



Mind the Gap!

**Physical Science Part 1
Physics**
Study Guide

Grade
12



basic education

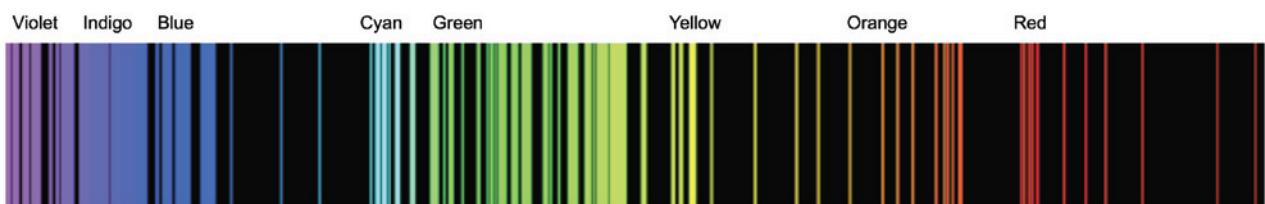
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Colour Diagrams: Emission and Absorption Spectra*



Colour Spectrum and White Light

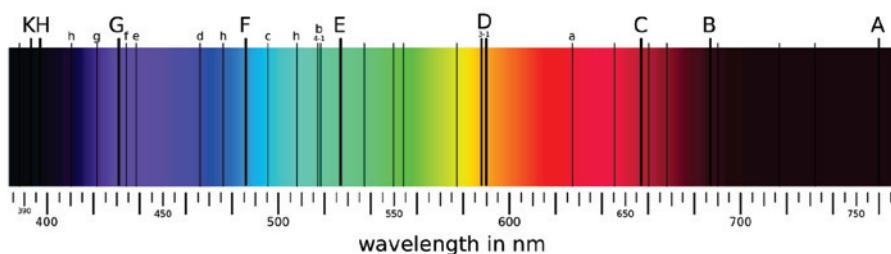
Emission spectrum of Iron (Fe)



Emission spectrum of Hydrogen (H)



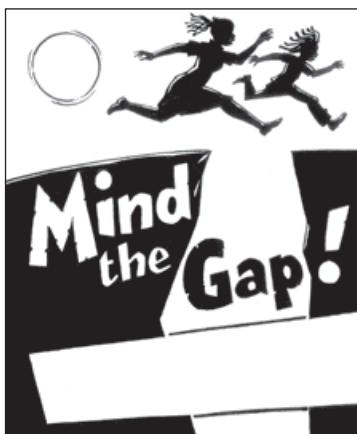
Emission Spectra



Absorption Spectrum

(Source: Wikimedia Commons)

* Refer to pages 144, 145 and 146



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**Curriculum and Assessment Policy Statement (CAPS) Grade 12
Mind the Gap study guide for Physical Science Part 1: Physics
ISBN 978-1-4315-1937-8**

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The first edition, published in 2012, for the Revised National Curriculum Statement (RNCS) Grade 12 Mind the Gap study guides for Accounting, Economics, Geography and Life Sciences; the second edition, published in 2014 , aligned these titles to the Curriculum and Assessment Policy Statement (CAPS) and added more titles to the series in 2015, including the CAPS Grade 12 **Mind the Gap study guide for Physical Science Part 1: Physics**.

ISBN 978-1-4315-1937-8

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Ministerial foreword

The Department of Basic Education (DBE) has pleasure in releasing the second edition of the *Mind the Gap* study guides for Grade 12 learners. These study guides continue the innovative and committed attempt by the DBE to improve the academic performance of Grade 12 candidates in the National Senior Certificate (NSC) examination.

The study guides have been written by teams of experts comprising teachers, examiners, moderators, subject advisors and coordinators. Research, which began in 2012, has shown that the *Mind the Gap* series has, without doubt, had a positive impact on grades. It is my fervent wish that the *Mind the Gap* study guides take us all closer to ensuring that no learner is left behind, especially as we celebrate 20 years of democracy.

The second edition of *Mind the Gap* is aligned to the 2014 Curriculum and Assessment Policy Statement (CAPS). This means that the writers have considered the National Policy pertaining to the programme, promotion requirements and protocols for assessment of the National Curriculum Statement for Grade 12 in 2014.

The CAPS aligned *Mind the Gap* study guides take their brief in part from the 2013 National Diagnostic report on learner performance and draw on the Grade 12 Examination Guidelines. Each of the *Mind the Gap* study guides defines key terminology and offers simple explanations and examples of the types of questions learners can expect to be asked in an exam. Marking memoranda are included to assist learners to build their understanding. Learners are also referred to specific questions from past national exam papers and examination memos that are available on the Department's website – www.education.gov.za.

The CAPS editions include Accounting, Economics, Geography, Life Sciences, Mathematics, Mathematical Literacy and Physical Sciences Part 1: Physics and Part 2: Chemistry. The series is produced in both English and Afrikaans. There are also nine English First Additional Language (EFAL) study guides. These include EFAL Paper 1 (Language in Context); EFAL Paper 3 (Writing) and a guide for each of the Grade 12 prescribed literature set works included in Paper 2. These are Short Stories, Poetry, *To Kill a Mockingbird*, *A Grain of Wheat*, *Lord of the Flies*, *Nothing but the Truth* and *Romeo and Juliet*. (Please remember when preparing for EFAL Paper 2 that you need only study the set works you did in your EFAL class at school.)

The study guides have been designed to assist those learners who have been underperforming due to a lack of exposure to the content requirements of the curriculum and aim to mind-the-gap between failing and passing, by bridging the gap in learners' understanding of commonly tested concepts, thus helping candidates to pass.

All that is now required is for our Grade 12 learners to put in the hours required to prepare for the examinations. Learners, make us proud – study hard. We wish each and every one of you good luck for your Grade 12 examinations.



Matsie Angelina Motshekga, MP
Minister of Basic Education

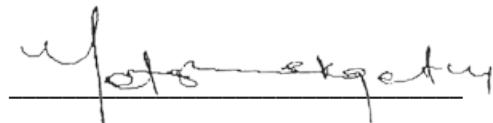

Matsie Angelina Motshekga, MP
Minister of Basic Education
2015

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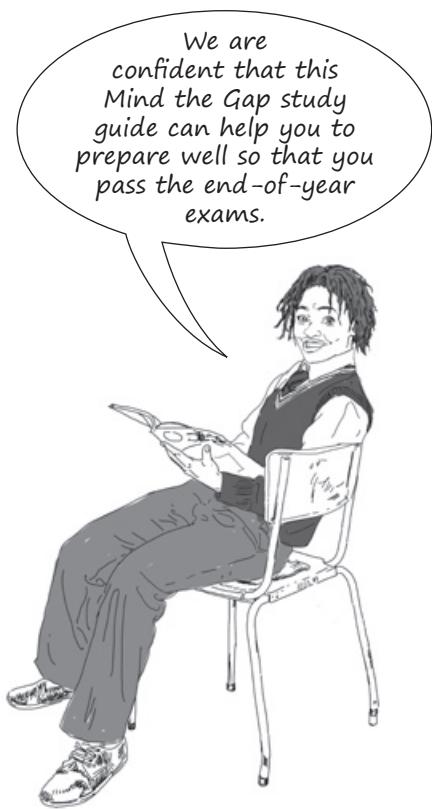
Dear Grade 12 learner

This *Mind the Gap* study guide helps you to prepare for the end-of-year CAPS Grade 12 exam.

The study guide does NOT cover the entire curriculum, but it does focus on core content of each knowledge area and points out where you can earn easy marks.

You must work your way through this study guide to improve your understanding, identify your areas of weakness and correct your own mistakes.

To ensure a good pass, you should also cover the remaining sections of the curriculum using other textbooks and your class notes.



Overview of the Grade 12 exam

The following topics make up each of the TWO exam papers that you write at the end of the year:

Cognitive level	Description	Paper 1 (Physics)
1	Remembering/Recall	15%
2	Understanding/Comprehension	35%
3	Applying and analysing	40%
4	Evaluating and creating (synthesis)	10%

Paper	Type of questions	Duration	Total	Date	Marking
1	Physics 10 multiple-choice questions – 20 marks Structured questions – 130 marks	3 hours	150	October/November	External

Paper 1: Physics Focus							
Content	Marks	Total	Duration	Weighting of cognitive levels			
Mechanics	63	150 marks	3 hours	15	35	40	10
Waves, sound and light	17						
Electricity and magnetism	55						
Matter and materials	15						



How to use this study guide

This study guide covers selected parts of the different topics of the CAPS Grade 12 curriculum in the order they are usually taught during the year. The selected parts of each topic are presented in the following way:

- An explanation of terms and concepts;
- Worked examples to explain and demonstrate;
- Activities with questions for you to answer; and
- Answers for you to use to check your own work.

	Pay special attention		Hints to help you remember a concept or guide you in solving problems		Worked examples
	Step-by-step instructions		Refers you to the exemplar paper		Activities with questions for you to answer

- The activities are based on exam-type questions. Cover the answers provided and do each activity on your own. Then check your answers. Reward yourself for things you get right. If you get any incorrect answers, make sure you understand where you went wrong before moving on to the next section.
- In these introduction pages, we will go through the mathematics that you need to know, in particular, algebra and graphs. These are crucial skills that you will need for any subject that makes use of mathematics. Make sure you understand these pages before you go any further.
- Go to www.education.gov.za to download past exam papers for you to practice.



Top 10 study tips

- 1.** Have all your materials ready before you begin studying – pencils, pens, highlighters, paper, etc.
- 2.** Be positive. Make sure your brain holds on to the information you are learning by reminding yourself how important it is to remember the work and get the marks.
- 3.** Take a walk outside. A change of scenery will stimulate your learning. You'll be surprised at how much more you take in after being outside in the fresh air.
- 4.** Break up your learning sections into manageable parts. Trying to learn too much at one time will only result in a tired, unfocused and anxious brain.
- 5.** Keep your study sessions short but effective and reward yourself with short, constructive breaks.
- 6.** Teach your concepts to anyone who will listen. It might feel strange at first, but it is definitely worth reading your revision notes aloud.
- 7.** Your brain learns well with colours and pictures. Try to use them whenever you can.
- 8.** Be confident with the learning areas you know well and focus your brain energy on the sections that you find more difficult to take in.
- 9.** Repetition is the key to retaining information you have to learn. Keep going – don't give up!
- 10.** Sleeping at least 8 hours every night, eating properly and drinking plenty of water are all important things you need to do for your brain. Studying for exams is like strenuous exercise, so you must be physically prepared.



If you can't explain it simply, you don't understand it well enough.

Albert Einstein

Mnemonics



A mnemonic code is a useful technique for learning information that is difficult to remember.

Here's the most useful mnemonic for Mathematics, Mathematical Literacy and Physical Science:

BODMAS:

B – Brackets

O – Of or Orders: powers, roots, etc.

D – Division

M – Multiplication

A – Addition

S – Subtraction

Throughout the book you will be given other mnemonics to help you remember information.

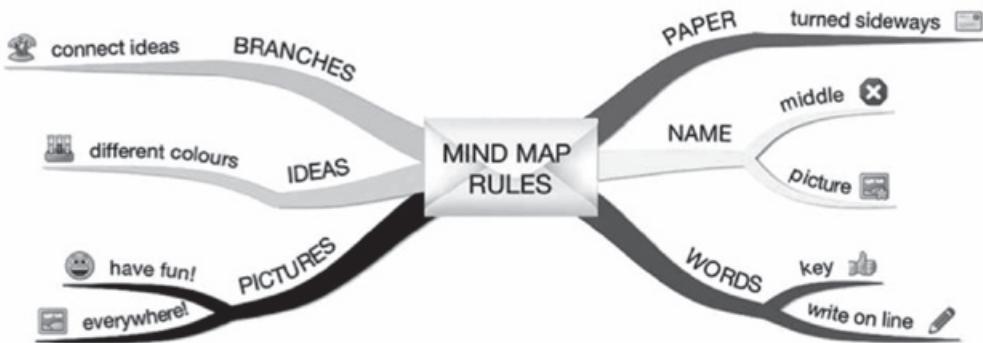
The more creative you are and the more you link your 'codes' to familiar things, the more helpful your mnemonics will be.

Education helps one cease being intimidated by strange situations.

Maya Angelou

Mind maps

There are several mind maps included in the Mind the Gaps guides, summarising some of the sections.



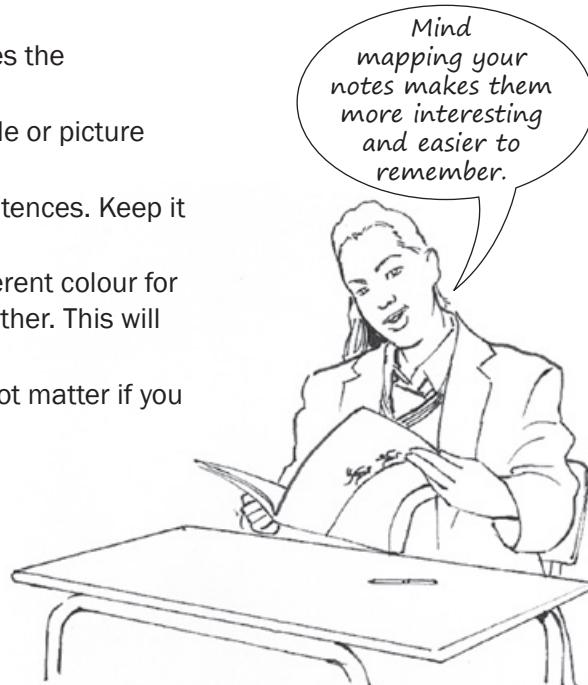
Mind maps work because they show information that we have to learn in the same way that our brains ‘see’ information.

As you study the mind maps in the guide, add pictures to each of the branches to help you remember the content.

You can make your own mind maps as you finish each section.

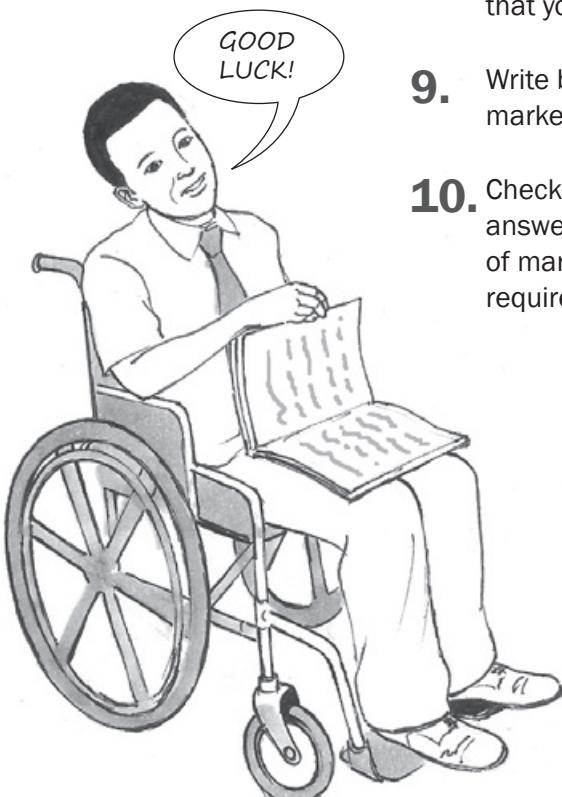
How to make your own mind maps:

1. Turn your paper sideways so your brain has space to spread out in all directions.
2. Decide on a name for your mind map that summarises the information you are going to put on it.
3. Write the name in the middle and draw a circle, bubble or picture around it.
4. Write only key words on your branches, not whole sentences. Keep it short and simple.
5. Each branch should show a different idea. Use a different colour for each idea. Connect the information that belongs together. This will help build your understanding of the learning areas.
6. Have fun adding pictures wherever you can. It does not matter if you can’t draw well.



On the day of the exam

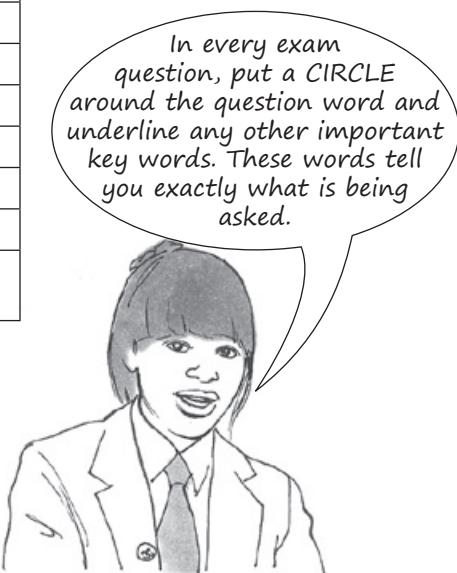
- 1.** Make sure you have all the necessary stationery for your exam, i.e. pens, pencils, eraser, protractor, compass, calculator (with new batteries). Make sure you bring your ID document and examination admission letter.
- 2.** Arrive on time, at least one hour before the start of the exam.
- 3.** Go to the toilet before entering the exam room. You don't want to waste valuable time going to the toilet during the exam.
- 4.** Use the 10 minutes reading time to read the instructions carefully. This helps to 'open' the information in your brain. Start with the question you think is the easiest to get the flow going.
- 5.** Break the questions down to make sure you understand what is being asked. If you don't answer the question properly you won't get any marks for it. Look for the key words in the question to know how to answer it. Lists of difficult words (vocabulary) is given a bit later on in this introduction.
- 6.** Try all the questions. Each question has some easy marks in it so make sure that you do all the questions in the exam.
- 7.** Never panic, even if the question seems difficult at first. It will be linked with something you have covered. Find the connection.
- 8.** Manage your time properly. Don't waste time on questions you are unsure of. Move on and come back if time allows. Do the questions that you know the answers for, first.
- 9.** Write big and bold and clearly. You will get more marks if the marker can read your answer clearly.
- 10.** Check weighting – how many marks have been allocated for your answer? Take note of the ticks in this study guide as examples of marks allocated. Do not give more or less information than is required.



Question words to help you answer questions

It is important to look for the question words (the words that tell you what to do) to correctly understand what the examiner is asking. Use the words in the table below as a guide when answering questions.

Question word/phrase	What is required of you
Analyse	Separate, examine and interpret
Calculate	This means a numerical answer is required – in general, you should show your working, especially where two or more steps are involved
Classify	Group things based on common characteristics
Compare	Point out or show both similarities and differences between things, concepts or phenomena
Define	Give a clear meaning
Describe	State in words (using diagrams where appropriate) the main points of a structure/process/phenomenon/investigation
Determine	To calculate something, or to discover the answer by examining evidence
Differentiate	Use differences to qualify categories
Discuss	Consider all information and reach a conclusion
Explain	Make clear; interpret and spell out
Identify	Name the essential characteristics PAY SPECIAL ATTENTION
Label	Identify on a diagram or drawing
List	Write a list of items, with no additional detail
Mention	Refer to relevant points
Name	Give the name (proper noun) of something
State	Write down information without discussion
Suggest	Offer an explanation or a solution
Tabulate	Draw a table and indicate the answers as direct pairs



Vocabulary

The following vocabulary consists of all the difficult words used in Mind the Gap Mathematics, Mathematical Literacy, and Physical Science. We suggest that you read over the list below a few times and make sure that you understand each term. Tick next to each term once you understand it so you can see easily where the gaps are in your knowledge.

KEY	
Abbreviation	Meaning
(v)	verb: doing-word or action word, such as "walk"
(n)	noun: naming word, such as "person"
(adj)	adjective: describing word such as "big"
(adv)	adverb: describing word for verbs, such as "fast"
(prep)	preposition: a word describing a position, such as "on", "at"
(sing)	singular: one of
(pl)	plural: more than one of
(abbr)	abbreviation

General terms

Term	Meaning
A	
abbreviate	(v). Make shorter.
account for	(v). Explain why.
adjacent	(adj). Next to something.
affect	(v). Make a difference to; touch the feelings of. Do not confuse with effect. See effect.
analyse	(v). Examine something in detail.
ante-	(prep). Before (e.g., ante-natal – before birth)
anti-	(prep). Against (e.g., anti-apartheid – against apartheid).
anti-clockwise	(adv. and adj.). In the opposite direction to the way a clock's hands move.
apparent	(adj). Clearly visible; the way something seems to be or the way it appears.
appear	(v). Come into sight; seem to be.

apply	(v). Make a formal application; be relevant to; work hard; place on.
approximate	(v. & adj.). Come close to (v); roughly, almost, not perfectly accurate, close but not exact. The verb is pronounced "approxi-may" and the adjective is pronounced "approxi-mitt".
ascending	(adj). Going up.
C	
cause	(v). Make something happen.
cause	(n). The person or thing that makes something happen; an aim or movement to which a person is committed.
causality	(n). Someone or something responsible for a result.
clockwise	(adj). In the direction a clock's hands move.
collide	(v). To crash into; to hit.
consecutive	(adj). One after another without any gaps or breaks.
consider	(v). Think about.
contrast	(v). Show the difference between; (n) something that is very different from what it is being compared with.
conversely	(adv). The opposite of.
counteract	(v). Act against something in order to stop it.
counter-clockwise	see anticlockwise.
D	
data (pl), datum (sing)	(n). Information given or found.
deduce	(v). To work something out by reasoning.
deduction	(n). Conclusion or idea that someone has worked out.
define	(v). Give the meaning of a word or words.
definition	(n). The meaning of a word or words.
deliver	(v). To bring and hand over.
denote	(v). To refer to or mean something.
descending	(adj). Going down.

determine	(v). Work out, usually by experiment or calculation.
direct	(v). Instruct or tell someone what to do; to control a process or movement; to go straight. Moving from one place to another in the straightest and quickest way (adj).
E	
effect	(n). Result.
effect	(v). Carry out, do, enact.
eject	(v). Force or throw something or someone out violently or suddenly.
elapse	(v). Pass by or finish, e.g., time.
establish	(v). Show or prove, set up or create.
excluding	(prep). Not including.
exclusive	(adj). Excluding or not admitting other things; reserved for one particular group or person.
exemplar	(n). A good or typical example.
exempt	(v). To free from a duty.
exempt	(adj). Be freed from a duty.
exemption	(n). Being freed from an obligation.
expel	(v). Force someone or something to leave a place. Eject.
extent	(n). The area covered by something.
F	
factor	(n). A circumstance, fact or influence that contributes to a result; a component or part.
factory	(n). A place where goods are made or put together from parts.
find	(v). Discover or locate.
find	(n). Results of a search or discovery.
finding	(n). Information discovered as the result of an inquiry.
fixed	(adj). Not able to move, attached; or repaired, not broken.
format	(n). Layout or pattern; the way something is laid out.
G	
global	(adj). Found all over the world (globe).

H	
horizontal	(adj). Across, from left to right or right to left. (From “horizon”, the line dividing the earth and the sky).
hover	(v). Float just above a surface.
hypothesis	(n). A theory or proposed explanation.
hypothetical	(adj). Theoretical or tentative; waiting for further evidence.
I	
identify	(v). Recognise or point out.
illustrate	(v). Give an example to show what is meant; draw.
impair	(v). Weaken or damage.
imply	(v). Suggest without directly saying what is meant.
indicate	(v). Point out or show.
inter-changeable	(adj). Can be swapped or exchanged for each other.
investigate	(v). Carry out research or a study.
issues	(v). Comes out of.
issues	(n). An important problem or a topic for debate.
M	
macroscopic	(adj). Visible without being made bigger.
magnitude	(adj). Size.
manipulate	(v). Handle or control (a thing or a person).
microscopic	(adj). Very small, not visible without being made bigger.
motivate	(v). Give someone a reason for doing something.
mount	(v). Attach, place upon (“mount the picture”); climb on (“mount the chair”); begin (“mount an attack”).
mount	(n). Mountain; frame or attachment that you can mount things on.
multiple	(adj). Many.
N	
negligible	(adj). Small and insignificant; can be ignored. From “neglect” (ignore).

numerical	(adj). Relating to or expressed as a number or numbers.	relative	(adj). Considered in relation to something else; compared to.
numerous	(adj). Many.	relative	(n). A family member.
O		represent	(v). Be appointed to act or speak for someone; amount to.
observe	(v). Look at; watch carefully.	resolve	(v). Finalise something or make it clear; bring something to a conclusion.
obtain	(v). Get.	respect	(v). Admire something or someone; consider the needs or feelings of another person.
occur	(v). Happen.	respectively	(adj). In regards to each other, in relation to items listed in the same order.
operate	(v). Work; drive; control.	S	
optimal	(adj). Best; most favourable.	simultaneously	(adv). At the same time.
optimum	(adj). Best; (n) the most favourable situation for growth or success.	site	(n). Place.
orientation	(n). Position or layout relative to other things or to compass points; getting used to the position or layout of things.	T	
P		tendency	(n). An inclination to do something in a particular way; a habit.
phenomenon	(n). A fact or situation that is seen to exist or happen.	transmission	(n). The act of sending (transmitting) something.
phenomena	(n). Plural of phenomenon.	transmit	(v). Send across.
prefix	(n). Part of a word that is attached to the beginning of many different words, changing their meaning, e.g., prehistoric – before written records were kept.	transverse	(adj). Extending across something.
principal	(n). Head of a school.	truncated	(adj). Cut short.
principal	(adj). Main or most important.	U	
principle	(n). A basic truth that guides the way a person behaves.	uniform	(n). Standardised clothing.
provide	(v). Make available for use; supply.	uniform	(adj). Remaining all the same at all times; unchanging.
Q		unimpeded	(adj). Free to move.
quality	(n). The standard of something compared to other similar things; a characteristic of someone or something.	universal	(adj). Found everywhere; true everywhere; applicable everywhere.
R		universe	(n). Everything that exists.
reciprocal	(adj). Given or done in return.	V	
record	(v). Make a note of something in order to refer to it later (pronounced ree-cord).	verify	(v). Show to be true; check for truth; confirm.
record	(n). A note made in order to refer to it later; evidence of something; a copy of something (pronounced rec-cord).	vice versa	(adv). The other way round.
		versus	(prep). Against. Abbreviated "vs" and sometimes "v".
		vertical	(adj). Upright; straight up; standing.
		via	(prep). By way of; by means of; travelling through.

