2.153 Adaptive Control
Lecture 3
Simple Adaptive Systems: Control & Stability

Daniel Wiese

dpwiese@mit.edu

Wednesday 18-Feb-2015

Introduction

Last time:

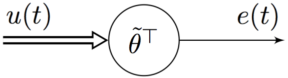
- Identification of multiple parameters in first order plant
    - Error model 1 and 3
    - Determining update law using Lyapunov functions
- Adaptive control

Today:

- Quick overview of error models 1 and 3
- Finish adaptive control of a first-order plant
- Using tuning gain in adaptive control
- Stability

Identification of a Vector Parameter - Error Model 1

Error Model 1: $e = \tilde{\theta}^\top u$



$\widetilde{\theta}$: parameter error

Adaptive law: $\dot{\widetilde{\theta}} = -eu$

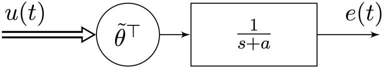
Lyapunov function: $V(\widetilde{\theta}) = \frac{1}{2}\widetilde{\theta}^\top \widetilde{\theta}$

$$\begin{aligned} \dot{V} &= \widetilde{\theta}^\top \dot{\widetilde{\theta}} \\ &= -\widetilde{\theta}^\top eu \\ &= -e^2 \leq 0 \end{aligned}$$

$\Rightarrow$ $\widetilde{\theta}(t)$ is bounded for all $t \geq t_0$

Error Model 3:

Error Model 3: $\dot{e} = -ae + \widetilde{\theta}^\top u$



$\widetilde{\theta}$: parameter error

Adaptive law: $\dot{\widetilde{\theta}} = -eu$

Lyapunov function: $V(e, \widetilde{\theta}) = \frac{1}{2}\left(e^2 + \widetilde{\theta}^\top \widetilde{\theta}\right)$

$$
\begin{aligned}
\dot{V} &= e\dot{e} + \widetilde{\theta}^\top \dot{\widetilde{\theta}} \\
&= -ae^2 + e\widetilde{\theta}^\top u + \widetilde{\theta}^\top \dot{\widetilde{\theta}} \\
&= -ae^2 + \widetilde{\theta}^\top (eu + \dot{\widetilde{\theta}}) \\
&= -ae^2 \leq 0
\end{aligned}
$$

$\Rightarrow e(t)$ and $\widetilde{\theta}(t)$ are bounded for all $t \geq t_0$

Stability using Error Models

So far our stability approach has been

- pick an "energy-like" function V
- take its time derivative
- choose a suitable update law that ensured $\dot{V} \leq 0$

What does this actually tell us about what happens?

- This we will talk more about today

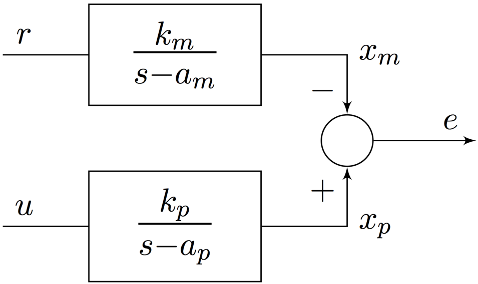
Also, what happened to error model 2?

- Similar to error model 3, but arises in situations where the entire system state is accessible
- We will see error model 4 later in the course

But first, we will revisit adaptive control of our first-order plant.

Model Reference Adaptive Control

This model-reference approach is represented by the following block diagram



Goal: Choose u so that $e(t) \to 0$ as $t \to \infty$.

- a_p unknown
- k_p unknown, but with known sign

Certainty Equivalence Principle

Step 1: Algebraic Part: Find a solution to the problem when parameters are known.

Step 2: Analytic Part: Replace the unknown parameters by their estimates. Ensure stable and convergent behavior.

The use of the parameter estimates in place of the true parameters is known as the *certainty equivalence principle*.

