A Framework for Vehicle Lateral Motion Control With Guaranteed Tracking and Performance*

Ankit Gupta[1*], Magnus Nilsson[2], Paolo Falcone[1], Emil Klintberg[2] and Lars Johannesson Mårdh[2]

Abstract— **In this paper, we propose a framework to design a controller for the vehicle lateral dynamics, which guarantees to meet desired safety and performance requirement. The strict safety requirement considered in this paper is a bound on the lateral deviation from the reference trajectory.**

The proposed control design relies on a mild assumption. That is, the trajectory planner generates a trajectory that is piece-wise clothoidal (PWC), with bounded curvature and curvature rate.

Closed-loop simulations using a linear quadratic regulator (LQR) and a model predictive control (MPC) controller, developed based on the proposed framework, show validate the proposed approach.

I. INTRODUCTION

Automated driving technologies, as well as many driver-assistance functionalities like lane keeping assist (LKA) and lane change assist (LCA) in semi-autonomous vehicles, are either in the test stage or already available [1]. These functionalities, in general, are built up of many subsystems. A commonly used structure is presented in Fig. 1. The lateral controller is an important subsystem in the overall vehicle control problem. The desired reference trajectory, which usually represents the path, is provided to the controller by a higher level path-planner. Based on a pre-fed control algorithm a steering action is decided by the controller which is then applied to the vehicle to keep it on the path.

As explained in [2], [3], for such safety-critical applications, the overall functionality built with many subsystems should always guarantee to meet the design specifications by construction (e.g. bounded deviation from the reference or lateral acceleration). A direct way could be to design these functionalities in a monolithic fashion (i.e. to design the path planner and the controller together) so that the overall design specification are met. A drawback of such an approach is the size of the design problem, which can be very large. Moreover, any modification in one of the subsystem involves redesigning the whole functionality.

Another approach could be to design each subsystem in a modular way. For this kind of compositional design and analysis, a popular framework is the *contract based design* (CBD) approach [4]. In the CBD approach, contracts are established between subsystems in a way that each subsystem guarantees satisfaction of its specification, assuming that the other subsystems do not violate their own specification. At the cost of potential conservatism, a large control problem is decomposed into smaller subproblems designed to meet their contract requirements. This kind of approach has already been applied in other automotive applications [5], [6].

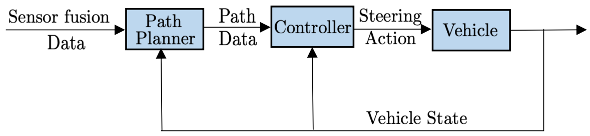

Fig. 1. Control Architecture

In an attempt to decouple the lateral controller design from the path planner, approaches in [2], [7], [8] assume that the reference trajectories generated by the path planner represent paths with bounded curvature. In these approaches, the controller guarantees to track the reference within some error bounds even when the change in the reference direction is arbitrary. It is known from [1] that the reference trajectory depends on the road geometry, which is often modelled as clothoids. The above-mentioned approach can be conservative from the design perspective since it is not able to utilize the path information and only able to meet the design specifications for the paths with a very small range of curvatures. Recently an MPC based lateral controller has been proposed in [9] which guarantees to track PWC trajectories within desired error bounds.

The objective of the present research is to develop a framework for the lateral control of a vehicle based on the CBD approach. The proposed scheme guarantees to meet the control specification while assuming that the path planner always generates a PWC reference trajectory with a predefined specification. The safety and performance specifications for the controller are formulated in terms of the bounded deviation from the reference and bounded steering input, and possibly other comfort requirements based on driving scenarios can be also included. The PWC trajectories with a bounded curvature and curvature rate are modelled as a simple integrator with constraints (we call it a path model). In [9], to overcome the issue related to state-dependent constraints arising due to path model (i.e. non-convex robust control invariant (RCI) set), a new algorithm is proposed to compute polytopic RCI set. In addition, an MPC control algorithm is designed by using the RCI set as a terminal constraint to guarantee constraint satisfaction.

Compared to [9], we present an alternative formulation

*This work is supported by the Vinnova FFI Complex Control Program, under the grant No. 2015-02309.

[1]A. Gupta and P. Falcone are with Electrical Engineering Department, Chalmers University of Technology, Gothenburg, Sweden.

[2]M. Nilsson, E. Klintberg and L. Johannesson are with Zenuity AB, Gothenburg, Sweden. Corresponding Author: ankit.gupta@chalmers.se

in which the initially assumed path model is modified in a way such that the new path model still represents all PWC trajectories which can be generated from the former. The main advantage of this approach is that the computation of the RCI set and the robust positive invariant (RPI) set (for a predefined control law) can be done using most of the existing approaches in the literature. To demonstrate the applicability of the proposed approach, we design an MPC and an LQR controller for simulation purpose.

