A General Statistical Model for Ultrasonic Backscattering from Tissues

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***Abstract*—The backscattered ultrasonic echo from tissue can be described in terms of Rayleigh distribution or K distribution. Even though both generalized K distribution and homodyned K distribution can account for some of the scattering conditions that exist in tissues, the analytical complexity involved with these distributions is significant. A much simpler generalized model based on the Nakagami distribution is proposed here. This model can describe the statistics of the envelope of the backscattered echo from an ensemble of scatterers with varying number densities, varying cross sections, and the presence or absence of regularly spaced scatterers. Computer simulations and experiments on tissue-mimicking phantoms have been undertaken to test the validity of the model. Results clearly show the versatility of the Nakagami distribution and its parameter to model the backscattered envelope from tissues. It is suggested that Nakagami distribution may be a good model for use in tissue characterization because of its simple analytical nature and ability to encompass different scattering conditions.**

I. Introduction

Ultrasonic tissue characterization methods are used extensively for the detection and identification of abnormalities in the breast, liver, kidney, etc. These methods depend on the model employed to describe the statistics of the backscattered ultrasonic echo from tissue, which were initially classified into pre-Rayleigh, Rayleigh, and post-Rayleigh, based on the properties of the backscattered echo [1], [2]. However, these simple models were limited in their ability to characterize the various tissue regions into normal and abnormal and to characterize further abnormal regions into benign and malignant lesions in breast, for example. Another model that was considerably more general than the Rayleigh model used K distribution [3]–[6] to describe the statistics of the backscattered envelope of the echo. The two parameters of the K distribution, namely the effective number of scatterers and the scaling parameter, together provided information on the number density of scatterers, the variation in the scattering amplitudes within the range cell, and the mean scattering amplitude [3]. The K distribution approximates to the Rayleigh distribution under conditions of high values of the effective number of scatterers. The K distribution, even though it is better than the Rayleigh, is not general enough to describe the statistics of the backscattered echo from a range cell containing a subresolvable or a resolvable periodic alignment of scatterers giving rise to post-Rayleigh statistics often represented by the Rician distribution. Yet another problem with the K distribution has been the computational difficulty related to the solution of the transcendental equations to estimate the parameter of the K distribution. The inability of the K distribution to account for the existence of the periodic scatterers was handled through the use of the generalized K distribution [3] and the homodyned K distribution [5]. Once again, the computational problems were compounded by the introduction of these distributions.

Hence, there is a need for a model that can possibly describe the statistics of the backscattered echo in a more general environment. The model must lead to parameters that are relevant for tissue characterization along with computational simplicity. Statistical models that can describe all of these different cases are available in radar and may be applicable to ultrasound. One of these models is based on the Nakagami distribution [7]. An effort to explore the applicability of the Nakagami distribution [7]–[9] to describe the ultrasonic backscattered echo from tissues is undertaken. Computer simulation and experiments on phantoms have been carried out to examine the statistical fit of this distribution and establish the relationship between the parameters of the distribution and the characteristics of the scattering regions. Limitations and extensions of the Nakagami distribution to model the ultrasonic echo have been described.

II. Theoretical Model

When an acoustic pulse travels through tissue or any medium, backscattering from the scatterers in the range cell contributes to the returned echo. This contribution to the echo from the scatterers in the range cell has been treated as a random walk because the locations of the scatterers are considered to be random. The backscattered echo, s(t), therefore, can be expressed as

$$s(t) = \sum_{n=1}^{N} \alpha_n \cos(\omega_0 t + \phi_n) \tag{1}$$

where ω_0 is the mean frequency of excitation and N is the number of scatterers in the range cell. The quantities α_n and ϕ_n are, respectively, the amplitude and phase of the n^{th} scatterer. In most of the analyses, the amplitude α_n is considered to be deterministic (even though this is not

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necessary), and the phase ϕ_n is considered to be uniformly distributed in the range $\{0, 2\pi\}$. Under conditions of large N, the backscattered echo, s(t), can be expressed as

$$s(t) = X\cos(\omega_0 t) - Y\sin(\omega_0 t). \tag{2a}$$

By virtue of the central limit theorem, X and Y are zero mean, identically distributed, Gaussian random variables and are given by

$$X = \sum_{n=1}^{N} \alpha_n \cos(\phi_n); Y = \sum_{n=1}^{N} \alpha_n \sin(\phi_n). \tag{2b}$$

The envelope of the backscattered echo, R, given by

$$R = \sqrt{X^2 + Y^2} \tag{3}$$

will be Rayleigh distributed. If there exists some subresolvable periodic alignment of scatterers, either X or Y will have a nonzero mean and R will be Rician distributed [2]. Allowing the amplitudes of the scatterers to be distributed randomly, the density function of the envelope R was shown to be K distributed. A generalized K distribution [3] was proposed to allow for any periodic alignment of scatterers within the range cell. Homodyned K distribution [5] was also proposed as a general model to describe the ultrasonic echo. The computational complexity involved in these models was quite significant, and the parameters of the distribution became less and less reliable [3]–[5], as the number density of scatterers increased.

It may be possible to achieve the goal of a simpler universal model for tissue characterization using the Nakagami distribution [7], which was proposed to describe the statistics of the returned radar echoes. We will not derive the Nakagami distribution; we will invoke the Nakagami distribution and show how well it fits within the ultrasonic imaging context. Note that the Nakagami distribution even allows partial correlation between the scatterers, which was not permitted under the Rayleigh or K-distributed scattering models. The probability density function of the envelope, $f(r)$, under the Nakagami model is given by

$$f(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{m}{\Omega} r^2\right) U(r) \tag{4}$$

where m is the Nakagami parameter and Ω is a scaling parameter. The parameter m [7] is constrained such that

$$m \geq \frac{1}{2}. \tag{5}$$

As we shall see later, this limit may be violated under certain conditions of interest in ultrasound. The Nakagami parameter m can be obtained from the moments of the envelope and can be expressed as

$$m = \frac{\left[E\left(R^2\right)\right]^2}{E\left[R^2 - E\left(R^2\right)\right]^2} \tag{6}$$

where $E(.)$ stands for the expectation.

