Lecture 8: Special deterministic self-tuning regulators

- Rejecting a constant disturbance.

- Example.

Two-degree of Freedom Linear Regulator

If we solve the equations

$$\begin{aligned} \boldsymbol{A}\,\boldsymbol{y}(t) &= \boldsymbol{B}\,\Big(\boldsymbol{u}(t) + \boldsymbol{v}(t)\Big) \\ \boldsymbol{R}\,\boldsymbol{u}(t) &= \boldsymbol{T}\,\boldsymbol{u_c}(t) - \boldsymbol{S}\,\boldsymbol{y}(t) \end{aligned}$$

with respect to $\boldsymbol{y}(t)$ and $\boldsymbol{u}(t)$, we obtain

Two-degree of Freedom Linear Regulator

If we solve the equations

$$A\,y(t) = B\,\Big(u(t) + v(t)\Big)$$

$$R\,u(t) = T\,u_c(t) - S\,y(t)$$

with respect to $y(t)$ and $u(t)$, we obtain

$$y(t) = \frac{B\,T}{A\,R + B\,S}\,u_c(t) + \frac{B\,R}{A\,R + B\,S}\,v(t)$$

$$u(t) = \frac{A\,T}{A\,R + B\,S}\,u_c(t) + \frac{B\,S}{A\,R + B\,S}\,v(t)$$

Constant input disturbance

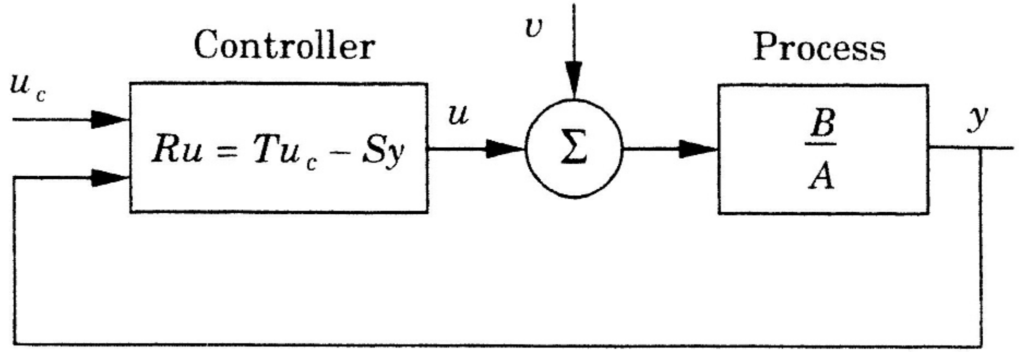

Figure 3.2 A general linear controller with two degrees of freedom.

Constant input disturbance

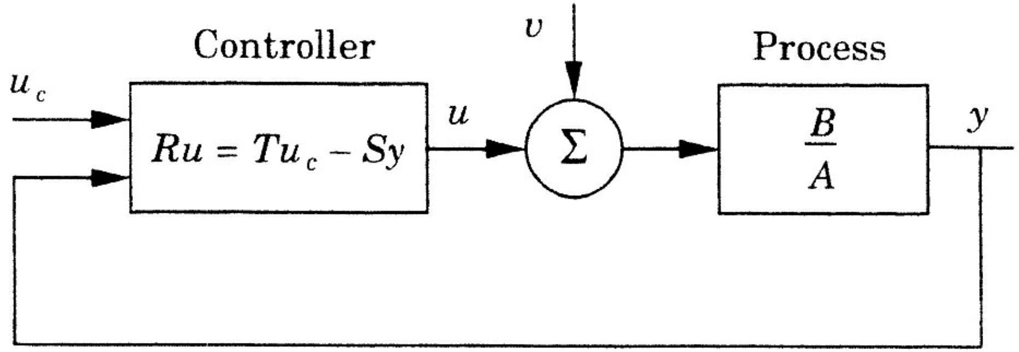

Figure 3.2 A general linear controller with two degrees of freedom.

$$y(t) = \cdots + \frac{B\,R}{\underbrace{A\,R + B\,S}_{A_c}}\,v(t)$$

Can a step disturbance $v(t) = \text{const} \neq 0$ for $t \geq t_c$ destroy performance of a self-tuning regulator?

