Organization of the course:

- Meetings in A206 10:15-12:00

March: 26, 30, 31;

April: 13, 14, 16, 20, 21, 23, 27, 28;

May: 11, 12, 14, 25, 26, 28;

June: 1, 2, 4

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- Meetings in A206 10:15-12:00

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- The textbook:

  K.J. Åström and B. Wittenmark

  Adaptive Control

  2nd Edition, Addison-Wesley Publishing Company, 1995

Lecture 1 topics:

- Discussion on difficulties and approaches for designing controllers for uncertain systems
    - What are control design techniques you know?
    - Are you ready to apply these techniques?

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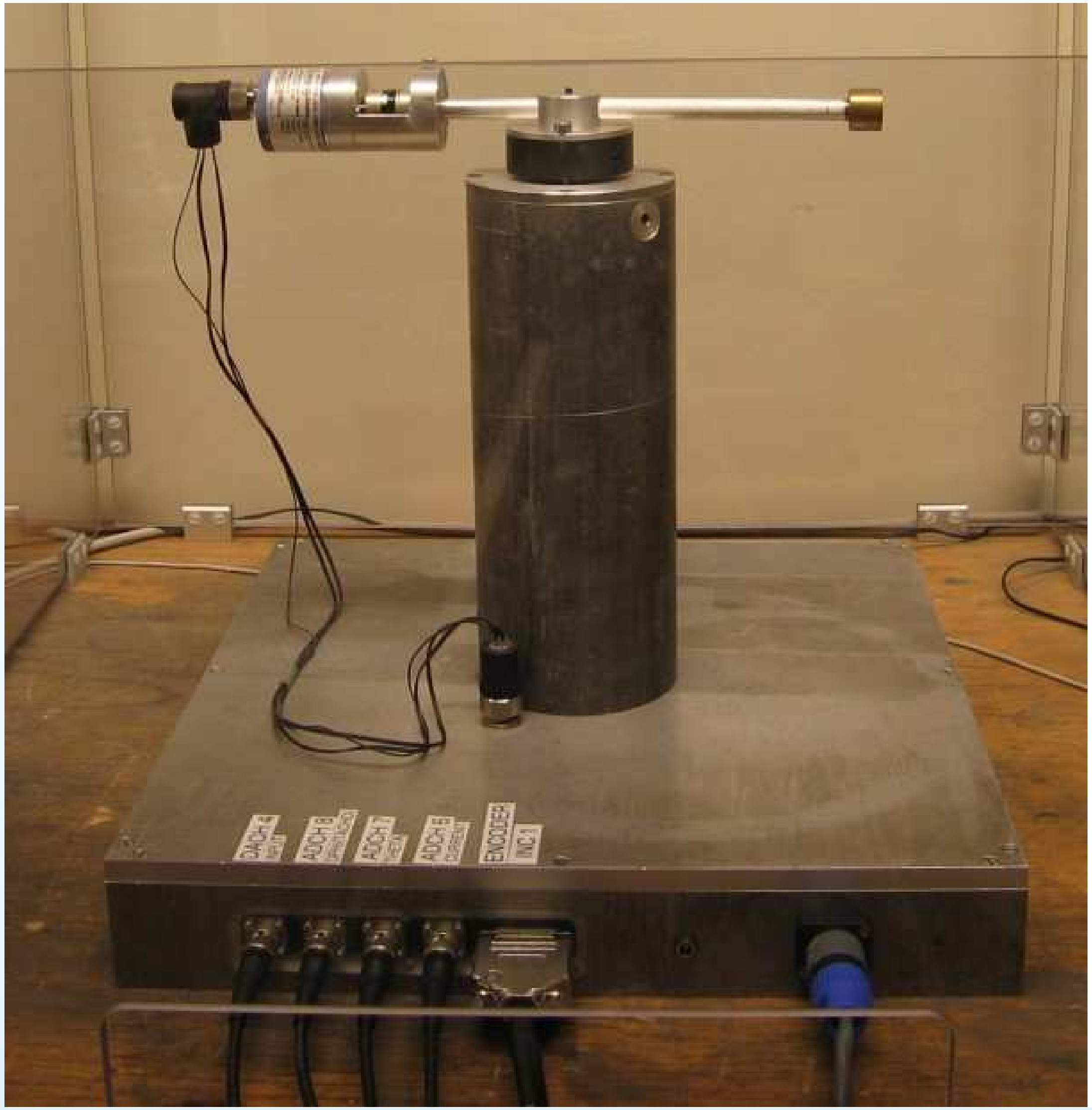
  - What is a good model?
- Motivation for the problem formulation of adaptive control design

How to design a controller for this system?



The arm of the Furuta Pendulum with the bob at its end

Build a Model for the Dynamics

According to Newton's law

$$\alpha \cdot \ddot{\phi} = \tau$$

where

- ϕ is the angle of the arm,
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The external torque τ consists of various components

$$\tau = u - F_{friction}(\cdot) - F_{dist.}(\cdot)$$

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Neglecting dynamics of voltage v and current i, we obtain

$$\tau = K_{DC} \cdot v - F_{friction}(\cdot) - F_{dist.}(\cdot)$$

Preliminary Model for Friction:

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The Coulomb friction model is often used for feedforward compensation of friction

$$F_C\left(\frac{d\phi}{dt}\right) = \begin{cases} F_+, & \frac{d\phi}{dt} > 0 \\ F_-, & \frac{d\phi}{dt} < 0 \end{cases}$$

Preliminary Model for Friction (Cont'd):

To identify parameters of the linear part of the model (α, K_{DC}), we need to compensate nonlinearities.

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Important Observation: we can estimate values v_+, v_-

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The experiments show that these values are

$$v_{Coulomb} \approx \begin{cases} \mathbf{0.032}, & \text{if } \frac{d}{dt}\phi > 0 \\ \mathbf{-0.033}, & \text{if } \frac{d}{dt}\phi < 0 \end{cases}$$

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Adding to control a feedforward signal $v_{Coulomb}$

$$v = v_{nominal} + v_{Coulomb},$$

removes strong nonlinearity, making the dynamics close to linear

$$\begin{aligned} \alpha \cdot \ddot{\phi}(t) &= K_{DC} \cdot v - F_{friction}(\cdot) - F_{dist.}(\cdot) \\ &= K_{DC} \cdot v_{nominal}(t) - \beta \cdot \dot{\phi}(t) + e(t, \cdot) \end{aligned}$$

Still Some Problems!

We have to choose the input signal $v_{nominal}$ for the system

$$\alpha\ddot{\phi}(t) = K_{DC}\, v_{nominal}(t) - \beta\dot{\phi}(t) + e(t), \quad y(t) = \phi(t) + e_m(t)$$

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- introducing a feedback action to stabilize the system,
- using a reference signal with a particular pass-band characteristic.

The simplest choice is the proportional feedback

$$v_{nominal}(t) = K_p\left(r(t) - y(t)\right) = K_p\left(r(t) - \{\phi(t) + e_m(t)\}\right)$$

where $r(t)$ is the reference signal.

