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Super-Resolution Reconstruction of Deformable Tissue from Temporal Sequence of Ultrasound Images

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***Abstract*—In this paper, the feasibility of super-resolution reconstruction of the deformable tissue from temporal sequences of ultrasound images is extensively studied. The proposed image observation model integrates the non-rigid motion and imaging formation process into a unified super-resolution frame. To facilitate the motion estimation, Lucas-Kanade optical flow is chosen for non-rigid motion estimation from mathematical point of view. Finally, optical flow based iterative back-projection algorithm is proposed for Super-resolution reconstruction, where flow driven diffusion method is developed to re-estimate the motion in the high-resolution coordinate, yielding accurate dense flow field. The efficiency of the proposed approach is demonstrated by simulated images and ultrasound carotid vessel images. Experimental results are provided and compared with existing super-resolution method, which indicate the proposed approach is highly efficient and promising.**

Keywords- super-resolution; optical flow; non-rigid motion estimation; temporal sequence; ultrasound image; carotid vessel

I. INTRODUCTION

Ultrasound image has low resolution and poor imaging quality due to the speckle noise produced by the interference of backscattered signals. Moreover, when the object is moving such as vessel beating, the image quality is worse. Therefore, it is of special importance to improve the resolution of ultrasound images. Inspired by the idea of Super-resolution (SR) image reconstruction, the feasibility of using SR method to improve the resolution for deformable tissue from temporal sequence of Ultrasound images is studied.

Super-resolution (SR) image reconstruction is a promising approach to overcome the inherent resolution limitations of the imaging system, since each low-resolution observation with different subpixel shifts from each other potentially contains novel information about the desired high-resolution image. It has a wide range of applications in medical imaging. It can greatly improve the image quality and also can zooming in the region of interest with high resolution, which is of particular value in diagnosis and therapy in clinic [1].

The goal of SR reconstruction is to infer the high-resolution image from a set of low-resolution images. Tsai and Huang were the first to address the problem of reconstructing a high resolution image from a sequence of low resolution undersampled images in frequency domain [2]. The extension of this approach for a blurred and noisy image was proposed by Kim et al [3]. Afterwards, other approaches are presented. See reference [4] and the reference therein. Iterative back-projection (IBP) SR algorithm is one of simple but powerful method [5]. It is similar to the back projection used in computer aided tomography.

However, most existing SR reconstruction approaches can only deal with rigid motion object reconstruction. Only few SR algorithms are proposed for non-rigid deformable object. In [6], super optical flow SR method is proposed for human face reconstruction. It did not consider any imaging formation model, thus it is only suitable for images with uniform background. Followed by their work, the optical flow based SR approach from probabilistic point of view is proposed, where non-rigid motion optical flow estimation and SR reconstruction is solved by EM algorithm [7]. But this approach involves heavy computational cost.

In this paper, a new approach is proposed to SR reconstruction of deformable tissue from temporal sequence of ultrasound images. The approach is different from existing method in that: first, the proposed image observation model integrates non-rigid motion equation, optical flow and imaging formation process into a unified super-resolution frame. Furthermore, optical flow based IBP algorithm is proposed for SR reconstruction that is efficient in accuracy and has fast convergence rate.

The organization of the paper is as follows: In section 2, the image observation model is proposed. In section 3, Lucas-Kanade optical flow is chosen for non-rigid motion estimation from mathematical point of view. In section 4, optical flow based IBP algorithm is developed for SR reconstruction; In section 5, experimental results are provided. Afterwards in section 6, we conclude with a summary and discussions.

II. IMAGE OBSERVATION MODEL

The proposed observation model supposes that the observation image can be thought of as the downsampling of the blurred version of the warping of ideal image.

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Let $f(x,y)$ denote the ideal image and g_k^H the non-rigid deformation of the ideal image. The motion field from f to g_k^H is presented as $(u_k(x,y), v_k(x,y))$. It can be described by a non-rigid deformation operator

$$T_{OPF,k} : f \to g_k^H$$

$$g_k^H(x,y) = f(x+u_k(x,y), y+v_k(x,y)) \qquad (1\text{-}1)$$

and the inverse operator is defined as

$$T_{OPF,k}^{-1} : g_k^H \to f$$

$$f(x,y) = g_k^H(x-u_k(x,y), y-v_k(x,y)) \qquad (1\text{-}2)$$

Let $g_k(m,n)$ denote the k-th low-resolution observation image. And T_k denote the mapping from the position on (x,y) in ideal image to the position on (m,n) in image g_k. The mapping is given by $T_k : (x,y) \to (m,n)$

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} (x+u_k(x,y))/s \\ (y+v_k(x,y))/s \end{pmatrix} \qquad (2\text{-}1)$$

and the inverse mapping is given by $T_k^{-1} : (m,n) \to (x,y)$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} sm - u_k(sm,sn) \\ sn - v_k(sm,sn) \end{pmatrix} \qquad (2\text{-}2)$$

where s is the scale factor. The typical value of $s = 2$.

