) and (14) in Theorem 1 are the condition for invariance. The conditions (15) and (16) are imposed to ensure that state and input constraints are satisfied by the set $\mathcal{C}_{low}$ and the obtained control gain K.

Since the intended EMPC controller should have desirably large feasible set, it implies that the volume of the set $\mathcal{C}_{low}$ should be as large as possible. An iterative algorithm is formulated using Theorem 1 to find $\mathcal{C}_{low}$ of desirably large volume. The volume maximization of set $\mathcal{C}_{low}$ is formulated in such a way that the consecutive solutions satisfy $volume(\mathcal{C}_{low}^{k+1}) \geq volume(\mathcal{C}_{low}^k)$. As proven in [10], this can be achieved by additionally imposing the condition

$$W^T W^k + (W^k)^T W - 2(W^k)^T W^k \succcurlyeq T \succ 0, \tag{20}$$

at each iteration. Thus an iterative algorithm to find LC-RCI set of desirably large volume can be formulated as

Algorithm 1: [$k+1$ step]

$$\max_{W,N,T,X_i,V_i,\Lambda_i,\Gamma_i,\phi_i} \quad \log\det(T)$$
$$\text{subject to:} \quad (13), (14), (15), (16) \text{ and } (20).$$

Initial Optimization to Compute W: Condition (20) is removed; (14) is imposed with $Y_i^0 \rightarrow I$ and $\log\det(T)$ is changed to $\log\det(W)$.

Algorithm 1 was proven to be recursively feasible for each iteration, hence at every iteration a set $\mathcal{C}_{low}$ of larger volume than the previous is calculated, until its volume converges to some local minima. The obtained K, in general, has low gain due to the cost maximized is the volume of set $\mathcal{C}_{low}$ and it is also independent of the LC-EMPC tuning matrices as opposed to FC-EMPC approach as stated in **Remark 1**.

To guarantee the stability of the controller, the terminal cost matrix P has to be selected based on K obtained from **Algorithm 1**. For selected $Q \succcurlyeq 0, R \succ 0$, and K, from [13], a sufficient condition for closed-loop stability is selecting terminal cost matrix P which satisfies

$$(A+BK)^T P(A+BK)-P \preccurlyeq -Q-K^T RK,\ P \succcurlyeq 0. \quad (21)$$

The computed state feedback gain K is instrumental in finding P for a selected Q and R, which are chosen to actually influence the performance.

Remark 2. *It is know that the matrix P selected in such a fashion will lead to a sub-optimal MPC controller. Nevertheless, the level of sub-optimality is often insignificant if prediction horizon N is long enough [11].*

Thus an LC-EMPC controller can be written as

Problem 2.

$$J(x(k)) = \min_{U_k} \|x_{k+N|k}\|_P^2 + \sum_{i=0}^{N-1} \|x_{k+i|k}\|_Q^2 + \|u_{k+i|k}\|_R^2$$

$$\begin{aligned} St:\ \ & x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + Ew_{k+i|k}, \\ & x_{k+i|k} \in \mathcal{X},\ w_{k+i|k} \in \mathcal{W} \\ & u_{k+i|k} \in \mathcal{U},\ \forall i = 0, \cdots, N-1 \\ & x_{k+N|k} \in \mathcal{C}_{low}, \\ & x_{k|k} = x(k). \end{aligned}$$

In the sequel we compare the FC-EMPC and LC-EMPC controllers using simulations.

IV. Simulation

Simulation was performed in MATLAB using the vehicle parameters presented in Table I. For a constant longitudinal velocity $V_x = 80\ km/hr$, we obtain system matrices in (4) using Euler discretization with a sampling time $T_s = 25\ ms$. We consider following bounds on the state and input

TABLE I

Vehicle parameters

Parameter	Description	Value
m	Mass	2164 $[kg]$
I_z	Yaw moment of inertia	4373 $[kg \times m^2]$
C_r	Rear cornering stiffness coeff.	228088 $[N/rad]$
C_f	Front cornering stiffness coeff.	142590 $[N/rad]$
l_r	Rear axle to CoG distance	1.6456 $[m]$
l_f	Front axle to CoG distance	1.3384 $[m]$

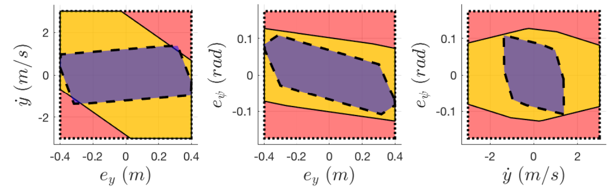

Fig. 1. Projections of RCI sets obtained using **Algorithm 1** (blue; dashed), Using LQ gain and geometric approach (yellow; solid) and admissible set (red; dotted)

constraints (5)-(7) to derive the sets $\mathcal{X}$, $\mathcal{U}$ in (8).

