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Estimation of Vehicle Lateral Velocity

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Abstract

Two techniques for the estimation of vehicle lateral velocity using state observers are considered. The first method uses a physical model of the vehicle handling. The physical model based observer produces noise free lateral velocity estimates, but can be sensitive to changes in the vehicle parameters. It produces reliable estimates in the vehicle linear handling region only. We show that the observer gain can be selected to make the observer insensitive to certain parameter variations. The second method uses a kinematic model relating longitudinal velocity, lateral velocity, longitudinal acceleration, lateral acceleration and yaw rate. This model contains no vehicle parameters, and hence the kinematic model based observer is unaffected by changes in the vehicle parameters. The observer produces reliable lateral velocity estimates throughout the linear and nonlinear handling regions, the estimates however are more noisy than those produced by the physical model based observer. The techniques are compared using simulated data for manoeuvers in the linear and nonlinear handling regions of the vehicle.

1. Motivation

The lateral velocity of a vehicle is that component of the vehicle velocity vector perpendicular to the vehicle heading, and parallel to the ground plane. No satisfactory method exists to measure vehicle lateral velocity on production vehicles. This paper demonstrates that the estimation of vehicle lateral velocity using state observers is a practical solution. Vehicle lateral velocity estimates are useful for enhanced vehicle handling control and four-wheel steering systems.

2. Introduction

State observers provide a formal means of reconstructing the states of a dynamic system from measurements of the system inputs and outputs. Two approaches to the estimation of vehicle lateral velocity using state observers are considered.

The first approach uses the following physical model of the vehicle handling[1] to estimate vehicle lateral velocity v (the notation is explained in full at the end of the paper).

$$\dot{v} = -\frac{C_0}{mV}v - \left(V + \frac{C_1}{mV}\right)r + \frac{2C_{\alpha f}}{m}\delta_f \quad (1)$$

$$\dot{r} = -\frac{C_1}{IV}v - \frac{C_2}{IV}r + \frac{2aC_{\alpha f}}{I}\delta_f \quad (2)$$

Where C_0, C_1, C_2 are the derived vehicle cornering stiffness coefficients:

$$C_0 = 2C_{\alpha f} + 2C_{\alpha r} \quad (3)$$

$$C_1 = 2aC_{\alpha f} - 2bC_{\alpha r} \quad (4)$$

$$C_2 = 2a^2C_{\alpha f} + 2b^2C_{\alpha r} \quad (5)$$

This model contains physical parameters such as the vehicle mass m, vehicle yaw moment of inertia I, and the tyre cornering stiffness coefficients $C_{\alpha f}$, $C_{\alpha r}$. The physical model based observer produces noise free estimates of vehicle lateral velocity, but is sensitive to changes in the above parameters. It only produces reliable estimates in the linear handling region (i.e. for lateral accelerations less than 3m.s^{-2}). The structure of the model permits the observer gain to be selected such that the observer is insensitive to certain parameter changes.

The second approach uses the following kinematic model of the vehicle:

$$\dot{u} = vr + a_x \quad (6)$$

$$\dot{v} = -ur + a_y \quad (7)$$

This model contains no physical parameters of the vehicle, and so the kinematic model based observer is unaffected by changes in them. The kinematic model based observer produces reliable estimates in the nonlinear handling region. The kinematic model is unobservable when the yaw rate r is zero, and so the resulting observer only functions when the yaw rate is non–zero. The estimates produced by the kinematic model based observer are more noisy than those produced by the physical model based observer.

3. Previous Work

Two approaches to the estimation of vehicle lateral velocity arise in the literature. The first approach, including work by Senger and Kortum[2], and Cao[3] uses a physical model of the car handling dynamics and tyres to estimate vehicle lateral velocity. The second approach, including work by Ray[4], uses a physical model of the car handling dynamics, but avoids using a tyre model by directly estimating the tyre forces.

