MAE241

Energy Analysis and Control Volumes

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Contents

1 The First Law

The total energy flowing in and out of the system is equal to the total change in internal energy change in the system. That is mathematically written as:

$$Q - W = \Delta E \tag{1}$$

$$(Q_{in} - Q_{out}) - (W_{out} - W_{in}) = \Delta E \tag{2}$$

Some definitions:

Isothermal Constant temperature

Isobaric Constant pressure

Isochoric Constant volume

2 Conservation of Mass

A control volume is a volume used to model and track physical processes. It is similar to a free body diagram. The control surface is the surface surrounding the volume.

The general conservation of mass equation is:

$$\frac{d}{dt}\int_{cv} \rho dV + \int_{cs} \rho(\mathbf{V}\cdot\mathbf{n})dA = 0 \tag{3}$$

Mass balance for a steady flow process:

$$\frac{dm_{cv}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m} \tag{4}$$

$$\dot{m}_1 = \dot{m}_2 \tag{5}$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \tag{6}$$

For incompressible fluids the density is constant, therefore the density eliminates from both sides.

Total energy for a flowing fluid, the energy per unit mass is:

$$\theta = \underbrace{PV}_{\text{flow energy}} + \underbrace{u}_{\text{internal energy}} + \underbrace{\frac{V^2}{2}}_{\text{kinetic energy}} + \underbrace{gz}_{\text{potential energy}} \tag{7}$$

Amount of energy transport:

$$E_{mass} = m\theta \tag{8}$$

$$= m(h + \frac{1}{2}V^2 + gz) \tag{9}$$

The rate of energy transport:

$$\dot{E}_{mass} = \dot{m}\theta \tag{10}$$

$$= \dot{m}(h + \frac{1}{2}V^2 + gz) \tag{11}$$

Applying energy balance for multiple a control volume with multiple inlets or outlets:

$$\dot{E}_{in} = \dot{E}_{out} \tag{12}$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\theta = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}\theta \tag{13}$$

2.0.1 Diffuser

A diffuser is a device that reduces the velocity of a flowing fluid. The governing equation for a diffuser is:

$$h_2 = h_1 + \frac{V_1^2}{2} \tag{14}$$

2.0.2 Nozzle

A nozzle is a device that increases the velocity of a flowing fluid. The governing equation is:

$$0 = \dot{Q} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}\right] \tag{15}$$

2.0.3 Turbine/Compressor

$$\dot{W} = \dot{m}(h_1 - h_2) \tag{16}$$

2.0.4 Throttling Valve

Reduces pressure

$$\dot{m}(h_1 - h_2) = 0 \tag{17}$$
$$h_1 = h_2 \tag{18}$$

3 Boundary Work

Boundary work is the work done on a system that increases the size of a control volume.

Boundary work at constant pressure:

$$W_b = \int_1^2 PdV \tag{19}$$
$$= P_0 \int_1^2 \tag{20}$$
$$= P_0(V_2 - V_1) \tag{21}$$
$$= mP_0(v_2 - v_1) \tag{22}$$

Boundary work at constant pressure:

$$W_b = \int_1^2 PdV \tag{23}$$
$$= 0 \tag{24}$$

Boundary work at constant temperature:

$$W_b = \int_1^2 PdV \tag{25}$$
$$= P_1 V_1 \ln(\frac{V_2}{V_1}) \tag{26}$$