Let h be a blur operator, and η_k be the noise. Let D_k be a downsampling quantity operator by a factor s. The k-th observation image g_k is obtained by downsampling of the blurred version of g_k^H. Thus we have

$$g_k(m,n) = D_k[h*(g_k^H(x,y)) + \eta_k] \qquad (3)$$

As $g_k^H(x,y) = T_{OPF,k}(f(x,y))$, then the observation model relating the high resolution image to the low resolution observed frames is given by

$$g_k(m,n) = D_k[h*T_{OPF,k}(f(x,y)) + \eta_k] \qquad (4)$$

It states that the observation image sequence is the downsampling of the blured version of a set of deformed versions of the ideal image.

SR image reconstruction is an inverse problem to solve the ideal image $f(x,y)$ from its observations in (4). It consists of two basic components as registration and interpolation. Registration is used to estimate the relative motion from all available low-resolution frames to a common reference frame. The second component refers to mapping the motion-compensated pixels onto a super-resolution grid.

III. LUCAS-KANADA OPTICAL FLOW FOR NON-RIGID MOTION ESTIMATION

An essential part of any SR reconstruction is registration or motion estimation, requiring the relative displacement in image sequences accurate and dense enough. Optical flow equation is widely used for motion estimation [8]. Let $E(x,y,t): R^2 \times R \to R$ denote the time-varying image brightness at point (x,y) on time t, the first order optical flow equation is derived by assuming that the brightness of a particular point in the pattern is constant. That is

$$\nabla E \cdot \vec{v} + E_t = 0 \qquad (5)$$

where $\vec{v} = (\partial x/\partial t, \partial y/\partial t) \stackrel{\Delta}{=} (v_1, v_2)$ is the velocity vector describing the motion field at the point (x,y).

As for non-rigid deformation, the motion equation should also be taken into account. In [9], non-rigid motion estimation is formulated as an optimization problem by solving the motion parameters. But it can not guarantee to be global optimal solution. In [10], the second order differential optical flow is derived for non-rigid motion estimation. That is

$$\begin{cases} E_{xx}v_1 + E_{xy}v_2 = -E_{xt} \\ E_{xy}v_1 + E_{yy}v_2 = -E_{yt} \end{cases} \qquad (6)$$

It states that for non-rigid deformation in a infinitesimal element, the gradient of the element brightness is stationary over time.

A weighted over-determined system of equations is formed by incorporating the first order optical flow equation (5) and the second order optical flow equation (6) as follows,

$$\begin{pmatrix} E_x & E_y \\ E_{xx} & E_{xy} \\ E_{xy} & E_{yy} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -\begin{pmatrix} E_t \\ E_{xt} \\ E_{yt} \end{pmatrix} \qquad (7)$$

Rewirte it as $A \quad \mathrm{v} = \mathrm{b}$, where

$$A = \begin{pmatrix} E_x & E_y \\ E_{xx} & E_{xy} \\ E_{xy} & E_{yy} \end{pmatrix}, \; b = -\begin{pmatrix} E_t \\ E_{xt} \\ E_{yt} \end{pmatrix}$$

Let λ_0, λ_1 be the weight for the first order equation and the second order equation respectively. Let $W = diag(\lambda_0, \lambda_1, \lambda_1)$, Premultiply $A^T W$ in both side of (7), and let $\lambda = \lambda_0/\lambda_1$, we can obtain

$$\begin{pmatrix} \lambda E_x^2 + E_{xx}^2 + E_{xy}^2 & \lambda E_x E_y + E_{xx}E_{xy} + E_{yy}E_{xy} \\ \lambda E_x E_y + E_{xx}E_{xy} + E_{yy}E_{xy} & \lambda E_y^2 + E_{xy}^2 + E_{yy}^2 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$
$$= -\begin{pmatrix} \lambda E_x E_t + E_{xx}E_{xt} + E_{yt}E_{xy} \\ \lambda E_y E_t + E_{xy}E_{xt} + E_{yy}E_{yt} \end{pmatrix} \qquad (8)$$

The motion field can be solved from (8). However, the numerical differentiation of spatial and temporal derivatives of the image brightness is an ill-posed problem unless regularization is performed. And also because of the sensitivity of numerical derivative, the second order derivatives cannot be accurate enough.

