Exercise 5 - Probabilistic models

First name: Brian

Last name: Schweigler

Matriculation number: 16-102-071

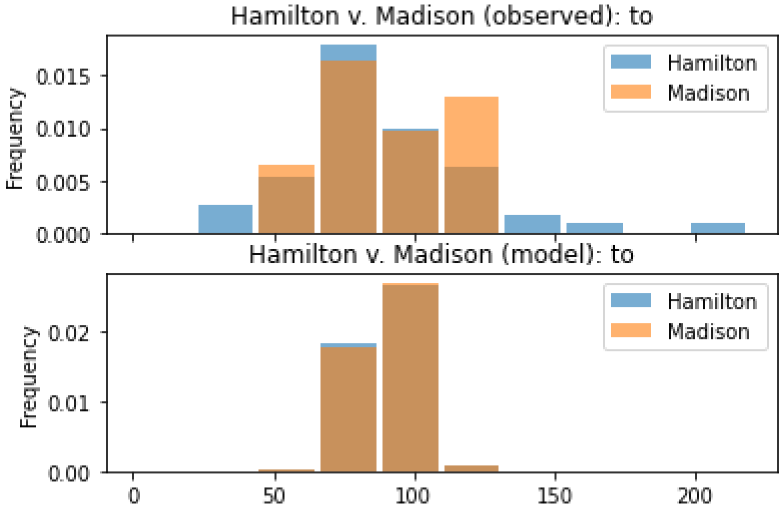
Q1: Considering the words “to”, “upon” and “would”, draw a graph representing the occurrences of those words in Hamilton and Madison’s articles

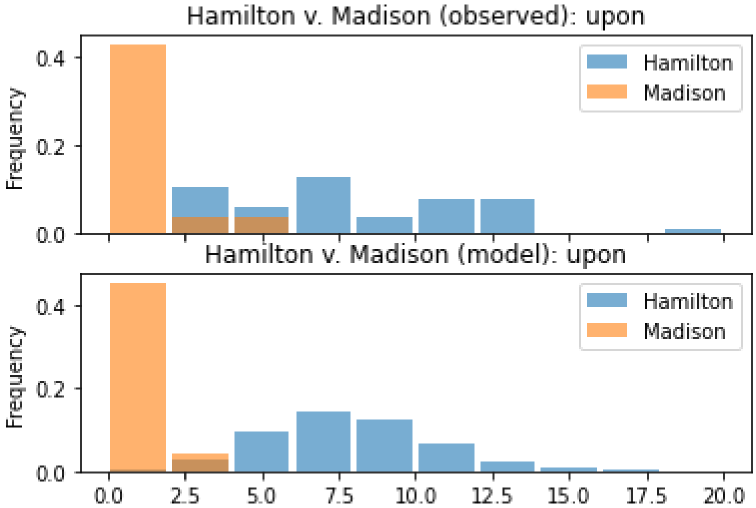
General imports and solving the question:

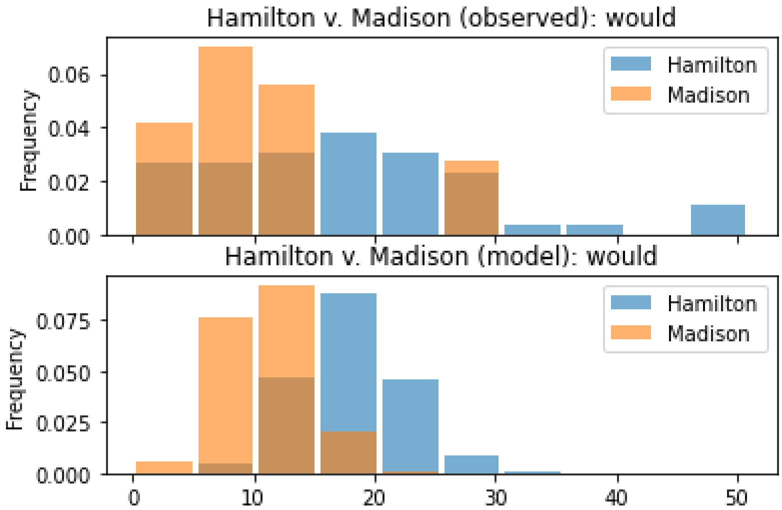
In [5]:

```
%load_ext autoreload
%autoreload 2
%matplotlib inline

import matplotlib.pyplot as plt
import pandas as pd
import re
import numpy as np
import lxml.etree
import os
from scipy import stats

np.random.seed(6)  # for reproducibility
df = pd.read_csv('Data/federalist-papersNew2.csv', index_col=0)
hamilton = df[df['AUTHOR'] == 'Hamilton']
madison = df[df['AUTHOR'] == 'Madison']

combined = pd.concat([hamilton, madison])
test_indices = [49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63]
test_set = df.loc[test_indices]

simulations = 100000


def binomial_samples(n, prob, size=None):
    return np.random.binomial(n, prob, size)


for token in ['to', 'upon', 'would']:
    values = combined.groupby('AUTHOR')[token].describe()[['max', 'mean', 'count']]
    fig, axes = plt.subplots(2, 1, sharex=True)
    combined.groupby('AUTHOR')[token].plot.hist(density=True, rwidth=0.9,
                                                alpha=0.6, legend=True, ax=axes[0], bins=10,
                                                range=(0, max(values['max'])))

    for author in ['Hamilton', 'Madison']:
        mean = values.loc[author]["mean"]
        count = values.loc[author]["count"]
        pd.Series(binomial_samples(1000, mean / 1000, size=simulations)).\
            plot.hist(
            label=author, density=True, rwidth=0.9, alpha=0.6, ax=axes[1],
            bins=10, range=(0, max(values["max"])), legend=True)

    axes[0].set_title('Hamilton v. Madison (observed): ' + token)
    axes[1].set_title('Hamilton v. Madison (model): ' + token)

    plt.show()
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```







Q2: Model these three words as a Binomial distribution, to reflect either occurrences in Hamilton or Madison's writing style (you only need to estimate the parameters p and n).

See the plots above.

Q3: If p = 0.001 and n = 5000, what is the probability (according to a Binomial) that we observe 5 occurrences of the underlying word-type?

Setup and solving the task

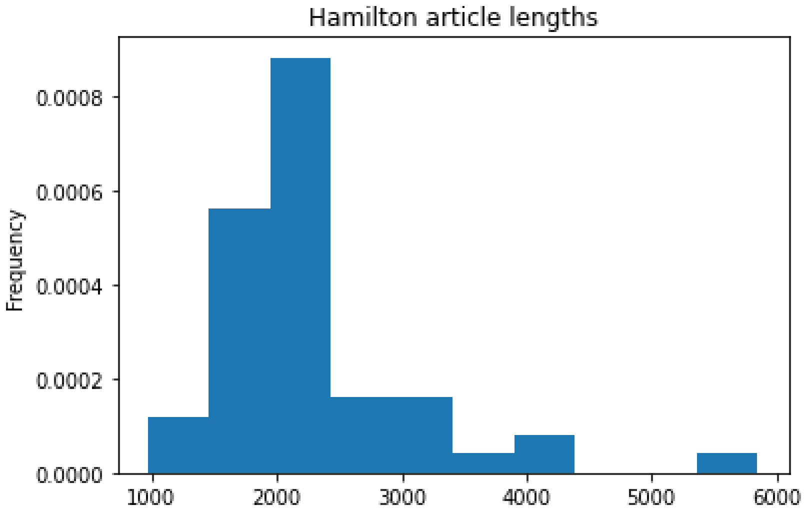
In [6]:
```
binomial_samples(5000, 0.001)
```

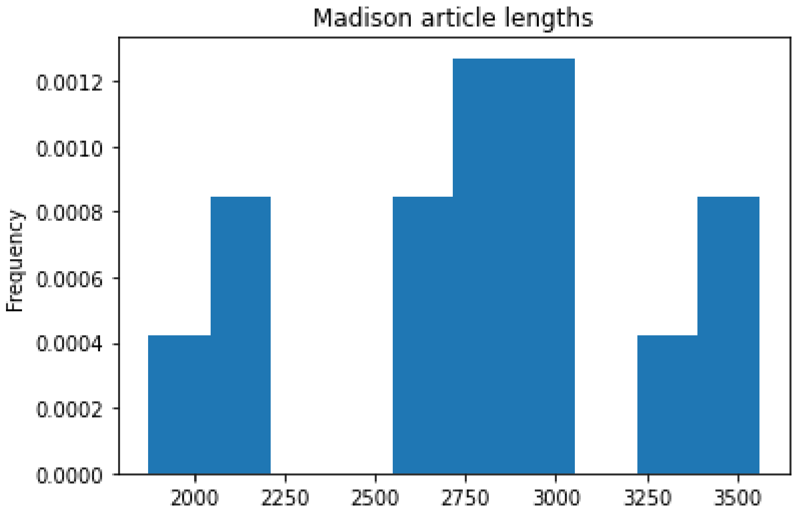
Out[6]:
```
3
```

Intuition says this should simply be 1'000?

Q4: Represent in a histogram the article lengths. Does it make sense to consider this distribution as a Gaussian one?

In [7]:
```python
authors = combined['AUTHOR'].copy()
lengths = combined.drop('AUTHOR', axis=1).sum(axis=1)
pd.Series(lengths[authors == 'Hamilton']).plot.hist(density=True, )
plt.title('Hamilton article lengths')
plt.show()
pd.Series(lengths[authors == 'Madison']).plot.hist(density=True, )
plt.title('Madison article lengths')
plt.show()
```





Hamilton could resemble a Gaussian distribution, (centered around a length of 2250), but for Madison it does not really match.

Q5: Compute the probabilities of obtaining the two sequences with and without Laplace smoothing.

In [8]:
```python
tokens = 5050
distinct_word_types = 555
unigrams = {"tri": 1000, "the": 1350, "big": 320, "increase": 10, "deal": 15,
            "today": 25, "bank": 8, "stock": 25, "will": 56, "is": 132}
bigrams = {"tri today": 3, "today the": 4, "the big": 210, "deal tri": 1,
           "the stock": 8, "stock is": 2, "tri the": 580, "big deal": 11}