Model Used in Experiments:

Combining

$$\alpha\ddot{\phi}(t) = K_{DC}\, v_{nominal}(t) - \beta\dot{\phi}(t) + e(t), \quad y(t) = \phi(t) + e_m(t)$$

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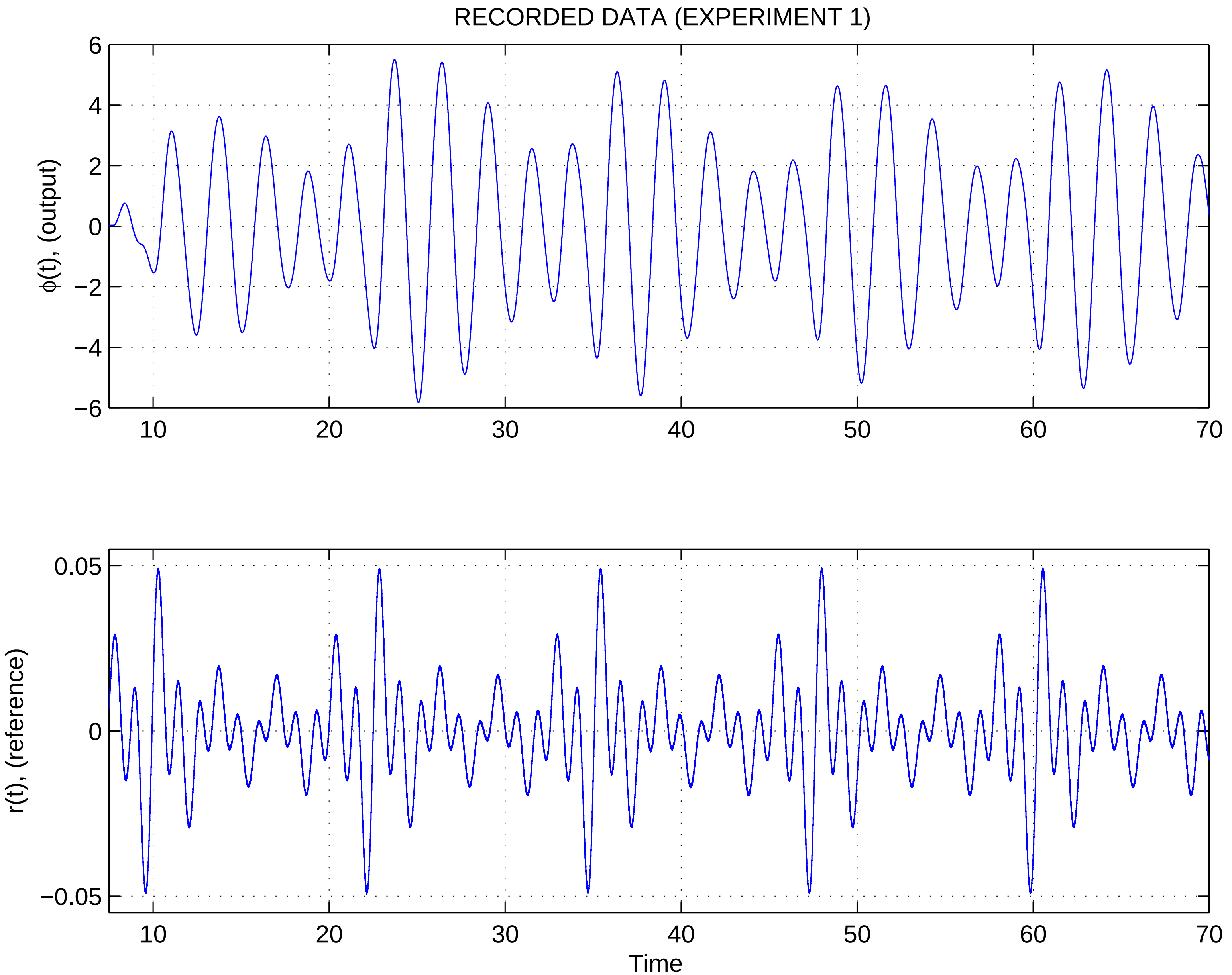
---

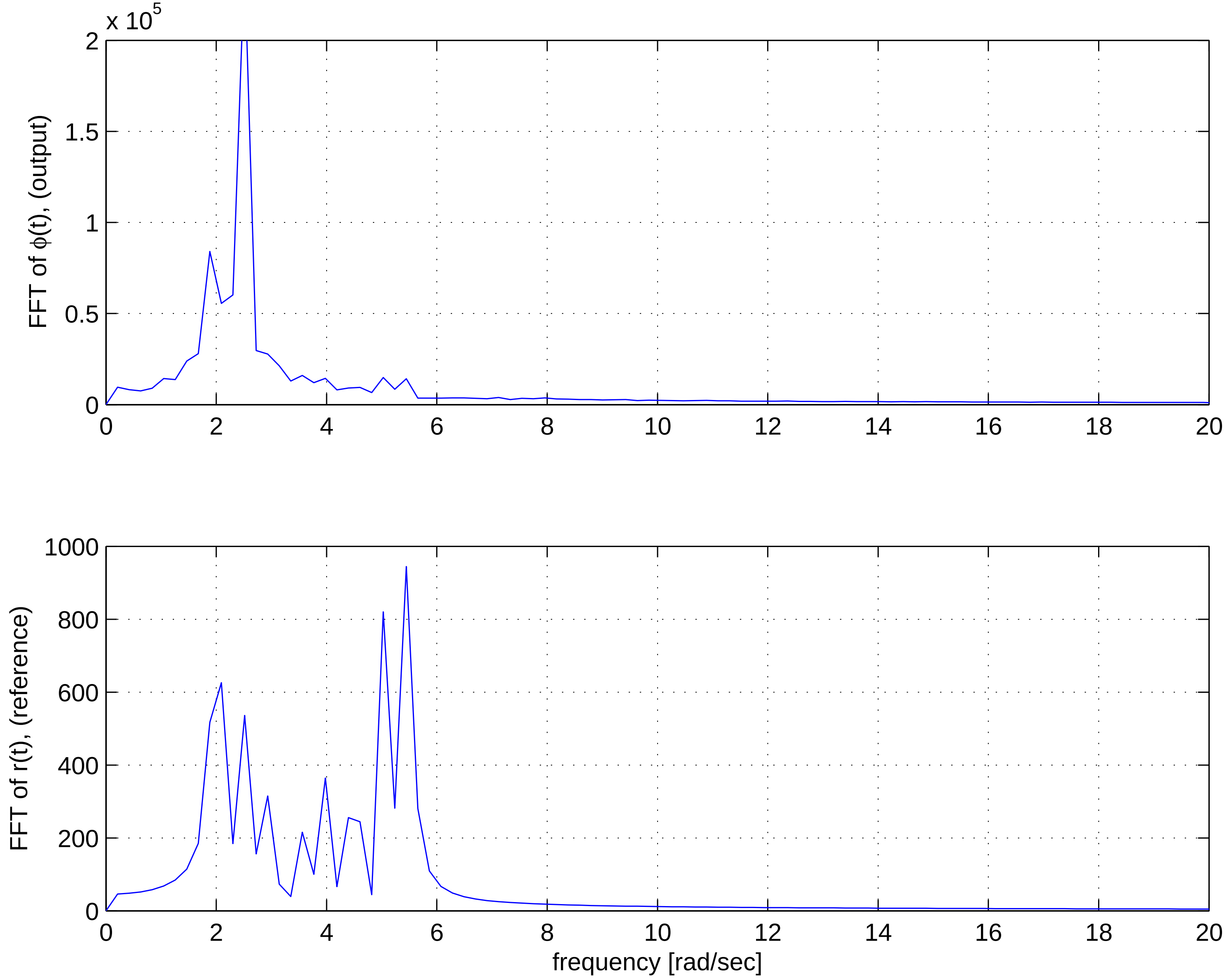
we get the input-output description ($r \to y$) with stable model

$$\begin{aligned} \alpha\cdot\ddot{\phi}(t) + \beta\cdot\dot{\phi}(t) + K_{DC}\cdot K_p\cdot\phi(t) &= \\ &= K_{DC}\cdot K_p \cdot r(t) + \Big\{ e(t) + K_p \cdot e_m(t) \Big\} \\ y(t) &= \phi(t) + e_m(t) \end{aligned}$$

It will be used in experiment!



