Heteroskedasticity

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Contents

Repeat

1. Model assumptions

Heteroskedasticity

1. Heteroskedasticity
2. Test visually
3. Statistical tests

Treatment

1. Robust standard error
2. Models

Model assumptions

1. **Linearity**
2. **No exact multicollinearity**
3. **Strong or strict exogeneity**
4. **Homoskedasticity**
5. **No autocorrelation**

In case of IRS

1. $f(X) = \beta_0 + \beta_1 X + \varepsilon$

2. The data matrix has full column rank.

   Variables cannot be written as a linear combination of each other

3. $\mathbb{E}(\varepsilon_i | X_i) = 0$

   The errors are independent of the explanatory variables

4. $\mathbb{D}^2(\varepsilon_i | X) = \sigma^2$

   The standard deviation of errors for different observations is constant

5. The errors for different observations are uncorrelated

Repeat | Heteroskedasticity | Treatment

http://real.mtak.hu/42944/1/dim.2016.07.pdf

Heteroskedasticity

The variance of the error term is not constant

Possible sources:

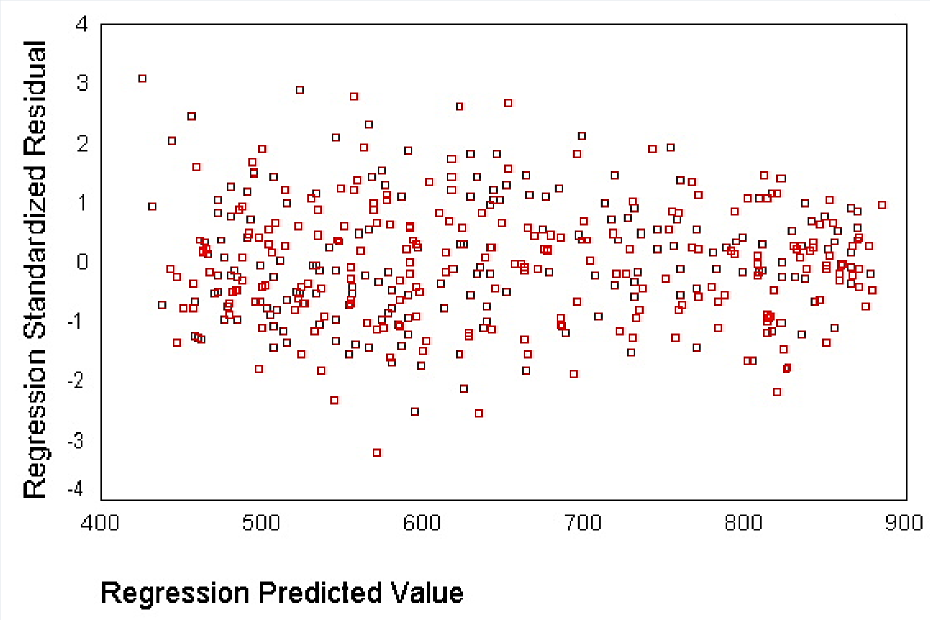
- Natural phenomenon – income, consumption -> expanding opportunities for the fulfilment of taste
- Grouped data – e.g. the observation unit is a household, company etc.
- Wrong functional form

