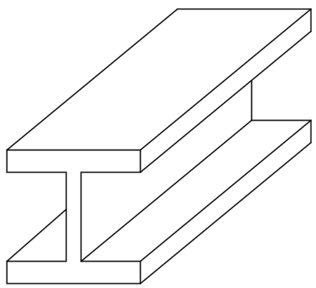
CEE421

Concrete Beam Design

Beam Design



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Updated: October 9, 2018

Contents

1 Beam Analysis

1.1 The Whitney Stress Block

The parameter $\beta_1$is calculated by equation 1 which is taken from ACI Table 22.2.2.4.3, where $f'_c$is taken to be in units of psi.

$$\beta_1 = s \begin{cases} 0.85 & f'_c \leq 4000 \\ 0.85 - 0.05\left[\dfrac{f'_c - 4000}{1000}\right] & 4000 \leq f'_c \leq 8000 \\ 0.65 & f'_c > 8000 \end{cases} \tag{1}$$

1.2 The Balance Condition

The balance condition is the condition which the concrete crushes at the same time that the steel yields. To solve for the neutral axis at the balance condition we use similar triangles.

$$\frac{c}{\varepsilon_{cu}} = \frac{d-c}{\varepsilon_s} \tag{2}$$

$$c\varepsilon_s = \varepsilon_{cu}(d-c) \tag{3}$$

$$c\varepsilon_s = d\varepsilon_{cu} - c\varepsilon_{cu} \tag{4}$$

$$c_{bal} = \frac{d\varepsilon_{cu}}{\varepsilon_s + \varepsilon_{cu}} \tag{5}$$

Using c_{bal} we can calculate the area of the steel $A_s$at the balance condition.

$$A_s = \frac{0.85 f'_c \beta_1 c_{bal} b}{f_y} \tag{6}$$

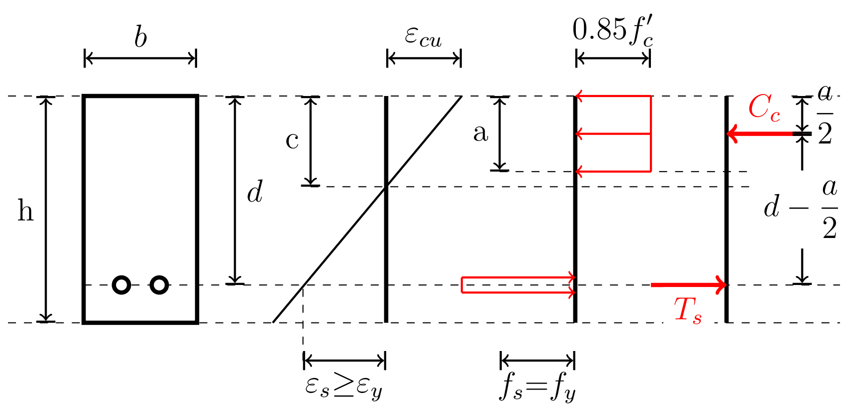
Next, we find the reinforcement ratio of the balance condition:

$$\rho_{bal} = \frac{0.85 f'_c \beta_1}{f_y}\left[\frac{\varepsilon_{cu}}{\varepsilon_s + \varepsilon_{cu}}\right] \tag{7}$$

$$= \frac{0.85 f'_c \beta_1}{f_y}\left[\frac{87}{87 + f_y}\right] \tag{8}$$

2 Beam Design

2.1 Deriving the Design Equations



Equation 9 is the basic design relationship for concrete beams.

$$\phi M_n \geq M_u \tag{9}$$

where $M_n$is the nominal moment in the beam and $M_u$is the ultimate moment in the beam.

The nominal moment of the beam is found by taking moments about the resultant compressive force. The only force left is the tension force in the steel.

$$\phi T_s z \geq M_u \tag{10}$$

Next, plug in the force in the tension steel and the distance from the compressive force to the tensile force. Note that the force in the tension steel is taken to be the area of the steel multiplied by the yield stress. This is because the steel is assumed to be yielding.

$$\phi \left(A_s f_y\right) \left(d - \frac{1}{2} a\right) \geq M_u \tag{11}$$

Now we will derive the height of the "Whitney Stress block" or the "equivalent stress block". From equilibrium we know the tensile force in the steel will be equal to the compressive force in the concrete. This fact will be used to isolate the height of the stress block

$$T_s = C_c \tag{12}$$
$$A_s f_y = 0.85 f'_c a b \tag{13}$$

Solve for a

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{14}$$

Next, plug in the value of a into equation 11.

$$\phi \left(A_s f_y\right) \left(d - \frac{1}{2} \frac{A_s f_y}{0.85 f'_c b}\right) \geq M_u \tag{15}$$

We want to get the equation in terms of the *reinforcement ratio*, ρ. We can accomplish this by dividing both sides of the equation by bd^2.

$$\phi \left(\frac{A_s}{bd} f_y\right) \left(\frac{d}{d} - \frac{1}{2} \frac{A_s f_y}{0.85 f'_c b d}\right) \geq \frac{M_u}{bd^2} \tag{16}$$

Substitute the reinforcement ratio into the equation

$$\phi \left(\rho f_y\right) \left(1 - \frac{1}{2} \frac{\rho f_y}{0.85 f'_c}\right) \geq M_u \tag{17}$$

