Lecture 7: Deterministic Self-Tuning Regulators

- Feedback Control Design for Nominal Plant Model via Pole Placement
- Indirect Self-Tuning Regulators
- Direct Self-Tuning Regulators

Indirect Self-Tuning Regulators

Consider a single input single output (SISO) system

$$\begin{aligned} y(t) + a_1 y(t-1) + \cdots + a_n y(t-n) \\ = b_0 u(t-d_0) + + \cdots + b_m u(t-d_0-m) \end{aligned}$$

Indirect Self-Tuning Regulators

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= b_0 u(t-d_0) + + \cdots + b_m u(t-d_0-m)
\end{aligned}
$$

It can be re written in the regression form

$$
\begin{aligned}
y(t) &= -a_1 y(t-1) - \cdots - a_n y(t-n) + b_0 u(t-d_0) + \\
&\quad + b_1 u(t-d_0-1) + \cdots + b_m u(t-d_0-m) \\
&= \phi(t-1)^T \theta^0
\end{aligned}
$$

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&\quad + b_1 u(t-d_0-1) + \cdots + b_m u(t-d_0-m) \\
&= \phi(t-1)^T \theta^0
\end{aligned}
$$

where $\theta^0 = \Big[a_1, \ldots, a_n, b_0, \ldots, b_m\Big]^T$

$$
\begin{aligned}
\phi(t-1) = \Big[& -y(t-1), \ldots, -y(t-n), \\
& u(t-d_0), \ldots, u(t-d_0-m)\Big]^T
\end{aligned}
$$

RLS Algorithm (discrete time)

Given the data $\left\{ y(t), \phi(t-1) \right\}_{t_0}^{N}$ defined by the model

$$y(t) = \phi(t-1)^T \theta^0, \qquad t = t_0,\, t_0+1,\, \ldots,\, N$$

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$$y(t) = \phi(t-1)^T\theta^0, \qquad t = t_0, t_0+1, \ldots, N$$

With the initial guess $\hat{\theta}(t_0)$ and a measure of our trust in this guess: $P(t_0) > 0$, we can use the RLS algorithm:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \phi(t-1)^T\hat{\theta}(t-1)\right)$$

$$K(t) = P(t)\phi(t-1) = \frac{P(t-1)\,\phi(t-1)}{\lambda + \phi(t-1)^T P(t-1)\phi(t-1)}$$

$$P(t) = \frac{1}{\lambda}\left(P(t-1) - \frac{P(t-1)\,\phi(t-1)\,\phi(t-1)^T\,P(t-1)}{\lambda + \phi(t-1)^T\,P(t-1)\,\phi(t-1)}\right)$$

RLS Algorithm (continuous time)

Given the data $\left\{y(\tau), \phi(\tau)\right\}_{t=t_0}^{t}$ defined by the model

$$\bar{y}(\tau) = \phi(\tau)^T \theta^0, \qquad \tau \in [t_0, t]$$

obtained from $\boxed{A(p)\, y(\tau) = B(p)\, u(\tau)}$ using filtered signals $y_f(\tau) = H_f(p)\, y(\tau)$ and $u_f(\tau) = H_f(p)\, u(\tau)$ as follows

$$\bar{y}(\tau) = p^n\, y_f(\tau), \qquad \phi(\tau)^T = \left[-p^{n-1}\, y_f(\tau),\ \ldots,\ p^{m-1}\, u_f(\tau)\right]$$

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With the initial guess $\hat{\theta}(t_0)$ and a measure of our trust in this guess: $P(t_0) > 0$, we can use the RLS algorithm:

$$\begin{aligned} \dot{\hat{\theta}}(t) &= P(t)\, \phi(t) \Big[\bar{y}(t) - \phi(t)^T \hat{\theta}(t)\Big] \\ \dot{P}(t) &= \alpha\, P(t) - P(t) \Big(\phi(t)\, \phi(t)^T\Big) P(t) \end{aligned}$$

Algorithm using RLS and MD Pole Placement

Off-line Parameters: Given polynomials B_m, A_m, A_o

Algorithm using RLS and MD Pole Placement

Off-line Parameters: Given polynomials $\boldsymbol{B_m}$, $\boldsymbol{A_m}$, $\boldsymbol{A_o}$

Step 1: Estimate the coefficients of A and $B = B^+ B^-$, i.e. θ^0, using the Recursive Least Squares algorithm.