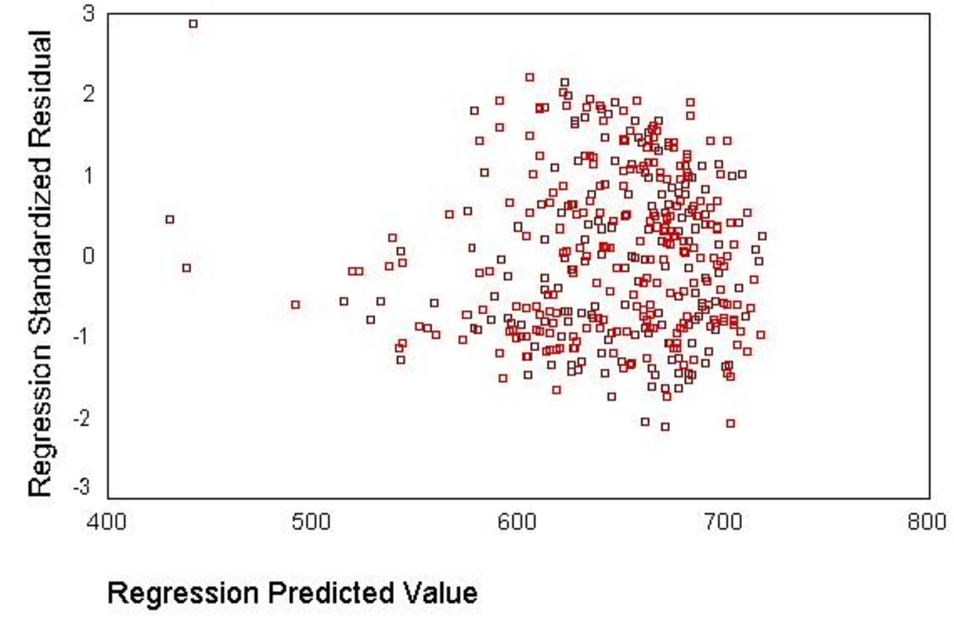
Heteroskedasticity

Consequences

- The estimation of parameters remains unbiased and consistent – but it is not BLUE
- The standard errors of the parameters are biased and inconsistent
- T test and F test do not follow T and F distribution (Even asymptotically)
- The tests and confidence intervals are not valid
- The prediction is unbiased, but it is not efficient

Heteroskedasticity – graphically testing





Heteroskedasticity – LM test

$$\boxed{H_0\text{: } E(u^2 | x_1, x_2, \ldots, x_k) = E(u^2) = \sigma^2}$$

$$\boxed{u^2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \ldots + \delta_k x_k + v}$$

$$\boxed{H_0\text{: } \delta_1 = \delta_2 = \ldots = \delta_k = 0}$$

$$\boxed{LM = n \cdot R^2_{\hat{u}^2}}$$

- Breusch-Pagan test for heteroskedasticity (BP test)
- Koenker test – robust version of BP test

Heteroskedasticity – White test

Idea: use everything that you have

For k=3 case, the model is the following:

$$\hat{u}^2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \delta_3 x_3 + \delta_4 x_1^2 + \delta_5 x_2^2 + \delta_6 x_3^2 + \delta_7 x_1 x_2 + \delta_8 x_1 x_3 + \delta_9 x_2 x_3 + error.$$

From that point, the test is similar to LM tests

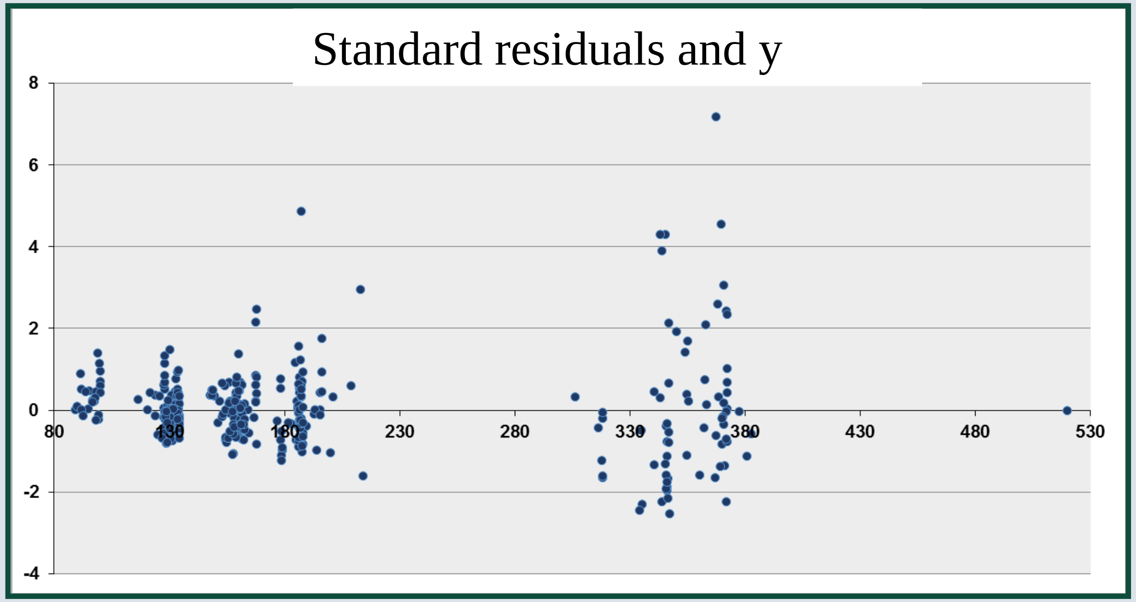
Check how the variables explain the square of estimated error term