The scaling factor Ω can be obtained as

$$\Omega = E\left(R^2\right). \tag{7}$$

The cumulative distribution $F(r)$ can be obtained as

$$F(r) = \int_0^r \frac{2m^m y^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{m}{\Omega} y^2\right) dy = P\left(\frac{m}{\Omega} r^2, m\right) \tag{8}$$

where $P(.,.)$ is the incomplete Gamma function. A plot of the Nakagami density function, $f(r)$, is shown in Fig. 1 for four different values of the Nakagami parameter, m. For m = 0.5, the density function is a half Gaussian, while for m = 1, the density function is Rayleigh. For m > 1, the density function appears to be similar to Rician. For values of 0.5 < m < 1, the density function can be described as pre-Rayleigh. We will now examine how this distribution can be used to describe various scattering conditions in ultrasound.

Consider a range cell containing a number of scatterers as shown in Fig. 2(a). For now, we assume that we have a large number of scatterers within the range cell and no correlation between the scatterers. Therefore, we can conclude that the conditions of central limit theorem are met that and the statistics will be Rayleigh. The Nakagami distribution becomes Rayleigh [7] when the Nakagami parameter m is equal to 1. Putting m = 1 in (4), we get

$$f(r) = \frac{2r}{\Omega} \exp\left(-\frac{r^2}{\Omega}\right) U(r) \tag{9}$$

with $E\left(R^2\right) = \Omega$. This is the simplest special case of the Nakagami distribution. Consider a slightly more complicated case shown in Fig. 2(b). The scattering region contains subresolvable periodic alignment of scatterers in addition to a collection of randomly located scatterers. Let us assume that the spacing is $\lambda/2$, where λ corresponds to the wavelength at the center frequency of insonation. Under these conditions, the backscattered signal s(t) can be expressed as [10], [11]

$$s(t) = \text{real part} \left\{ \exp(j\omega_0 t) \left\{ \sum_{n=1}^{N} \alpha_n \exp(j\phi_n) + \sum_{l=1}^{L} \alpha_l \exp(j2\pi) \right\} \right\} \tag{10}$$

where the second summation is a result of the presence of the half wavelength-spaced scatterers (total of L) assumed to have similar scattering amplitudes as the randomly located ones. Under this condition, envelope R can be expressed as

$$R = \sqrt{X_1^2 + Y^2} \tag{11}$$

where X_1 is a Gaussian random variable, by virtue of the central limit theorem but has a nonzero mean. It can be shown that R will be Rician distributed with a density function given by

$$f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + R_0^2}{2\sigma^2}\right) I_0\left(\frac{rR_0}{\sigma^2}\right) U(r). \tag{12}$$

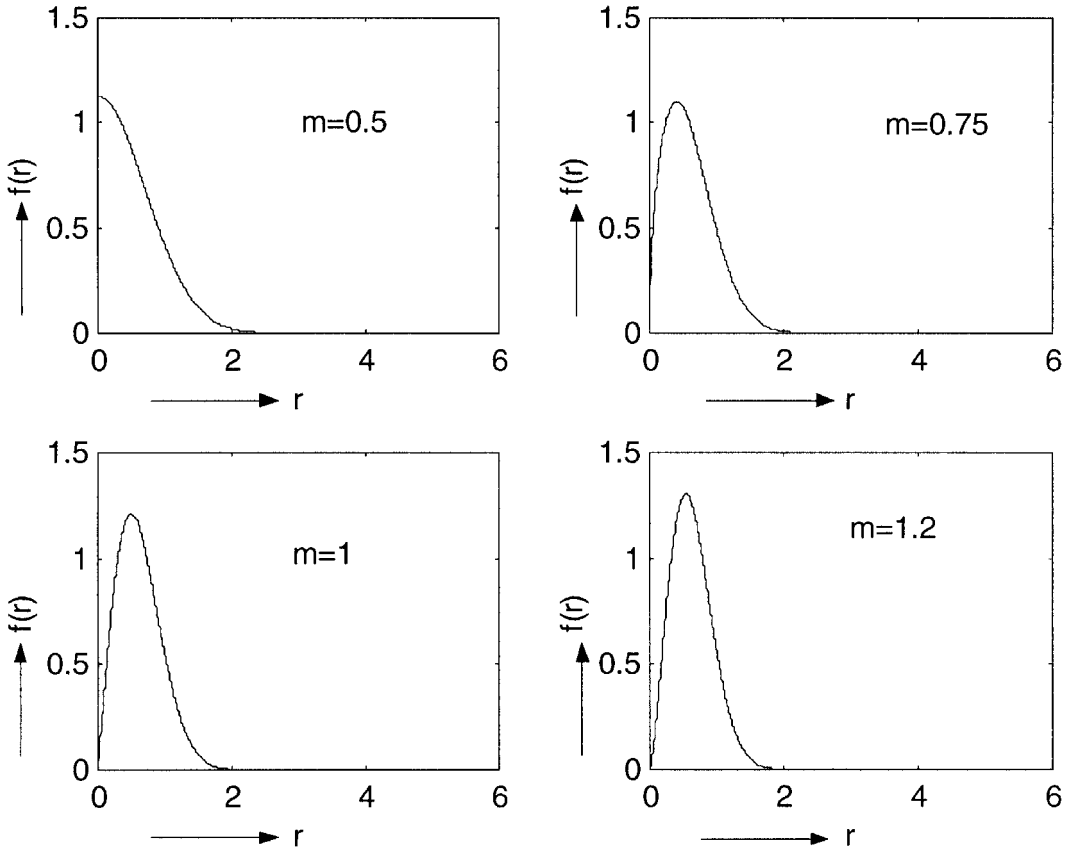

Fig. 1. Plots of the Nakagami probability density function for four different values of m. All curves were obtained for $\Omega = 0.5$.

