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ECON 676
Problem Set 2

Question 1

1. Below is the table of the weights of each industry under the MVP and tangent portfolios.

Industries	**NoDur**	**Durbl**	**Manuf**	**Enrgy**	**HiTec**	**Telcm**	**Shops**	**Hlth**	**Utils**	**Other**
Weights of **MVP** (%)	76.88	-5.73	-13.72	21.83	-10.56	54.94	-5.58	7.13	7.32	-32.52
Weights of **Tangent Portfolio** (%)	83.77	7.99	-17.61	32.05	3.19	33.32	-4.70	28.18	-3.43	-62.76

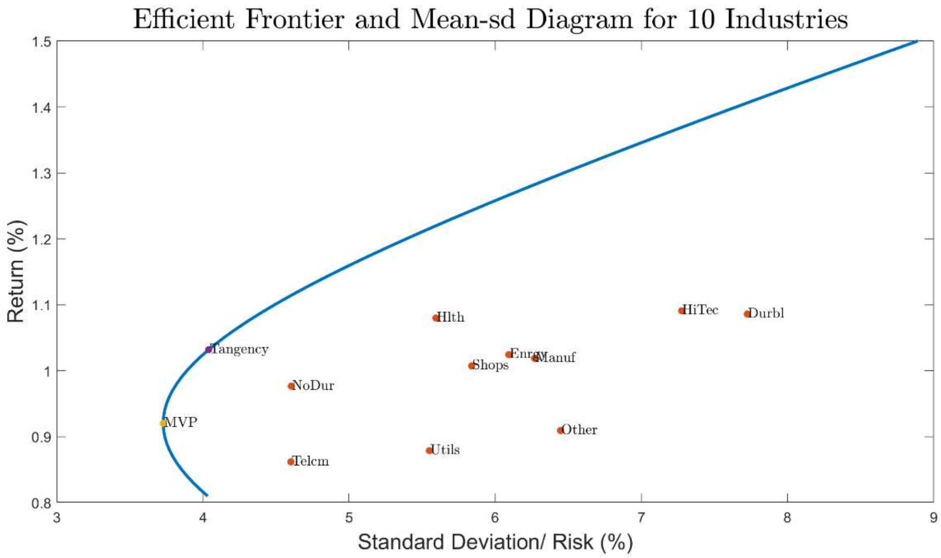
Notice: the numbers in black are positive, and the numbers in red are negative.

- R is the mean of returns on 10 industries ($N \times 1$ matrix), σ is the standard deviation of returns on 10 industries ($N \times N$ matrix), V is the covariance matrix ($N \times N$ matrix) and $\mathbf{1}$ is (1 1 ... 1)' ($N \times 1$ matrix).
- For the MVP, the weights for individual assets are $W_{MVP} = \frac{V^{-1}\mathbf{1}}{\mathbf{1}'V^{-1}\mathbf{1}}$
- For the tangent portfolio, the weights for individual assets are $W_{TP} = \frac{V^{-1}(R-r_f\mathbf{1})}{\mathbf{1}'V^{-1}(R-r_f\mathbf{1})}$
- Based on the formulas given above, we obtained the results shown in the table 1. Both portfolios have heavy weights on Non-Durables industry. Compared to the MVP, the tangent portfolio has a positive weight on Durables and High-Technology industries where MVP has negative weights. In addition, Tangent portfolio has a negative weight on Utilities where MVP has a positive weight.

a) The mean and standard deviation of the MVP and the Tangent Portfolio are displayed below.

	Mean (%)	Standard Deviation (%)
MVP	0.9204	3.7286
Tangent Portfolio	1.0320	4.0395

- The MVP is the portfolio with the minimum variance. It has a smaller standard deviation and a smaller mean return relative to the tangent portfolio.



