Combinational Circuits Lab

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Problem 1:

Design a circuit that has three inputs and two outputs. Output 1 indicates when the 3 bit unsigned input is odd and output 2 indicates when the 3 bit unsigned input value is even. Provide the initial equations that describe your circuit and minimize your output equations using k-maps. Draw your circuits from the minimized equations using AND, OR, and NOT gates. Now, given that you have two outputs, two initial equations, two k-maps, and two solutions, you should have two circuits:

1. These circuits should be minimal, but, what does this mean?
2. If possible, draw a (third) simpler circuit that will give both outputs.

Solution:

Truth Table:

Inputs			Outputs	
A	B	C	X (odd)	Y (even)
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

Initial Equations:

$$X = A'B'C + A'BC + AB'C + ABC$$

$$Y = A'B'C' + A'BC' + AB'C' + ABC'$$

Note: For all K-Maps, highlighted cells indicate groupings.

K-Map For Equation X:

	AB				
		00	01	11	10
C	0	0	0	0	0
	1	1	1	1	1

K-Map For Equation Y:

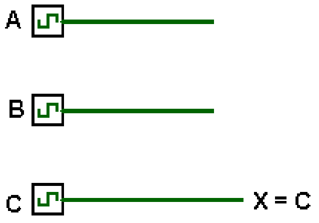
	AB				
		00	01	11	10
C	0	1	1	1	1
	1	0	0	0	0

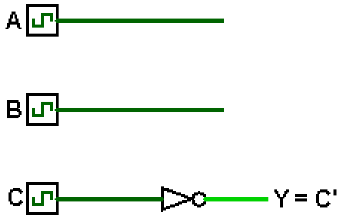
Simplified Equations:

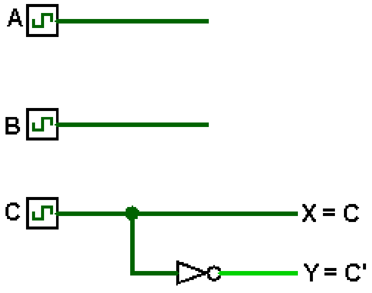
$$X = C$$

$$Y = C'$$

Circuits:


Circuit for X Equation


Circuit For Y Equation


Combined Circuit For Both Outputs

Description:

To solve this problem, I used all of the methods specified in the prompt and discussed in class. First, by creating a truth table displaying the 3 inputs and the 2 outputs for each input. Then, using the truth table, I set up the initial equations in sum of products form. Then I designed Karnaugh maps for each equation. Then, using the karnaugh maps, I simplified the two equations to get $X = C$ and $Y = C'$. Finally, I designed the two circuits for both of these simplified equations and designed a simpler circuit combining those two that would give outputs for both. The circuits were quite simple after simplifying the equations. For the output X which indicates that the input was odd, it ended up with the output being equal to the input. For the output Y which indicates that the input was even, it ended up with the output being equal to the negation of the input. For the combined circuit, we have the input leading straight to output X and leading into a NOT gate which leads to output Y. As for the question at the bottom of the problem prompt, the circuits should be minimal because I converted the truth table to equations and simplified them with K-maps. This means that the circuits have the smallest number of components necessary.

Problem 2:

Design a circuit that has four inputs and three outputs. Output one indicates whether the two's complement signed 4 bit input is larger than 5, output two indicates if the signed 4 bit input is smaller than -5, and output three indicates that the signed 4 bit input is equal to zero. Provide the initial equations that describe your circuit and minimize your output equations using k-maps. Draw your circuits from the minimized equations using AND, OR, and NOT gates.

1. Redraw your equations using NAND gates only.
2. You should have three NAND gate circuits. Think about whether or not you can reuse common segments of your NAND only circuits. If so, draw a new common circuit that is as minimal as you can achieve and describe your process for creating your circuit. If not, discuss why it's not possible.