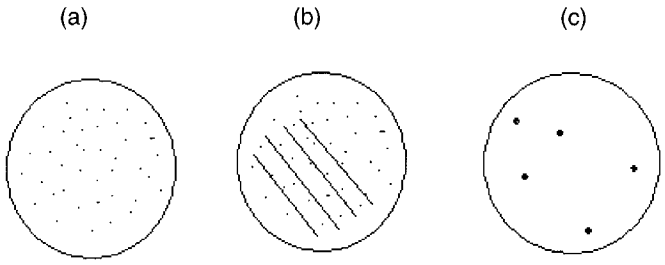

Fig. 2. Three separate scattering conditions are shown: a) completely random alignment of a large number of scatterers, b) randomly located scatterers along with regularly spaced scatterers and c) a small number of scatterers.

R_0 is the mean value of the inphase component X_1, and $I_0()$ is the zeroth-order modified Bessel function. The quantity σ^2 is the variance of the inphase or the quadrature component. The Nakagami distribution becomes Rician when m > 1 as shown by Nakagami [7; p. 16]. The Nakagami parameter m and the Rician distribution parameters are related through [7; p. 17, Eq (54)]

$$\frac{1}{m} = 1 - \frac{R_0^4}{\left(2\sigma^2 + R_0^2\right)^2}. \tag{13}$$

Consider now another case of subresolvable periodic alignment of scatterers. Assume that the regular spacing corresponds to $\lambda/4$ or lower. The equation for the backscattered echo s(t) can now be expressed as

$$s(t) = \text{real part} \left\{ \exp(j\omega_0 t) \left\{ \sum_{n=1}^{N} \alpha_n \exp(j\phi_n) + \sum_{l=1}^{L} \alpha_l \exp\left(j\frac{\pi}{\varepsilon}\right) \right\} \right\} \tag{14}$$

where ε can take values of 1,2, ... depending on $\lambda/4$, $\lambda/8$ spacing, etc. L is once again the total number of regularly spaced scatterers in the range cell. In this case, it is possible to have the inphase and quadrature components of the envelope X and Y to have almost zero means but of unequal variances, σ_1^2 and σ_2^2, respectively. Under these conditions, the density function of the envelope can be shown to be [12], [13]

$$f(r) = \frac{r}{\sigma_1\sigma_2} e^{-\left(\frac{r^2}{4\sigma_1^2} + \frac{r^2}{4\sigma_2^2}\right)} I_0\left[\frac{r^2}{4}\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2}\right)\right] U(r) \tag{15}$$

where $I_0()$ is the zeroth-order modified Bessel function. This density function can be identified as the generalized Rician [2], [13]. Envelope that has a distribution given by (15) has the property that its signal-to-nose ratio SNR_E, given by

$$SNR_E = \frac{mean(R)}{std.dev(R)}, \tag{16}$$

will be less than 1.91 [2]. Although the value of SNR_E for the case of Rayleigh is 1.91, the SNR_E value is larger than 1.91 in the case of Rician-distributed envelope in (12). The Nakagami distribution becomes generalized Rician for values of $0.5 < m < 1$ as shown [7; p. 17]. Note that when the ratio of the number of periodic scatterers to the number of randomly located scatterers becomes smaller and smaller, m will approach 1. Another aspect of both Rician and generalized Rician is the fact the phase of the backscattered echo will not be uniform in the range $\{0,\ 2\pi\}$; Rayleigh envelope implies uniformity of the phase.

We also need to look at another case that exists in ultrasonic tissue characterization where the range cell contains very few scatterers with random locations as shown in Fig. 2(c). It is not possible to obtain an analytical expression for the pdf of the envelope corresponding to this case. However, we can see whether the Nakagami distribution can account for this case in addition to the two cases discussed earlier. As will be shown later, this will correspond to a special case of Nakagami distribution with m less than 0.5. To see if such a possibility exists, we can look at the functional form of the Nakagami distribution. It is possible to see that the Nakagami distribution can be identified as belonging to the class of density functions such as Gamma distributions [7; p. 8]. If we define a new random variable $Y = R^2$, the probability density function of Y, $f(y)$ can be shown to be [12]

$$f(y) = \frac{m^m y^{m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{m}{\Omega}\right) U(y), m > 0. \qquad (17)$$

Eq. (17) is the Gamma density function with a parameter m that takes values in the range $0 < m < \infty$ instead of the limitation on m that $m \geq 0.5$ given in (5). If we use the Gamma density function in place of the Nakagami distribution, the lower limit of the value of m can be below 0.5. Thus, it is possible to have values of m lying between 0 and 0.5.

We will now explore all of these scattering conditions along with the others by undertaking simple simulations using random number generators. (Note that this simulation does not use any characteristic elements of ultrasonic imaging such as pulse shapes, center frequency, etc. That simulation is done in the next section.)

All three different scattering conditions were recreated to see whether the Nakagami parameter m takes on values as suggested based on the types of random variables encountered in ultrasonic scattering. These results are shown in Table I. Variable 1 corresponds to the case of a few scatterers (five), each with Gamma-distributed scattering cross sections; variables 2, 3, and 4 correspond to the cases in which the range cell contains a large number of scatterers, leading to Gaussian pdf for the inphase and quadrature components. The values in variables 2, 3, and 4 correspond to zero mean and unequal variances (variable 2), zero mean and equal variances (variable 3), and non-zero mean and equal variance (variable 4) cases of Gaussian random variables. In other words, column 1 corresponds to the case of five Gamma-distributed random variables; and the other columns correspond to the case of two random variables each. The envelope, R, given by

$$R = \sqrt{\sum_{k=1}^{N} X_k^2} \qquad (18)$$

was calculated to achieve the values of m, using (6).