L-K optical flow is a special case of the second order method [11]. It solves the weighted least squares approach to the first order optical flow estimation. By assuming that the velocity is constant over a finite region, yields the following over-determined system:

$$\begin{pmatrix} E_{x1} & E_{y1} \\ E_{x2} & E_{y2} \\ \dots & \dots \\ E_{xN} & E_{yN} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -\begin{pmatrix} E_{t1} \\ E_{t2} \\ \dots \\ E_{tN} \end{pmatrix} \qquad (9)$$

Where $\{(xi,yi), i = 1,2,\dots,N\}$ is the pixel set falling in a window centered on the current pixel (x_0, y_0).

Rewrite it as $B \quad \mathrm{v} = \mathrm{d}$, where

$$B = [\nabla E(X_1), \dots, \nabla E(X_n)]^T \;, \quad d = -(E_t(X_1), \dots E_t(X_n))^T$$

Let $W = diag[w(X_1 - X_0),...,w(X_n - X_0)]$

Where $w(X)$ denotes the rectangle window function. Premultiply $B^T W$ to both side of (9), thus we have

$$\begin{pmatrix} \sum w(x-x_0,y-y_0)E_x^2(x,y) & \sum w(x-x_0,y-y_0)E_x(x,y)E_y(x,y) \\ \sum wx-x_0,y-y_0)E_x(x,y)E_y(x,y) & \sum w(x-x_0,y-y_0)E_y^2(x,y) \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -\begin{pmatrix} \sum w(x-x_0,y-y_0)E_x(x,y)E_t(x,y) \\ \sum w(x-x_0,y-y_0)E_y(x,y)E_t(x,y) \end{pmatrix} \quad (10)$$

It only involves in the first order derivative. When $B^T WB$ is nonsingular we have the unique solution of motion field

$$V = (B^T WB)^{-1} B^T Wd \quad (11)$$

It can be proved that (10) is the special case of (8) when $\lambda = \sum w(x,y) / \sum w(x,y)x^2$ [11]. Therefore, non-rigid motion equation can be accurate estimated by the L-K algorithm thus facilitate the motion estimation greatly.

IV. Optical flow based IBP algorithm for super resolution reconstruction

IBP algorithm is similar to the back projection used in computer aided tomography. The SR image is estimated by minimizing the mean squared error between the observed and simulated low-resolution (LR) images iteratively. The advantage of this approach is that it is numerically similar to common iterative methods for solving sets of linear equations and therefore has rapid convergence.

It is originally proposed for SR reconstruction with rigid motion, where the motion parameter is invariant to scales. However for non-rigid deformation, the motion in different scales cannot be described by the same parameters. Therefore, the motion in the high-resolution coordinate must be reestimated from the low resolution ones. To solve this problem, a flow driven diffusion method is developed.

Let v^L be the optical flow in the low-resolution coordinate, which is the motion field between the observation image g_k and the common reference image. And let v_0 be the optical flow in the higher resolution coordinate by bilinear interpolation of v^L, and v the corresponding accurate optical flow in the high resolution coordinate describing the relative motion between the high-resolution image g_k^H and the ideal image f. Based on the assumption of continuity of tissue deformation and inspired by the diffusion theory [13], the flow driven diffusion method is proposed. The optical flow v is estimated by minimizing the following variational function:

$$E(v) = \int_\Omega (|v - v_0|^2 + \alpha\psi(\sum_{i=1}^{2} |\nabla v_i|^2)) dxdy \quad (12)$$

where α is a weight for smoothing item, $\psi(x^2)$ is a penalizing function that is differentiable in its arguments and convex in x.

From variational method, the flow driven diffusion equation is derived as follows:

$$\begin{cases} \dfrac{\partial v_i}{\partial t} = \alpha div(g(\sum_{j=1}^{2} |\nabla v_j|^2) \cdot \nabla v_i) & on \quad \Omega \\ \partial_n v_i| = 0; & on \quad \partial\Omega \\ v(\cdot,0) = v_0(\cdot) & on \quad \Omega \end{cases} \quad (13)$$

where Ω denots a rectangular image domain, ∂_n is the derivative in normal direction, and $g(x^2) = \psi'(x^2)$. $g(\cdot)$ is the diffusivity which is a decreasing function in its argument. The diffusivity g is chosen in this way such that $g(0) \to 1$ and $g(\infty) \to 0$, leading to edge-preserving diffusion. The numerical implementation of flow driven diffusion equation use the AOS scheme[13], which is highly efficient.