Senger and Kortum's work assumes that the tyres are operating in the linear region, with known tyre cornering stiffnesses, and that the vehicle parameters (mass, yaw moment of inertia etc) are known. The resulting observer will only function correctly in the linear handling region under design tyre and loading conditions. Cao overcomes these restrictions by using a combined parameter and state estimation approach, however, it is not known how successful this is as he gives no results.

Ray describes a 17 state extended Kalman filter for the estimation of tyre longitudinal and lateral forces, vehicle longitudinal velocity, lateral velocity, and yaw rate. While this provides comprehensive information, and is only dependent on two vehicle parameters (vehicle mass, and vehicle yaw moment of inertia), it only achieves this at great computational expense.

The physical model based observer presented in this paper is based on Senger and Kortum's observer. Senger and Kortum's observer uses measurements of wheel speed, steer angle and lateral acceleration. The physical model based observer presented in this paper uses these measurements, and in addition uses a measurement of vehicle yaw rate. The extra measurement improves the quality of the estimates, and allows a range of observers to be designed with different sensitivity to measurement noise and vehicle parameter variations.

The kinematic model based observer presented in this paper is an observer implementation of the extended Kalman filter used by Ray. It can be shown that the kinematic model based observer produces results comparable to the extended Kalman filter. However the observer approach described in this paper is simpler to understand, design and implement. We also show that the observer is stable under all operating conditions.

4. State Observers

The vehicle physical and kinematic models described in previous sections can be expressed in the standard state space form:

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t) \tag{8}$$

$$\boldsymbol{y}(t) = \mathbf{C}\boldsymbol{x}(t) + \mathbf{D}\boldsymbol{u}(t) \tag{9}$$

The system is 'observable' if the observability matrix $\mathcal{O}$ has rank equal to the number of states n. Where the observability matrix is:

$$\mathcal{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix} \tag{10}$$

A state observer for this system is defined by the following state equation:

$$\dot{\hat{\boldsymbol{x}}}(t) = (\mathbf{A} - \mathbf{KC})\hat{\boldsymbol{x}}(t) + (\mathbf{B} - \mathbf{KD})\boldsymbol{u}(t) + \mathbf{K}\boldsymbol{y}(t) \tag{11}$$

Where $\mathbf{K}$ is an $n \times r$ gain matrix. The state error $\tilde{\boldsymbol{x}}(t)$ between the estimated state $\hat{\boldsymbol{x}}(t)$ and the true state $\boldsymbol{x}(t)$ is:

$$\tilde{\boldsymbol{x}}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t) \tag{12}$$

It can be shown that $\tilde{\boldsymbol{x}}(t)$ obeys the following state equation:

$$\dot{\tilde{\boldsymbol{x}}}(t) = (\mathbf{A} - \mathbf{KC})\tilde{\boldsymbol{x}}(t) \tag{13}$$

Thus if the eigenvalues of the matrix $\mathbf{A} - \mathbf{KC}$ are stable and faster than the eigenvalues of $\mathbf{A}$, $\tilde{\boldsymbol{x}}(t)$ will rapidly decay to zero and the estimates $\hat{\boldsymbol{x}}(t)$ will converge to the true states $\boldsymbol{x}(t)$. State observers are generally designed for time invariant systems, so the gain $\mathbf{K}$ is constant. Both the physical model, and kinematic model discussed in this paper are time variant, and so a time varying gain matrix is required to achieve a suitable response. This results in a slight additional computational overhead. The state observer approach remains significantly simpler to implement than other methods such as the extended Kalman filter.

5. The Physical Model based Observer

The physical model based state observer uses the steer angle δ_{f} (which is the input to the physical model) and measurements of lateral acceleration $a_{\mathrm{y(meas)}}$, and yaw rate $r_{\mathrm{(meas)}}$ to produce estimates of lateral velocity v and yaw rate r. The model is shown to be observable. Two techniques for selecting the observer gain matrix are examined. The first technique uses the steady state

Kalman gain, and provides near optimal results under known conditions of process and measurement noise. The second technique uses a gain selected such that the resulting observer is insensitive to changes in the rear tyre cornering stiffness $C_{\alpha r}$.