Certainty Equivalence Principle- Step 1

Step 1: Algebraic Part: Propose the control law

$$u(t) = \theta_c x_p + k_c r$$

and choose θ_c, k_c so that closed-loop transfer function matches the reference model transfer function.

$$\begin{aligned}\dot{x}_p &= a_p x_p + k_p u \\ &= a_p x_p + k_p(\theta_c x_p + k_c r) \\ &= (a_p + k_p \theta_c) x_p + k_p k_c r\end{aligned}$$

Now compare this to the reference model equation

$$\dot{x}_m = a_m x_m + k_m r$$

Desired Parameters: $\theta_c = \theta^*$ and $k_c = k^*$ must satisfy the *matching condition*

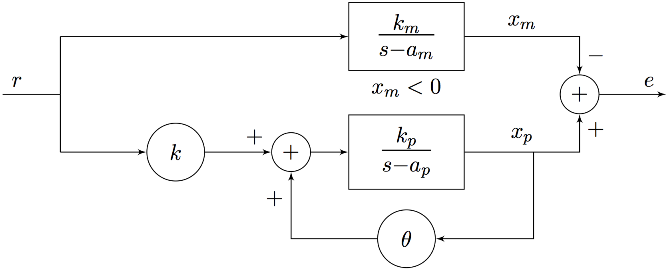
$$a_p + k_p \theta^* = a_m \qquad \text{and} \qquad k_p k^* = k_m$$

Certainty Equivalence Principle- Step 1

Solve for the nominal or ideal parameters

$$\theta^* = \frac{a_m - a_p}{k_p} \qquad \text{and} \qquad k^* = \frac{k_m}{k_p}$$

This is represented with the following block diagram



Certainty Equivalence Principle - Step 2

Step 2: Analytic Part: Replace the unknown parameters by their estimates. Ensure stable and convergent behavior. From Step 1, we have

$$u(t) = \theta^* x_p + k^* r, \qquad \theta^* = \frac{a_m - a_p}{k_p}, \qquad k^* = \frac{k_m}{k_p}$$

Replace θ^* and k^* by their estimates $\theta(t)$ and $k(t)$ and determine stable update laws.

Certainty Equivalence Principle - Step 2

Adaptive control input:

$$u(t) = \theta(t)x_p + k(t)r$$

Define the parameter errors as

$$\widetilde{\theta}(t) = \theta(t) - \theta^*$$
$$\widetilde{k}(t) = k(t) - k^*$$

Plug the control law into the plant equation (note that : $\theta(t) = \widetilde{\theta}(t) + \theta^*$ and $k(t) = \widetilde{k}(t) + k^*$):

$$\begin{aligned}
\dot{x}_p &= a_p x_p + k_p u(t) \\
&= a_p x_p + k_p \big[\theta(t)x_p + k(t)r\big] \\
&= a_p x_p + k_p \big[\widetilde{\theta}(t)x_p + \theta^* x_p + \widetilde{k}(t)r + k^* r\big] \\
&= \big[a_p + k_p\theta^*\big]x_p + k_p\widetilde{\theta}(t)x_p + k_p k^* r + k_p\widetilde{k}(t)r \\
&= a_m x_p + k_p\widetilde{\theta}(t)x_p + k_m r + k_p\tilde{k}(t)r
\end{aligned}$$

Certainty Equivalence Principle - Step 2

Plant with control law substituted in, and after some algebra:

$$\dot{x}_p = a_m x_p + k_p \widetilde{\theta}(t) x_p + k_m r + k_p \tilde{k}(t) r$$

Reference Model:

$$\dot{x}_m = a_m x_m + k_m r$$

Define the tracking error as

$$e = x_p - x_m$$

Error model 3:

$$\begin{aligned} \dot{e} &= a_m e + k_p \widetilde{\theta}(t) x_p + k_p \tilde{k}(t) r \\ &= a_m e + k_p \widetilde{\overline{\theta}}^{\top}(t) \omega \end{aligned}$$

