1. Aufgabe

1.1 $J = \dfrac{1}{2}\sum_{k=0}^{N-1}(x_k^2 + u_k^2)$ u.B.v. $x_{k+1} = x_k + u_k\,;\quad x_0 = 2\,;\quad x_N = x_4 = 0$

Stufe $N = 4$:

$J_4^* = 0$

Stufe $N - 1 = 3$:

$J_3^* = \min\limits_{u_3}\left\{\dfrac{1}{2}(x_3^2 + u_3^2)\right\}$ u.B.v. $x_4 = x_3 + u_3 = 0$

$\longrightarrow u_3^* = -x_3$ $\quad J_3^* = x_3^2$

Stufe $N - 2 = 2$:

$J_2^* = \min\limits_{u_2}\left\{x_3^2 + \dfrac{1}{2}(x_2^2 + u_2^2)\right\}$ u.B.v. $x_3 = x_2 + u_2$

$\longrightarrow u_2^* = -\dfrac{2}{3}x_2$ $\quad J_2^* = \dfrac{5}{6}x_2^2$

Stufe 1:

$J_1^* = \min\limits_{u_1}\left\{\dfrac{5}{6}x_2^2 + \dfrac{1}{2}(x_1^2 + u_1^2)\right\}$ u.B.v. $x_2 = x_1 + u_1$

$\longrightarrow u_1^* = -\dfrac{5}{8}x_1$ $\quad J_1^* = \dfrac{13}{16}x_1^2$

Stufe 0:

$J_0^* = \min\limits_{u_0}\left\{\dfrac{13}{16}x_1^2 + \dfrac{1}{2}(x_0^2 + u_0^2)\right\}$ u.B.v. $x_1 = x_0 + u_0$

$\longrightarrow u_0^* = -\dfrac{13}{21}x_0$ $\quad J_0^* = \dfrac{17}{21}x_0^2$

1.2 $u_0^* = -\dfrac{13}{21}x_0 = -\dfrac{26}{21} \approx -1{,}24$

$x_1^* = x_0 + u_0^* = 0{,}76$

$u_1^* = -\dfrac{5}{8}x_1^* = -0{,}475$

$x_2^* = x_1^* + u_1^* = 0{,}285$

$u_2^* = -\dfrac{2}{3}\cdot 0{,}285 = -0{,}19$

$x_3^* = 0{,}285 - 0{,}19 = 0{,}095$

$u_3^* = -0{,}095$

$x_4^* = 0$

1.3 <u>Stufe 3</u>:

$u_3^* = -x_3 \qquad \mathcal{X}_3 = \{x_3 \mid 0 \le x_3 \le 2\}$

$x_4 = x_3 + u_3^* = 0 \in \mathcal{X}_4$

<u>Stufe 2</u>:

$u_2^* = -\frac{2}{3}x_2 \qquad \mathcal{X}_2 = \{x_2 \mid 0{,}4 \le x_2 \le 2\}$

$x_3 = x_2 - \frac{2}{3}x_2 = \frac{1}{3}x_2 \longrightarrow \frac{4}{30} \le x_3 \le \frac{2}{3} \subset \mathcal{X}_3$

<u>Stufe 1</u>:

$J_1^* = \min_{u_1}\left\{\frac{5}{6}x_2^2 + \frac{1}{2}(x_1^2 + u_1^2)\right\}$ u.B.v. $x_2 = x_1 + u_1$; $0{,}4 \le x_2 \le 2$

Einsetzverfahren liefert Problem mit UNB $\longrightarrow$ Lagrange-Funktion mit Kuhn-Tucker-Bedingungen; Fallunterscheidung für $\mu_1 > 0$, $\mu_1 = 0$!

$$\longrightarrow \quad u_1^* = \begin{cases} \frac{2}{5} - x_1 & \text{für } \frac{4}{5} \le x_1 \le \frac{16}{15} \\ -\frac{5}{8}x_1 & \text{für } \frac{16}{15} \le x_1 \le 2 \end{cases}$$

$$J_1^* = \begin{cases} x_1^2 - \frac{2}{5}x_1 + \frac{16}{75} & \text{für } \frac{4}{5} \le x_1 \le \frac{16}{15} \\ \frac{13}{16}x_1^2 & \text{für } \frac{16}{15} \le x_1 \le 2 \end{cases}$$

<u>Stufe 0</u>:

$J_0^* = \min_{u_0}\left\{J_1^* + \frac{1}{2}x_0^2 + \frac{1}{2}u_0^2\right\}$ u.B.v. $x_1 = x_0 + u_0$; $\frac{4}{5} \le x_1 \le 2$; $x_0 = 2$

Fall 1: $\frac{4}{5} \le x_1 \le \frac{16}{15} \qquad \longrightarrow -\frac{6}{5} \le u_0 \le -\frac{14}{15}$

$$J_{01}^* = (u_0^* + 2)^2 - \frac{2}{5}(u_0^* + 2) + 0{,}213 + \frac{1}{2}u_0^{*2} + 2$$

u.B.v. $-\frac{6}{5} \le u_0 \le -\frac{14}{15}$

Fall 1.a: $u_{01a}^* = -\frac{6}{5} \qquad \longrightarrow J_{01a}^* = 3{,}25$

Fall 1.b: $u_{01b}^* = -\frac{14}{15} \qquad \longrightarrow J_{01b}^* = 3{,}36$

Fall 2: $\frac{16}{15} \le x_1 \le 2$

$J_{02}^* = \frac{13}{16}(u_0^* + 2)^2 + \frac{1}{2}u_0^{*2} + 2$ u.B.v. $-\frac{14}{15} \le u_0 \le 0$

Fall 2: $u_2 = -\frac{14}{15} \qquad \longrightarrow J_{02}^* \approx 3{,}4$

$$\longrightarrow \quad u_0^* = -\frac{6}{5}$$

$$J_0^* = 3{,}25$$