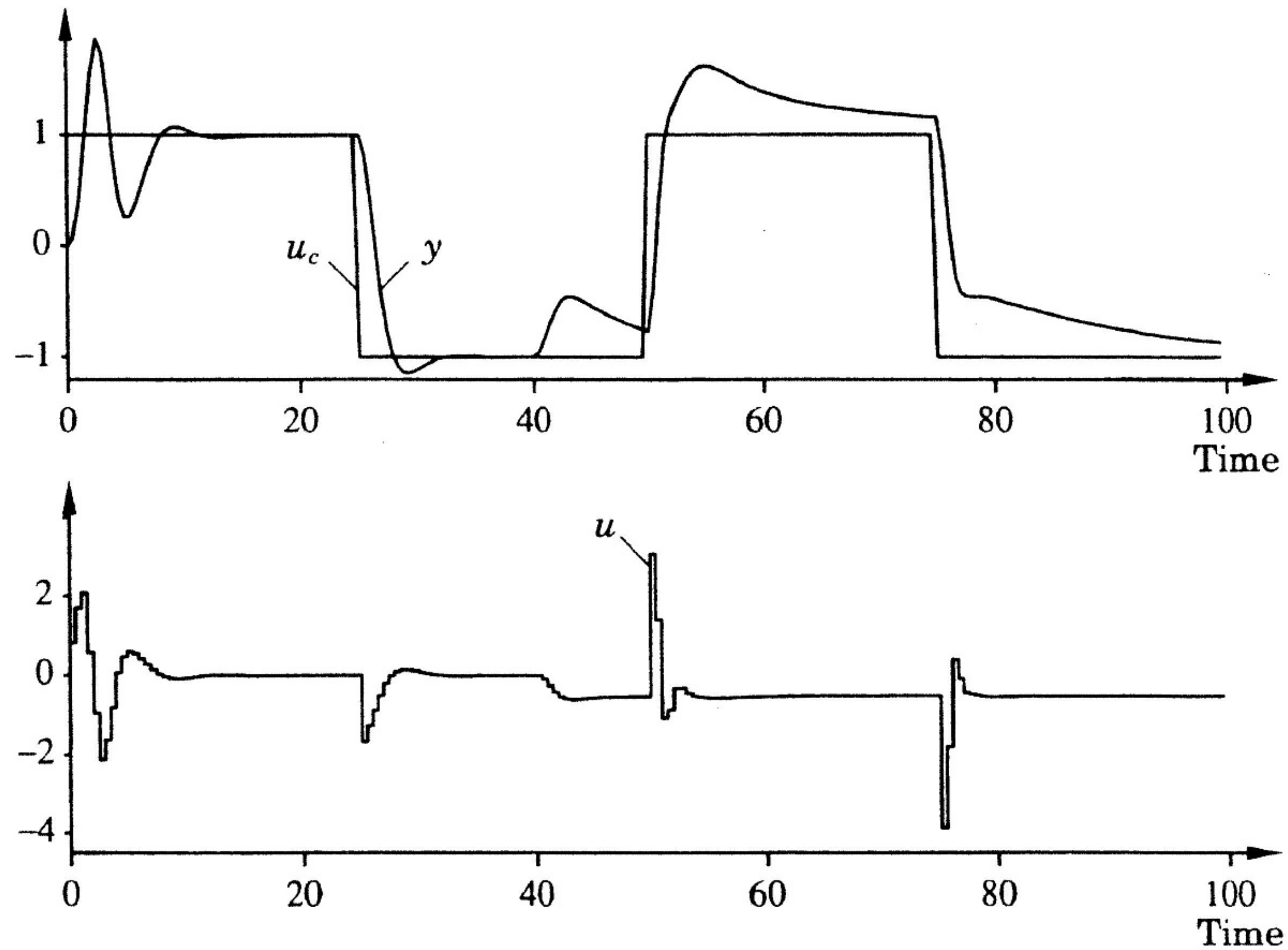

Figure 3.14 Output and control signal when for a system with an indirect self-tuner without zero canceling when there is a load disturbance in the form of a step at the process input at time $t = 40$.

What is the time when a step disturbance is applied?

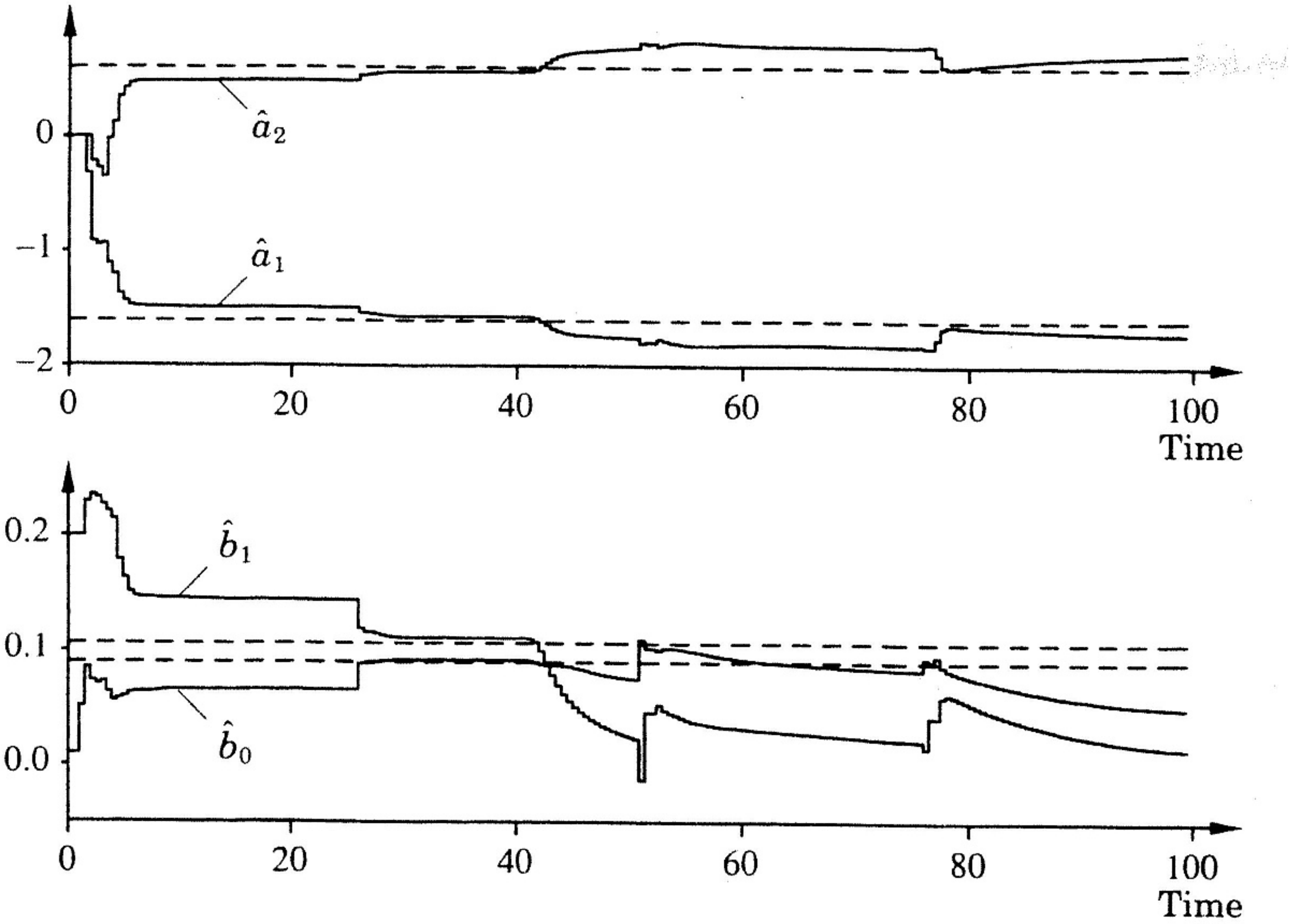

Figure 3.15 Parameter estimates corresponding to Fig. 3.14.