Certainty Equivalence Principle - Step 2

Plant:

$$\dot{x}_p = a_m x_p + k_p \widetilde{\theta}(t) x_p + k_m r + k_p \tilde{k}(t) r$$

Reference Model:

$$\dot{x}_m = a_m x_m + k_m r$$

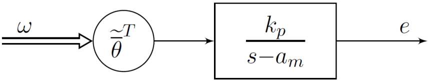
Define the tracking error as

$$e = x_p - x_m$$

Error model 3:

$$\begin{aligned} \dot{e} &= a_m e + k_p \widetilde{\theta}(t) x_p + k_p \tilde{k}(t) r \\ &= a_m e + k_p \tilde{\bar{\theta}}^{\top}(t) \omega \end{aligned}$$

$$\omega = \begin{bmatrix} x_p \\ r \end{bmatrix}$$



$$\tilde{\bar{\theta}} = \begin{bmatrix} \widetilde{\theta}(t) \\ \tilde{k}(t) \end{bmatrix}$$

Certainty Equivalence Principle - Step 2

This is where we left off last time.

$$\begin{aligned} u(t) &= \theta(t)x_p + k(t)r \\ \dot{e} &= a_m e + k_p \overline{\widetilde{\theta}}^\top(t)\omega \end{aligned}$$

- Let's finish proving stability
- Note one difference versus when we last saw error model 3: the presence of k_p in the error model!

Propose a slightly modified candidate Lyapunov function

$$V = \frac{1}{2}\left(e^2 + |k_p|\overline{\widetilde{\theta}}^\top \overline{\widetilde{\theta}}\right)$$

Certainty Equivalence Principle - Step 2

$$\begin{aligned} \text{error model:} \qquad \dot{e} &= a_m e + k_p \widetilde{\overline{\theta}}^\top(t)\omega \\ \text{Lyapunov function:} \qquad V &= \frac{1}{2}\left(e^2 + |k_p| \widetilde{\overline{\theta}}^\top \widetilde{\overline{\theta}}\right) \end{aligned}$$

Take the time derivative of V

$$\begin{aligned} \dot{V} &= e\dot{e} + \widetilde{\overline{\theta}}^\top \dot{\widetilde{\overline{\theta}}} \\ &= a_m e^2 + k_p e \widetilde{\theta}^\top \omega + |k_p| \widetilde{\overline{\theta}}^\top \dot{\widetilde{\overline{\theta}}} \\ &= a_m e^2 + \widetilde{\theta}^\top (k_p e \omega + |k_p| \dot{\overline{\widetilde{\theta}}}) \end{aligned}$$

Propose the update law: $\dot{\widetilde{\theta}} = -\text{sign}(k_p) e\omega$

$$\dot{V} = a_m e^2 \leq 0$$

$\Rightarrow e(t)$ and $\widetilde{\theta}(t)$ are bounded for all $t \geq t_0$

Adaptive Gain γ

- We have reduced the adaptive control problem to error model 3, and proved stability using a Lyapunov function
- Now that we have shown stability, what about performance?
- To provide an extra degree of freedom to tune the closed-loop performance, we add an additional term, an *adaptive gain* γ
- We will look at performance quantitively later on - for now we will just look at it qualitatively

Adaptive Gain γ

Same plant, reference model, and control las as before give the same error model:

$$\dot{e} = a_m e + k_p \widetilde{\theta}(t) x_p + k_p \tilde{k}(t) r$$

Now introduce a gain γ in the update law

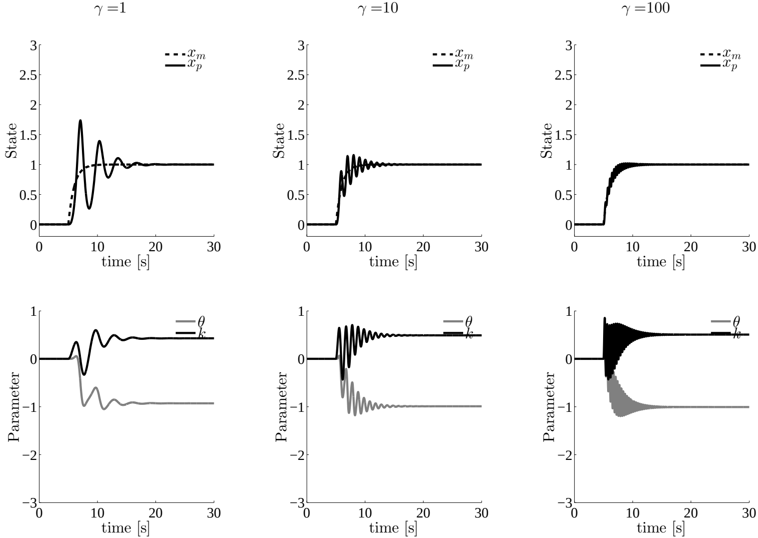
$$\begin{aligned} \dot{\theta}(t) &= -\gamma \mathsf{sign}(k_p) e x_p \\ \dot{k}(t) &= -\gamma \mathrm{sign}(k_p) e r \end{aligned}$$

Choose the following candidate Lyapunov function and differentiate

$$\begin{aligned} V &= \frac{1}{2}\left(e^2 + \frac{|k_p|}{\gamma} \overline{\widetilde{\theta}}^\top \overline{\widetilde{\theta}}\right) \\ \dot{V} &= e\dot{e} + \overline{\widetilde{\theta}}^\top \dot{\overline{\widetilde{\theta}}} \\ &= a_m e^2 + \overline{\widetilde{\theta}}^\top \left(k_p e \phi + \frac{|k_p|}{\gamma} \dot{\overline{\widetilde{\theta}}}\right) \\ &= a_m e^2 \le 0 \end{aligned}$$

Adaptive Gain Example

Simulation Parameters: $a_m = -1,\ k_m = 1,\ a_p = 1,\ k_p = 2$



Adaptive Gain Example

These simulations show (qualitatively) the effect that γ has on the performance of the system

- The rate at which the parameters is adjusted increases
- The state follows the reference model more closely
- This results in increased control rates and fast oscillations

There are ways to reduce the oscillations, which we will see later in the course.

We will look at this from a quantitative view in coming lectures as well

For now, we will go through stability in a little more depth.

Equilibrium Points

Consider the following dynamical system

$$\begin{aligned} \dot{x}(t) &= f(x(t), t) \\ x(t_0) &= x_0 \end{aligned} \tag{1}$$

Definition: equilibrium point (pg 45) The state x_{eq} is an *equilibrium point* of (1) if it satisfies:

$$f(x_{\text{eq}}, t) = 0 \tag{2}$$

for all $t \geq t_0$.

Definition: autonomous (pg 45) If the RHS of (1) does not depend on t, the equation is called *autonomous*.

Equilibrium Points

Definition: isolated equilibrium (pg 45) If x_{eq} is the only constant solution in the neighborhood of x_{eq}, it is called an *isolated equilibrium*.

Both linear and nonlinear systems can have multiple equilibrium points

- Linear systems will have a single equilibrium point, or an infinity of non-isolated equilibrium points e.g. mass

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f$$

- Nonlinear systems can have an infinity of isolated equilibrium points e.g. pendulum

Note: when we talk about stability, we talk about the stability of a particular equilibrium *point*.

Stability Definitions

Definition 2.1: stable (pg 51) The equilibrium state x_{eq} of (1) is said to be *stable* if for every $\epsilon > 0$ and $t_0 \geq 0$, there exists a $\delta(\epsilon, t_0) > 0$ such that $\|x_0\| < \delta$ implies that $\|x(t; x_0, t_0)\| < \epsilon \; \forall t \geq t_0$.

