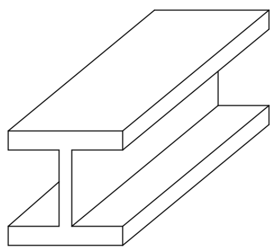
CEE421

Concrete Beam Design

Beam Design



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1 Beam Analysis

1.1 Ultimate Capacity

Concrete beams are analyzed at the *ultimate state* to determine the ultimate strength or ultimate capacity of the beam. When a concrete beam is under a distributed load top fiber in compression bottom fiber in compression. The bottom part of the beam below the neutral axis must be reinforced with steel to prevent cracking from the bottom fibers propagating upward causing failure of the whole beam.

At this state the concrete at the top of the beam is *just* beginning to crush. ACI states that the compressive strain of concrete occurs at $\varepsilon_c = \varepsilon_{cu} = 0.003$. This failure condition will be used to determine the needed tensile reinforcement. The strain profile of the cross section is linear. The compressive side of the beam has a non-linear stress profile. The tensile side of the beam is assumed to only be supported by the tensile force of the reinforcing steel.

1.2 The Whitney Stress Block

The *Whitney Stress Block* also referred to as the *equivalent stress block* is the solution for the non-linear stress profile of concrete in compression.

The parameter $\beta_1$is calculated by equation 1 which is taken from ACI Table 22.2.2.4.3, where $f'_c$is taken to be in units of psi.

$$\beta_1 = \begin{cases} 0.85 & f'_c \leq 4000 \\ 0.85 - 0.05\left[\dfrac{f'_c - 4000}{1000}\right] & 4000 \leq f'_c \leq 8000 \\ 0.65 & f'_c > 8000 \end{cases} \tag{1}$$

1.3 The Balance Condition

The balance condition is the condition which the concrete crushes at the same time that the steel yields. To solve for the neutral axis at the balance condition we use similar triangles.

$$\frac{c}{\varepsilon_{cu}} = \frac{d-c}{\varepsilon_s} \tag{2}$$

$$c\varepsilon_s = \varepsilon_{cu}(d-c) \tag{3}$$

$$c\varepsilon_s = d\varepsilon_{cu} - c\varepsilon_{cu} \tag{4}$$

$$\boxed{c_{bal} = \frac{d\varepsilon_{cu}}{\varepsilon_s + \varepsilon_{cu}}} \tag{5}$$

Using c_{bal} we can calculate the area of the steel $A_s$at the balance condition. This is the area of steel needed for the steel to yield at the same time that the concrete crushes.

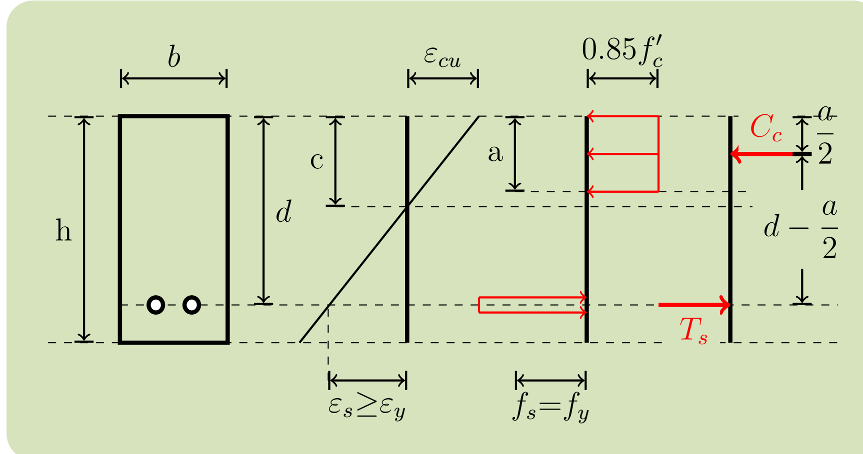
$$A_{s,bal} = \frac{0.85 f'_c \beta_1 c_{bal} b}{f_y} \tag{6}$$

