Problem Set 3

Instructions: Work in a group of 1-3 people. Each group hands in one electronic copy of their answers. Be brief and to the point, but be sure to explain your logic. Do not print data, entire spreadsheets, or programs – instead, copy the relevant statistics to a table. All tables and charts should have legends and explanations. Answers (excluding tables and figures) should be typed and a maximum of ten pages. Exceeding these limits will draw a penalty.

In this problem set you will evaluate and test the CAPM using the cross-sectional framework**.**

In order to proceed you need Microsoft Excel and the file "Problem_Set3.xls". This file contains three spreadsheets:

1) monthly returns of 49 value-weighted industry portfolios;

2) monthly returns of 25 size and BE/ME sorted portfolios;
Remember to screen out any missing data, which are denoted by a -99.99 or -999 (basically, any return < -1 is a missing value).
The average size and book to market ratio (BE/ME) of the portfolios are also provided in the spreadsheet. Size is on a monthly basis. BE/ME is on an annual basis.

3) monthly returns of a proxy for the market portfolio (R_M).

Part I: For the 49 industry portfolio spreadsheet:

a) Consider the cross-sectional regression,
(1) $R_i = \gamma_0 + \gamma_M \beta_{iM} + \eta_i$,
where γ_0 and γ_M are regression parameters and $\beta_{iM} = \text{cov}(R_i, R_M)/\sigma^2(R_M)$. If the CAPM holds, then what should γ_0 and γ_M equal (for both the Sharpe/Lintner and Black versions)?

b) Estimate γ_0 and γ_M using the approach pioneered by Fama and MacBeth. The following is a brief outline of the procedure:

1. Estimate β_{iM} for each portfolio (denote the estimate b_{iM}). Assume that the betas do not change over time; hence, you can estimate the betas using full-period OLS regressions.
2. Each month estimate the cross-sectional regression, $R_{it} = \gamma_{0t} + \gamma_{Mt} b_{iM} + n_{it}$. Note: the estimated beta (b_{iM}) is the same for every time period (i.e., it has no time subscript).
3. Compute the time series average of the estimates of γ_0 and γ_M. In addition, compute the standard error and *t*-stat of the time series averages.

Can you reject the hypothesis that the proxy for the market portfolio is mean variance efficient? Why or why not?

c) Estimate the cross-sectional regression, $ave(R_i) = \gamma_0 + \gamma_M b_{iM} + n_i$. Are the estimates of γ_0 and γ_M different than the average estimates in part b? Are the standard errors of the estimates of γ_0 and γ_M different than the standard errors of the average estimates in part b? Why or why not? Which method is superior? Why?

d) Plot the $ave(R_i)$ for the portfolios against their b_{iM}. Does the plot resemble a positive relationship? What should the plot look like?

e) Now, Consider the cross-sectional regression,
(2) $R_i = \gamma_0 + \gamma_M \beta_{iM} + \gamma_{size} \ln(size) + \gamma_{B/M} \ln(BE/ME) + \eta_i$,
where γ_0, γ_M, γ_{size}, and $\gamma_{B/M}$ are regression parameters, $\beta_{iM} = cov(R_i, R_M)/\sigma^2(R_M)$, and size and BE/ME are the average size and book to market ratio of the portfolio.

Note that size is provided on a monthly basis, but BE/ME is provided on an annual basis. For simplicity, assume BE/ME within each year is constant so that you have 12 constant BE/ME numbers within each year.

<u>Please answer the following:</u>

If the CAPM holds, then what should γ_{size} and $\gamma_{B/M}$ equal?

f) Estimate γ_0, γ_M, γ_{size} and $\gamma_{B/M}$ using the Fama MacBeth procedure. The following is a brief outline of the procedure:

1. Estimate β_{iM} for each portfolio. Assume that the betas do not change over time; hence, you can estimate the betas using full-period OLS regressions. (Note: you have already done this.)

2. Each month estimate the regression,
$R_{it} = \gamma_{0t} + \gamma_{Mt} b_{iM} + \gamma_{sizet} \ln([size]_{t-1}) + \gamma_{B/Mt} \ln([BE/ME]_{t-1}) + n_{it}$.

Note: You need to lag size and BE/ME appropriately in the regression. This means that size and BE/ME need to be measured ex ante and therefore must be the previous month's size and previous year's BE/ME.

Why is it important that the size and BE/ME characteristics should be lagged values?

3. Compute the time series average of the estimates of γ_0, γ_M, γ_{size}, and $\gamma_{B/M}$. In addition compute the standard error and *t*-stat of the time series averages.

Can you reject the hypothesis that the proxy for the market portfolio is mean variance efficient? Why or why not?

Part II: For the 25 Size and BE/ME portfolio spreadsheet:

1) Repeat parts b), c), d), and f) for the 25 size and BE/ME portfolios instead of the industries.

2) Repeat parts c) and d) above for the 25 size and BE/ME portfolios using the tangency portfolio (computed in sample using the sample mean and covariance matrix of the 25 portfolios over the entire sample) of the 25 size and BE/ME portfolios instead of the market portfolio.
    a. First, compute the tangency weights for the 25 portfolios.
    b. Then reconstruct the returns of the tangency portfolio over time and use this for R_M to estimate the betas.
    c. How do the results highlight the Roll (1977) critique of tests of the CAPM?

3) Repeat parts c) and d) above for the 25 size and BE/ME portfolios, only this time use the following tangency portfolio of the 25 size and BE/ME portfolios, where we split the data in half and use one half to estimate the portfolio weights and the other half to measure portfolio returns using those weights.

    First, split the sample into two halves: (1) take all observations from *odd months* in *even years* and *even months* in *odd years* (i.e., if starting in 1980, this would look like: 01/1980, 03/1980, . . . , 11/1980, 02/1981, 04/1981, . . . 12/1981, 01/1982, 03/1982, . . .); and (2) the opposite (take all observations from *even months* in *even years* and *odd months* in *odd years*).
    a. Then, compute the tangency portfolio weights for the 25 portfolios using only the moments estimated from the first sample and apply them to returns in the second sample.
    b. Then, do the reverse---compute the tangency portfolio weights from the second sample and apply those to returns in the first sample.
    c. The resulting return series is your new tangency portfolio returns which you will use to repeat the cross-sectional tests.

4) How are the results different between 2) and 3) and why? How does the difference in results between 2) and 3) showcase the Roll (1977) critique?