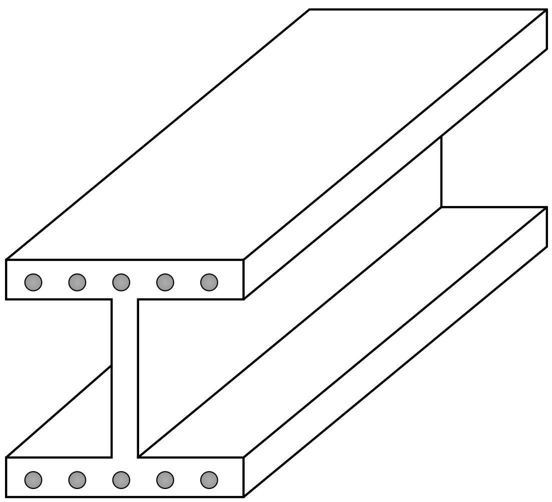
CEE421

Doubly Reinforced Concrete Beam

Beam Design



Arizona State University

Brian Badahdah

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1 Introduction

The goal of this document is to introduce the idea of double reinforced beams and why compression reinforcement is necessary in concrete design.

One goal of concrete design is reduce material costs, and one way to do that is ensuring a tension controlled section. A tension controlled section can use a moment reduction factor of 0.9. However, to ensure tension control, the steel must yield before the concrete will crush. Which means there is a maximum amount of tension reinforcement that can be added before tension steel will not yield before the concrete crushes. Adding compression reinforcement allows the addition of tension reinforcement.

2 Moment Capacity of Singly Reinforced Beams

We can calculate the maximum possible moment strength of a beam at the balance condition (when the tension steel yields at the same time the concrete crushes).

$$\rho_{bal} = 0.85\beta_1 \left[\frac{f'_c}{f_y}\right]\left[\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005}\right] \tag{1}$$

Next, calculate the area of steel:

$$A_s = \rho_{bal} bd \tag{2}$$

Calculate the depth of the Whitney stress block:

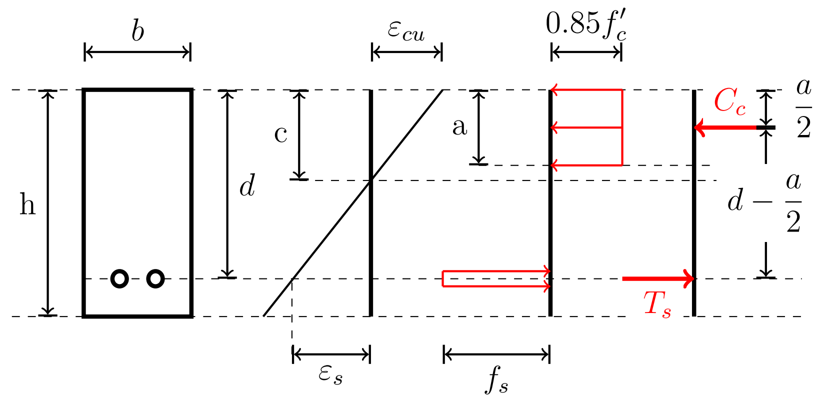
$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{3}$$

Finally, calculate the maximum moment strength for the tension controlled, singly reinforced concrete beam, using moment equilibrium:

$$M_n = A_s f_y \left[d - \frac{a}{2}\right] \tag{4}$$

If this moment strength is not high enough, and the cross sectional area cannot be increased, then a doubly reinforced beam will be necessary to increase the moment capacity.

3 Piecewise Stress



The stress the compression and tension steel is defined in the following equations.

$$f_s = \begin{cases} E\varepsilon_{cu}\left[\frac{d-c}{c}\right] & \varepsilon_s < \varepsilon_y \\ f_y & \varepsilon_s > \varepsilon_y \end{cases} \tag{5}$$

$$f'_s = \begin{cases} E\varepsilon_{cu}\left[\frac{c-d'}{c}\right] & \varepsilon'_s < \varepsilon_y \\ f'_y & \varepsilon'_s > \varepsilon_y \end{cases} \tag{6}$$

In order to determine the stress in both the tension and compression steel we must first determine the strains. They can be calculated using similar triangles from the strain profile of the cross section.

$$\varepsilon_s' = \varepsilon_{cu}\left[\frac{c-d'}{c}\right] \tag{7}$$

$$\varepsilon_s = \varepsilon_{cu}\left[\frac{d-c}{c}\right] \tag{8}$$

Therefore, to calculate the stress, the strain must be known.

4 Determining the Neutral Axis

The depth to the neutral axis, c, must also be known to compute the stress and strain in the steel. Using force equilibrium of the section the following equation can be found:

$$0 = f_s A_s - f_s' A_s' - 0.85 f_c' \beta_1 c b \tag{9}$$

The neutral axis depth is in each term of this equation, by virtue of the stress piecewise equation, and the bounds of the piecewise as determined by the strain in the steel, which is calculated using the depth of the neutral axis.

If both the compression and tension steel are yielding, then the stress in the steel can be substituted from the previous equations, as the yield stress of the steel.

However, often the compression steel will not yield, and the equation becomes non-linear, and piecewise.

Using Newton's method can efficiently solve the equation for the depth of the neutral axis. With a good initial guess of c, a guess of the strain values of the steel can be determined, followed by the stress, and a better estimate of the neutral axis can be determined.

Taking the derivative of the force equation, with respect to the neutral axis:

$$0 = \frac{\partial f_s}{\partial c} A_s - \frac{\partial f_s'}{\partial c} A_s' - 0.85 f_c' \beta_1 b \tag{10}$$

where, the derivatives of the stress are piecewise functions defined by the following equations:

$$\frac{\partial f_s}{\partial c} = \begin{cases} E\varepsilon_{cu}\left[-\frac{d}{c^2}\right] & \varepsilon_s < \varepsilon_y \\ 0 & \varepsilon_s > \varepsilon_y \end{cases} \tag{11}$$

$$\frac{\partial f_s'}{\partial c} = \begin{cases} E\varepsilon_{cu}\left[\frac{d}{c^2}\right] & \varepsilon_s' < \varepsilon_y' \\ 0 & \varepsilon_s' > \varepsilon_y' \end{cases} \tag{12}$$

References