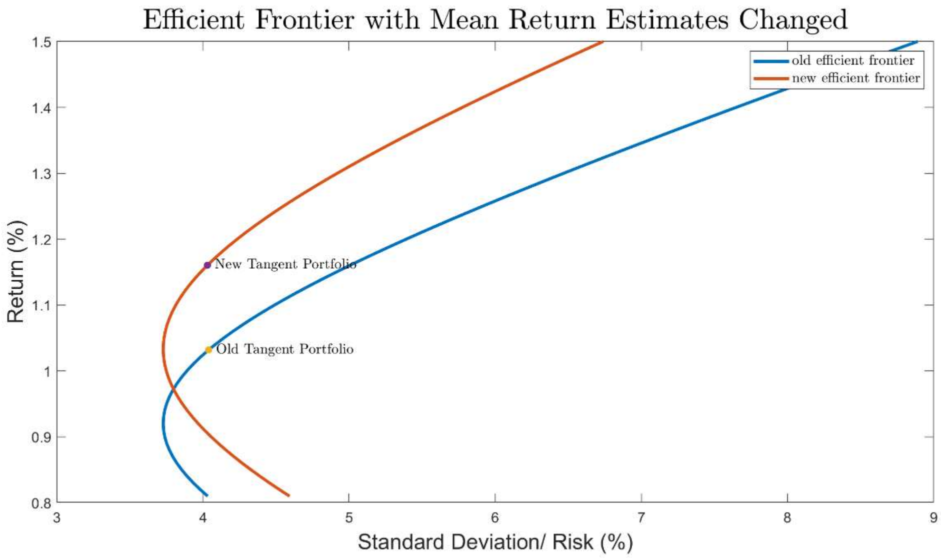
- The graph above displays the efficient frontier. The efficient frontier is parabola, and both the MVP and the tangent portfolio land on the efficient frontier, but the 10 industries are on the right side of the efficient frontier, which means they are less desirable than the portfolios on the efficient frontier.
- The shape of the efficient frontier can be explained by its equation. $\sigma^2 = (r_p\ \mathbf{1})A^{-1}(r_p\ \mathbf{1})'$ where $A = (R\ \mathbf{1})V^{-1}(R\ \mathbf{1})$. Suppose $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, then $A^{-1} = \frac{1}{ac-\ ^{2}}\begin{bmatrix} c & -b \\ -b & a \end{bmatrix}$. Therefore, $\sigma^2 = (r_p\ \mathbf{1})A^{-1}(r_p\ \mathbf{1})' = \frac{a-2br_p+cr_p^2}{ac-b^2}$. Based on this equation, the efficient frontier is parabola.

b)

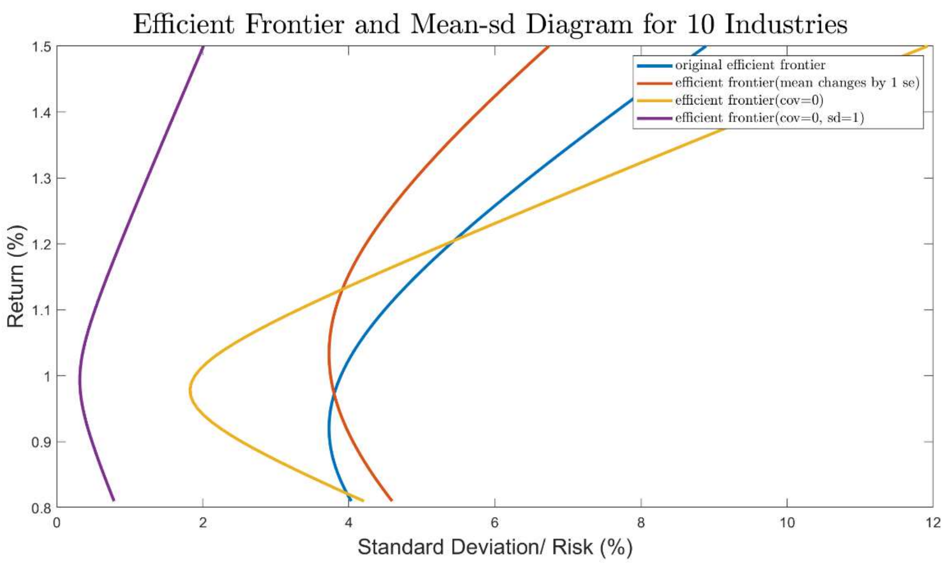
- The reliability of the mean return estimates is relatively high. The table below is the mean returns and the standard error of 10 industries. The standard error are around 20% of the mean returns, indicating the confidence interval of the mean returns of each industry is narrow.

