Problem Set 5

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Part 1: Business Cycle Variation

(a) The table below displays the average returns on RMRF, SMB, HML, and UMD during recessions only. Here we ran regression of the portfolio returns on the indicator function. The average return during the recession is $\beta_0 + \beta_1$.

$$R_i = \beta_0 + \beta_1 \cdot 1_{\{recession\ happened\}} + \epsilon_i \quad (i = RMRF, SMB, HML\ and\ UMD)$$

- The results tell me that, during the recessions, the market portfolio on average generated a negative return, but SML, HML and UMD portfolios on average generated positive returns.
- According to their β_1's t-stats, only the market portfolio shows a significantly comove with the business cycle. However, the SML, HML and UMD portfolios show no significant sensitivity to the business cycle. I believe this insignificance can be the proof that SML, HML and UMD are the risk factors that unrelated to the market risk, but they may relate to other risks.

Portfolio	RMRF	SMB	HML	UMD
Intercept	0.9045 (5.03)	0.2484 (2.30)	0.4436 (3.77)	0.7589 (4.76)
$1\{recession\ happened\}$	-1.3806 **(-3.29)**	-0.1944 (-0.77)	-0.2444 (-0.89)	-0.5311 (-1.42)
Average return during recessions (%)	-0.0476	0.0540	0.1992	0.2278

(b) The table below displays the average returns on the smallest growth stocks, smallest value stocks, largest growth stocks, and largest value stocks during the recessions only. On average, the large firms and the value stocks had better performance during recessions. The t-stats indicate that the smallest value stocks and the largest growth stocks have significant cyclical relationships.

Portfolio	Smallest Growth	Smallest Value	Largest Growth	Largest Value
Intercept	0.8358 (2.03)	1.7438 (5.59)	0.8346 (4.65)	1.1937 (4.15)
$1\{recession\ happened\}$	-1.4835 (-1.54)	-2.0686 **(-2.84)**	-1.2184 **(-2.91)**	-1.2727 (-1.89)
Average return during recessions (%)	-0.6477	-0.3248	-0.3838	-0.0790

The table below displays the average returns on the smallest losers, smallest winners, largest losers, and largest winners during recessions only. On average, the small firms and the winners had better performance during recessions. The t-stats indicate that both the smallest and the largest winners have significant cyclical relationships.

Portfolio	Smallest Losers	Smallest Winners	Largest Losers	Largest Winners
Intercept	0.8289 (2.28)	1.9635 (6.63)	0.5424 (1.90)	1.2395 (6.38)
$1\{recession\ happened\}$	-1.1037 (-1.29)	-1.9742 **(-2.85)**	-1.2893 (-1.92)	-1.6632 **(-3.66)**

Average return during recessions (%)	-0.2748	-0.0107	-0.7469	-0.4237

Part 2: Characteristics vs. Covariances

- **25 Size and BE/ME portfolios**

(c) 1. The table below displays the 25 betas for the market portfolio, the SMB portfolio and the HML portfolio.

Portfolio	β_{iM}	β_{iSMB}	β_{iHML}
1	1.29	1.43	0.43
2	1.08	1.53	0.23
3	1.05	1.24	0.52
4	0.94	1.22	0.58
5	0.98	1.30	0.92
6	1.08	1.12	-0.22
7	1.02	0.97	0.14
8	0.98	0.83	0.35
9	0.97	0.81	0.57
10	1.06	0.91	0.89
11	1.12	0.81	-0.23
12	1.02	0.51	0.05
13	0.99	0.44	0.31
14	0.98	0.46	0.55
15	1.12	0.59	0.88
16	1.08	0.33	-0.35
17	1.02	0.23	0.09
18	1.02	0.20	0.35
19	1.02	0.19	0.57
20	1.21	0.29	0.94
21	1.02	-0.15	-0.26
22	0.98	-0.20	0.02
23	0.96	-0.25	0.34
24	1.04	-0.19	0.65
25	1.18	-0.15	1.02

2. Applying the betas above to cross-sectional regressions, we obtained 1085 γ_0, γ_M, γ_{size}, $\gamma_{B/M}$, γ_{SMB} and γ_{HML}.

3. The table below displays the average estimates of γ_0, γ_M, γ_{size}, $\gamma_{B/M}$, γ_{SMB} and γ_{HML} for 3 equations.

Equation 1	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{B/M}}$
Estimates	1.9633	-0.6293	-0.0460	0.2273
Standard error	0.41	0.45	0.03	0.06
t-stat	4.77	-1.41	-1.39	**3.81**

Equation 2	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{HML}}$
Estimates	2.0548	-1.0604	0.1222	0.4157

Standard error	0.40	0.42	0.10	0.11
t-stat	5.14	-2.54	1.17	**3.78**

Equation 3	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{B/M}}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{HML}}$
Estimates	1.8122	0.0407	-0.0976	0.4671	-0.1387	-0.3941
Standard error	0.39	0.38	0.05	0.11	0.20	0.23
t-stat	4.70	0.11	**-1.96**	**4.16**	-0.71	-1.74

(d) From the results of the equation 1 and 2, we can find the characteristic BE/ME and the covariance HML have a significant explanatory power on the 25 portfolios' returns. However, based on the results of the equation (3), we can see that characteristics have a stronger explanatory power than the covariances. The argument is that the t-stat of $\overline{\gamma_{size}}$ and $\overline{\gamma_{B/M}}$ are significant while the t-stats of other variables are not.