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Step 1: Estimate the coefficients of A and $B = B^+ B^-$, i.e. θ^0, using the Recursive Least Squares algorithm.

Step 2: Apply the Minimum Degree Pole Placement algorithm

$$(\deg A = \deg A_m, \quad \deg B = \deg B_m, \quad \deg A_o = \deg A - \deg B^+ - 1, \quad B_m = B^- B_{pm})$$

$$R = R_p B^+, \quad T = A_o B_{pm}, \quad S, R_p : \; A R_p + B^- S = A_o A_m$$

with estimates for A and B taken from the previous Step.

Algorithm using RLS and MD Pole Placement

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Repeat Steps 1, 2, 3 (until the performance is satisfactory).

Example 3.4 (Recursive Modification of Example 3.1)

Given a continuous time system

$$\ddot{y} + \dot{y} = u,$$

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its pulse transfer function with sampling period $h = 0.5$ sec is

$$G(q) = \frac{b_1 q + b_2}{q^2 + a_1 q + a_2} = \frac{0.1065q + 0.0902}{q^2 - 1.6065q + 0.6065}$$

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The task is to synthesize a 2-degree-of-freedom controller such that the complementary sensitivity transfer function is

$$\frac{B_m(q)}{A_m(q)} = \frac{0.1761q}{q^2 - 1.3205q - 0.4966}$$

We will take $u_c(t) = 1$ for $t \geq 0$ except for

$u_c(t) = -1$ for $15 \leq t < 30$, $u_c(t) = 0$ for $50 \leq t \leq 70$

Example 3.4 (Recursive Modification of Example 3.1)

We have found the controller via MD pole placement

$$R(q)u(t) = T(q)u_c(t) - S(q)y(t)$$

with

$$\begin{aligned} R(q) &= q + 0.8467 \\ S(q) &= 2.6852 \cdot q - 1.0321 \\ T(q) &= 1.6531 \cdot q \end{aligned}$$

that stabilizes the discrete plant. Will it stabilize the continuous-time system as well?