Experiment 1: $K_p = 0.05$ and $r(t)$ is the sum of sinusoids



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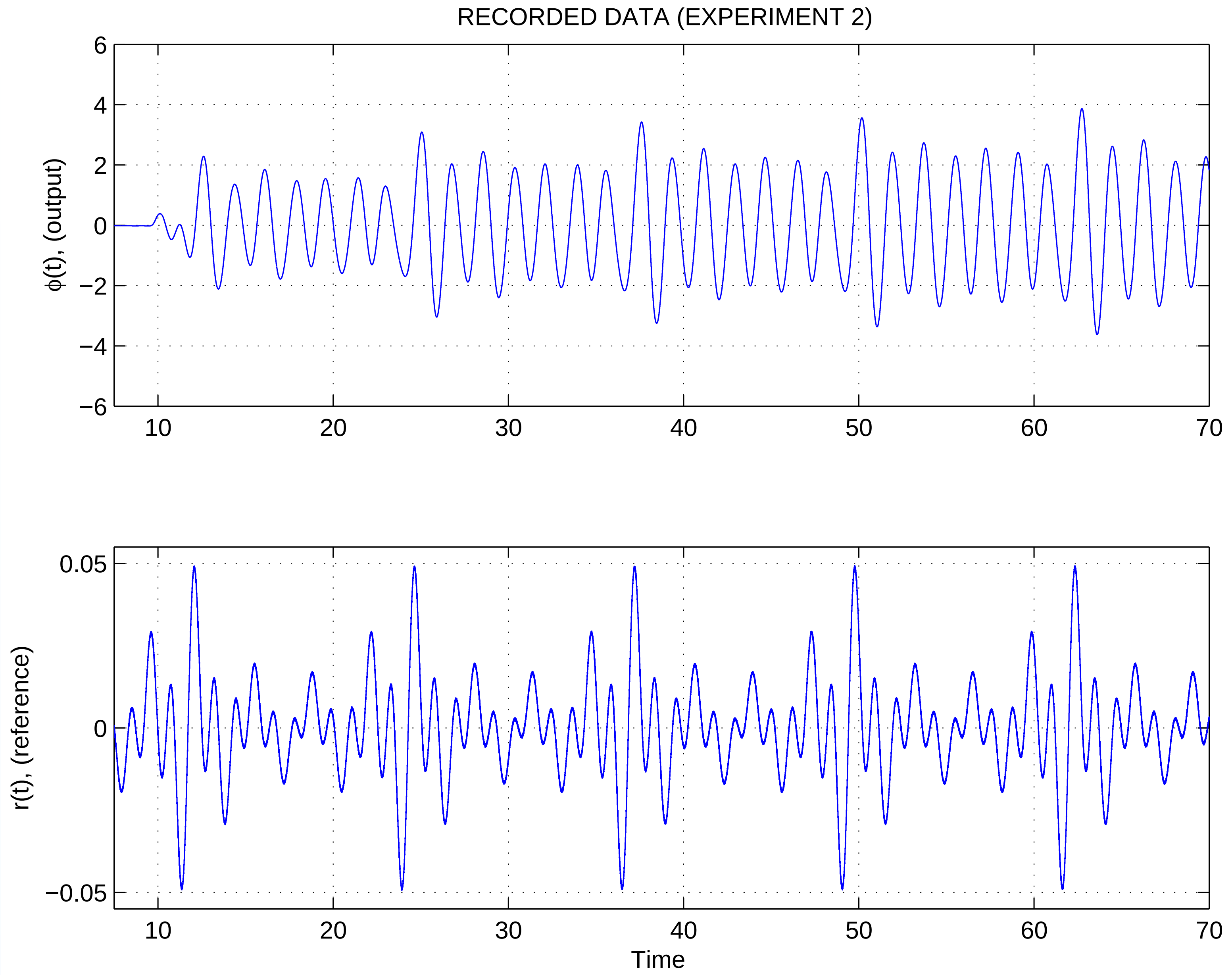

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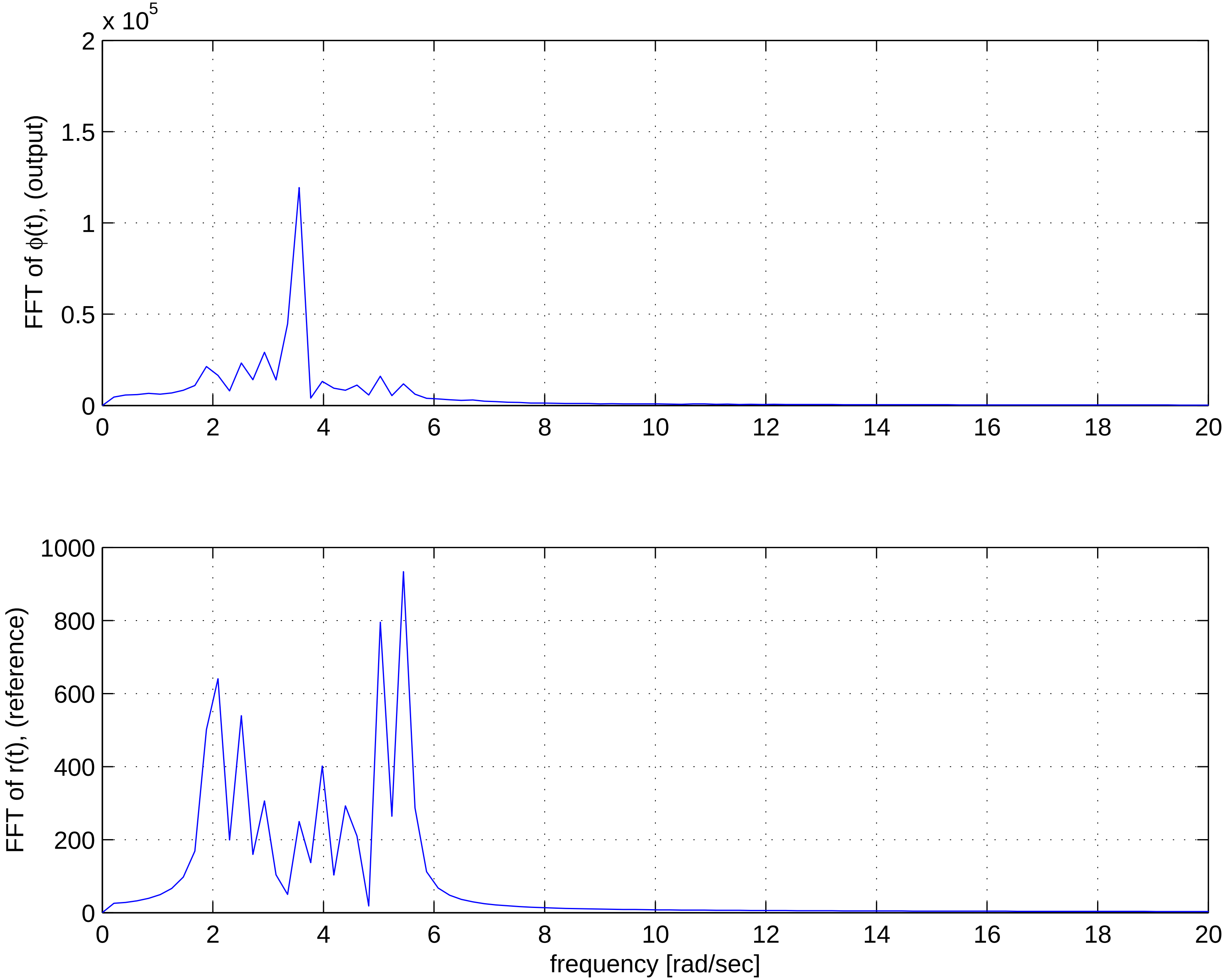