The paper is structured as follows. We start with the preliminaries in section II. The vehicle lateral dynamics, constraints and the problem statement are presented in section III. The section IV presents a reformulation of the problem and the proposed control framework. In section V, the closed-loop simulation results are shown. Concluding remarks with the discussion of possible scope for future consideration have been include in sectionVI.

II. Preliminaries

For completeness, we present some definitions from [10], [11] which will be useful in the later parts.

Definition 1. *Given a dynamical system $x(k+1) = f(x(k), u(k), w(k))$ where $x(k) \in \mathcal{X}$, $u(k) \in \mathcal{U}$, $w(k) \in \mathcal{W}$ are the state, input and disturbance vectors, a set $\Omega \subseteq \mathcal{X}$ is a robust control invariant (RCI) set if*

$$x(k) \in \Omega \Rightarrow \exists u(k) \in \mathcal{U} : x(k+1) \in \Omega, \forall w(k) \in \mathcal{W} \quad (1)$$

Definition 2. *Given a dynamical system $x(k+1) = f(x(k), w(k))$ where $x(k) \in \mathcal{X}$, $w(k) \in \mathcal{W}$ are the state and disturbance vectors, a set $\mathcal{C} \subseteq \mathcal{X}$ is a robust positive invariant (RPI) set if*

$$x(k) \in \mathcal{C} \Rightarrow x(k+1) \in \mathcal{C}, \forall w(k) \in \mathcal{W} \quad (2)$$

Definition 3. *Given a dynamical system $x(k+1) = f(x(k), u(k))$ where $x(k)$ and $u(k) \in \mathcal{U}$ are the state and input vectors. A set $\mathcal{R}(S)$ is a one-step forward reachable set from a set S if*

$$x(k) \in S \Rightarrow \exists u(k) \in \mathcal{U} : x(k+1) \in \mathcal{R}(S) \quad (3)$$

III. Modeling and Problem Formulation

A. Path Model

Consider a vehicle travelling at a constant longitudinal velocity V_x on a path with constant curvature ρ. The resulting yaw rate of the vehicle is given by $\dot{\psi}_d = \rho V_x$. We assume that, based on the road geometry or the manoeuvre, the path planner in Fig.1 generates a time-varying PWC reference trajectory with bounded curvature and curvature rate. Hence, $\dot{\psi}_d$ should vary with a bounded rate as well and, similarly to [9], we model $\dot{\psi}_d$ as

$$\dot{\psi}_d(k+1) = \dot{\psi}_d(k) + \gamma(k), \quad (4)$$
$$|\dot{\psi}_d(k)| \leq \theta, \ |\gamma(k)| \leq \overline{\gamma}. \quad (5)$$

Where $\gamma(k)$ is the variation of $\dot{\psi}_d(k)$ between consecutive sampling time instances. We assume that the $\gamma(k)$ is decided by the path-planner algorithm to generate the PWC reference trajectories for the vehicle to track.

Remark 1. *Here θ and $\overline{\gamma}$ are the constant parameters whose values are decided based on the driving scenario or the performance requirements (e.g. bounded lateral acceleration or lateral jerk). The requirements on the path-planner to generate reference trajectories according to such specifications are used in the lateral controller design.*

B. Vehicle Dynamics and Constraints

For controller design purposes, we consider the following standard bicycle model of the vehicle lateral dynamics with a constant longitudinal velocity V_x, [1]

$$\begin{bmatrix} \dot{e}_y \\ \ddot{y} \\ \dot{e}_\psi \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & V_x & 0 \\ 0 & \frac{-(C_f+C_r)}{mV_x} & 0 & -V_x - \frac{(l_f C_f - l_r C_r)}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-(l_f C_f - l_r C_r)}{I_z V_x} & 0 & \frac{-(l_f^2 C_f + l_r^2 C_r)}{I_z V_x} \end{bmatrix} \underbrace{\begin{bmatrix} e_y \\ \dot{y} \\ e_\psi \\ \dot{\psi} \end{bmatrix}}_{z(t)} + \begin{bmatrix} 0 \\ \frac{C_f}{m} \\ 0 \\ \frac{l_f C_f}{I_z} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \dot{\psi}_d, \quad (6)$$

where m, I_z are the vehicle mass and inertia. C_f, C_r are the front and rear cornering stiffness coefficients and l_f, l_r are the distance of the front and rear wheel axle from the vehicle centre of gravity, respectively. The vehicle states e_y, e_ψ are the vehicle lateral and orientation error, w.r.t. a predefined path and $\dot{y}$, $\dot{\psi}$ are the lateral velocity and the yaw rate of the vehicle, while δ is the steering input to the vehicle and $\dot{\psi}_d$ is the rate of change of the path orientation (the desired vehicle yaw rate), generated by the path planner according to (4). Note that $\dot{\psi}_d$ can be seen as a bounded input disturbance, with bounded rate, to the vehicle dynamics (6) and is used next in the paper to establish a link between path planner and the later controller, through the vehicle model.