Technical terms

A	
absorption	(n). To take into; the process of taking something in.
accelerate	(v). To increase speed (or velocity) per second, measured in metres per second per second ($\text{m}\cdot\text{s}^{-2}$ or m/s^2). See also velocity, speed, decelerate.
algebra	(n). A mathematical system where unknown quantities are represented by letters, which can be used to perform complex calculations through certain rules.
ammeter	(n). A device to measure amperage. See amp, amperage.
amperage	(n). The number of amperes.
amp, ampere	(n). One coulomb of charge passing one point in one second. See coulomb for more. Technically, 1 ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 metre apart in a vacuum, would produce between these conductors a force of $2 \times 10^{-7} \text{ N/m}$.
amplitude	(n). The size of something (common usage). Physics: the maximum extent of a vibration in the direction perpendicular to the direction of propagation; or, in simpler language: the furthest a vibration moves (left/right) when a wave is travelling forward.
angel	(n). In Abrahamic religions, a messenger from God. Note the spelling.
angle	(n). The difference in position between two straight lines which meet at a point, measured in degrees. Note the spelling.
anions	(n). A negative ion. See cation, ion.
apparatus	(n). Equipment; parts of a scientific experiment.
area	(n). Length x breadth (width). In common usage: a place.
armature	(n). Part of an electrical engine or generator (dynamo); the part which has coils of wire wrapped around it, which is attached to a central axle (rod).

atm	(n). Abbreviation: atmospheres of pressure (1 atm = 101,3 kPa). The pressure of the air at sea level. Same as “bar” (barometric pressure).
ATM	(n). Abbreviation: automatic teller machine.
atmosphere	(n). The air or the gases surrounding a planet; the sky; as a unit of measurement, see atm.
atmospheric	(adj). To do with the atmosphere.
atom	(n). The smallest unit of a chemical element, which, if broken down further, no longer behaves in the same way chemically. Consists of a nucleus or centre part which is positively charged, and an electron cloud (negatively charged) which surrounds the nucleus. See nuclear.
attract	(v). To bring something closer.
average	(n). Mathematics: The sum of parts divided by the quantity of parts. In common use: neither very good, strong, etc., but also neither very weak, bad, etc; the middle. In Physical Science and Mathematics: if you are asked to find the average, you always have to calculate it using the information you have. For example, the average of (1;2;3) is 2, because $(1+2+3)/3 = 2$. See also mean, median and mode.
axis (sing), axes (pl, pronounced “akseez”)	(n). A line along which points can be plotted (placed), showing how far they are from a central point, called the origin. See origin. “Vertical axis” or “y-axis” refers to how high up a point is above the origin (or how far below). “Horizontal axis” or “x-axis” refers to how far left or right a point is away from the origin.
B	
battery	(n). A collection of cells connected in series (end-to-end). See cell. In common use, “battery” is used to mean the same as “cell” (e.g. a penlight or AA cell), but this use is incorrect except for a car battery, which consists of a series of cells.

bias	(n). To be inclined against something or usually unfairly opposed to something; to not accurately report on something; to favour something excessively.	Cartesian	(adj). Anything believed or proposed by Rene Descartes. In particular, the x-and-y axis coordinate system.
bi-	(prefix). Two.	cation	(n). A positively charged ion. See anion, ion.
BODMAS	(abbr.). Brackets, of/orders (powers, squares, etc), division, multiplication, addition, subtraction. A mnemonic (reminder) of the correct order in which to do mathematical operations.	cell	(n). An apparatus that generates electricity using electrochemistry. An AA or Penlight battery, as it is commonly called, is a cell. A car battery consists of a number of cells inside a single container.
body	(n). Physics: any object.	CFL	(n). Compact Fluorescent Light; a small fluorescent tube curled up inside a standard lightbulb shape.
boil	(v). Physics: to cause a liquid's vapour pressure to exceed the pressure of the gas in the container, usually by heating it, but it can be done by lowering the pressure of the gas in the container, too. See vapour pressure. In common usage, to make a liquid hot until it bubbles.	charge	(n). Chemistry: having too many or too few electrons (most commonly), resulting in a substance ionising. A positive charge results from too few electrons, and a negative charge from too many electrons. Physics: a basic feature of all physical electromagnetic particles, except, e.g. neutrons and photons, which have zero charge. All protons have a positive charge, all electrons have a negative charge.
bond	(n). A connection. In physics and chemistry, between atoms and molecules.	chart	(v). To draw a diagram comparing values on Cartesian axes.
brake vs break	(n & v). To brake means to stop. To break means to destroy. Brakes are devices on vehicles which cause the vehicle to stop.	circuit	(n). A track or pathway which meets end-to-end. Electricity: a closed series of wires or cabling which starts at a power source and ends at the same power source which enables the flow of current.
breadth	(n). How wide something is. From the word "broad".	circumference	(n). The distance around the outer rim of a circle.
brush	(n. & v.). Anything that contacts anything else lightly in a sweeping motion (n); to sweep over something whilst touching lightly (v). Electrodynamics: the item that contacts the armature (the part of the motor that turns). See armature.	coefficient	(n). A constant value placed next to an algebraic symbol as a multiplier. Same as constant (see below). Or: a multiplier or factor that measures a property, e.g. coefficient of friction.
C		coil	(n). Electricity: a loop of wire or series of loops of wire used to create a magnetic field. In a car engine, it refers to the device which sends a spark of electricity to the spark plugs. General use: any loop of any wire, rope or string.
calibrate	(v). To adjust a measuring tool or measurement against a known accurate measurement to ensure that the measuring tool or measurement is accurate; to check a measurement or measuring tool's accuracy; to mark with accurate measurements using a standard scale like cm, mm, mℓ, etc. Common use: to assess or evaluate carefully.	commutator	(n). The connectors or rings which contact the brushes in an electrical engine.
		compressed	(adj). Subjected to pressure, squashed.

condensation	(n). When a vapour or gas cools down and starts to collect into larger droplets; changing phase from vapour or gas to liquid. Condensation reaction: to produce a larger molecule from two smaller ones.
conditions (STP)	(n). Physics and Chemistry: how the environment is: temperature and pressure. STP (Standard Temperature and Pressure) is 25 °C and 1 atm).
conduct	(v). Electricity: to allow electricity through a substance. Thermodynamics: to allow heat through a substance.
conductivity	(n). How well a substance allows heat or electricity through it; the opposite of resistance.
conductor	(n). A substance that allows electricity through. A poor conductor obstructs or resists electricity.
conservation	(n). A law which describes something that does not change. E.g. the conservation of matter-energy says that matter-energy cannot be created or destroyed, only transformed from one form into another. There are a number of other conservations, e.g. momentum and torque.
constant	(n). See coefficient. Means “unchanging”.
continuous	(adj). Mathematics: having no breaks between mathematical points; an unbroken graph or curve represents a continuous function. See function.
control	(n. and v.). To ensure something does not change without being allowed to do so (v); an experimental situation to which nothing is done, in order to compare to a separate experimental situation, called the ‘experiment’, in which a change is attempted. The control is then compared to the experiment to see if a change happened.
control variable	(n). A variable that is held constant in order to discover the relationship between two other variables. “Control variable” must not be confused with “controlled variable” (see independent variable).
conventional	(adj). In a standard way. Electricity: conventional current flows from + to -. In reality, electrons flow from - to +.

coordinate	(n). The x or y location of a point on a Cartesian graph, given as an x or y value. Coordinates (pl) are given as an ordered pair (x, y).
correlate	(v). To see or observe a relationship between two things, without showing that one causes the other.
correlation	(n). That there is a relationship between two things, without showing that one causes the other.
correspond	(v). To pair things off in a correlational relationship. For two things to agree or match. E.g. A corresponds to 1, B corresponds to 2, C corresponds to 3, etc.
coulomb	(n). A measure of charge, the quantity of electrons carried by a 1-amp current past a point in one second. $6,2415 \times 10^{18}$ electrons.
counteract	(v). Oppose or resist.
crest	(n). The top of a wave in a transverse wave. See transverse, trough, wave.
cubed	(adj). The power of three; multiplied by itself three times.
cubic	(adj). Shaped like a cube; having been multiplied by itself three times.
current	(n). Flowing electrons.
cylinder	(n). A tall shape with parallel sides and a circular cross-section – think of a log of wood, for example, or a tube. See parallel. The formula for the volume of a cylinder is $\pi r^2 h$.
D	
decelerate	(v). To slow down. Opposite of accelerate.
denominator	(n). See divisor. In popular speech: a common factor.
density	(n). How much mass is contained in a particular volume; how compact something is. Measured in kg/m^3
depend	(v). To be controlled or determined by something; to require something to happen or exist first.

dependent (variable)	(adj/n). A variable whose value depends on another; the thing that comes out of an experiment, the effect; the results. See also independent variable and control variable. The dependent variable has values that depend on the independent variable, and we plot it on the vertical axis.	Doppler (effect)	(n). The compression of a wave (increase in its frequency) as an object approaches an observer and the spreading out of a wave (or lowering of frequency) as the object moves away from an observer.
determine(s) (causation)	(v). To cause; to ensure that; to set up so that; to find out the cause of.	ductile	(adj). Able to be stretched, usually into a thin wire. Said of metals.
di-	(prefix). Two.	durable	(adj). Tough; something that can endure.
diagonal	(adj. & n.). A line joining two opposite corners of an angular shape.	dynamic	(adj). Changing often. Relating to forces that produce motion. Opposite of static. See static and electrostatic.
diameter	(n). The line passing through the centre of a shape from one side of the shape to the other, esp. a circle. Formula: $d = 2r$. See radius, radii, circumference.	dynamo	(n). A mechanical device structured the same way as an electric motor, however, instead of taking in electrical energy so as to turn, mechanical energy is used to turn it and it generates electricity. A machine that converts mechanical energy into electrical by rotating metal coils (usually copper) within a magnetic field. Same as generator, motor.
difference	(n). Mathematics: subtraction. Informally: a dissimilarity. How things are not the same.		
diffraction	(n). The process of spreading out light or splitting light into its component wavelengths. See wavelength.	E	
dimension	(n). A measurable extent, e.g. length, breadth, height, depth, time. Physics, technical: the base units that make up a quantity, e.g. mass (kg), distance (m), time (s).	earth	(n., v.). The planet upon which we live (n); Electricity: to connect an electrical circuit to the earth (v); the wire that connects an electrical circuit to the earth, having zero potential difference (n). Used to prevent circuit overload in the case of excess current.
diode	(n). A semiconductor device with two terminals (electrodes), usually allowing current to flow in one direction only.	ecliptic	(n). A circle in outer space, representing the sun's apparent path during the year.
discharge	(v., n.). Release (v); something that has been released (n). Electricity: to release a charge (v).	E_k	(n, abbr.). Kinetic energy. The energy of motion.
displace	(v). To move or relocate something.	elastic	(adj). Able to stretch and return to its original shape. Physics: a collision which does not transfer E_k from one object involved in the collision, to the other object(s).
displacement	(n). A distance moved; the process of being moved.	electric	(adj). Containing electricity (electrons).
distance	(n). See displacement.	electrode	(n). General use: the point where electrons enter or exit a power source or a circuit. Specifically (Electrochemistry): Part of a circuit dipped into a solution to receive or release electrons. See anode and cathode.
distribution	(n). How something is spread out. Mathematics: the range and variety of numbers as shown on a graph.		
divisor	(n). The number below the line in a fraction; the number that is dividing the other number above the fraction line. See numerator, denominator.		
domain	(n). The possible range of x-values for a graph of a function. See range.		

electro-dynamics	(n). The study of motion caused by electricity, or electricity caused by motion.	empirical	(adj). Relating to the senses or to things that you can see, touch, taste, etc. Chemistry: empirical formula: a formula giving the proportions of the elements present in a compound but not the actual numbers or arrangement of atoms; the lowest ratio of elements without giving structure or quantities.
electro-magnetic	(adj). That electricity causes magnetism and vice versa; the relationship between electricity and magnetism.	energetic	(adj). Having a lot of energy; performing a lot of work.
electro-magnet	(n). A coil of wire that becomes magnetised when electricity passes through it.	energy	(n). Work or the ability to do work. There are various forms of energy: motion (E_k), light energy (photons), electrical energy, heat, etc. Energy can neither be created nor destroyed, but only converted from one form to another. See conservation.
electromotive	(adj). Usually electromotive force or emf. The potential difference caused by electromagnetism, which causes current to flow. Producing a current with electromagnetism. See emf.	engine	(n). A machine that transfers or converts energy, or converts power (electrical, chemical) into motion.
electron	(n). A fundamental physical particle bearing a negative charge, weighing approximately 9×10^{-28} g, which is found around atomic nuclei in areas called 'orbitals'. Responsible for electricity and chemical reactions. Symbol e-. See proton, nucleus.	E_p	(n, abbr). Potential energy. See potential and E_k . Energy that a system has due to torsion (twisting), extension (stretching), or gravitation (being placed at a height above a large body). When a system which is under these conditions is released, it releases the E_p in the form of E_k . Examples: a compressed spring, someone about to jump.
electroscope	(n). A device to measure the strength of a charge of static electricity or ionisation of the air. See static.	estimate	(n., v.). To give an approximate value close to an actual value; an imprecise calculation.
electrostatic	(adj). Relating to electrons or electric fields which are not flowing as current. See static.	evaporate	(v). To change phase of matter from liquid to gas. Compare sublimate and boil.
element	(n). Mathematics: part of a set of numbers. Physics: a pure substance made only of atoms of one type, with the same number of protons in each nucleus. An element cannot be broken down further without losing its chemical properties. Each element has a unique atomic number which is the number of protons in the nucleus. See nucleus, atom, isotope. Popular use: part of.	excited	(n). The state of being in a higher energy level (higher than ground state).
elevation	(n). Science: height above the ground or sea level. Architecture: a face of a building as viewed from a certain direction on an architect's plan of the building. See plan.	exert	(v). To impose or place pressure or force upon something; to make an effort.
emf	(abbr). Same as electromotive force. Always written in lowercase (small letters).	exponent	(n). When a number is raised to a power, i.e. multiplied by itself as many times as shown in the power (the small number up above the base number). So, 2^3 means $2 \times 2 \times 2$. See also cubed.
emission	(n). Something released, e.g. gas, light, heat.	exponential	(adj). To multiply something many times; a curve representing an exponent.
emit	(v). To release.		

extrapolation	(n). To extend the line of a graph further, into values not empirically documented, to project a future event or result. In plain language: to say what is going to happen based on past results which were obtained (gotten) by experiment and measurement. If you have a graph and have documented certain results (e.g. change vs time), and you draw the line further in the same curve, to say what future results you will get, that is called 'extrapolation'. See predict.	frequency	(n). How often. Physics: the number of crests and troughs of a wave passing a point per second. See wavelength, crest, trough. Experienced by humans as high- or low-pitched sounds (high or low frequency), or different colours (visible light). See also Doppler effect.
F		friction	(n). The act of rubbing two things together; heat generated or lost due to objects rubbing together; a force; the resistance between two surfaces moving whilst in contact.
		function	(n). Mathematics: when two attributes or quantities correlate. If y changes as x changes, then $y = f(x)$. See correlate, graph, Cartesian, axis, coordinate. Also: a relation with more than one variable (mathematics). Chemistry: functional group: part of a molecule that gives the substance its chemical properties in common with other similar chemicals.
fahrenheit	(n). A temperature scale based on human body temperature. Water freezes at 32°F and boils at 212°F under standard conditions. The conversion formula to centigrade/celcius is: $(^{\circ}\text{F} - 32) \times \frac{5}{9} = ^{\circ}\text{C}$	fuse	(n., v.). Electricity: a wire that melts when an excessive current passes through it (n). The rating of the fuse in amps tells you what the maximum current can be before the fuse melts and breaks the circuit. Common use: to join or merge two things (v).
field	(n). Physics: an area in which a force is experienced (felt) and is caused by either electricity, gravity, or magnetism. E.g. an electric field is one in which a test charge (a free particle) would move spontaneously. A magnetic field is one in which iron filings automatically orient themselves along regular lines.	G	
fluid	(n). Any substance that can flow and take the shape of a container; liquid, some gels, and gas.	gas	(n). The third phase of matter. When a solid is heated it turns into liquid, and when a liquid is heated it turns into gas.
flux	(n). A flow of energy or ions; a substance used to lower another substance's melting point so that it can be shaped or used more easily, e.g. glass and lead.	gaseous	(adj). In a gas form.
force	(n). The exertion of energy. An influence tending to change the motion of a body or produce motion or stress in a stationary body. The size of such an influence is often calculated by multiplying the mass of the body by its acceleration ($F = ma$). See energy, exert.	generator	(n). See dynamo.
		gradient	(n). A slope. An increase or decrease in a property or measurement. Also the rate of such a change. In the formula for a line graph, $y = mx + c$, m is the gradient.
		gradually	(adv). To change or move slowly.
		graph	(n). A diagram representing experimental or mathematical values or results. See Cartesian.
		graphic	(n., adj.). A diagram or graph (n). Popular use: vivid or clear or remarkable (adj.).

graphically	(adv). Using a diagram or graph. Popular use: to explain very clearly.
gravitation	(n). A force of attraction exerted on all bodies by all bodies. The strength of the force is proportional to the mass of the body; the more massive the body, the larger the gravity. $F = \frac{G(m_1 m_2)}{r^2}$. See body, force.
gravity	(n). Same as gravitation. Popular use: seriousness.
ground	(n). Same as earth.
H	
heat	(n). Physics: a measure of the average kinetic energy of the molecules or atoms in a substance; enthalpy; the energy of an object as molecular motion. Alternatively, infra-red radiation (heat radiation) coming off a body. See body.
heavy	(adj). Massive; having a great mass; weighty. See weight, mass.
hertz	(n). A measure of frequency.
hf	(equation). $E = hf$ is a measure of the energy of a photoelectron (see photoelectron). h is Planck's constant, $6,626 \times 10^{-34} \text{ Js}$. f is the frequency of incident light (see incident). hf standing alone can also represent light, a photon or photoelectron. See photon.
horsepower	(n). An old unit of power, approximately 750 W. See watts, power.
Huygens' principle	(n). Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wave-front is the tangential surface to all of these secondary wavelets. Used to explain refraction.
hyperbola	(n). Mathematics: a graph of a section of a cone with ends going off the graph; a symmetrical (both sides the same) open curve.
hypotenuse	(n). The longest side of a right-angled triangle.

I	
ideal	(adj). Not as seen in real life; theoretical. Ideal gas: a hypothetical gas whose molecules occupy negligible space and have no interactions, and which consequently obeys the gas laws ($PV = nRT$) exactly.
ignition	(n). The start of a combustion reaction. Common use: to start a car (which has an internal combustion engine). See engine, combustion.
illuminate	(v). To explain or light up.
impact	(n., v.). To hit hard (v); a strong influence or blow (n).
impair	(v). Prevent; hinder; slow down.
impulse	(n). Physics: A shock wave or impact which acts on a body for a short period of time, resulting in a change in the body's momentum. Effectively the same as momentum. Or: a jolt of electricity. Popular use: to have a sudden careless urge to do something.
incandescent	(adj). Giving off light as a result of being heated.
incidence, incident	(n., adj.). An event, something that happened (n). As of light, falling onto a surface (adj). "Incident light".
incline	(n. & v.). Slope. See gradient (n); to lean (v).
independent (variable)	(n). The things that act as input to the experiment, the potential causes. Also called the controlled variable. The independent variable is not changed by other factors, and we plot it on the horizontal axis. See control, dependent variable.
indigo	(n). The colour between violet and blue; purplish-blue.
induce	(v). To cause or start. Electricity: to cause a current to flow by moving a conductor (metal) in a magnetic field.
induction	(n). The process of inducing a current.
inelastic	(adj). Opposite of elastic; a collision in which energy is lost or transferred.

inertia	(n). Physics: The same as momentum; the unwillingness of an object to move or change direction when moving.
infrared	(n., adj.). Electromagnetic radiation (light) with a frequency just lower than visible red light. Heat radiation. 800 nm to about 1 mm. Not visible to the naked eye.
insulate	(v). To separate or isolate from. Electricity: to prevent the flow of electricity.
insulation	(n). The state of being separated from. Electricity: a substance or surface that stops the flow of electricity.
intensity	(n). How strong a force or field is; how bright a light is.
interference	(n). When two forces or things interact in a way that prevents the other from acting. Physics: specifically when two waves interact and either reinforce or cancel each other out.
interval	(n). Gap. A difference between two measurements.
intra-molecular	(adj). Within or inside a molecule. See molecule, intermolecular.
inverse	(n). The opposite of. Mathematics: one divided by. E.g. $\frac{1}{2}$ is the inverse of 2.
inversion	(n). Chemistry: turning something upside down.
ion	(n). An atom or molecule or part of a molecule which has an electrical charge due to gaining or losing one or more electrons.
ionic (bond)	(adj.). A bond in which electrons have been transferred from one side of the molecule to another resulting in a cation and anion, which then attract. E.g. NaCl .
ionisation	(n). The process of ionising. See ionise.
ionise	(v). To turn into an ion. See ion.
isotope	(n). An element which has a different number of neutrons from the usual number of neutrons in the element. E.g. ^{12}C has 6 protons and 6 neutrons, but ^{14}C has 8 neutrons and 6 protons, and is radioactive.

J	
joule	(n). Unit of energy.
K	
kelvin	(n). Unit of temperature, with absolute zero being the point where no molecular motion occurs, at $-273,15^{\circ}\text{C}$. Hence, the freezing point of water is $273,15\text{ K}$. Note that there is no degree sign before K.
kinetic	(adj). Pertaining to motion. (About movement).
kWh	(abbr). Unit of power (kilowatt hours) that electricity suppliers charge for. See power, watt. 1000 watts used in 1 hour = $1\text{kWh} = 1$ unit. So e.g. a 2000 W heater uses 2 units per hour.
L	
law	(n). In Physical Science, a formula or statement, deduced (discovered) from observation (watching). The formula or statement will then predict that under the same conditions the same thing will always happen. E.g. the first law of thermodynamics says that matter and energy cannot be destroyed, but only changed from one form to another.
LED	(abbr). Light emitting diode. A type of light bulb, used to make computer screens and modern torches.
linear	(adj). In a line. Mathematics: in a direct relationship, which, when graphed with Cartesian coordinates, turns out to be a straight line.
logarithm	(n). Mathematics: a quantity representing the power by which a fixed number (the base) must be raised to produce a given number. The base of a common logarithm is 10, and that of a natural logarithm is the number e (2,7183...). A log graph can turn a geometric or exponential relationship, which is normally curved, into a straight line.

longitude	(n). Lines running north to south on the earth, measuring how far east or west one is, in degrees, from Greenwich in the UK. “Longitudinal” (adj) means from north to south, or top to bottom. Running lengthwise. Physics: a wave whose vibrations move in the direction of propagation (travel). Example: sound. Statistics: a study in which information is gathered about the same people or phenomena over a long period of time.	medium (sing)	(n). Common use: the average. Science: the substance that transmits something else (e.g. glass, air), or allows something to pass through it, e.g. light, information, etc. Plural is media.
M		metal	(n). A substance which is malleable (can be hammered flat), is ductile (can be drawn into a wire), which conducts electricity and heat well and which is reflective (most light striking it is emitted again). Most elements are metals except the few on the right hand side of the periodic table starting at Boron (B) and running diagonally down to Astatine (At).
		meter	(n). A device used to measure something. You might see this spelling used in American books for metre. See metre.
mach	(n). A measure of speed. Mach 1 is the speed of sound, approximately 340 m/s.	metre	(n). The SI unit of length, 100 cm.
macroscopic	(adj). Large enough to be visible to the unaided human eye; big enough to be seen.	metric	(adj). A measurement system, using a base of 10 (i.e. all the units are divisible by 10). The USA uses something known as the Imperial system, which is not used in science. The Imperial system is based on 12. Examples: 2,54 cm (metric) = 1 inch (imperial). 1 foot = 12 inches = approx. 30 cm; 1 metre = 100 cm. 1 Fl.Oz (fluid ounce) = approx 30 ml.
magenta	(n). A bright purple/pink colour.	microscopic	(adj). Too small to be seen by the unaided human eye.
magnitude	(n). Size.	microwave	(n). Electromagnetic radiation (light) with a lower frequency than red light, and lower than infrared. Wavelength between 1 mm and 30 cm. Used to heat food and in cellphone towers.
manipulate	(v). To change, or rearrange something. Usually in Mathematics it means to rearrange a formula to solve for (to get) an answer.	modal	(adj). Pertaining to the mode, or method. Can mean: about the mathematical mode or about the method used. See mode.
mass	(n). The amount of substance in a body. Do not confuse with weight, which is the amount of force that a mass exerts on a surface. The mass of an object is constant everywhere in the universe (as the amount of substance does not change). However, the weight of an object changes according to the strength of the gravitational field on it. E.g. you would weigh about 38% of your weight on Mars, and about $\frac{1}{6}$ on the moon, because these bodies are smaller than earth. (Advanced point: mass increases as you approach light speed).	mode	(n). Mathematics: the most common number in a series of numbers. See also mean, median.
matter	(n). Substance; stuff. Opposite of vacuum (nothing).	model	(n). A general or simplified way to describe an ideal situation, in science, a mathematical description that covers all cases of the type of thing being observed. A representation.
mean	(n). See average.		
mechanical	(n). By means of physical force.		
media (pl)	(n). More than one medium, or way of transmitting or sending.		
median	(n). Mathematics: the number in the middle of a range of numbers written out in a line or sequence.		

momentum (sing), momenta (pl)	(n). Same as impulse; roughly how much force something can exert when it collides with something else, or, more technically, how much inertia it has (how unwilling it is to move or change direction). Specifically, where p is momentum: $p = mv$ (mass \times velocity). See velocity, mass, inertia, impulse. Momentum is conserved; if one object collides with another, the momentum of both objects before and after the collision is the same.
moon	(n). Any small spherical body that orbits a planet. See planet.
N	
neutron	(n). A subatomic particle with no charge, mass approximately the same as a proton, found in the nucleus of an atom. Symbol n^0 . If there are too many neutrons in a nucleus, the substance will be radioactive as it releases alpha particles (helium nuclei, $2p^+ + 2n^0$).
newton	(n). Unit of force, symbol N, equal to the force that would give a mass of one kilogram an acceleration of one metre per second per second ($1 \text{ kg}\cdot\text{m/s}^2$). 1 kg mass exerts a force of 9,8 N on earth.
nichrome	(n). An alloy of nickel and chrome, with some iron; a wire of high resistance.
nonconservative	(adj). Not conserving; see conservation.
normal	(n., adj.). Mathematics and Physics: a force, vector or line that acts at right angles to another force, vector or line or object. (n.). Common use: Regular or standard (adj.).
nucleus (sing), nuclei (pl), nuclear (n.)	(n.) The centre of something (generally), specifically the centre of an atom, consisting of at least one proton (hydrogen), or two protons and two neutrons (helium). Plural nuclei is pronounced “noo-klee-eye”.
numerator	(n). The opposite of a denominator; the number on top in a fraction.

O	
ohm	(n). Unit of resistance, symbol Ω , written as R in a formula.
ohmic	(adj). Follows Ohm's law ($V = IR$).
omega	(n). Last letter of the Greek alphabet, Ω . Symbol for ohms.
opaque	(adj). Not transparent; not allowing sharp vision through, but allowing light through.
optical	(adj). Pertaining to light or vision or the eye.
optimal	(adj). Best, most.
orbit	(n). The (approximate) circle traced by planets travelling around a star (sun). Biology: the eye socket.
origin	(n). Mathematics: the centre of a Cartesian coordinate system. General use: the source of anything, where it comes from.
outlier	(n). Statistics: a data point which lies well outside the range of related or nearby data points.
P	
packet	(n). Physics: a quantum.
parallel	(adj). Keeping an equal distance along a length to another item (line, object, figure). Mathematics: two lines running alongside each other which always keep an equal distance between them.
particle	(n). Any small part, e.g. a proton, an atom, a molecule.
particular	(adj). A specific thing being pointed out or discussed; to single out or point out a member of a group.
pascal	(n). The unit of pressure, abbreviated Pa, units: N/m^2
perimeter	(n). The length of the outer edge; the outer edge of a shape.
period	(n). The time between crests or troughs of waves; the time gap between events; a section of time.
periodic	(adj). Regular; happening regularly.
perpendicular	(adj). Normal; at right angles to (90°).

phase	(n). Time, period; a state of matter (solid, liquid, gas); the relationship in time between the cycles of a system (such as an alternating electric current or a light or sound wave) and either a fixed reference point or the states or cycles of another system with which it may or may not be synchronised (simultaneous). I.e. if two systems vibrate at the same time at the same rate, they're "in phase".	power	(n). Power is the rate at which (how fast) work is done or energy is transferred or transformed (work/time or J/s). It is a scalar quantity, measured in watts (W). Mathematics: an exponent. See exponent.
phi	(n). The Greek letter Φ (f) used to represent a photon (particle of light).	predict	(v). General use: to foresee. Physical Science: to state what will happen, based on a law. See law.
photocell	(n). A device which captures light energy and converts it to electrical energy. Makes use of the photoelectric effect. See photoelectric.	pressure	(n). A continuous force exerted on an object over a certain area, in pascals, Pa. N/m ² . See pascal.
photoelectric	(adj). The effect in which high-energy light, such as UV, can cause electricity to flow in a metal, by ejecting electrons from the metal as photoelectrons.	probability	(n). How likely something is. See likely. Probability is generally a mathematical measure given as a decimal, e.g. [0] means unlikely, but [1,0] means certain, and [0,5] means just as likely versus unlikely. [0,3] is unlikely, and [0,7] is quite likely. The most common way to express probability is as a frequency, or how often something comes up. E.g. an Ace is $\frac{1}{13}$ or 0,077 likely, in a deck of cards, because there are 4 of them in a set of 52 cards.
photoelectron	(n). An electron ejected from a metal by light.	project	(n. & v.). A project (n., pronounced PRODJ-ekt) is a plan of action or long-term activity intended to produce something or reach a goal. To project (v., pronounced prodj-EKT), is to throw something, or to guess or predict (a projection). To project a result means to predict a result. See extrapolate.
photon	(n). A light particle.	projectile	(n). An object that has been projected (thrown or shot). Projectiles will usually start at velocity (speed) 0 m/s, and accelerate, then fall to earth at the same rate as they accelerated.
pi	(n). π , the Greek letter p, the ratio of the circumference of a circle to its diameter. A constant without units, value approximately 3,14159.	propagate	(v). Send or deliver, spread. Said of roots, information, and light or fields.
plane	(n). A flat surface.	propagation	(n). The state of travelling or spreading. "Direction of propagation" means the direction in which something is travelling or spreading.
planet	(n). A large spherical astronomical body which orbits a star (sun). See moon.	proportion	(n). To relate to something else in a regular way, to be a part of something in relation to its volume, size, etc; to change as something else changes. See correlate and respectively.
plot	(v). To place points on a Cartesian coordinate system; to draw a graph.		
polarity	(n). To have poles or distinct ends, e.g. positive and negative (electricity), north and south (magnetism); having two opposite and contradictory properties.		
positive	(adj). Having many protons not paired with electrons; a lack of electrons.		
potential	(n). Having the ability to do work, in particular, E_p (potential energy, the tendency to fall or start moving, as in a spring), or emf (voltage). General use: potential exists when there is an energy difference between two points, e.g. due to gravity or electrical charge. In the context of electricity, read it as "voltage".		

proton	(n). The positively-charged particle that forms the centre of an atomic nucleus, weighing 1 836 times as much as an electron, but having the same and opposite charge. Symbol p^+ . See also nucleus, neutron, electron.	radiate	(v). Physics: To send out radiation (electromagnetic emissions, light, or radioactivity). See emit. Mathematics: To spread out from a central point. See radius.
pull	(n). A force towards the source of the force. Measured in newtons, N.	radiation	(n). Radioactivity or electromagnetic emissions. See radioactive.
pulse	(n). A single vibration or wave crest/trough.	radio	(n). Electromagnetic radiation with a wavelength larger than microwaves.
pump	(n). A machine that uses energy to transfer a fluid from one place to another. In Biology one finds cellular pumps, which are biological machines for transferring ions and nutrients.	radioactive	(adj). Giving out harmful radiation, especially alpha (helium nuclei), beta (high-energy electrons) or gamma (high-frequency electromagnetic radiation).
push	(n). The opposite of a pull.	radius (sing), radii (plur)	(n). The distance between the centre of an object, usually a circle, and its circumference or outer edge. Plural is pronounced "ray-dee-eye."
Pythagoras's Theorem	(n). The square on the hypotenuse is equal to the sum of the squares on the other two sides of a right-angled triangle. Where h is the hypotenuse, a is the side adjacent to the right angle, and b is the other side: $h^2 = a^2 + b^2$.	rainbow	(n). A visible spectrum produced by sunlight diffracting through raindrops. See diffraction.
Q		random	(n). Unpredictable, having no cause or no known cause. Done without planning.
qualitative	(adj). Relating to the quality or properties of something. A qualitative analysis looks at changes in properties like colour, that can't be put into numbers. Often contrasted with quantitative.	range	(n). Mathematics: the set of values that can be supplied to a function. The set of possible y-values in a graph. See domain.
quantitative	(adj). Relating to, or by comparison to, quantities. Often contrasted with qualitative. A quantitative analysis is one in which you compare numbers, values and measurements.	rate	(n). How often per second (or per any other time period). Physics: number of events per second; see frequency.
quantity	(n). Amount; how much.	ratio	(n). A fraction; how one number relates to another number; exact proportion. If there are five women for every four men, the ratio of women to men is 5:4, written with a colon (:). This ratio can be represented as the fraction $\frac{5}{4}$ or $1\frac{1}{4}$ or 1.25; or we can say that there are 25% more women than men.
quantum	(n). The smallest amount of energy possible in a wave of a particular frequency, a discrete quantity of energy E proportional in magnitude to its frequency. In a formula, $E = hf$. See hf.	ray	(n). Physics: A single straight line of electromagnetic radiation. Mathematics: a line from amongst a set of lines passing through the same central point. See radius.
R		reflective	(adj). Giving off all light shone upon it. Popular use: someone who thinks a lot.
radar	(n). A device which uses radio to detect moving vehicles, especially aircraft.		

refraction	(n). Bending light when it travels from one medium (e.g. air) into another medium (e.g. water or glass). Changing the direction of propagation of any wave as a result of its travelling at different speeds at different points along the wave front. See Huygens' principle, diffraction.
repel	(v). To push away from. Physics: like charges repel, unlike charges attract.
repulsion	(n). The process of repelling something.
resistance	(n). How much a conductor (usually metal) slows down or impedes the flow of current. See current, impede, ohm.
resistor	(n). A device which resists current.
rest	(n). Physics: A state of lowest energy.
resultant	(n). Physics: A force, velocity, or other vector quantity which is equivalent to the combined effect of two or more component vectors acting at the same point. The result of mechanical forces pulling in different directions. See vector. It is calculated using trigonometry, with two of the vectors being sides of a right-angled triangle, and the resultant being the third side.
revolution	(n). Physics: A turn; to turn over. Hence common usage in politics: to overturn the government.
revolve	(v). To turn around a point; usually said of something moving at a distance from the central point or another object. Compare to rotate.
rhombus	(n). A quadrilateral (four sided) figure (diagram or shape) which has equal sides, but no right-angles (90° angles).
rms	(abbr). Root mean square. Always lowercase. The square root of the arithmetic mean (average) of the squares of a set of values. See average. Used to get the average voltage of an AC current.
rotate	(v). Same as revolve, but usually said of a solid object turning around its own centre point.

