Post Hoc Analysis – Tukey's Test

Step 0: Verify Assumptions
Tukey's test has five assumptions.

1. The k samples are each obtained using simple random sampling.
2. The k samples data independent of each other within and among the samples.
3. The k populations are normally distributed.
4. The k populations have equal variances.
5. A decision to reject the null hypothesis that $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$ was made during the one-way ANOVA.

Step 1: State the Hypothesis
A claim is made regarding pairs of population means. This claim is used to determine the following null and alternative hypotheses.

H_0: $\mu_i = \mu_j$
H_1: $\mu_i \neq \mu_j$

Step 2: Select a Level of Significance
The level of significance α is determined by the one selected in the one-way ANOVA.

Step 3: Calculate the Test Statistic
The test statistic for Tukey's test is given by:

$$q = \frac{\bar{x}_j - \bar{x}_i}{\sqrt{\frac{s^2}{2}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$

where $\bar{x}_j > \bar{x}_i$,
s^2 is the mean square error estimate of σ^2 (MSE) from ANOVA,
n_i is the sample size from population i, and
n_j is the sample size from population j.

Step 4: Determine the Decision Criterion

The Classical Approach: Find the Critical Value for Tukey's Test
The critical value for Tukey's test using a familywise error rate α is given by

$q_{\alpha,v,k}$

where

$v = n - k$, the degrees of freedom due to error from ANOVA, and
k is the total number of mean being compared.

Step 5: Make a Decision
Reject the null hypothesis if $q \geq q_{\alpha,v,k}$.
Do not reject the null hypothesis if $q < q_{\alpha,v,k}$.

Step 6: State the Conclusion
State the conclusion of the hypothesis test based on the decision made and with respect to the pairwise claim.

Reject H_0	There is sufficient evidence (at the α level) to conclude that the means of populations *i* and *j* are significantly different.
Do Not Reject H_0	There is not sufficient evidence (at the α level) to conclude that the means of populations *i* and *j* are significantly different.