By discretizing (6) and redefining the input as $\Delta\delta(k) = \delta(k) - \delta(k-1)$, we obtain an augmented system

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k) + \hat{E}\dot{\psi}_d(k), \quad (7)$$

where $\hat{x}(k) = [z(k)^T \ \ \delta(k-1)]^T$ and $u(k) = \Delta\delta(k)$.

In this paper, the safety requirements impose a bound on the lateral deviation of the vehicle from the reference PWC trajectory as

$$|e_y| \leq \overline{e}. \quad (8)$$

Furthermore, the state and input vector in (7) are subject to a set of physical and design constraints to meet the a set of performance specifications [2], [7]. These constraints along with (8) can be compactly written as

$$\hat{\mathcal{X}} \triangleq \{\hat{x} | \ \hat{H}\hat{x}(k) \leq \mathbf{1}\}, \ \mathcal{U} \triangleq \{u | \ Gu(k) \leq \mathbf{1}\}, \quad (9)$$

where $\mathbf{1}$ is a vector of ones of a compatible dimension.

The necessary definitions and notation have been introduced to formulate the following

Problem 1. *Given the path model (4) and (5), construct a stabilizing controller for (7), which guarantees to satisfy the safety and performance requirements (9) at all times.*

Problem 1 could be solved with MPC schemes, including a RCI set for the system (7), calculated according to the disturbance model (4) and (5). Computing such RCI set for the extended system is not straight forward because the system (4) is not stabilizable. By treating (4) and (5) together as state-dependent disturbance, approach mentioned in [9], [12] can be used to calculate the RCI set. In particular, the RCI set obtained from [12] for such a system is non-convex, while the RCI set from [9] is polytopic and its use is restricted to MPC control strategies.

In the next section, we propose an alternative approach. We first introduce a new path model which is stabilizable by construction while still representing trajectories which can be generated by (4) and (5). We then show how this model can be used in a MPC and a LQR control design scheme, which solves **Problem 1**.

IV. Lateral Controller With Guaranteed Performance

A. New Path Model

To facilitate the controller design, we consider a new path model as

$$\dot{\psi}_p(k+1) = \alpha\dot{\psi}_p(k) + \beta v(k),\ v(k) \in [-1, 1], \quad (10)$$

where $|\alpha| < 1,\ \beta$ are the parameters defining the new path model. Moreover, $|\alpha| < 1,\ \beta$ should be chosen such that any reference trajectory generated by (4) and (5) can be also generated by (10) by selecting $v(k)$ appropriately within the given bounds. A sufficient condition for this is that $\mathcal{R}^d(\dot{\psi}_d(k)) \subseteq \mathcal{R}^p(\dot{\psi}_p(k))$ for all $|\dot{\psi}_d(k)| \leq \theta$ and $\dot{\psi}_p(k) = \dot{\psi}_d(k)$, where $\mathcal{R}^d(\dot{\psi}_d(k))$ and $\mathcal{R}^p(\dot{\psi}_p(k))$ are the one-step forward reachable sets (**Definition 3**) from some point $\dot{\psi}_d(k)$ and $\dot{\psi}_p(k)$, respectively.

In the next lemma we propose an approach to select the parameters α and β such that the model (10) meets such requirement. Note that *w.l.o.g.* we assume $\alpha \geq 0$ since (10) is a first order system and the constraints are symmetric.

Lemma 1. *$\mathcal{R}^d(\dot{\psi}_d(k)) \subseteq \mathcal{R}^p(\dot{\psi}_p(k))$ for all $|\dot{\psi}_d(k)| \leq \theta$ and $\dot{\psi}_p(k) = \dot{\psi}_d(k)$ if $\alpha \geq 0$ and $\beta > 0$ are selected as*

$$\alpha = \frac{\theta - \epsilon}{\theta},\ \beta = \bar{\gamma} + \epsilon,$$

for some $\epsilon \in (0,\ \theta]$.