S	
satellite	(n). Any body that orbits any other body. Astronomy: a moon is a satellite of a planet. A planet is a satellite of a star. Engineering: a man-made machine that orbits the earth to provide telecommunications or military services.
scalar	(n). A quantity that does not have a direction, only a magnitude (size). Compare vector. Example: mass.
scale	(n). A system of measurement, with regular intervals or gaps between units (subdivisions) of the scale.
series	(n). Physics: components of a circuit arranged end-to-end without any branches, so that current passes through all items in the circuit one after the other. Chemistry: a range of elements on the periodic table with common properties.
SI	(abbr). Système International. The international system of metric units used by scientists. See metric, IUPAC.
simplify	(v). To make simpler. Mathematics: to divide throughout by a common factor (number or algebraic letter) that will make the equation easier to read and calculate.
solve	(v). To come up with a solution (answer). Show your working.
sound	(n). Vibrations that are transmitted through air or another medium, which one can hear (detect with the ear). It travels at approximately mach 1. See mach. Old use: to be safe or intact, as in "safe and sound" (compare "asunder", and "sundry", meaning in parts).
space	(n). Astrophysics: the vacuum between planets, stars, etc.
spectroscope	(n). A device used to split light up into a spectrum, to analyse the components (elements) that make up the light source. Used to tell what elements make up stars.

spectrum (sing), spectra (pl)	(n). A series of continuous wavelengths which make up a portion of the range of electromagnetic radiation. More simply, a section of electromagnetic radiation, or light. E.g. the microwave spectrum, the radio spectrum, the visible light spectrum, etc. A rainbow is a spectrum of visible light. See rainbow.	substitution	(n). The process of substituting. Mathematics: to replace an algebraic symbol in a formula with a known value or another formula, so as to simplify the calculation. See simplify. Chemistry: to cause a substituent to bond to a substance.
speed	(n). How much distance is covered in a certain amount of time; the scalar of velocity. Measured in metres per second (m/s). See velocity.	sum	(n., v.). To add things up. Represented by Greek Sigma (Σ): Σ or the plus sign (+).
sphere	(n). A perfectly round three-dimensional shape. A ball.	sun	(n). Our nearest star. See star.
square	(n). Mathematics: a shape or figure with four equal sides and only right angles; the exponent 2 (e.g. the square of 4 is $4^2 = 16$).	system	(n). Any closely associated and inter-related or inter-dependent group of things; a set of things working together. Chemistry: a vessel (container) which contains a chemical reaction.
squared	(adj). Having been multiplied by itself, put to the exponent 2. See square.	T	
stable	(adj). Chemistry and Nuclear Physics: not likely to break down or react further.	tangent	(n). Mathematics: a straight line touching a curve only at one point, indicating the slope of the curve at that point; the trigonometric function of the ratio of the opposite side of a triangle to the adjacent side of a triangle in a right-angled triangle; a curve that goes off the chart.
star	(n). A large spherical body in outer space undergoing continuous nuclear fusion. E.g. the sun. See sun. The second-nearest star to us is Alpha Centauri, at 4,7 light-years.	temperature	(n). A measurement of heat. See heat, enthalpy. SI unit is kelvin.
static	(adj). Not in motion, opposite of dynamic. Also short for static electricity.	tensile (strength)	(adj). How strong a material is when stretched; how much force it can withstand before breaking.
stationary	(n). Not moving.	tension	(n). A force of stretching in a material.
stationery	(n). Pens, pencils, paper, ink, files, folders, etc.	terminal	(n). Final; end point.
statistics	(n). The mathematics of chance and probability.	theory	(n). A usually mathematical representation of an explanation for something in the sciences, which does not depend on the thing being explained. A theory differs from a law in that theories are prone to empirical (visible or measurable) refutation (rejection); meaning that they can be discarded if evidence comes in that they are wrong. See law.
steam	(n). Water vapour, microscopic droplets of water. Not a gas, a suspension of water droplets in air. See suspension, gas, liquid, phase, aerosol.	threshold	(n). Physical Science: the magnitude or limit of something, which, if exceeded, will cause something else, e.g. release of radiation, a chemical reaction, etc; the minimum amount of energy required to cause something. Medicine: the maximum safety level of a dose.
STP	(abbr). Standard temperature and pressure; 101,3 kPa and 25 °C.		
sublimate	(v). To change phase of matter from solid straight to gas without the intermediate phase of liquid. See the case of dry ice (CO ₂).		
substance	(n). Matter. Physical things.		
substitute	(v). To replace.		

thrust	(n). See push.
tides	(n). Motion of the earth's oceans due to the gravitational pull of the moon.
torque	(n). Angular momentum; a force that causes rotation; the amount of force delivered through a rotation. See momentum.
trajectory	(n). The path that a projectile will take. See projectile, project (v).
transformer	(n). Physics: a device used to increase (step-up) or decrease (step-down) the voltage of an AC current.
transistor	(n). Physics: a device which can hold current.
transparent	(adj). Allowing most light through. Compare opaque.
transverse (wave)	(adj). A wave whose vibrations move perpendicular to the direction of propagation (left/right). Example: light. Compare longitudinal.
travel	(n). Movement.
trends	(n). Mathematics: regular patterns within data.
trigonometry	(n). Mathematics: the relationship and ratios between sides and angles within a right-angled triangle.
trough	(n). The low point in a transverse wave. See crest, wave, transverse. "Trough" is pronounced "troff".
tungsten	(n). An element (metal) used in incandescent (hot) light bulbs, symbol W.
U	
ultrasound	(n). Sound with too high a frequency for the human ear to detect, used in scanning the body, e.g. the heart for obstructions for blood flow, or foetuses to image (draw, depict) them on a scanning machine.
ultraviolet	(n). Light with a higher frequency than violet light, not visible to the human eye. Present in sunlight, causes sunburn. Can be stopped by Ozone (O_3) in the atmosphere. In darkness it reflects off white clothing as violet light. Also called "black light". Wavelength 400 nm - 10 nm.

unit	(n). A subdivision of a scale. See scale.
unstable	(n). Chemistry or Nuclear Physics: prone to disintegrating or reacting.
UV	(abbr). See ultraviolet.
V	
vacuum	(n). The absence of matter. See matter. Vacuums do not have suction powers. Suction is really a result of a higher pressure pushing objects into the area of low pressure (vacuum).
vapour pressure	(n). The pressure above a liquid caused by molecules evaporating from the surface of its liquid form, when in phase equilibrium (i.e. as many molecules leaving the liquid surface are condensing back into the liquid).
variable	(n., adj.). A letter used to represent an unknown quantity in algebra (n); a quantity that changes (n); subject to change (adj).
vector	(n). A mathematical quantity with direction and magnitude (size). Example: velocity. Compare speed, scalar.
velocity	(n). The amount of distance covered per unit time, in a specified direction. Compare speed. Units: m/s.
visible	(adj). Able to be seen by the human eye, opposite of invisible. Compare microscopic, macroscopic.
viscosity	(n). The thickness of a fluid. A viscous fluid flows slowly, e.g. syrup. Pronounced "viss-KOSS-ittee" and "viss-k's".
volatility	(n). How easily something evaporates. E.g. Ether (C_2H_5OC) is more volatile than water.
volt	(n). Unit of potential difference in electricity. The difference of potential (E_p) that would carry one ampere of current against one ohm resistance. Same as emf. See emf, resistance, ohm, ampere.
voltage	(n). The measurement of volts.
volume	(n). A measure of the space occupied by an object, equal to length x breadth x height.

W	
watt	(n). Unit of power. See power. $P = \frac{W}{t}$ (joules per second).
wattage	(n). The amount of power being used, usually rated in kWh. See kWh.
wave	(n). A series of increasing and decreasing energy concentrations, e.g. sound, light, heat radiation, movements on liquid surfaces, vibrations. A periodic vibration either in a substance (e.g. air, water, solids) which disturbs or moves the particles in that substance, or a vibration or fluctuation (regular change) in electromagnetism. Waves have compressions and rarefactions (dense and looser areas) in air or water or solids when acting as sound. When acting on the surface of a liquid, or in light or electromagnetic waves, waves have crests and troughs (high points and low points). See trough, crest, propagate. Waves have a period (time range between crests/troughs), a frequency (how many crest/trough combinations pass a point per second), as well as a wavelength. See wavelength below.
wavelength	(n). The distance between crests/troughs, written as Greek L (lambda): λ .

weber	(n). The SI unit of magnetic flux, causing the emf of 1 volt in a circuit of one coil when generated or removed in one second.
weight	(n). The force exerted on the mass of a body by a gravitational field. See mass, gravity, force, acceleration. $W = mg$, where $g = 9,8 \text{ m/s}^2$, m is the mass of the body.
work	(n). Energy, measured in joules. Usually in the sense of the amount of energy required to move an object or change its course (direction of motion).
X	
x-rays	(n). Electromagnetic radiation of higher energy than UV but lower energy than Gamma radiation. Wavelength 0,01 to 10 nm. Used to see into the body, but emitted by stars as well.
Y	
yard	(n). Old Imperial measurement of length, approximately equal to a metre (1,09 m).

The maths you need

This section gives you the basic mathematical skills that you need to pass any subject that makes use of mathematics. Whether you're studying for Physical Science, Mathematics, or Mathematical Literacy, these basic skills are crucial. Do not go any further in this book until you have mastered this section.

1. Basic Pointers

- If a formula does not have a multiplication (\times) sign or a dot-product (\cdot), and yet two symbols are next to each other, it means “times”. So, $m_1 m_2$ means mass 1 times mass 2. You can also write it as $m_1 \times m_2$, or $m_1 \cdot m_2$
- Comma means the same as decimal point on your calculator (i.e. $4,5 = 4.5$). Do not confuse the decimal point with dot product (multiply): $4.5 = 4\frac{1}{2}$ but $4 \cdot 5 = 20$. Rather avoid using the dot product for this reason.
- In science it is common to write divisors with an exponent. This means, for example, that 0.5 metres per second is usually written $0.5 \text{ m}\cdot\text{s}^{-1}$ rather than 0.5 m/s . Both notations are perfectly correct, however, and you may use either. It is important, however, that you either use -1 or $/$. If you just put 0.5 ms , that means 0.5 milliseconds, which is not a velocity (speed in a direction); it is a *time*.
- A variable is something that varies (means: changes). So, for example, the weather is a variable in deciding whether to go to the shops. Variables in science and mathematics are represented with letters, sometimes called algebraic variables. The most common you see in maths is x , probably followed by y, z . In science, variables are given their letter symbols specifically depending on what they stand for; so, for example, M or m are used for mass (amount of substance in kilograms); v is used for velocity (speed in a certain direction); a is used for acceleration (change in velocity), etc. You can guess for the most part what a variable is for by what its letter is; so V is voltage, R is resistance, P is pressure, and so on.

2. Subject of Formula or Solving For

Very often in science and mathematics you have to “make something the subject of a formula” or “solve for something”. This refers to finding the value of an unknown quantity if you have been given other quantities and a formula that shows the relationship between them.



Worked example 1

If John has 5 apples, and he gives some to Joanna, and he has two apples left, how many did he give to Joanna? Well, the formula would be something like this:

$$5 - x = 2$$

To solve for x , we simply have to swap the x and the 2. What we’re actually doing is adding “ x ” to both sides:

$$5 - x + x = 2 + x$$

this becomes:

$$5 = 2 + x$$

then we subtract 2 from both sides to move the 2 over:

$$5 - 2 = 2 - 2 + x$$

$$5 - 2 = x$$

$3 = x$... so John gave Joanna three apples.

The same procedures apply no matter how complex the formula looks. Just either add, subtract, square, square root, multiply, or divide throughout to move the items around.



Worked example 2

Let’s take an actual example from Electricity: $V = IR$. This means, the voltage in a circuit is equal to the current in the circuit times the resistance.

Suppose we know the voltage is 12 V, and the resistance is 3 Ω . What is the current?

$$V = IR$$

$$12 = 3 \times I$$

divide throughout by 3 so as to isolate the I

$$\frac{12}{3} = \left(\frac{3}{3}\right) I$$

remember that anything divided by itself is 1, so:

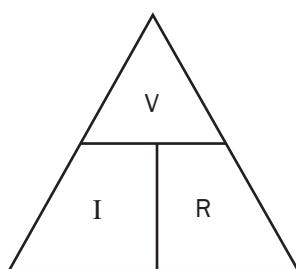
$$\frac{12}{3} = (1) \times I \dots \text{and } \frac{12}{3} = 4 \dots \text{so}$$

$$4 = I \text{ or}$$

$I = 4 A$... The circuit has a current of 4 amperes.

It is possible to remember how to solve for these equations using a triangle mnemonic as shown:

If you’re solving for V , cover V with your hand. Then, I next to R means times R , or IR . So, $V = IR$. If you’re solving for R , cover R with your hand. V is over I . So $R = \frac{V}{I}$. While this is an easier way to do it, remember that many formulas do not consist of only three parts, so it is better to know how to make something the subject of a formula, or solve for something.





Worked example 3

Here's a more tricky example. Given

$$K_c = 4,5$$

$$[\text{SO}_3] = 1,5 \text{ mol/dm}^3$$

$$[\text{SO}_2] = 0,5 \text{ mol/dm}^3$$

$$[\text{O}_2] = \frac{(x - 48)}{64} \text{ mol/dm}^3$$

solve for x .

$$K_c = \frac{[\text{SO}_3]^2}{[\text{SO}_2]^2 [\text{O}_2]} \quad \therefore 4,5 = \frac{(1,5)^2}{(0,5)^2 \frac{(x - 48)}{64}}$$

$$\therefore x = 176 \text{ g}$$

How did we get that answer?



Step by Step

Let's see how it works.

First, solve for the exponents (powers):

$$4,5 = \frac{2,25}{(0,25) \frac{(x - 48)}{64}}$$

Now, we can see that 2,25 and 0,25 are similar numbers (multiples of five), so let's divide them as shown.

$$4,5 = (2,25 / 0,25) \times ((x - 48) / 64)$$

That leaves us with

$$4,5 = 9 \times ((x - 48) / 64)$$

But if we're dividing a divisor, that second divisor can come up to the top row. Here's a simple example:

$$\begin{aligned} 1 \div (2 \div 3) &= \frac{1}{\frac{2}{3}} \\ &= \frac{1 \times 3}{2} \\ &= \frac{3}{2} = 1,5 \end{aligned}$$

If you doubt this, try it quickly on your calculator: $1 \div (2 \div 3)$... this means, one, divided by two-thirds. Well, two-thirds is 0,6667, which is almost one. So how many "two-thirds" do you need to really make up one? The answer is one and a half "two-thirds" ... i.e. $0,6667 + (0,6667 \div 2) = 1$. Hence the answer is 1,5.

So, back to the original problem, we can bring the 64 up to the top line and multiply it by nine:

$$4,5 = 9 \times ((x - 48) / 64)$$

$$4,5 = \frac{9 \times 64}{x - 48}$$

$$4,5 = \frac{576}{x - 48}$$

Now we can inverse the entire equation to get the x onto the top:

$$\frac{1}{4,5} = x - \frac{48}{576}$$

Now we multiply both sides by 576 to remove the 576 from the bottom row

$$\frac{576}{4,5} = \frac{(x - 48) 576}{576}$$

and we cancel the 576's on the right hand side as shown above. Now, if $576 \div 4,5 = 128$, then

$$128 = x - 48$$

Now we add 48 to both sides to move the 48 across

$$128 + 48 = x - 48 + 48 \quad \dots \text{ hence, } 128 + 48 = x = 176.$$

3. Statistics

Many experiments in science and many reports in economics make use of statistics. You should therefore at least know the following:

Dependent variable: The thing that comes out of an experiment, the effect; the results.

Independent variable(s): The things that act as input to the experiment, the potential causes. Also called the controlled variable.

Control variable: A variable that is held constant in order to discover the relationship between two other variables. “Control variable” must not be confused with “controlled variable”.

It is important to understand that correlation does not mean causation. That is, if two variables seem to relate to each other (they seem to correlate), it doesn't mean that one causes the other. A variable only causes another variable if one of the variables is a function $f(x)$ of the other. We will see more about this when we look at graphs, below.

Mean: The average. In the series 1, 3, 5, 7, 9, the mean is $1 + 3 + 5 + 7 + 9$ divided by 5, since there are 5 bits of data. The mean in this case is 5.

Median: The datum (single bit of data) in the precise middle of a range of data. In the series 1, 3, 5, 7, 9, the median value is 5.

Mode: The most common piece of data. In the series 1, 1, 2, 2, 3, 3, 3, 4, 5, the mode is 3.

Often in scientific formulas it is said that things are proportional to each other. However, we cannot calculate the value of a force or energy output or mass etc., if we only know what things are in proportion (i.e. which things correlate).

Let's take momentum for example. Momentum (how forcefully something moves, more or less), is proportional to velocity (speed in a direction). So the faster something's moving, the more momentum it has. But p (momentum) can't be calculated if we only know velocity; we need to know mass as well. Why? Because momentum is also proportional to mass; the more massive something is, the more momentum it has. Thus, to get rid of the proportionality sign (\propto), we have to come up with a formula.

Many experiments in science serve to find out what the relationship is between two variables, i.e. if they're merely correlated – proportional – or if they're causally related. In the case of momentum, it's easy, because there are no further variables: $p = mv$. However, in the case of gravity or

electric or magnetic field strengths, it's not that easy. In those cases, we have to introduce something called a "constant". A constant is a fixed value that is always multiplied into an equation. Constants are often written k. However, some specific constants, such as in the Law of Gravity, have their own symbol, in this case, G. These constants are given in the tables later in this book.

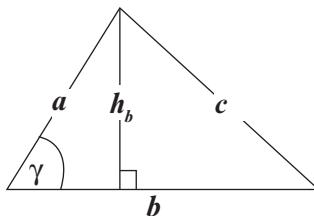
4. Triangles

Science makes a lot of use of triangles, e.g. vector force diagrams, areas under graphs, and so on.

The area of a triangle is half the base times the height: $A = \frac{b}{2}(h)$. A triangle with a base of 5 cm and a height of 3 cm will have an area of $2,5 \times 3 = 7,5 \text{ cm}^2$. This is useful when considering graphs of motion/acceleration.

$$A = 7,5$$

b	Base	5
h_b	Height	3



Lengths of Triangle Sides

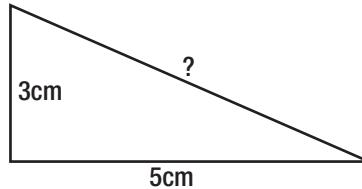
You can calculate the lengths of sides of right-angled triangles using Pythagoras' Theorem. The square of the hypotenuse is equal to the sum of the squares of the other two sides: In this diagram, b = base, h_b = height, and c = the hypotenuse: $c^2 = h_b^2 + b^2$.

e.g.

Worked example 4

In the triangle shown, the hypotenuse, marked "?", can be obtained by squaring both sides, adding them, and then square-rooting them for the length of the hypotenuse. That is:

$3^2 + 5^2 = 9 + 25 = 34$. Since in this case $34 = \text{hyp}^2$ it follows that the square root of 34 gives the value of "?", the hypotenuse. That is, 5,83 cm.



This is useful when you get to vector addition, e.g. if the known sides were vectors and we wanted to know the value and strength of the force, and the direction in which it would go.

5. Trigonometry

You can use trigonometry to calculate triangle sides' sizes if you do not have enough information, e.g. you do not have the sizes of at least two sides (but do have the angle).

sin = opposite / hypotenuse

sin = O/H

cos = adjacent / hypotenuse

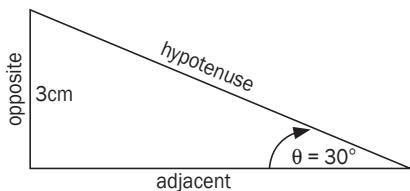
cos = A/H

tan = opposite / adjacent

tan = O/A

It's probably easiest to remember this as SOHCAHTOA (soak a toe or soccer toe).

Hypotenuse is the longest side next to to the angle, usually represented with a theta (θ). “Opposite” means the side of the triangle directly opposite the angle. “Adjacent” means the side adjacent to (next to) the angle, that is not the hypotenuse.



Worked example 5

In this triangle, the side opposite the angle θ is 3cm long. The side adjacent to the angle θ , and the hypotenuse, are unknown. Theta, the angle, is 30 degrees.

How do we calculate the hypotenuse? Well,

$$\sin \theta = \frac{O}{H} = 3 \text{ cm} \div H.$$

$\sin 30^\circ = 0,5$ (you can get this from your calculator, or memorise it).

thus

$$0,5 = 3 / H$$

solving for H, we multiply throughout by H to make H the subject of the formula:

$$H \times 0,5 = 3 \times H \div H$$

$$H \times 0,5 = 3$$

now divide throughout by 0,5 to isolate the H:

$$H \times 0,5 \div 0,5 = 3 \div 0,5$$

$$H = 3 \div 0,5 \therefore H = 6 \text{ cm}$$

Let's try work out what the adjacent side is equal to, assuming we don't know the hypotenuse.

$$\tan \theta = O/A$$

$$\tan 30^\circ = 3 \text{ cm} \div A$$

$$0,57735 = 3 \div A$$

$$A \times 0,57735 = 3 \times A \div A$$

$$A \times 0,57735 = 3$$

$$A = 3 \div 0,57735$$

$$A = 5,196 \text{ cm} \approx 5,2 \text{ cm.}$$

Let's check this with Pythagoras. Suppose we want to prove that the opposite side is 3 cm. We have H = 6 and A = 5,2. So, Pythagoras tells us that $A^2 + O^2 = H^2$. So,

$$5,22 + 0^2 = 6^2$$

$$0^2 = 6^2 - 5,22$$

$$0^2 = 36 - 27$$

$$0^2 = 9$$

So the square root of O^2 will give us O namely, O = 3 cm. The trigonometric calculation is correct.

Lastly, there are three other operations you can do in trigonometry, but they're just inverses of the first three: *cosecant*, *secant*, and *cotangent*. *Cosec*, sometimes abbreviated *csc*, is the reciprocal (inverse) of sine. *Sec* is the inverse of *cosine*. And *cot* is the inverse of tangent. So this means if $\sin = O/H$, then $\text{cosec} = H/O$, and so on.



Worked example 6

Earth orbits the sun at a distance of 149 597 870 700 metres or 149 597 870,7 km (one hundred and forty nine million km). This distance is called the AU, or astronomical unit. The flat disc that corresponds to earth's orbit is called the 'ecliptic'. Suppose that on 21 December, an unknown object is observed at an angle of 88° to the ecliptic, and that on June 21 the same object is observed at 92° . How far is the unknown object in AU?



Step by Step

Step 1. Ignore extra information. Since the earth orbits the sun, the angle to the unknown object relative to the earth is the same in both cases; it's just that on one date, the earth is on one side of the unknown object, and on the other date, it is on the other side.

From the angles given, you can tell that the unknown object is at 90° to the sun relative to the earth.

Step 2. We know the angle to the unknown object, and the distance to the sun. So, if we draw a triangle where the sun is the right-angle, the earth is at the tip of the hypotenuse, and the distant unknown object is opposite the sun, we get the following triangle.

So, we want the hypotenuse. We know that triangles add up to 180° , so the difference between θ and the given angle of 88° is 2° . That means that the angle the unknown object makes in reference to earth is 2° . Thus:

$$\sin \theta = \frac{H}{R}$$

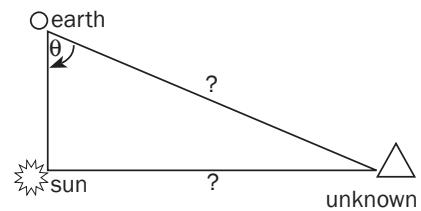
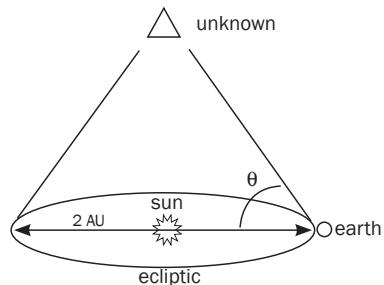
$$\sin 2^\circ = 1 \text{ AU} \div R = 149 597 870,7 \text{ km} \div R$$

$$0,035 = 149 597 870,7 \text{ km} \div R$$

$$R = 149 597 870,7 \text{ km} \div 0,035$$

$$R = 4 286 533 756,4964 \text{ km} = 28,6 \text{ AU}$$

This means that the unknown object is 4,2 billion km away, or 28,6 AU away.



6. Graphs

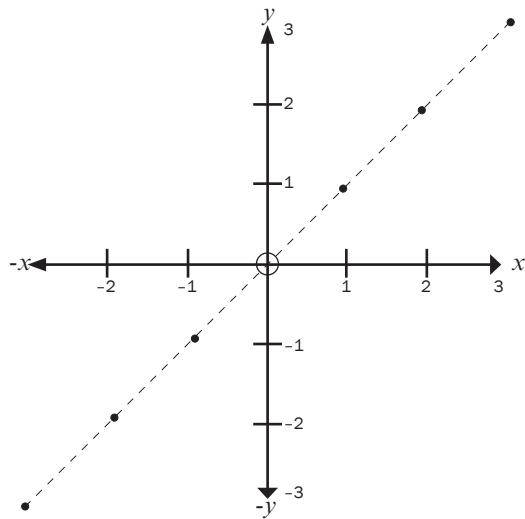
A lot of work in science involves interpreting graphs. You get graphs of motion, graphs of rates of chemical reactions, graphs of distance-relative strengths of force fields, and so on. Before you can understand these graphs, it's probably best to start from scratch with Cartesian Coordinates.

"Coordinates" are numbers that refer to the distance of a point along a line, or on a surface, or in space, from a central point called the "origin". Graphs that you will use have only two dimensions (directions). The positions of points on these graphs are described using two coordinates: how far across (left-to-right) the point is, called the x -coordinate, and how far up-or-down on the page the point is, called the y -coordinate.



Worked example 7

Consider the following graph. It shows six points in a straight line.



The coordinates shown can be described using what are called “ordered pairs”. For example, the furthest point in this graph is 3 units across on the “ x -axis” or horizontal line. Likewise, it is also 3 units up on the y -axis, or vertical (up and down) line. So, its coordinates are $(3;3)$. The point just below the midpoint or “origin”, is one unit down of the x -axis, and one unit left of the y -axis. So its coordinates are $(-1;-1)$. Note that anything to the left or below of the origin (the circle in the middle), takes a minus sign. In most cases in science, you'll only have graphs showing positive axes (plural of axis, pronounced aks-eez), since most graphs are of time.

This series of dots look like they're related to each other, because they're falling on a straight line. If you see a result like this in an experimental situation, it usually means that you can predict what the next dot will be, namely, $(4;4)$. This kind of prediction is called “extrapolation”. If you carry out the experiment, and find that the result is $(4;4)$, and then $(5;5)$, you've established that there is a strong relation or correlation. You can therefore start thinking about a formula to describe your findings. For example, this might be a graph showing a measurement of voltage (x) against a measurement of resistance (y).

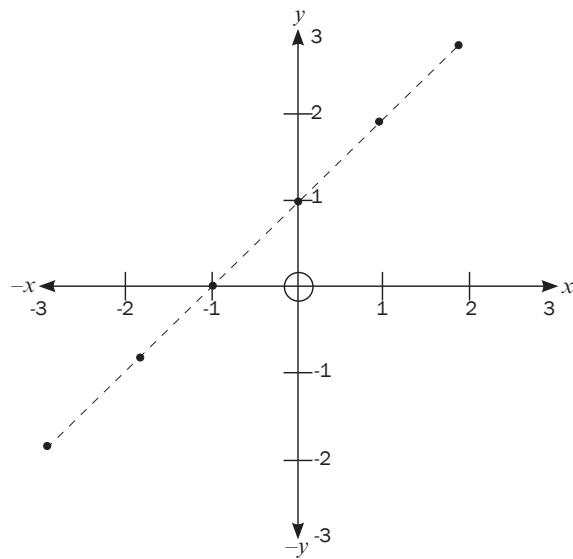
Now, another way of saying that x relates to y , or x is proportional to y , is to say that y is a function of x . This is written $y = f(x)$. So, in the example given above, voltage is a function of resistance. But how is y related to x in this graph? Well, it seems to be in a 1 to 1 ratio: $y = x$. So the formula for this graph is $y = x$. In this case, we're only dealing with two factors; x and y . In other graphs you'll find that sometimes more factors are involved, such as acceleration graphs, which have units of m/s^2 . Don't worry about that; you treat them the same way (for example, m/s^2 vs. time).



