PCS Final Project: Simulations of an analytical model of multihop connectivity of inter-vehicle communication systems

B10902039 Pang-Wei Huang,
B10507008 Liang-Yeh Shen

Abstract

In the field of Inter-Vehicle Communication (IVC), the performance of multihop connectivity is a critical issue. However, obtaining the associated performance metrics is often challenging. While some research provides analytical formulas for these metrics, they typically focus on specific distributions of communication nodes or rely on simulation-based methods. The reference paper proposes an analytical model that offers a analytical, real-time solution. This model considers exact vehicle positions, enabling efficient analysis of multihop connectivity under arbitrary traffic patterns.

Note: We redid some experiments in the 16th week of the semester, so some of the statistics here differ from those we presented on December 17th, 2024.

1 Introduction

In the 2000s, the proliferation of wireless communication devices made it possible and affordable to implement decentralized communication networks between vehicles. Such networks can support incident response systems, potentially preventing thousands of accidents daily worldwide and saving lives.

To analyze the performance of such networks, it is important to evaluate the connectivity between nodes on a road given a specific distribution of communication nodes. Unfortunately, previous studies have primarily focused on analytical models that rely on simplified assumptions, such as uniform or Poisson distributions of vehicles. These models often fail to capture the complexity of real-world traffic patterns. Other research, by contrast, has focused on simulation-based methods. While flexible, these methods are computationally expensive and time-consuming, making them unsuitable for real-time analysis.

In the reference paper, the authors introduce an innovative analytical model that addresses these challenges. Since it considers the exact positions of vehicles, it is flexible enough to be generalized to many scenarios. Moreover, it is computationally inexpensive, making it suitable for real-time analysis.

2 Conceptual Framework

2.1 Assumptions

In the reference paper, the authors make the following two assumptions.

The first assumption is that the transmission time between communication nodes can be ignored—in other words, the transmission is instantaneous. This assumption is supported by another paper, which shows that it takes 110 ms to transmit a message of typical size. In such a short time period, the distribution of vehicles does not change significantly. Therefore, the assumption is reasonable.

[1] Our code: https://github.com/BrianHuangNTUCSIE/PCS_Final
[2] Reference paper: https://ieeexplore.ieee.org/document/5374054

The second assumption is that transmission nodes send messages as far as possible. In other words, information propagates in the fashion of "most forwarded within range" (MFR). This assumption does not affect the computation of connectivity between two vehicles, as they are considered connected as long as an MFR communication path exists between them.

2.2 Definitions

For a general traffic stream, assuming stationarity, we label the information source as vehicle 0, and the subsequent vehicles in the direction of information flow are indexed as $1, 2, \ldots$. The position of vehicle n is represented by $x(n)$, where $x(k+1) \geq x(k)$. Without loss of generality, we restrict our consideration to a continuous traffic stream of K vehicles (excluding the information source) such that the gap between any two consecutive vehicles does not exceed the transmission range R.

Within the transmission range R of the information source at $x(0)$, the farthest vehicle n_1 can be identified, and the region $(x(0), x(n_1)]$ is referred to as cell 1. Similarly, the farthest vehicle n_2 within the transmission range of vehicle n_1 is located, defining cell 2 as $(x(n_1), x(n_2)]$. By repeating this process, the traffic stream can be divided into multiple cells, characterized by a sequence of boundary vehicles $n_0, n_1, \ldots$, where $x(n_c) \leq cR$. The cell containing vehicle k is denoted by $C(k)$. Thus, $C(k) = c$ if and only if $x(n_{c-1}) < x(k) \leq x(n_c)$.

Further, we define the farthest upstream and downstream (in the direction of information propagation) vehicles within the transmission range of vehicle $k > 0$ as $\underline{k}$ and $\overline{k}$, respectively. That is, $\underline{k} \geq 0$ is the minimum k satisfying $|x(k) - x(\underline{k})| \leq R$, and $\overline{k}$ is the maximum k satisfying $\left|x(\overline{k}) - x(k)\right| \leq R$.

Note that the event of whether a vehicle is equipped is random, with a probability of the market penetration rate, denoted by μ (where $\nu = 1 - \mu$). All events associated with vehicles in a traffic stream are assumed to be independently and identically distributed (iid) and form Bernoulli trials.

3 An Analytical Model of Multihop Connectivity of an IVC System in a Traffic Stream

Here, we define the notations for the probability of three events:

1. $P(k; h)$: The probability that vehicle k is node h.

2. $\overline{P}(k; h)$: The probability that vehicle k is node h and also the end node.

3. $\mathscr{P}(k, l; h)$: The joint probability that vehicle k is node $h-1$ and vehicle l is node h.