Industries	**NoDur**	**Durbl**	**Manuf**	**Enrgy**	**HiTec**	**Telcm**	**Shops**	**Hlth**	**Utils**	**Other**
Mean (%)	0.98	1.09	1.02	1.02	1.09	0.86	1.01	1.08	0.88	0.91
Standard Error (%)	0.14	0.23	0.19	0.19	0.22	0.14	0.18	0.17	0.17	0.20
SE/Mean	0.14	0.22	0.19	0.18	0.20	0.16	0.18	0.16	0.19	0.22

- The graph below is the original frontier efficient and the frontier efficient given the mean returns plus one standard deviation. As the graph shows, when we artificially increased the mean return estimates by a one standard error, the efficient frontier will shift up by 0.15%, which is small, and the tangent portfolio will shift up by approximately same amount. Therefore, even if our mean return estimates deviate from the real value, the efficient frontier and the tangent portfolio will not change much.



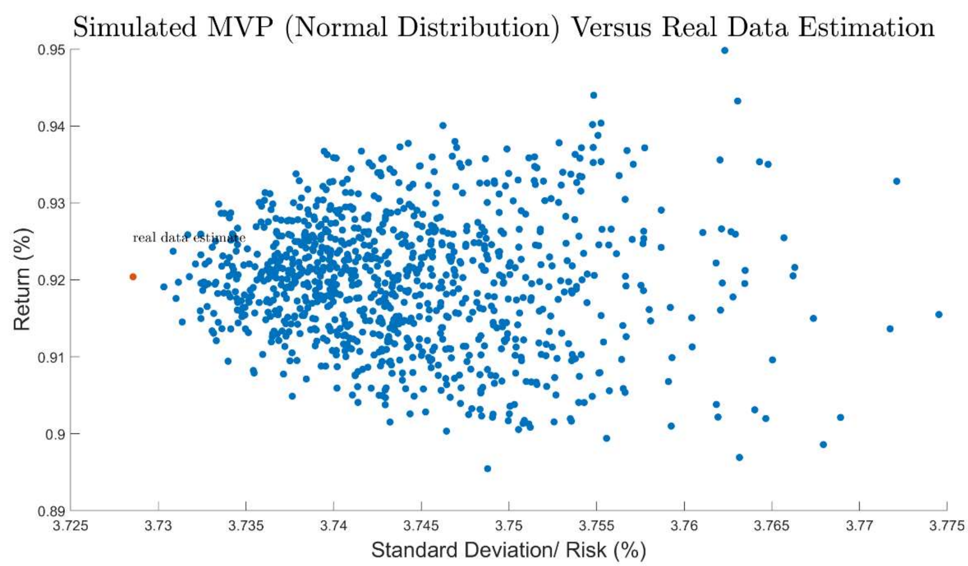
c)

