# Problem Set 2

*Instructions: Work in a group of 1-3 people. Each group hands in one electronic copy of their answers. Be brief and to the point, but be sure to explain your logic. Do not print data, entire spreadsheets, or programs – instead, copy the relevant statistics to a table. All tables and charts should have legends and explanations. Answers (excluding tables and figures) should be typed and a maximum of five pages. Exceeding these limits will draw a penalty.*

This problem set provides both computational and theoretical practice on forming mean-variance efficient portfolios and employing mean-variance mathematics.

In order to proceed you need Microsoft Excel and file "Problem_Set2.xls". This file contains data for 10 value-weighted industry portfolios, using monthly returns over an 90-year period. You will be computing mean-variance efficient portfolios for these 10 industries.

The following is a quick review of some of the tools you will need for this assignment:

Follow the instructions at the top of the spreadsheet.

In Matlab, the function to invert the matrix X is "inv(X)" and multiplying two matrices together is "X*Y."

Questions:

1. Find the minimum variance and tangency portfolios of the industries. (hint: you will need to compute the means (arithmetic average), standard deviations, variances, and covariance matrix of the industries. The risk-free rate is given in the spreadsheet.) Comment on the different weights applied to each industry under the MVP and Tangent portfolios.

    a) Compute the means and standard deviations of the MVP and Tangent portfolios. Plot the efficient frontier of these 10 industries and plot the 10 industries as well on a mean-standard deviation diagram. Why does the efficient frontier exhibit the shape that it does (i.e., why is it a parabola)?

    b) Comment on the reliability of the mean return estimates for each industry. Then, artificially change the mean return estimates of each industry by a one standard error increase. How much does the Tangent portfolio change? Does the efficient frontier change a lot or a little?

    c) Comment on the reliability of the covariance matrix estimate. First, assume that all covariances are zero and recompute the efficient frontier using the diagonal matrix of variances as the covariance matrix. Then, assume very simply that the covariance matrix is just the identity matrix (i.e., a matrix of ones along the diagonal and zeros everywhere else). Does the mean-variance frontier change a lot or a little, relative to b)? How important are the covariance terms relative to the variance terms?

d) Run some simulations similar to what Jorion did in his study. Using the mean and covariance matrix you calculated in sample from the historical returns, use these parameters to simulate data under a multivariate normal distribution.

- Draw a random sample of 10 (*N*) returns from this distribution *T* times (*T* = the number of months). This gives you one simulation.
- Calculate the tangency and minimum variance portfolio weights from these simulated data. *Then, apply these weights to the actual (NOT SIMULATED) returns on the industries* (e.g., the weights come from the simulated returns, but they are applied to true/actual returns on the industries).
- Then repeat 1,000 times and save the mean and standard deviation of each MVP and Tangency portfolio you calculated under each simulation of data used to get the weights and applied to actual returns.
- On two separate plots of mean-standard deviation space, plot the simulated MVP and Tangency portfolios relative to the ones calculated using weights estimated from the real data. (One plot for MVP and one for Tangency portfolios, each plot will contain 1001 data points).
- These plots indicate the estimation error (under a normal distribution) of the Tangency and MVP weights. Which portfolio (MVP or Tangency) is estimated with less error? Why?

e) Now run some simulations under the empirical distribution of returns rather than the normal distribution. This is called a block *bootstrap simulation*.

- Draw a random sample of 10 (*N*) returns from the empirical distribution *T* times (*T* = the number of months) with replacement. The way to do this is to randomly sample a particular month by selecting a number (integer) at random from 1 to *T*. Pick the 10 industry returns corresponding to the month chosen randomly between 1 and *T*. These 10 returns become your first data point of 10 industry returns (effectively this becomes month *t=1* in the simulation). Then, pick another number from 1 to *T*, even if it is the same number (this is what resampling with replacement means), and repeat. This becomes month *t=2* in the simulation. Repeat *T* times and this gives you one simulation.
  (Hint: in Matlab you can create *T* random numbers between 1 and *T* and then simply pull off the rows in the industry return matrix (which is *T* X 10) that correspond to the *T* random numbers chosen. A Matlab file is posted to help you out.)

- Calculate the tangency and minimum variance portfolio weights from these simulated data. *Then, apply these weights to the actual (NOT SIMULATED) returns on the industries* (e.g., the weights come from the simulated returns, but they are applied to true/actual returns on the industries).
- Then repeat 1,000 times and save the mean and standard deviation of each MVP and Tangency portfolio you calculated under each simulation of data used to get the weights and applied to actual returns.
- On two separate plots of mean-standard deviation space, plot the simulated MVP and Tangency portfolios relative to the ones calculated using weights estimated from the real data. (One plot for MVP and one for Tangency portfolios, each plot will contain 1001 data points).
- These plots indicate the estimation error (under the empirical distribution) of the Tangency and MVP weights. How does the estimation error compare under the empirical simulations versus the normal distribution simulations of question d)?

2. Solve by hand the following. There are three securities A, B, C with mean returns of 17%, 13%, and 9%, respectively. Furthermore, their standard deviations are 20%, 40%, and 15%, respectively. The correlation between A and B is 0.50, between B and C is 0.30, and between A and C is zero. The risk-free rate is 5%.

a) Find the MVP and Tangent portfolios of these three assets, and calculate each of the portfolio return means and standard deviations.

b) Write the equation for the efficient frontier of these three assets.

c) Find the portfolio of A, B, C that gives the lowest possible variance for a return of 13%, and find the portfolio that gives the highest possible return for a standard deviation of 15%. Calculate the Sharpe ratios of these two portfolios.

d) Repeat c) allowing an investor to also invest in the riskless asset. Find the portfolio that gives the lowest possible variance for a return of 13%, and find the portfolio that gives the highest possible return for a standard deviation of 15%. Again, calculate the Sharpe ratios of these two portfolios. Compare your answers to those in c). Illustrate graphically (in a mean-standard deviation diagram) what is going on.