The number of samples used was 5000 for every random number listed. For every scenario, 50 such iterations were performed to obtain the average value of m shown in Table I. The properties of the random variables used in the simulations are also given. We see that, indeed, it is possible to have m values less than 0.5. Nakagami [7; p. 8] refers to this dichotomy in his work where it is stated that, for $m < 0.5$, the distribution can be identified as a form of Gamma distribution. Therefore, we will refer to the case of $m < 0.5$ as Nakagami-Gamma to distinguish it from the standard Nakagami distribution for which $m \geq 0.5$. Such a definition is in conformity with reference to Rician as Nakagami-Rice. The other values of m can also be realized using appropriate sets of random variables. A typical set of histograms obtained through this trial is shown in Fig. 3. All of these trials were conducted using MATLAB (The Math Works Inc., Natick, MA) on a PC.

It appears that the Nakagami distribution can encompass most of the scattering conditions that exist within the tissue or the scattering region. The parameters can be computed easily from the moments of the envelope. Computer simulation and experiments on phantoms conducted to test the validity of the model will now be described.

III. Computer Simulation

Computer simulation was carried out to verify the hypotheses advanced in the previous section. A one-dimensional discrete scattering model was used in our simulation. It was assumed that the locations of the scatterers were uniformly distributed. An A-line of 3.7 cm was chosen, leading to the location of the scatterers being uniformly distributed in the interval $\{0, 3.7$ cm$\}$. The transmitted pulse was modeled [6] as

$$P(t) = -t \exp\left(-4\beta^2 t^2\right) \sin(2\pi f_0 t) \qquad (19)$$

where f_0 is the center frequency and β (0.8 MHz) is the bandwidth. The received echo signal, s(t), is a superposition of echoes from all of the scatterers present in the scattering volume:

$$s(t) = \sum_{n=1} \alpha_n P\left(t - 2\frac{x_n}{c}\right) \qquad (20)$$

where α_n is the amplitude of the n^{th} scatterer assumed to have a Gamma distribution, x_n is the location of the n^{th} scatterer from the origin, and c is the velocity of sound (1446 m/s).

A transmit frequency of 3.5 MHz was used. Several simulations were carried out with number densities of randomly located scatterers varying from 5 to 15/mm. In addition to the randomly located scatterers, scatterers with

TABLE I

THE CHARACTERISTICS OF THE RANDOM VARIABLES USED IN THE SIMULATIONS ARE SHOWN. VARIABLE 1 CORRESPONDS TO THE CASE OF A FEW SCATTERERS (FIVE), EACH WITH GAMMA-DISTRIBUTED SCATTERING CROSS SECTIONS; VARIABLES 2, 3, AND 4 CORRESPOND TO THE CASES IN WHICH THE RANGE CELL CONTAINS A LARGE NUMBER OF SCATTERERS, LEADING TO GAUSSIAN PDF FOR THE INPHASE AND QUADRATURE COMPONENTS. THE CASES SHOWN IN 2, 3, AND 4 CORRESPOND RESPECTIVELY, TO ZERO MEAN AND UNEQUAL VARIANCES (VARIABLE 2), ZERO MEAN AND EQUAL VARIANCES (VARIABLE 3), AND NON-ZERO MEAN EQUAL VARIANCE (VARIABLE 4) CASES OF GAUSSIAN RANDOM VARIABLES.

Gamma density function: $f(x) = \frac{x^{a-1}}{\Gamma(a)b^a} \exp\left(-\frac{x}{b}\right) U(x)$

Gaussian density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\lfloor -\frac{1}{2\sigma^2}(x-\mu)^2 \rfloor$

$$R = \sqrt{\sum_{k=1}^{N} X_k^2}$$

Variable 1	a	b	Variable 2	μ	σ	Variable 3	μ	σ	Variable 4	μ	σ
Gamma	0.5	2	Gaussian	0	2	Gaussian	0	2.1	Gaussian	1	0.5
Gamma	0.4	2	Gaussian	0	4.1	Gaussian	0	2.1	Gaussian	0	0.5
Gamma	0.5	2									
Gamma	0.3	2									
Gamma	0.4	2									
m_{average}	0.3713			0.7381			0.9986			1.4542	