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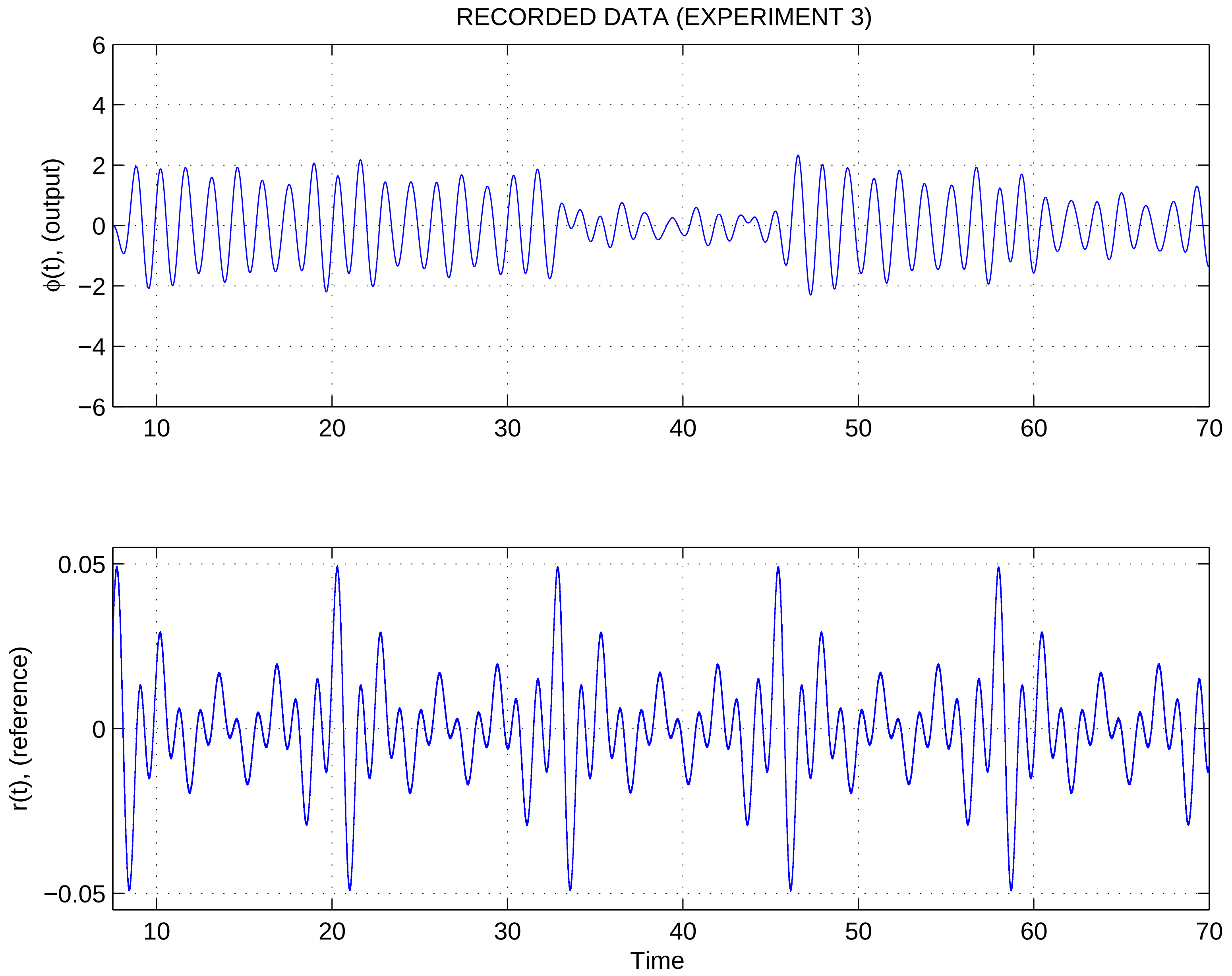

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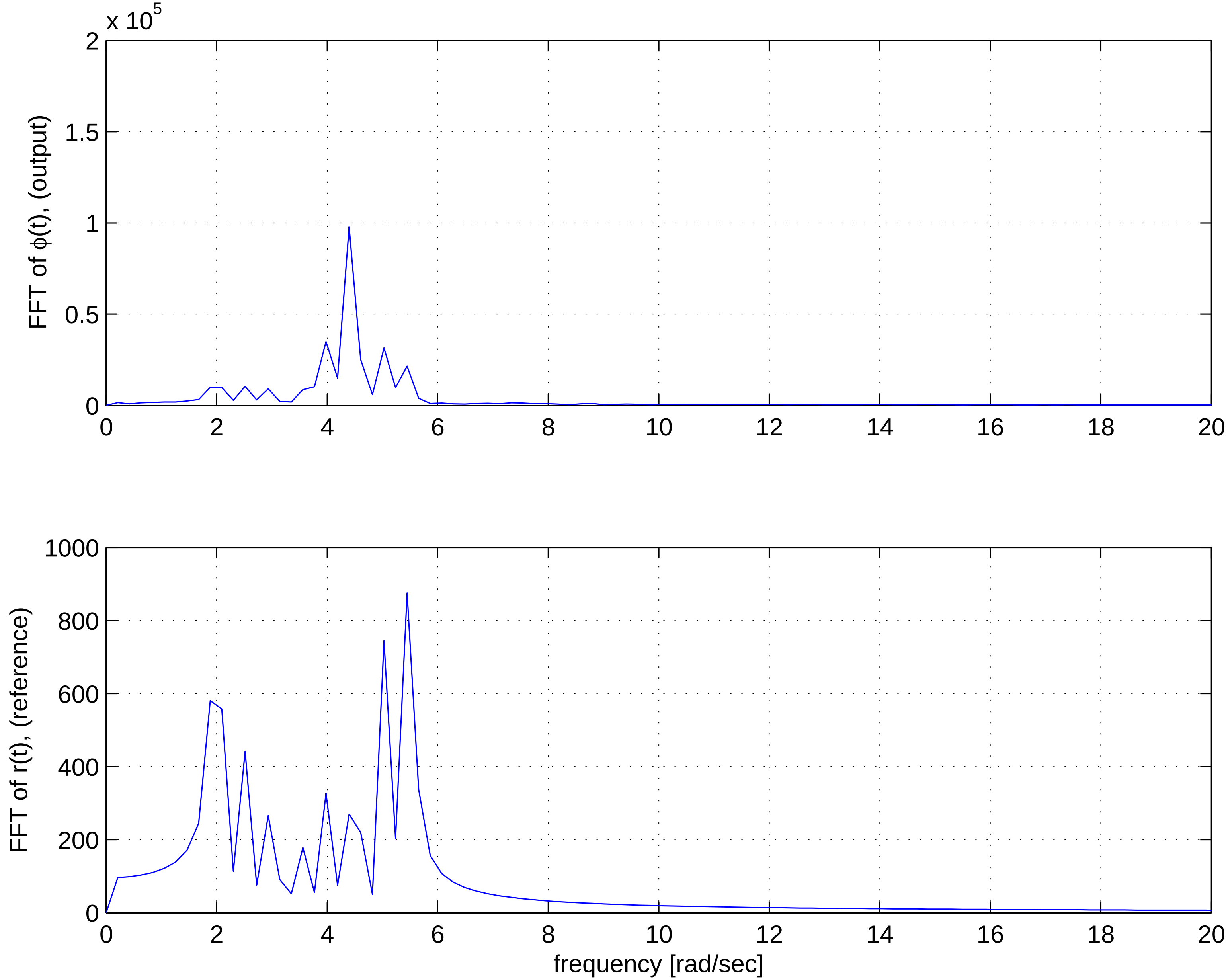

