Math 1301 Practice Final Exam Version 2

Vanderbilt University

8 December 2025

Name: ____________________

Please do not open the exam until instructed to do so.
You are allowed a one-page (double-sided) formula sheet.
Your instructor may ask to see your formula sheet.
No calculators, phones, computers, smart watches, etc. are permitted.

The Vanderbilt Honor Code applies.

Part 1. *(20 points)*

Q1 $\dfrac{d}{dx}\left[\log_9(5^{\sin(e^x)}) + \cos^{-1}(\ln(\sqrt[50]{x}))\right] =$ $\boxed{}$

Q2 $\dfrac{d}{dx}\ln^{5^x}(x) =$ $\boxed{}$

Q3 $\displaystyle\lim_{x\to 0^+}\left(\frac{\sin(x)}{x}\right)^{1/x^2} =$ $\boxed{}$

Q4 The value of k such that the equation $e^{2x} = k\sqrt{x}$ has *one* solution is $\boxed{}$

Part 2. (*10 points*) Consider the two power series given by

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2} \qquad \text{and} \qquad J_1(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$$

These are called Bessel Functions of the first kind.

Q1 Determine the radii of convergence of $J_0(x)$ and $J_1(x)$.

Q2 Show that $y(x) = J_0(x)$ solves the differential equation

$$x^2 y''(x) + xy'(x) + x^2 y(x) = 0.$$

Part 3 (*15 points*) Evaluate the following integrals.

Q1 $\int_0^1 \tan(\sin^{-1}(x)) =$ $\boxed{}$

Q2 $\int \sqrt{1-x^2}\ dx =$ $\boxed{}$

Q3 $\int \dfrac{\cot(x)}{\ln(\sin(x))}\ dx =$ $\boxed{}$

Part 4. (*15 points*)

Q1 If $f(x) = \sum_{n=1}^{\infty} a_n x^n$, then $\sum_{n=1}^{\infty} n^2 a_n x^n =$ $\boxed{}$

Q2 The values of p and q such that $\sum_{n=3}^{\infty} \frac{1}{n^p \ln^q(n)}$ converges are $\boxed{}$

Q3 Evaluate the following integral: $\int \frac{\ln(\ln(x))}{x}\, dx =$ $\boxed{}$

Part 5. (*15 points*)

Q1 Consider the parametric equations

$$x(t) = e^t \cos(t) \quad y(t) = e^t \sin(t)$$

Eliminate the parameter and find $\dfrac{dy}{dx}$ in terms of t.

Q2 Find the length of the polar curve of $r = \dfrac{1}{\theta}$ from $0 \leq \theta \leq 2\pi$

Q3 Eliminate the parameter of the parametric equations $x = 2\cos(\theta)$, $y = 1 + \sin(\theta)$. Find the points of the horizontal and vertical tangent lines.

Part 6. (*10 points*)

Q1 The general solution to $\dfrac{dy}{dx} = \dfrac{3^x y\left(\ln^2(y) + 3\ln(y) + 2\right)}{1 + 9^x}$ is $\boxed{}$

Q2 Use power series to evaluate: $\lim_{x\to 0} \dfrac{1 - \cos(x)}{1 + x - e^x} = \boxed{}$

Part 7 (*20 points*) Determine if the following series converge or diverge. Show your work and state any necessary hypotheses.

Q1 $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

Q2 $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

Q3 $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^2}{2^x}$ (Use the alternating series test)

Q4 $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{6}\right)$

Part 8 (*20 points*) Define the function $f(x) = \displaystyle\int_0^x \frac{1}{(t+1)\sqrt{1-\ln^2(t+1)}}\,dt$

Q1 The domain of $f(x)$ is $\boxed{}$

Q2 $f(x)$ has an inverse on $\boxed{}$

Q3 $f\left(\displaystyle\sum_{n=1}^{\infty} \frac{(3)^{n/2}}{n!\;2^n}\right) = \boxed{}$

Q4 $(f^{-1})'(1/2) = \boxed{}$

Part 9 (*20 points*) Suppose that $y = f(x)$ is a solution to the differential equation

$$\frac{dy}{dx} = yx\ln(x) + k\sin(x)$$

where $k > 0$ and $f(1) = 4$.

Q1 The second order Taylor polynomial of $f(x)$ at $x = 1$ is given by

Q2 If $k = 0$, then the solution to the initial value problem is given by

Q3 The domain of $f(x)$ is

Part 10 (*25 points*) Define the function $g(x) = xe^x$.

Q1 Calculate $\int_0^1 g(x)\,dx$ in two ways: an integration techique and a power series technique.

Q2 $y = g(x)$ satisfies the following differential equation $\boxed{}$.

Q3 The above differential equation is **SEPARABLE** or **NON-SEPARABLE**.

Q4 The domain of $h(p) = \displaystyle\int_1^\infty \frac{g(x)}{e^{px}}\,dx$ is $\boxed{}$.

Q5 $g(x)$ has an inverse on the domain $\boxed{}$.

Q6 $\displaystyle\int_0^e g^{-1}(x)\,dx = \boxed{}$. (Hint: what is an inverse?)

Part 11 *Extra space for work*

End of Exam. Check your work!