Yet, one may argue that we cannot determine whether characteristics have better explanatory power than that of covariance, given true values. SMB and HML are proxies for size and book-to-market ratio respectively; however, they do not perfectly explain characteristics. First, our estimation of β_{SMB} and β_{HML} in the time-series regression may contain noise. Second, this noise contained in $\beta's$, when fed into the cross-section regression, may result in the insignificant t-stats of γ_{SMB} and γ_{SMB}. Yet, since this proxy is all that we can observe and we cannot observe the true value of the beta's, it is possible that we cannot say which better captures returns. Maybe both of them do capture the returns, but the noise embedded in the covariances weakens their explanatory power.

(e) 1. The table below displays the 25 betas for the market portfolio, the SMB portfolio and the HML portfolio.

Portfolio	β_{iM}	β_{iSMB}	β_{iHML}
1	1.10	1.36	-0.28
2	0.97	1.31	0.05
3	0.93	1.08	0.30
4	0.88	1.06	0.44
5	0.95	1.07	0.68
6	1.13	0.99	-0.37
7	1.01	0.88	0.12
8	0.96	0.76	0.39
9	0.96	0.70	0.57
10	1.07	0.88	0.79
11	1.10	0.73	-0.42
12	1.03	0.54	0.17
13	0.99	0.43	0.42
14	0.97	0.41	0.60
15	1.08	0.55	0.80
16	1.08	0.39	-0.41
17	1.06	0.20	0.19
18	1.04	0.17	0.43
19	1.00	0.20	0.55

20	1.16	0.26	0.80
21	0.96	-0.24	-0.37
22	0.99	-0.21	0.09
23	0.95	-0.26	0.31
24	1.03	-0.23	0.64
25	1.11	-0.10	0.81

2. Applying the betas above to cross-sectional regressions, we obtained 1085 $\gamma_0, \gamma_M, \gamma_{size}, \gamma_{B/M}, \gamma_{SMB}$ and γ_{HML}.

3. The table below displays the average estimates of $\gamma_0, \gamma_M, \gamma_{size}, \gamma_{B/M}, \gamma_{SMB}$ and γ_{HML} for 3 equations.

Equation 1	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{B/M}}$
Estimates	1.9635	-0.5551	-0.0401	0.2340
Standard error	0.31	0.35	0.04	0.08
t-stat	6.42	-1.61	-1.07	**3.08**

Equation 2	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{HML}}$
Estimates	1.6135	-0.6741	0.1688	0.4078
Standard error	0.26	.0.31	0.12	0.11
t-stat	6.23	-2.16	1.36	**3.59**

Equation 3	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{B/M}}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{HML}}$
Estimates	1.8593	-0.6840	0.0242	0.4723	0.1818	-0.3265
Standard error	0.41	0.35	0.06	0.13	0.23	0.23
t-stat	4.55	-1.95	0.43	**3.60**	0.80	-1.41

The results above further prove that the BE/ME is a characteristic that better capture the variation of returns than the covariances, because the t-stat of $\overline{\gamma_{B/M}}$ is significant at 95% confidence level while other variables are not.

- **25 Size and Momentum portfolios**

(f) 1. The table below displays the 25 betas for the market portfolio, the SMB portfolio and the UMD portfolio.

Portfolio	β_{iM}	β_{iSMB}	β_{iHML}
1	1.06	1.48	-0.76
2	1.01	1.31	-0.46
3	0.96	1.24	-0.31
4	1.04	1.26	-0.05
5	1.09	1.32	0.23
6	1.16	1.00	-0.71
7	1.02	0.90	-0.40
8	0.99	0.76	-0.17

9	1.01	0.89	0.04
10	1.15	1.01	0.33
11	1.18	0.61	-0.78
12	1.06	0.52	-0.40
13	1.01	0.49	-0.20
14	0.99	0.48	0.08
15	1.12	0.68	0.43
16	1.22	0.26	-0.83
17	1.09	0.15	-0.41
18	1.01	0.21	-0.17
19	1.05	0.19	0.08
20	1.11	0.41	0.46
21	1.13	-0.08	-0.75
22	0.98	-0.19	-0.45
23	0.99	-0.17	-0.18
24	1.00	-0.20	0.13
25	1.10	-0.04	0.46

2. Applying the betas above to the cross-sectional regressions, we obtained 1085 $\gamma_0, \gamma_M, \gamma_{size}, \gamma_{ret212}, \gamma_{SMB}$, and γ_{UMD}.
3. The table below displays the average estimates of $\gamma_0, \gamma_M, \gamma_{size}, \gamma_{ret212}, \gamma_{SMB}$, and γ_{UMD} for 3 equations.