The physical model of equations (1) and (2) in state space form is:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \mathbf{A} \begin{bmatrix} v \\ r \end{bmatrix} + \mathbf{B}\delta_f \tag{14}$$

Where:

$$\mathbf{A} = \begin{bmatrix} -\frac{C_0}{mV} & -\left(V + \frac{C_1}{mV}\right) \\ -\frac{C_1}{IV} & -\frac{C_2}{IV} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \frac{2C_{\alpha f}}{m} \\ \frac{2aC_{\alpha f}}{I} \end{bmatrix} \tag{15}$$

Measurements of lateral acceleration $a_{y(meas)}$ and yaw rate $r_{(meas)}$ are used. The measurement model is:

$$\begin{bmatrix} a_{y(meas)} \\ r_{(meas)} \end{bmatrix} = \mathbf{C} \begin{bmatrix} v \\ r \end{bmatrix} + \mathbf{D}\delta_f \tag{16}$$

Where:

$$\mathbf{C} = \begin{bmatrix} -\frac{C_0}{mV} & -\frac{C_1}{mV} \\ 0 & 1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \frac{2C_{\alpha f}}{m} \\ 0 \end{bmatrix} \tag{17}$$

The matrix $\mathbf{C}$ is rank 2 provided C_0 is non–zero. This is always the case, so the physical model is observable. The physical model based state observer is:

$$\begin{bmatrix} \dot{\hat{v}} \\ \dot{\hat{r}} \end{bmatrix} = (\mathbf{A} - \mathbf{KC}) \begin{bmatrix} \hat{v} \\ \hat{r} \end{bmatrix} + (\mathbf{B} - \mathbf{KD})\delta_f + \mathbf{K} \begin{bmatrix} a_{y(meas)} \\ r_{(meas)} \end{bmatrix} \tag{18}$$

The observer gain matrix $\mathbf{K}$ is selected to meet a number of (often conflicting) criteria, among them observer tracking, measurement noise rejection, and insensitivity to variations in the vehicle model. Observer tracking is obtained by ensuring that the eigenvalues of the matrix $\mathbf{A} - \mathbf{KC}$ are sufficiently fast. Measurement noise rejection is obtained by ensuring that the eigenvalues of $\mathbf{A} - \mathbf{KC}$ are not so fast that excessive noise propagates from the measurements to the state estimates. Observer insensitivity can sometimes be achieved by using the structure of the model to make the observer estimates insensitive to changes in some parameters. The selection of the observer gain matrix $\mathbf{K}$ is examined more closely in the following paragraphs. Two approaches are examined, the first uses a Kalman gain. The second uses a gain that makes the observer insensitive to changes in the rear tyre cornering stiffness coefficient.

Kalman Gain Matrix

Near optimal measurement noise rejection can be provided using a fixed gain Kalman filter. The Kalman gain is determined by solving the Ricatti equation:

$$\mathbf{AP}_\infty + \mathbf{P}_\infty\mathbf{A}^\mathbf{T} + \mathbf{Q} - \mathbf{P}_\infty\mathbf{C}^\mathbf{T}\mathbf{R}^{-1}\mathbf{CP}_\infty = 0 \tag{19}$$

Where $\mathbf{Q}$ is the process noise covariance matrix, and $\mathbf{R}$ is the measurement noise covariance matrix. The Kalman gain is:

$$\mathbf{K}_\infty = \mathbf{P}_\infty\mathbf{CR}^{-1} \tag{20}$$

The fixed gain Kalman filter is implemented using this value of gain as the observer gain $\mathbf{K}$. Because the $\mathbf{A}$ and $\mathbf{C}$ matrices are functions of the vehicle speed V, the resulting observer gain is a function of speed.

