Problem Set 4

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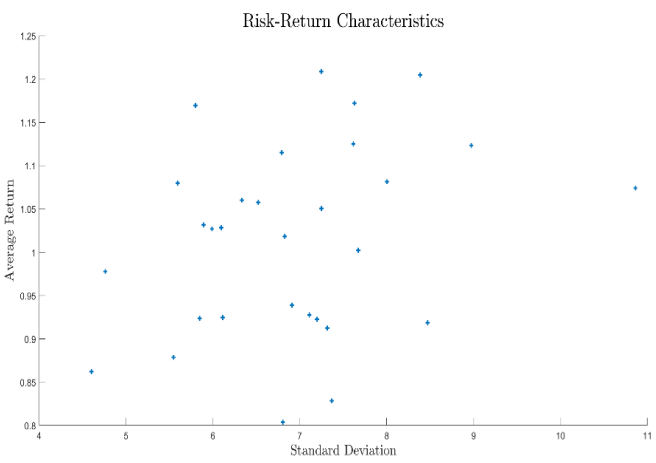
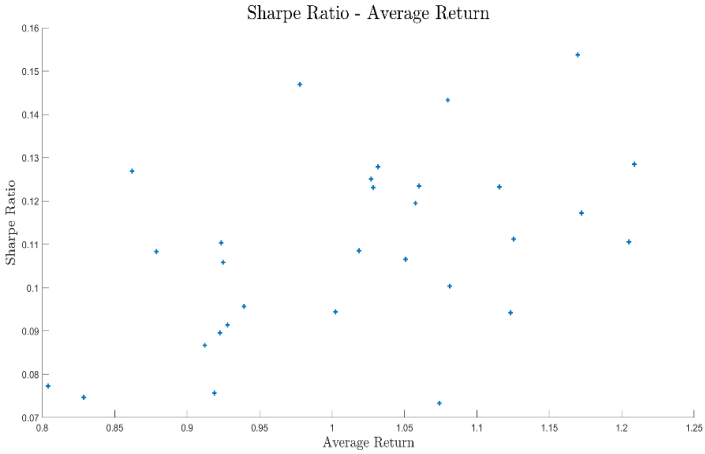
Part 1: 30 Value-Weight Industry portfolios

a) The table below summarizes the statistics of the 30 value-weighted industry portfolios.

Industry	Mean	Standard Deviation	Sharpe Ratio
Food	0.9778	4.7614	0.1470
Beer	1.2087	7.2452	0.1285
Smoke	1.1698	5.7979	0.1538
Games	1.1234	8.9731	0.0942
Books	0.9227	7.1970	0.0896
Hshld	0.9236	5.8481	0.1104
Clths	0.9248	6.1112	0.1058
Hlth	1.0799	5.5942	0.1433
Chems	1.0601	6.3346	0.1235
Txtls	1.0026	7.6750	0.0944
Cnstr	0.9392	6.9108	0.0957
Steel	0.9188	8.4729	0.0756
FabPr	1.0508	7.2501	0.1066
ElcEq	1.1723	7.6269	0.1173
Autos	1.0813	8.0051	0.1003
Carry	1.1254	7.6173	0.1112
Mines	0.9122	7.3175	0.0867
Coal	1.0742	10.8620	0.0733
Oil	1.0285	6.0951	0.1231
Util	0.8789	5.5496	0.1083
Telcm	0.8620	4.6019	0.1269
Servs	1.2051	8.3869	0.1105
BusEq	1.1155	6.7947	0.1233
Paper	1.0319	5.8925	0.1279
Trans	0.9279	7.1083	0.0914
Whlsl	0.8287	7.3691	0.0747
Rtail	1.0271	5.9903	0.1250
Meals	1.0577	6.5241	0.1195
Fin	1.0187	6.8279	0.1085
Other	0.8039	6.8061	0.0773

The scatter plots below exhibit the *standard deviation – average return* and *average return – Sharpe Ratio* pairs of the 30 industry portfolios. As illustrated, we do not observe a discernible pattern.





b) The table below summarizes the time-series regression used to perform Gibbons-Ross-Shanken (GRS) test.