Definition 2.2: attractive (pg 51) The equilibrium state x_{eq} of (1) is said to be *attractive* if for some $\rho > 0$ and every $\eta > 0$ and $t_0 > 0$, there exists a number $T(\eta, x_0, t_0)$ such that $\|x_0\| < \rho$ implies that $\|x(t; x_0, t_0)\| < \eta \; \forall t \geq t_0 + T$.

- Attractivity: all trajectories starting in a neighborhood of x_{eq} eventually approach x_{eq}.
- Attractivity and stability are independent concepts

Definition 2.3: asymptotically stable (pg 51) The equilibrium state x_{eq} of (1) is said to be *asymptotically stable* if it is both stable and attractive.

Stability Definitions (Continued)

Definition 2.4: uniformly stable (pg 52) The equilibrium state x_{eq} of (1) is said to be *uniformly stable* if in Definition 2.1 δ is independent of initial time.

Definition: globally stable (pg 52) If $\lim_{\epsilon \to \infty} \delta(\epsilon) = \infty$ in the definitions above, then the equilibrium stat is said to be *globally stable*.

See page 52 for more stability definitions.

Lyapunov's Stability Methods

A motivating example: determine the stability of the origin for the following scalar system

$$\dot{x}(t) = ax(t)$$

Can determine the stability of the origin by solving the differential equation:

$$x(t) = e^{at}$$

The *origin* is globally exponentially stable.
What about the Van der Pol oscillator?

$$\ddot{x} = -x + (x^2 - 1)\dot{x}$$

which has an equilibrium point at the origin.

Lyapunov's methods allow us to determine the stability of an equilibrium for such a system without solving the differential equation!

Lyapunov's Indirect (1st) Method

- Also called Lyapunov's linearization method
- Concerned with *local* stability of an equilibrium point of a nonlinear system
- Idea: all physical systems are nonlinear, but a linearized approximation is valid in a neighborhood of an equilibrium

Formally:

$$\dot{x} = f(x)$$

- In general, the nonlinear system can be non-autonomous, and the resulting linearized system will be time varying.

Define perturbations about an equilibrium point x_{eq} as

$$x = x_{\text{eq}} + \delta x$$

Differentiating

$$\dot{x} = \dot{x}_{\text{eq}} + \dot{\delta x}$$

Lyapunov's Indirect (1^{st}) Method (Continued)

Recalling the definition of an equilibrium point, we obtain

$$\dot{\delta x} = f(x) = f(x_{\text{eq}} + \delta x)$$

Now perform a Taylor Series expansion of $f(x_{\text{eq}} + \delta x)$

$$f(x_{\text{eq}} + \delta x) = f(x_{\text{eq}}) + \left.\frac{\partial f}{\partial x}\right|_{\text{eq}} \delta x + \varepsilon(x)$$

since $f(x_{\text{eq}}) = 0$ we have

$$\dot{\delta x} = \left.\frac{\partial f}{\partial x}\right|_{\text{eq}} \delta x + \varepsilon(x)$$

which we write, after neglecting the higher order terms, as

$$\dot{\delta x} = A\delta x$$

Lyapunov's Indirect (1st) Method (Continued)

We evaluate $A = \frac{\partial f}{\partial x}$ as

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_2} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad \text{where} \qquad f = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}$$

This is called the *Jacobian matrix* of f.

Theorem: Lyapunov's linearization method When a nonlinear system f is linearized about an equilibrium point x_{eq} with Jacobian matrix A:

- If A is strictly stable, then x_{eq} is asymptotically stable for f
- If A is unstable, then x_{eq} is unstable for f
- If A is marginally stable, then we can draw no conclusion about stability of x_{eq} for f

Lyapunov's Indirect (1st) Method (Continued)