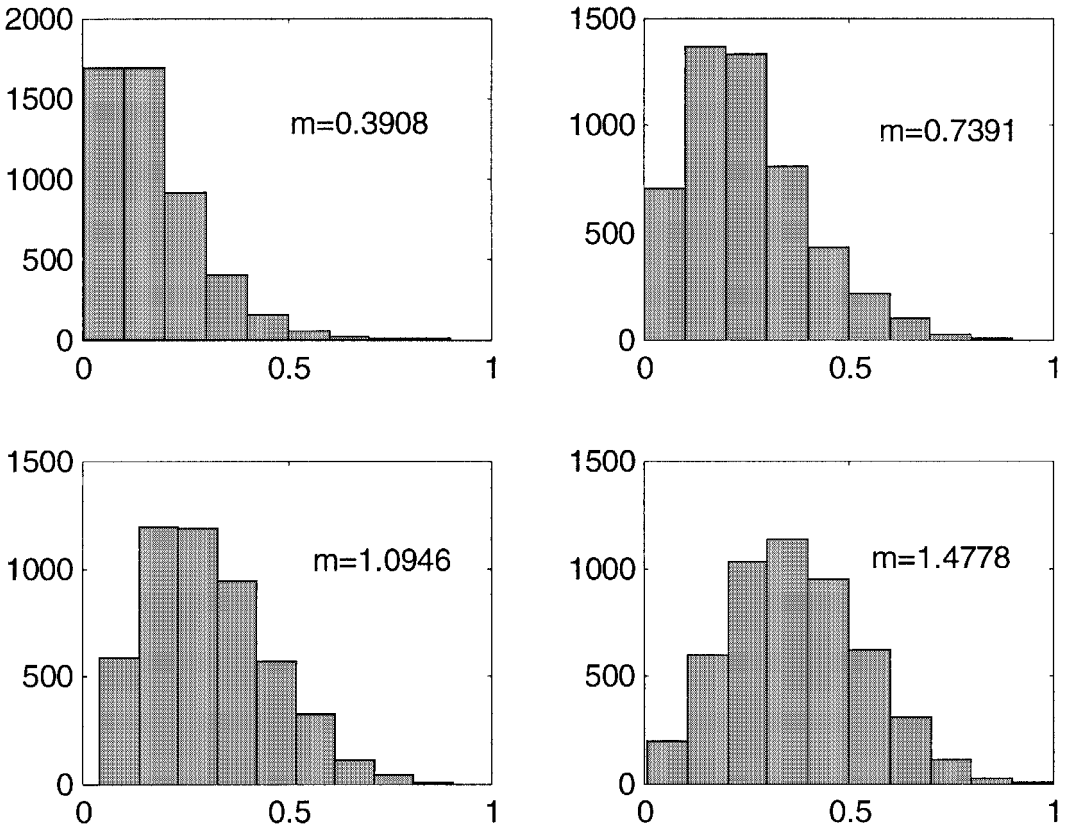

Fig. 3. Plots of the histograms of the envelope obtained from one set each of the simulations described in Table I. The m values are indicated.

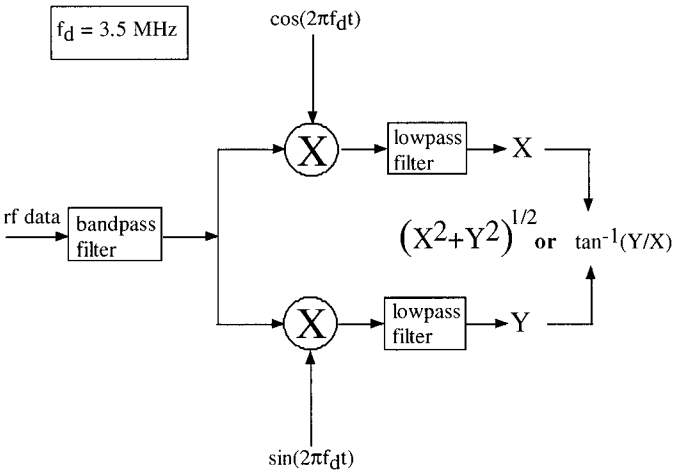

Fig. 4. Block diagram of the demodulator. The envelope $\sqrt{X^2+Y^2}$ and phase arctan $\left(\frac{Y}{X}\right)$ can be obtained from the rf data using the demodulator set-up shown.

a regular spacing were also considered. Thus, the summation in (20) contains contributions from both randomly located scatterers as well as regularly spaced scatterers. The spacing of the regular structure was considered to be a Gaussian-distributed random variable with a mean spacing of λ, $\lambda/2$, and $\lambda/4$ and standard deviation of about 0.01%. The signal-to-noise ratio of the scattering amplitudes, $\mathrm{SNR_a}$,

$$SNR_a = \frac{\langle a \rangle}{\sqrt{\langle a^2 \rangle - \langle a \rangle^2}}, \tag{21}$$

was varied from a low value of 0.5 to a high value 20 with the scattering amplitudes treated as Gamma-distributed random variables.

The backscattered echo was demodulated using the set-up shown in Fig. 4. The envelope and the phase were calculated as shown. In each simulation, the rf backscattered signal was sampled at 20 MHz, corresponding to a sampling window size of 3.7 cm. The resulting backscattered signal amplitude or envelope was used for statistical processing to compute the parameters of the Nakagami distribution. Of the 1024 samples for each A-line, every other sample was used for the computation of the Nakagami parameter. This was done to ensure that the samples were uncorrelated. Each set of simulations was repeated seven times, and the average and standard deviation of the parameters were calculated. The simulations and processing were done using MATLAB on a PC.

Results of these simulations are shown in Fig. 5. As suggested in the previous section, if the signal-to-noise ratio of the amplitudes is low, the Nakagami parameter m is less than 0.5 in the absence of any periodicity [Fig. 2(c)]. These m values correspond to the very low values of the effective number, M, of the K distribution [3], which materialize when the amplitudes have a high degree of randomness indicated by very low values of SNR_a [3]. However, as the SNR_a increases, the m value goes up and approaches the limiting value of 1 for the Rayleigh distribution. The presence of regularly spaced scatterers on the order of wavelength does not change the characteristics of the m parameter. However, the presence of half wavelength spacing of the regularly spaced scatterers pushes the value of m above 1 as soon as the SNR_a is higher than 1.5, possibly indicating that the envelope is Rician distributed. This aspect will be explored in the next paragraph. As suggested in the previous section, quarter wavelength spacing of the periodic scatterers seems to lower the value of m. The periodicity or regularity of scatterers does not seem to produce m values larger than 2 except in one or two cases. This aspect is again explored toward the end of this section (22). Hence, m can be used as a parameter to separate the cases of presence and absence of regularly spaced scatterers when $m > 1$.