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Impulse Response for Non-parametric Identification:

If the system is linear and the input signal is impulse

$$\boldsymbol{r(t) = \Big\{r(0),\, r(t_s),\, r(2t_s),\, \ldots,\, r(nt_s), \ldots\Big\} = \Big\{{\color{red}\rho},\, 0,\, 0,\, 0,\, \ldots\Big\}}$$

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then the output is

$$\begin{aligned} y\,(t = k \cdot t_s) &= \sum_{i=1}^{+\infty} g(i \cdot t_s) r(t - i \cdot t_s) + e(t) \\ &= g(t_s) r(t - t_s) + g(2t_s) r(t - 2t_s) + \ldots + g(kt_s) r(t - kt_s) + \ldots \\ &= g(t_s) \cdot 0 + g(2t_s) \cdot 0 + \cdots + g(kt_s) \cdot \rho + \cdots + e(t) \\ &= g(kt_s) \cdot \rho + e(t) \end{aligned}$$

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Estimates $\left\{ \hat{g}(kt_s) \right\}$ used for guessing delay, gain, stability. . .

Spectral Analysis for Non-parametric Identification:

Computing fast Fourier transforms

$$Y_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} y(t) e^{j\omega t}, \quad R_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} r(t) e^{j\omega t}$$

can be used for estimation the transfer function of the system

$$\hat{G}_N(e^{j\omega}) = Y_N(\omega)/R_N(\omega) \qquad \left[\approx G(e^{j\omega}) \right]$$

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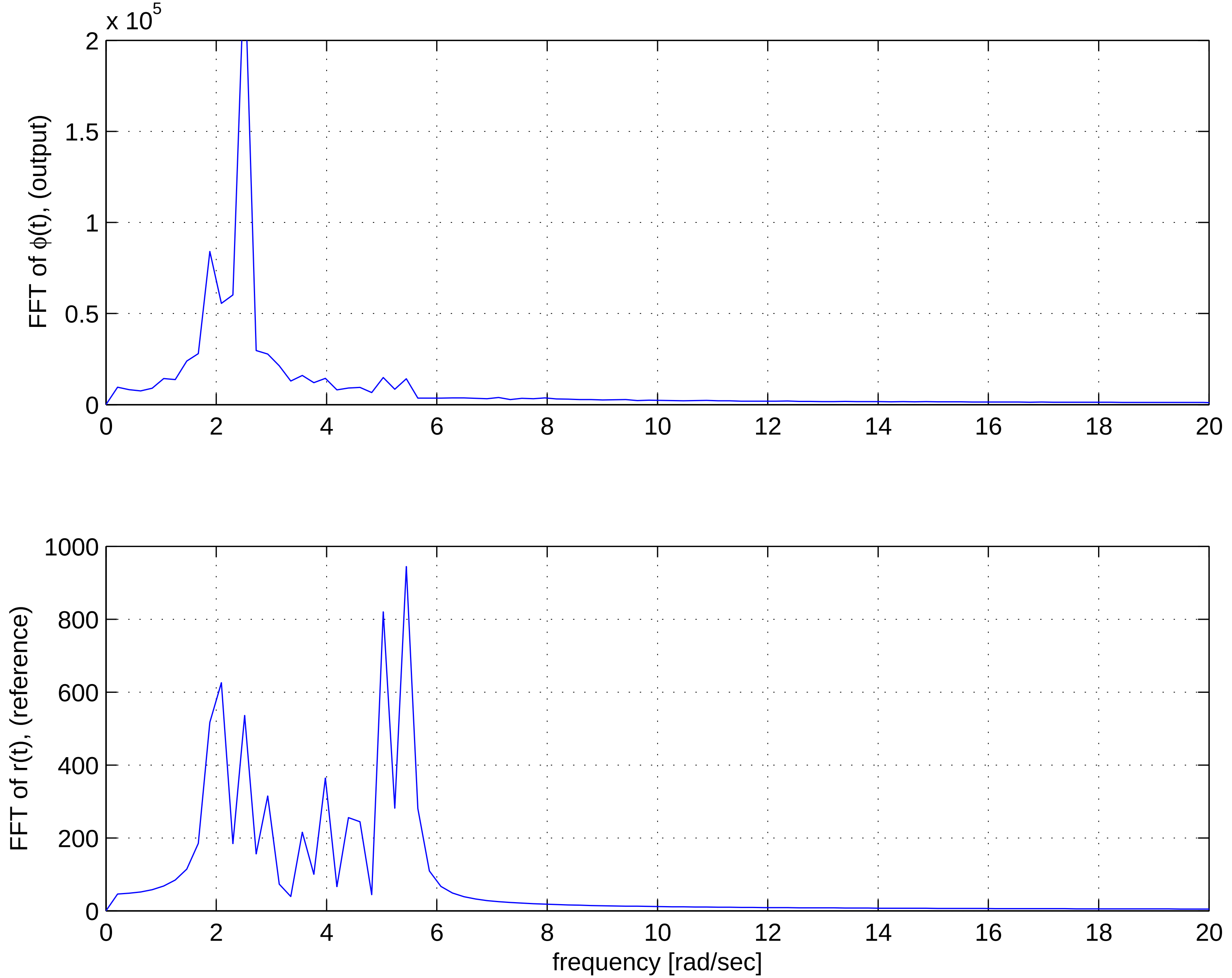
Such estimate can be computed from input and cross spectrum

$$\begin{aligned} \Phi_{yr}^N(\omega) &= \int_{-\pi}^{\pi} W(\xi - \omega) Y_N(\xi) \bar{R}_N(\xi) d\xi \\ \Phi_{r}^N(\omega) &= \int_{-\pi}^{\pi} W(\xi - \omega) R_N(\xi) \bar{R}_N(\xi) d\xi \end{aligned}$$

as

$$\hat{G}_N(e^{j\omega}) = \frac{\Phi_{yr}^N(\omega)}{\Phi_{r}^N(\omega)} \qquad \left[\approx G(e^{j\omega}) \right]$$