Insensitive Observer Design

Variation of vehicle handling conditions (loading, tyre pressures etc) results in a situation whereby the actual vehicle handling model matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ differ from those that were used to design the observer, denoted $\mathbf{A_N}$, $\mathbf{B_N}$, $\mathbf{C_N}$, $\mathbf{D_N}$. The difference between the actual and design values are denoted: $\Delta\mathbf{A}$, $\Delta\mathbf{B}$, $\Delta\mathbf{C}$, $\Delta\mathbf{D}$, where:

$$\Delta\mathbf{A} = \mathbf{A} - \mathbf{A_N} \qquad \Delta\mathbf{B} = \mathbf{B} - \mathbf{B_N} \tag{21}$$

$$\Delta\mathbf{C} = \mathbf{C} - \mathbf{C_N} \qquad \Delta\mathbf{D} = \mathbf{D} - \mathbf{D_N} \tag{22}$$

The observer error dynamics are given by:

$$\dot{\tilde{\boldsymbol{x}}} = (\mathbf{A_N} - \mathbf{KC_N})\tilde{\boldsymbol{x}} - (\Delta\mathbf{A} - \mathbf{K}\Delta\mathbf{C})\boldsymbol{x} - (\Delta\mathbf{B} - \mathbf{K}\Delta\mathbf{D})\boldsymbol{u} \tag{23}$$

If, for a given parameter variation, $\mathbf{K}$ is selected such that $\Delta\mathbf{A} - \mathbf{K}\Delta\mathbf{C} = 0$ and $\Delta\mathbf{B} - \mathbf{K}\Delta\mathbf{D} = 0$ then the observer will be insensitive to that parameter variation. We consider the design of an observer that is insensitive to changes in the rear cornering stiffness $C_{\alpha r}$. The difference between the actual and design values of rear tyre cornering stiffness is denoted $\Delta C_{\alpha r}$. In this case $\Delta\mathbf{B} - \mathbf{K}\Delta\mathbf{D} = 0$ for any value of $\mathbf{K}$ and:

$$\Delta\mathbf{A} - \mathbf{K}\Delta\mathbf{C} = \begin{bmatrix} \frac{2\Delta C_{\alpha r}}{mV}(k_{11} - 1) & -\frac{2b\Delta C_{\alpha r}}{mV}(k_{11} - 1) \\ \frac{2\Delta C_{\alpha r}}{mV}\left(k_{21} + \frac{bm}{I}\right) & -\frac{2b\Delta C_{\alpha r}}{mV}\left(k_{21} + \frac{bm}{I}\right) \end{bmatrix} \tag{24}$$

Thus selecting $k_{11} = 1$ and $k_{21} = -\frac{bm}{I}$ gives $\Delta\mathbf{A} - \mathbf{K}\Delta\mathbf{C} = 0$ as required. The remaining elements of the gain matrix can be selected to achieve a suitable observer response.

A similar procedure can be used to design an observer that is insensitive to changes in the front cornering stiffness.

6. The Kinematic Model based State Observer

The kinematic model based state observer uses the longitudinal and lateral accelerations a_x, a_y, yaw rate r (which are the inputs to the kinematic model) and measurements of the vehicle longitudinal velocity $v_{(\text{meas})}$ to produce estimates of the vehicle longitudinal and lateral velocities u, v. The model is shown to be observable when the vehicle yaw rate is non-zero. The eigenvalues of the kinematic model are located on the imaginary axis at $\lambda = \pm ir(t)$. The observer gain is selected such that the observer eigenvalues are located at $\lambda = -\alpha|r(t)|$, where α is a positive constant. This selection is found to perform well, and have good stability properties.