Proof: given some $\dot{\psi}_p(k)$ and $\dot{\psi}_d(k)$, from **Definition 3**, (4), (5) and (10),

$$\mathcal{R}^d(\dot{\psi}_d(k)) = [\dot{\psi}_d(k) - \overline{\gamma}, \dot{\psi}_d(k) + \overline{\gamma})], \quad (11)$$

$$\mathcal{R}^p(\dot{\psi}_p(k)) = [\alpha\dot{\psi}_p(k) - \beta, \alpha\dot{\psi}_p(k) + \beta]. \quad (12)$$

For now, lets assume $\dot{\psi}_d(k) = \dot{\psi}_p(k) = 0$. From (11) and (12), $\mathcal{R}^d(\dot{\psi}_d(k)) = [-\overline{\gamma}, \overline{\gamma}]$ and $\mathcal{R}^p(\dot{\psi}_p(k)) = [-\beta, \beta]$. Hence by selecting $\beta = \overline{\gamma} + \epsilon$ (where $\epsilon \geq 0$) we obtain the desired property, i.e $\mathcal{R}^d(\dot{\psi}_d(k)) \subseteq \mathcal{R}^p(\dot{\psi}_p(k))$. By Substituting β in (12), the set $\mathcal{R}^p(\dot{\psi}_p(k))$ can now be rewritten as

$$\mathcal{R}^p(\dot{\psi}_p(k)) = [\alpha\dot{\psi}_p(k) - \overline{\gamma} - \epsilon,\ \alpha\dot{\psi}_p(k) + \overline{\gamma} + \epsilon]. \quad (13)$$

When $\dot{\psi}_d(k) = \dot{\psi}_p(k) \neq 0$, sufficient conditions for $\mathcal{R}^d(\dot{\psi}_d(k)) \subseteq \mathcal{R}^p(\dot{\psi}_p(k))$ obtained by comparing upper bound and lower bounds in (11) and (13) are given by

$$(1-\alpha)\dot{\psi}_d(k) \leq \epsilon,\ \forall\, |\dot{\psi}_d(k)| \leq \theta, \quad (14)$$

$$(\alpha-1)\dot{\psi}_d(k) \leq \epsilon,\ \forall\, |\dot{\psi}_d(k)| \leq \theta. \quad (15)$$

For simplicity, we consider following two cases, $i)\ \dot{\psi}_p(k) = \dot{\psi}_d(k) > 0$ and $ii)\ \dot{\psi}_p(k) = \dot{\psi}_d(k) < 0$. In the first case, we have $|\alpha| < 1$ and (14) satisfied if $\alpha \geq \frac{\theta-\epsilon}{\theta}$ where $\epsilon > 0$. As we initially assumed $\alpha \geq 0$ and it is desirable to have α to be as small a possible, hence we select $\alpha = \frac{\theta-\epsilon}{\theta}$ and $\epsilon \in (0,\ \theta]$. Note that the α and β selected in this way also satisfy (15). For the second case, using the similar arguments as before, we arrive at the same conditions. Finally, from steady state analysis of the (10), we note that $|\dot{\psi}_p(k)| \leq \frac{\beta}{1-\alpha} = \frac{\overline{\gamma}+\epsilon}{\epsilon}\theta = \bar{\theta}$. Clearly $\theta < \bar{\theta}$ and thus the statement in the **Lemma 1** holds.

Next, we bound the state variable in (10) with $|\dot{\psi}_p(k)| \leq \bar{\theta}$. As a consequence, thanks to **Lemma 1**, replacing (4) with (10) in the control design does not affects the path planner design because a controller robust to all desired yaw rate (trajectories) generated by (10) is also robust to (4).

For convenience, from (7) and (10) we define a new extended system

$$x(k+1) = Ax(k) + Bu(k) + Ev(k), \quad (16)$$

where $x(k) = [\hat{x}(k)^T \quad \dot{\psi}_p(k)]^T$, $u(k) = \Delta\delta(k)$, and $\left[\begin{array}{c|c|c} A & B & E \end{array}\right] = \left[\begin{array}{cc|c|c} \hat{A} & \hat{E} & \hat{B} & 0 \\ 0 & \alpha & 0 & \beta \end{array}\right]$. The state and input constraints for the system (16), resulting from constraints (9) and (10) can be rewritten as

$$\mathcal{X} \triangleq \{x|\ Hx(k) \leq \mathbf{1}\},\ \mathcal{U} \triangleq \{u|\ Gu(k) \leq \mathbf{1}\}, \quad (17)$$
$$v(k) \in [-1,\ 1].$$

We treat $v(k)$ as an uncontrolled input which depends on the trajectory of $\dot{\psi}_d(k)$ provided by the path-planner. The new extended system (16) is stabilizable since the eigenvalue corresponding to the uncontrolled state $\dot{\psi}_p(k)$ is $\alpha \in [0,\ 1)$. Hence most of the commonly used algorithms to compute the RCI set can be now directly applied to (16) and (17).

