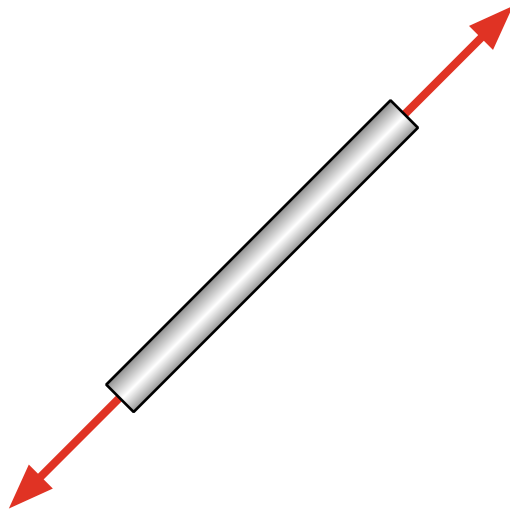


Structural Analysis with the Direct Stiffness Method

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1 Truss Direct Stiffness Method

The *Direct Stiffness Method* is a *Finite Element Method* of analysis that models structural elements as springs. We will begin with deriving the axial stiffness of a structural element. [Felippa, 2000]

1.1 Axial Stiffness

Members in a truss are modelled as springs with only axial deformation. The axial force in the spring is modelled by Hooke's law:

$$F = ku \quad (1)$$

where F is the axial force, k is the axial stiffness, and u is the axial deformation. To define the axial stiffness we will look at a member of constant cross-sectional area with only elastic deformation. The stress in such a member is defined by the stress-strain relationship:

$$\sigma = E\varepsilon \quad (2)$$

where σ is the stress, E is the modulus of elasticity, and ε is the strain. Multiplying both sides of the equation by the cross-sectional area, A , will convert the stress into a force.

$$\sigma A = E\varepsilon A \quad (3)$$

$$F = E\varepsilon A \quad (4)$$

The engineering definition of strain will be used here. Strain is defined as:

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{u}{L} \quad (5)$$

where L is the original length of the member and ΔL is the change in length of the member. Then substituting in the definition of engineering strain into the previous equation:

$$F = EA \frac{u}{L} \quad (6)$$

Therefore the axial stiffness from the equation is $\frac{AE}{L}$ for any structural member. The axial force of a structural element can be rewritten as:

$$F = \frac{AE}{L}u \quad (7)$$

1.2 Local Element Stiffness

Each element in a structural system will have its own local element stiffness equations.

1.3 Global Element Stiffness

1.4 Global Structural Stiffness

References

[Felippa, 2000] Felippa, C. A. (2000). A historical outline of matrix structural analysis: A play in three acts.