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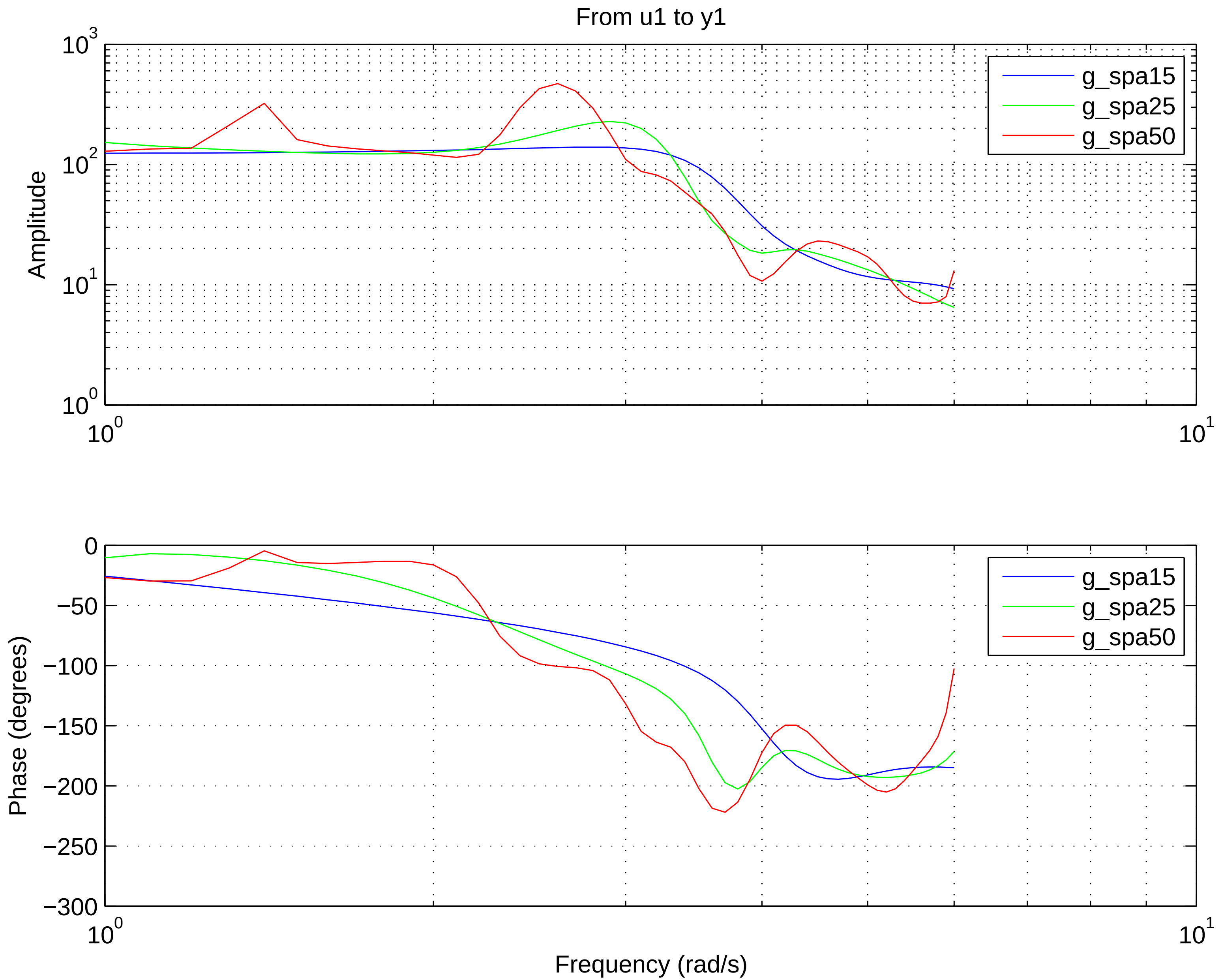
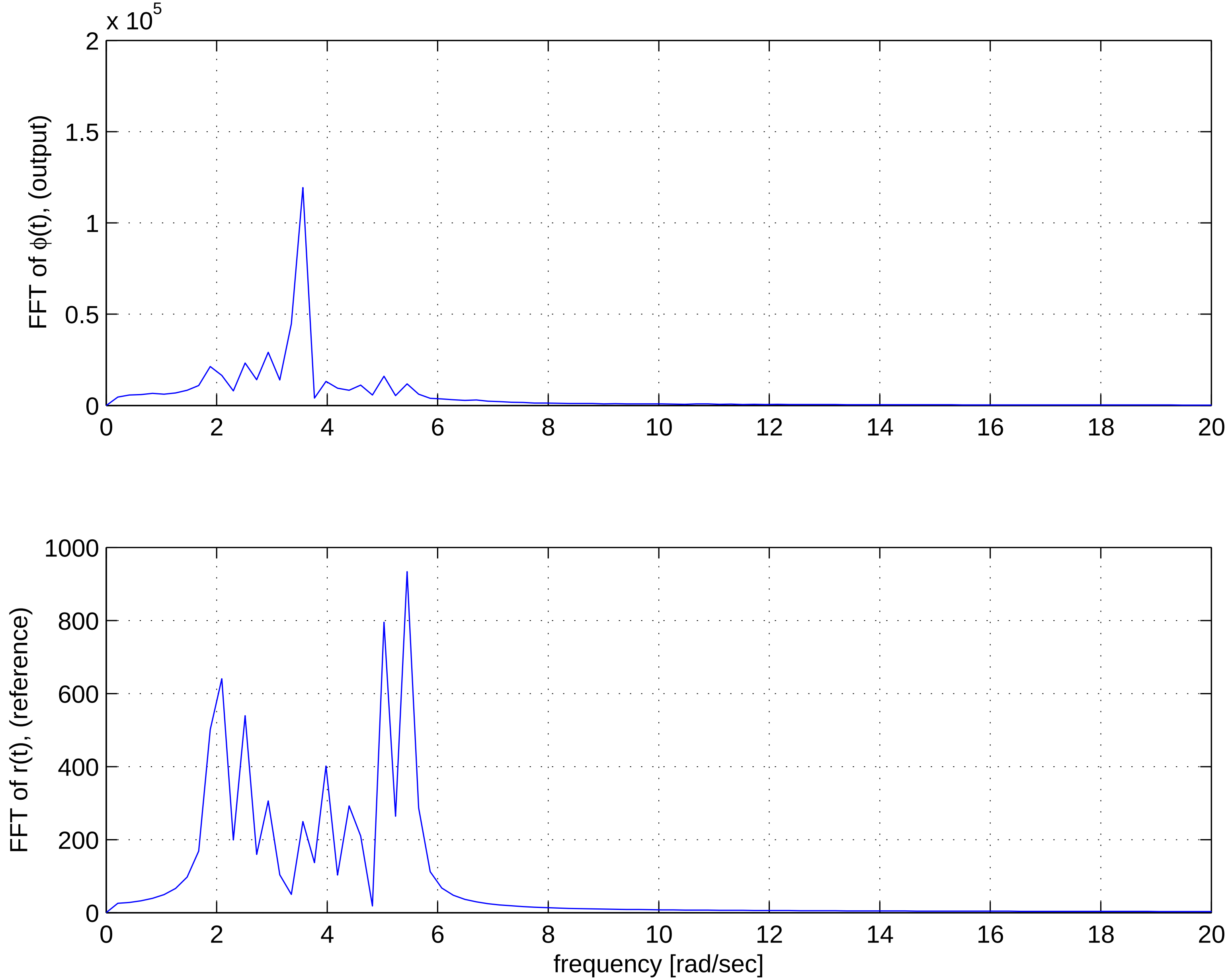