$$\begin{cases} e_{y_{max}} = -e_{y_{min}} = 0.4\ [m], \\ \dot{y}_{max} = -\dot{y}_{min} = 3\ [m/s], \\ e_{\psi_{max}} = -e_{\psi_{min}} = 10\frac{\pi}{180}\ [rad], \\ \delta_{max} = -\delta_{min} = 5\frac{\pi}{180}\ [rad]. \end{cases} \quad (22)$$

We assume the side wind velocity V_w to be bounded between $[-10, 10]\ m/s$ which translates to

$$D = 0.01. \quad (23)$$

In reality, the constraints (22) are less restrictive than considered, we purposely keep constraints tight due to the highway like scenario. Thus controller designing become even more challenging under considered bound.

We next calculate (W, K) pair defining LC-RCI set using **Algorithm 1** in CVX [12] for system (4) with constraints (22) and input disturbance (23).

$$\left[\frac{W}{K}\right] = \left[\begin{array}{cccc} 0.33007 & -0.03055 & -0.02703 & 0.01232 \\ 0.19543 & 1.07430 & 0.09127 & 0.18256 \\ -0.04113 & -0.01854 & 0.02422 & -0.00305 \\ 0.17859 & 0.19348 & -0.14139 & 0.19695 \\ \hline -0.18673 & 0.01569 & -3.31030 & -0.43399 \end{array}\right].$$

For comparison purpose we also calculate maximal RCI sets as explained in Remark 1 using MPT toolbox [14]. The obtained maximal RCI set was represented by 34 hyperplanes compared to 8 hyperplanes in case of the LC-RCI set. Fig. 1 shows the LC-RCI set, maximal RCI set and admissible set.

We design FC-EMPC and LC-EMPC controller with tuning matrices $Q = diag([10\ \ 1\ \ 1\ \ 1])$ and $R = 1$ as mentioned in [16]. For LC-EMPC controller, matrix P was calculated using (21) and controller gain K obtained from **Algorithm 1**. EMPC was implemented using MPT toolbox [16]. In Table II, the complexity of the EMPC schemes are presented for different values of prediction horizon. It can be observed that throughout, the overall complexity of the LC-EMPC scheme is approximately one-third of the FC-EMPC scheme with same prediction horizon. The feasibility set for the two controllers are shown in Fig. 2. It can be seen that the LC-EMPC scheme has feasibility set comparable to the feasibilty set of FC-EMPC.

For simulation purpose we consider a scenario in which car is driven along a straight path with some initial offset from the reference. The side wind is considered as an input

TABLE II
PREDICTION HORIZON VS NUMBER OF CRITICAL REGIONS IN EMPC

N	FC-EMPC	LC-EMPC
2	187	53
3	556	189
4	1242	498
5	2375	951
6	4038	1597
7	6110	2657

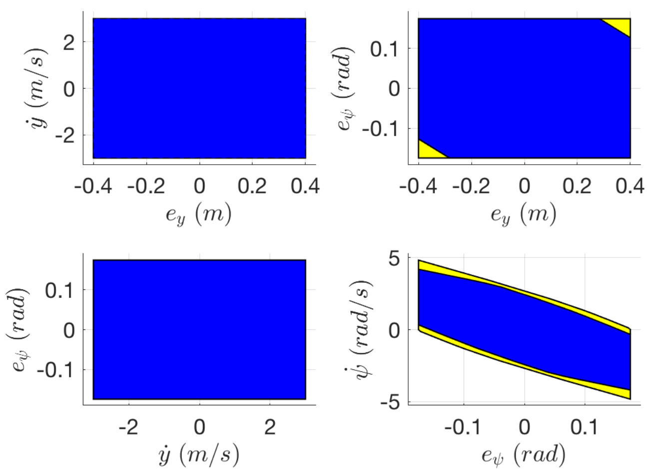

Fig. 2. Projections of feasibility sets of LC-EMPC (blue) and FC-EMPC (yellow) for $N = 7$.