Return to the Van der Pol oscillator:

$$\ddot{X} = -X + (X^2 - 1)\dot{X}$$

write as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} X_2 \\ -X_1 + (X_1^2 - 1)X_2 \end{bmatrix}$$

where $X_1 = X_2 = 0$ is an equilibrium point. The nonlinear system is represented as

$$\dot{X} = f(X)$$

We want to linearize about the origin to get

$$\dot{x} = \frac{\partial f}{\partial x}\Big|_{\text{eq}} x = Ax$$

Lyapunov's Indirect (1st) Method (Continued)

With

$$\begin{aligned} f_1 &= X_2 \\ f_2 &= -X_1 + (X_1^2 - 1)X_2 \end{aligned}$$

We linearize by taking partial derivatives and obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 + 2X_1X_2 & X_1^2 - 1 \end{bmatrix}\Bigg|_{\mathrm{eq}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Evaluating the Jacobian at the equilibrium point we obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which is stable.

Using Lyapunov's Indirect Method, we can conclude that the origin of the nonlinear system $\ddot{X} = -X + (X^2 - 1)\dot{X}$ is stable.

Lyapunov's Direct (2^{nd}) Method

- Lyapunov's first method does not work if the linearization is marginally stable.
- Need a new tool: Lyapunov's second, or direct method
- Again, we will have another tool which gives us information about the stability of an equilibrium point for a nonlinear system, without solving the differential equation
- Lyapunov's direct method is motivated by the observation that as physical systems dissipate energy, they eventually settle to an equilibrium point.
- Idea: determine a scalar, positive definite energy-like function and look at how this quantity changes with time.

Lyapunov's Direct (2^{nd}) Method (Continued)

For the system

$$\dot{x} = f(x)$$

Let

(i) $V(x) > 0, \ \forall x \neq 0, \text{and } V(0) = 0$

(ii) $\dot{V}(x) = \left(\frac{\partial V}{\partial x}\right)^T f(x) < 0$

(iii) $V(x) \to \infty \text{ as } \|x\| \to \infty$

Then $x = 0$ is asymptotically stable. If instead of (ii), we have

(ii-b) $\dot{V} \leq 0$

Then $x = 0$ is stable.

- Now that we have learned more about stability, we know what this means
- But what more can we say?

Asymptotic Convergence of $e(t)$ to Zero

- Stability is an important and necessary first step, but recall our control goal of ensuring $e(t) \to 0$ as $t \to \infty$
- In order to prove that this will in fact be the case, we require some additional tools
- We will soon introduce Barbalat's Lemma and a corollary
- To understand these new tools, we first introduce some notation regarding the norms of signals

Norms

- Given two real numbers, the notion of the "size" of these numbers is apparent
- However, given quantity such as a vector, matrix, or time-varying signal, we may by interested in how "big" they are when compared to another vector, matrix, or time-varying signal respectively
- A norm is a non-negative measure of the magnitude of a given quantity that satisfy three basic properties
- We will not go into the details of norms in lecture, but will post a short handout online

Signal Norms

- Given a signal (either vector-valued or scalar), we wish to determine some measure of its magnitude
- Perhaps the signal is very large at one instance of time, and small everywhere else, or maybe it is moderately large for all time
- Whatever the case may be, we wish to have some way to quantify these varying degrees of the "largeness" of a signal
- Often signal norms are denoted using the capital letter $\mathcal{L}_p$
- For the vector valued signal $x(t)$ the following norms are given

Signal Norms

$\mathcal{L}_p$ **Norm** The general definition of the $\mathcal{L}_p$ norm is the following, where $p \in \mathbb{N}^+$.

$$\|x(t)\|_{L_p} = \left(\int_0^t \|x(\tau)\|^p d\tau\right)^{\frac{1}{p}}$$

$\mathcal{L}_1$ **Norm**

$$\|x(t)\|_{L_1} = \int_0^t \|x(\tau)\| d\tau$$

$\mathcal{L}_2$ **Norm**

$$\|x(t)\|_{L_2} = \sqrt{\int_0^t \|x(\tau)\|^2 d\tau}$$

$\mathcal{L}_\infty$ **Norm**

$$\|x(t)\|_{L_\infty} = \sup_t \|x(t)\|$$

In this class we will mostly be using the $\mathcal{L}_2$ and $\mathcal{L}_\infty$ norms.