3.1 Properties of and Relationships between the Three Probabilities

3.1.1 Basic Properties

1. $P(k; h) = 0$ if $C(k) \leq h \leq 2C(k) - 1$. However, $P(0; 0) = 1$.

2. $\mathscr{P}(k, l; h) = 0$ if $j < \underline{k}$ or $k > \overline{j}$.

3. If $C(k) = 1$, $\mathscr{P}(k, l; h) = 0$ for $j = 1, \ldots, k-1$.

4. If $C(k) = 1$, $\mathscr{P}(0, k; 1) = \mu \cdot \nu^{n_1 - k}$.

5. If $C(k) = 1$, $P(k; 1) = \mu \cdot \nu^{n_1 - k}$

3.1.2 Relationships between the three probabilities

To compute the three probabilities, we can use the following recursive equations. Note that the termination conditions are listed in the previous subsection.

$$P(k; h) = \sum_{j=\underline{k}}^{k-1} \mathscr{P}(j, k; h) \tag{1}$$

$$\mathscr{P}(k,l;h) = \left(P(k;h-1) - \sum_{j=\underline{l}}^{k-1} \mathscr{P}(j,k;h-1) \right) \mu\nu^{\overline{k}-l} \tag{2}$$

$$\overline{P}(k;h) = P(k;h) - \sum_{l=k+1}^{\overline{k}} \mathscr{P}(k,l;h+1) \tag{3}$$

3.2 Implementation Details

While the above properties and recursive equations are sufficient to solve the three probabilities, the computational cost remains overwhelming. To obtain the answer in a reasonable time, we employ a programming technique called dynamic programming (DP).

Dynamic programming is an optimization approach that solves problems by breaking them down into smaller overlapping subproblems. It stores the results of these subproblems in a table structure to avoid redundant computations. By reusing previously computed values, DP significantly reduces the time complexity of solving recursive problems. This approach is particularly effective for problems with overlapping subproblems and optimal substructure properties.

Specifically, in this context, we store the results after computing $P(k;h)$, $\mathscr{P}(k,l;h)$, and $\overline{P}(k;h)$. By doing so, we avoid recalculating the same probabilities multiple times.

Note: This technique is not mentioned in the reference paper. The authors do not provide their source code, pseudo-code, or any implementation details.

3.3 Performance Measurements

From the probability $\overline{P}(k;h)$, we can define the following performance measurements, with vehicle K as the last vehicle in a traffic stream.

1. The probability for information to stop at k, regardless of the number of hops, is

$$\overline{P}(k) = \sum_{h=C(k)}^{2C(K)} \overline{P}(k;h),$$

   which is also the probability for $x(k)$ to be the longest propagation distance.