Industry	Intercept	Beta	Industry	Intercept	Beta
Food	0.2182	0.7394	Carry	0.0732	1.1887
Beer	0.3171	0.9421	Mines	0.0427	0.9082
Smoke	0.4822	0.6287	Coal	-0.0505	1.2998
Games	-0.0591	1.3886	Oil	0.1850	0.8681
Books	-0.0770	1.1080	Util	0.0940	0.7783
Hshld	0.0578	0.9023	Telcm	0.1532	0.6615
Clths	0.1171	0.8132	Servs	0.3976	0.8128
Hlth	0.2553	0.8393	BusEq	0.1357	1.0775
Chems	0.1030	1.0426	Paper	0.1321	0.9545
Txtls	-0.0173	1.1389	Trans	-0.0912	1.1378
Cnstr	-0.1024	1.1724	Whlsl	-0.1587	1.0891
Steel	-0.2431	1.3570	Rtail	0.1205	0.9650
FabPr	-0.0360	1.2416	Meals	0.1638	0.9455
ElcEq	0.0579	1.2841	Fin	-0.0193	1.1667
Autos	-0.0122	1.2521	Other	-0.1677	1.0648

GRS F-statistic: 1.8895, p-value: 0.0028

Based on the test statistic and the p-value, we should reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (30 industry portfolios). That is, the portfolio used as the market proxy is not mean-variance efficient and the CAPM is rejected.

c) If we have T periods, N assets, and K factors, the test statistic of the GRS test is computed as:

$$\widehat{W} = \frac{T}{N} \times \frac{T-N-K}{T-K-1} \times \frac{\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}}{1 + \widehat{\mu_m^2} \Big/ \widehat{\sigma_m^2}}$$

The null hypothesis of the test is:

$$H_0: \widehat{W} = 0 \quad equivalently \quad H_0: \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} = 0 \quad equivalently \quad H_0: \hat{\alpha} = 0$$

where $\hat{\alpha} = [\widehat{\alpha_1}, \dots, \widehat{\alpha_N}]'$.

That is, the null hypothesis states that we cannot improve the Sharpe Ratio of the portfolio by adding some combination of the set of N risky assets to the portfolio used as the market proxy. This is equivalent to arguing that CAPM is true, since, if the market proxy is mean-variance efficient, the model would correctly price the asset, and

we would have a vector of intercepts with zeros, or, in other words, observe an alpha of zero.
The time-series regression here estimates the beta risk premium by regressing the excess return on each portfolio $r_{portfolio} - r_f$ against the excess return on the market, $r_m - r_f$.

d) The table below lists the estimated intercept, its t-stat and the p-values of the industry portfolios. The signs of the intercepts vary, with magnitudes less than 0.5. Yet, we observe that 3 of them – on Food, Smoke and Hlth – are statistically significant. In these industry portfolios, we can say that we have statistical evidence that CAPM has difficulty pricing them.

Industry	Intercept	t-stat	p-value	Industry	Intercept	t-stat	p-value
Food	**0.2182**	**2.7185**	**0.0067**	Carry	0.0732	0.5726	0.5670
Beer	0.3171	1.9954	0.0462	Mines	0.0427	0.2549	0.7988
Smoke	**0.4822**	**3.3518**	**0.0008**	Coal	-0.0505	-0.1982	0.8429
Games	-0.0591	-0.3860	0.6995	Oil	0.1850	1.5364	0.1247
Books	-0.0770	-0.6215	0.5344	Util	0.0940	0.8418	0.4001
Hshld	0.0578	0.5742	0.5659	Telcm	0.1532	1.7119	0.0872
Clths	0.1171	0.8932	0.3719	Servs	0.3976	1.8128	0.0701
Hlth	**0.2553**	**2.5144**	**0.0121**	BusEq	0.1357	1.2351	0.2171
Chems	0.1030	1.1272	0.2599	Paper	0.1321	1.4747	0.1406
Txtls	-0.0173	-0.1213	0.9034	Trans	-0.0912	-0.8171	0.4141
Cnstr	-0.1024	-1.1638	0.2448	Whlsl	-0.1587	-1.1544	0.2486
Steel	-0.2431	-1.8293	0.0676	Rtail	0.1205	1.3048	0.1922
FabPr	-0.0360	-0.4076	0.6837	Meals	0.1638	1.3020	0.1932
ElcEq	0.0579	0.5791	0.5627	Fin	-0.0193	-0.2313	0.8171
Autos	-0.0122	-0.0915	0.9271	Other	-0.1677	-1.4763	0.1402