sentence_1 = "today the big deal".split(" ")
sentence_2 = "the stock is decreasing".split(" ")


def conditional_probability(word, last):
    return max(unigrams.get(word, 0) / tokens, bigrams.get(last + " " + word, 0
                                                            ) / unigrams.get(word, 0.00000000000(


def laplace_conditional_probability(word, last, n = distinct_word_types):
    return max((unigrams.get(word, 0) + 1) / (tokens + n),
               (bigrams.get(last + " " + word, 0) + 1) / (unigrams.get(word, 0) + n))
```

```
def laplace_conditional_probability_lower(word, last, n=9):
    return max((unigrams.get(word, 0) + 1) / (tokens + n),
               (bigrams.get(last + " " + word, 0) + 1) / (unigrams.get(word, 0) + n))


def probability(sentence_array, last="tri", prob_f=None):
    if len(sentence_array) == 1:
        return prob_f(sentence_array[0], last)
    return prob_f(sentence_array[0], last) * probability(sentence_array[1:], sentence_array[0],


print("Normal probability for sentence 1: ", probability(sentence_1, prob_f=conditional_probabi
print("Laplace probability for sentence 1: ",
      probability(sentence_1, prob_f=laplace_conditional_probability))
print("laplace prob lower n",
      probability(sentence_1, prob_f=laplace_conditional_probability_lower))

print("Normal probability for sentence 2: ", probability(sentence_2, prob_f=conditional_probabi
print("Laplace probability for sentence 2: ",
      probability(sentence_2, prob_f=laplace_conditional_probability))
print("Laplace prob lower n",
      probability(sentence_2, prob_f=laplace_conditional_probability_lower))
```

```
Normal probability for sentence 1:  0.015438118811881188
Laplace probability for sentence 1:  8.43902983951003e-06
laplace prob lower n 0.010074611449724291
Normal probability for sentence 2:  0.0
Laplace probability for sentence 2:  2.0233858528910923e-07
Laplace prob lower n 0.0003305709761121411
```

I am unsure if I have chosen the correct approach. I tried to follow the lecture, but there were no python examples there. As far as I know, the maximum principle is to just pick the higher probability between bigrams and unigrams.

Q6: Provide an example of one drawback of applying the direct estimation as suggested by Mary. Provide as well two drawbacks related to Laplace smoothing.

With Mary's approach we may assign the probability of 0 to something we do not know.

With Laplace we tend to overestimate the probability of words we do not see; which is especially a problem once the sample gets too large. The probability of frequent n-grams are underestimated. But its (supposdly) a simple technique.