The kinematic model of equations (6) and (7) in state space form is:

$$\overset{\bullet}{\begin{bmatrix} u \\ v \end{bmatrix}} = \mathbf{A} \begin{bmatrix} u \\ v \end{bmatrix} + \mathbf{B} \begin{bmatrix} a_x \\ a_y \end{bmatrix} \tag{25}$$

Where:

$$\mathbf{A} = \begin{bmatrix} 0 & r(t) \\ -r(t) & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{26}$$

We use measurements of vehicle speed $u_{(\text{meas})}$. The measurement model is:

$$u_{(\text{meas})} = \mathbf{C} \begin{bmatrix} u \\ v \end{bmatrix} \tag{27}$$

Where:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{28}$$

The observability matrix for the kinematic model is:

$$\mathcal{O} = \begin{bmatrix} 1 & 0 \\ 0 & r \end{bmatrix} \tag{29}$$

Hence the kinematic model is observable when the yaw rate is non-zero. The kinematic model based state observer is:

$$\overset{\bullet}{\begin{bmatrix} \hat{u} \\ \hat{v} \end{bmatrix}} = (\mathbf{A} - \mathbf{KC}) \begin{bmatrix} \hat{u} \\ \hat{v} \end{bmatrix} + \mathbf{B} \begin{bmatrix} a_x \\ a_y \end{bmatrix} + \mathbf{K} r_{(\text{meas})} \tag{30}$$

The eigenvalues of $\mathbf{A}$ are located at $\lambda = \pm ir(t)$. We select $\mathbf{K}$ to put the eigenvalues of $\mathbf{A} - \mathbf{KC}$ at $\lambda = -\alpha\,|r(t)|$. Solving gives:

$$\mathbf{K} = \begin{bmatrix} 2\alpha\,|r(t)| \\ (\alpha^2 - 1)\,r(t) \end{bmatrix} \tag{31}$$

The observer of equations (30) and (31) represents a nonlinear dynamical system. The design method does not guarantee stability, however it is shown in appendix A that this observer will be stable. This represents a significant advantage over the extended Kalman filter used by Ray[4].

7. Results

This section contains results produced by the three observers discussed (the physical model based observer using a Kalman gain, the physical model based observer using a gain insensitive to rear tyre cornering stiffness changes, and the kinematic model based observer) operating on simulated data. The simulation consists of a three state handling model and a nonlinear tyre model[6]. We simulate three manoeuvers. The first is a double 'figure of 8' at 40km/h with normal tyre stiffnesses. The second is a double 'figure of 8' at 40km/h with low rear tyre stiffness. The third is a severe step steer manoeuver at 60km/h with normal tyre stiffnesses. During the first two manoeuvers the vehicle remains in the linear handling region. In the third manoeuver the vehicle enters the nonlinear handling region.

The results produced by the physical model based observer using a Kalman gain are shown in figure 1. The observer produces noise free lateral velocity estimates, however the estimates are erroneous under low rear tyre stiffness and severe cornering conditions.

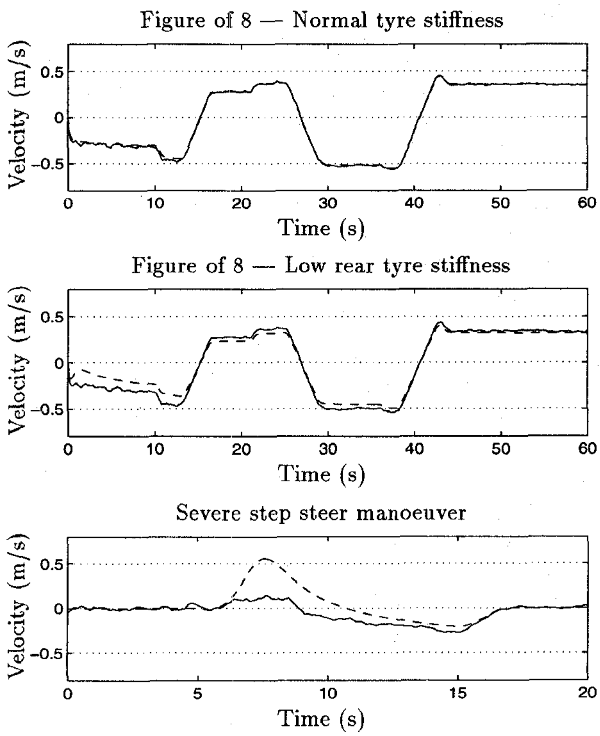

Figure 1: Lateral velocity estimates (solid lines) produced by the physical model based observer using the Kalman gain. Actual lateral velocities are indicated with dashed lines.