2. The probability for information to travel to or beyond k, regardless of the number of hops, is

$$s(k) = \sum_{i=k}^{K} \overline{P}(i).$$

3. The probability for information to stop at h hops, regardless of the final position, is

$$\overline{P}(h) = \sum_{k=1}^{K} \overline{P}(k;h).$$

4. The expected number of hops is

$$E(h) = \sum_{h=1}^{\infty} h\overline{P}(h).$$

5. The expected information propagation distance is

$$E(x) = \sum_{k=1}^{K} x(k)\overline{P}(k).$$

6. The connectivity between information source and an equipped vehicle k is

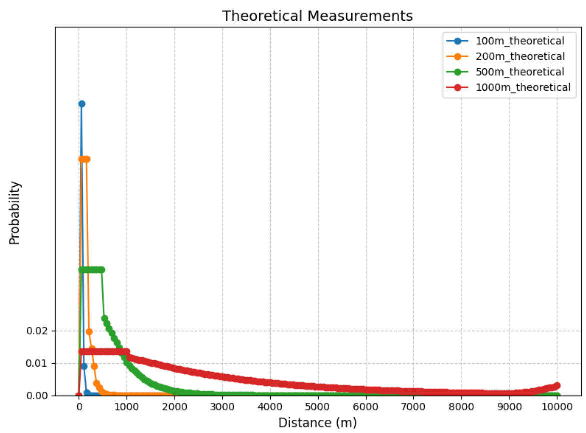
$$\overline{s}(k) = s(\underline{k}).$$

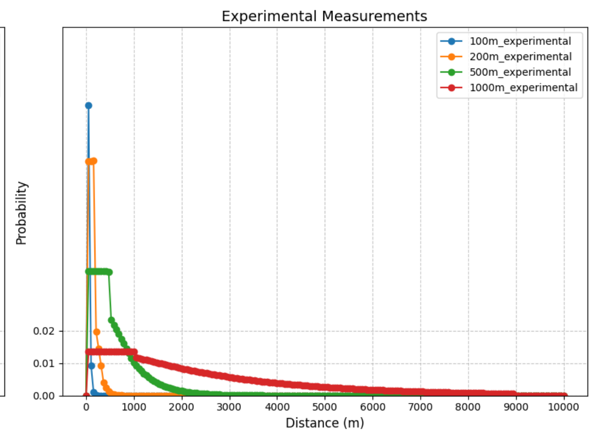
That is, equipped vehicle k receives information whenever its farthest upstream vehicle $\underline{\mathrm{k}}$ does. Here the connectivity $\overline{s}(k)$ can also be defined for road-side stations, as long as they are connected through IVC

4 Experiments

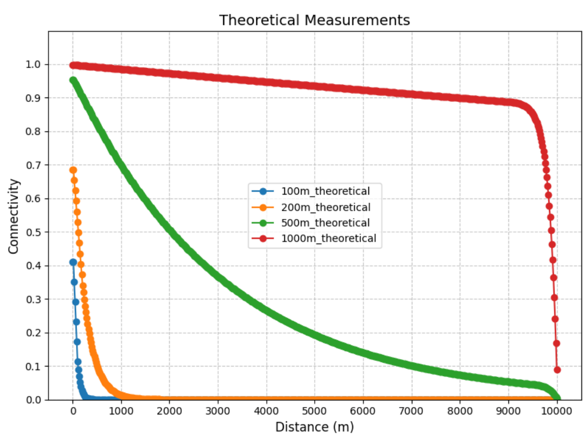
In the following experiments, we set the distance between each two cars = 52.631578947 and $\mu = 0.1$, which is the same as the second experiment in the reference paper. However, in measurement 6, since we want to compare the behavior between different distances, the distance in measurement 6-1 = 17.24137931.

