

Math 2400 Practice Exam 1

Vanderbilt University

February 4, 2026

Name: _____

Please do not open the exam until instructed to do so.
You are allowed a non-CAS calculator and a notes sheet provided in class.
No phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. (50 points) Identify the following differential equations (separable, linear, Bernoulli, homogeneous, or exact) and then solve.

Q1 $\frac{dy}{dx} = x^2 - 2xy + y^2$

Q2 $\frac{dy}{dx} = y + y^4$

Q3 $\frac{dy}{dx} = 3(y + 7)x^2$

Q4 $x \frac{dy}{dx} = y + \sqrt{x^2 - y^2}$

Q5 $(\cos(x) + \log(y))dx + \left(\frac{x}{y} + e^y\right)dy = 0$

Q6 Solve the following initial value problem.

$$\begin{aligned}(x^2 + 1) \frac{dy}{dx} + 3xy &= x \\ y(0) &= 1\end{aligned}$$

Part 2. (10 points) Consider the autonomous first order ODE

$$\frac{dy}{dt} = y^2 - y$$

Q1 Find the equilibrium solutions.

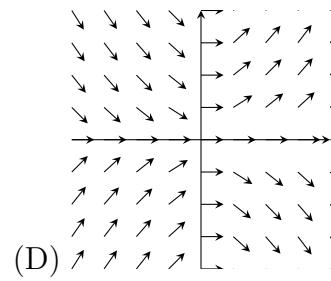
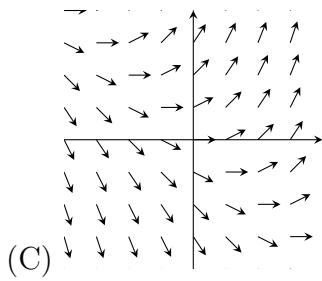
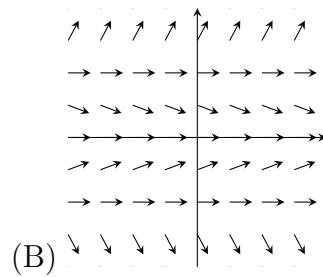
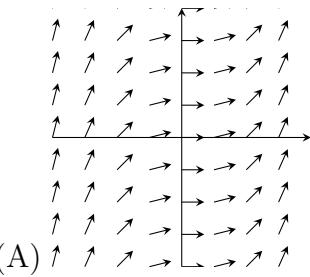
Q2 Determine the stability of the equilibrium solutions.

Q3 Sketch the solution curves.

Part 3 (10 points)

Q1 Match the following differential equations with their slope fields.

$$\frac{dy}{dx} = x + y \quad \frac{dy}{dx} = y^3 - y \quad \frac{dy}{dx} = x^2 \quad \frac{dy}{dx} = y^{1/3}x^{1/3}$$



Q2 Which of the above ODEs fails the existence and uniqueness theorem?

Part 4. (10 points) A tank initially contains 80 gal of brine containing 40 lb of salt. Brine containing 2 lb of salt per gallon enters the tank at the rate of 2 gal/min and the perfectly mixed brine in the tank flows out at the rate of 4 gal/min. Let $S(t)$ be the amount of salt at a time t in minutes. Write down the initial value problem for $S(t)$.

Part 5. (10 points) Use the elimination method to decide whether the following system is consistent or inconsistent. Then, find the solution set.

$$\begin{aligned}x - 2y + z &= 2 \\2x - y - 4z &= 13 \\x - y - z &= 5\end{aligned}$$

Part 6. (10 points) Write the solution as an augmented matrix and then solve it by computing the echelon form.

$$\begin{aligned}x_1 + 2x_2 + x_3 &= 4 \\3x_1 + 8x_2 + 7x_3 &= 20 \\2x_1 + 7x_2 + 9x_3 &= 23\end{aligned}$$