Next, we find the reinforcement ratio of the balance condition. The reinforcement ratio is the ratio of the area of steel to the area above the tension steel (bd). Therefore to calculate the reinforcement ratio take equation 6 as the area of steel. This results in equation 7.

$$\rho_{bal} = \frac{0.85 f'_c \beta_1}{f_y} \left[\frac{\varepsilon_{cu}}{\varepsilon_s + \varepsilon_{cu}} \right] \tag{7}$$

This equation can be specialized by plugging in the value of $\varepsilon_{cu}$and multiply the numerator and denominator by the modulus of elasticity of steel $E_s = 29000$ ksi the result is shown in equation 8. The value for $f_y$that is plugged in must be in ksi.

$$\rho_{bal} = \frac{0.85 f'_c \beta_1}{f_y} \left[\frac{87}{87 + f_y} \right] \tag{8}$$



2 Beam Design

2.1 Deriving the Design Equations

Equation 9 is the basic design relationship for concrete beams.

$$\phi M_n \geq M_u \tag{9}$$

where $M_n$is the nominal moment capacity of the beam and $M_u$is the ultimate moment in the beam.

The nominal moment of the beam is found by taking moments about the resultant compressive force. The only force left is the tension force in the steel.

$$\phi T_s z \geq M_u \tag{10}$$

Next, plug in the force in the tension steel and the distance from the compressive force to the tensile force. Note that the force in the tension steel is taken to be the area of the steel multiplied by the yield stress. This is because the steel is assumed to be yielding.

$$\phi\left(A_s f_y\right)\left(d - \frac{1}{2}a\right) \geq M_u \tag{11}$$

Now we will derive the height of the "Whitney Stress block" or the "equivalent stress block". From equilibrium we know the tensile force in the steel will be equal to the compressive force in the concrete. This fact will be used to isolate the height of the stress block

$$T_s = C_c \tag{12}$$
$$A_s f_y = 0.85 f_c' a b \tag{13}$$

Solve for a

$$a = \frac{A_s f_y}{0.85 f_c' b} \tag{14}$$

Next, plug in the value of a into equation 11.

$$\phi\left(A_s f_y\right)\left(d - \frac{1}{2}\frac{A_s f_y}{0.85 f_c' b}\right) \geq M_u \tag{15}$$

We want to get the equation in terms of the *reinforcement ratio*, ρ. We can accomplish this by dividing both sides of the equation by bd^2.

$$\phi\left(\frac{A_s}{bd} f_y\right)\left(\frac{d}{d} - \frac{1}{2}\frac{A_s f_y}{0.85 f_c' b d}\right) \geq \frac{M_u}{bd^2} \tag{16}$$

Substitute the reinforcement ratio into the equation

$$\phi\left(\rho f_y\right)\left(1 - \frac{1}{2}\frac{\rho f_y}{0.85 f_c'}\right) \geq M_u \tag{17}$$

$$\rho_{design} = \frac{0.75\rho_b}{2} \tag{18}$$

$$\phi R \geq \frac{M_u}{bd^2} \tag{19}$$

where

$$R(f'_c, f_y, \rho) = \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c}\right) \tag{20}$$

To solve for the depth of the neutral axis:

$$\begin{align} T &= C \tag{21}\\ A_s f_s &= 0.85 f'_c ab \tag{22}\\ A_s E_s \varepsilon_s &= 0.85 f'_c \beta_1 cb \tag{23}\\ A_s E_s \frac{\varepsilon_{cu}(d-c)}{c} &= 0.85 f'_c \beta_1 cb \tag{24}\\ A_s E_s (d-c)\varepsilon_{cu} &= 0.85 f'_c \beta_1 c^2 b \tag{25} \end{align}$$

Solving for the neutral axis c is a matter of using the quadratic formula. This requires putting the equation into the following form:

$$0.85 f'_c \beta_1 c^2 b + A_s E_s \varepsilon_{cu} c - A_s E_s \varepsilon_{cu} d = 0 \tag{26}$$

2.2 Spacing Requirements

When designing beams you must ensure that there is enough horizontal space to fit the steel bars. The width of the beam must fit the diameters of the bars, ACI required spacing between the bars, and cover on the left and right of the bars. The minimum spacing between bars is the maximum of the following three items as per ACI 25.2.1.