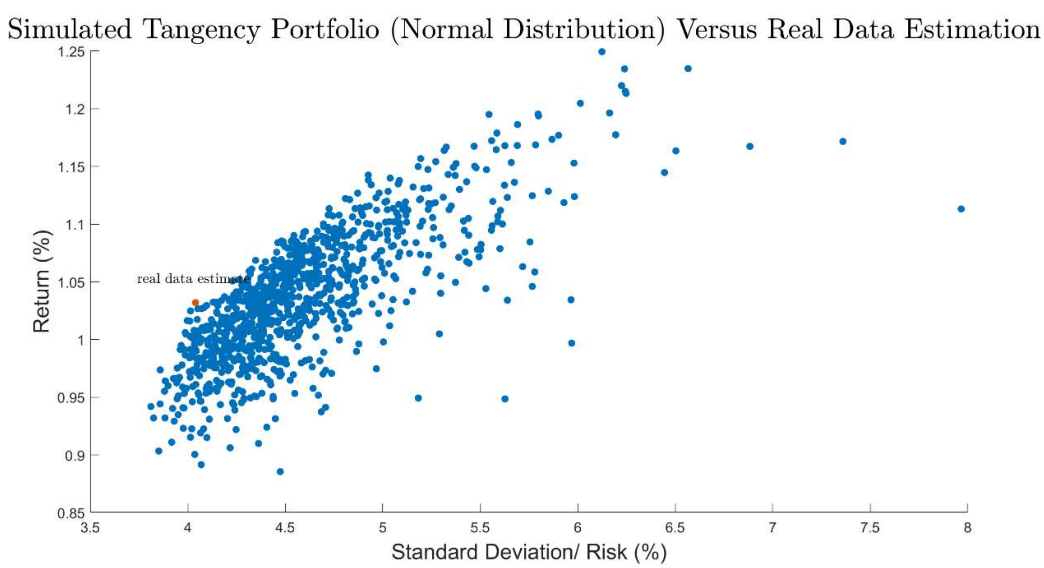


- The shape and the location of the frontier on mean-volatility space changes a lot when we have different estimated values in the covariance matrix.
- The efficient frontier given the covariance matrix is a diagonal matrix (the yellow curve) changes a lot relative to what we observed in (b) (the red curve).
- The efficient frontier given the covariance matrix is an identity matrix (the purple curve) also changes a lot relative to what we observed in (b) (the red curve).
- In the graph, the comparison between the yellow curve and the blue curve indicates the sensitivity of the efficient frontier to the covariance change, and the comparison between the purple curve and the yellow curve indicates the sensitivity of the efficient frontier to the variance change. We cannot tell whether covariance terms are more important than the variance, since we cannot directly measure the amount of change in each case. However, theoretically, as more assets are added to a portfolio,

covariances become more important than variances. This is because the idiosyncratic risks are diversified away in a well-diversified portfolio, while the systematic risks, which are reflected in the covariances, play a more important role in the portfolio's variance. In this case, the portfolio constructed by 10 assets may not be well-diversified, so the efficient frontier change greatly when we get rid of the covariances and when we change the variances.

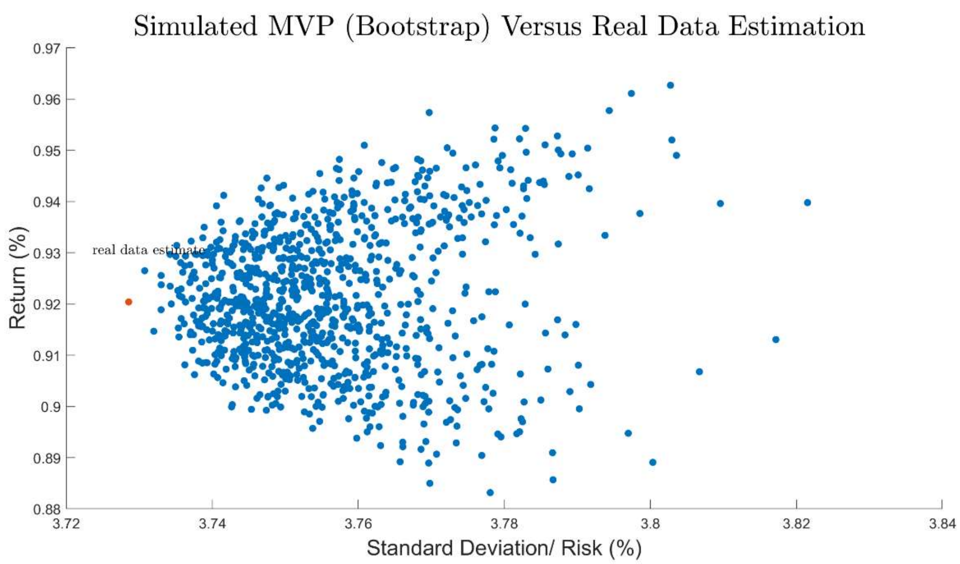
d) The two graphs below use simulated data under a multivariate normal distribution. Each plot contains 1001 data points including the ones calculated using weights estimated from real data (red points).