Part 2: 10 Past Return portfolios

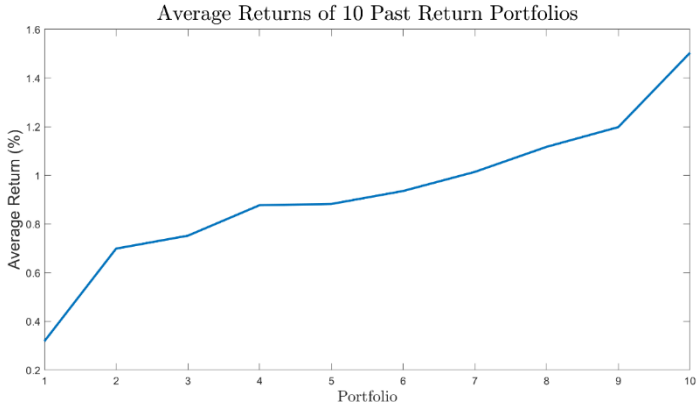
e)

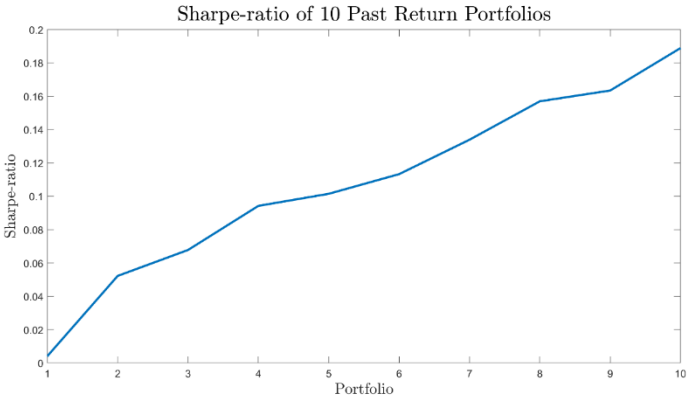
Repeat a):

The table below summarizes the statistics of the 10 past-return portfolios.

Past Return Portfolios	Mean	Standard Deviation	Sharpe Ratio
Past Loser (1)	0.3184	9.7899	0.0041
2	0.6991	8.0535	0.0523
3	0.7522	6.9883	0.0678
4	0.8774	6.3559	0.0943
5	0.8825	5.9518	0.1016
6	0.9358	5.8029	0.1133
7	1.0143	5.4951	0.1340
8	1.1172	5.3433	0.1570
9	1.1983	5.6280	0.1635
Past Winner (10)	1.5038	6.4857	0.1890

The plots below exhibit the average returns and the Sharpe Ratios of the 10 portfolios lined up from past losers to past winners. As illustrated, we observe that past winners tend to continue performing well, while past losers continue to not do well. This may be attributed to the momentum factor, or just that firms that performed well in the past are the strong and healthy firms, so they simply continue to do better than the others.





Repeat b)

The table below summarizes the time-series regression used to perform Gibbons-Ross-Shanken (GRS) test.

Past Return Portfolios	Intercept	Beta
Past Loser (1)	-0.9690	1.5585
2	-0.4394	1.3284
3	-0.2874	1.1757
4	-0.1089	1.0935
5	-0.0681	1.0383
6	-0.0100	1.0309
7	0.1082	0.9697
8	0.2341	0.9341
9	0.2956	0.9644
Past Winner (10)	0.5626	1.0238

GRS F-statistic: 6.4758, p-value: 9.3276e-10

Based on the test statistic and the p-value, we should reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (all assets divided into the 10 bins

based on their past performances). That is, the portfolio used as the market proxy is not mean-variance efficient and CAPM is rejected.