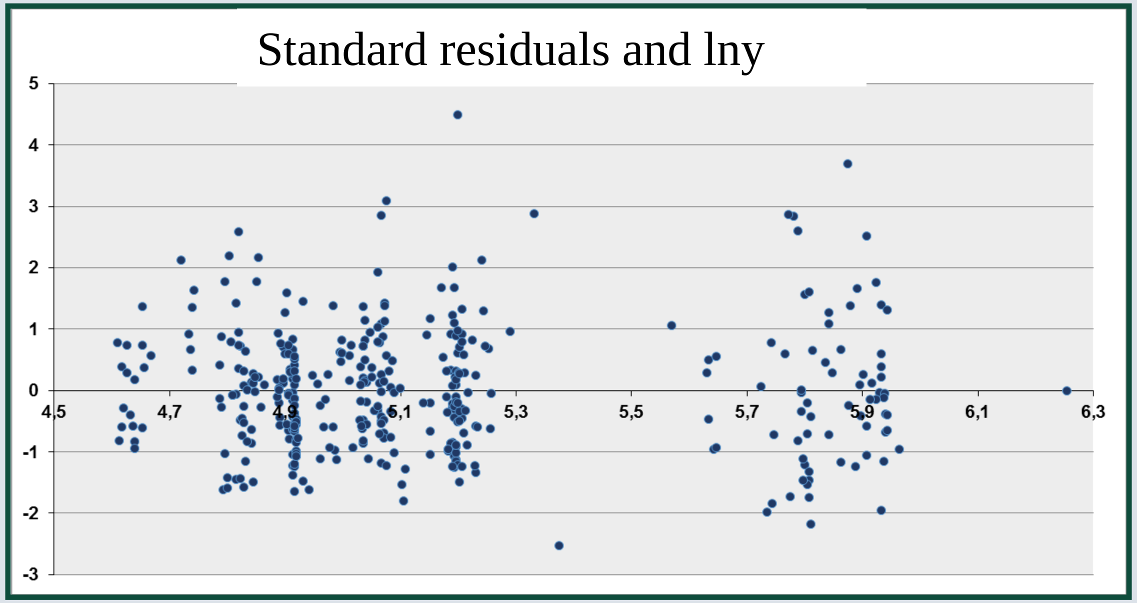
We can use quadratic form or quadratic+interaction

General Treatment

General solutions:

- Use of intensity ratios
- Changing functional form:
  - Logarithm transformation
  - Quadratic effects
  - Interactions





Repeat
Heteroskedasticity
Treatment

Robust standard error

Situation: B parameters are unbiased, standard error is biased
Idea: try to correct the standard errors
Solution: Heteroscedasticity Consistent Covariance Matrix, HCCM

$$(\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top \hat{\boldsymbol{\Omega}} \boldsymbol{X} (\boldsymbol{X}^\top \boldsymbol{X})^{-1},$$

$$\hat{\boldsymbol{\Omega}} = \mathrm{diag}(\hat{u}_1^2, \hat{u}_2^2, \ldots, \hat{u}_n^2);$$

Several solutions: White, Eicker etc
White (1980):
https://www.jstor.org/stable/1912934?seq=13#metadata_info_tab_contents

Models

Generalized Least Squares (GLS) - In practice FGLS (feasible)

$$\widehat{\beta_{\mathrm{GLS}}} = \left(\mathbf{X}^T \mathbf{\Omega}^{-1} \mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{\Omega}^{-1} \mathbf{y}$$

WLS

Weighted Least Squares

We should know the original weights, these are not estimated!

Thank you for your attention!