Rejecting constant input disturbances

$$\boldsymbol{y}(\boldsymbol{t}) = \cdots + \frac{\boldsymbol{B}\,\boldsymbol{R}}{\boldsymbol{A}\,\boldsymbol{R} + \boldsymbol{B}\,\boldsymbol{S}}\,\boldsymbol{v}(\boldsymbol{t})$$

Rejecting constant input disturbances

$$y(t) = \cdots + \frac{B\,R}{A\,R + B\,S}\,v(t)$$

To reject a constant or step disturbance

- in continuous time: s should be a factor of $R(s)$
- in discrete time: $z - 1$ should be a factor of $R(z)$

Rejecting constant input disturbances

$$y(t) = \cdots + \frac{B\,R}{A\,R + B\,S}\,v(t)$$

To reject a constant or step disturbance

- in continuous time: s should be a factor of $R(s)$
- in discrete time: $z - 1$ should be a factor of $R(z)$

Suppose we have found R^0 and S^0 solving

$$A\,R^0 + B\,S^0 = A_c^0 = A_0\,A_m\,B^+$$

Note that for any X and Y:

$$R = X\,R^0 + Y\,B, \qquad S = X\,S^0 - Y\,A$$

satisfy $\quad A\,R + B\,S = X\,A_c^0 \quad \leftarrow$ must be stable

Rejecting constants in discrete time

In discrete-time systems we want

$$R(z) = X(z)\, R^0(z) + Y(z)\, B(z) = (z-1)\, R_1(z)$$

Rejecting constants in discrete time

In discrete-time systems we want

$$R(z) = X(z)\, R^0(z) + Y(z)\, B(z) = (z-1)\, R_1(z)$$

Take

$$X(z) = z + x_0, \qquad Y(z) = y_0, \qquad \boxed{|x_0| < 1}$$

Rejecting constants in discrete time

In discrete-time systems we want

$$\boldsymbol{R(z)} = X(z)\,\boldsymbol{R^0(z)} + Y(z)\,B(z) = (z-1)\,R_1(z)$$

Take

$$X(z) = z + x_0, \qquad Y(z) = y_0, \qquad \boxed{|x_0| < 1}$$

and solve

$$(z-1)\,R_1(z) = (z + x_0)\,R^0(z) + y_0\,B(z)$$

Rejecting constants in discrete time

In discrete-time systems we want

$$R(z) = X(z)\,R^0(z) + Y(z)\,B(z) = (z-1)\,R_1(z)$$

Take

$$X(z) = z + x_0, \qquad Y(z) = y_0, \qquad \boxed{|x_0| < 1}$$

and solve

$$(z-1)\,R_1(z) = (z + x_0)\,R^0(z) + y_0\,B(z)$$

substituting $z = 1$

$$0 = (1 + x_0)\,R^0(1) + y_0\,B(1)$$

Rejecting constants in discrete time

In discrete-time systems we want

$$\color{red}{R(z)} = X(z)\,\color{red}{R^0(z)} + Y(z)\,B(z) = (z-1)\,R_1(z)$$

Take

$$X(z) = z + x_0, \qquad Y(z) = y_0, \qquad \boxed{|x_0| < 1}$$

and solve

$$(z-1)\,R_1(z) = (z+x_0)\,R^0(z) + y_0\,B(z)$$

substituting $z = 1$ we obtain

$$y_0 = -\frac{(1+x_0)\,R^0(1)}{B(1)}$$

Rejecting constants in discrete time

In discrete-time systems we want

$$R(z) = X(z)\, R^0(z) + Y(z)\, B(z) = (z-1)\, R_1(z)$$

Take

$$X(z) = z + x_0, \qquad Y(z) = y_0, \qquad \boxed{|x_0| < 1}$$

and solve

$$(z-1)\, R_1(z) = (z + x_0)\, R^0(z) + y_0\, B(z)$$

Finally,

$$y_0 = -\frac{(1 + x_0)\, R^0(1)}{B(1)} \quad \Rightarrow \quad \begin{cases} R(z) = (z + x_0)\, R^0(z) + y_0\, B(z) \\ S(z) = (z + x_0)\, S^0(z) - y_0\, A(z) \end{cases}$$

Rejecting constants in continuous time

In continuous-time systems we want

$$\boldsymbol{R(s)} = \boldsymbol{X(s)\,R^0(s) + Y(s)\,B(s) = s\,R_1(s)}$$

Rejecting constants in continuous time

In <u>continuous-time</u> systems we want

$$\boldsymbol{R(s)} = X(s)\, \boldsymbol{R^0(s)} + Y(s)\, B(s) = s\, R_1(s)$$

Take

$$X(s) = s + x_0, \qquad Y(s) = y_0, \qquad \boxed{x_0 > 0}$$

Rejecting constants in continuous time

In continuous-time systems we want

$$R(s) = X(s)\, R^0(s) + Y(s)\, B(s) = s\, R_1(s)$$

Take

$$X(s) = s + x_0, \qquad Y(s) = y_0, \qquad \boxed{x_0 > 0}$$

and solve

$$s\, R_1(s) = (s + x_0)\, R^0(s) + y_0\, B(s)$$

Rejecting constants in continuous time

In continuous-time systems we want

$$\boldsymbol{R(s)} = X(s)\, \boldsymbol{R^0(s)} + Y(s)\, B(s) = s\, R_1(s)$$

Take

$$X(s) = s + x_0, \qquad Y(s) = y_0, \qquad \boxed{x_0 > 0}$$

and solve

$$s\, R_1(s) = (s + x_0)\, R^0(s) + y_0\, B(s)$$

substituting $s = 0$

$$0 = x_0\, R^0(0) + y_0\, B(0)$$

Rejecting constants in continuous time

In continuous-time systems we want

$$\boldsymbol{R(s) = X(s)\, R^0(s) + Y(s)\, B(s) = s\, R_1(s)}$$

Take

$$\boldsymbol{X(s) = s + x_0, \qquad Y(s) = y_0, \qquad \boxed{x_0 > 0}}$$

and solve

$$\boldsymbol{s\, R_1(s) = (s + x_0)\, R^0(s) + y_0\, B(s)}$$

substituting $\boldsymbol{s = 0}$ we obtain

$$\boldsymbol{y_0 = -\frac{x_0\, R^0(0)}{B(0)}}$$

Rejecting constants in continuous time

In <u>continuous-time</u> systems we want

$$\boldsymbol{R(s) = X(s)\, R^0(s) + Y(s)\, B(s) = s\, R_1(s)}$$

Take

$$\boldsymbol{X(s) = s + x_0, \qquad Y(s) = y_0, \qquad \boxed{x_0 > 0}}$$

and solve

$$\boldsymbol{s\, R_1(s) = (s + x_0)\, R^0(s) + y_0\, B(s)}$$

Finally,

$$\boldsymbol{y_0 = -\frac{x_0\, R^0(0)}{B(0)} \quad \Rightarrow \quad \begin{cases} R(s) = (s + x_0)\, R^0(s) + y_0\, B(s) \\ S(s) = (s + x_0)\, S^0(s) - y_0\, A(s) \end{cases}}$$