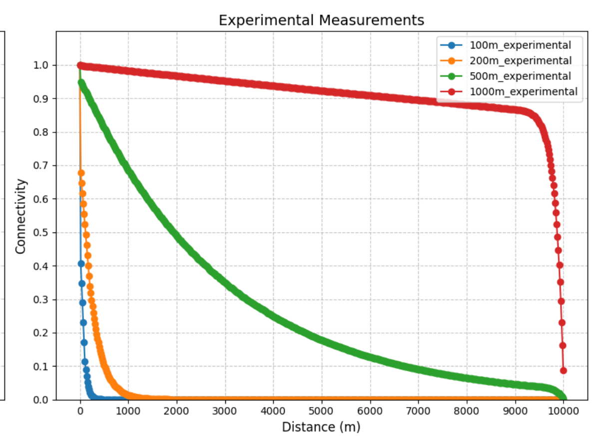
4.1 Measurement 1



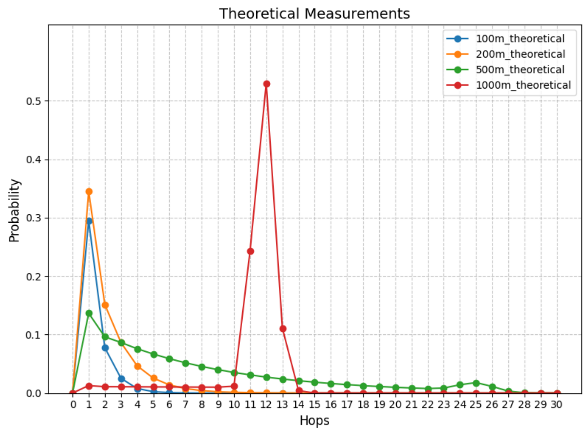


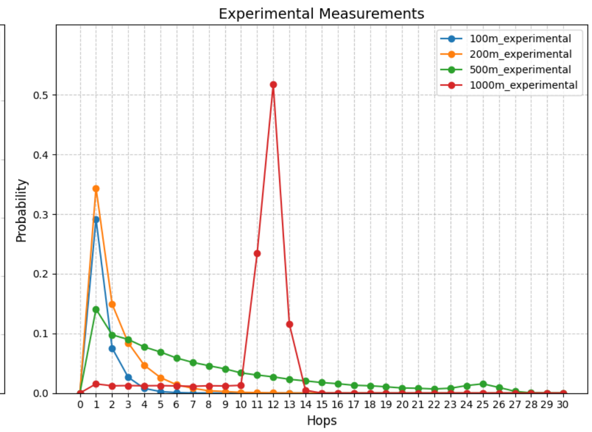
4.2 Measurement 2





4.3 Measurement 3





4.4 Measurement 4

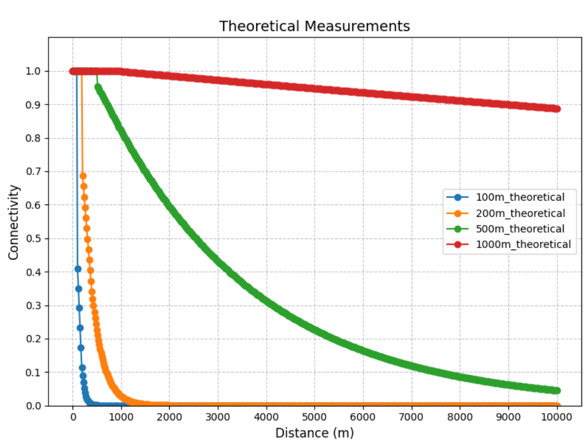
	Theoretical	Experimental	Error Rate (%)
100m	0.576924	0.5792	0.3945060354570196
200m	1.44483	1.43259	0.8471584892340295
500m	7.46646	7.12215	4.611422280438109
1000m	11.134145	11.04636	0.7884305440606371

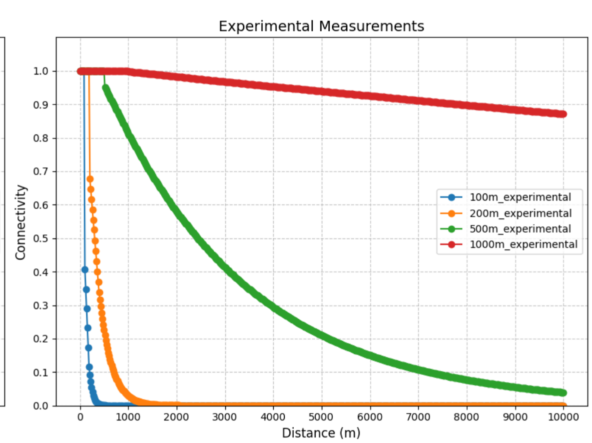
4.5 Measurement 5

	Theoretical	Experimental	Error Rate (%)
100m	33.363583	33.146034	0.6520552663663198
200m	187.350956	185.786379	0.8351048926593071
500m	2863.949278	2709.507586	5.392612682995983
1000m	9225.974776	9116.067586	1.1912799749453802

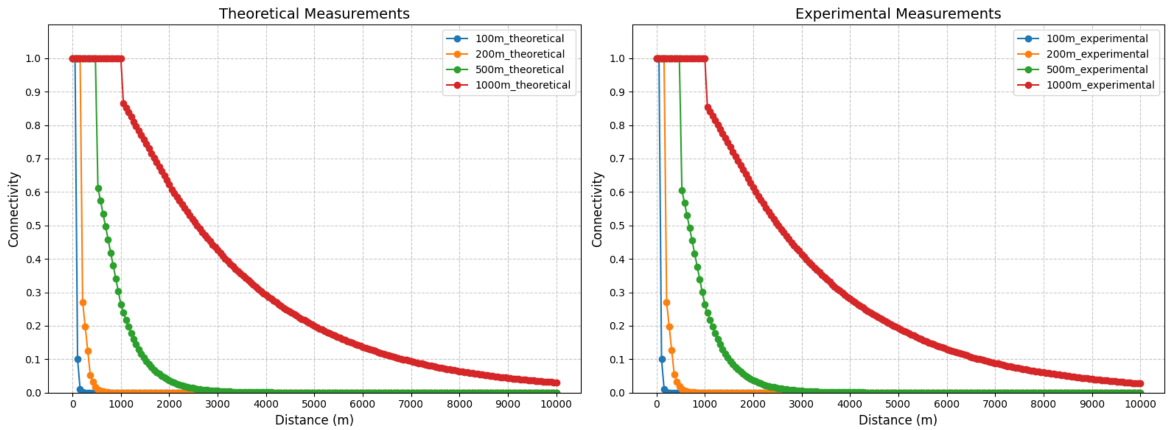
4.6 Measurement 6

4.6.1 Measurement 6-1





4.6.2 Measurement 6-2



We obtained results from the simulations that were very close to those of the analytical model, verifying its validity.

5 Conclusion

In the reference paper, a recursive model was proposed for computing end-node probabilities. After studying examples of multihop connectivity, we verified the model through our simulations. This model can be applied to any 1-D mobile ad hoc network with arbitrary distribution patterns. To expand the application scenario, it can also be extended to include communication between vehicles, vehicle-to-infrastructure, vehicle-to-home, and other real-time transportation systems. Due to its inexpensive computation, it may also be applicable for measuring connectivity in Low Earth Orbit satellite communications.