Repeat d)

Past Return Portfolios	Intercept	T-Stat	p-value
Past Loser (1)	**-0.9690**	**-6.2215**	**0.0000**
2	**-0.4394**	**-3.8410**	**0.0001**
3	**-0.2874**	**-3.1441**	**0.0017**
4	-0.1089	-1.4650	0.1432
5	-0.0681	-1.0664	0.2865
6	-0.0100	-0.1881	0.8509
7	**0.1082**	**2.0055**	**0.0452**
8	**0.2341**	**4.1686**	**0.0000**
9	**0.2956**	**4.4012**	**0.0000**
Past Winner (10)	**0.5626**	**5.3425**	**0.0000**

The signs of the intercepts tend to be negative for the past losers and positive for the past winners, with magnitudes small for the firms that performed mediocre in the past. We observe that the intercepts become more statistically significant for the group of firms in the two ends: past losers and past winners. In these portfolios, we can say that we have statistical evidence that CAPM has difficulty pricing them.

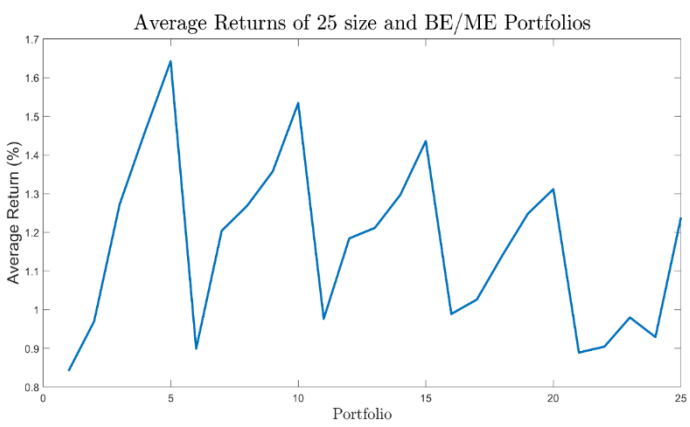
Part 3: 25 Size and BE/ME portfolios

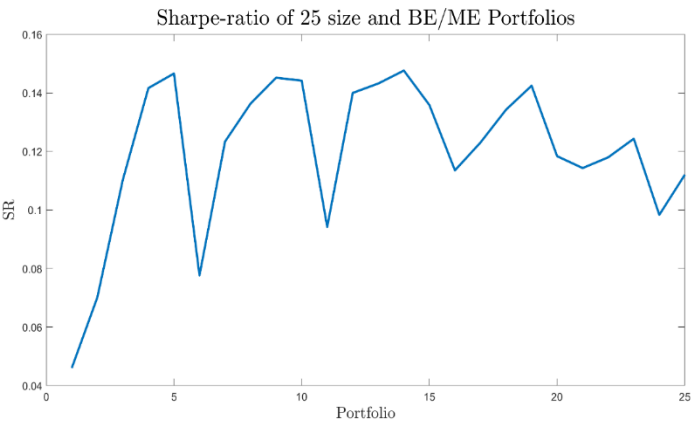
f) Repeat a):

The table below summarizes the statistics of the 25 size-BE/ME portfolios.

Size	BE/ME	Mean	Standard Deviation	Sharpe Ratio
Small (1)	Low (1)	0.8417	12.2542	0.0460
Small (1)	2	0.9699	9.8670	0.0701
Small (1)	3	1.2724	9.0129	0.1103
Small (1)	4	1.4614	8.3545	0.1416
Small (1)	High (2)	1.6424	9.3054	0.1466
2	Low (1)	0.8991	7.9958	0.0777
2	2	1.2041	7.5062	0.1234
2	3	1.2690	7.2675	0.1364
2	4	1.3575	7.4350	0.1452
2	High (2)	1.5345	8.7141	0.1442
3	Low (1)	0.9767	7.4143	0.0942
3	2	1.1845	6.4746	0.1400
3	3	1.2115	6.5183	0.1432
3	4	1.2970	6.9022	0.1476
3	High (2)	1.4362	8.5250	0.1359
4	Low (1)	0.9891	6.2624	0.1135
4	2	1.0262	6.0802	0.1231
4	3	1.1404	6.4230	0.1343
4	4	1.2483	6.8111	0.1424
4	High (2)	1.3118	8.7323	0.1184
Big (5)	Low (1)	0.8892	5.3460	0.1143
Big (5)	2	0.9045	5.3094	0.1180
Big (5)	3	0.9800	5.6455	0.1243
Big (5)	4	0.9293	6.6224	0.0983
Big (5)	High (2)	1.2383	8.5694	0.1121