Rejecting constants in the estimator

Let us modify the regressor to make it insensitive to $v(t)$

$$A\,y(t) = B\left(u(t) + v(t)\right) \quad \Rightarrow \quad \bar{y}(t) = \phi(t)^{T}\,\theta$$

Rejecting constants in the estimator

Let us modify the regressor to make it insensitive to $v(t)$

$$A\,y(t) = B\left(u(t) + v(t)\right) \quad \Rightarrow \quad \bar{y}(t) = \phi(t)^T\,\theta$$

Introducing a filter H_f

$$H_f\,A\,y(t) = H_f\,\left(B\left(u(t) + v(t)\right)\right)$$

Rejecting constants in the estimator

Let us modify the regressor to make it insensitive to $v(t)$

$$A\, y(t) = B\left(u(t) + v(t)\right) \quad \Rightarrow \quad \bar{y}(t) = \phi(t)^T\, \theta$$

Introducing a filter H_f

$$A\, H_f\, y(t) = B\left(H_f\, u(t) + H_f\, v(t)\right)$$

Rejecting constants in the estimator

Let us modify the regressor to make it insensitive to $v(t)$

$$A\, y(t) = B\,\Big(u(t) + v(t)\Big) \quad \Rightarrow \quad \bar{y}(t) = \phi(t)^T\, \theta$$

Introducing a filter H_f

$$A\, \underbrace{H_f\, y(t)}_{=y_f(t)} = B\,\Big(\underbrace{H_f\, u(t)}_{=u_f(t)} + \underbrace{H_f\, v(t)}_{=0}\Big)$$

which removes the disturbance: $\boxed{A\, y_f(t) = B\, u_f(t)}$.

Rejecting constants in the estimator

Let us modify the regressor to make it insensitive to $v(t)$

$$A\,y(t) = B\left(u(t) + v(t)\right) \quad \Rightarrow \quad \bar{y}(t) = \phi(t)^T\,\theta$$

Introducing a filter H_f

$$A\,\underbrace{H_f\,y(t)}_{=y_f(t)} = B\left(\underbrace{H_f\,u(t)}_{=u_f(t)} + \underbrace{H_f\,v(t)}_{=0}\right)$$

which removes the disturbance: $\boxed{A\,y_f(t) = B\,u_f(t)}$.

To reject a constant disturbance in the estimator

- in discrete time: take $H_f(z) = \dfrac{z-1}{z}$
- in continuous time: take $H_f(s) = \dfrac{s}{s^2 + 2\,\zeta\,\omega\,s + \omega^2}$

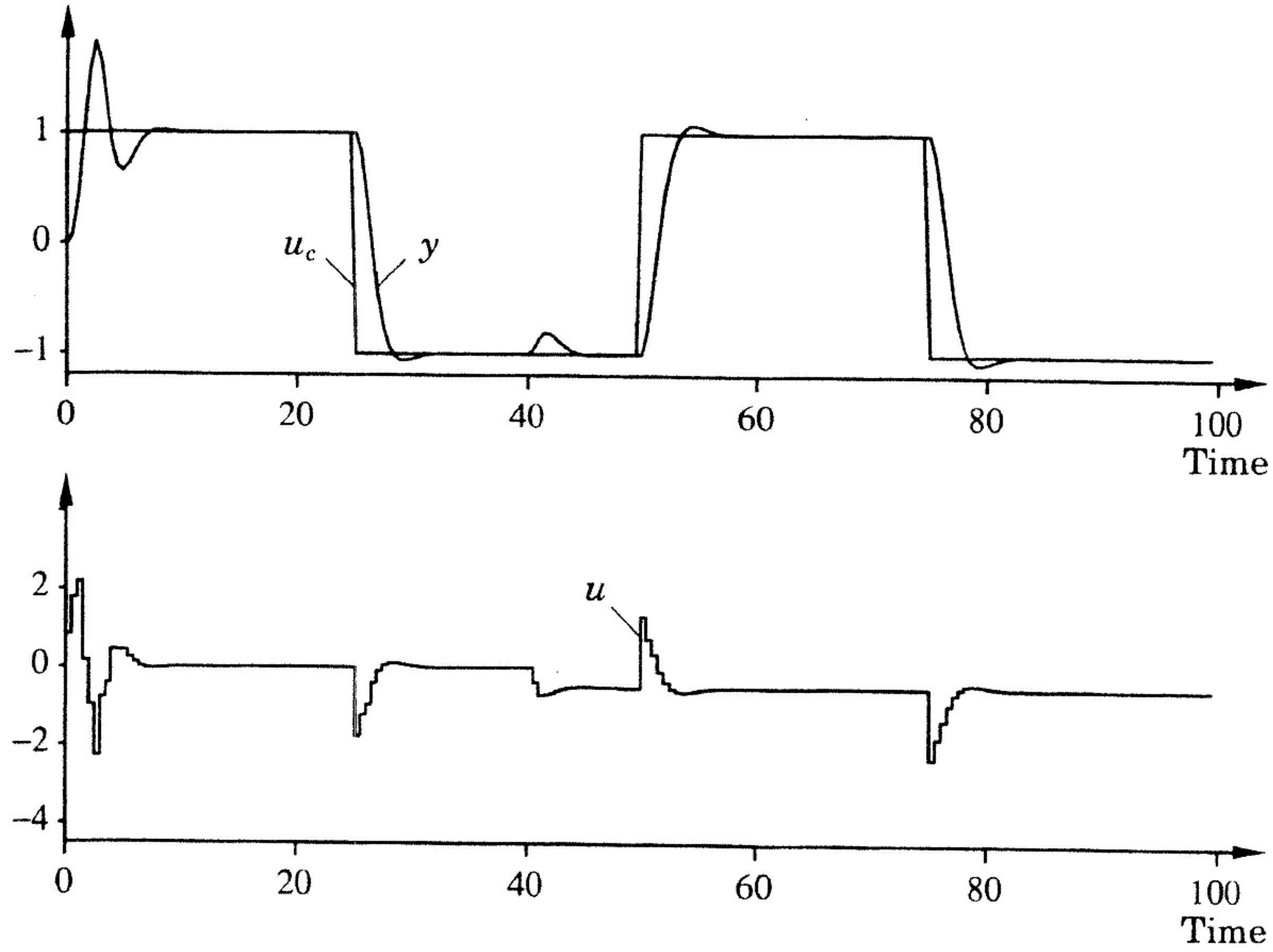

Figure 3.16 Output and control signal with an indirect self-tuner with integral action and a modified estimator.