Existence of Signals in Normed Spaces

- In proving stability for adaptive systems, we are initially concerned with showing boundedness of signals, such as the error $e(t)$
- If this error signal is bounded for all time, we write this formally as

$$e \in \mathcal{L}_\infty$$

  where this implies the $\mathcal{L}_\infty$ norm of $e(t)$ exists and is finite
- Similarly, if the $\mathcal{L}_2$ norm of a signal $e(t)$ exists and is finite, we write

$$e \in \mathcal{L}_2$$

- Using this notation, we will now present Barbalat's Lemma

Barbalat's Lemma

Lemma 2.12 (Barbalat's) page 85

(i) If $f : \mathbb{R}^+ \to \mathbb{R}$ is uniformly continuous for $t \geq 0$

(ii) And if $\lim_{t\to\infty} \int_0^t |f(\tau)| d\tau$ exists and is finite

Then $\lim_{t\to\infty} f(t) = 0$

What does this say about the function f?

Corollary If $g \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, and $\dot{g}$ is bounded, then $\lim_{t\to\infty} g(t) = 0$.

Notice:

- Replace g with e in the corollary, and that is what we need to prove!
- So we need to show $e \in \mathcal{L}^2 \cap \mathcal{L}^\infty$ for the adaptive systems we have seen so far

Using Barbalat's Lemma

- Recall the error model 3 stability proof we did on slide 17
- We left off with $\dot{V} \leq 0 \Rightarrow e \in \mathcal{L}^\infty$ and $\widetilde{\theta} \in \mathcal{L}^\infty$
- For all bounded inputs r, x_m will be bounded, as it is the output of our stable reference model
- Since $x_m \in \mathcal{L}^\infty$ and $e \in \mathcal{L}^\infty \Rightarrow x_p \in \mathcal{L}^\infty$
- Now to show $e \in \mathcal{L}^2$

Using Barbalat's Lemma

Note that

$$\int_0^t \dot{V}(\tau)d\tau = V(t) - V(0)$$

Since V is non increasing and positive definite, $V(0) - V(t) \leq V(0)$. This gives

$$-\int_0^t \dot{V}(\tau)d\tau \leq V(0)$$

Substituting in our expression for $\dot{V} = a_m e^2$, remembering that $a_m < 0$

$$|a_m| \int_0^t e^2(\tau)d\tau \leq V(0)$$

which is equivalent to

$$|a_m| \int_0^t \|e(\tau)\|^2 d\tau \leq V(0) < \infty$$

Using Barbalat's Lemma

$$|a_m| \int_0^t \|e(\tau)\|^2 d\tau \leq V(0) < \infty$$

simplifies to

$$\sqrt{\int_0^t \|e(\tau)\|^2 d\tau} < \infty$$

Recognize that this is just $\|e(t)\|_{L_2} < \infty$ we write $e \in \mathcal{L}_2$

Finally, we need to show the boundedness of $\dot{e}$ so we can apply Barbalat's lemma

$$\dot{e} = a_m e + k_p \overline{\tilde{\theta}}^\top(t)\omega$$

Everything on the right side is bounded $\Rightarrow \dot{e}$ is bounded

Thus, the conditions to apply Barbalat's lemma (corollary) are met, so we conclude $\lim_{t\to\infty} e(t) = 0$

Brief Summary

Today we:

- Reviewed and finished stability for the first-order adaptive control problem from last lecture
- Introduced the adaptive gain γ and showed qualitatively some of its effects on the performance of the system
- Covered lots of different stability definitions
- Went over Lyapunov's first and second methods for proving stability for nonlinear systems
- Stated what signal norms were, and discussed the existence of signals in normed spaces
- Gave Barbalat's Lemma and showed how to use it to show $e \to 0$