Experiment 1 ($K_p = 0.05$), we expect a pick around 2.5-3 rad/s



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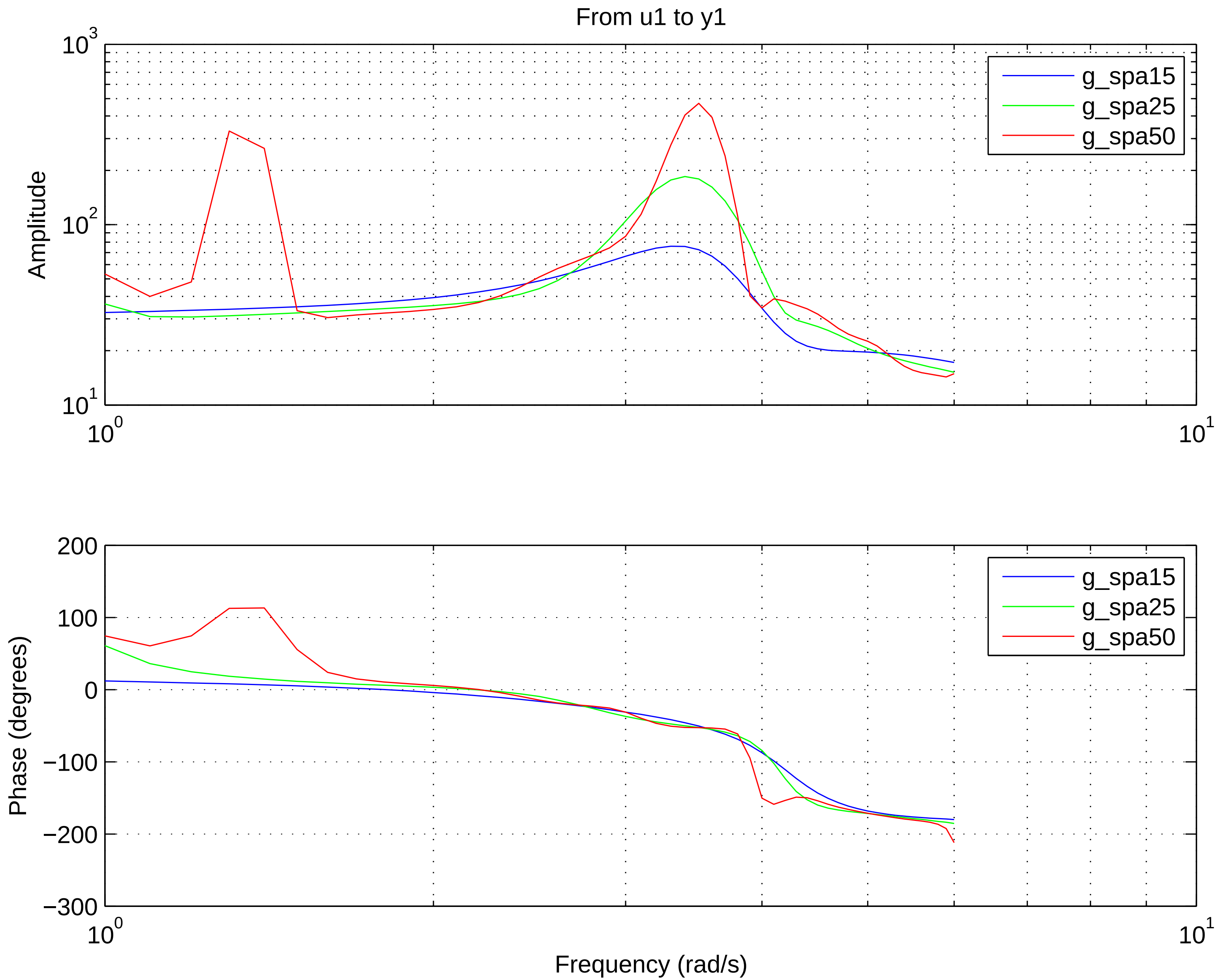

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Prediction Error Method:

Both the model of the system obtained from basic principles and the spectral analysis of the data suggest

- the order of the model equals to 2
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for the 2nd order system $y(t) = \phi(t) + e_m(t)$,

$$\alpha \cdot \ddot{\phi}(t) + \beta \cdot \dot{\phi}(t) + K_{DC} \cdot K_p \cdot \phi(t) =$$

$$= K_{DC} \cdot K_p \cdot r(t) + \left\{ e(t) + K_p \cdot e_m(t) \right\}$$

Guesses:

$$b_1 \approx 0, \qquad b_0 \approx a_0, \qquad a_1 \approx \frac{\beta}{\alpha}, \qquad a_0 \approx \frac{K_{DC} K_p}{\alpha}$$

Comments on PEM:

Prior to start searching for parameters, one should take into account the following.

- The spectra of data was within interval $[\mathbf{2},\ \mathbf{6}]$ [rad/sec] $\Rightarrow$ an estimate for $\boldsymbol{b_0}$ of our model

$$\hat{G}(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

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  will be unreliable and of a large variance,

- The result of estimation might be an unstable system. The viscous friction in the system is small; hence, one should expect to have two resonance poles close to imaginary axis, which can be identified wrongly.

  An appropriate projection ensuring stability for an estimated system should be implemented in such a case.

Results of PEM:

Two guesses from the first principle modeling are wrong for all the estimated models:

- the estimate for b_1 differs from zero;
- the estimate for b_0 is 10 times bigger that the estimate of a_0.

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Experiment 1: $\quad a_0/K_p = 121.37, \quad \sigma = 1.15$

Experiment 2: $\quad a_0/K_p = 126.16, \quad \sigma = 0.347$

Experiment 3: $\quad a_0/K_p = 124.69, \quad \sigma = 0.236$

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These mean values and their variances are used to obtain

$$\frac{K_{DC}}{\alpha} \approx 124.86, \quad \sigma = 0.125$$

What is Left?

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$$\frac{K_{DC}}{\alpha} \approx 124.86$$

- How to obtain estimates for K_{DC} and α?
- Suggestion: Change the inertia α by adding known value!
- Denote it by J_a, then the same procedure will result in estimation of

$$\frac{K_{DC}}{\alpha + J_a} \approx A$$

Two equations will be enough for estimating K_{DC} and α!

Summary

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Summary

- To choose a reasonable *linear* mathematical model for control design one have to reduce effect of nonlinearities such as friction.
- Finding reliable parameters for a model is a very challenging task.
- It is not clear neither
  - how accurately parameters must be estimated nor
  - how precise we need to know them to succeed with a particular control design (see Examples in Chapter 1 of the book!).
- Perhaps, one can attempt to estimate parameters on-line, *adapting* the accuracy to what is enough to achieve a particular control goal.