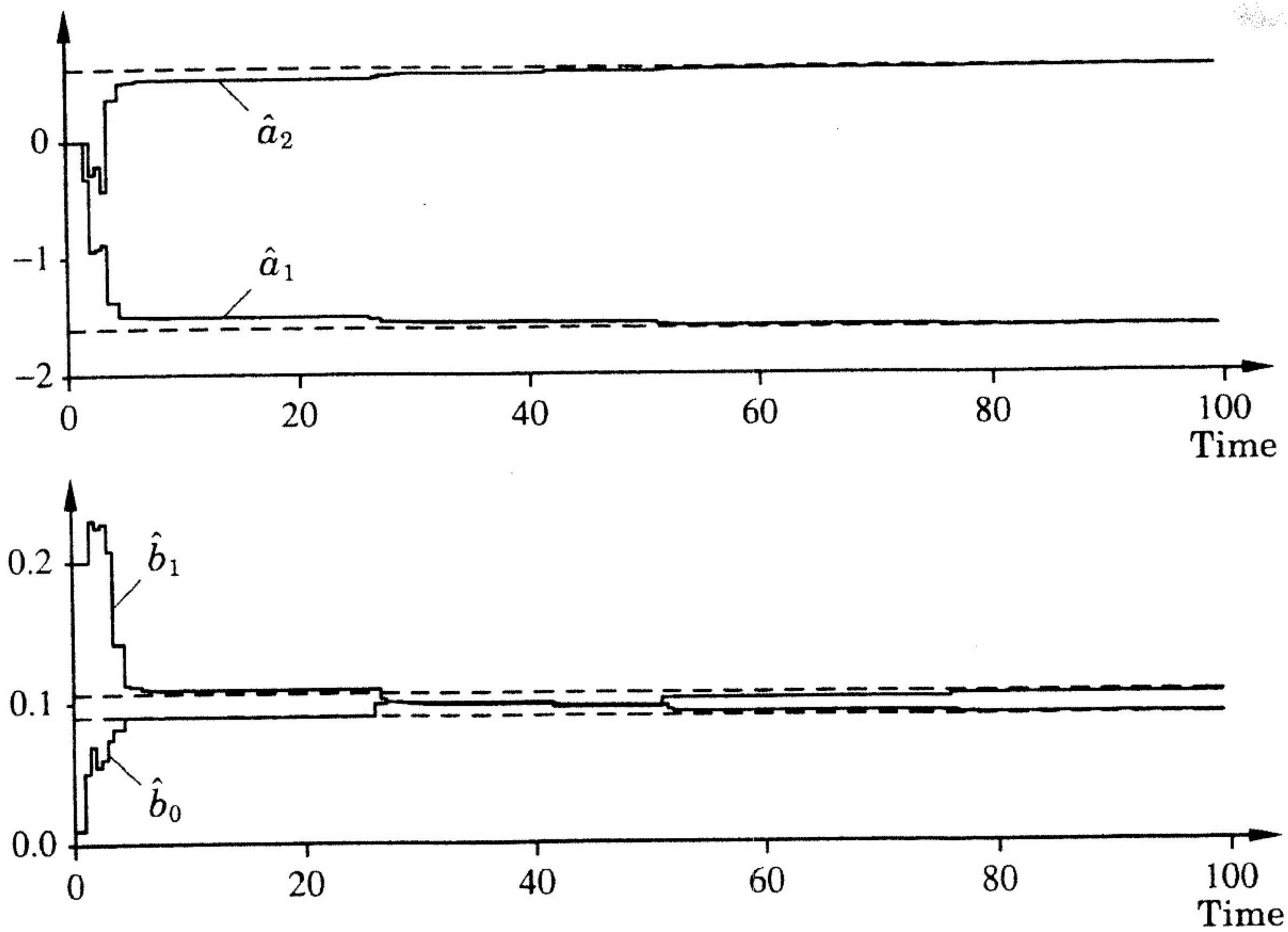

Figure 3.17 Parameter estimates corresponding to Fig. 3.16.

Example: problem formulation

A first order plant with delay is represented by

$$G(s) = \frac{k\,e^{-\tau\,s}}{s + a}$$

where a, k, and τ are unknown positive constants. In a digital control system with sampling period $h > \tau$, the discrete-time equivalent of the plant (preceded by zero-order hold) is

$$G(z) = \frac{b_0\,z + b_1}{z^2 + a_1\,z}$$

where

$$a_1 = -e^{-ah}, \quad b_0 = \frac{k}{a}\left(1 - e^{-a(h-\tau)}\right), \quad b_1 = \frac{k}{a}\left(e^{-a(h-\tau)} - e^{-ah}\right)$$

Design a discrete-time, indirect, self-tuning regulator so that the closed-loop system follows the model

$$G_m(z) = \frac{2\,z + 1}{3\,z^2}$$

Assume there is a constant disturbance at the plant input.

Example: Is it possible to follow the model?

$$G(z) = \frac{b_0\, z + b_1}{z^2 + a_1\, z} \quad \Longrightarrow \quad \begin{cases} B(z) = b_0 \left(z + \frac{b_1}{b_0}\right) = B^{-}(z) B^{+}(z) \\ A(z) = z^2 + a_1\, z \end{cases}$$

Example: Is it possible to follow the model?

$$G(z) = \frac{b_0\, z + b_1}{z^2 + a_1\, z} \quad \Longrightarrow \quad \begin{cases} B(z) = b_0 \left(z + \frac{b_1}{b_0} \right) = B^-(z) B^+(z) \\ A(z) = z^2 + a_1\, z \end{cases}$$

$$G_m(z) = \frac{2\, z + 1}{3\, z^2} \quad \Longrightarrow \quad \begin{cases} B_m(z) = 0.\bar{6}\, (z + 0.5) \\ A_m(z) = z^2 \end{cases}$$

Example: Is it possible to follow the model?

$$G(z) = \frac{b_0\, z + b_1}{z^2 + a_1\, z} \quad \Longrightarrow \quad \begin{cases} B(z) = b_0 \left(z + \frac{b_1}{b_0}\right) = B^{-}(z) B^{+}(z) \\ A(z) = z^2 + a_1\, z \end{cases}$$

$$G_m(z) = \frac{2\, z + 1}{3\, z^2} \quad \Longrightarrow \quad \begin{cases} B_m(z) = 0.\bar{6}\, (z + 0.5) \\ A_m(z) = z^2 \end{cases}$$

If
$$\frac{b_1}{b_0} = \frac{e^{-a(h-\tau)} - e^{-ah}}{1 - e^{-a(h-\tau)}} = \frac{e^{a\tau} - 1}{e^{ah} - e^{a\tau}} \neq 0.5$$

to have exact model following the zero at $z = -b_1/b_0$ must be canceled.