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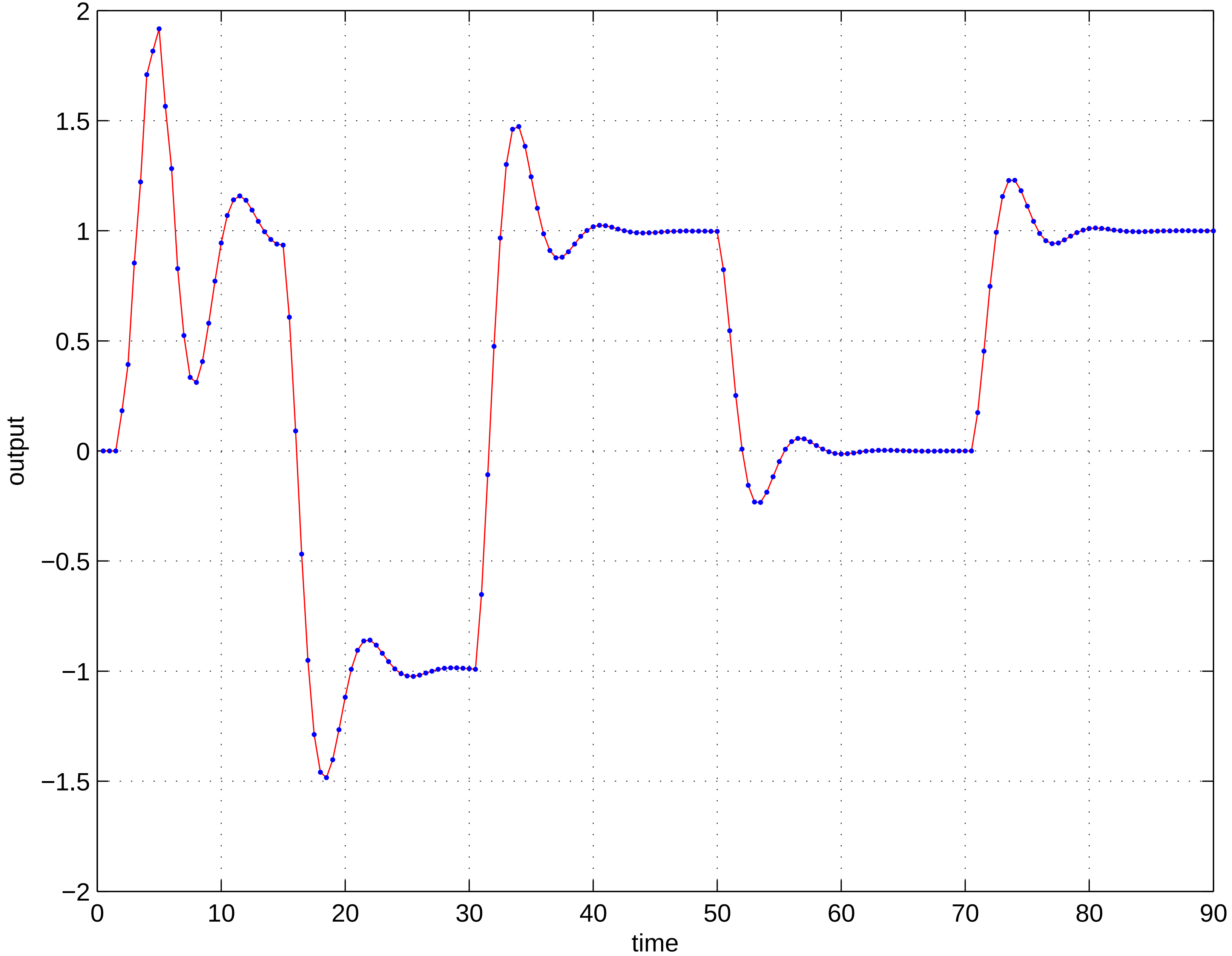
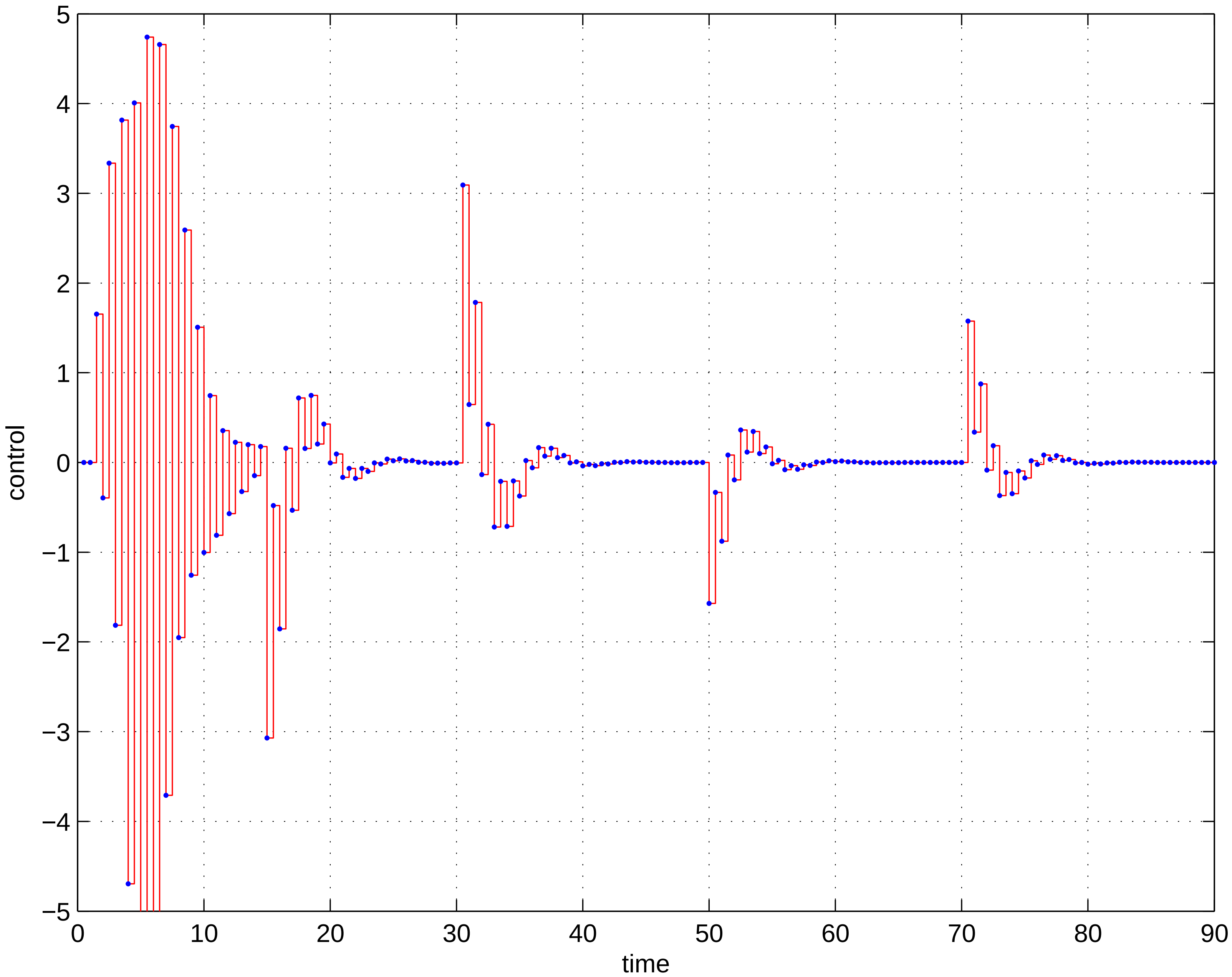
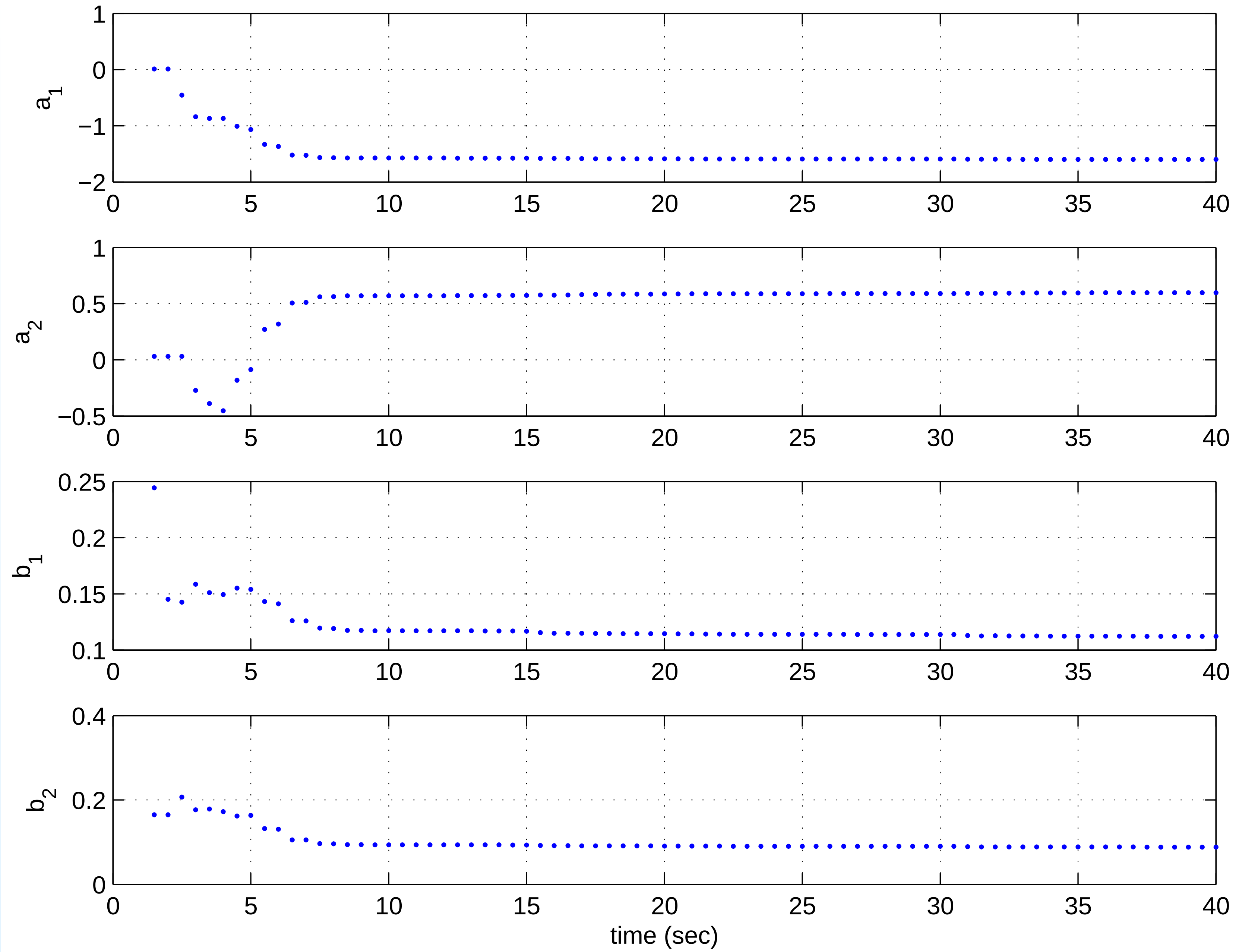
Let us design the adaptive controller which would recover the performance for the case when parameters of the plant - the polynomials $A(q)$ and $B(q)$ – are not known!