Once the relative motion in the high coordinate is estimated, SR reconstruction can be performed iteratively by minimizing the mean squared error between the observed and simulated low-resolution (LR) images as formulated by (4).

The optical flow based IBP algorithm for SR reconstruction is proposed as follows.

Step 1 registration

From all available low resolution images, choose the image in the middle frame as the common reference image.

For each image in the sequence in turn

1.2.1 estimate the relative motion between this image with the reference images by L-K optical flow method;

1.2.2 re-estimate the optical flow in the high-resolution coordinate according to the flow driven diffusion approach proposed in this section.

Step 2 reconstruction

While (not convergence), iterate the following two steps

2.1 simulate the image forming process according to the following equation

$$g_k^{(t)}(m,n) = D_k[h^{PSF} * (T_{OPF,k}(f^{(t)}(x,y)))] \quad (14)$$

2.2 update the high-resolution image by the following equation

$$f^{(t+1)}(x,y) = f^{(t)}(x,y) + \frac{1}{K}\sum_{k=1}^{K} T_{OPF,k}^{-1}[U_k(g_k(m,n) - g_k^{(t)}(m,n)) * h^{BP}] \quad (15)$$

End

where $f^{(t)}$ is the t-th approximation of ideal super-resolution image f. $g_k^{(t)}$ is the k-th simulated low resolution image created by $f^{(t)}$ according to image formation model as described in eq.(14). h^{BP} is a back projection kernel function. U_k denotes any upsampling operator by a factor s, which can be chosen as bilinear interpolation operator. And K is the number of low-resolution images in a sequence.

V. Experiments

To evaluate the presented approach, experiments on synthetic images and ultrasound carotid vessel images are performed. First, the efficiency of the proposed flow driven diffusion method for reestimating the optical flow in the

TABLE I
MEASUREMENT OF THE ESTIMATED OPTICAL FLOWS

Method	MSE	AAE
Flow driven diffusion	0.4123	26.0251°
the approach in [15]	0.6055	28.1506°

high-resolution coordinate is evaluated. And the results are compared with that from the bilinear interpolated images. Second, the proposed optical flow based IBP SR algorithm is demonstrated and compared with the super optical flow SR approach in [6] where SR image is estimated using a robust mean of the registered interpolated images and with the classical bilinear interpolation methods.

Fig 1(a) and (b) show the ideal synthetic image of an ellipsoid undergoing non-rigid deformation. In the experiments, the optical flow is computed by the L-K algorithm. Fig 1 (c) show the optical flow between the two ideal images that can be considered as the ground truth optical flow. Then the ultrasound imaging process is simulated by adding speckle noise and then downsampling to obtain the low-resolution observation images. The corresponding low-resolution observation images are shown in fig2 (a) and (b). Fig2(c) shows the optical flow between the two low-resolution observation images. Fig3 (a) shows the optical flow in the high resolution coordinate estimated by the proposed flow driven diffusion approach. It shows that the coherent deformation behavior agrees with the deformation continuity in biological tissue. Fig 3(b) shows the optical flow in the high resolution coordinate estimated from the bilinear interpolation images, which show disorder and discontinuity in some regions.

To quantitatively evaluate the errors between the estimated flow and the ground truth flow field, three different measures are used, which are mean square error (MSE), the average angular error (AAE), and the improved signal noise ratio (ISNR) [14].

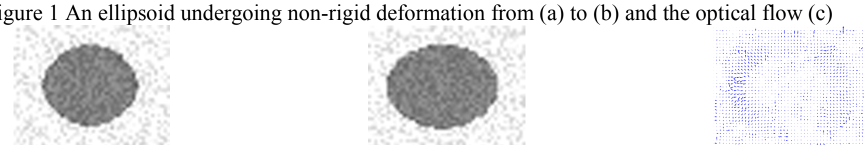
Let v denote the ground truth optical flow, and $\hat{v}$ the estimated optical flow, then we define

$$MSE = \|v - \hat{v}\|_2^2 / \|v\|_2^2$$
$$\mathrm{AAE} = \frac{1}{N}\sum_{i,j} \arccos((v_{ij} \cdot \hat{v}_{ij}) / (\|v_{ij}\| \cdot \|\hat{v}_{ij}\|)) \quad (16)$$