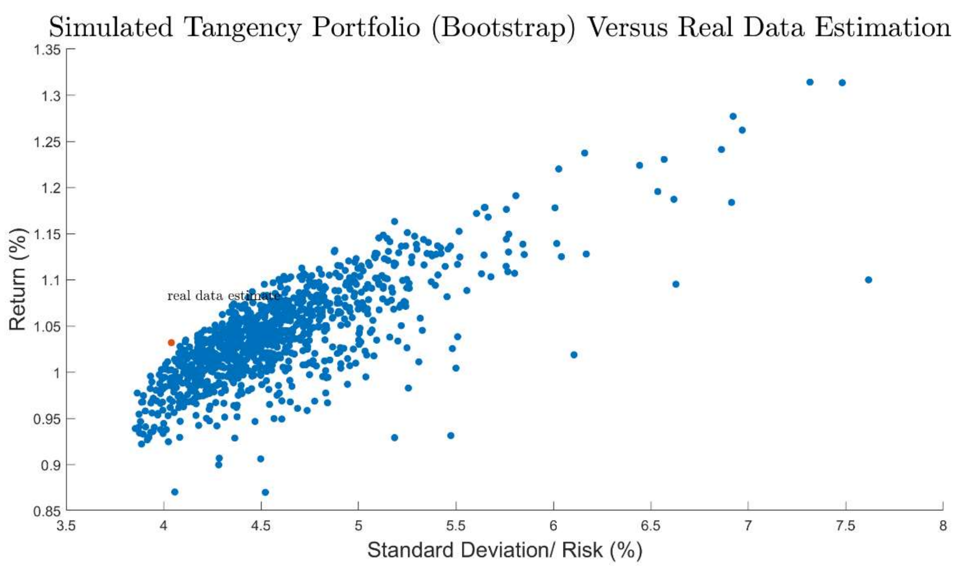




- In the two graphs, the MVP is estimated with less error, as the denser blue dots indicate. The weights of the MVP are $\frac{V^{-1}\mathbf{1}}{\mathbf{1}'V^{-1}\mathbf{1}}$, which involves no mean return, while the weights of the tangent portfolio, $\frac{V^{-1}(R-r_f\mathbf{1})}{\mathbf{1}'V^{-1}(R-r_f\mathbf{1})}$, involves mean returns. Because the covariance estimation is more precise than the return estimation, the MVP has a smaller estimation error.

e) The two plots below use simulated data under a block bootstrap simulation. Each plot contains 1001 data points including the ones calculated using weights estimated from real data (red points).





- Compared to the normal distribution simulations of question d, bootstrap has a higher estimation error because the points are less dense. This is because the bootstrap uses the real data, which has a fatter tail than the multivariate normal distribution. These fat-tail events will incur a less accurate estimate, leading to the more scattered dots and a high estimation error.

Question 2

a)

	MVP	**Tangent portfolio**
Weight on A	43.14%	69.17%
Weight on B	-10.03%	-12.81%
Weight on C	66.89%	43.64%
Mean	12.05%	14.02%
Standard Deviation	11.51%	13.02%