The plots below exhibit the average returns and the Sharpe Ratios of the 25 portfolios lined up from Small-Low BE/ME ratio to Big-High BE/ME ratio. As illustrated, portfolios with high BE/ME ratios tend to have greater return and greater Sharpe Ratios. Moreover, smaller firms tend to exhibit higher returns, but when adjusted for their risk, they have lower Sharpe ratio. This is consistent with what Fama and French discovered in 1992.





Repeat b)

The table below summarizes the time-series regression used to perform GRS test.

Size	BE/ME	Intercept	Beta
Small (1)	Low (1)	-0.4981	1.6303
Small (1)	2	-0.2259	1.4090
Small (1)	3	0.1002	1.3729
Small (1)	4	0.3566	1.2694
Small (1)	High (2)	0.4657	1.3798
2	Low (1)	-0.2034	1.2659
2	2	0.1273	1.2264
2	3	0.2109	1.1977
2	4	0.2897	1.2125
2	High (2)	0.3585	1.3787
3	Low (1)	-0.1123	1.2451
3	2	0.1733	1.1256
3	3	0.2015	1.1238
3	4	0.2636	1.1596
3	High (2)	0.2607	1.3779
4	Low (1)	-0.0002	1.0919
4	2	0.0450	1.0797
4	3	0.1360	1.1153
4	4	0.2185	1.1541
4	High (2)	0.1086	1.4204
Big (5)	Low (1)	-0.0111	0.9553
Big (5)	2	0.0078	0.9498
Big (5)	3	0.0713	0.9684
Big (5)	4	-0.0707	1.1084
Big (5)	High (2)	0.1058	1.3119

GRS F-statistic: 3.5368, p-value: 1.3515e-08

Based on the test statistic and the p-value, we should reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (all assets divided into a 5-by-5 bins based on their size and BE/ME ratio). That is, the portfolio used as the market proxy is not mean-variance efficient and CAPM is rejected.

Repeat d)

Size	BE/ME	Intercept	t-stat	p-value
Small (1)	Low (1)	-0.4981	-1.8954	0.0583
Small (1)	2	-0.2259	-1.1644	0.2445
Small (1)	3	0.1002	0.6292	0.5294
Small (1)	**4**	**0.3566**	**2.4041**	**0.0164**
Small (1)	**High (2)**	**0.4657**	**2.6941**	**0.0072**
2	Low (1)	-0.2034	-1.5693	0.1169
2	2	0.1273	1.1477	0.2513
2	**3**	**0.2109**	**2.0249**	**0.0431**
2	**4**	**0.2897**	**2.6271**	**0.0087**
2	**High (2)**	**0.3585**	**2.5400**	**0.0112**

3	Low (1)	-0.1123	-1.1344	0.2569
3	**2**	**0.1733**	**2.4126**	**0.0160**
3	**3**	**0.2015**	**2.6446**	**0.0083**
3	**4**	**0.2636**	**2.8741**	**0.0041**
3	**High (2)**	**0.2607**	**2.0074**	**0.0450**
4	Low (1)	-0.0002	-0.0028	0.9977
4	2	0.0450	0.7854	0.4324
4	3	0.1360	1.8953	0.0583
4	**4**	**0.2185**	**2.5107**	**0.0122**
4	High (2)	0.1086	0.8326	0.4053
Big (5)	Low (1)	-0.0111	-0.2356	0.8137
Big (5)	2	0.0078	0.1705	0.8647
Big (5)	3	0.0713	1.0507	0.2936
Big (5)	4	-0.0707	-0.7950	0.4268
Big (5)	High (2)	0.1058	0.7071	0.4797

The signs of the intercepts tend to be positive for those with high BE/ME ratio. We observe that the intercepts become more statistically significant for the group of firms with high BE/ME ratio and mid-cap firms. In these portfolios, we can say that we have statistical evidence that CAPM has difficulty pricing them.

g) The table below summarizes the time-series regression used to perform GRS test.