The smaller error between the estimated optical flow and the ground truth flow is, the more accurate of the estimation is. Table 1 show the MSE and AAE measures. From it we can see that the estimated optical flow by our approach is closer to the ground truth flow as compared with that from the bilinear interpolated images. In addition, the improved SNR is used to measure the improvement by our approach, which is defined by $ISNR = 10\lg\|v - v_2\|^2 / \|v - v_1\|^2$, where V_1 denote the estimated optical flow by our approach and V_2 the estimated flow from bilinear interpolated images. We compute the ISNR=1.6691db. It also indicates that the proposed flow driven diffusion method for reestimating the optical flow in the high-resolution coordinate is more accurate and reliable.

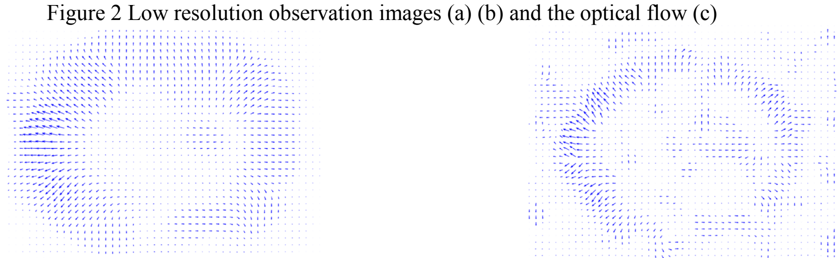
(a) ideal image 1 (b) ideal image 2 (c) the optical flow between (a) and (b)

Figure 1 An ellipsoid undergoing non-rigid deformation from (a) to (b) and the optical flow (c)



(a) observation image 1 (b) observation image 2 (c) the optical flow between (a) and (b)

Figure 2 Low resolution observation images (a) (b) and the optical flow (c)



(a) by the proposed flow driven diffusion method (b) from the bilinear interpolated images

Figure 3 Estimated optical flow in the high-resolution coordinate

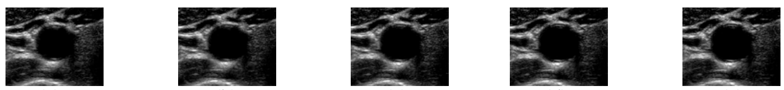
Second, optical flow based IBP SR algorithm is performed on temporal sequence of human carotid ultrasound images. The carotid undergoes non-rigid deformation. The temporal cross-sectional image sequence of carotid vessel in one cardiac cycle is obtained from Philip iE33 ultrasound equipment by free-hand scanning at a given position. The frame acquisition rate is 30 fps. In the experiment, the blur operator h is chosen to be Gaussian function and the back projection kernel h^{BP} is chosen to be $h^{BP} = h$.

Fig 4 shows five carotid vessel images in a temporal sequence. The third frame is considered as the reference image. Fig5 (a) shows the SR image by the proposed optical flow based IBP algorithm, fig 5(b) is by the super-resolution optical flow SR method proposed in [6], and fig 5(c) is by the bilinear interpolation method.

The carotid vessel has three layers from inside to outside. However, due to the limitation of ultrasound imaging system and motion artifact, the ultrasound vessel wall image always show discontinuity and inhomogeneity. From fig5 we can see that fig5 (a) has the best visual quality for its smooth and coherent structure in vessel wall, whereas fig5(c) and fig5 (d) show heterogeneity.

To further see clearly the layer structures, a region of interest is selected as shown in fig6 (a). Corresponding to fig5 (a)-(c), the magnification of the region of interest in the SR images are shown in fig6 (b)-(d). In fig6 (b), we could clearly see the tunica intima, tunica media, and tunica adventitia structures as indicated by the arrow A, arrow B and arrow C respectively, which is consistent with the true vessel structures. However in fig6(c) and fig6(d), three layer structures can not be clearly identified. Experiment indicates that SR image by our approach has been improved obviously than the SR method proposed in [6] and bilinear interpolation methods, as our method considers the image formation process into SR reconstruction.

Furthermore, to test the robustness of the proposed approach, it is also applied to a lower and poorer quality ultrasound image sequences. The temporal cross-sectional image sequence of carotid vessel in systole cycle is obtained from Asisz ASU-3500 ultrasound equipment by free-hand scanning at a given position.