Worked example 8

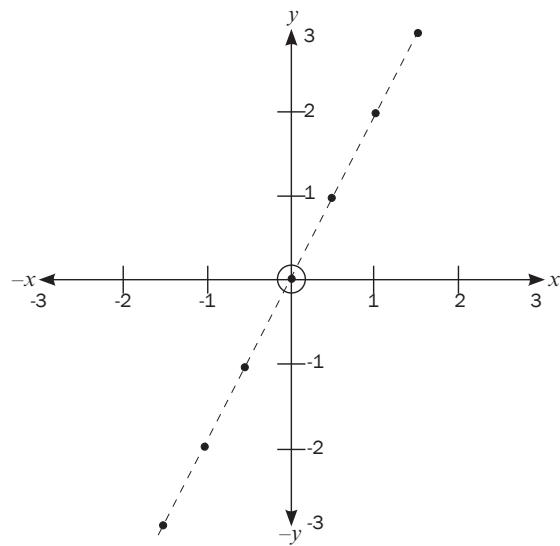
Now, let's take a slightly more complex case, illustrated below.

In this next graph, we see that wherever x is equal to something, y is one more. So, trace your finger from the bottom left dot upwards. It meets the x -axis at the point -3 . Do the same for the same point towards the y -axis. You'll see it meets the y -axis at -2 . You'll see the next coordinates are $(-2;-1)$, then $(-1;0)$, then $(0;1)$, $(1;2)$, and finally $(2;3)$. From this we can see that whatever x is, y is one more. So, $y = x + 1$ is the formula for this line.



Worked example 9

Let's take another case. In this next case, we see the following values: where x has a certain value, y has double that value. Let's tabulate it.



x	y
1,5	3
1	2
0,5	1
0	0
-0,5	-1
-1	-2
-1,5	-3

So, when x is 1,5, y is 3, when x is 1, y is 2. Thus, the formula for this line is: $y = 2x$. This value next to x is called the “gradient” or “slope” of the line. The larger the value next to x is, i.e. the larger the gradient, the steeper the slope. The gradient is usually abbreviated as “ m ” when it is unknown.

Now, how this applies to science is simple: if we are looking, for example, at a case of a graph of a chemical reaction, we will usually have the x -axis as time. And the y -axis will usually be the quantity (amount) of substances produced. So, if we have a graph of a chemical reaction with a large gradient, it means that the reaction is fast; a lot of substance (y) is produced in a short time (x). If, for example, we heated the reaction, and saw that the gradient increased even more, that would show that the chemical reaction was sped up by heat, or, that reaction rate is proportional to heat. Likewise, if the gradient sloped downwards, it would show that the reaction slowed down over time, because y , the amount of substance produced, was decreasing, as x (time) increased, e.g. because the reactants were being used up.

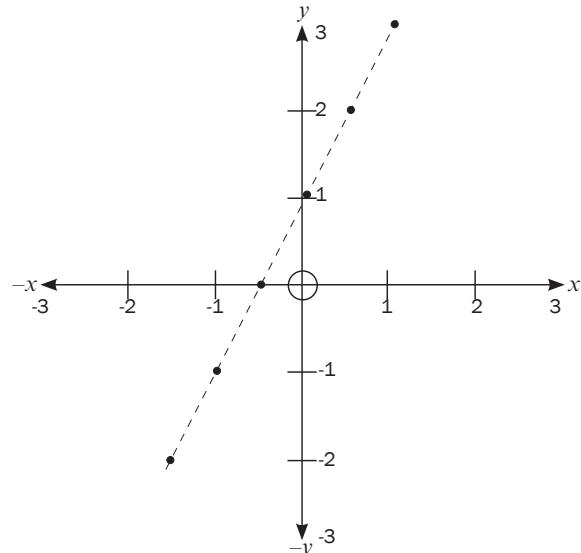
e.g.

Worked example 10

Let's do one more case. In this case, we can see that y is a function of x , since it's a straight line graph. However, it's not that easy to see the relationship between x and y . We can see that the slope is the same as the previous graph, so it must be something like $y = 2x$. However, it doesn't quite make sense, since $2(-1,5)$ is not -2 . We see that where x is zero (at the origin), y is at 1. But the slope is the same, so it must be $y = 2(0) + 1$. So the formula is: $y = 2x + 1$.



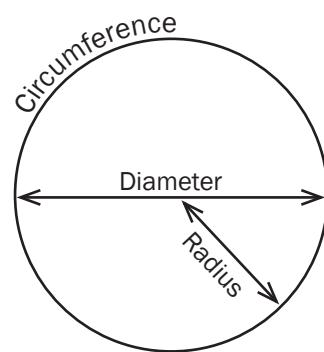
As you can see, this is where mathematicians get the general equation for a straight line of $y = mx + c$. (“ c ” stands for “constant”).



x	y	$2x + 1$
-1,5	-2	$2(-1,5)+1 = -3+1 = -2$
-1	-1	$2(-1)+1 = -2+1 = -1$
-0,5	0	$2(-0,5) + 1 = -1+1 = 0$
0	1	$2(0)+1 = 0+1 = 1$
0,5	2	$2(0,5)+1 = 1+1 = 2$
1	3	$2(1)+1 = 2+1 = 3$

7. Circles

- Diameter is the width of a circle ($2r$); radius is half the diameter ($\frac{d}{2}$). The edge of a circle is called the “circumference”. “Diameter” means to “measure across”. Compare “diagonal” which means an angle across a square or rectangle, so “dia-” means “across” (Greek). “Circumference” means to “carry in a circle” (Latin); think of how the earth carries us in a circle or orbit around the sun. To remember the difference between these things, just remember that the sun’s rays *radiate out from the sun in every direction*, so the *radius* is the distance from the centre of a circle, e.g. the sun, to the outer edge of a circle surrounding it, e.g. earth’s orbit (the circumference).
- Area of a circle = πr^2
- Circumference = $2 \pi r$
- You can use the above to solve for radius or diameter.

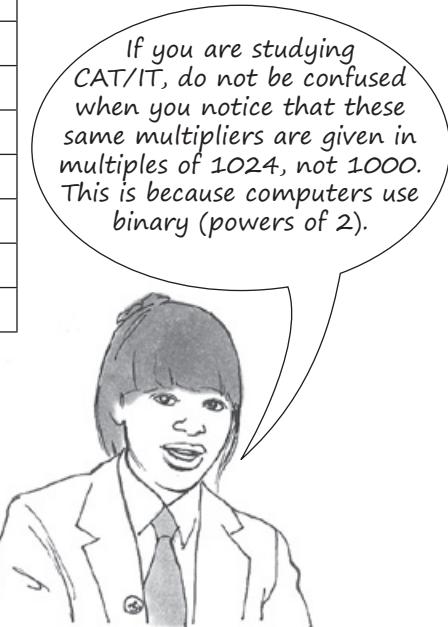


Resource Sheets

The following information sheets will be supplied to you in the exam. You do not need to memorise them.

SI Units: Multipliers

Prefix	Symbol	Value	Value written in full
tera	T	10^{12}	1 000 000 000 000
giga	G	10^9	1 000 000 000
mega	M	10^6	1 000 000
kilo	k	10^3	1 000
hecto	h	10^2	100
deka	da	10^1	1
deci	d	10^{-1}	0,1
centi	c	10^{-2}	0,01
milli	m	10^{-3}	0,001
micro	μ	10^{-6}	0,000 001
nano	n	10^{-9}	0,000 000 001
pico	p	10^{-12}	0,000 000 000 001
femto	f	10^{-15}	0,000 000 000 000 001



Constants

Formulas

Forces and Motion:

u, v_0	=	velocity (initial)
v, v_1	=	velocity (final)
s, x	=	displacement/distance travelled
t	=	time
a	=	acceleration
Δ	=	change in
p	=	momentum
r	=	distance between bodies

$$F = ma$$

$$F = G(m_1 m_2)/r^2 \quad (\text{Newton's Law of Gravitation})$$

$$a = \Delta v/t \quad v = s/t$$

$$v = u + at \quad \text{or} \quad v_1 = v_0 + a\Delta t$$

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad \Delta x = v_0\Delta t + \frac{1}{2}a\Delta t^2$$

$$v^2 = u^2 + 2as \quad \text{or} \quad v_1^2 = v_0^2 + 2as$$

$$p = mv \quad p_1 v_1 = p_2 v_2$$

Replace "a" with g (9.8 m/s^2) if falling to earth.

Energy, Work, Power, Waves:

E, W	=	energy or work
E_k	=	kinetic energy
W_0	=	work function
P	=	power
H	=	height off the ground
h	=	Planck's constant
λ	=	wavelength (Greek letter L)
f	=	frequency
c	=	speed of light
m	=	mass
v	=	velocity
T	=	time/period

$$E = W \quad P = \frac{W}{t} \quad E_p = mgH$$

$$W = F\Delta x \cos\theta$$

$$E_k = \frac{1}{2}mv^2 \quad (\text{at speeds much less than light})$$

$$E = mc^2 \quad (\text{light speed, nuclear reactions})$$

$$E = hf$$

$$v = f\lambda \quad (\text{at light speed, use } c = f\lambda)$$

$$f = \frac{1}{T}$$

Doppler Effect:

$$f_L = \frac{V \pm V_L(f_s)}{V \pm V_s} \quad \text{OR} \quad f_L = \frac{V(f_s)}{(V - V_s)}$$

Electricity and Transformers:

$$E = \text{energy (joules)}$$

$$q, Q = \text{charge}$$

$$v, V = \text{voltage}$$

$$F = \text{force}$$

$$I = \text{current in amperes}$$

$$R = \text{resistance in ohms } (\Omega)$$

$$s/p = \text{secondary/primary coils}$$

$$E = \frac{V}{d} \quad E = F/q$$

$$E = \frac{kQ}{r^2}$$

$$F = \frac{k(q_1 q_2)}{r^2} \quad (\text{compare to Newton's Law of Gravitation})$$

$$F = k(I_1 I_2) L/r \quad (\text{where in this case, } k = 2 \times 10^{-7})$$

$$V = IR \quad I = Q/\Delta t$$

$$P = IV = \frac{QV}{t} \quad E = VIt = \frac{Vt}{R}$$

$$R = r_1 + r_2 + r_3 \dots \quad (\text{in series})$$

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \dots \quad (\text{in parallel})$$

$$V_p I_p = V_s I_s \quad \frac{V_s}{V_p} = N_s/N_p$$

Notes:

In most cases, a subscript of "1" means "before" and a subscript of "2" means "after", so, $p_1 v_1 = p_2 v_2$ means "momentum at time 1 times velocity at time 1 (before collision), is equal to momentum at time 2, times velocity at time 2 (after collision)".

To convert from km/h to m/s, divide by 3.6.

Notes

Mechanics: Force and Newton's Laws

Summary

This section covers important aspects from the Grade 11 work. Make sure that you revise it well so that you can apply this knowledge to your Grade 12 work. In order to be successful in this section, you need to revise trigonometry and Pythagoras' theorem.

You must know how to:

- Draw a sketch of parallel and perpendicular vectors.
- Determine the resultant vector graphically using the head to tail method as well as by calculation.
- Resolve a vector into its parallel and perpendicular components.

You must remember:

- Newton's Laws and how to apply them.
- Different kinds of forces.
- Force diagrams and free body diagrams.

Definitions and Laws you must remember:

1. A **force** is a push or pull upon an object resulting from the object's interaction with another object.
2. **Gravitational force** is the force of **attraction** that objects exert on other objects in virtue of having mass. It is the force that makes all things fall and causes tides in the ocean. The greater the mass of an object, the greater its gravitational pull.
3. The **normal force** is a **perpendicular** force that a surface exerts on an object with which it is in contact.
4. The **resultant (net) force** acting on an object is the **vector sum** of all the forces acting on the object. The **vector sum** is the sum of all vectors (all the forces added up, taking their directions into account).
5. **Newton's First Law of Motion:** An object will remain at rest or continue moving at a constant velocity (or at constant speed in a straight line) unless acted upon by a non-zero **external** resultant force.
6. **Newton's Second Law of Motion:** If a **resultant (net) force** acts on an object, the object will accelerate in the direction of the resultant force. The acceleration is directly proportional to the resultant force and inversely proportional to the mass of the object.
7. **Newton's Third Law of Motion:** When object A exerts a force on object B, object B **simultaneously** exerts a force on object A, which is **of equal magnitude** but **opposite in direction**.
8. **Newton's Law of Universal Gravitation:** A force of gravitational attraction exists between any two particles or objects anywhere in the universe. The magnitude of this force is **directly proportional** to the product of the objects' masses and is **inversely proportional** to the square of the distance between their centres.



hint

"Universal" means the statement is valid for any object in the universe.

1.1 Revision: Vectors

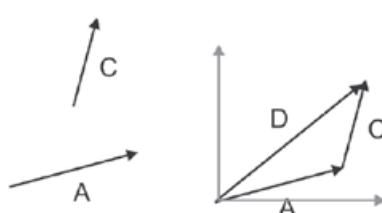
A vector is a quantity that has both **magnitude** (size) and **direction**.

- We can use bold type to represent a vector, \mathbf{R} , or an arrow above the letter \vec{R} .
- Vectors may be added or subtracted graphically by laying them head to tail / head to head on a set of axes.



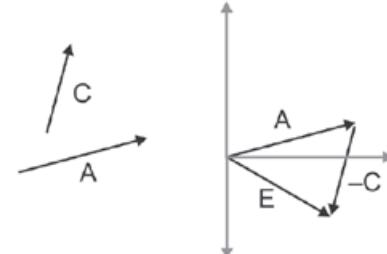
Worked example 1

Addition



$$\vec{A} + \vec{C} = \vec{D}$$

Subtraction



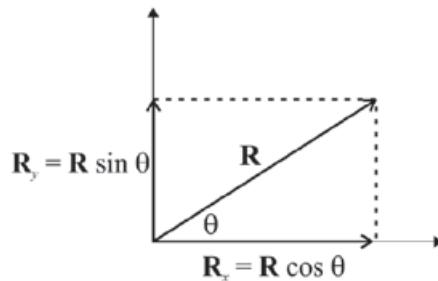
$$\vec{A} - \vec{C} = \vec{E}$$

- Just as one vector is the sum of two vectors, we can also find two vectors to make up one vector.
- In mechanics, it is often useful to break up a vector into two component vectors, one horizontal and the other vertical. We use basic trigonometry to find the components.



Worked example 2

Example: Vector \vec{R} makes an angle θ with the x -axis. \vec{R} is broken into component vectors \vec{R}_x and \vec{R}_y .



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{R_x}{R}$$

$$\therefore R \cos \theta = R_x$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{R_y}{R}$$

$$\therefore R \sin \theta = R_y$$

$\vec{R}_x = \vec{a} \cos \theta$ and $\vec{R}_y = \vec{a} \sin \theta$, where \vec{R}_y is the angle between \vec{a} and the x -axis (\vec{R}_x).



Worked example 3

If vector \vec{R} has a magnitude of 5 and is at an angle of $\theta = 36,86^\circ$, the components are $\vec{F}_x = 5 \cos 36,86^\circ = +4$ and $\vec{R}_y = 5 \sin 36,86^\circ = +3$.

1.2 What is force?

When objects interact with each other, they exert forces on each other.

If a force acts on an object, it can cause a change to the object. Some of the possible changes are:

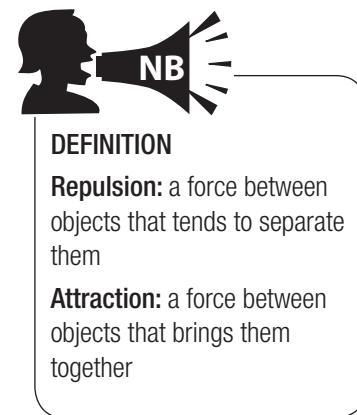
- the shape of an object
- the object's state of rest
- the velocity of the object
- the direction in which the object moves
- the object's acceleration.

Force (\vec{F}) is a **vector quantity**. This means it has magnitude and direction.

- It may be represented by an arrow in a vector diagram. The length of the arrow shows its **magnitude** and the angle shows its **direction**.
- It is measured in the SI unit **newton (N)**.

We show the force vector using \vec{F} .

F without an arrow represents the size of the force vector only.



Example

represents a force (\vec{F}) of 12 N to the right.
12 N

- Objects exert push (**repulsion**) or pull (**attraction**) forces on each other.
- A force can be classified as either a **contact force** or a **non-contact force**
- Objects can exert a force on each other when they are in contact (touching each other) e.g. friction and normal forces

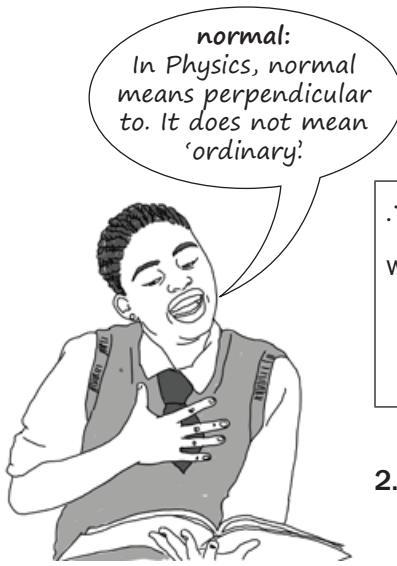
OR

- Objects can exert a force on each other when they are **not in contact** (i.e. are apart from each other) e.g. magnetic, electrostatic and gravitational forces.

1.3 Different types of forces

We study these different forces:

1. Gravitational force or weight (\vec{F}_g or \vec{w})
2. Normal forces (\vec{F}_N or \vec{N})
3. Frictional forces (\vec{F}_f)
4. Applied forces (push or pull)
5. Tension (\vec{F}_T or \vec{T})



1. Gravitational force (\vec{F}_g or \vec{a}):

- Gravitational force is the force of **attraction** that the Earth exerts on an object above its surface.
- Gravitational force acts **downwards** towards the centre of the Earth.
- The weight (\vec{w}) of an object is the same as the gravitational force (\vec{F}_g) on the object, so $\vec{F}_g = \vec{w}$
- The weight of an object is the product of the **mass** and the **gravitational acceleration** of the Earth.
so $\vec{w} = m\vec{g}$ where m is mass and \vec{g} is the acceleration due to gravity.

$$\therefore \vec{F}_g = \vec{w} = m\vec{g}$$

where \vec{F}_g is gravitational force
 \vec{w} is the weight of an object
 $m\vec{g}$ is mass \times gravitational acceleration

2. Normal force (\vec{F}_N or \vec{N}):

When an object rests on a surface, the surface exerts a force on the object, called a **normal force**.

It is a **contact** force that acts at a right angle (90°) upwards from the surface.

In the diagrams below, you will see a free body diagram and a force diagram. In a force diagram, you show the object that is experiencing forces. The forces act on the body at its "centre of gravity". In a free body diagram, you do not show the object that is experiencing forces; i.e. you treat the object as a single point.

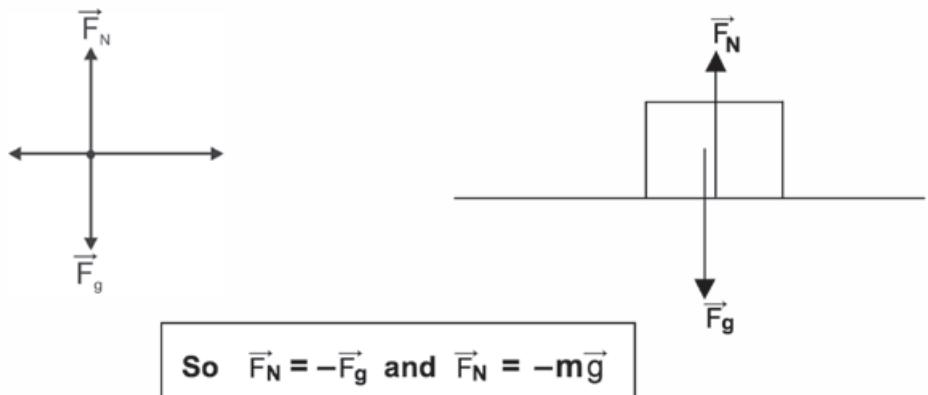


DEFINITION

A "centre of gravity" is a point from which the weight of a body or system may be considered to act. In uniform gravity it is the same as the center of mass.

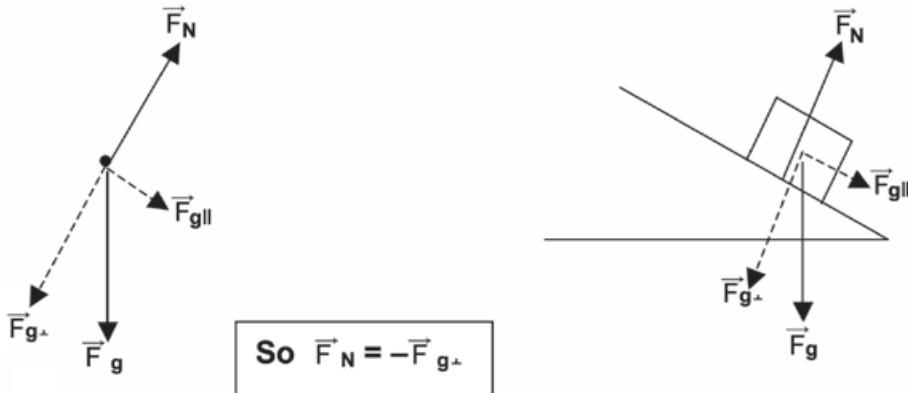
2.1. When an object is resting or moving on a horizontal surface the normal force will have the same magnitude, but an opposite direction to the weight of the object or gravitational force.

An object resting on a horizontal surface



- 2.2. When an object is resting or moving on an inclined plane (surface), the normal force will have the same magnitude, but an opposite direction to the perpendicular component of the weight of the object or gravitational force.

An object resting on an inclined plane (surface)



3.1 Frictional force (\vec{F}_f or f):

- Frictional force opposes motion. So it works against the movement of an object.
- Frictional force acts in the **opposite direction** to an object's motion or intended motion.
- The rougher the surface, the more friction there is between the object and the surface.

The less rough the surface, the less friction there is between the object and the surface.

This means that the greater the magnitude of the normal force acting on the object, the greater the magnitude of the frictional force. Think of grinding something here. The harder you press, the more "normal" (perpendicular) force there is. Hence, when you are grinding something, e.g. crushing maize for making pap, it experiences strong normal (perpendicular) forces and thus strong frictional forces; hence it is ground up.

- If an object is at rest, then there is a **static** frictional force.
- If the object is moving, then there is **kinetic** frictional force.

3.2 The coefficient of friction (μ)

The **coefficient of friction** depends on the material of the two surfaces that are in contact.

Examples

- Steel on wet ice has a low coefficient of friction (slides easily).
- Rubber on tar has a higher coefficient of friction (more grip, less sliding).

- When an object is at rest on a horizontal surface and **no force is applied** to it, then there is no static friction.
- When a small force is applied to an object at rest, then the force of static friction increases as the applied force increases.



- As the force increases, the static friction continues to increase.
- This continues until the static friction reaches a maximum value – it cannot increase further. Eventually maximum static friction force is exceeded and the object moves.
- The friction then decreases to a smaller value called the kinetic friction (f_k).
- The kinetic friction remains constant while the object moves at a constant speed.
- The kinetic friction remains smaller than the maximum static friction. $f_s \leq \mu_s F_N$ and $f_k = \mu_k F_N$



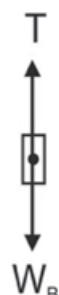
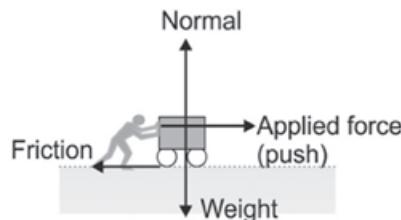
When an object moves along a surface inclined at an angle θ , the normal force is multiplied by the kinetic coefficient of friction to find the frictional force.

The kinetic coefficient is calculated using $\cos \theta$:

$$\vec{F}_N = \vec{F}_{g\perp} = m\vec{g} \cdot \cos \theta \quad f_k = \mu_k \vec{F}_N$$

4. Applied forces

An applied force is a force that a person or object applies to another object. If a person is pushing a cart along the ground, then there is an applied force acting upon the object.



5. Tension (\vec{F}_T or \vec{T}):

When an object is pulled by a rope (or string or cable), or hanging from a ceiling, the rope applies a force on the object. This force is called **tension**. It is a **contact force** and acts in the opposite direction to the ‘pull’. If an object hangs from a rope, the direction of the tension is always **upwards** in the rope. This force complies with Newton’s Third Law, i.e. it is the reaction to the action of the pulling.

1.4 Force diagrams and free body diagrams



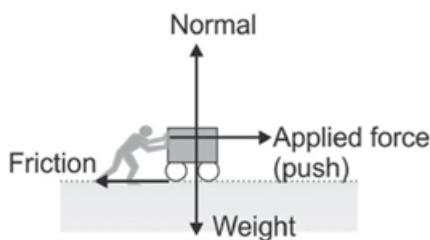
- In force and free body diagrams we consider forces acting on ONE (the same) object
- When you answer questions about force, you must:
 - name the forces
 - state which object exerts a force on which object
 - state the directions of the forces.

Example

A man pushes a loaded trolley along a horizontal floor.

We can identify the following forces that are acting on the trolley:

- the weight or \vec{F}_g of the trolley and load, i.e. the force exerted downwards on the trolley by gravity/ the earth
- the normal force, \vec{F}_N , exerted upwards on the trolley by the floor
- the applied force that the man exerts on the trolley, which acts in a forwards direction
- the frictional force, \vec{F}_f , in the opposite direction to the motion.



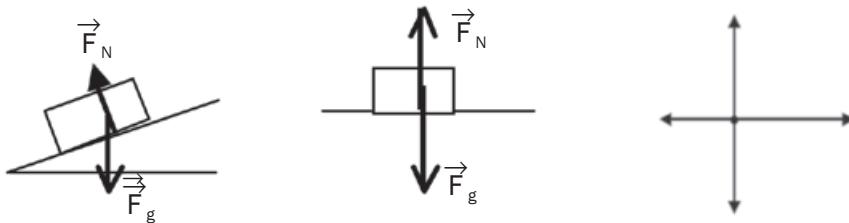
Forces acting on an object can be represented by **force diagrams** or by **free body diagrams**.

Force diagram

- The object is represented as a block and the forces as vectors.
- The vectors start at the point of application.
- Weight is drawn from the object's centre of gravity, downwards.

Free body diagram

- The object is represented as a dot and the forces as vectors.
- The vectors start at the dot and they all point away from the dot.
- If the object is on an inclined surface, the weight vector can be resolved into two component vectors.

**Force symbols in diagrams**

We use these symbols to help represent forces in force diagrams and free body diagrams:

- \vec{F} or \vec{F}_{applied} : applied force, in the direction applied
- \vec{F}_f or F : friction force, surface on object, opposite to direction of motion
- \vec{F}_g or \vec{w} : gravitational force or weight, force exerted by the earth on object, downwards
- \vec{F}_N or N : normal force, surface on object; perpendicularly upwards from the surface
- \vec{F}_T or T : tension, cable or rope on object, in direction of motion.

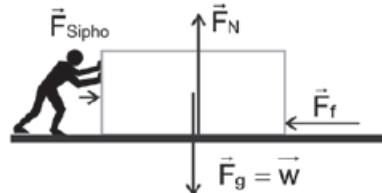


Steps for drawing force or free body diagrams

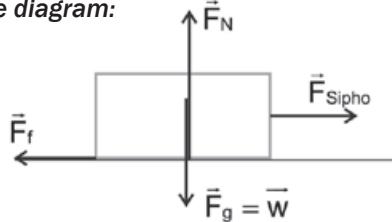
Follow the steps in this example.

Example: An object on a horizontal surface (plane):

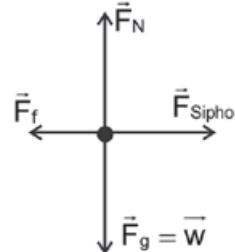
Sipho exerts a force to the right while pushing a car along a rough, flat road. Draw a force diagram and a free body diagram to represent the situation.



Force diagram:



Free body diagram:



Step 1. If there is a surface, draw a line to represent it.

Step 2. Force diagram: draw a block to represent the object.

Step 3. Free body diagram: draw a dot to represent the object.

Step 4. Draw a vector to represent the weight (\vec{F}_g) of the object.

Step 5. If the body rests on a surface draw an arrow to represent the normal force, upwards from and perpendicular to the surface. (\vec{F}_N)

Step 6. Draw an arrow to represent each applied force.

Step 7. Draw an arrow to represent friction (if there is friction).

Step 8. If you are drawing a free body diagram, erase the line representing the surface.



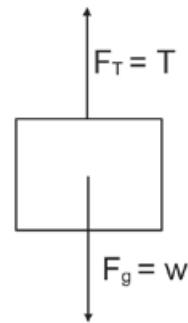
Worked example 4

Draw a force diagram and a free body diagram for an object hanging from a rope or a cable.



Solution

Force diagram:



Free body diagram:



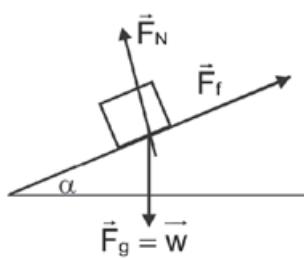
Component vectors in free body diagrams

Free body diagrams are useful for showing all the forces involved in a situation.