It may seem difficult to differentiate between the presence of the periodic scatterers and the absence of them when the value of m is less than 1. To explore this, a chi-square test [12], [13] was conducted to check whether the phase of the backscattered echo matches with the case of the phase being uniformly distributed in the range $\{0,\ 2\pi\}$. The phase values were obtained from the inphase and quadrature components after demodulation (Fig. 4). These results are shown in Fig. 6. A bin size of 20 was used for the test. It is clear that uniformity of the phase is accepted when there are no regular scatterers present. Uniformity of the phase is also accepted even for quarter wavelength periodicity for a high signal-to-noise ratio and high number densities. This is to be expected because the presence of quarter wavelength spacing will only introduce additional variation in the phase, and a half wave periodicity will make the phase highly nonuniform by tilting the average phase to 2π as discussed in the previous section. Note that the presence of periodic spacing affects the phase characteristics to a significant extent when the number density is small. The departure from the uniformity of the phase is less pronounced even for half wavelength periodicity case for high number density of scatterers. This clearly demonstrates that even when $m < 1$, the presence of regularly spaced scatterers $(\lambda/4, \lambda/8, \ldots)$ can be concluded by performing a chi-square test for uniform distribution.

Chi-square tests were also conducted to test the hypothesis that envelope was Nakagami distributed. Once again, the tests were conducted on seven sets of data, and the average values of chi-square test statistics are shown in Fig. 7. The Nakagami hypothesis seems acceptable within the limits of error except for the case of 5 scatterers at $\lambda/2$ spacing. The reasons for this are as follows. It has been suggested that the Nakagami distribution at higher values of m does not represent accurately the Rician at the tails [14]. The other reason is that the bin size has been fixed, and this will have a serious impact on the Chi-square test applied to heavy tailed distributions [14], [15]. The chi-square test was also conducted (not shown here) to test the hypothesis that the envelope is Rayleigh. The chi-square test statistic values for the Rayleigh case were in thousands, except for the case of high number densities with no regular spacing.

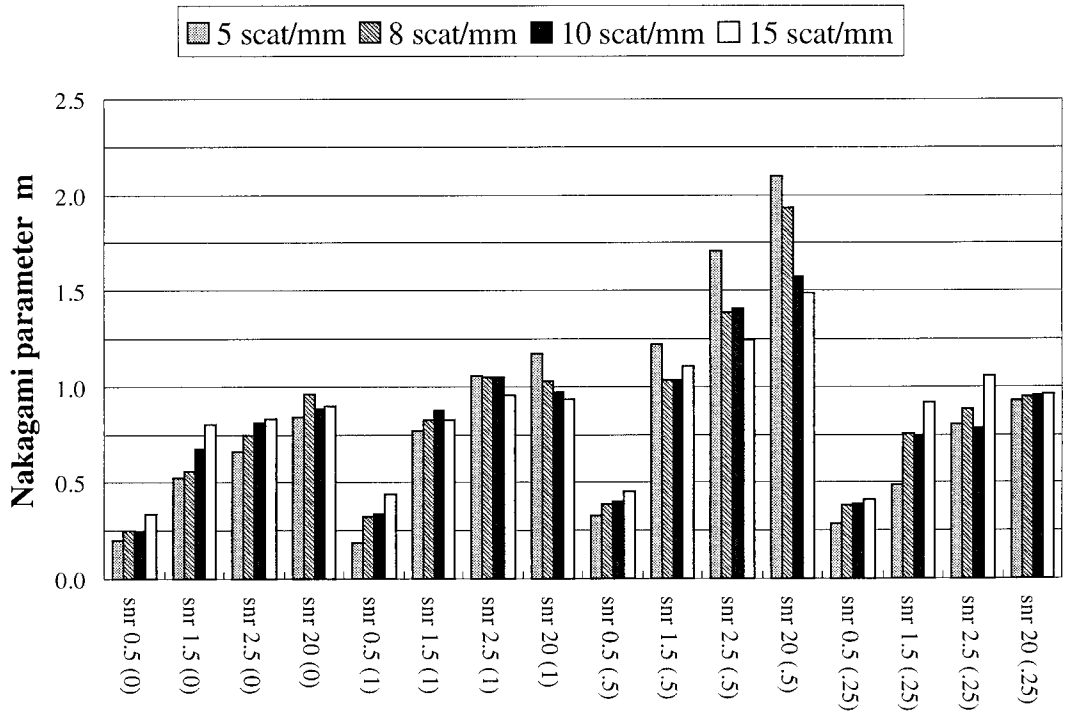

Fig. 5. Chart showing the values of m obtained through simulation. The signal-to-noise ratio of the amplitudes (SNR_a) is indicated by snr. The numbers within the brackets along the X-axis (0, 1, 0.5, and 0.25) refer to the spacing of the regularly spaced scatterers that are present along with the randomly located scatterers. The value of zero corresponds to the absence of any regularly spaced scatterers, and the other values refer to λ-, $\lambda/2$-, and $\lambda/4$-spaced scatterers along with the randomly located scatterers.