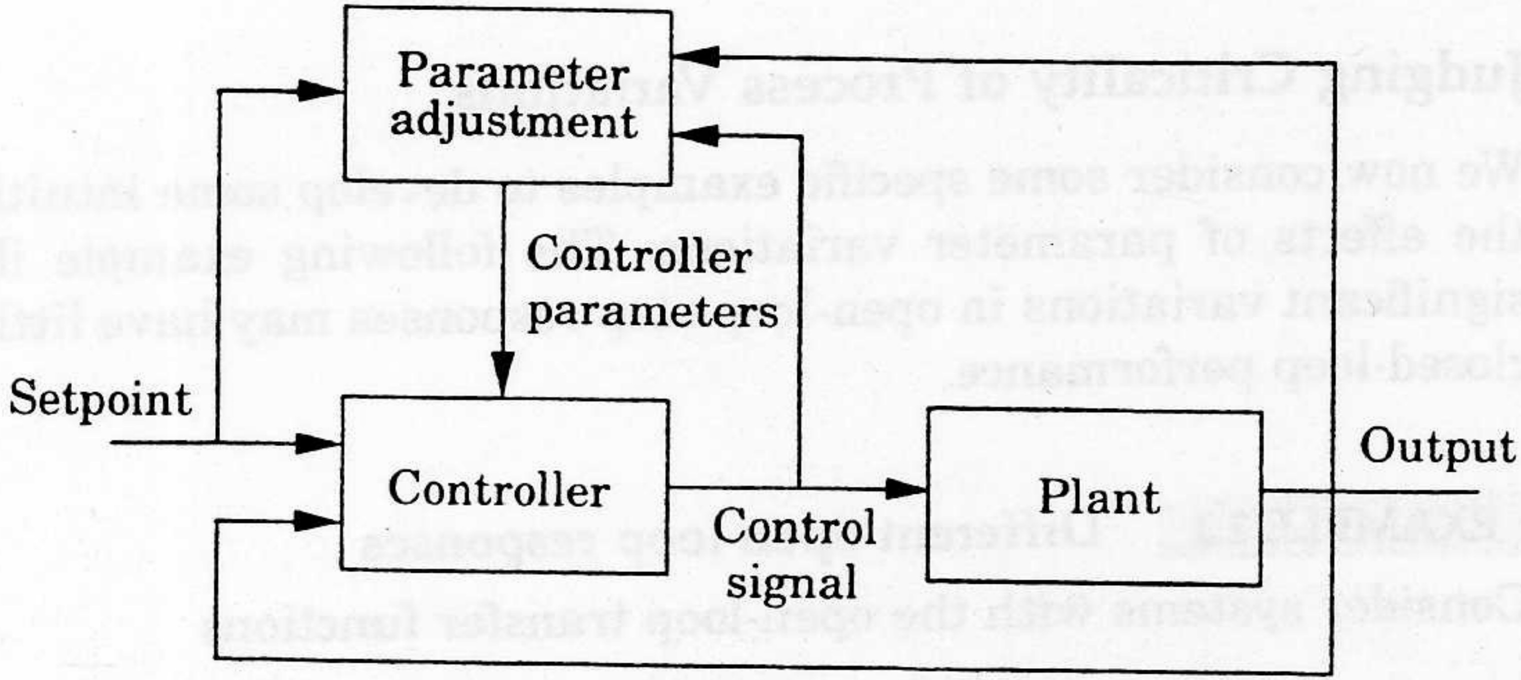

Figure 1.1 Block diagram of an adaptive system.

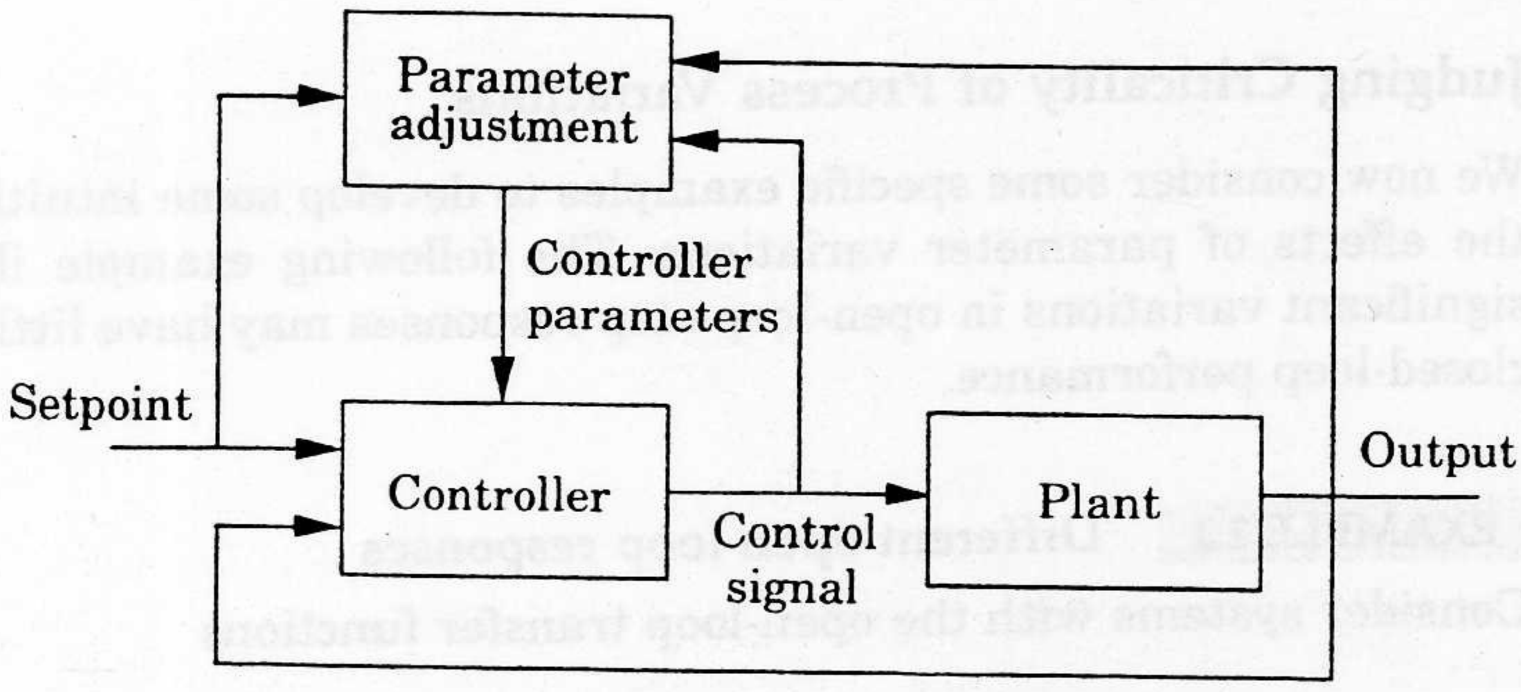

Figure 1.1 Block diagram of an adaptive system.

Adaptive control has two components:

- on-line estimating of some parameters
- simultaneous redesigning of the control law.

Note: a parametric model is needed and a knowledge on how to design a controller for each possible set of parameters.

Next Lecture / Assignments:

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