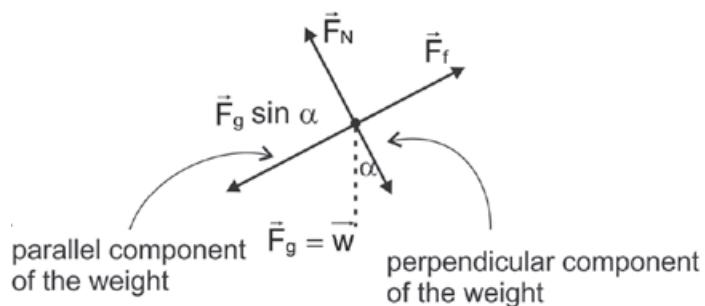
When an object rests on an inclined plane, the force due to gravity may be shown by two vectors:

- one representing the component parallel to the surface
- the other representing the component perpendicular to the surface.

Force diagram:



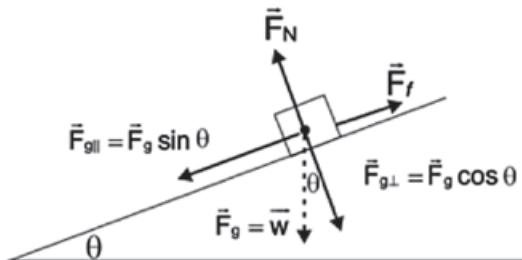
Free body diagram:





Worked example 5

If an object with a mass of 40 kg slides down a surface which has a coefficient of kinetic friction $\mu_k = 0,14$, and a slope of 15° , what is the net force on the object as it slides down the surface? Use the diagram to help you.



Solution

Weight component down the slope

$$= \vec{F}_g \sin \theta = mg \sin 15^\circ = 40 \times 9,8 \times 0,26 = 101,92 \text{ N.}$$

Frictional force up the slope

$$\vec{F}_g \cdot \mu_k \cos \theta = 40 \times 9,8 \times 0,14 \times 0,96 = 52,68 \text{ N}$$

\therefore Net force down the slope is:

$$101,92 + (-52,68) = 49,24 \text{ N.}$$



Direction of frictional force:

The friction acts against the object to prevent it from sliding down the slope so it acts upwards parallel to the slope.



Worked example 6

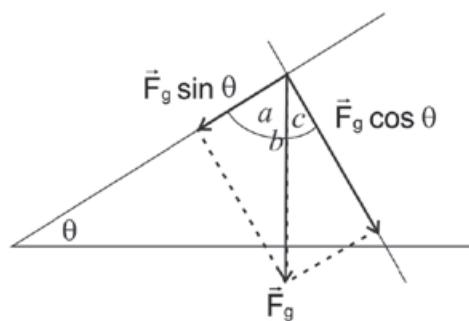
Calculate the components of the weight of an object when it is resting on a surface which slopes at an angle of θ .

Solution

$$\theta + a = 90^\circ \text{ and } a + c = 90^\circ. \text{ So } c = \theta.$$

The component of the weight perpendicular to the surface with a slope of θ is

$$\vec{F}_{g\perp} = \vec{F}_g \cos \theta \text{ and parallel to the surface is } \vec{F}_{g\parallel} = \vec{F}_g \sin \theta.$$



1.5 Resultant (net) force

When a number of forces act on an object, we need to determine the resultant or net force acting on the object.



The **resultant (net) force** acting on an object is the **vector sum** of all the forces acting on the object.

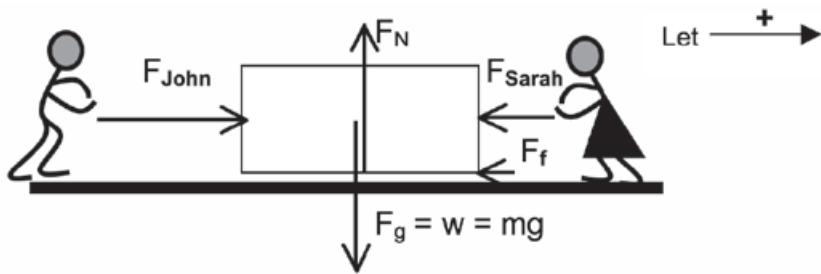
$$\vec{F}_{\text{net}} = \vec{F}_{\text{res}} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \dots$$

$\sum \vec{F}$ is the sum of all the forces acting on the object



Worked example 7

John exerts a force of 100 N to the right on a box resting on a rough, horizontal surface. Sarah exerts a force of 50 N to the left on the box. The friction between the box and the surface is 5 N. Draw a force diagram and calculate the resultant force acting on the box.



Solution

$$\begin{aligned}\vec{F}_{\text{net}} &= \sum \vec{F} = (+\vec{F}_{\text{John}}) + (-\vec{F}_{\text{Sarah}}) + (-\vec{F}_f) \\ &= \vec{F}_{\text{John}} - \vec{F}_{\text{Sarah}} - \vec{F}_f \\ &= 100 - 50 - 5 = 45 \text{ N} \therefore \vec{F}_{\text{net}} = 45 \text{ N to the right}\end{aligned}$$

Remember:

- Force is a vector.
- Indicate the direction of the force with a + or – sign.
- Interpret the answer in words as the final step in your solution

Note: Since the force applied by Sarah and the frictional force are opposite in direction to the force applied by John, $\vec{F}_{\text{Sarah}} = -50 \text{ N}$ and $\vec{F}_f = -5 \text{ N}$

Now consider a situation where a box slides down a slope. The force that makes the box slide down the slope is the component of the box's weight that acts parallel to the slope.

$\therefore \vec{F}_{g\parallel} = mg \cdot \sin \alpha$ where α is the angle between the slope and the horizontal.
Always calculate this force first.





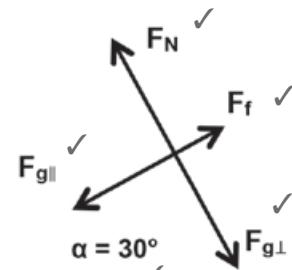
Activity 1

A box of mass 100 kg slides down a rough slope which forms an angle of 30° to the horizontal. The friction that acts on the box is 20 N. Draw a **free body diagram** representing all the forces acting on the object and calculate the resultant force acting on the box and causing it to slide. Perpendicular forces may be ignored. (12)

Solution

Let the direction down the slope be positive.

$$\begin{aligned} \text{Then } \vec{F}_f &= -20 \text{ N} \checkmark \\ \vec{F}_{\text{net}} &= \sum \vec{F} = (+ \vec{F}_{g\parallel}) + (\vec{F}_f) \checkmark \\ &= \vec{F}_{g\parallel} + \vec{F}_f \\ &= mg \cdot \sin \alpha + \vec{F}_f \checkmark \quad \checkmark \\ &= (100)(9,8)(\sin 30^\circ) + (-20) \quad \checkmark \\ &= 490 - 20 = 470 \text{ N} \\ \therefore 470 \text{ N, down the slope} &\checkmark \end{aligned}$$



[12]



Activity 2

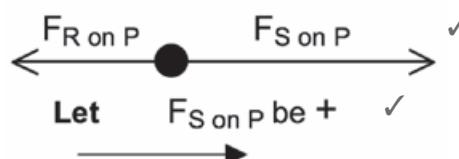
R and S are two positively charged spheres. P is a negatively charged sphere. Sphere R exerts an electrostatic force of 0,2 N on P and sphere S exerts a force of 0,6 N on sphere P.



Draw a free body diagram for sphere P and then calculate the resultant force on sphere P. (5)



Solution



$$\begin{aligned} \vec{F}_{\text{net}} &= \sum \vec{F} = (\vec{F}_{S \text{ on } P}) + (\vec{F}_{R \text{ on } P}) \checkmark \\ &= 0,6 + (-0,2) \checkmark \\ &= 0,4 \text{ N} \quad \therefore 0,4 \text{ N towards S} \checkmark \end{aligned}$$

Note: $\vec{F}_R = -0,2 \text{ N}$

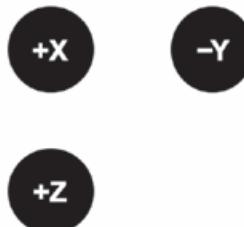
[5]



Activity 3

Three identical spheres X, Y and Z are in the same horizontal plane. Spheres X and Z are both positive and sphere Y is negative. Sphere Y exerts an electrostatic force of 450 N on sphere X and sphere Z exerts an electrostatic force of 350 N on sphere X.

1. Draw a free body diagram for sphere X and indicate the electrostatic forces acting on it. (2)
2. Calculate the magnitude of the resultant electrostatic force on sphere X. (8) [10]



X and Z are positive
 \therefore Z repels X and
 Y is negative and Z is positive
 \therefore Y attracts X

Solutions

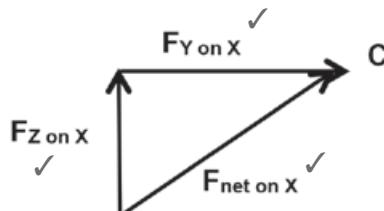
1. Free body diagram:



2.
$$\vec{F}_{net}^2 = \vec{F}_{Z\ on\ X}^2 + \vec{F}_{Y\ on\ X}^2 \quad (\text{pythagoras})$$

$$= 350^2 + 450^2 = 325\ 000$$

$$= \therefore \vec{F}_{net} = \sqrt{325\ 000} = 570,09 \text{ N}$$



(8)
[10]

1.6 Newton's First Law of Motion (Law of Inertia)



DEFINITION

Resists:

opposes, prevents, works against



Inertia

- **Inertia** is the property of an object that resists any change in the state of rest or uniform motion.
If the object is at rest, it resists any change to a state of motion.
If it is in motion, it resists any change to the speed and direction of its motion.
- Inertia is determined by the object's mass. The greater an object's mass, the greater its inertia.

Example

A box lying in the boot of a car will move forwards when the car brakes.

The box's inertia resists the change in movement and allows the box to continue moving in the direction in which the car was moving before it stopped. This is why you must wear seatbelts!



Newton's 1st Law of Motion

An object will remain at rest or continue moving at a constant velocity (in a straight line) unless acted upon by a non-zero external resultant force.

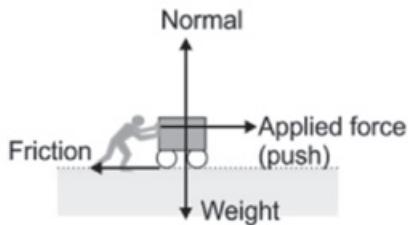
$$F_{\text{net}} = 0 \text{ N} \therefore \vec{a} = 0 \text{ m}\cdot\text{s}^{-2}$$



Worked example 8

The diagram shows the forces on a trolley moving with constant velocity.

1. A man pushes a loaded trolley with constant velocity along a horizontal floor. The trolley and load have a mass of 56 kg and the friction of the moving trolley is 2,1 N. Calculate the force the man exerts to push the trolley along the floor.
2. If he then pushes the trolley with a force of 2,5 N to the right, calculate the acceleration of the trolley.



Solutions

1. The acceleration = 0, so the net force is equal to zero. The force the person pushes with is equal and opposite to the force of kinetic friction on the trolley.

$$F_{\text{push}} = F_{\text{trolley}} = 2,1 \text{ N}$$

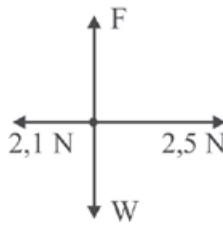
2. The diagram shows the free body diagram.

The net force is 0,4 N in the forward direction.

The trolley accelerates forward:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{0,4}{56} = 7,14 \times 10^{-3} \text{ m}\cdot\text{s}^{-2}$$

forward / to the right.



1.7 Velocity and acceleration: Revision



- Velocity (v) is the rate of change in position (displacement). It is a vector. Speed is a scalar.
 $\vec{V}_{\text{average}} = \frac{\Delta \vec{x}}{\Delta t}$... Δx is the displacement; rate is shown by change in time Δt
- Acceleration (\vec{a}) is the rate of change of velocity.
 $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$... Δv is change in velocity: final velocity (v_f) – initial velocity (v_i)
- Equations of motion: In Grade 10 you learnt these equations that describe the relationships between velocity, acceleration, displacement and time:
 - $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$
 - $\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta x$
 - $\Delta \vec{x} = \vec{v}_i \Delta t + \frac{1}{2}\Delta t^2$

1.8 Newton's Second Law of Motion in terms of acceleration

When the resultant force acting on an object is NOT zero, the object's state of motion will change.

It may:

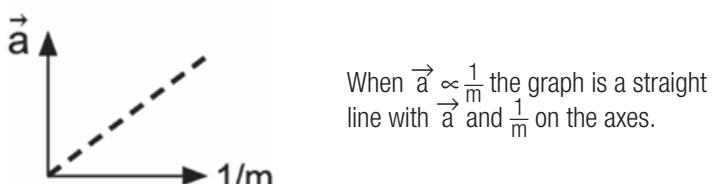
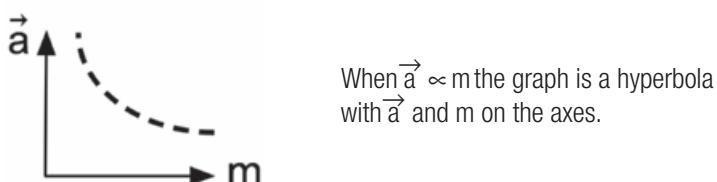
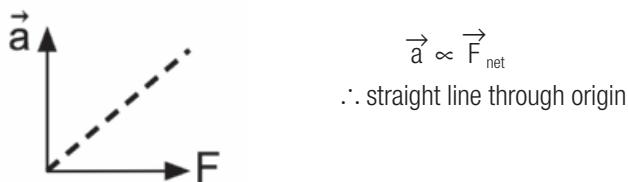
- start moving (then $\vec{v}_i = 0 \text{ m}\cdot\text{s}^{-1}$ and $\vec{v}_f \neq 0 \text{ m}\cdot\text{s}^{-1}$);
- stop moving (come to rest, then $\vec{v}_f = 0 \text{ m}\cdot\text{s}^{-1}$);
- move faster (accelerate); move slower (decelerate); or
- the direction in which it moves will change.



Newton's Second Law of Motion

If a **resultant (net) force** acts on an object, the object will accelerate in the direction of the resultant force. The acceleration produced is directly proportional to the resultant force and inversely proportional to the mass of the object. In other words, acceleration is the amount of change in speed (or velocity), per second, hence, it is metres per second change per second, or $\text{m}\cdot\text{s}^{-2}$

For any object $\vec{a} \propto \vec{F}_{\text{net}}$ and $\vec{a} \propto \frac{1}{m}$ $\therefore \vec{F}_{\text{net}} = m\vec{a}$
where \vec{a} is acceleration ($\text{m}\cdot\text{s}^{-2}$), \vec{F} is force (N) and m is mass (kg)



If *different* forces are applied to the same object and its mass stays constant, then $\vec{a} \propto \sum \vec{F}$. The bigger the net resultant force acting on the object, the more the object will accelerate.



Worked example 9

A resultant force \vec{F} is applied to an object of mass m and the object accelerates at \vec{a} . What will the object's acceleration be if the resultant force acting on the object is tripled?

Solution

m is constant $\therefore \vec{a} \propto \vec{F}$ and if the force is tripled (from \vec{F} to $3\vec{F}$), the acceleration will also triple \therefore the object will accelerate at $3\vec{a}$.

NOTE:

If a constant non-zero resultant force is applied to **two** objects, then $\vec{a} \propto \frac{1}{m}$. The object with the smaller mass will accelerate more than the object with the bigger mass. Think about it: it's easier to make a lighter object move further and faster.



Worked example 10

A constant resultant force \vec{F} is applied to objects of masses m and $2m$. If the object of mass m accelerates at \vec{a} , what will the acceleration of the other object be?

Solution

$$\vec{F} \text{ is constant} \therefore \vec{a} \propto \frac{1}{m}$$

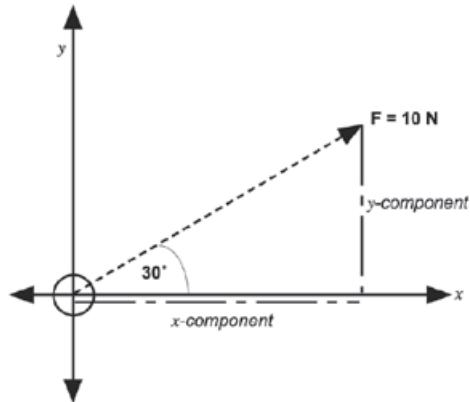
If the mass doubles (from m to $2m$), the acceleration will halve
 \therefore the object of mass $2m$ accelerates at $\frac{1}{2}\vec{a}$.



Steps to solve problems on Newton's Laws

hint

Sometimes the forces you are given in a diagram are not at right angles to each other, yet when you draw a force diagram on the Cartesian Plane, the forces must be drawn at right angles to each other. In order, then, to find out what the vertical and horizontal parts of a force are, when that force is at an angle, we have to use trigonometry.



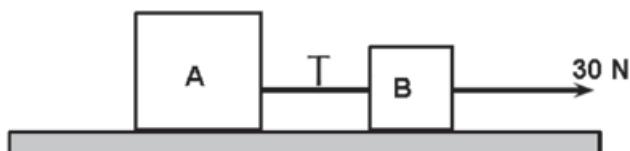
Well, the y -component is opposite the angle, and the hypotenuse (10 N) is known, so since sine is O/H, $\sin 30^\circ \times 10 \text{ N} =$ the y -component: 5 N. Likewise, the x -component is adjacent to the angle, so since cosine is A/H, $\cos 30^\circ \times 10 \text{ N} = 8,67 \text{ N}$. So your x -component is 8,67 N and your y -component is 5 N.

- Step 5:** List all the given information and convert the units if necessary.
- Step 6:** Determine which physical principle (law) can be applied to solve the problem.
- Step 7:** Use the principle (law) to answer the question, often by substituting numerical values into an appropriate equation.
- Step 8:** Check that the question has been answered and that the answer makes sense.



Activity 4

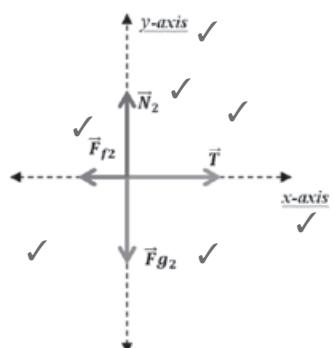
Two boxes, **A** and **B**, are lying on a table and are connected by a piece of string. The mass of box **A** is 3 kg and the mass of box **B** is 2 kg. Assume that the mass of the string is very small, so we can ignore it. A 30 N pulling force, pointing to the right, is applied to box **B**, causing the two boxes to move. The surface acts with a frictional force of 5,9 N on box **A** and 4,1 N on box **B**.



1. Calculate the acceleration of boxes A and B. (14)
 2. Calculate the magnitude of the tension on the string. (5)
- [19]

Solution

1. We are going to take the whole system as a unit.



Data:

$m_A = 3 \text{ kg}$, $m_B = 2 \text{ kg}$ ✓✓
 $m_T = m_A + m_B = 3 \text{ kg} + 2 \text{ kg} = 5 \text{ kg}$
 $F_A = 30 \text{ N}$ to the right
 $F_{fA} = 5,9 \text{ N}$ to the left
 $F_{fB} = 4,1 \text{ N}$ to the left
 $F_{fT} = F_{fA} + F_{fB} = 5,9 + 4,1 = 10 \text{ N}$ to the left
 $a = ?$ to the right
 $T = ?$

(9)

$$F_{RTx} = m_T a \text{ (from } F = ma\text{)} \quad \checkmark \checkmark$$

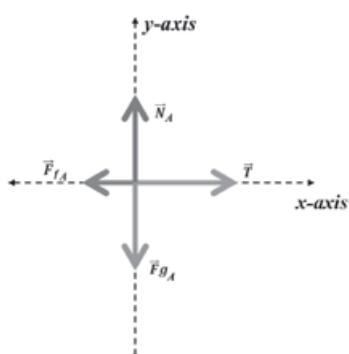
$$F_A + F_{fT} = m_T a$$

$$(30) + (-10) = 5a$$

$$a = 4 \text{ m/s}^2 \text{ to the right} \quad \checkmark \quad (5)$$

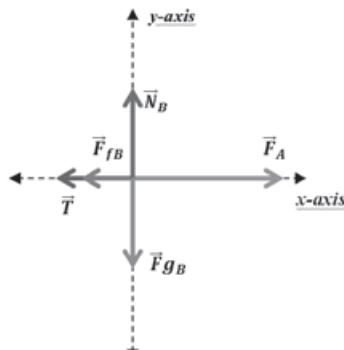
2. To calculate the tension you may use box A or box B

Tension using box A



$F_{RAx} = m_A a \quad \checkmark$
 $T + F_{fA} = m_A a \quad \checkmark$
 $T - 5,9 = (3)(4) \quad \checkmark \checkmark$
 $T = 12 + 5,9$
 $T = 17,9 \text{ N} \quad \checkmark$

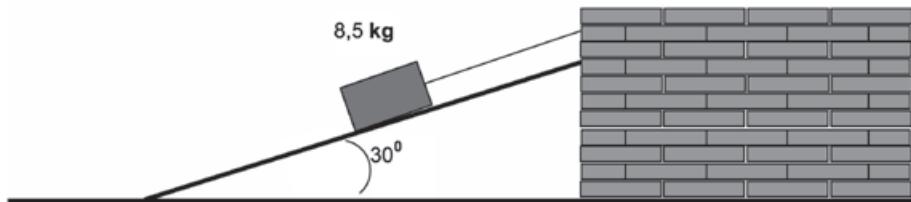
(5)

Tension using box B

$$\begin{aligned} F_{RBx} &= m_B a \checkmark \\ F_A + T + F_{fB} &= m_B a \checkmark \\ 30 - T - 4,1 = (2)(4) &\quad \checkmark \checkmark \\ -T + 25,9 &= 8 \\ -T &= -17,9 \\ T &= 17,9 \text{ N } \checkmark \end{aligned}$$

(5)
[19]**Activity 5**

The sketch below shows a block of 8,5 kg at equilibrium on an inclined (sloping) plane (surface).



Calculate:

1. The magnitude of the tension in the cord. (12)
2. The magnitude of the normal force acting on the block. (6)
3. The magnitude of the block's acceleration, If the cord is cut. (4)

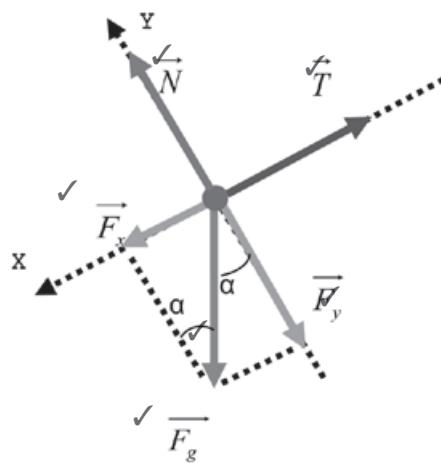
[22]

Solution**1. Data**

$$m = 8,5 \text{ kg} ; \alpha = 30^\circ ; v_i = 0 ; a_i = 0$$

Let's make the free body diagram of forces.

(6)



The gravitational force is not in the direction of any axis then we have to determine its components on the x -axis and y -axis.

1. Applying Newton's First Law

$$\sum \vec{F} = 0$$

$$\vec{F}_g + \vec{T} = 0 \quad \checkmark$$

Working with the projections of the forces on the x -axis we get:

$$F_{gx} - T = 0$$

$$F_g \cdot \sin \alpha - T = 0 \quad \checkmark$$

$$m \cdot g \cdot \sin 30^\circ - T = 0 \quad \checkmark$$

$$8,5 \times 9,8 \times 0,5 - T = 0$$

$$41,65 - T = 0$$

$$T = 41,65 \text{ N} \quad \checkmark$$

(6)

2. Working on the y -axis

$$N - F_{gy} = 0$$

$$N - (F_g \cdot \cos \alpha) = 0 \quad \checkmark$$

$$N - (m \cdot g \cdot \cos 30^\circ) = 0 \quad \checkmark$$

$$N - (8,5 \times 9,8 \times 0,866) = 0$$

$$N - 73,1 = 0$$

$$N = 73,1 \text{ N} \quad \checkmark$$

(6)

3. Applying Newton's Second Law.

$$\sum \vec{F}_x = m \vec{a}_x \quad \checkmark$$

If the cord is cut there is no tension force acting on the block and there is only one force acting on the direction of the x -axis, causing acceleration to the block.

Working with the projections

$$F_{gx} = m \cdot a$$

$$F_g \cdot \sin \alpha = m \cdot a$$

$$mg \cdot \sin 30^\circ = m \cdot a \quad \checkmark$$

Simplifying:

$$g \cdot \sin 30^\circ = a$$

$$a = g \cdot \sin 30^\circ$$

$$a = 9,8 \times 0,5 \quad \checkmark$$

$$a = 4,9 \text{ m} \cdot \text{s}^{-2} \quad \checkmark$$

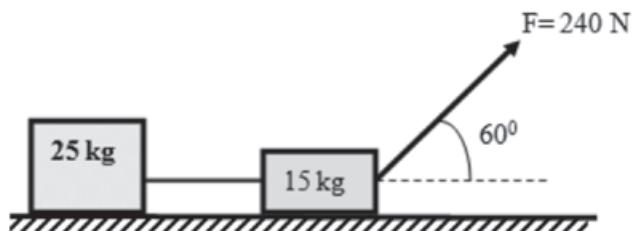
(4)

[22]



Activity 6

Two blocks of 25 kg and 15 kg are connected by a light string on a horizontal surface. Assume that the string cannot stretch. A force of magnitude 240 N is applied to the block of 15 kg forming an angle of 60° with the horizontal as shown in the sketch below. The coefficient of kinetic friction is 0,20.

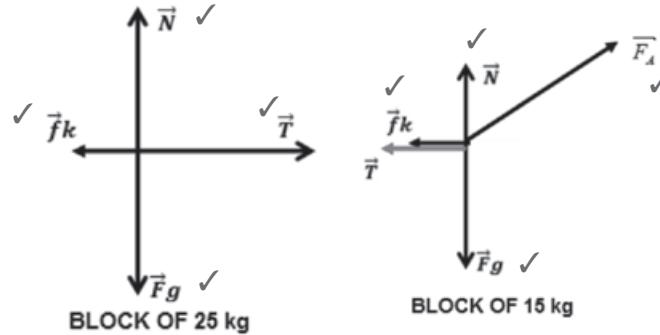


1. State Newton's Second Law of Motion in words. (7)
2. Draw a free body diagram for each block. (8)
3. Calculate the magnitude of the acceleration of the blocks. (14)
[29]

Solutions

1. If a resultant force ✓ acts on a body, it will cause the body to accelerate ✓ in the direction of the resultant ✓ force. The acceleration of the body will be directly ✓ proportional to the resultant ✓ force and inversely ✓ proportional to the mass ✓ of the body. (7)

2.



(8)

3.

Option 1

Taking the objects as a system

$$F_{Rx} = ma \quad \checkmark$$

$$F_x + F_{ff} = ma_x \quad \checkmark$$

$$F_x - F_{ff} = m_1 a_x$$

$$F_x - (F_{f1} + F_{f2}) = (m_1 + m_2)a_x \quad \checkmark$$

$$F_x - (\mu N_1 + \mu N_2) = (m_1 + m_2)a_x$$

Note: In this series of solutions we have omitted (taken out) the vector arrow above F and a; this is just to make it easier to read the solution.

We have to calculate the normal force for both blocks

$$N_1 = F_g = m_1g$$

$$N_2 = m_2g - F \cdot \sin 60^\circ \quad \checkmark$$

$$F \cdot \cos 60^\circ - [\mu m_1 g + \mu(m_2 g - F \cdot \sin 60^\circ)] = (m_1 + m_2)a_x$$

$$(240 \cdot \cos 60^\circ) - [(0,2)(25)(9,8) + (0,2)[(15)(9,8)] - 240 \cdot \sin 60^\circ] = (25+15)a_x \quad \checkmark \quad \checkmark$$

$$120 - [49 + (0,2)(147 - 207,85)] = 40a_x \quad \checkmark$$

$$83,17 = 40a_x \quad \checkmark$$

$$a_x = 2,08 \text{ m/s}^2 \quad \checkmark$$

Option 2

Applying Newton's Second Law of motion to each object individually

$$F_R = m_a$$

For object 1:

$$T = F_{f1} = m_1a_x \quad \checkmark$$

$$T - F_{f1} = m_1a_x$$

$$T - \mu m_1 g = m_1 a_x \quad \checkmark$$

For object 2:

$$F_{R2x} = m_2a \quad \checkmark$$

$$F_x + T + F_{f2} = m_2a_x \quad \checkmark$$

$$F_x - T - F_{f2} = m_2a_x \quad \checkmark$$

$$F \cdot \cos 60^\circ - T - \mu N_1 = m_1 a_x$$

$$F \cdot \cos 60^\circ - T - \mu(m_2 g - F \cdot \sin 60^\circ) = m_2 a_x \quad \checkmark$$

Adding equation (1) and (2).

$$T - \mu m_1 g + F \cdot \cos 60^\circ - T - \mu(m_2 g - F \cdot \sin 60^\circ) = m_1 a_x + m_2 a_x$$

$$\text{Taking out } T \text{ and } a_x: -\mu m_1 g + F \cdot \cos 60^\circ - \mu(m_2 g - F \cdot \sin 60^\circ) = (m_1 + m_2)a_x$$

$$[-(0,2)(25)(9,8)] + [240 \cdot \cos 60^\circ - (0,2)[(15)(9,8)] - (240 \cdot \sin 60^\circ)] = (25 + 15)a_x \quad \checkmark \quad \checkmark$$

$$(-49 + 120) - (0,2)(147 - 207,85) = 40a_x$$

$$71 + 12,17 = 40a_x \quad \checkmark$$

$$83,17 = 40a_x$$

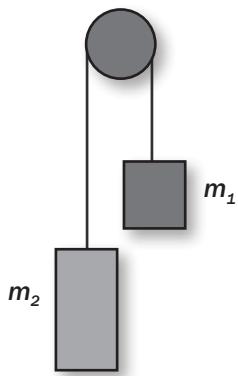
$$a_x = 2,08 \text{ m/s}^2 \quad \checkmark$$

(14)

[29]



Activity 7



The sketch below shows two blocks connected by a string of negligible mass that passes over a frictionless pulley also of negligible mass. The arrangement is known as *Atwood's machine*. One block has mass $m_1 = 2 \text{ kg}$ and the other has mass $m_2 = 4 \text{ kg}$.