Size	BE/ME	Intercept	Beta
Small (1)	Low (1)	-0.0075	0.2221
Small (1)	2	-0.0037	0.2705
Small (1)	3	-0.0052	0.3887
Small (1)	4	-0.0041	0.4618
Small (1)	High (2)	-0.0049	0.5325
2	Low (1)	-0.0063	0.2440
2	2	-0.0045	0.3619
2	3	-0.0037	0.3868
2	4	-0.0031	0.4210
2	High (2)	-0.0037	0.4900
3	Low (1)	-0.0056	0.2739
3	2	-0.0037	0.3539
3	3	-0.0038	0.3645
3	4	-0.0035	0.3976
3	High (2)	-0.0028	0.4514
4	Low (1)	-0.0042	0.2781
4	2	-0.0044	0.2927
4	3	-0.0035	0.3367
4	4	-0.0033	0.3786
4	High (2)	-0.0030	0.4031
Big (5)	Low (1)	-0.0036	0.2390
Big (5)	2	-0.0029	0.2447
Big (5)	3	-0.0032	0.2742
Big (5)	4	-0.0024	0.2542
Big (5)	High (2)	-0.0035	0.3748

GRS F-statistic: 1.1765e-04, p-value: 1

Based on the test statistic and the p-value, we strongly fail to reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (all assets divided into a 5-by-5 bins based on their size and BE/ME ratio).

This is not surprising since the target portfolio we used as the market proxy is the portfolio that has already obtained the maximum Sharpe Ratio using the 25 portfolios, by mathematical definition.

h) The table below summarizes the time-series regression used to perform GRS test.

Size	BE/ME	Intercept	Beta
Small (1)	Low (1)	0.1145	0.1977
Small (1)	2	0.1981	0.2173
Small (1)	3	0.2823	0.3133
Small (1)	4	0.3596	0.3625
Small (1)	High (2)	0.3666	0.4390
2	Low (1)	0.2002	0.1852
2	2	0.2749	0.2865
2	3	0.3202	0.2951
2	4	0.3629	0.3153
2	High (2)	0.3908	0.3809
3	Low (1)	0.2092	0.2154

3	2	0.3229	0.2567
3	3	0.3223	0.2689
3	4	0.3299	0.3031
3	High (2)	0.3669	0.3482
4	Low (1)	0.2539	0.2011
4	2	0.2702	0.2103
4	3	0.2699	0.2607
4	4	0.3275	0.2828
4	High (2)	0.3210	0.3136
Big (5)	Low (1)	0.1886	0.1859
Big (5)	2	0.2242	0.1770
Big (5)	3	0.2434	0.2018
Big (5)	4	0.2208	0.1894
Big (5)	High (2)	0.2725	0.3026

GRS F-statistic: 2.6370, p-value: 2.3261e-05

Based on the test statistic and the p-value, we should reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (all assets divided into a 5-by-5 bins based on their size and BE/ME ratio). That is, the portfolio used as the market proxy is not mean-variance efficient and CAPM is rejected.

This is not surprising. As we have observed in the last problem set, this out-of-sample tangency portfolio constructed is quite different from the in-sample tangency portfolio. This is because (1) the weights on the assets oscillate between two values every other month, and (2) the weights computed through Markowitz optimization is very sensitive to slight changes in the inputs. Therefore, our out-of-sample tangency portfolio is not mean-variance efficient when we applies the optimal weights of the in-sample data to the out-of-sample data.

i) The table below summarizes the time-series regression used to perform GRS test.