Remark 2. *If we select $\epsilon = 0$ then the obtained new path model (10) is the same as the disturbance model (4) with $|\dot{\psi}_p| \leq \infty$ thus in this case RCI set for (16) and (17) does not exist. If $\epsilon = \theta$ is selected then the new path model represents a path with $|\dot{\psi}_p| \leq \overline{\gamma} + \theta$ changing arbitrarily within the bound. For this, the RCI set may not exist or will be very conservative. Thus it is desirable to select $\epsilon \in (0,\ \theta]$, which leads to an RCI set with the largest possible volume. This can be done by doing line search on ϵ.*

In the next section, we present the control scheme which solves **Problem 1**

B. Lateral Controller

As mentioned before, for control design purposes, we assume that the path planner provides a reference trajectory which meets the specification (4) and (5). Let the M-step trajectory provided by the path-planner at the time instant k be

$$\dot{\Psi}_d(k) = [\dot{\psi}_d(k+1),\, \dot{\psi}_d(k+2), \ldots,\, \dot{\psi}_d(k+M)]^T. \quad (18)$$

The trajectory (18) can be reconstructed from (10) by selecting

$$v(k) = \frac{\dot{\psi}_d(k+1) - \alpha\dot{\psi}_d(k)}{\beta},\ \dot{\psi}_p(k) = \dot{\psi}_d(k). \quad (19)$$

For the presentation, let $\dot{\Psi}_p(k) = [\dot{\psi}_p(k+1),\, \dot{\psi}_p(k+2), \ldots,\, \dot{\psi}_p(k+M)]^T$ and $V(k) = [v(k),\, v(k+1), \ldots,\, v(k+M-1)]^T$ be the trajectories generated from (10), (18) and (19).

Next we present the main result of the paper.

Lemma 2. *The PWC trajectories represented by (18) can be tracked within the desired error bound (8) if the RCI set or the RPI set (for some predefined control law) for the system (16) and (17) exist.*

Proof: From **Lemma 1** it is known that the trajectory of $V(k)$ is always bounded between $[-1,\,1]$ at all times. Hence if the RCI/RPI set exists, from the **Definition 1** and the **Definition 2**, there always exists a control input trajectory which keeps the system within the constraints and hence satisfies (8).

Lemma 2 states a result, which is crucial from the overall design perspective. It shows that the trajectory-planner and controller can be designed independently, once the specifications on the trajectory model (4) and (5) between the planner and the controller are agreed.

The proposed control framework is rather flexible, as it is not tailored to a specific control structure. Next, we show how to use the result in **Lemma 2** to solve **Problem 1** with two different control schemes.

The overall controller structure is shown in Fig. 2.

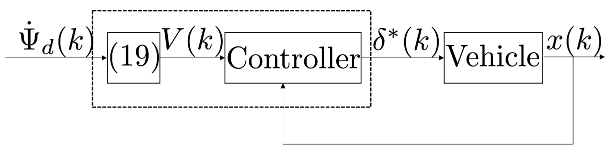

Fig. 2. Closed-loop control scheme

1) LQR: In this case we consider $M = 1$ in (18) and (19). For the desired tuning, an LQR controller can be designed for the system (16). Thus from **Lemma 2**, the requirement in **Problem 1** are met if the closed-loop system is operated in the corresponding maximal RPI set. This approach is useful when the on-board computational resource in the vehicle is limited. An obvious drawback of using an LQR controller can be a very small operational domain (i.e size of the RPI set) and a smaller set of PWC trajectories which can be tracked within a desired error bounds.

2) MPC: For large operational domain (i.e the feasibility set), an MPC controller can be designed for the (16) and (17). We consider an MPC controller, formulated as follows

$$J(x(k)) = \min_{U_k} \|x_{k+N|k}\|_P^2 + \sum_{i=0}^{N-1} \|x_{k+i|k}\|_Q^2 + \|u_{k+i|k}\|_R^2$$

$$\begin{aligned}
\text{St: } \ & x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + Ev_{k+i|k}, \\
& x_{k+i|k} \in \mathcal{X},\ v_{k+i|k} = v(k+i), \\
& u_{k+i|k} \in \mathcal{U},\ \forall i = 0, \cdots, N-1 \\
& x_{k+N|k} \in \mathcal{C}, \\
& x_{k|k} = x(k). \qquad (20)
\end{aligned}$$

where $U_k = \left[u_{k|k}, \ldots, u_{k+N-1|k}\right]$, N is prediction horizon, $P,\ Q \succeq 0$ and $R \succ 0$ are the tuning matrices of appropriate dimensions. The set $\mathcal{C}$ is a RCI or a RPI set. The above scheme is referred as dual-mode MPC controller in [13], which is always guaranteed to be recursively feasible and stable by construction. If the preview of the trajectory (18) is available, by setting $N = M$, it can be used to compute $v(k+i)$ from (19) to improve the controller performance.

