

Math 2400 Practice Exam 1

Vanderbilt University

February 4, 2026

Name: _____

Please do not open the exam until instructed to do so.
You are allowed a non-CAS calculator and a notes sheet provided in class.
No phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. (*50 points*) Identify the following differential equations (separable, linear, Bernoulli, homogeneous, or exact) and then solve.

Q1 $\frac{dy}{dx} = x^2 - 2xy + y^2$

Q2 $\frac{dy}{dx} = y + y^4$

Q3 $\frac{dy}{dx} = 3(y + 7)x^2$

Q4 $x \frac{dy}{dx} = y + \sqrt{x^2 - y^2}$

Q5 $(\cos(x) + \log(y))dx + \left(\frac{x}{y} + e^y\right)dy = 0$

Q6 Solve the following initial value problem.

$$\begin{aligned}(x^2 + 1) \frac{dy}{dx} + 3xy &= x \\ y(0) &= 1\end{aligned}$$

Part 2. (*10 points*) Consider the autonomous first order ODE

$$\frac{dy}{dt} = y^2 - y$$

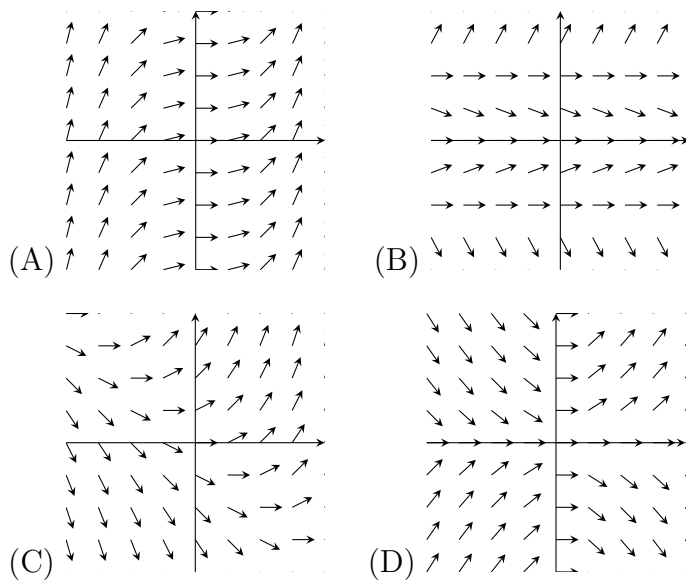
Q1 Find the equilibrium solutions.

Q2 Determine the stability of the equilibrium solutions.

Q3 Sketch the solution curves.

Part 3 (10 points)**Q1** Match the following differential equations with their slope fields.

$$\frac{dy}{dx} = x + y \quad \frac{dy}{dx} = y^3 - y \quad \frac{dy}{dx} = x^2 \quad \frac{dy}{dx} = y^{1/3}x^{1/3}$$

**Q2** Which of the above ODEs fails the existence and uniqueness theorem?

Part 4. (*10 points*) A tank initially contains 80 gal of brine containing 40 lb of salt. Brine containing 2 lb of salt per gallon enters the tank at the rate of 2 gal/min and the perfectly mixed brine in the tank flows out at the rate of 4 gal/min. Let $S(t)$ be the amount of salt at a time t in minutes. Write down the initial value problem for $S(t)$.

Part 5. (*10 points*) Use the elimination method to decide whether the following system is consistent or inconsistent. Then, find the solution set.

$$\begin{aligned}x - 2y + z &= 2 \\2x - y - 4z &= 13 \\x - y - z &= 5\end{aligned}$$

Part 6. (*10 points*) Write the solution as an augmented matrix and then solve it by computing the echelon form.

$$\begin{aligned}x_1 + 2x_2 + x_3 &= 4 \\3x_1 + 8x_2 + 7x_3 &= 20 \\2x_1 + 7x_2 + 9x_3 &= 23\end{aligned}$$

End of Exam. Check your work!