Example: Is it possible to follow the model?

$$G(z) = \frac{b_0\, z + b_1}{z^2 + a_1\, z} \quad \Longrightarrow \quad \begin{cases} B(z) = b_0 \left(z + \frac{b_1}{b_0}\right) = B^-(z) B^+(z) \\ A(z) = z^2 + a_1\, z \end{cases}$$

$$G_m(z) = \frac{2\, z + 1}{3\, z^2} \quad \Longrightarrow \quad \begin{cases} B_m(z) = 0.\bar{6}\,(z + 0.5) \\ A_m(z) = z^2 \end{cases}$$

If $$\frac{b_1}{b_0} = \frac{e^{-a(h-\tau)} - e^{-ah}}{1 - e^{-a(h-\tau)}} = \frac{e^{a\tau} - 1}{e^{ah} - e^{a\tau}} \neq 0.5$$

to have exact model following the zero at $z = -b_1/b_0$ must be canceled. This can be done only if it is stable:

$$\left| \frac{e^{a\tau} - 1}{e^{ah} - e^{a\tau}} \right| < 1$$

Example: Is it possible to follow the model?

$$G(z) = \frac{b_0\, z + b_1}{z^2 + a_1\, z} \quad \Longrightarrow \quad \begin{cases} B(z) = b_0 \left(z + \frac{b_1}{b_0} \right) = B^-(z) B^+(z) \\ A(z) = z^2 + a_1\, z \end{cases}$$

$$G_m(z) = \frac{2\, z + 1}{3\, z^2} \quad \Longrightarrow \quad \begin{cases} B_m(z) = 0.\bar{6}\, (z + 0.5) \\ A_m(z) = z^2 \end{cases}$$

If $$\frac{b_1}{b_0} = \frac{e^{-a(h-\tau)} - e^{-ah}}{1 - e^{-a(h-\tau)}} = \frac{e^{a\tau} - 1}{e^{ah} - e^{a\tau}} \neq 0.5$$

to have exact model following the zero at $z = -b_1/b_0$ must be canceled. This can be done only if it is stable:

$$\left| \frac{e^{a\tau} - 1}{e^{ah} - e^{a\tau}} \right| < 1$$

which (with $h > \tau$) gives: $2e^{a\tau} - 1 < e^{ah}$ and $h > 2\tau$ for $h \ll 1$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\, R + B\, S = A_o\, A_m\, B^+ \;\Rightarrow\; R = R_p\, B^+, \; A\, R_p + B^-\, S = A_o\, A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\, R + B\, S = A_o\, A_m\, B^+ \;\Rightarrow\; R = R_p\, B^+,\; A\, R_p + B^-\, S = A_o\, A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

$$(z^2 + a_1\, z)\, 1 + b_0\, (s_0\, z + s_1) = z^2$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\, R + B\, S = A_o\, A_m\, B^+ \;\Rightarrow\; R = R_p\, B^+,\; A\, R_p + B^-\, S = A_o\, A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

$$(z^2 + a_1\, z)\, 1 + b_0\, (s_0\, z + s_1) = z^2 \;\Rightarrow\; z^2 + (a_1 + b_0\, s_0)\, z + b_0\, s_1 = z^2$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\, R + B\, S = A_o\, A_m\, B^+ \;\Rightarrow\; R = R_p\, B^+,\; A\, R_p + B^-\, S = A_o\, A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

$$(z^2 + a_1\, z)\, 1 + b_0\, (s_0\, z + s_1) = z^2 \;\Rightarrow\; z^2 + (a_1 + b_0\, s_0)\, z + b_0\, s_1 = z^2$$

$$h \neq \tau \;\Rightarrow\; b_0 \neq 0 \;\Rightarrow\; s_1 = 0, \quad s_0 = -a_1/b_0$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\, R + B\, S = A_o\, A_m\, B^+ \;\Rightarrow\; R = R_p\, B^+,\; A\, R_p + B^-\, S = A_o\, A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

$$(z^2 + a_1\, z)\, 1 + b_0\, (s_0\, z + s_1) = z^2 \;\Rightarrow\; z^2 + (a_1 + b_0\, s_0)\, z + b_0\, s_1 = z^2$$

$$h \neq \tau \;\Rightarrow\; b_0 \neq 0 \;\Rightarrow\; s_1 = 0, \quad s_0 = -a_1/b_0$$

$$\boxed{S(z) = -(a_1/b_0)\, z}, \qquad \boxed{R(z) = z + (b_1/b_0)}$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\, R + B\, S = A_o\, A_m\, B^+ \;\Rightarrow\; R = R_p\, B^+,\; A\, R_p + B^-\, S = A_o\, A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

$$(z^2 + a_1\, z)\, 1 + b_0\, (s_0\, z + s_1) = z^2 \;\Rightarrow\; z^2 + (a_1 + b_0\, s_0)\, z + b_0\, s_1 = z^2$$

$$h \neq \tau \;\Rightarrow\; b_0 \neq 0 \;\Rightarrow\; s_1 = 0, \quad s_0 = -a_1/b_0$$

$$\boxed{S(z) = -(a_1/b_0)\, z}, \qquad \boxed{R(z) = z + (b_1/b_0)}$$

$$\frac{B_m}{A_m} = \frac{B\, T}{A\, R + B\, S} = \frac{B^+\, B^-\, T}{A_o\, A_m\, B^+} = \frac{B^-\, T}{A_o\, A_m} \;\Rightarrow\; T(z) = \frac{B_m}{B^-}$$