Equation 1	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{ret212}}$
Estimates	2.4609	-1.1704	-0.1036	0.0090
Standard error	0.41	0.40	0.03	0.00
t-stat	5.99	-2.94	**-3.16**	**4.19**

Equation 2	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{UMD}}$
Estimates	1.7784	-0.6782	0.4133	0.6843
Standard error	0.36	0.37	0.11	0.15
t-stat	5.00	-1.82	**3.67**	**4.65**

Equation 3	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{ret212}}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{UMD}}$
Estimates	2.9000	-1.0298	-0.0947	0.0096	0.0307	0.0150
Standard error	0.49	0.40	0.05	0.00	0.20	0.22
t-stat	5.96	-2.61	**-2.00**	**3.34**	0.15	0.07

(g) From the results of the equation 1 and 2, we can find the characteristics and the covariances have a significant explanatory power on the 25 portfolios' returns. However, based on the results of the equation (3), we can see that characteristics have a stronger explanatory power than the covariances. The argument is that the t-stats of $\overline{\gamma_{size}}$ and $\overline{\gamma_{ret212}}$ are significant while the t-stats of $\overline{\gamma_{SMB}}$ and $\overline{\gamma_{UMD}}$ are not.

Yet, one may argue that we cannot determine whether characteristics have better explanatory power than that of covariance, given true values. SMB and HML are proxies for size and book-to-market ratio respectively; however, they do not perfectly explain characteristics. First, our estimation of β_{SMB} and β_{UMD} in the time-series regression may contain noise. Second, this noise contained in $\beta's$, when fed into the cross-section regression, may result in the insignificant t-stats of γ_{SMB} and γ_{UMD}. Yet, since this proxy is all that we can observe and we cannot observe the true value of the beta's, it is possible that we cannot say which better captures returns. Maybe both of them do capture the returns, but the noise embedded in the covariances weakens their explanatory power.

(h) 1. The table below displays the 25 betas for the market portfolio, the SMB portfolio and the UMD portfolio.

Portfolio	β_{iM}	β_{iSMB}	β_{iHML}
1	1.03	1.20	-0.73
2	0.83	0.91	-0.32
3	0.80	0.82	-0.14
4	0.83	0.86	0.03
5	1.01	1.12	0.26
6	1.17	0.94	-0.74
7	0.93	0.72	-0.36
8	0.89	0.62	-0.11
9	0.91	0.70	0.04
10	1.13	0.94	0.34
11	1.14	0.61	-0.77
12	0.96	0.43	-0.36
13	0.90	0.41	-0.18
14	0.93	0.38	0.05
15	1.13	0.70	0.40
16	1.17	0.31	-0.81
17	1.02	0.14	-0.41
18	0.95	0.12	-0.18
19	0.97	0.12	0.06
20	1.11	0.43	0.44
21	1.16	-0.12	-0.75
22	0.92	-0.21	-0.43
23	0.93	-0.22	-0.11
24	0.95	-0.25	0.16
25	1.09	-0.04	0.48

2. Applying the betas above to the dataset, we obtain 1085 $\gamma_0, \gamma_M, \gamma_{size}, \gamma_{ret212}, \gamma_{SMB}, and\ \gamma_{UMD}$.
3. The table below displays the average estimates of $\gamma_0, \gamma_M, \gamma_{size}, \gamma_{ret212}, \gamma_{SMB}, and\ \gamma_{UMD}$ for 3 equations.

Equation 1	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{ret212}}$
Estimates	1.8647	-0.8038	-0.0425	0.0090
Standard error	0.29	0.35	0.03	0.00
t-stat	6.35	-2.33	-1.30	**4.79**

Equation 2	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{UMD}}$
Estimates	1.5037	-0.4181	0.2532	0.7367
Standard error	0.26	0.30	0.13	0.17
t-stat	5.87	-1.38	1.95	**4.39**

Equation 3	$\overline{\gamma_0}$	$\overline{\gamma_M}$	$\overline{\gamma_{size}}$	$\overline{\gamma_{ret212}}$	$\overline{\gamma_{SMB}}$	$\overline{\gamma_{UMD}}$

Estimates	1.9970	-0.6907	-0.0610	0.0103	-0.0857	-0.0935
Standard error	0.34	0.39	0.05	0.00	0.24	0.23
t-stat	5.95	-1.76	-1.14	**4.73**	-0.35	-0.41

The t-stats results after January, 1963 prove that the momentum characteristic still strongly capture the cross-section variation of returns while the size characteristic turns out not to work really well, but still the covariances do not significantly capture the variation of returns because their t-stats are between -1.96 and 1.96. This result makes me more strongly about my answer to the question in general.