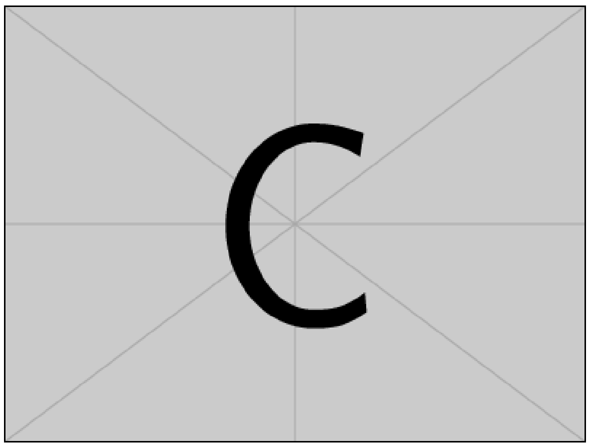
CEE384

Interpolation

Interpolation of discrete datasets



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Updated: October 12, 2018

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1 Quadratic Splines

A *quadratic spline* is a set of piecewise quadratic functions which intersect all points in a dataset. There are three conditions that mathematically govern these functions. The first condition is that the functions must be *continuous*. The second condition is that the functions must be *smooth*, meaning that the derivatives are continous.

There are $3n$ unknowns, where n is the number of data points in the dataset. This is because there is an a_i, b_i, and c_i, for each i data point. We will look at a particular example with 4 points (shown in table 1), and three connecting splines.

Table 1: Example dataset for quadratic spline

x	x_0	x_1	x_2	x_3
y	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$

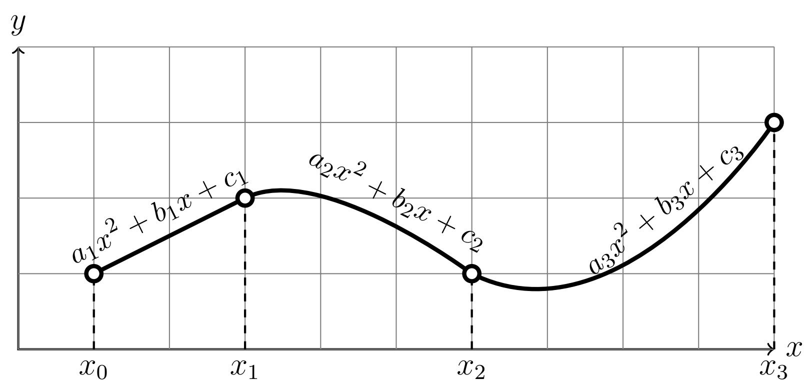

Figure 1: Example dataset plot with splines

1.1 Continuous Functions

The spline equations should be continuous and go through all the points in the dataset. This means that the

$$f(x) = \begin{cases} a_1x^2 + b_1x + c_1 & x_0 < x < x_1 \\ a_2x^2 + b_2x + c_2 & x_1 < x < x_2 \\ a_3x^2 + b_3x + c_3 & x_2 < x < x_3 \end{cases} \tag{1}$$

The first spline equation must be equal at the start of the interval, x_1, to the value of dataset $f(x_0)$. As an equation this is:

$$f(x_0) = a_1x_0^2 + b_1x_0 + c_1 \tag{2}$$

At the point, x_1, the first spline equation must be equal to the output value from the dataset, $f(x_1)$. The second equation must *also* be equal to this. Therefore this generates two additional equations:

$$f(x_1) = a_1x_1^2 + b_1x_1 + c_1 \tag{3}$$
$$f(x_1) = a_2x_1^2 + b_2x_1 + c_2 \tag{4}$$

At the second data point, x_2, the second and thrid spline must be equal to $f(x_2)$. This generates two additional equation.

$$f(x_2) = a_2x_2^2 + b_2x_2 + c_2 \tag{5}$$
$$f(x_2) = a_3x_2^2 + b_3x_2 + c_3 \tag{6}$$

Finally, the last spline must go through the last point, generating one more continuity equation:

$$f(x_3) = a_3x_3^2 + b_3x_3 + c_3 \tag{7}$$

This results in 6 equations total leaving 3 more unknowns that must be found. To generalize this pattern: for every n points the continuity condition generates $2(n-1)$ equations.

1.2 Smooth Functions

Pseudocode

1. Define dataset or take dataset as input if programmed as a stand alone function
2. Initialize **A** as a matrix of zeros (3n,3n)
3. Loop through number of points and use continuous function information
   (a) Set values for first equation
4. Loop through number of points and use continuous derivatives information
   (a) Use interior points