The blocks have just this instant been released from rest.

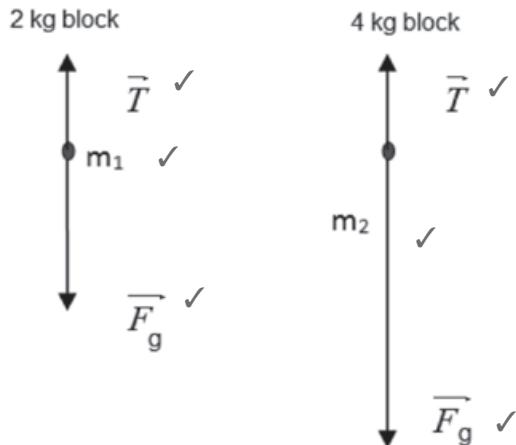
1. Draw a free body diagram of all the forces acting on each block. (6)
2. Calculate the magnitude of the acceleration of the system. (7)
3. Calculate the magnitude of the tension in the string. (4)
4. Compare the magnitude of the net force on m_1 with the net force on m_2 . (1)

Write down only GREATER THAN, SMALLER THAN or EQUAL TO.

5. Will the pulley rotate clockwise or anticlockwise? (1)
- [19]

Solutions

1. See diagram below:



Note: In this series of solutions we have omitted (taken out) the vector arrow above F and a ; this is just to make it easier to read the solution.

2. $\sum \vec{F}_{\text{net}} = m \vec{a} \quad \checkmark$

For the 2 kg block (+ upwards)

$$T - F_{g1} = m_1 a$$

$$T - m_1 g = m_1 a \quad \checkmark$$

$$T - 2 \times 9,8 = 2a \quad \checkmark$$

For the 4 kg block (+ downwards)

$$-T + m_2 g = m_2 a$$

$$-T + 4 \times 9,8 = m_2 a \quad \checkmark$$

Solving the system of equations

$$T - 2 \times 9,8 - T + 4 \times 9,8 = (2 + 4)a$$

$$2 \times 9,8 = 6a$$

$$a = + 3,27 \text{ m}\cdot\text{s}^{-2} (\text{upwards})$$

$$a = 3,27 \text{ m}\cdot\text{s}^{-2} \quad \checkmark$$

(6)

(7)

3. Option 1

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2} \right) \times g \quad \checkmark \checkmark$$

$$T = \left(\frac{2 \times 2 \times 4}{2 + 4} \right) \times 9,8$$

$$T = 26,13 \text{ N} \quad \checkmark \checkmark$$

Option 2

$$T - F_{g1} = m_1 a \quad \checkmark \quad \text{OR} \quad T = m_1(a+g) \quad \checkmark$$

$$T = 2(3,27 + 9,8) \quad \checkmark$$

$$T = 26,14 \text{ N} \quad \checkmark$$

Option 3

$$-T + m_2 g = m_2 a \quad \checkmark \quad \text{OR} \quad -T = m_2 a - m_2 g \quad \checkmark \quad \text{OR} \quad T = -m_2 a + m_2 g$$

$$T = 4(-3,27 + 9,8) \quad \checkmark \checkmark$$

$$T = 26,12 \text{ N} \quad \checkmark \quad (4)$$

4. Smaller than. \checkmark (1)

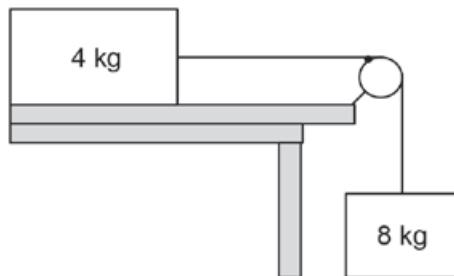
5. Anticlockwise. \checkmark (1)

[19]



Activity 8

A 4 kg block on a horizontal, rough surface is connected to a 8 kg block by a light string that passes over a frictionless pulley as shown below. Assume that the string cannot stretch. The coefficient of kinetic (dynamic) friction between the block of 4 kg and the surface is 0,6.

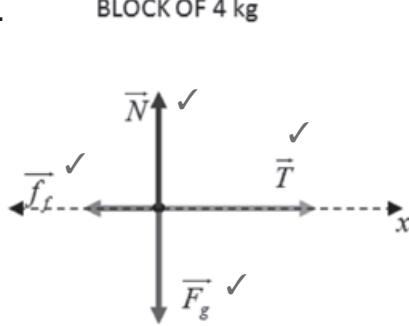


1. Draw a free body diagram of all the forces acting on both blocks. (6)
2. Calculate the acceleration of the system. (10)
3. Calculate the magnitude of the tension in the string. (3)
4. Calculate the magnitude of the frictional force that acts on the 4 kg block. (4)
5. Calculate the apparent weight of the 8 kg block. (4)
6. How does the apparent weight of the 8 kg block compare with its true weight? Write down only GREATER THAN, EQUAL TO or LESS THAN. (1)
7. How does the apparent weight of the 4 kg block compare with its true weight? Write down only GREATER THAN, EQUAL TO or LESS THAN. (1)

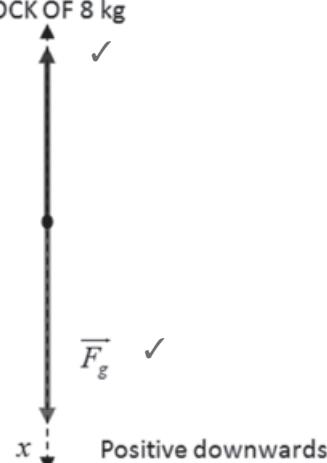
[29]

Solutions

1. BLOCK OF 4 kg



BLOCK OF 8 kg



(6)

Let's take the direction of motion as positive

2. Let's apply Newton's Second Law of Motion to each block.

$$\sum \vec{F} = m \vec{a} \quad \checkmark$$

Block of 4 kg (A)

In the x direction (horizontal)

$$T - f_f = m_A a$$

$$T - \mu N = m_A a \quad \checkmark$$

$$T - \mu m_A g = m_A a \text{ (call this Equation 1)}$$

Note: In this series of solutions we have omitted (taken out) the vector arrow above F and a ; this is just to make it easier to read the solution.

Block of 4 kg (A)

In y direction (up/down)

$$N - F_g = 0 \text{ (not moving horizontally)}$$

$$N = F_g = mg \quad \checkmark$$

Block of 8 kg (B)

In the x direction (horizontal)

$$-T + F_g = m_B a \quad \checkmark$$

$$-T + m_B g = m_B a \text{ (call this Equation 2)}$$

Solving the system of equations (adding Equation 1 and 2)

$$T - \mu m_A g - T + m_B g = m_A a + m_B a \quad \checkmark \quad \checkmark$$

Removing T and isolating a:

$$-\mu m_A g + m_B g = (m_A + m_B)a$$

$$-(0,6)(4)(9,8) + (8)(9,8) = (4 + 8)a$$

$$54,88 = 12a$$

$$a = 4,57 \text{ m/s}^2 \quad \checkmark$$

(10)

3. Using Equation 2

$$-T + m_B g = m_B a$$

$$-T = 8 \times 4,57 - (8 \times 9,8)$$

$$T = 41,84 \text{ N} \quad \checkmark$$

(3)

Using Equation 1

$$T - \mu m_A g = m_A a$$

$$T - (0,6)(4)(9,8) = (4)(4,57)$$

$$T = (0,6)(4)(9,8) + (4)(4,57)$$

$$T = 41,8 \text{ N } \checkmark$$

(3)

4. $f_f = \mu N$

$$N = mg$$

$$f_f = \mu mg$$

$$f_f = 0,6 \times 4 \times 9,8$$

$$f_f = 23,52 \text{ N } \checkmark$$

(4)

5. $-T + m_B g = m_B a$ ✓ ✓

$$-T = -8 \times 4,57 + (8 \times 9,8)$$

$$\text{Apparent weight} = T = 41,84 \text{ N } \checkmark$$

(4)

6. Less than ✓

(1)

7. Equal to ✓

(1)

[29]

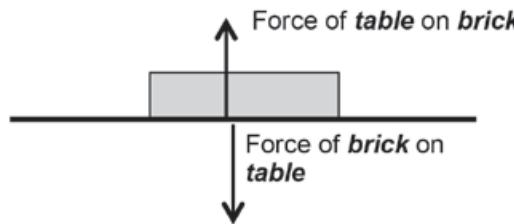
1.9 Newton's Third Law of Motion

For a third law forces pair:

- the forces are **equal in magnitude**
- the forces act in the same straight line but in **opposite directions** on **different objects**
- the forces do *not* cancel each other, as they act on **different objects**.

Example

The force diagram shows the pair of forces when a brick rests on a table. (Note: these are the *contact forces*)



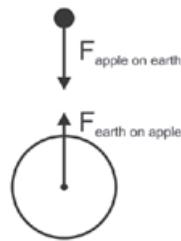
When pairs of objects interact they exert forces on each other. If object A exerts a force on object B, object B will exert an equal force on object A but in the opposite direction.

For any two objects A and B;
 $\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$

Opposite direction is indicated by a negative (-) sign.

Example

The **reaction force** of the weight of an object is the force that the object exerts on the earth, upwards. (These are not contact forces, they act at a distance.)



1.10 Newton's Law of Universal Gravitation



Newton's Law of Universal Gravitation states that:

Each body in the universe **attracts every other body** with a force that is **directly proportional** to the product of their masses and is **inversely proportional** to the square of the distance between their centres.

$$\text{For any two objects: } F \propto m_1 \cdot m_2 \quad \text{and} \quad F \propto \frac{1}{r^2} \quad \therefore F = G \frac{m_1 m_2}{r^2}$$

F: magnitude of force (N)

m: mass (kg)

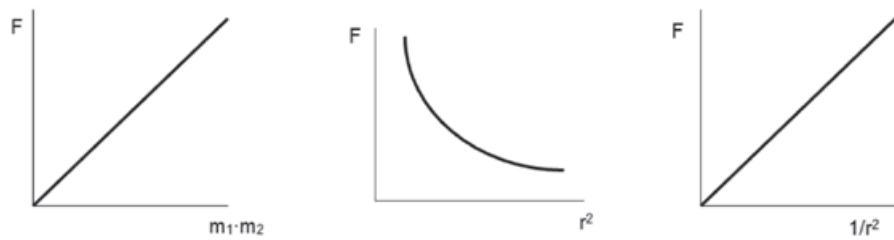
r: distance between centres of the objects (m)

G: universal gravitation constant ($6,67 \times 10^{-11} \text{ N}\cdot\text{m}^2\cdot\text{kg}^{-2}$)



DEFINITION

Universal means that the statement is valid anywhere in the universe.

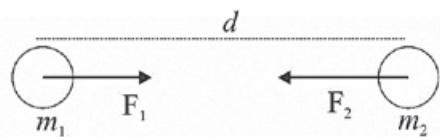


- 1) $F \propto m_1 \cdot m_2$
 \therefore straight line
 through the
 origin

- 2) $F \propto \frac{1}{r^2}$
 \therefore hyperbola



Worked example 11



A **force of gravitational attraction** exists between the earth with mass m_1 and a person with mass m_2 . The force on m_1 is \vec{F}_1 and the force on m_2 is \vec{F}_2 .

Compare the magnitudes (sizes) of these forces and state the name of the law which explains your answer.

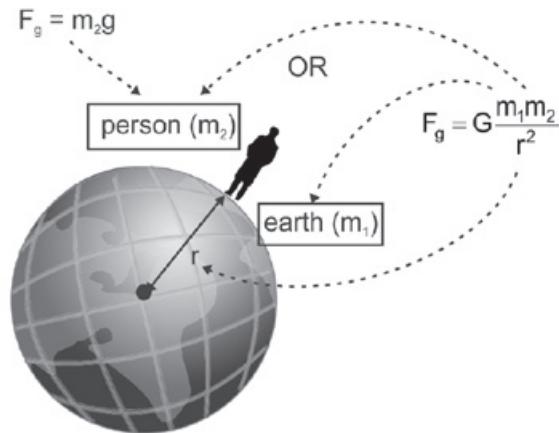
Solution

$\vec{F}_1 = -\vec{F}_2$ according to Newton's Third Law of Motion:

The force between the earth (m_1) and a person (m_2) standing on its surface:

$$\therefore m_2g = G \frac{m_1m_2}{r^2}$$

$$\therefore g = G \frac{m_1}{r^2}$$



The gravitational acceleration on earth (or on any planet) is:

- dependent on the **mass of the earth (planet)** (m_1)
- dependent on the **distance** between the centre of the object and the centre of the earth (planet)
- independent of the **mass of the object** on the planet on which the force acts. On a different planet, acceleration due to gravity is different.

Application of the law of Universal Gravitation: This law enables us to calculate the size (mass) of astronomical bodies like planets, stars, etc.



It is important to understand and be able to compare mass and weight.

1.11 The difference between mass and weight

Mass

- Mass is the amount of **matter** in an object.
- Mass determines the object's **inertia**.
- Mass remains **constant**.
- Mass is measured in **kilograms** (kg).
- Mass is a **scalar** quantity (with magnitude, but not direction).

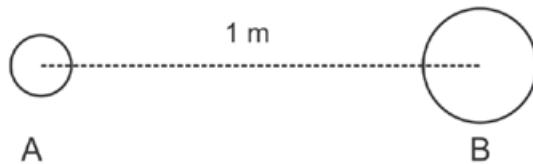
Weight

- Weight is determined by the **force of attraction** the earth exerts on the object.
- Weight depends on the object's **distance** from the centre of the earth.
- Weight depends on the **masses** of the earth (planet) and the object.
- Weight is measured in **Newton** (N).
- Weight is a **vector** quantity, so it has magnitude and direction.
 $\vec{F}_g = m \cdot \vec{g}$ or $= m \cdot \vec{g}$ where
 \vec{g} = gravitational acceleration (9,8 m·s⁻² on earth).

e.g.

Worked example 12

The diagram shows a ball A of mass 0,01 kg which is 1 m (measured from centre to centre) from another ball B of mass 520 g. Calculate the magnitude of the force of ball A on ball B.



Solution

$$F = G \frac{m_1 m_2}{r^2} = \frac{6,67 \times 10^{-11} \times 0,01 \times 0,52}{1^2}$$

$$= 3,57 \times 10^{-14} \text{ N}$$



Worked example 13

An object weighs 720 N on earth. It orbits the Earth in a satellite at a height equal to the earth's diameter, above the surface of the Earth. What does the object weigh on the satellite?

Hint: diameter = 2 × radius



Step by step

Step 1. Determine the number of radii from the centre of the Earth.

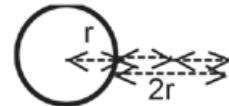
→ On the Earth's surface, the object is 1 radius from the Earth's centre.

Step 2. Determine how many times the distance between the object and the centre of the Earth has increased.

→ In orbit, the object is 1 diameter = 2 radii above the surface.

Step 3. Square this number (multiply it by itself).

→ So the object is 3 radii from the centre.
 $(3)^2 = 9$



Step 4. The force has decreased this number of times because

$$\vec{F} \propto \frac{1}{r^2}$$

→ ∴ the gravitational force on the object has decreased 9 times because

$$\vec{F} \propto \frac{1}{r^2}$$

Step 5. Divide the value of the force (or weight) by the value calculated in step 3.

→ ∴ its weight on the satellite is
 $720 \text{ N} \div 9 = 80 \text{ N}$

Gravitational acceleration on planets other than earth

Newton's universal law of gravitation can be used to calculate the acceleration due to the force of gravity on any planet.

If the mass and radius of a planet are known, we can calculate \vec{g} for that planet.



Worked example 14

The Mars Rover is an automated vehicle that has been sent to explore the surface of the planet Mars.

If the value of acceleration due to gravity on the planet Mars is $\vec{g}_{\text{Mars}} = 3,7 \text{ m}\cdot\text{s}^{-2}$. Calculate the weight of the Mars rover on Mars if it has a mass of 174 kg.

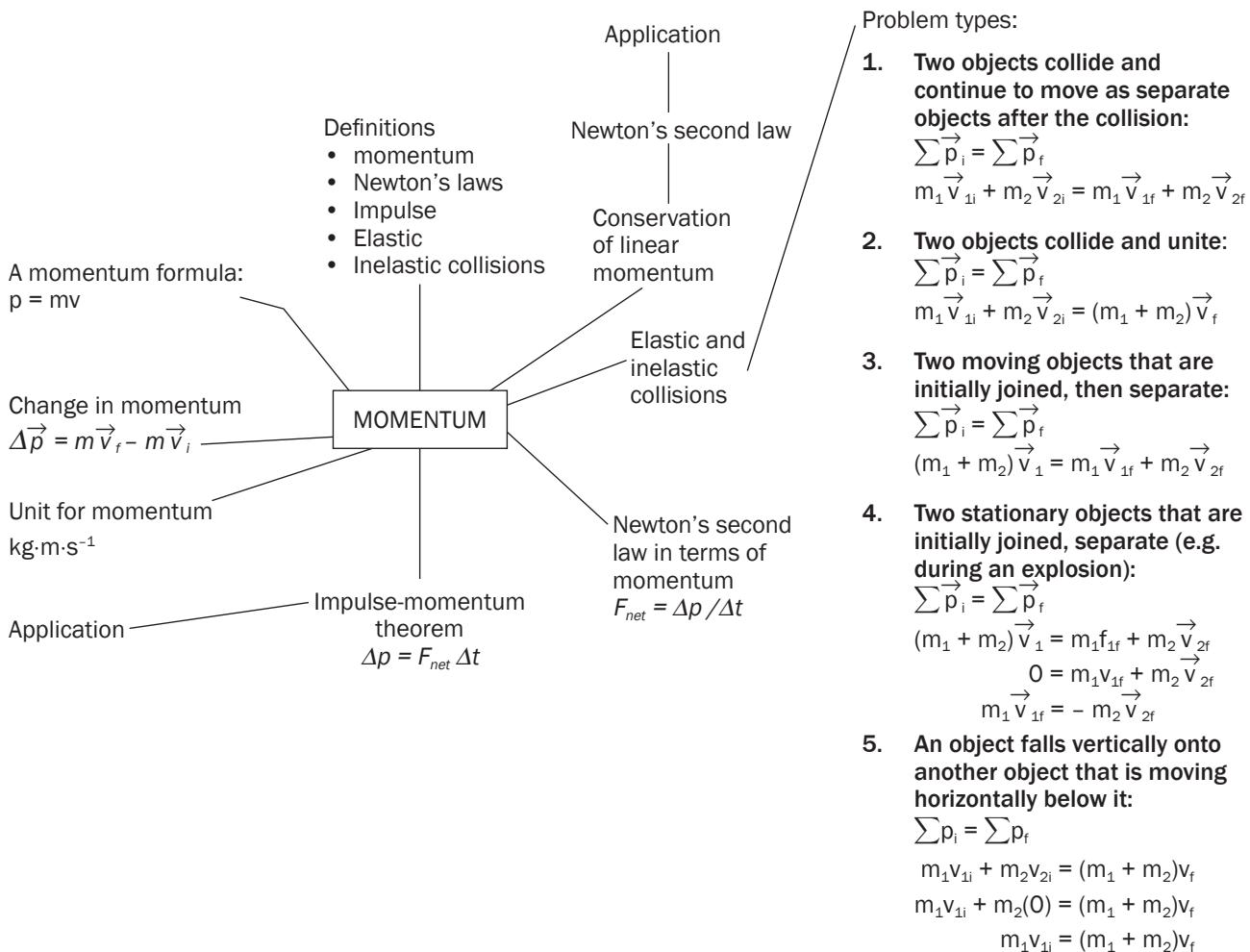
Solution

$$\vec{w}_{\text{Mars}} = \vec{g}_{\text{Mars}} \times m_{\text{object}} = 3,7 \times 174 = 643,8 \text{ N towards the centre of the planet Mars.}$$



Momentum and impulse

Summary



2.1 Momentum

Momentum is a vector quantity with the same direction as the object's velocity.

You need to remember the differences between speed and velocity.

Speed

- Speed is the distance covered per unit time
- Speed is a scalar quantity (thus has magnitude and no direction).
- Symbol: v

Velocity

- Velocity is the rate at which an object is displaced.
- Velocity is a vector (thus has magnitude and direction).
- Symbol: \vec{v}



The formula for momentum is: $p = mv$

- where
- p = momentum
- m = mass
- v = velocity
- mass is measured in kilograms (kg)
- velocity is measured in $\text{m}\cdot\text{s}^{-1}$
- the unit of momentum is: $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$

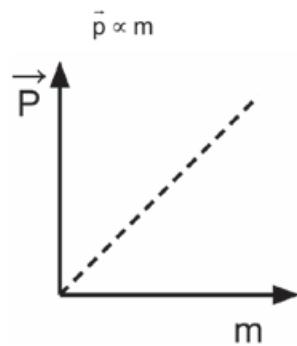
Conservation means to keep things (forces) the same (in a closed system).

Linear momentum refers to the momentum of objects in a straight line.
A closed system is a system that does not experience any external forces.

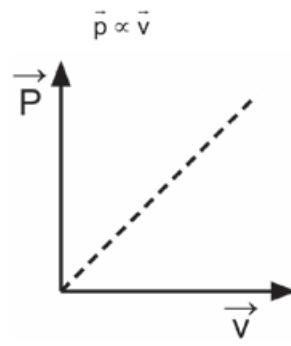


The **momentum** of an object is defined as the product of its mass and velocity ($\vec{p} = m \vec{v}$)

Momentum is **directly proportional** to the **mass** of the object:



Momentum is also **directly proportional** to the **velocity** of the object:



Examples of momentum:

The motion that is a result of objects colliding with each other, an object exploding, or a bullet being fired is described by momentum.

2. Impulse:

Impulse is the product of the **net force** acting on an object and the time that the force is applied to an object. (**Impulse = FΔt**). Think of the term “impulsive” or “having an impulse to do something”. This might help you to remember what it means.



Definitions and principles or laws have certain key words that should not be left out. These are written in **bold** in each of the above definitions.

3. Newton's second law of motion in terms of momentum:

The **net (resultant)** force acting on an object is equal to the object's **rate** of change of momentum. In a formula: $F_{\text{net}} = \frac{\Delta p}{\Delta t}$

4. The law of conservation of linear momentum:

The **total** linear momentum of **an isolated (closed)** system remains **constant (is conserved)**.

2.2 Change in momentum

When an object's velocity changes in **magnitude (size)** or **direction**, its momentum will also change. Since an object's mass remains constant during a collision (assuming it does not break up or approach light speed), it follows that the change in its velocity is what causes a change in its momentum.

We *only* study objects moving in straight lines, for example, backwards and forwards, left and right or up and down.

Change in velocity	Change in momentum
$\Delta \vec{v} = \vec{v}_f - \vec{v}_i$	$\Delta \vec{p} = \vec{p}_f - \vec{p}_i$
where	
$\Delta \vec{v}$: change in velocity in $\text{m}\cdot\text{s}^{-1}$	$\Delta \vec{p} = m \vec{v}_f - m \vec{v}_i$
\vec{v}_f : final velocity in $\text{m}\cdot\text{s}^{-1}$	$\Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)$
\vec{v}_i : initial velocity in $\text{m}\cdot\text{s}^{-1}$	where
	$\Delta \vec{p}$: change in momentum in $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$.
	\vec{p}_f : final momentum in $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$.
	\vec{p}_i : initial momentum in $\text{kg}\cdot\text{m}\cdot\text{s}^{-1}$
	m: mass in kg

Momentum is a vector quantity, so you must give the direction in all your answers.



Steps to follow when solving problems

1. Make a sketch (on your rough work page) of the situation.
2. Always choose and indicate direction and write it down clearly. It is recommended that you choose a positive direction (e.g. to the right is positive).
3. Write down the information in symbols. Remember to include the correct signs for the directions of the initial and final velocity.
4. Choose the correct formula from the information sheet.
5. Substitute the values into the formula.
6. Solve for the unknown variable.

e.g.

Worked example 1

1. A car has a momentum of $20\ 000 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$. What will the car's new momentum be if its mass is doubled (by adding more passengers and a greater load) and it travels at the same velocity?
2. What will the velocity be if the momentum is $60\ 000 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ and the mass of the car is $2\ 000 \text{ kg}$?
3. A truck has a mass of $6\ 000 \text{ kg}$ and travels at $80 \text{ km}\cdot\text{h}^{-1}$. How does the momentum change if the truck is loaded with $1\ 200 \text{ kg}$ and then travels at $60 \text{ km}\cdot\text{h}^{-1}$?

Solutions

1. The formula for momentum is $\vec{p} = m\vec{v}$, so the momentum will double and will be equal to $40\ 000 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ in the same direction as before.
2. $\vec{p} = m\vec{v}$
 $60\ 000 = 2\ 000 \times \vec{v}$
 $\therefore \vec{v} = 30 \text{ m}\cdot\text{s}^{-1}$ in the same direction as the momentum.
3. Convert both velocities to $\text{m}\cdot\text{s}^{-1}$:
 $\vec{v}_i = 22,22 \text{ m}\cdot\text{s}^{-1}$ & $\vec{v}_f = 16,67 \text{ m}\cdot\text{s}^{-1}$
 $\vec{p}_i = m_i \vec{v}_i = (6000 \times 22,22) = 133\ 320 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$
 $\vec{p}_f = m_f \vec{v}_f = (7200 \times 16,67) = 120\ 024 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$
 $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 133\ 320 - 120\ 024 = 13\ 296 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$

2.3 Newton's Second Law of Motion in terms of momentum



Newton's Second Law of Motion can be used to find the object's acceleration due to the net force, and the object's change in **momentum** due to the net force.

We know that the object's **change in momentum** is always:

- directly proportional to the net force acting on the object $\vec{\Delta p} \propto F_{\text{net}}$
- directly proportional to the time that the net force acts on the object $\vec{\Delta p} \propto \Delta t$ in the direction of the net force acting on the object.

Newton's Second Law of Motion states that: **The resultant/net force acting on object is equal to the rate of change of momentum and the change is in the direction of the resultant/net force.**



Worked example 2

Why is it less painful for a high jumper to land on foam-rubber carpet than on the ground?

Solution

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t}$$

The \vec{F}_{net} needed to bring the jumper to rest ($\vec{v}_f = 0 \text{ m}\cdot\text{s}^{-1}$) depends on $\vec{\Delta p}$ and Δt . When he lands on the foam-rubber, he comes to rest over a longer period of time (Δt) than if he lands on the ground.

- So time taken Δt to change his momentum *increases*
- \vec{F}_{net} decreases ($\vec{F}_{\text{net}} \propto \frac{1}{\Delta t}$)
- The magnitude of \vec{F}_{net} determines the amount of pain experienced, so it is less painful to land on foam-rubber.

NOTE:

The same reasoning explains why a cricket player would draw his hands back to catch a fast ball and why modern motor vehicles are designed with air bags and crumple zones.



Worked example 3

- A spaceship has a mass of 1 000 kg. The rocket engines discharge for 5 s and increase the rocket's velocity from 25 to 30 $\text{m}\cdot\text{s}^{-1}$. Calculate the force exerted by the engines to cause this change in momentum.
- Assume the direction of the initial velocity is positive and the answer you obtain in the above problem is negative, what would be the direction of the exerted force?

Solutions

- Let the direction of the initial velocity be positive.

$$\vec{F}_{\text{net}} = \frac{\vec{\Delta p}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(1000)(30) - (1000)(25)}{5} \\ = 1\ 000 \text{ N in the initial direction of motion.}$$

- The same i.e. in the initial direction of motion.

NOTE:

- As the question is asking for a vector quantity (force) the answer must have both magnitude and direction.
- Since the answer is positive and it was decided the direction of the initial velocity is positive the direction of the force exerted is the same as that of the initial velocity.



Activity 1

Study the diagrams below showing the movement of a 150 g baseball thrown at a wall at right angles.

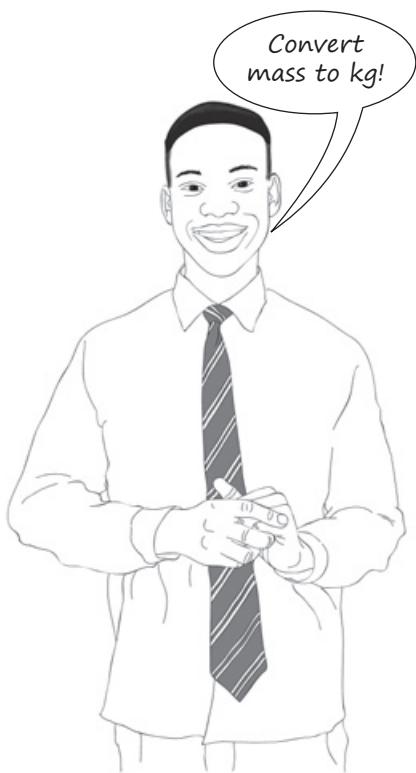


Diagram A

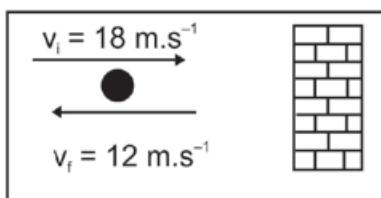


Diagram B

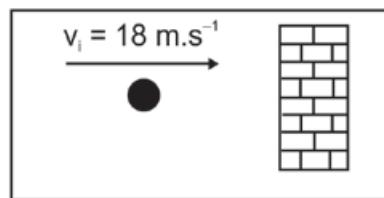


Diagram A:

$$m = 150 \div 1000 = 0,15 \text{ kg}$$

$$\vec{v}_i = +18 \text{ m}\cdot\text{s}^{-1}$$

$$\vec{v}_f = -12 \text{ m}\cdot\text{s}^{-1}$$

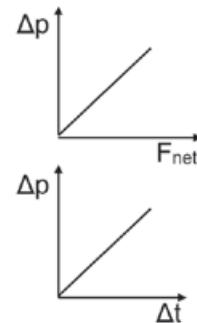
Diagram B:

$$m = 0,15 \text{ kg}$$

$$\vec{v}_i = +18 \text{ m}\cdot\text{s}^{-1}$$

$$\vec{v}_f = 0 \text{ m}\cdot\text{s}^{-1}$$

Let the direction **towards** the wall be **positive**.