In case of unavailability of the preview trajectory, $v(k+i)$ is treated as a disturbance input arbitrarily varying between $[-1,\,1]$. In this case, many existing robust MPC schemes e.g [14], [15] can be used to guarantee recursive feasibility and stability.

In the next section, we perform a closed-loop simulation of the proposed controllers and compare them.

V. Simulation

Simulations have been performed in MATLAB using the vehicle parameters presented in Table I. For this, we consider a constant longitudinal velocity $V_x = 80\ km/h$ and a sampling time $T = 25\ ms$. The bounds on the state and

TABLE I

Vehicle parameters

Parameter	Description	Value
m	Mass	2164 $[kg]$
I_z	Yaw moment of inertia	4373 $[kg \times m^2]$
C_r	Rear cornering stiffness coeff.	228088 $[N/rad]$
C_f	Front cornering stiffness coeff.	142590 $[N/rad]$
l_r	Rear axle to CoG distance	1.6456 $[m]$
l_f	Front axle to CoG distance	1.3384 $[m]$

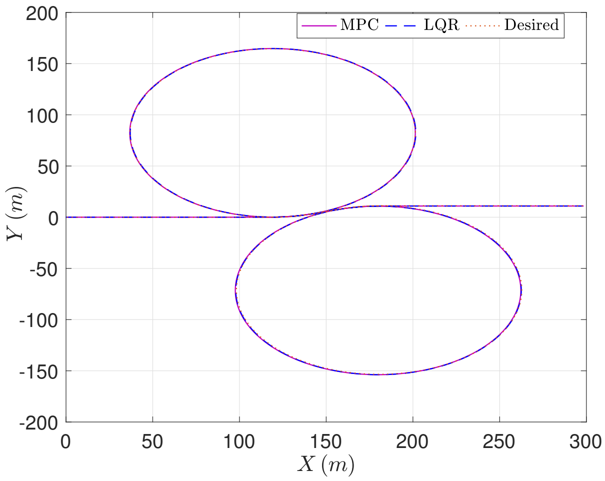

Fig. 3. Desired path in the global frame (brown, dotted), path tracked using the MPC controller (magenta, solid) and the LQR controller (blue,dashed).

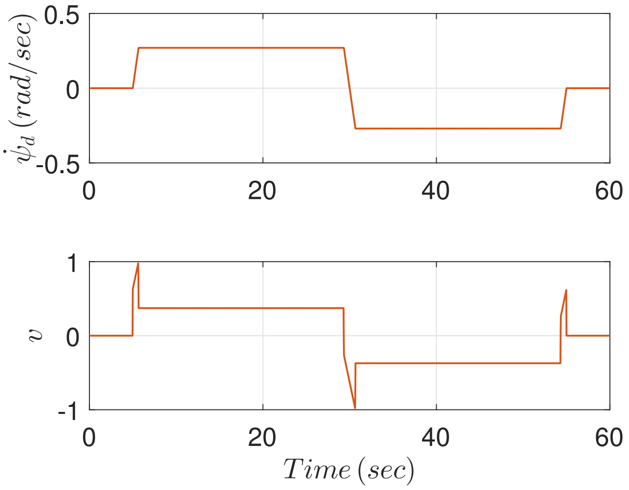

Fig. 4. Time trajectory of the desired yaw rate of the path (top) and the corresponding uncontrolled input (19) (bottom).

input variables were considered to be

$$\begin{cases} |e_y| <= 0.3,\ |\dot{y}| <= 3, \\ |e_\psi| <= 10\frac{\pi}{180},\ |\dot{\psi}| <= 1, \\ |\delta| <= 10\frac{\pi}{180},\ |\Delta\delta| <= 0.0125. \end{cases} \tag{21}$$

We assume that the path trajectory $\Psi_d(k)$ is generated by (4) and (5) with

$$\theta = 0.27,\ \bar{\gamma} = 0.0101. \tag{22}$$

With this specification, the path planner is allowed to generate reference PWC trajectories with a minimum radius of 82.3 m. Here we like to clarify that the bounds in (22) are used to demonstrate the controller capability to track the worst-case trajectories. As indicated in **Remark 1**, in practice, they are set based on many factors e.g bounded lateral acceleration or jerk.