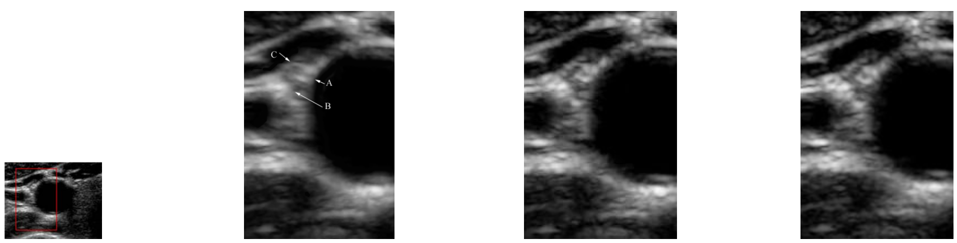


(a) (b) (c) (d) (e)

Figure 4 five carotid vessel images in a temporal sequence

(a) (b) (c)

Figure 5 SR image reconstruction from five low resolution obervation images. (a) By the proposed optical flow based IBP SR algorithm;(b) By super optical flow SR approach and (c) By bilinear interpolation method



(a) Region of interest (ROI) (b) ROI from fig.5(a) (c) ROI from fig.5(b) (d) ROI from fig.5(c)

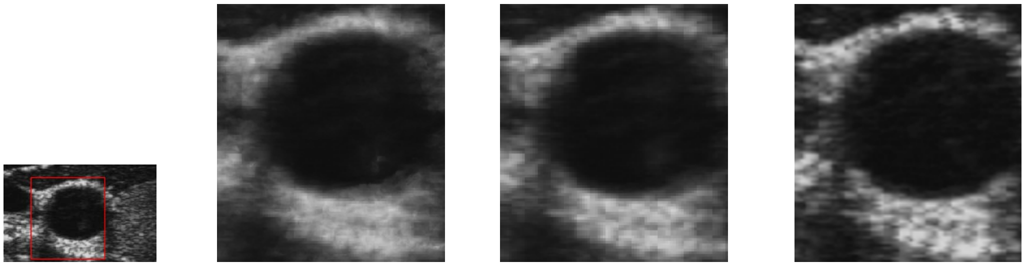
Figure 6 Magnification of region of interest corresponding to the SR images in fig. 5

(a) (b) (c) (d) (e) (f) (g)

Figure 7 Seven lower and poorer resolution carotid vessel images in a temporal sequence

(a) (b) (c)

Figure 8 SR image reconstruction from seven low resolution obervation images. (a) By the proposed optical flow based IBP SR algorithm; (b) By super optical flow SR approach and (c) By bilinear interpolation method



(a) Region of interest(ROI) (b) ROI from fig.8(a) (c) ROI from fig.8(b) (d) ROI from fig.8(c)

Figure 9 Magnification of region of interest corresponding to the SR images in fig. 8

Fig 7 shows seven carotid vessel images in a temporal sequence. Fig 8(a) shows the SR image by the proposed optical flow based IBP algorithm. Whereas fig8 (b) is by super optical flow SR method proposed in [6]. And fig8(c) is by bilinear interpolation method. Comparison indicates that fig8 (a) show best visual quality. Whereas fig 8(b) and fig 8 (c) appear mosaic blocks in some regions.

Furthermore, a region of interest is selected as shown in fig9 (a). Corresponding to fig8 (a)-(c), magnification of the region of interest in SR image is shown in fig9 (b)-(d).

Comparison of fig9 (b)-fig9 (d), we can see that fig9 (b) is most clear and smooth in vessel wall. Fig9(c) and fig9 (d) show mosaic blocks and heterogeneity. It further demonstrates that the proposed SR approach is highly efficient and could also be applied to poor quality images.

VI. CONCLUSION

In this paper, an efficient approach to SR reconstruction of carotid vessel from temporal sequences of ultrasound images is proposed. Being considered the imaging formation into the observation model, the proposed optical flow based IBP SR algorithm is highly efficient and robust. In addition, L-K optical flow is chosen for non-rigid motion estimation from mathematical point of view, thus greatly facilitate the numerical solution. The efficiency of the proposed approach is demonstrated by synthetic images and carotid ultrasound images. Quantitative evaluation show that the propose flow driven diffusion method for re-estimating the optical flow in the high-resolution coordinate is more accurate than that from bilinear interpolated images. Experiments on carotid ultrasound images show that our SR approach is more efficient and reliable as compared to other SR methods. It has potential to offer a way to attain higher spatial resolution and could be applied to SR reconstruction of deformable tissue for many other medical images.

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