To calculate the reinforcement ratio at the balance condition:

$$c_{bal} = \left[\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right] d \tag{18}$$

$$\rho_{bal} = \frac{0.85 f'_c \beta_1}{f_y} \left[\frac{87}{f_y + 87}\right] \tag{19}$$

$$\rho_{design} = \frac{0.75 \rho_b}{2} \tag{20}$$

$$\phi R \geq \frac{M_u}{bd^2} \tag{21}$$

where

$$R(f'_c, f_y, \rho) = \rho f_y \left(1 - \frac{f_y}{1.7 f'_c \rho}\right) \tag{22}$$

To solve for the depth of the neutral axis:

$$T = C \tag{23}$$
$$A_s f_s = 0.85 f'_c ab \tag{24}$$
$$A_s E_s \varepsilon_s = 0.85 f'_c \beta_1 cb \tag{25}$$
$$A_s E_s \frac{\varepsilon_{cu}(d-c)}{c} = 0.85 f'_c \beta_1 cb \tag{26}$$
$$A_s E_s (d-c)\varepsilon_{cu} = 0.85 f'_c \beta_1 c^2 b \tag{27}$$
$$\tag{28}$$

Solving for the neutral axis c is a matter of using the quadratic formula. This requires putting the equation into the following form:

$$0.85 f'_c \beta_1 c^2 b + A_s E_s \varepsilon_{cu} c - A_s E_s \varepsilon_{cu} d = 0 \tag{29}$$

Table 1 shows the dimensions of reinforcing steel bars.

2.2 Automating beam design

Taking the fundamental design equation from equation 21 and rearranging for the bd^2 term yields:

$$bd^2 = \frac{M_u}{\phi R(f'_c, f_y, \rho)} \tag{30}$$

Now, we can take this equation and divide both sides d^3. This puts the left side as the ratio of b/d and the right side is only dependent on d^3.

$$\frac{b}{d} = \frac{M_u}{d^3 \phi R(f'_c, f_y, \rho)} \tag{31}$$

Table 1: Areas, Weights, and Dimensions of Reinforcing Bars

Bar Size (No.)	Weight (lb/ft)	Diameter(in.)	Area (in.^{2})
3	0.376	0.375	0.11
4	0.668	0.500	0.20
5	1.043	0.625	0.31
6	1.502	0.750	0.44
7	2.044	0.875	0.60
8	2.67	1.000	0.79
9	3.40	1.128	1.00
10	4.30	1.270	1.27
11	5.31	1.410	1.56
14	7.65	1.693	2.25
18	13.60	2.257	4.00

The ideal ratio is between 0.4 and 0.6. This can be substituted on the left side of the equation as a constant value which can be specified as an input. Furthermore the equation is rewritten such that all terms are on the right side and the equation is equal to zero.

$$g(d) = \frac{M_u}{d^3 \phi R(f'_c, f_y, \rho)} - \lambda_1 \tag{32}$$

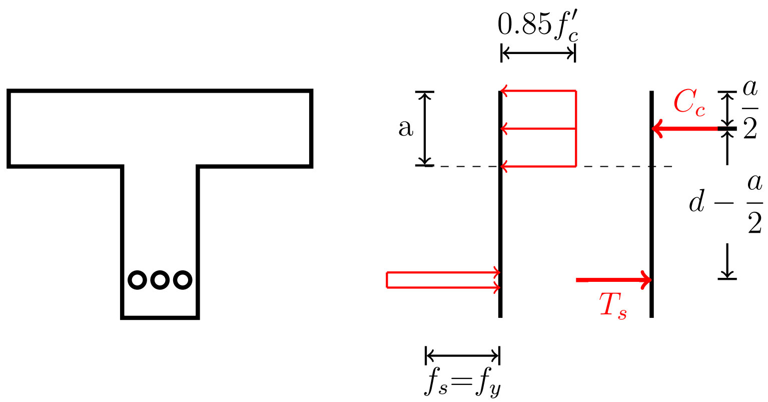
The tangent to this residual equation is:

$$A(d) = -\frac{3}{d^4} \frac{M_u}{\phi R(f'_c, f_y, \rho)} \tag{33}$$

The root of the residual equation can be solved using Newton's method.

Pseudocode

1. Input physical quantities (f'_c,f_y,etc...)
2. Define relevant functions
3. Set-up and execute Newton's method to solve for d
   (a) Use the residual $g(d)$ and tangent $A(d)$ from equations 32 and 33, respectively.
   (b) Calculate b given the value of d once the solution has converged
4. Round up values of b and d
5. Loop through possible bar sizes in table 1
   (a) Calcualte number of bars required for each size
   (b) Calculate the width required for that bar No.
   (c) If it meets the spacing requirements add it to a list of usable candidates
6. Compute neutral axis from equation 29
7. Check that the steel is yielding



3 T-Beams

Variables Definitions

A_s Area of tension reinforcing steel. 3

M_n Nominal moment strength. 3

M_u Factored ultimate moment in the beam. 3

. 3

ρ_b The reinforcement ratio of concrete at the balance condition. 3

ε_s Strain of tension steel. 3

ε_y Yield strain of steel. 3

ε_{cu} Ultimate compressive strain of concrete. 3

b Width of compression zone in a beam. 3

d Distance from the extreme fiber in compression. 3

f'_c Yield stress of concrete. 3

f_s Stress of the steel. 3

f_y Yield stress of steel. 3