Size	BE/ME	Intercept	Beta
Small (1)	Low (1)	0.1145	0.1977
Small (1)	2	0.1981	0.2173
Small (1)	3	0.2823	0.3133
Small (1)	4	0.3596	0.3625
Small (1)	High (2)	0.3666	0.4390
2	Low (1)	0.2002	0.1852
2	2	0.2749	0.2865
2	3	0.3202	0.2951
2	4	0.3629	0.3153
2	High (2)	0.3908	0.3809
3	Low (1)	0.2092	0.2154
3	2	0.3229	0.2567
3	3	0.3223	0.2689
3	4	0.3299	0.3031
3	High (2)	0.3669	0.3482
4	Low (1)	0.2539	0.2011
4	2	0.2702	0.2103
4	3	0.2699	0.2607
4	4	0.3275	0.2828
4	High (2)	0.3210	0.3136

Big (5)	Low (1)	0.1886	0.1859
Big (5)	2	0.2242	0.1770
Big (5)	3	0.2434	0.2018
Big (5)	4	0.2208	0.1894
Big (5)	High (2)	0.2725	0.3026

GRS F-statistic: 3.6831, p-value: 3.8024e-09

Based on the test statistic and the p-value, we should reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (all assets divided into a 5-by-5 bins based on their size and BE/ME ratio). That is, the Markowitz-optimized 30-industry portfolio used as the market proxy is not mean-variance efficient and CAPM is rejected.

This is somewhat unexpected since we originally thought that the 30 industry portfolios well-represents the true market portfolio. It may be the case that the 30 portfolios were not sufficiently diversified, or each portfolio is not optimized with its assets inside.

j) The table below summarizes the time-series regression used to perform GRS test.

Size	BE/ME	Intercept	Beta
Small (1)	Low (1)	-0.1860	0.4383
Small (1)	2	-0.0017	0.4127
Small (1)	3	0.4200	0.3444
Small (1)	4	0.5683	0.3618
Small (1)	High (2)	0.7881	0.3363
2	Low (1)	-0.1799	0.4669
2	2	0.2306	0.4105
2	3	0.3082	0.4003
2	4	0.4368	0.3824
2	High (2)	0.5732	0.3977
3	Low (1)	-0.0587	0.4476
3	2	0.1389	0.4470
3	3	0.2462	0.4059
3	4	0.3450	0.3888
3	High (2)	0.4626	0.4080
4	Low (1)	-0.1081	0.4775
4	2	0.0233	0.4227
4	3	0.1573	0.4128
4	4	0.2944	0.3940
4	High (2)	0.2892	0.4361
Big (5)	Low (1)	-0.1161	0.4242
Big (5)	2	-0.0764	0.4042
Big (5)	3	0.0672	0.3710
Big (5)	4	0.0103	0.3702
Big (5)	High (2)	0.2988	0.3876

GRS F-statistic: 3.3203, p-value: 8.6609e-08

Based on the test statistic and the p-value, we should reject the null hypothesis that we cannot increase the Sharpe Ratio of the market portfolio by adding some combination of the risky assets (all assets divided into a 5-by-5 bins based on their size and BE/ME ratio). That is, the Markowitz-optimized 30-industry portfolio used as the market proxy is not mean-variance efficient and CAPM is rejected.

This is, again, somewhat unexpected since we originally thought that the 10 portfolios grouped by their past

performances contain all the assets in the market, and therefore well-represent the true market portfolio. It may be the case that the each group is not optimized to have the maximum Sharpe Ratio, and therefore the optimized portfolio using the 10 groups is not mean-variance efficient.

k) The table below exhibits the correlations between the three tangency portfolios constructed by 30 value-weighted industry groups, 10 past performance groups, and 25 size-BE/ME ratio groups. As we can see, the correlations between the tangency portfolios are quite small. This is consistent with the results we observed in the previous questions. If they all well-represented the market portfolios, and GRS tests not rejected, they would have a high correlation, since theoretically, they should be the same. Yet, since we observed the opposite above, it makes sense to have such low correlations.

	30 VW Industries	10 past return portfolios	25 size and BE/ME portfolios
30 VW Industries	1	0.3690	0.2258
10 past return portfolios	0.3690	1	0.2436
25 size and BE/ME portfolios	0.2258	0.2436	1