The response of the closed-loop system with adaptive controller



The control signal



Adaptation of parameters defining polynomials $A(q)$ and $B(q)$

Lecture 6: Deterministic Self-Tuning Regulators

- Feedback Control Design for Nominal Plant Model via Pole Placement
- Indirect Self-Tuning Regulators
- Direct Self-Tuning Regulators

Direct Self-Tuning Regulators

The previous algorithm consists of two steps

Step 1: Computing estimates for the polynomials $A(q)$, $B(q)$

Direct Self-Tuning Regulators

The previous algorithm consists of

Step 1: Computing estimates $\hat{A}(q)$, $\hat{B}(q)$ for $A(q)$, $B(q)$

Step 2: Computing polynomials defining the controller

$$R(q), \qquad T(q), \qquad S(q)$$

based on $\hat{A}(q)$, $\hat{B}(q)$.

Direct Self-Tuning Regulators

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Such procedure has potential drawbacks like

- Redundant step in design: why to know $A(q)$ and $B(q)$?

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- Control design methods (pole placement, LQR, ...) are sensitive to mistakes in $\hat{A}(q)$, $\hat{B}(q)$. Is there a way to avoid such poorly conditioned numerical computations?

Direct Self-Tuning Regulators

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The procedure has potential drawbacks like

- Redundant step in design: why to know $A(q)$ and $B(q)$?
- Control design methods (pole placement, LQR, ...) are sensitive to mistakes in $\hat{A}(q)$, $\hat{B}(q)$. Is there a way to avoid such poorly conditioned numerical computations? E.g.:

$$\hat{A}(q) = q(q-1), \qquad \hat{B}(q) = q + \varepsilon$$

Direct Self-Tuning Regulators

The idea of another approach comes from the observation that for the plant

$$A(q)\, y(t) = B^+(q)\, B^-(q)\, u(t)$$

and for the target system (the model to follow)

$$A_m(q)\, y(t) = B_{mp}(q)\, B^-(q)\, u_c(t)$$

the Diophantine equation

$$A_o(q)\, A_m(q) = A(q)\, R_p(q) + B^-(q)\, S(q)$$

can be seen as a regression to estimate coefficients of R and S

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$$\boldsymbol{A_o(q) A_m(q) y(t) = R_p(q) B(q) u(t) + B^{-}(q) S(q) y(t)}$$

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$$A_o(q) A_m(q) y(t) = R_p(q) \cdot B^{+}(q) \cdot B^{-}(q) u(t) + B^{-}(q) S(q) y(t)$$

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the Diophantine equation

$$A_o(q)\,A_m(q) = A(q)\,R_p(q) + B^{-}(q)\,S(q)$$

can be seen as a regression to estimate coefficients of R and S

$$A_o(q)A_m(q)y(t) = B^{-}(q)\left(R(q)\cdot u(t) + S(q)y(t)\right)$$

Direct Self-Tuning for Minimum-Phase Systems

Suppose that $B(q)$ is stable and can be canceled, i.e.

$$\begin{aligned} A(q)y(t) &= B(q)u(t) \\ &= b_0 u(t-d_0) + b_1 u(t-d_0-1) + \cdots + b_m u(t-d_0-m) \\ &= B^+(q)B^-(q)u(t) = B^+(q)b_0 u(t) \end{aligned}$$