1. Calculate the momentum of the baseball as it strikes the wall in Diagram A. (3)
2. Calculate the momentum of the baseball as it leaves the wall in Diagram A. (3)
3. Calculate the change in momentum during the collision in Diagram A. (3)
4. Calculate the force exerted by the wall on the baseball in Diagram A and in Diagram B, if each collision lasts 0,1 s. (6)
5. Draw a vector diagram to illustrate the relationship between the initial momentum (\vec{p}_i), the final momentum (\vec{p}_f) and the change in momentum ($\Delta \vec{p} = \vec{p}_f - \vec{p}_i$) for the baseball in Diagram A. (6)

[21]

Solutions

1. $\vec{p}_i = m \vec{v}_i$ ✓
 $m = 150 \text{ g} = 0,15 \text{ kg}$
 $\vec{v}_i = 18 \text{ m}\cdot\text{s}^{-1}$
 $\vec{p}_i = m \vec{v}_i = (0,15)(18)$ ✓
 $\vec{p}_i = 2,7 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ towards the wall (3)

2. $\vec{p}_f = m \vec{v}_f$ ✓
 $m = 150 \text{ g} = 0,15 \text{ kg}$
 $\vec{v}_f = -12 \text{ m}\cdot\text{s}^{-1}$
 $\vec{p}_f = m \vec{v}_f = (0,15)(-12)$ ✓
 $\vec{p}_f = 1,8 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ away from the wall (3)

3. Here are two ways to answer:

$$\Delta \vec{p} = m \cdot \Delta \vec{v}$$
 ✓
 $m = 0,15 \text{ kg}$
 $\Delta \vec{p} = (0,15)(-12 - 18)$ ✓
 $\Delta \vec{p} = -4,5 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$
 $\therefore \Delta \vec{p}$ is $4,5 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ away from the wall

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i$$
 ✓
 $\Delta \vec{p} = -1,8 - 2,7$ ✓
 $\Delta \vec{p} = -4,5 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$
 $\therefore \Delta \vec{p}$ is $4,5 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ away from the wall (3)

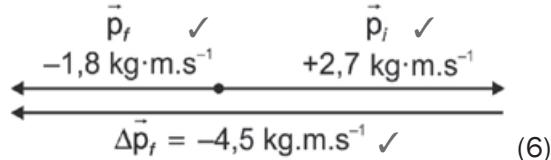
4. Diagram A: $F_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m \vec{v}_f - m \vec{v}_i}{\Delta t} = \frac{(0,15)(-12) - (0,15)(18)}{0,1} = -45 \text{ N}$ ✓

Force exerted by the wall is 45 N away from the wall

Diagram B: $F_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m \vec{v}_f - m \vec{v}_i}{\Delta t} = \frac{(0,15)(0) - (0,15)(18)}{0,1} = -27 \text{ N}$ ✓

Force exerted by the wall is 27 N away from the wall (6)

5. $\vec{p}_f = -1,8 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ away from the wall ✓
 $\vec{p}_i = 2,7 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ towards the wall ✓
 $\Delta \vec{p}_f = 4,5 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$ away from the wall ✓



[21]

2.4 Impulse

Impulse is another way to define momentum. Impulse is a measure of the amount of force applied to an object, for a certain period of time. Think of it as a measure of the shock experienced by an object when another object collides with it.

The formula for Impulse is: Impulse = $F\Delta t$ where

F is force in newtons N

Δt is change in time in seconds

Impulse and momentum are in fact the same thing. We can show this by dimensional analysis, that is, by working out what the units of impulse are, and comparing the units to the units of momentum.

$$F = ma$$

$$\therefore a = \frac{F}{m}$$

$$a = \frac{\Delta s}{t^2}$$

$$\therefore F = \frac{\Delta s}{t^2} \times m$$

$$\text{Impulse} = \frac{\Delta s}{t^2} \times m \times \Delta t$$

$$\text{Impulse} = \frac{\Delta s}{t} \times m$$

$$p = mv = vm$$

$$p = \frac{\Delta s}{t} \times m$$

$$\therefore p = \text{Impulse}$$



Activity 2

A cricket ball of mass 175 g is thrown horizontally towards a player at $12 \text{ m}\cdot\text{s}^{-1}$. It is hit back in the opposite direction with a velocity of $30 \text{ m}\cdot\text{s}^{-1}$. The ball is in contact with the bat for a period of 0,05 s. Calculate:

1. The impulse of the ball. (4)
2. The force exerted on the ball by the bat. (3)
3. The force exerted on the bat by the ball. Motivate your answer by referring to a Law of Motion. (5)

[12]

Solutions

1. impulse = $\vec{F}\Delta t = m\Delta\vec{v} = (0,175)[(-30)-(12)] = -7,35 \text{ N}\cdot\text{s}$
therefore 7,35 N·s away from the bat ✓ (4)
2. $\vec{F}\Delta t = -7,35 = F(0,05) \therefore F = \frac{-7,35}{0,05} = -147 \text{ N}$
therefore 147 N away from the bat ✓ (3)
3. 147 N towards the bat. According to Newton's Third Law of Motion the force of the bat on the ball is equal to the force of the ball on the bat, but in the opposite direction, ✓
 $F_{\text{bat on ball}} = -F_{\text{ball on bat}}$ ✓ (5)

[12]

2.5 The principle of conservation of linear momentum



The principle of conservation of linear momentum states that: **The total linear momentum in a closed system remains constant (is conserved)**



Steps for solving problems on conservation of linear momentum

- Step 1. Choose a direction as positive.
- Step 2. Sketch the situation – draw a block to represent each object.
- Step 3. Write down the equation for the Conservation of Momentum:
$$\sum \vec{p}_i = \sum \vec{p}_f$$
- Step 4. Expand this equation according to the type of collision.
- Step 5. Substitute the known values into the equation. Remember to check the direction of the objects' velocities and to use the correct signs for the directions.
- Step 6. Calculate the answer.
- Step 7. Write the answer, include units and indicate the direction.

VERY IMPORTANT

- Always remember to include units in your answer
- Remember that the +/- signs represent direction

We can solve problems about the conservation of linear momentum according to the nature of the collision or separation (explosion) of the objects involved. We usually solve problems in which two objects are involved.

2.6 Problem types

1. Two objects collide and continue to move as separate objects after the collision:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

2. Two objects collide and unite:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$$

3. Two moving objects that are initially joined, then separate:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$(m_1 + m_2) \vec{v}_1 = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

4. Two stationary objects that are initially joined, separate (e.g. during an explosion):

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$(m_1 + m_2) \vec{v}_1 = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 \vec{v}_{1f} = - m_2 \vec{v}_{2f}$$

5. An object falls vertically onto another object that is moving horizontally below it:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$m_1 v_{1i} + m_2 (0) = (m_1 + m_2) v_f$$

$$m_1 v_{1i} = (m_1 + m_2) v_f$$



This looks scary, but it's not really! It's saying that the sum of the momenta remains the same, that is, before collision and after collision, the total momentum before and after is the same. So, $\sum p_i$ is the sum of all the initial momenta. $\sum p_f$ is the sum of all final momenta. To calculate the initial momentum sum, just add up the momenta of all the objects.

Problem Type 1: Two objects collide and continue to move as separate objects after the collision

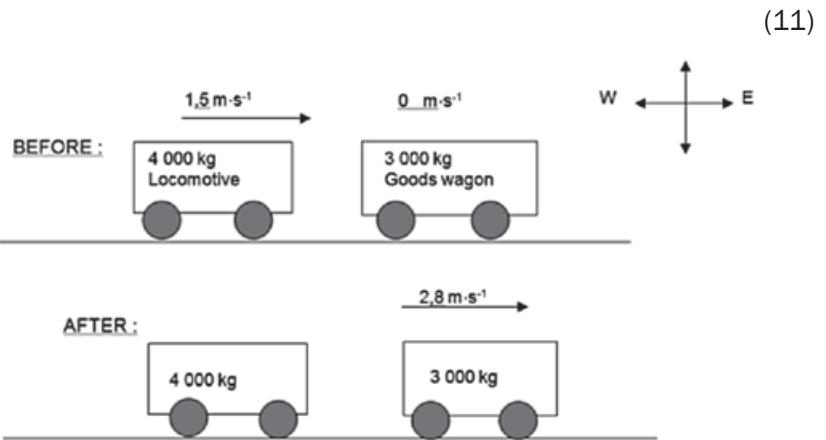


Activity 3

In a railway shunting yard, a locomotive (train engine) of mass 4 000 kg, travels due east at a velocity of $1,5 \text{ m}\cdot\text{s}^{-1}$. The train driver tries to link it to a stationary wagon of mass 3 000 kg by letting them collide. Instead, the wagon moves due east with a velocity of $2,8 \text{ m}\cdot\text{s}^{-1}$.

Calculate the magnitude and direction of the velocity of the locomotive immediately after the collision.

Remember:
If an object is stationary,
its velocity is $0 \text{ m}\cdot\text{s}^{-1}$;
"stationary" means "not
moving".



Solution

Let the locomotive be object 1 and the wagon be object 2.

Let motion to the east be positive

Then:

$$\begin{aligned} m_1 &= 4 000 \text{ kg} & \text{and} & \quad m_2 = 3 000 \text{ kg} \\ v_{1i} &= +1,5 \text{ m}\cdot\text{s}^{-1} & v_{2i} &= 0 \text{ m}\cdot\text{s}^{-1} \\ v_{1f} &= ? & v_{2f} &= +2,8 \text{ m}\cdot\text{s}^{-1} \end{aligned}$$

$$\begin{aligned} \sum \vec{p}_i &= \sum \vec{p}_f & \checkmark \\ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} &= m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} & \checkmark \\ (4000)(1,5) + (3000)(0) &= (4000)v_{1f} + (3000)(2,8) & \checkmark \\ 4000 \vec{v}_{1f} &= 6000 - 8400 & \\ \vec{v}_{1f} &= 0,6 \text{ m}\cdot\text{s}^{-1} & \checkmark \end{aligned}$$

$\therefore 0,6 \text{ m}\cdot\text{s}^{-1}$ to the west

[11]

Problem Type 2: Two objects collide and unite



Activity 4

A boy of mass 40 kg runs at $5 \text{ m}\cdot\text{s}^{-1}$ east and jumps onto a skateboard of mass 2 kg moving at $3 \text{ m}\cdot\text{s}^{-1}$ east.

Calculate the speed at which the boy and skateboard move together. (7)

Solution

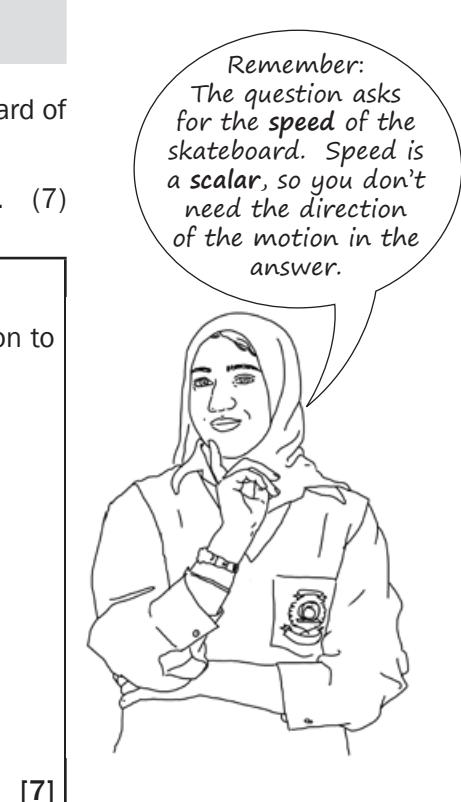
Let the boy be object 1 and the skateboard be object 2. Let motion to the east be positive.

Then:

$$\begin{aligned}m_1 &= 40 \text{ kg} & \text{and} & \quad m_2 = 2 \text{ kg} \\v_{1i} &= 5 \text{ m}\cdot\text{s}^{-1} & v_{2i} &= 3 \text{ m}\cdot\text{s}^{-1} \\v_{(1+2)f} &=?\end{aligned}$$

$$\begin{aligned}\sum p_i &= \sum p_f && \checkmark \\(40)(5) + (2)(3) &= (40 + 2)v_f && \checkmark \\42v_f &= 206 && \\v_f &= 4,9 \text{ m}\cdot\text{s}^{-1} && \checkmark\end{aligned}$$

\therefore the boy and skateboard move together at $4,9 \text{ m}\cdot\text{s}^{-1}$ ✓



[7]

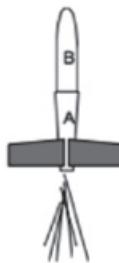
Problem Type 3: Two moving objects that are initially joined, then separate



Activity 5

Hendrik is an amateur rocket builder. He launches a two-stage rocket as shown in the diagram. Section A (stage 1) contains the rocket engine and fuel. Section B (stage 2) has a mass of 2 kg.

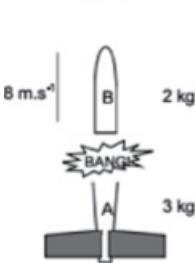
1. Hendrik says that Newton's Third Law of Motion is used to explain why the rocket moves upwards during flight. Identify one action-reaction pair of forces involved with the rocket's motion. (1)



At a certain height, when the rocket has a velocity of $5 \text{ m}\cdot\text{s}^{-1}$ upwards, the last fuel is used up, and section A has a mass of 3 kg. To get section B even higher, a small explosion separates section B from section A at this point and increases the upwards velocity of section B to $8 \text{ m}\cdot\text{s}^{-1}$.

2. State the Law of Conservation of Linear Momentum in words. (4)
3. Calculate the velocity of section A after the explosion. (11)

[16]



Solutions

1. The force of the rocket on the expelled (pushed out) gases and the force of the expelled gases on the rocket. ✓ (1)
2. The total linear momentum of a closed system remains constant in magnitude and direction. ✓ (4)
3. Let upwards be positive.

For section A: $m_A = 3 \text{ kg}$ and $v_{Af} = ?$

For section B: $m_B = 3 \text{ kg}$ and $v_{Bf} = 8 \text{ m}\cdot\text{s}^{-1}$

Before the explosion:

$$V_{(A+B)i} = 5 \text{ m}\cdot\text{s}^{-1}$$

$$\sum p_i = \sum p_f \quad \checkmark$$

$$(m_A + m_B)v_i = m_A v_{Af} + m_B v_{Bf} \quad \checkmark$$

$$(3 + 2)(5) = (3)(v_{Af}) + (2)(8) \quad \checkmark$$

$$3 v_{Af} = 25 - 16 \quad \checkmark$$

$$\therefore v_{Af} = \frac{9}{3} = 3 \text{ m}\cdot\text{s}^{-1} \quad \checkmark$$

$$\therefore 3 \text{ m}\cdot\text{s}^{-1} \text{ upwards} \quad \checkmark$$

(11)

[16]

Problem Type 4: Two stationary objects that are initially joined, and then separated (e.g. during an explosion)

When two objects are forced apart by an explosion or as a result of a compressed spiral spring released between them, they move in opposite directions after the explosion e.g. when a gun fires a bullet, the bullet moves forwards and the gun moves backwards.



Activity 6

A gun of mass 1 kg is attached to a trolley of mass 4 kg and is loaded with a bullet of mass 2 g. The system is at rest on a frictionless horizontal surface. The gun is fired by remote control and the bullet has a muzzle velocity of $350 \text{ m}\cdot\text{s}^{-1}$.

Calculate the velocity of the trolley and gun after the bullet has been fired. (8)

Solution

Let the direction of the bullet's motion be positive.

Let the trolley and gun be object 1:

$$m_1 = 1 + 4 = 5 \text{ kg}$$

$$v_{1f} = ?$$

$$\sum p_i = \sum p_f \quad \checkmark$$

Let the bullet be object 2:

$$(m_1 + m_2)v_i = m_1 v_{1f} + m_2 v_{2f} \quad \checkmark$$

$$m_2 = 2 \text{ g} = 0,002 \text{ kg}$$

$$0 = m_1 v_{1f} + b_2 v_{2f}$$

$$v_{1f} = 350 \text{ m}\cdot\text{s}^{-1}$$

$$m_1 v_{1f} = - m_2 v_{2f} \quad \checkmark$$

Before the explosion:

$$(5)v_{1f} = -(0,002)(350) \quad \checkmark$$

$$v_{(1+2)i} = 0 \text{ m}\cdot\text{s}^{-1}$$

$$v_{1f} = \frac{-(0,002)(350)}{5} = -0,14 \text{ m}\cdot\text{s}^{-1} \quad \checkmark$$

Therefore the gun and trolley move at $0,14 \text{ m}\cdot\text{s}^{-1}$ in the direction opposite to that of the bullet, after the explosion. ✓ [8]

Problem Type 5: An object falls vertically onto another object that is moving horizontally below it



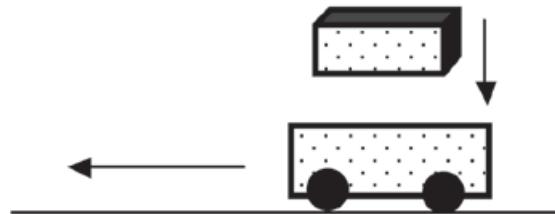
Activity 7

A trolley of mass 3 kg moves at $4 \text{ m}\cdot\text{s}^{-1}$, west along a frictionless horizontal path. A brick of mass 1 kg drops vertically onto it. The brick lands on the trolley at a vertical velocity of $0,5 \text{ m}\cdot\text{s}^{-1}$. Calculate the velocity of the brick and trolley system after the collision.



If an object falls vertically, its horizontal velocity is zero.

[13]



Solution

The brick strikes the trolley vertically at $0,5 \text{ m}\cdot\text{s}^{-1}$. So the brick's horizontal velocity is zero ($0 \text{ m}\cdot\text{s}^{-1}$). ✓ Momentum is conserved in a straight line. ✓ So the brick's vertical velocity is ignored when applying the Law of Conservation of Linear Momentum. ✓ (3)

Let motion west be positive

Let the trolley be object 1:

$$m_1 = 3 \text{ kg}$$

$$v_{1i} = +4 \text{ m}\cdot\text{s}^{-1}$$

Let the brick be object 2:

$$m_2 = 1 \text{ kg}$$

$$v_{2i} = 0 \text{ m}\cdot\text{s}^{-1}$$

After the collision: $v_1 + v_2 = ?$

$$\sum p_i = \sum p_f \quad \checkmark$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$(3)(4) + (1)(0) = (3 + 1)v_f \quad \checkmark$$

Zero, because the brick is not moving horizontally

$$12 = (4)v_f$$

$$v_f = 3 \text{ m}\cdot\text{s}^{-1} \quad \checkmark$$

\therefore brick and trolley system have a velocity of $3 \text{ m}\cdot\text{s}^{-1}$ west (horizontally) (10)

[13]

2.7 Elastic and inelastic collisions

2.7.1 Revision

Linear momentum is always conserved in a closed system. Kinetic energy is, however, always conserved, and is often transformed into other forms of energy, like heat and sound, or potential energy.

2.7.2 Differentiating between elastic and inelastic collisions

hint

Collisions are classified as either **elastic collisions** or as **inelastic collisions**.

Elastic Collisions:

- linear momentum is conserved
- colliding objects remain separate and are not changed in any way
- total kinetic energy is conserved:

$$\sum E_{ki} = \sum E_{kf}$$
- E_k before collision = E_k after collision
- the initial kinetic energy is not transformed into any other forms of energy.

Inelastic Collisions:

- linear momentum is conserved
- colliding objects are joined or change their shapes
- total kinetic energy is not conserved:

$$\sum E_{ki} > \sum E_{kf}$$
- E_k before collision > E_k after collision
- some of the initial kinetic energy is transformed into other forms of energy e.g. heat, light, sound.

Remember that for objects moving much below the speed of light (e.g. bullets, trains, people, bricks),

$$E = \frac{1}{2}mv^2$$

thus, if $p = mv$, then $E = \frac{1}{2}pv$



Steps for solving problems on elastic and inelastic collisions

Step 1. Calculate the sum of the kinetic energies of all the objects before the collision

$$\sum E_{ki} = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2$$

Step 2. Calculate the sum of the kinetic energies of all the objects after the collision

$$\sum E_{kf} = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

Step 3. Compare the total kinetic energy of the system before the collision to the total kinetic energy of the system after the collision.

Step 4. If $\sum E_{ki} = \sum E_{kf}$ $\therefore \sum E_k$ before the collision = $\sum E_k$ after the collision therefore the collision was elastic

If $\sum E_{ki} \neq \sum E_{kf}$ $\therefore \sum E_k$ before the collision $\neq \sum E_k$ after the collision therefore the collision was inelastic



Activity 8

Collisions between vehicles take place on the roads in our country daily. In one of these collisions, a car of mass 1 600 kg, travelling at a speed of $30 \text{ m}\cdot\text{s}^{-1}$ to the left, collides head-on with a minibus of mass 3 000 kg, travelling at $20 \text{ m}\cdot\text{s}^{-1}$ to the right. The two vehicles move together as a unit in a straight line after the collision.

1. Calculate the velocity of the two vehicles after the collision. (8)
 2. Do the necessary calculations to show that the collision was inelastic. (13)
 3. New cars have a crumple zone to help minimise injuries during accidents. Air bags and padded interiors can also help to reduce the chance of death or serious injury. Use principles in Physics to explain how crumple zones and air bags can reduce the chance of death or injury. (9)
- [30]

Solutions

1. Let the motion to the left be positive, and treating the minibus as object 1 and the car as object 2:



$$\begin{aligned} p_{\text{before collision}} &= p_{\text{after collision}} \\ m_1 v_{1i} + m_2 v_{2i} &= (m_1 + m_2) v_f \\ (1\,600)(30) + (3\,000)(-20) &= (1\,600 + 3\,000) v_f \\ 48\,000 - 60\,000 &= (4\,600) v_f \\ v_f &= -2,6 \text{ m}\cdot\text{s}^{-1} \\ \therefore v_f &= 2,6 \text{ m}\cdot\text{s}^{-1} \text{ to the right } \checkmark \end{aligned} \quad (8)$$

2. Before the collision:

$$\begin{aligned} \sum K_i &= \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \\ &= \frac{1}{2}(1\,600)(30)^2 + \frac{1}{2}(3\,000)(20)^2 \\ &= 720\,000 + 600\,000 = 1,32 \times 10^6 \text{ J } \checkmark \end{aligned}$$

After the collision:

$$\begin{aligned} \sum K_f &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \\ &= \frac{1}{2}(1\,600 + 3\,000)(2,6)^2 \\ &= 15\,548 \text{ J } \checkmark \end{aligned}$$

$$\begin{aligned} \sum K_i &> \sum K_f & \checkmark \\ \therefore E_k \text{ before collision} &> E_k \text{ after collision} & \therefore \text{the collision is inelastic} \end{aligned} \quad (13)$$

Remember:
Velocity is a vector quantity. ∴ include the direction in your answer.



3. Crumple zones in a car ensure that the car comes to rest over a longer period of time (Δt) during an accident, while air bags ensure that the driver / passenger comes to rest over a longer period of time inside the car. ✓
 ✓
 ∵ Δt to change the momentum of the car and of the driver OR passenger increases
 ∴ \vec{F}_{net} decreases ($\vec{F}_{\text{net}} \propto \frac{1}{\Delta t}$) and ✓
 ✓
 the magnitude of \vec{F}_{net} determines the extent of the passengers' injuries ✓
 ✓
 ∴ crumple zones and air bags decrease the extent of injuries during accidents. ✓ ✓ (9)

[30]



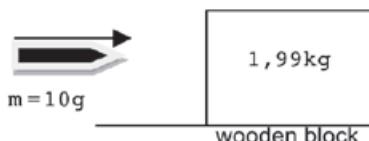
Activity 9

A bullet of mass 10 g, moving at a velocity of $300 \text{ m}\cdot\text{s}^{-1}$, strikes a wooden block of mass 1,99 kg resting on a flat horizontal surface as shown in the diagram below. The bullet becomes embedded in the block. Ignore the effects of air friction.

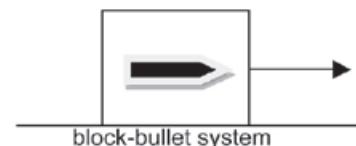
BEFORE COLLISION

$$v = 300 \text{ m}\cdot\text{s}^{-1}$$

$m = 10 \text{ g}$



AFTER COLLISION



1. Write down in words the principle of conservation of linear momentum. (2)
2. Calculate the speed of the block-bullet system immediately after the collision. (4)
3. Is this collision elastic or inelastic? Give a reason for the answer. (2)
 The floor exerts a constant frictional force of 8 N on the block-bullet system as it comes to rest.
4. Calculate the distance that the block-bullet system moves after the collision. (5)

[13]

Solutions

1. The total (linear) momentum remains constant/is conserved ✓ in an isolated/a closed system/the absence of external forces. ✓ (2)

2. To the right as positive

$$\sum p_{\text{before}} = \sum p_{\text{after}} \checkmark$$

$$(0,01)(300) \checkmark + (1,99)(0) = (0,01 + 1,99)v_{f2} \checkmark$$

$$v_{f2} = 1,5 \text{ m}\cdot\text{s}^{-1} \checkmark \quad (4)$$

3. Inelastic ✓

Kinetic energy is not conserved.✓ (2)

4. $F_{\text{net}} = ma \checkmark$

$$\therefore (-8) = 2a \checkmark \therefore a = -4 \text{ m}\cdot\text{s}^{-2}$$

$$V_{f2} = V_{i2} + 2a\Delta x \checkmark$$

$$0^2 = (1,5)^2 + 2(-4)\Delta x \checkmark$$

$$\Delta x = 0,28 \text{ m} \checkmark \quad (5)$$

[13]



Vertical projectile motion in one dimension

Summary

You must remember that:

acceleration, velocity and displacement (change in position or place) of a projectile occurs if it is:

- dropped from a certain height i.e. an object falling freely from rest;
- projected (thrown) upwards and then falls back to the same level as the original level;
- projected upwards and then falls back to a level below the original level;
- a falling object that bounces on a surface.

You need to be able to:

- Describe the motion for the different types of projectiles mentioned above;
- Draw and interpret graphs of the rate of change of position, velocity and acceleration;
- Use the graphs to calculate displacement or acceleration (using a velocity-time graph).

You must remember:

- **displacement** = moving to another place
- **velocity** = speed in a specific direction
- **acceleration** = change in velocity within a certain time.

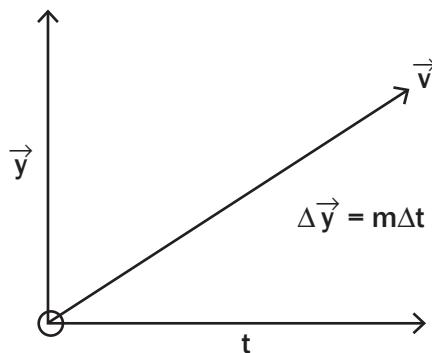


Learn the summary very well.

3.1 Revision: Graphs of velocity, acceleration and displacement

1. The slope of a displacement-time graph gives the velocity of an object:

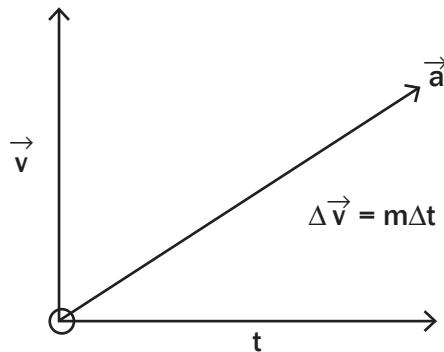
$$\text{slope} = \frac{\Delta \vec{y}}{\Delta t} = \vec{v} \text{ (m/s)}$$



where m is the velocity

2. The slope of a velocity-time graph gives the acceleration of the object:

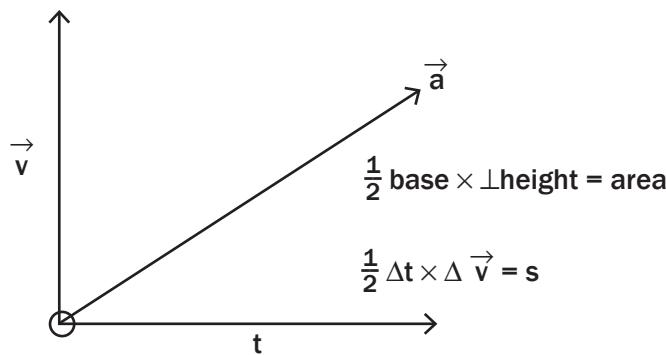
$$\text{slope} = \frac{\Delta \vec{v}}{\Delta t} = \vec{a} \text{ (m/s}^2\text{)}$$



where m is the acceleration

3. The area below a velocity-time graph gives the displacement of the object:

$$\text{Area of Triangle} = \frac{1}{2} \text{ base} \times \perp(\text{perpendicular})\text{height}$$



where s is displacement

3.2 Free fall

hint

projectile: an object (e.g. stone, ball or bullet) that travels through the air while gravity is the only force acting on it.

unimpeded: without being opposed or obstructed or disturbed.