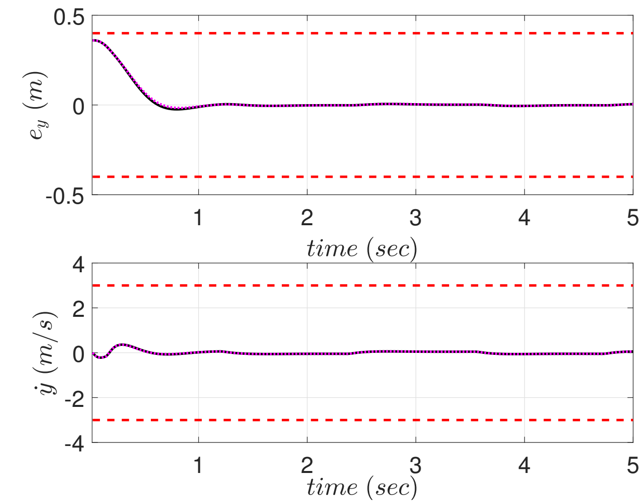

Fig. 3. Time trajectories of e_y and $\dot{y}$ with LC-EMPC (black;solid line), FC-EMPC (magenta;dotted line), and constraints (red; dashed line).

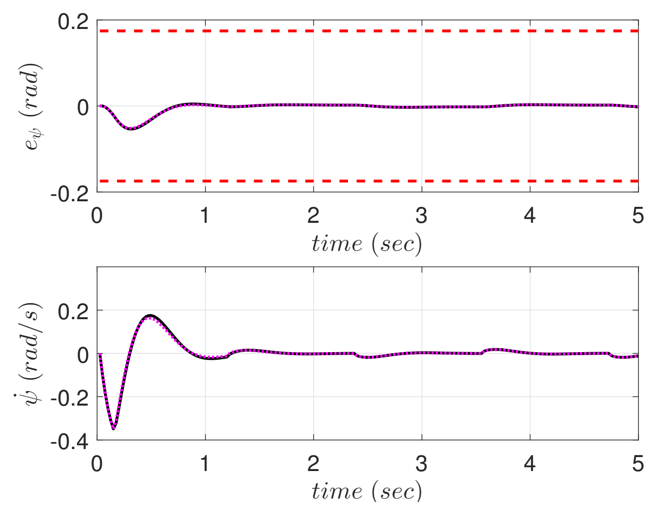

Fig. 4. Time trajectories of e_ψ and $\dot{\psi}$ with LC-EMPC (black;solid line), FC-EMPC (magenta;dotted line), and constraints (red; dashed line).

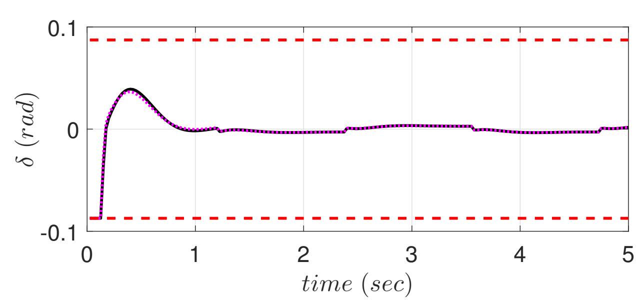

Fig. 5. Time trajectories of δ with LC-EMPC (black;solid line), FC-EMPC (magenta;dotted line), and constraints (red; dashed line).

disturbance affecting the car lateral motion. The wind profile used in the simulation is shown in Fig. 6. The closed-loop simulations of the compared strategies with $N = 7$ are shown in Fig. 3-5. As mentioned in **Remark 2**, it can be observed that the performance of the two controllers are similar while the LC-EMPC controller being almost three times faster than FC-EMPC controller. This is because of the one-third reduction in the number of critical regions in the LC-EMPC (see, Table II) controller which drastically reduces the search time to find the optimal control law. Lastly, the proposed LC-MPC controller is guaranteed to be recursively feasible and stable by design. Thus, both the controllers always satisfy the desired safety and performance constraints mentioned in Section II-A which can be also seen in Fig. 3-5 and hence meets the ASIL-D related design requirements.

V. CONCLUSION

This paper presents a simulation results of an EMPC controller for vehicle lateral motion. The proposed controller was able to meet the ASIL D type of requirements for safety and desired performance criteria. The proposed approach had performance similar to the existing approach while having complexity one-third of it and slightly reduced feasibility set.

Although preliminary, this work forms the basis of our further research to design an ASIL D compliant control algorithm for vehicle motion control in the face of model

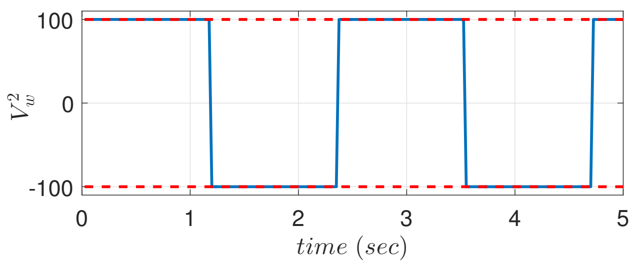

Fig. 6. Considered wind profile V_w^2 (blue, solid line) and bounds (red, dashed line).

uncertainty and measurement noise while being less complex.

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