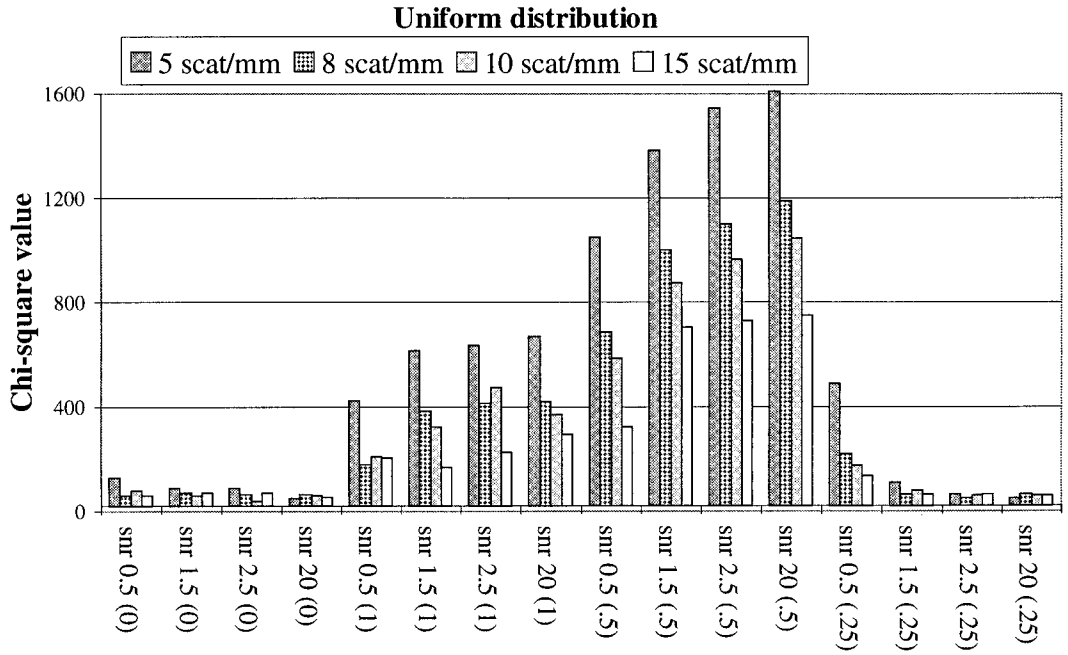

Fig. 6. Chart showing the results of the Chi-square test for phase match with uniform distribution. The X-axis values are the same as those in Fig. 5.

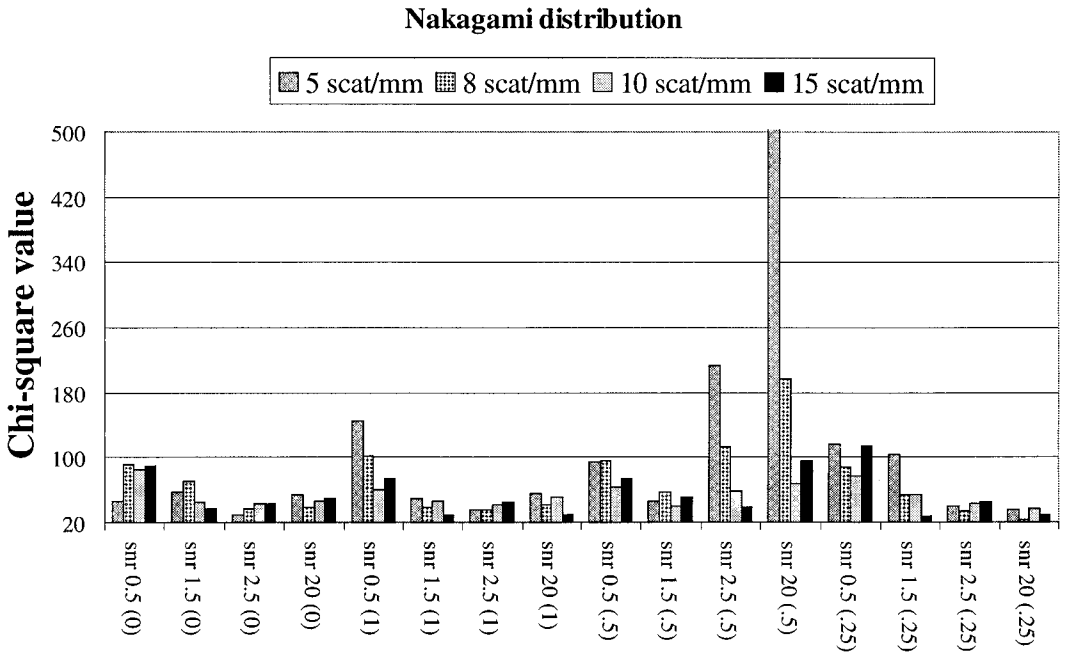

Fig. 7. Chart showing the results of the Chi-square test for the Nakagami probability density function. The X-axis values are the same as those in Fig. 5.

It is also of interest to note here that the typical values of the ratio of the steady component to the diffuse component in tissues is less than 1 [5], and, in radar, it may be much higher than 1. Therefore, we can calculate theoretically what values of m are to be expected in medical ultrasound by comparing the Nakagami distribution with Rician distribution.

Once again, noting that $\mathrm{R}_0 = \mathrm{s}$ and $\Omega = E(R^2) = 2\sigma^2 + R_0^2$, m can be expressed in terms of $k = \frac{s}{\sigma}$ [5]. From (13), we get

$$\frac{I}{m} = 1 - \frac{1}{\left(\frac{2}{k^2} + 1\right)^2}. \tag{22}$$

For the case of completely random locations of scatterers, $k = 0$ and m given in (24) becomes 1. In tissues, the typical values of k [5] are less than 1 and taking $k = 1$, m becomes 9/8, which is much smaller than 2. Thus, we see that for realistic cases of the presence of regularly spaced scatterers, m is less than 1.5, and the Nakagami pdf is acceptable based on the Chi-square test for values of m less than 1.5. Thus, for the values of m of interest in medical ultrasound, the Nakagami distribution is acceptable on the basis of the Chi-square test.

IV. Experimental Results on Phantoms

The efficacy of the Nakagami distribution to model the backscattered echo from tissues was explored using a tissue-mimicking phantom. The ATS 532 is a commercially available phantom (ATS Laboratories Inc., Bridgeport, CT) in which seven cones of progressively higher scatterer concentrations are embedded in a uniform background. Targets 1 through 7 had scatterer densities of 2, 4, 8, 16, 64, 128, and 256 scatterers/mm^3, respectively, embedded in a background of 32 scatterers/mm^3. The phantom was scanned using an Advanced Technology Laboratories (ATL, Bothell, WA) Ultramark 9 ultrasound unit. Targets 1 through 7 along with the background of the ATS phantom were scanned with transmit focus set to correspond approximately with the center of the target. The Ultramark 9 has dynamic receive focus. The scan plane of the data was held at a right angle to the axis of the cones.