To obtain the parameters for the new path model (10), according to **Remark 2** we select $\epsilon = 0.006$. Thus from **Lemma** 1 we obtain $\alpha = 0.9778$, $\beta = 0.0161$ and $\bar{\theta} = 0.7256$. Using the above parameters we construct the extended system (16) and (17). We also add integral of the lateral error as an additional state variable to achieve zero lateral deviation at the steady state.

We first design a LQR controller with the following tuning matrices,

$$Q = \text{diag}([1, 0, 0.1, 0, 0.1, 0, 1]),\ R = 1. \tag{23}$$

The obtained control law is then used to compute the RPI set for the closed-loop system.

For the MPC controller design, we assume that the path planner provides a preview trajectory for 10 steps. Using the same tuning matrices as in the case of the LQR controller and setting the previously obtained RPI set as a terminal constraint, we design an MPC controller (20) with $N = 10$.

For the simulation, we consider a scenario in which the vehicle is driven along the path shown in Fig. 3. The vehicle initially placed at (X,Y)=(0,0) has to trace the upper curved path followed by the lower curved path before exiting. The corresponding trajectories of $\dot{\psi}(k)$ and $v(k)$ in (19) are shown in Fig. 4 and are provided to the controllers for tracking. It can be seen that the desired reference trajectory excites the peak yaw rate which can be handled by the designed controllers.

Using the proposed controllers we perform the closed-loop simulations where continuous time model (6) was used to simulate the vehicle. The results of the simulations are shown in Fig. 3, 5 and 6. Fig. 3 shows the tracking performance of both the controllers. It can be observed that the desired tracking error specification is met in both cases. As seen in Fig. 5-6, the closed-loop response of the system always satisfies the desired safety and performance constraints. From Fig. 5 it can be observed that with the MPC controller the lateral deviation from the desired path is relatively less compared to when the LQR controller is used. It is due to the MPC controller ability to use the preview information which can be seen in the Fig.6. The MPC controller starts applying the steering input detecting the oncoming curve from the preview data, whereas the LQR controller only reacts when the curve starts. Thus an MPC controller with the ability to use path trajectory information was able to achieve a control objective over a larger set of initial condition with better overall performance. While the LQR controller can be a good choice if the computational efficiency and simplicity are the main concern.

Lastly, we like to mention that the considered bounds in (22) were the worst-case bounds for which we were able to compute an RPI set using proposed the LQR controller and used the obtained RPI set in the MPC controller as a terminal constraint for a fair comparison. For the larger path bounds, RCI set can be still computed and used in the MPC controller as a terminal constraint to guarantee constraint satisfaction.

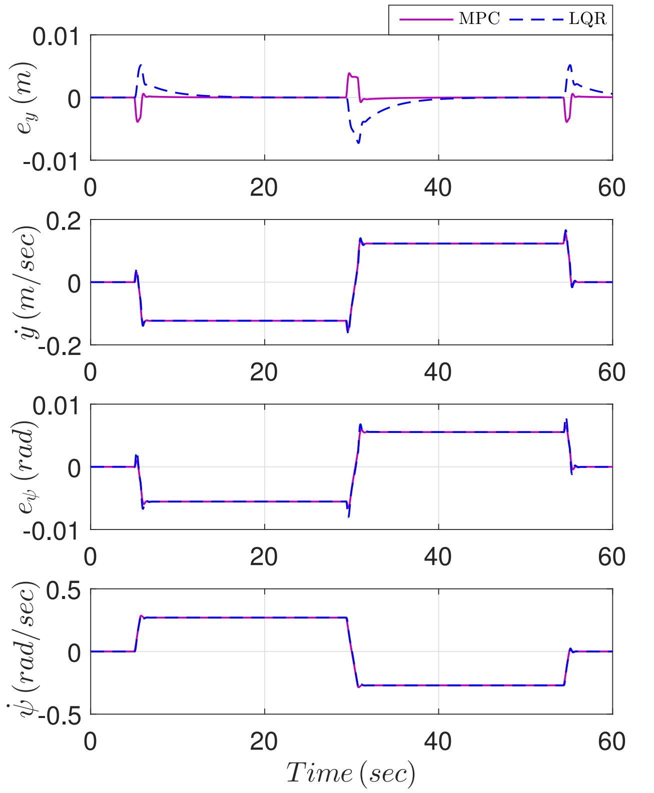

Fig. 5. Time trajectories of the state variables obtained using the MPC controller (magenta, solid) and the LQR controller (blue, dashed).