The results produced by the physical model based observer using a gain insensitive to rear tyre stiffness changes are shown in figure 2. The observer produces

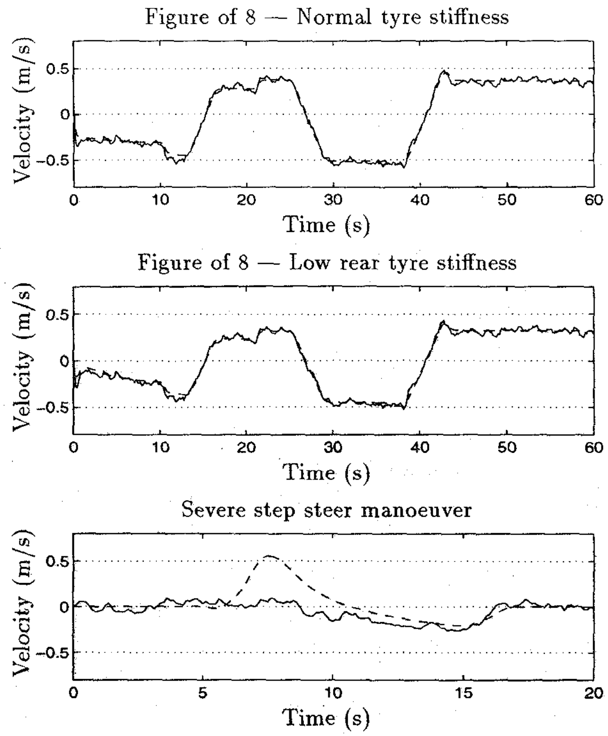

Figure 2: Lateral velocity estimates (solid lines) produced by the physical model based observer using a gain insensitive to rear tyre stiffness changes. Actual lateral velocities are indicated with dashed lines.

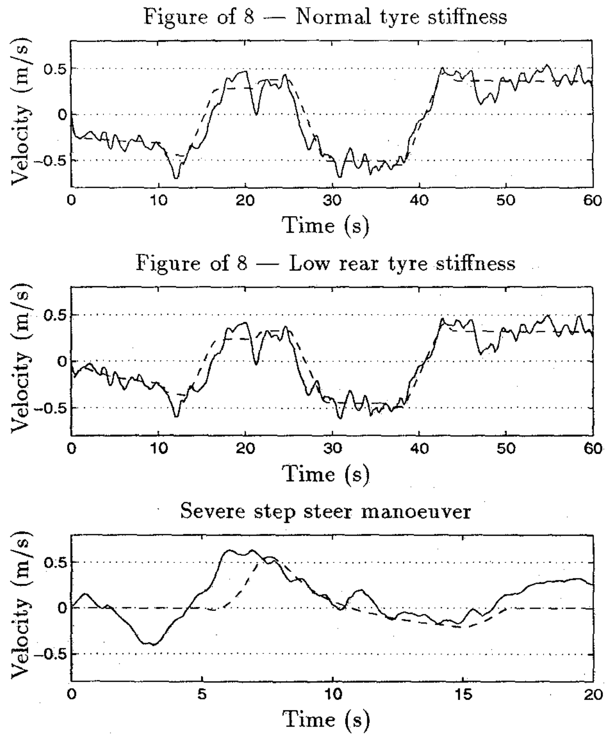

Figure 3: Lateral velocity estimates (solid lines) produced by the kinematic model based observer. Actual lateral velocities are indicated with dashed lines.

slightly more noisy lateral velocity estimates. The estimates are good for normal and low rear tyre stiffness conditions, but are erroneous under severe cornering conditions.

The results produced by the kinematic model based observer are shown in figure 3. The observer produces noisy estimates, however the estimates are not affected by the low rear tyre stiffness or severe cornering conditions. The kinematic model based observer estimates drift significantly at the start and end of the step steer manoeuver because the yaw rate is zero. When the yaw rate is zero the kinematic model becomes unobservable, and the observer gain is zero. The drift is caused by the integration of the acceleration measurements. This can be prevented by resetting the observer when the yaw rate is zero.