The data were acquired at Thomas Jefferson University Hospital and analyzed in laboratories at Drexel University. The rf data collected were transferred to Drexel University for processing. The processing was similar to that described in the previous section except for the demodulation frequency of 6 MHz. The image of one of the targets (# 1) is shown in Fig. 8. Note that no smoothing algorithm was applied, and the image was a two-dimensional display of the envelope. All of the target images were similar in these features. Once again, all of the processing was done using MATLAB on a PC. The statistics toolbox and the signal processing toolbox were used throughout to obtain the results described in this paper.

The number of samples chosen for the study of m was 2000 of which every other sample was used for the computation of m. This was done to avoid the correlated nature of samples caused by the oversampling done during the original data collection. Each sample set comprised 20 scan lines and 100 rf samples in each scan line. The values of m and other parameters that were calculated were averaged over the sample sets. This meant that the averaging was done over at least six sets for the information at the target sites, and averaging over 20 sets was performed at the background (32/mm^3).

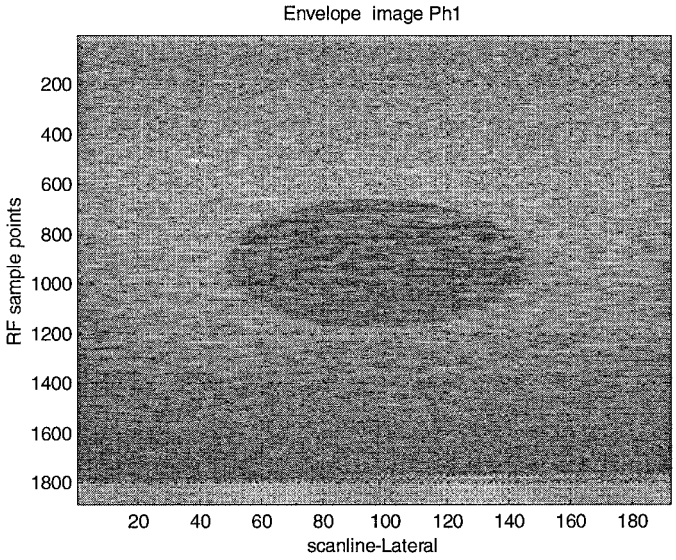

Fig. 8. Envelope image of one of the targets.

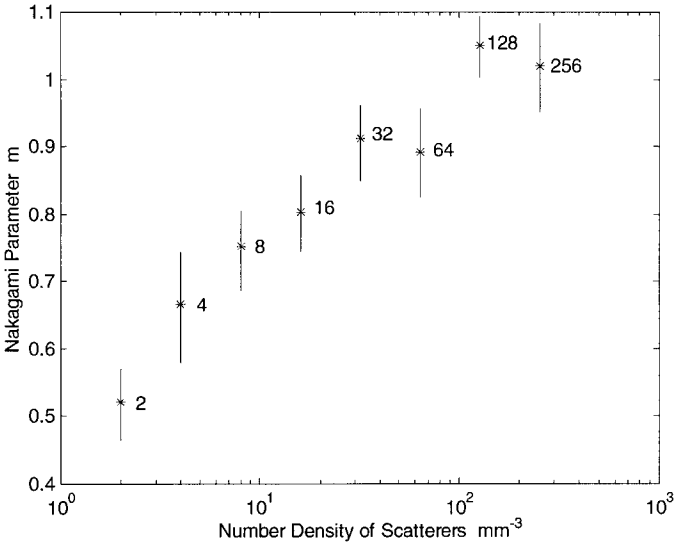

Fig. 9. The Nakagami parameter for the targets of the ATS phantom. The number densities are 2, 4, 8, 32(background), 64, 128, and 256 scatterers/mm^3.

The Nakagami parameter m is plotted in Fig. 9 for the seven targets plus the background. The parameter appears to have the ability to distinguish between the different scatterer number densities corresponding to the various targets. As the number density of the target goes beyond 32 scatterers/mm^3, the ability to separate the different scatterer number densities becomes poor. This is due to the fact that we are approaching the Rayleigh regime, and the m values will be approaching unity.

The trend in terms of increase in m as the number density goes up seems obvious. If we compare these results with those based on K distribution, the parameter M of K distribution appears to lose its capability [4]–[6] as the number densities reach the Rayleigh regime. The Nakagami parameter appears to retain the ability to distinguish these different scattering scenarios for number densities of up to 32 000/cm^3.

Comparing the parameters of the K distribution and the Nakagami distribution, we see that the parameter m can be thought of as a clustering parameter with a limited dynamic range. Although the value of the effective number M of the K distribution [3] can go from 0 to ∞, m only goes up to 1.5 or less in ultrasound. (In radar, where the steady component can exceed the strength of the diffuse component by many orders of magnitude (k of [5] $\gg 1$), m can be much higher, ~ 5 to 10.) In the absence of any periodic scatterers, m goes up as the number density increases and becomes equal to 1. Because, in ultrasonic tissue characterization, the number densities of scatterers are low (800/cm^3), the m parameter retains its physical meaning at these densities. The presence of any regularly spaced scatterers can be concluded using a phase analysis, which is possible because the demodulation provides the means to compute the phase along with the envelope.

V. Conclusions

The model based on the Nakagami distribution has been explored as a viable means to describe the backscattered ultrasonic echo from tissues. Even though the conventional Nakagami distribution is applicable only for values of the parameter m ≥ 0.5, in the ultrasound context, it is possible to have values of m less than 0.5. The m parameter seems to be able to separate different scattering conditions, and, combined with the phase analysis, the Nakagami distribution seems capable of modeling almost all of the scattering conditions. The parameters, both m and Ω (which was not used in these studies; it is simply a measure of the average power) are relatively easy to calculate. Therefore, the model is analytically simple and straightforward compared with the K distribution. The fact that Nakagami distribution can even incorporate Rician and generalized Rician distribution as special cases makes it a very attractive model in ultrasonic imaging.

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