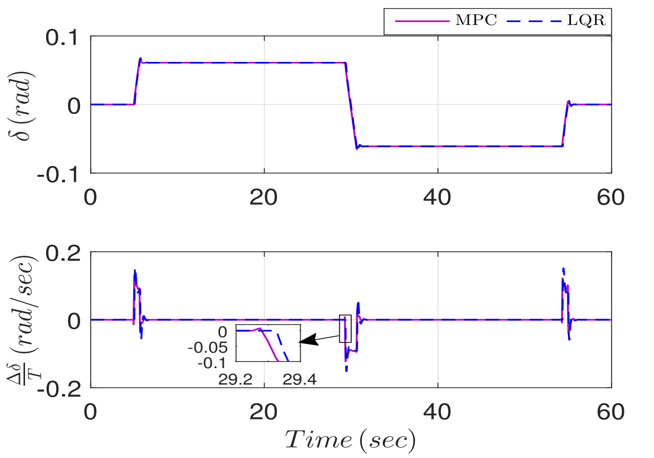

Fig. 6. Time trajectory of the steering angle (top) and the steering rate (bottom) obtained using the MPC controller (magenta, solid) and the LQR controller (blue; dashed).

VI. Conclusion

The main result of the paper showed how the lateral controller can be designed for the tracking of PWC trajectory of a predefined specification. It is proved that the guaranteed tracking can be achieved within the desired error bounds provided certain conditions are met.

Although preliminary, this work forms the basis of our further research to design a lateral control algorithm for vehicle motion control in the face of model uncertainty and measurement noise.

Another promising direction of research could focus on developing a path planner algorithm that can generate path trajectories with given specifications.

References

[1] R. Rajamani, "Vehicle dynamics and control," *Springer*, 2012.

[2] L. Ni, A. Gupta, P. Falcone, and L. Johannesson, "Vehicle lateral motion control with performance and safety guarantees," *IFAC-PapersOnLine*, vol. 49, no. 11, pp. 285 – 290, 2016.

[3] A. Gupta and P. Falcone, "Low-complexity explicit mpc controller for vehicle lateral motion control," *IEEE Conference on Intelligent Transportation Systems (ITSC)*, pp. 2839–2844, Nov 2018.

[4] A. Sangiovanni-Vincentelli, W. Damm, and R. Passerone, "Taming dr. frankenstein: Contract-based design for cyber-physical systems*," *European Journal of Control*, vol. 18, no. 3, pp. 217 – 238, 2012.

[5] S. W. Smith, P. Nilsson, and N. Ozay, "Interdependence quantification for compositional control synthesis with an application in vehicle safety systems," *IEEE Conference on Decision and Control (CDC)*, pp. 5700–5707, Dec 2016.

[6] X. Xu, J. W. Grizzle, P. Tabuada, and A. D. Ames, "Correctness guarantees for the composition of lane keeping and adaptive cruise control," *IEEE Transactions on Automation Science and Engineering*, vol. 15, no. 3, pp. 1216–1229, July 2018.

[7] J. Guldner, H.-S. Tan, and S. Patwardhan, "Analysis of automatic steering control for highway vehicles with look-down lateral reference systems," *Vehicle System Dynamics*, vol. 26, no. 4, pp. 243–269, 1996.

[8] E. J. Rossetter and J. C. Gerdes, "Lyapunov based performance guarantees for the potential field lane-keeping assistance system," *Journal of Dynamic Systems, Measurement, and Control*, vol. 128, no. 3, pp. 510–522, 2005.

[9] S. D. Cairano, U. V. Kalabić, and K. Berntorp, "Vehicle tracking control on piecewise-clothoidal trajectories by mpc with guaranteed error bounds," *IEEE Conference on Decision and Control (CDC)*, pp. 709–714, Dec 2016.

[10] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747 – 1767, 1999.

[11] F. Borrelli, A. Bemporad, and M. Morari, *Predictive Control for Linear and Hybrid Systems*. Cambridge University Press, 2017.

[12] S. V. Rakovic, E. C. Kerrigan, D. Q. Mayne, and J. Lygeros, "Reachability analysis of discrete-time systems with disturbances," *IEEE Transactions on Automatic Control*, vol. 51, no. 4, pp. 546–561, April 2006.

[13] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789 – 814, 2000.

[14] J. Lee and Z. Yu, "Worst-case formulations of model predictive control for systems with bounded parameters," *Automatica*, vol. 33, no. 5, pp. 763 – 781, 1997.

[15] S. V. Raković, B. Kouvaritakis, R. Findeisen, and M. Cannon, "Homothetic tube model predictive control," *Automatica*, vol. 48, no. 8, pp. 1631 – 1638, 2012.