Solution:

Truth Table:

Inputs				Outputs		
A	B	C	D	X (>5)	Y (<-5)	Z (== 0)
0	0	0	0	0	0	1
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	0	1	0	0	0

0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	0	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	0	0	0

Initial Equations:

$X = A'BCD' + A'BCD$

$Y = AB'C'D' + AB'C'D + AB'CD'$

$Z = A'B'C'D'$

K-Map For Equation X:

		AB			
		00	01	11	10
CD	00	0	0	0	0
	01	0	0	0	0
	11	0	1	0	0
	10	0	1	0	0

K-Map For Equation Y:

Note: For this K-map, there are two groupings both with two cells. It just is not possible for me to highlight both groupings in two different colors. The first grouping is with the two 1’s in the top right corner. The second grouping is with the 1 in the top right corner and the 1 in the bottom right corner.

		AB			
		00	01	11	10
CD	00	0	0	0	1
	01	0	0	0	1
	11	0	0	0	0
	10	0	0	0	1

K-Map For Equation Z:

		AB			
		00	01	11	10
CD	00	1	0	0	0
	01	0	0	0	0
	11	0	0	0	0
	10	0	0	0	0

Simplified Equations:

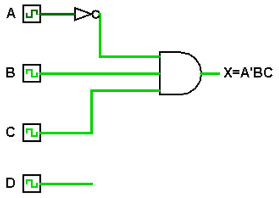
$X = A'BC$

$Y = AB'C' + AB'D'$

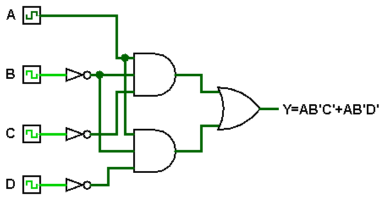
$Z = A'B'C'D'$

Circuits:

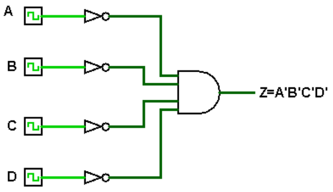
Circuit For X Equation



Circuit For Y Equation

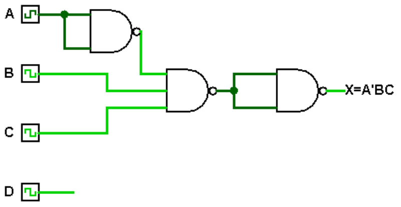


Circuit For Z Equation

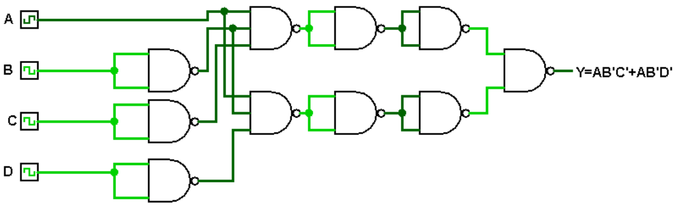


Circuits Using Only NAND Gates

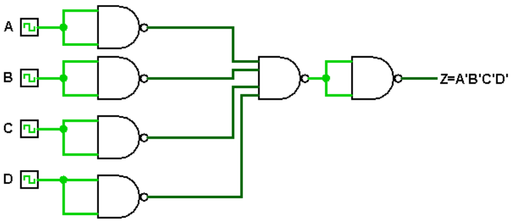
For X Equation



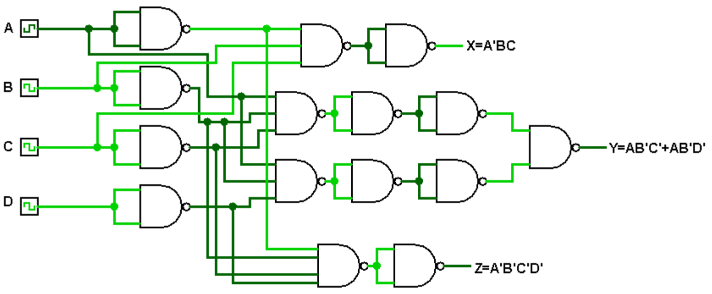
For Y Equation



For Z Equation



Common Circuit Reusing Segments of Previous Circuits



Description:

To solve this problem, I created a truth table for the inputs and outputs. I then used the truth table to create 3 initial equations: $X = A'BCD' + A'BCD, Y = AB'C'D' + AB'C'D + AB'CD'$, and $Z = A'B'C'D'$. With these, I designed a K-map for each equation and then simplified each equation using the K-maps to get: $X = A'BC, Y = AB'C' + AB'D'$, and $Z = A'B'C'D'$. Finally, I designed circuits for each of those simplified equations. The output X, indicating that the input is greater than 5, outputs 1 when both inputs B and C are 1 and A is 0. Output Y, indicating the input is less than -5, outputs 1 when either A is 1 and B and C are 0 or A is 1 and B and D are 0. Output Z, indicating the input is equal to 0, outputs 1 when all 4 inputs A, B, C, and D are 0.

Problem 3:

Consider an apartment that has two bedrooms and a shared hallway to a common area. There is a single hall light and a switch in each room to turn on the hall light. No matter what position each switch is in, flipping either switch turns the hall light on if off, or off if on. Your goal is to design a logic circuit to represent this problem. Define the inputs and outputs and then provide the initial equations that describe your circuit and minimize your output equations using k-maps. Draw your circuits from the minimized equations using AND, OR, and NOT gates.

1. Now that you've designed this circuit can you reduce the number of gates? You may use any kind of gates that we have discussed in class.

2. Your landlord is thinking of adding a third bedroom. Can your design be easily modified to accommodate this? Why or why not?

Solution:

Truth Table:

Inputs		Output
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

K-Map For Equation:

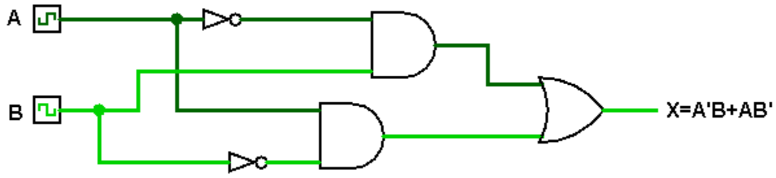
		A	
B		0	1
	0	0	1
	1	1	0

Simplified Equation:

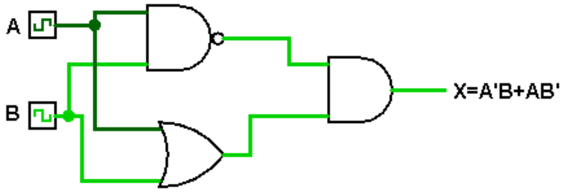
$X = A'B + AB'$

Circuits:

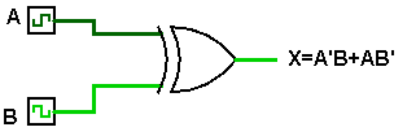
Circuit For Equation X



Reduced Circuit For Equation For X



Reduced Circuit For Equation X Implementing XOR Gate



Description:

To solve this problem, the methods I used were similar to previous problems. First, I created a truth table for the inputs and output. Then, using the truth table, I set up the initial equation in sum of products form. Then I designed a K-map for the equation. Then, using the karnaugh map, I simplified the equation to get $X = A'B + AB'$. Finally, I designed the circuit for that equation and then formed two reductions to that circuit using a NAND gate in one and a XOR gate in the other. For the output X indicating the light being on or off, it was equal to 1 if either A or B was 1 (ie. if either light switch was turned on). When reducing the circuit, I found that the simplified equation could easily be implemented using a single XOR gate. As for the question at the bottom of the problem prompt, if a third bedroom was added, the logic would become more complex because the circuit relies on the fact that there are only two inputs. It wouldn’t be as simple as just adding another switch into the circuit. The logic would change and so would the circuit.