- 1 inch
- $\frac{4}{3}$ diameter of largest aggregate
- diameter of the reinforcing bar

2.3 Automating beam design

Taking the fundamental design equation from equation 19 and rearranging for the bd^2 term yields:

$$bd^2 = \frac{M_u}{\phi R(f'_c, f_y, \rho)} \tag{27}$$

Now, we can take this equation and divide both sides d^3. This puts the left side as the ratio of b/d and the right side is only dependent on d^3.

$$\frac{b}{d} = \frac{M_u}{d^3 \phi R(f'_c, f_y, \rho)} \tag{28}$$

The ideal ratio is between 0.4 and 0.6. This can be substituted on the left side of the equation as a constant value which can be specified as an input. Furthermore the equation is rewritten such that all terms are on the right side and the equation is equal to zero.

$$g(d) = \frac{M_u}{d^3 \phi R(f'_c, f_y, \rho)} - \lambda_1 \tag{29}$$

The tangent to this residual equation is:

$$A(d) = -\frac{3}{d^4} \frac{M_u}{\phi R(f'_c, f_y, \rho)} \tag{30}$$

The root of the residual equation can be solved using Newton's method.

$$d_{i+1} = d_i - \frac{g(d_i)}{A(d_i)} \tag{31}$$

Pseudocode

1. Input physical quantities (f'_c, f_y, etc...)
2. Define relevant functions
3. Set-up and execute Newton's method to solve for d
   (a) Use the residual $g(d)$ and tangent $A(d)$ from equations 29 and 30, respectively.
   (b) Calculate b given the value of d once the solution has converged
4. Round up values of $b$and d
5. Calculate the neeeded $A_s$from ρ_{design}
6. Loop through possible bar sizes in table 1
   (a) Calcualte number of bars required for each size
   (b) Calculate the width required for that bar No.
   (c) If it meets the spacing requirements add it to a list of usable candidates
7. Compute neutral axis from equation 26
8. Check that the steel is yielding
9. Print and plot results

3 T-Beams

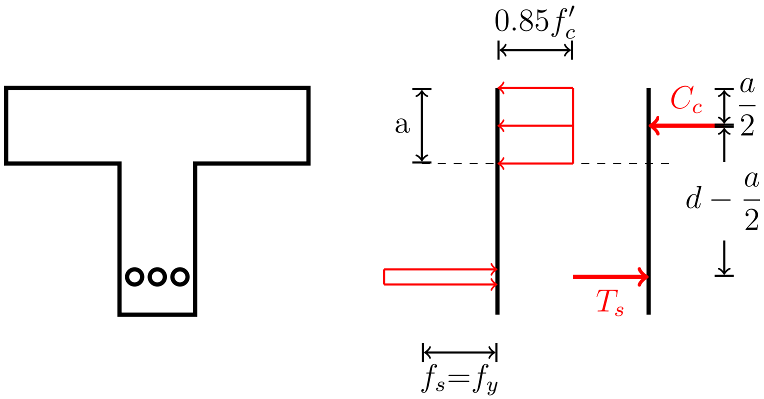


Table 1 shows the dimensions of reinforcing steel bars.

Table 1: Areas, Weights, and Dimensions of Reinforcing Bars

Bar Size (No.)	Weight (lb/ft)	Diameter(in.)	Area (in.^{2})
3	0.376	0.375	0.11
4	0.668	0.500	0.20
5	1.043	0.625	0.31
6	1.502	0.750	0.44
7	2.044	0.875	0.60
8	2.67	1.000	0.79
9	3.40	1.128	1.00
10	4.30	1.270	1.27
11	5.31	1.410	1.56
14	7.65	1.693	2.25
18	13.60	2.257	4.00