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\end{aligned}
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then Diophantine equation

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leads to the regression

$$A_o(q)A_m(q)y(t) = b_0\left(R_p(q)u(t) + S(q)y(t)\right)$$

which is in the form $\boxed{\eta(t) = \phi(t)^T \theta}$ with

$$A_o(q)A_m(q)y(t) = \eta(t), \quad \phi(t)^T\theta = b_0\left(R(q)u(t) + S(q)y(t)\right)$$

Direct Self-Tuning for Minimum-Phase Systems

In the formula

$$\eta(t) = b_0 R(q) u(t) + b_0 S(q) y(t) = \phi(t)^T \theta$$

the vector of regressors is

$$\phi(t) = [u(t), \ldots, u(t-l), y(t), \ldots, y(t-l)]^T$$

and the vector of parameters is

$$\theta = [b_0 r_0, \ldots, b_0 r_l, b_0 s_0, \ldots, b_0 s_l]^T$$

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$$\theta = [b_0 r_0, \ldots,\ b_0 r_l,\ b_0 s_0, \ldots,\ b_0 s_l]^T$$

Suppose θ is estimated by $\hat{\theta}$, then

$$R(q) = q^l + \frac{\hat{\theta}_2}{\hat{\theta}_1} q^{l-1} + \ldots, \qquad S(q) = q^l + \frac{\hat{\theta}_{l+2}}{\hat{\theta}_1} q^{l-1} + \ldots$$

Direct Self-Tuning for Minimum-Phase Systems

Some modification to the regression model

$$A_o(q)A_m(q)y(t) = \eta(t) = b_0 R(q)u(t) + b_0 S(q)y(t) = \phi(t)^T\theta$$

can be made if we introduce the signals

$$u_f(t) = \frac{1}{A_o(q)A_m(q)}u(t), \quad y_f(t) = \frac{1}{A_o(q)A_m(q)}y(t)$$

Direct Self-Tuning for Minimum-Phase Systems

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Then the regression model is

$$\begin{aligned} y(t) &= b_0R(q)u_f(t) + b_0S(q)y_f(t) \\ &= \mathcal{R}(q)u_f(t) + \mathcal{S}(q)y_f(t) \\ &= \phi(t)^T\theta \end{aligned}$$

Direct Self-Tuning Regulators: Summary

Given

- polynomials $A_m(q)$, $B_m(q)$, $A_o(q)$
- the relative degree d_0 of the plant $(\deg A_o = d_0 - 1)$

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Step 1: Estimate coefficients of polynomials $R(q)$ and $S(q)$ from the regression model

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Step 2: Compute the control signal by

$$R(q)u(t) = T(q)u_c(t) - S(q)y(t)$$

where for $B_m(q) = q^{d_0} A_m(1)$ (minimal delay and unit static gain):

$$T(q) = A_o(q) A_m(1)$$

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$$T(q) = A_o(q) A_m(1)$$

Repeat Steps 1 and 2

Direct ST Reg. for Non-Minimum-Phase Systems

If $B(q)$ is not stable or cannot be canceled, i.e.

$$\begin{aligned} A(q)\,y(t) &= B(q)\,u(t) \\ &= b_0 u(t-d_0) + b_1 u(t-d_0-1) + \cdots + b_m u(t-d_0-m) \\ &= B^+(q)B^-(q)u(t), \quad B^-(q) \not\equiv \text{const} \end{aligned}$$

Direct ST Reg. for Non-Minimum-Phase Systems

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&= B^+(q)B^-(q)u(t), \quad B^-(q) \not\equiv \text{const}
\end{aligned}
$$

then Diophantine equation

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A_o(q)A_m(q) = A(q)R(q) + B^-(q)S(q)
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Repeat Steps 1 and 2

Next Lecture / Assignments:

Next meeting (April 26, 13:00-15:00, in A205Tekn).

Homework problems: Consider the process:

$G(s) = \dfrac{1}{s(s+a)}$, where a is an unknown parameter. Assume that the desired closed-loop system is

$G_m(s) = \dfrac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$. Construct

- continuous-time indirect algorithm,
- discrete-time indirect algorithm,
- continuous-time direct algorithm,
- discrete-time direct algorithm.