Math 1301 Practice Final Exam Version 1

Vanderbilt University

8 December 2025

Name: ____

Please do not open the exam until instructed to do so.
You are allowed a one-page (double-sided) formula sheet.
Your instructor may ask to see your formula sheet.
No calculators, phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. (*20 points*)

Q1 $\dfrac{d}{dx}\left[5^{\sqrt{x}} + \log_4(x^x) + \sin^{-1}(4x)\right] = \boxed{}$

Q2 $\dfrac{d}{dx}\sec(x)^{\cos^{-1}(x)} = \boxed{}$

Q3 $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{\tan^{-1}(x)}\right) = \boxed{}$

Q4 The point of horizontal tangents for the curve $y = \ln^2(x+4)$ is $\boxed{}$

Part 2. (*10 points*) Find the interval of convergence for the following power series.

$$p(x) = \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}(x+1)^n$$

Part 3 (*15 points*) Evaluate the following integrals.

Q1 $\displaystyle\int \left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right) \sin(e^x)\,(\ln(e^e))^x \; dx =$

Q2 $\displaystyle\int \ln(x^2+x+1)\; dx =$

Q3 $\displaystyle\int \tan(x)\ln(\cos(x))\; dx =$

Part 4. (*15 points*)

Q1 The value of $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^n(x)\cos(\pi n)}{3^n}$ is

Q2 The values of p such that $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)[\ln(\ln(n))]^p}$ converges are

Q3 The limit of the sequence defined by $a_n = \begin{cases} 2 & n=1 \\ \frac{1}{3-a_{n-1}} & n>1 \end{cases}$ is

Part 5. (*15 points*)

Q1 Find a parametric equation for the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and find the slope of the tangent line at an arbitrary point using the parametric equations.

Q2 Find the length of the polar curve of $r = e^{\theta/2}$ from $0 \leq \theta \leq \pi/2$

Q3 Find the values of theta that the polar curve $r = 1+\cos(\theta)$ has vertical and horizontal tangent lines.

Part 6. (*10 points*)

Q1 The general solution to $\cos(y)\dfrac{dy}{dx} = xe^{x^2+\ln(1+\sin^2(y))}$ is $\boxed{}$

Q2 The integral $\displaystyle\int_1^{\infty} \frac{\cos(x)}{x}\, dx$ **CONVERGES** or **DIVERGES**. (explain your response)

Part 7 (*20 points*) Determine if the following series converge or diverge. Show your work and state any necessary hypotheses.

Q1 $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{(2n)!}$

Q2 $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2+\sin(n)}$

Q3 $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{\pi n}\right)^{n^2}$

Q4 $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^3+\cos^2(n)n^2+3^n n+4}$

Part 8 (*20 points*) Define the function $f(x) = \arctan(\ln(x+1)) - \arccos(\ln(\sqrt[4]{x^2+2x+1}))$

Q1 The domain of $f(x)$ is $\boxed{}$

Q2 $f'(x) = \boxed{}$

Q3 $f(x)$ has an inverse on $\boxed{}$ (justify your answer)!

Q4 $f\left(e^{\sqrt{3}} - 1\right) = \boxed{}$

Part 9 (*20 points*) Define the function $g(x) = a^x$.

Q1 Use logarithmic differentiation to prove $g'(x)$.

Q2 Find the first four terms of the Taylor series of $g(x)$ centered at $x = 3$.

Q3 Using **Q1**, write down a differential equation that $g(x)$ satisfies. Is the differential equation separable?

Q4 The values of a such that $\int_0^\infty g(x)\,dx$ converges are $\boxed{}$.

Part 10 (*25 points*) Consider the power series $h(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$.

Q1 The radius of convergence of $h(x)$ is $\boxed{}$.

Q2 The power series $h(x)$ is given by the function $\boxed{}$.

Q3 The 41 partial sum of $h(1/2)$ is an **OVERESTIMATE** or **UNDERESTIMATE** of $h(1/2)$.

Q4 The number of terms required for to estimate $g(0.1)$ with an error less than $\frac{1}{100}$ is $\boxed{}$.

Q5 Using **Q1**, $\int_0^1 h(x)\ dx =$ $\boxed{}$.

Part 11 *Extra space for work*

End of Exam. Check your work!