Example: Step 1

Control design assuming known parameters and no disturbances:

$$\deg\{A_o\} = \deg\{A\} - \deg\{B\} - 1 = 2 - 1 - 1 = 0 \quad \Rightarrow \quad \boxed{A_o(z) = 1}$$

$$\deg\{S\} = \deg\{A\} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad S(z) = s_0\, z + s_1$$

$$A_c^0 = A\,R + B\,S = A_o\,A_m\,B^+ \;\Rightarrow\; R = R_p\,B^+,\; A\,R_p + B^-\,S = A_o\,A_m$$

$$\deg\{R_p\} = \deg\{A_o\} + \deg\{A_m\} - \deg\{A\} = 0 - 2 + 2 = 0 \;\Rightarrow\; R_p(z) = 1$$

$$(z^2 + a_1\,z)\,1 + b_0\,(s_0\,z + s_1) = z^2 \;\Rightarrow\; z^2 + (a_1 + b_0\,s_0)\,z + b_0\,s_1 = z^2$$

$$h \neq \tau \;\Rightarrow\; b_0 \neq 0 \;\Rightarrow\; s_1 = 0, \quad s_0 = -a_1/b_0$$

$$\boxed{S(z) = -(a_1/b_0)\,z}, \qquad \boxed{R(z) = z + (b_1/b_0)}$$

$$\frac{B_m}{A_m} = \frac{B\,T}{A\,R + B\,S} = \frac{B^+\,B^-\,T}{A_o\,A_m\,B^+} = \frac{B^-\,T}{A_o\,A_m} \;\Rightarrow\; \boxed{T(z) = \frac{0.\bar{6}}{b_0}(z + 0.5)}$$

Example: Step 2

Control redesign to deal with constant disturbances:

The previously designed controller

$$R_0(z) = z + (b_1/b_0), \quad S_0(z) = -(a_1/b_0)\, z, \quad T_0(z) = \frac{0.\bar{6}}{b_0}(z + 0.5)$$

should be modified

Example: Step 2

Control redesign to deal with constant disturbances:

The previously designed controller

$$R_0(z) = z + (b_1/b_0), \quad S_0(z) = -(a_1/b_0)\, z, \quad T_0(z) = \frac{0.\bar{6}}{b_0}(z + 0.5)$$

should be modified to ensure $R(1) = 0$ and $S(1) \neq 0$.
Take x_0 s.t. $x_0 < 1$, e.g. $x_0 = 0$ and

$$y_0 = -\frac{(1 + x_0)\, R^0(1)}{B(1)} = -\frac{(1 + x_0)\,(1 + (b_1/b_0))}{b_0 + b_1} = -\frac{1 + x_0}{b_0}$$

Example: Step 2

Control redesign to deal with constant disturbances:

The previously designed controller

$$R_0(z) = z+(b_1/b_0), \quad S_0(z) = -(a_1/b_0)\, z, \quad T_0(z) = \frac{0.\bar{6}}{b_0}(z+0.5)$$

should be modified to ensure $R(1) = 0$ and $S(1) \neq 0$.
Take x_0 s.t. $x_0 < 1$, e.g. $x_0 = 0$ and

$$y_0 = -\frac{(1+x_0)\, R^0(1)}{B(1)} = -\frac{(1+x_0)\,(1+(b_1/b_0))}{b_0+b_1} = -\frac{1+x_0}{b_0}$$

$$R(z) = (z+x_0)R_0(z)+y_0B(z) = (z+x_0)\left(z+\frac{b_1}{b_0}\right)-\frac{1+x_0}{b_0}(b_0z+b_1)$$

$$S(z) = (z+x_0)S_0(z)+y_0A(z) = (z+x_0)\left(-\frac{a_1}{b_0}z\right)+\frac{1+x_0}{b_0}(z^2+a_1z)$$

Example: Step 2

Control redesign to deal with constant disturbances:

The previously designed controller

$$R_0(z) = z+(b_1/b_0), \quad S_0(z) = -(a_1/b_0)\, z, \quad T_0(z) = \frac{0.\bar{6}}{b_0}(z+0.5)$$

should be modified to ensure $R(1)=0$ and $S(1)\neq 0$.
Take x_0 s.t. $x_0 < 1$, e.g. $x_0 = 0$ and

$$y_0 = -\frac{(1+x_0)\, R^0(1)}{B(1)} = -\frac{(1+x_0)\,(1+(b_1/b_0))}{b_0+b_1} = -\frac{1+x_0}{b_0}$$

$$R(z) = (z+x_0)R_0(z) + y_0 B(z) = \left(z+\frac{b_1}{b_0}\right)(z-1)$$

$$S(z) = (z+x_0)S_0(z) + y_0 A(z) = \frac{1+x_0-a_1}{b_0}z^2 + \frac{1}{b_0}z$$

Example: Step 2

Control redesign to deal with constant disturbances:

The previously designed controller

$$R_0(z) = z+(b_1/b_0), \quad S_0(z) = -(a_1/b_0)\, z, \quad T_0(z) = \frac{0.\bar{6}}{b_0}(z+0.5)$$

should be modified to ensure $R(1) = 0$ and $S(1) \neq 0$.
Take x_0 s.t. $x_0 < 1$, e.g. $x_0 = 0$ and

$$y_0 = -\frac{(1+x_0)\, R^0(1)}{B(1)} = -\frac{(1+x_0)\,(1+(b_1/b_0))}{b_0+b_1} = -\frac{1+x_0}{b_0}$$

$$R(z) = (z+x_0)R_0(z) + y_0 B(z) = \left(z + \frac{b_1}{b_0}\right)(z-1)$$

$$S(z) = (z+x_0)S_0(z) + y_0 A(z) = \frac{1+x_0-a_1}{b_0} z^2 + \frac{1}{b_0} z$$

$$\frac{B_m}{A_m} = \frac{B\,T}{A\,R + B\,S} = \frac{B^-\,T}{A_o\,A_m\,(z+x_0)} \Rightarrow T(z) = \frac{0.\bar{6}}{b_0}(z+0.5)(z+x_0)$$

Example: Step 3

Including a parameter estimation scheme:

The feedback controller

$$R(q)\,u(t) = T(q)\,u_c(t) - S(q)\,y(t)$$

should use estimates of parameters:

$$R(z) = \left(z + \frac{\hat{b}_1(t)}{\hat{b}_0(t)}\right)(z-1), \quad S(z) = \frac{1 + x_0 - \hat{a}_1(t)}{\hat{b}_0(t)} z^2 + \frac{1}{\hat{b}_0(t)} z,$$

$$T(z) = \frac{0.\bar{6}}{\hat{b}_0(t)}(z + 0.5)(z + x_0)$$

Example: Step 3

Including a parameter estimation scheme:

The feedback controller

$$R(q)\,u(t) = T(q)\,u_c(t) - S(q)\,y(t)$$

should use estimates of parameters:

$$R(z) = \left(z + \frac{\hat{b}_1(t)}{\hat{b}_0(t)}\right)(z-1), \quad S(z) = \frac{1 + x_0 - \hat{a}_1(t)}{\hat{b}_0(t)} z^2 + \frac{1}{\hat{b}_0(t)} z,$$

$$T(z) = \frac{0.\bar{6}}{\hat{b}_0(t)}(z+0.5)(z+x_0)$$

We will use the filtered signals:

$$y_f(t) = \frac{q-1}{q}\,y(t), \qquad u_f(t) = \frac{q-1}{q}\,u(t)$$

Example: Step 3 (cont'd)

With (using $\frac{q-1}{q} = 1 - q^{-1}$)

$$y_f(t) = y(t) - y(t-1), \qquad u_f(t) = u(t) - u(t-1)$$

Example: Step 3 (cont'd)

With (using $\frac{q-1}{q} = 1 - q^{-1}$)

$$y_f(t) = y(t) - y(t-1), \qquad u_f(t) = u(t) - u(t-1)$$

we rewrite the model

$$A(q)\, y(t) = B(q)\, u(t) + w, \qquad w = \text{const}$$

in the form

$$(q^2 + a_1\, q)\, y_f(t) = (b_0\, q + b_1)\, u_f(t) + \frac{q-1}{q}\, w$$

Example: Step 3 (cont'd)

With (using $\frac{q-1}{q} = 1 - q^{-1}$)

$$y_f(t) = y(t) - y(t-1), \qquad u_f(t) = u(t) - u(t-1)$$

we rewrite the model

$$A(q)\, y(t) = B(q)\, u(t) + w, \qquad w = \text{const}$$

in the form

$$(1 + a_1\, q^{-1})\, y_f(t) = \left(b_0\, q^{-1} + b_1\, q^{-2}\right) u_f(t)$$

Example: Step 3 (cont'd)

With (using $\frac{q-1}{q} = 1 - q^{-1}$)

$$y_f(t) = y(t) - y(t-1), \qquad u_f(t) = u(t) - u(t-1)$$

we rewrite the model

$$A(q)\, y(t) = B(q)\, u(t) + w, \qquad w = \text{const}$$

in the form

$$y_f(t) = -a_1\, q^{-1}\, y_f(t) + b_0\, q^{-1}\, u_f(t) + b_1\, q^{-2}\, u_f(t)$$

Example: Step 3 (cont'd)

With (using $\frac{q-1}{q} = 1 - q^{-1}$)

$$y_f(t) = y(t) - y(t-1), \qquad u_f(t) = u(t) - u(t-1)$$

we rewrite the model

$$A(q)\, y(t) = B(q)\, u(t) + w, \qquad w = \text{const}$$

in the form

$$y_f(t) = -a_1\, y_f(t-1) + b_0\, u_f(t-1) + b_1\, u_f(t-2)$$

Example: Step 3 (cont'd)

With (using $\frac{q-1}{q} = 1 - q^{-1}$)

$$y_f(t) = y(t) - y(t-1), \qquad u_f(t) = u(t) - u(t-1)$$

we rewrite the model

$$A(q)\, y(t) = B(q)\, u(t) + w, \qquad w = \text{const}$$

in the form

$$y_f(t) = -a_1\, y_f(t-1) + b_0\, u_f(t-1) + b_1\, u_f(t-2)$$

Finally, for the RLS algorithm we have

$$y_f(t) = \phi(t-1)^T\, \theta^0, \qquad \hat{\theta}(t) = \left[\hat{a}_1(t),\ \hat{b}_0(t),\ \hat{b}_1(t)\right]$$

$$\phi(t-1)^T = \left[-y_f(t-1),\ u_f(t-1),\ u_f(t-2)\right]$$

Parameter Projection

Parameter projection is a useful modification of parameter estimation algorithms, which ensures the estimates always stay inside known (correct) regions. Typically,

$$\theta^0 \in \Omega = \{\theta : \ \underline{\theta}_i \le \theta_i \le \bar{\theta}_i\}$$

or

$$\theta^0 \in \Omega = \left\{\theta : \ \sum_{i=1}^{n} \theta_i^2 \le R\right\}$$

Parameter Projection

Parameter projection is a useful modification of parameter estimation algorithms, which ensures the estimates always stay inside known (correct) regions. Typically,

$$\theta^0 \in \Omega = \{\theta : \ \underline{\theta}_i \le \theta_i \le \bar{\theta}_i\}$$

or

$$\theta^0 \in \Omega = \left\{\theta : \ \sum_{i=1}^{n} \theta_i^2 \le R\right\}$$

Then, the following modification for RLS algorithm is useful

$$\hat{\theta}_p(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \phi(t-1)^T \hat{\theta}(t-1)\right)$$

$$\hat{\theta}(t) = \begin{cases} \hat{\theta}_p(t), & \text{if} \quad \hat{\theta}_p(t) \in \Omega \\ \arg\min\limits_{\theta \in \Omega} \left[\theta - \hat{\theta}_p(t)\right]^T P(t)^{-1} \left[\theta - \hat{\theta}_p(t)\right], & \text{otherwise} \end{cases}$$

Next Lecture / Assignments:

Next meeting (April 28, 10:00-12:00, in A206Tekn): Stochastic and Predictive Self-Tuning Regulators.

Homework problems: Report is due on May 12, 2010.

Consider the process with $B(s) = 1$ and $A(s) = s(s + a)$, where a is an unknown parameter. Assume that the desired closed-loop system is defined by $B_m(s) = \omega^2$ and $A_m(s) = s^2 + 2\zeta\omega s + \omega^2$. Design

1. continuous-time indirect regulator,
2. discrete-time indirect regulator,
3. continuous-time direct regulator,
4. discrete-time direct regulator.

Take $a = 1$, $\omega = 1$, $\zeta = 0.7$, $A_o(s) = s + 2$, $A_o(z) = 1$, and sampling time $h = 0.5$. Independently on your design, the plant should be simulated as a continuous-time system preceded and followed by zero-order hold blocks.