Problem 4:

A given circuit has four inputs. Two of the inputs are considered the fractional portion of a binary number while the other two inputs are considered the integral portion of the binary number. The outputs of this circuit should represent a 2-bit binary number associated with the 4-bit input but with rounding up and down. In other words, if the input is greater or equal to 0.5, the output should represent the input rounded up. Otherwise, its output should represent the input rounded down to the nearest integer. Note: For this exercise you may assume that the input will never be larger than can be represented by the output, in other words, it will not *overflow.* You may treat any overflow input conditions as don't cares. Provide a truth table, a reduced Boolean equation and circuit diagram for your solution using AND, OR, and NOT gates.

Solution:

Truth Table:

Inputs				Outputs	
A	B	C	D	X	Y
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	0	1
0	1	0	1	0	1
0	1	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	**X**	**X**
1	1	1	1	**X**	**X**

Initial Equations:

$X = A'BCD' + A'BCD + AB'C'D' + AB'C'D + AB'CD' + AB'CD + ABC'D' + ABC'D$

$Y = A'B'CD' + A'B'CD + A'BC'D' + A'BC'D + AB'CD' + AB'CD + ABC'D' + ABC'D$

K-Map For X Equation:

Note: The “don’t cares” are in two groupings, the green and the yellow.

		AB			
		00	01	11	10
CD	00	0	0	1	1
	01	0	0	1	1
	11	0	1	**X** (1)	1
	10	0	1	**X** (1)	1

K-Map For Y Equation:

		AB			
		00	01	11	10
CD	00	0	1	1	0
	01	0	1	1	0
	11	1	0	**X** (0)	1
	10	1	0	**X** (0)	1

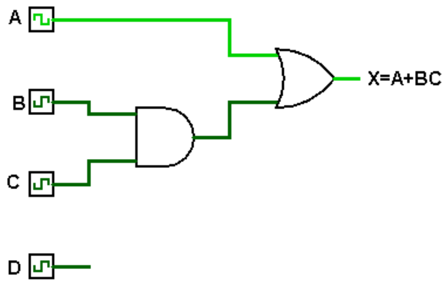
Simplified Equations:

$X = A + BC$

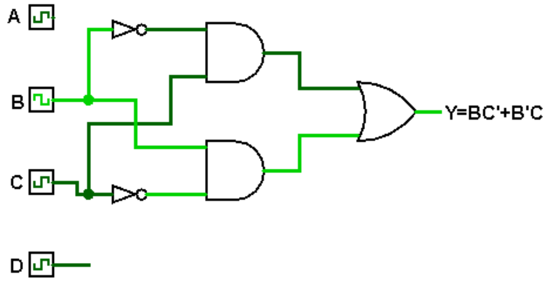
$Y = BC' + B'C$

Circuits:

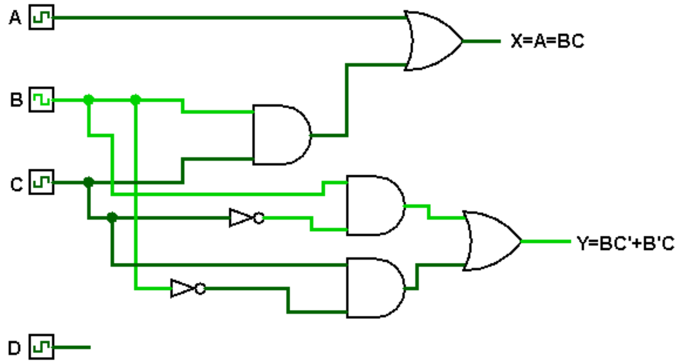
Circuit For Equation X



Circuit For Equation Y



Combined Circuit



Description:

To solve this problem, I first created a truth table for all the inputs and outputs. Then, using the truth table, I created the initial equations for the problem X and Y. Then, I designed K-maps for each equation. Then, using the K-maps, I simplified the equations to get $X = A + BC$ and $Y = BC' + B'C$. Finally, I designed the circuit for each equation and combined those two circuits to make one simple circuit including both outputs. There were two “don’t care” values for each output indicating that the input would result in an overflow. This was very helpful when reducing the equations with Karnaugh maps as I could choose either 0 or 1 to be put into the cell.