We conclude that while the physical model based observer using a Kalman gain produces noise free estimates, its sensitivity to parameter changes and its restriction to operation in the linear handling region limits its application as a technique for lateral velocity estimation. The physical model based observer using a gain insensitive to rear tyre stiffness change provides a useful compromise between estimate noise and sensitivity to parameter changes. The kinematic model based observer produces more noisy lateral velocity estimates, but is unaffected by vehicle parameter changes, and operates equally well in the linear and nonlinear handling regions.

8. Conclusions

We have shown that state observers can be used to estimate vehicle lateral velocity using both physical and kinematic models of the vehicle. Physical model based observers are sensitive to vehicle parameter changes, but produce low noise estimates. The sensitivity to parameter changes can be reduced by a suitable choice of gain. Kinematic model based observers are not sensitive to vehicle parameter changes, but produce noisy estimates, and drift when the yaw rate is zero. The kinematic model based observer described in this paper is shown to have some useful stability properties.

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A. Stability of the Kinematic Model Based Observer

The error dynamics of the kinematic model based observer of equation (30) with observer gain of equation (31) are given by:

$$\begin{bmatrix} \dot{\tilde{u}} \\ \dot{\tilde{v}} \end{bmatrix} = \begin{bmatrix} -2\alpha|r| & r \\ -\alpha^2 r & 0 \end{bmatrix} \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix} \tag{32}$$

The error dynamics are a function of the 'input' $r(t)$ and are not guaranteed to be stable except when $r(t)$ is a non–zero constant.

Stability can be proven by considering the Lyapunov function [7]:

$$V\left(\begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}\right) = \frac{\alpha^2\tilde{u}^2 + \tilde{v}^2}{2} \tag{33}$$

Differentiating with respect to time and using equation (32) to eliminate derivatives of $\tilde{u}$, $\tilde{v}$ gives:

$$\dot{V}\left(\begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}\right) = -2\alpha^3|r|\tilde{u}^2 \tag{34}$$

Now:

$$V > 0 \quad \text{for} \quad \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{35}$$

$$V = 0 \quad \text{for} \quad \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{36}$$

And:

$$\dot{V} < 0 \quad \text{for} \quad \tilde{u} \neq 0 \tag{37}$$

But $\tilde{u} = 0$ is not a trajectory of equation (32). Hence the dynamical system given by equation (32) is asymptotically stable for all non–zero $r(t)$.

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Notation

t	Time
δ_f	Front steer angle
u, v	Longitudinal and lateral components of vehicle velocity
V	Resultant vehicle velocity
r	Vehicle yaw rate
a_x, a_y	Forward and lateral components of vehicle acceleration
m	Vehicle mass
I	Vehicle yaw moment of inertia
a	Distance from vehicle centre of mass to front axle
b	Distance from vehicle centre of mass to rear axle
$C_{\alpha f}$, $C_{\alpha r}$	Front and rear tyre cornering stiffness coefficients
C_0, C_1, C_2	Derived vehicle cornering stiffness coefficients
$\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$, $\boldsymbol{y}(t)$	State, input, and measurement vectors
$\hat{\boldsymbol{x}}(t)$, $\tilde{\boldsymbol{x}}(t)$	State estimate, and state estimate error vectors
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$	State space matrices
$\mathbf{A_N}$, ..., $\mathbf{D_N}$	Nominal state space model matrices
$\Delta\mathbf{A}$, ..., $\Delta\mathbf{D}$	Difference between actual and nominal model matrices
$\mathbf{K}$	Estimator gain matrix
α	Kinematic model based observer design parameter
$\mathbf{Q}$, $\mathbf{R}$	Process and measurement noise covariance matrices
$\mathbf{P}_\infty$, $\mathbf{K}_\infty$	Steady state Kalman filter error covariance, and gain matrices