- R = $[0.17 \quad 0.13 \quad 0.09]'$, $r_f = [0.17 \quad 0.13 \quad 0.09]'$, V = $\begin{bmatrix} 0.04 & 0.04 & 0 \\ 0.04 & 0.16 & 0.018 \\ 0 & 0.018 & 0.0225 \end{bmatrix}$
- We use the formulas $W_{MVP} = \frac{V^{-1}\mathbf{1}}{\mathbf{1}'V^{-1}\mathbf{1}}$ and $W_{TP} = \frac{V^{-1}(R - r_f\mathbf{1})}{\mathbf{1}'V^{-1}(R - r_f\mathbf{1})}$ to calculate the weights of each portfolio and use the formulas $Mean = W'R$ and $Var = W'RW$.
- Compared to MVP, Tangent portfolio has a much higher weight on A and a much lower weight on C. The mean and risk of tangent portfolio are higher.

b)

The equation of efficient frontier:

$$\mathrm{a} - 2\mathrm{b} * \mathrm{r_p} + \mathrm{c} * \mathrm{r_p^2} = (\mathrm{ac} - \mathrm{b}^2) * \sigma_\mathrm{p}^2$$

where:

$$\mathrm{a} = \mathrm{R'V^{-1}R} = 1.2013$$
$$\mathrm{b} = 1'\mathrm{V^{-1}R} = 0.0910$$
$$\mathrm{c} = 1'\mathrm{V^{-1}}1 = 0.0076$$

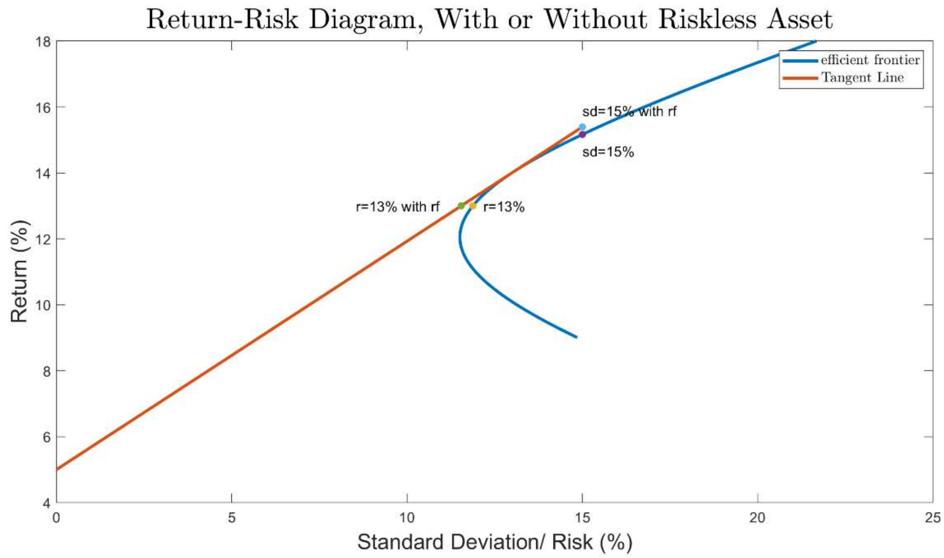
c)

	Portfolio (r=13% without rf)	Portfolio 2 (sd=15% without rf)
Weights on A	55.69%	84.29%
Weights on B	-11.37%	-14.42%
Weights on C	55.69%	30.13%
Mean	13.00%	15.17%
Standard Deviation	11.88%	15.00%
Sharpe Ratio	0.67	0.68

d)

	Portfolio (r=13% with rf)	Portfolio 2 (sd=15% with rf)
Weights on Tangent portfolio	88.68%	115.23%

Weights on Riskless Asset	11.32%	-15.23%
Mean	13.00%	15.39%
Standard Deviation	11.54%	15.00%
Sharpe Ratio	0.69	0.69



- Compared to the answers in c, the portfolio with a fixed return has a lower standard deviation, and the portfolio with a fixed standard deviation has a higher return. Both portfolios have higher Sharpe ratio than the ones in c.
- The reason is that here two portfolios both stay on the tangent line (CAL), which includes a list of portfolios mixing tangent portfolio and riskless asset. And all the portfolios on the line have the maximum Sharpe ratio, so for any given risk or return, the portfolios on the line have the highest return or the lowest risk.