DEFINITIONS:

- **Free fall** is the unimpeded motion of an object in the absence of air friction (resistance) where only gravitational force influences the object.
- **Gravitational acceleration:**
 - Gravitational acceleration (\vec{g}) is the constant acceleration of a free falling object due to gravity.
 - All objects experience the same gravitational acceleration (if we ignore the effects of air resistance). Hence, if there were no air a feather would fall at the same speed as a stone. See http://www.youtube.com/watch?v=5C5_d0EyAfk (where O is "oh", not zero).
 - **It is always directed downwards.**
 - On earth it is $\vec{g}_{\text{earth}} = 9,8 \text{ m}\cdot\text{s}^{-2}$ and on the moon: $\vec{g}_{\text{moon}} = 1,6 \text{ m}\cdot\text{s}^{-2}$

Formulas

All the following formulas are useful when you calculate projectile motion:

- $\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$
- $\vec{v}_f^2 = \vec{v}_i^2 + 2 \vec{a} \cdot \Delta \vec{y}$
- $\Delta \vec{y} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$
- $\Delta \vec{y} = \frac{\vec{v}_1 + \vec{v}_2}{2} \times \Delta t$

where:

\vec{v}_i is initial velocity ($\text{m}\cdot\text{s}^{-1}$)
 \vec{v}_f is final velocity ($\text{m}\cdot\text{s}^{-1}$)
 $\Delta \vec{y}$ is displacement (m)
 \vec{a} is acceleration ($\text{m}\cdot\text{s}^{-2}$)
 Δt is time (s)

Tips for calculations:

- Ignore air resistance for all calculations in Grade 12 unless the question states that there is air resistance.
- Free falling objects experience a **constant downward** acceleration equal to the gravitational acceleration, ($\vec{g} = 9,8 \text{ m}\cdot\text{s}^{-2}$).
- Choose a direction (downwards or upwards) as positive and keep this unchanged throughout the problem.
- Indicate the direction you have chosen as positive clearly at the start of your answers.

e.g. \downarrow \uparrow

REMEMBER:

Gravitational acceleration " \vec{g} " is constant and always directed downwards even when the object is moving upwards.

Therefore:

- If upwards direction is chosen as positive gravitational acceleration (\vec{g}) will be negative.
- If downwards direction is chosen as positive gravitational acceleration (\vec{g}) will be positive.

3.3 Graphs of Projectile Motion

Type 1: Dropping a projectile

When a projectile is **dropped (from rest)** from a certain height, then:

- Initial velocity $\vec{v}_i = 0 \text{ m}\cdot\text{s}^{-1}$
- The velocity increases as the object falls downwards
- The velocity is a maximum (\vec{v}_f) as the object hits the ground.

If you choose downwards as positive

$$\begin{aligned}\vec{v}_i &= 0 \\ \vec{a} &= +9,8 \\ \Delta \vec{y} &+? \\ \Delta t &=? \\ \Downarrow \quad \vec{v}_f &= + \text{max}\end{aligned}$$

If you choose upwards as positive

$$\begin{aligned}\vec{v}_i &= 0 \\ \vec{a} &= -9,8 \\ \Delta \vec{y} &-? \\ \Delta t &=? \\ \Downarrow \quad \vec{v}_f &= - \text{max}\end{aligned}$$

NOTE:

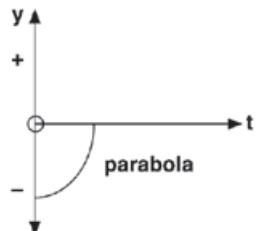
If the object is **thrown downwards** (not dropped) from a certain height, then initial velocity is not zero.

$$\vec{v}_i \neq 0$$

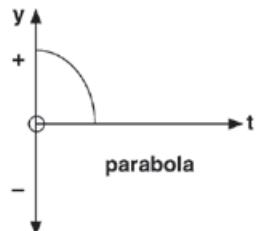
\vec{v}_i = the velocity at which the object is thrown.

Graphs and Projectile Motion: Type 1 – Dropping a projectile

Downwards as positive



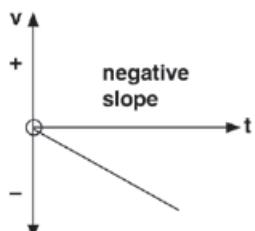
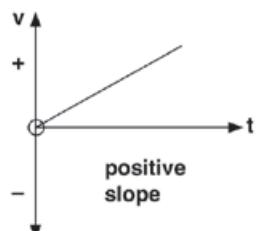
Upwards as positive



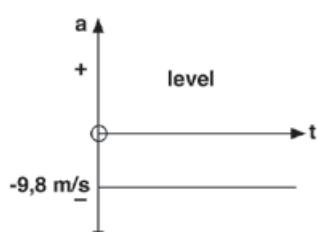
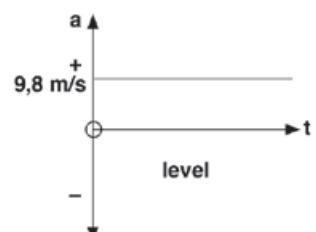
The distance (y) starts increasing slowly and then more quickly.

NOTE:

Note that the ground is taken as the zero reference for these graphs.



The velocity (v) increases at the same rate from start to finish.



The acceleration is constant at $9,8 \text{ m/s}^2$ whether it is in a positive direction or not. Only the sign changes.

hint

The mass of a falling object is irrelevant during free fall. Ignore the given value! The value of the mass is only relevant if you're asked to calculate the momentum with which it strikes the ground. Remember the feather and hammer.

**Activity 1**

A ball of mass 200 g is dropped from the roof of a 100 m high building. Ignore air resistance and calculate:

1. the velocity of the ball when it hits the ground. (4)

2. how long the ball is in the air before it hits the ground. (5)

[9]

Solutions

(Calculations for ‘down positive’ and for ‘up positive’ are provided. You only need to do one way!)

Let direction of motion down be positive

$$\vec{v}_i = 0 \text{ m}\cdot\text{s}^{-1} \quad \vec{v}_f = ? \text{ (a)}$$

$$\Delta \vec{y} = +100 \text{ m} \quad \vec{a} = +9,8 \text{ m}\cdot\text{s}^{-2}$$

$$\Delta t = ? \text{ (b)}$$

$$1. \vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot \Delta \vec{y} \quad \checkmark$$

$$= 0^2 + (2)(9,8)(100)$$

$$= 1960$$

$$\vec{v}_f = \sqrt{1960} = +44,27 \text{ m}\cdot\text{s}^{-1}$$

$$\therefore 44,27 \text{ m}\cdot\text{s}^{-1}, \text{ downwards} \quad \checkmark$$

(4)

$$2. \vec{v}_f = \vec{v}_i + \vec{a} \Delta t \quad \checkmark$$

$$44,27 = 0 + (9,8) \Delta t$$

$$\Delta t = \frac{44,27}{9,8} = 4,52 \text{ s}$$

$$\therefore \text{the ball is in the air for } 4,52 \text{ s} \quad \checkmark$$

(5)

[9]

If we let direction of motion up to be positive the solution is the same, only the sign changes.

This example shows that projectiles can have their motion described by a single set of equations for both upward and downward motion. It is not necessary to set motion in two directions for the same question. However it is important that you are able to solve problems using both approaches i.e. downwards - positive OR upwards - positive.

3.4 Graphs of Projectile Motion

Type 2: Projectile shot up, then falls down

Thinking about Physics

A gun fires a bullet up into the air and the bullet leaves the gun at mach 1 (the speed of sound). Suppose the bullet flies up to a certain height and then falls down to earth. Will it hit the ground at mach 1, or will it reach a certain limiting velocity and not achieve mach 1 again?

Answer: It will hit the ground at approximately mach 1 (ignoring air resistance). The reason is that as it flies up, it decelerates (gets gradually slower), until it reaches 0 m/s at the peak of its travel. It then has the same E_p (potential energy) as it did when it left the gun with the original kinetic energy (E_k).

Thus, as it falls back to earth, the E_p is converted to E_k again. Since energy is conserved, the amount of energy with which it strikes the ground, must be the same as that energy that it had when it left the gun, namely, enough energy to reach mach 1.

The deceleration of the bullet from mach 1 to 0 m/s is entirely due to gravity and air resistance; hence, when it falls back, its acceleration will be entirely due to gravity.

NB: These three graphs below are of the SAME EVENT: an object thrown upwards, then falling back down.

<p>Displacement - time graph</p>	<p>The displacement (y) increases in the positive direction until it reaches a maximum, and then it decreases. It becomes negative if it moves below the starting point.</p> <p>NB: This graph tracks the position of an object thrown up, then falling, in time. It DOES NOT mean that the object is thrown in an arc.</p>
<p>Velocity - time graph</p> <p>negative slope</p>	<p>The velocity (v) decreases until it reaches zero at the maximum height. Then the magnitude of the velocity increases, but in a negative direction, hence it goes below the t axis.</p> <p>NB: This is NOT a rock rolling down a hill. This is an object being thrown or shot up, reaching a maximum height, then falling back down again.</p>
<p>Acceleration - time graph</p>	<p>The acceleration is constant at $9,8 \text{ m/s}^2$ throughout, since we chose upwards as positive.</p> <p>NB: This is NOT an object travelling in a straight line; it is an object experiencing uniform negative acceleration (deceleration) due to gravity.</p>



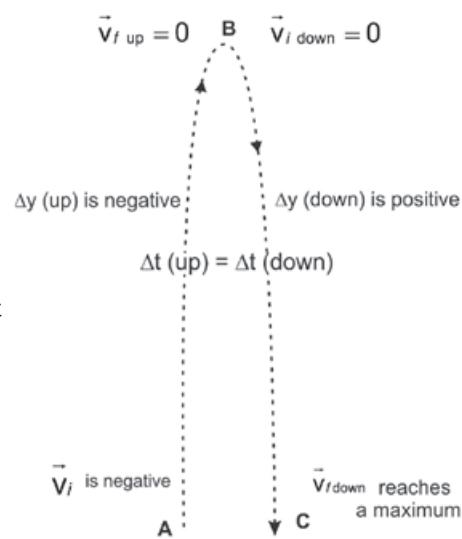
For the three graphs illustrated above

- Upwards is positive. If downwards is taken as positive the graphs will be inverted (upside-down).
- The original position is taken as the reference point.

3.5 Type 2a: A projectile projected vertically upwards which falls back to the same level

- We choose downwards as positive
- Initial velocity (\vec{v}_i) at A. The object leaves the starting point in an upwards direction.
- \vec{v}_i is negative, as it is moving upwards.
- The magnitude of the velocity decreases as the object rises
- The velocity is zero when the object reaches the highest point at B.
- \vec{v}_f (up) at B = 0
= \vec{v}_i (down) at B
- The velocity increases as the object falls down towards the ground.
- The initial velocity up is equal in magnitude to the final velocity down.
- \vec{v}_i (up) at A = \vec{v}_f (down) at C

If downwards is chosen as positive
 $\vec{g} = +9,8 \text{ m}\cdot\text{s}^{-2}$ ALWAYS.



NOTE: v_i (up) at A = v_f (down) at C
because level A is the same as level C

- The time taken to rise from A to B = time taken to return from B to the original position C
 t (up) AB = t (down) BC
- The total time taken through AB to C = time to rise from A to B + time to return from B to original position C.
- The object's displacement is zero (as it returns to its original position).
- The velocity (\vec{v}_f) down is a maximum as the object hits the ground at C.
- The acceleration of the object is constant at
 $\vec{g} = 9,8 \text{ m}\cdot\text{s}^{-2}$ downwards throughout the motion.

If upwards is chosen as positive
 $\vec{g} = -9,8 \text{ m}\cdot\text{s}^{-2}$ ALWAYS.



Activity 2

A ball is thrown vertically upwards at $4 \text{ m}\cdot\text{s}^{-1}$ and returns to the thrower's hand.

Let the direction of motion downwards be positive.

Calculate:

1. The maximum height reached by the ball. (4)
2. The time taken for the ball to reach the highest point in its trajectory. (5)
3. The total time that the ball is in the air. (3)
4. The ball's total displacement during the motion. (1)

[13]

Solutions

Let the direction of motion down be positive

$$\vec{v}_i = -4 \text{ m}\cdot\text{s}^{-1}$$

$$\vec{v}_f = 0 \text{ m}\cdot\text{s}^{-1}$$

$$\Delta \vec{y} = ? \text{ (a) m}$$

$$\vec{a} = +9,8 \text{ m}\cdot\text{s}^{-2}$$

$$\Delta t = ? \text{ (b)}$$

$$1. \vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot \Delta \vec{y} \quad \checkmark$$

$$0 = (-4)^2 + (2)(9,8) \vec{y} \Delta$$

$$19,6 \Delta \vec{y} = -16$$

$$\Delta \vec{y} = -0,82 \text{ m} \quad \checkmark$$

\therefore the ball reaches a height of 0,82 m above the starting level (4)

$$2. \vec{v}_f = \vec{v}_i + \vec{a} \Delta t \quad \checkmark$$

$$0 = (-4) + (9,8) \Delta t$$

$$\Delta t = \frac{4}{9,8} = 0,41 \text{ s}$$

\therefore the ball takes 0,41 s to reach the highest point in its trajectory (5)

$$3. \text{ Time upwards} = \text{time downwards} \quad \checkmark$$

\therefore total time in the air is

$$(2)(0,41) = 0,82 \text{ s} \quad \checkmark \quad (3)$$

$$4. \text{ Total displacement} = \Delta \vec{y} = 0 \text{ m} \quad \checkmark \quad (1)$$

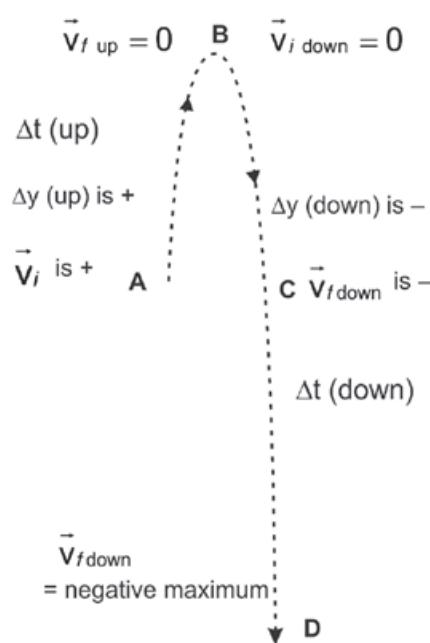
Displacement is measured in a straight line from the initial position (the thrower's line from the original to the final position (**the thrower's hand is the initial and final position**)).



[13]

3.6 Type 2b: A projectile projected vertically upwards which falls below the original level

- We choose upwards as positive.
- Initial velocity (\vec{v}_i) at A
- is velocity as object leaves the starting point in an upwards direction. It is positive.
- The magnitude of the velocity decreases as the object rises.
- The velocity is zero when the object reaches the highest point at B i.e. \vec{v}_f (up) at B = 0
= \vec{v}_i (down) at B.
- The magnitude of the velocity increases as the object falls downwards.
- The magnitude of the initial velocity upwards at A is equal to magnitude of velocity down at the starting level, C.
- \vec{v}_i (up) at A = $-\vec{v}_f$ (down at starting level) at C.



- The time taken to rise from A to B = time taken to return from B to the original position C.
- The total time taken to move though ABC to D = time to rise from A to B + time to return from B to original position C + time from C to the ground D.
- The object's displacement is downwards (as it passes the starting point and falls further downwards).
- The object's displacement (A to D) is equal in magnitude to the height from which it was released.
- The final velocity (\vec{v}_f) is a maximum as the object hits the ground at D.
- The acceleration of the object is constant, $\vec{g} = 9,8 \text{ m}\cdot\text{s}^{-2}$ downwards throughout the motion.

If downwards is chosen as positive
 $\vec{g} = +9,8 \text{ m}\cdot\text{s}^{-2}$ ALWAYS.

If upwards is chosen as positive
 $\vec{g} = -9,8 \text{ m}\cdot\text{s}^{-2}$ ALWAYS.



hint

- Use a ruler to draw the axes and any straight lines!

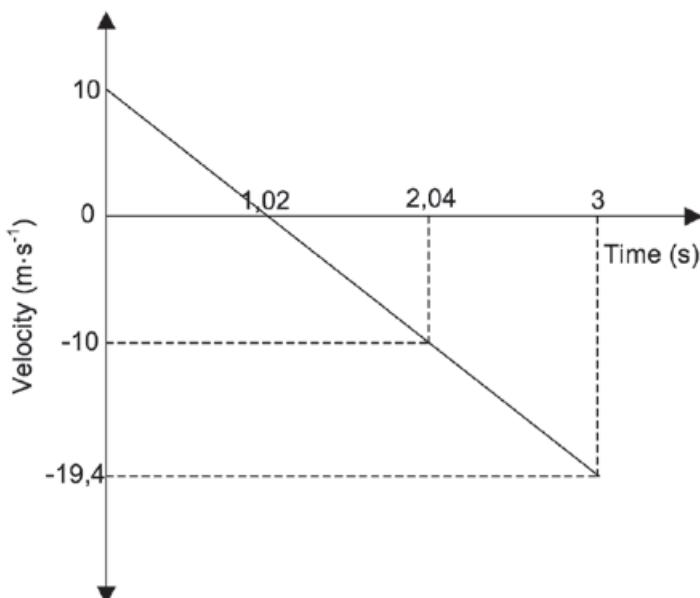
Drawing a sketch graph

- The graph does not have to be to scale, but it must have the correct shape
- The graph must show the physical quantity in words followed by the abbreviated unit (in brackets) e.g. velocity ($\text{m}\cdot\text{s}^{-1}$)
- The graph must include the values asked for in the question.



Activity 3

Lerato throws a stone vertically into the air from the top of a cliff. The stone strikes the ground below after 3 s. The velocity vs. time graph below shows the motion of the stone. **Ignore the effect of air resistance.**



1. How long does the stone take to fall from the height of the cliff to the ground below? (2)
2. What is the maximum height that the stone reaches above the ground? (Hint: calculate the height the stone reaches above the cliff, then calculate the height of the cliff, and add these two numbers). (4)
3. Draw a graph of position versus time. Use upwards as **negative**. (6)
[12]

Solutions

1. $3 - 2,04\checkmark = 0,96 \text{ s}\checkmark$ (2)

2. Option 1

$$\begin{aligned}\Delta y &= v_i \Delta t + \frac{1}{2} a \Delta t^2 \checkmark \\ &= (10)(3) + \frac{1}{2} (-9,8)(3)^2 \checkmark \\ &= 14,1 \text{ m}\end{aligned}$$

$\Delta y = 14,1 \text{ m}$ below the starting point

$$\begin{aligned}v_f^2 &= v_i^2 + 2a\Delta y \\ 0 &= 100 + 2(-9,8) \Delta y \checkmark\end{aligned}$$

$$\Delta y = 5,1 \text{ m}$$

$$\text{Maximum height above the ground} = 5,1 + 14,1 = 19,2 \text{ m} \checkmark$$

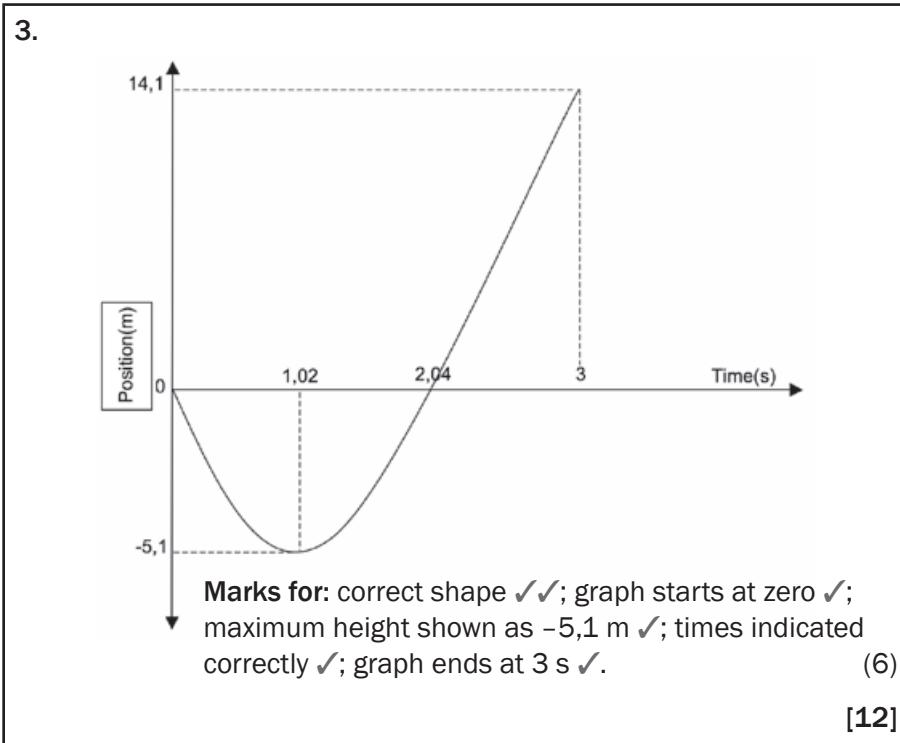
Option 2

$$\begin{aligned}\Delta y &= v_i \Delta t + \frac{1}{2} a \Delta t^2 \checkmark \\ &= 0 \checkmark + \frac{1}{2} (-9,8)(3 - 1,02)^2 \checkmark \\ &= -19,21 \text{ m}\end{aligned}$$

$$\Delta y = 19,21 \text{ m} \text{ (maximum height above the ground)} \checkmark$$

Option 3

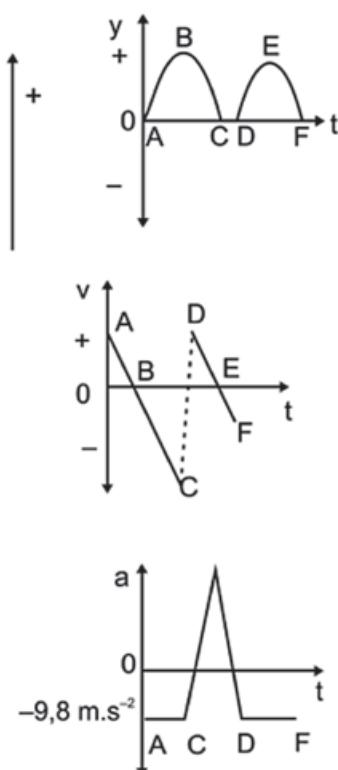
$$\begin{aligned}v_f^2 &= v_i^2 + 2a\Delta y \checkmark \\ (-19,4)^2 \checkmark &= 0 + 2 (-9,8) \Delta y \checkmark \\ \Delta y &= 19,2 \text{ m} \text{ (maximum height above the ground)} \checkmark\end{aligned}\quad (4)$$



3.7 Graphs of Projectile Motion Type 3: A bouncing ball

When a ball falls freely through the air, gravity is the only force that acts on it. The resultant force on the ball is downwards and it accelerates in the direction of the resultant force (Newton's Second Law of Motion).

- Consider a ball that bounces up from the ground (at A). Let *direction of motion upwards* be positive.
- The ball rises to a maximum height while slowing down (gravity accelerates it downwards at $9.8 \text{ m}\cdot\text{s}^{-2}$)
- At the highest point in its path (at B) its velocity is $0 \text{ m}\cdot\text{s}^{-1}$
- It still accelerates downwards at $9.8 \text{ m}\cdot\text{s}^{-2}$ due to gravity.
- It falls to the ground and its velocity increases until it strikes the ground (at C).
- From C to D the ball is in contact with the ground.
- The ground exerts an upward force on it which is greater than the force of gravity.



- The resultant force on the ball is therefore upwards and the ball accelerates upwards, in other words it slows down and stops.
- While still in contact with the ground, it starts to move upwards and (at D) leaves the ground.
- The collisions with the ground are inelastic and some of the ball's kinetic energy is transformed into e.g. sound and heat every time it strikes the ground.
- So the velocity as the ball leaves the ground ($\vec{v}_{i \text{ upwards}}$) is less than the velocity at which it hits the ground.
- After each bounce, the height reached by the ball is less than during the previous bounce.



Activity 4

A hot-air balloon is rising upwards at a constant velocity of 5 ms^{-1} . When the balloon is 60 m above the ground, a boy drops a ball from it and the ball falls freely.

Assume that the balloon continues to move upwards at the same constant velocity. When the ball hits the ground, it bounces vertically upwards to a height of 8 m above the ground. It falls back to the ground and bounces again to reach a height of 5 m. Take **upwards as positive**.

1. Describe the motion of the ball from the moment it is dropped until it hits the ground. (8)
2. Why does the ball not reach the same height during the second bounce as during the first? (7)
3. What is the magnitude and direction of the ball's velocity at the moment when it is dropped? (1)
4. Calculate maximum height reached by the ball. (5)
5. Where is the ball after 3 seconds? (6)
6. How far apart will the ball and the balloon be after 3 seconds? (7)
7. Calculate the time taken for the ball to reach the ground. (7)
8. Calculate the time the ball takes to reach the height of 8 m above the ground after its first bounce. (10)
9. Calculate the velocity at which the ball hits the ground the after the first bounce. (4)
10. Draw a sketch graph of velocity vs. time for the ball from the moment it is dropped until it reaches the height of 5 m after its first bounce. (5)
11. Draw a sketch graph of position vs. time for the ball for the same time as in (10). Use the position of the ball when it is dropped as the point of reference. (5)
12. Draw a sketch graph of acceleration vs time for ball for the same time as in (10). (4)

[69]



The point of reference for a position-time graph is placed on the time axis, where $y = 0 \text{ m}$.

Solutions

1. Initially the ball and hot-air balloon will both move upwards at a constant velocity. ✓
When the ball is dropped it continues to move upwards ✓ but decelerates constantly (at $9,8 \text{ m}\cdot\text{s}^{-2}$) ✓ due to the gravitational ✓ attraction force of the earth and slows down until it reaches the highest point in its trajectory (path) ✓. It stops momentarily ($\vec{v} = 0$) ✓ and then starts to accelerate downwards constantly ✓ (at $9,8 \text{ m}\cdot\text{s}^{-2}$). Its speed increases until it hits the ground at a maximum velocity ✓. (8)
2. The collision between the ball and the ground is inelastic. Some of the ball's kinetic energy is converted into heat ✓ and sound energy and the ball is deformed during the collision. ✓ The upward force of the ground on ✓ the ball causes it to bounce upwards but the kinetic energy is less than before the collision, so the velocity ✓ at which the ball leaves the ground is less than ✓ the velocity at which it hit the ground and the height reached ✓ is lower ✓ than the previous bounce. (7)

3. 5 ms^{-1} upwards ✓ (1)

4. $\vec{v}_i = +5 \text{ ms}^{-1}$ $\vec{v}_i = 0 \text{ ms}^{-1}$ $\vec{a} = -9,8 \text{ ms}^{-2}$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot \Delta \vec{y} \quad \checkmark$$

$$0^2 = 5^2 + 2(-9,8)\Delta \vec{y}$$

$$-25 = -19,6\Delta \vec{y}$$

$$\Delta \vec{y} = \frac{-25}{-19,6} = 1,28 \text{ m} \quad \checkmark$$

\therefore the ball will reach a maximum height of $(60 + 1,28) = 61,28 \text{ m}$ above the ground. (5)

5. $\vec{v}_i = +5 \text{ ms}^{-1}$ $\Delta t = 3 \text{ s}$ $\vec{a} = -9,8 \text{ ms}^{-2}$

$$\Delta \vec{y} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2 \quad \checkmark$$

$$\therefore \Delta \vec{y} = (5)(3) + \frac{1}{2} (-9,8)(3)^2 \quad \checkmark$$

$$\therefore \Delta \vec{y} = -29,1 \text{ m} \quad \checkmark$$

\therefore the ball is 29,1 m below the point from where it was released, or $(60 - 29,1) = 30,9 \text{ m}$ above the ground. ✓ (6)

6. The hot-air balloon moved upwards at a constant velocity.

$$\Delta \vec{y} = v_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2 \quad \checkmark$$

$$\Delta \vec{y} = (5)(3) + 0 \quad \checkmark$$

$$\Delta \vec{y} = 15 \text{ m} \quad \checkmark$$

\therefore After 3 s the hot-air balloon will be 15 m ✓ above the starting point.

We know from Question 4 that the ball will be 29,1 m below the starting point after 3 s. ✓

\therefore after 3 s the hot-air balloon and the ball will be $(15 + 29,1) = 44,1 \text{ m}$ apart. ✓ (7)

7. **NOTE:** Always calculate the velocity at which the ball hits the ground first.

$$\vec{v}_i = +5 \text{ ms}^{-1} \Delta y = -60 \text{ m} \quad \vec{a} = -9,8 \text{ ms}^{-2}$$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot \Delta \vec{y} \quad \checkmark$$

$$\vec{v}_f^2 = (5)^2 + 2(-9,8)(-60) \quad \checkmark$$

$$\therefore \vec{v}_f = 34,66 \text{ ms}^{-1} \text{ downwards} \quad \checkmark$$

$$\text{then } \vec{v}_f = \vec{v}_i + \vec{a} \cdot \Delta t$$

$$-34,66 = 5 + (-9,8)\Delta t$$

$$\therefore \Delta t = 4,05 \text{ s} \quad \checkmark$$

(7)

8. NOTE: The up and down displacement of the ball from the first to the second contact with the ground, is the same in magnitude (size) and $t_{\text{up}} = t_{\text{down}}$.

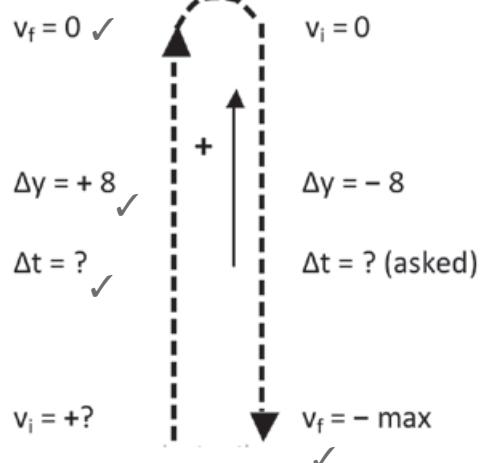
Consider the downward motion as negative as in the previous calculations. For the downward part of the bounce:

$$\begin{aligned}\vec{v}_i &= 0 \text{ ms}^{-1} \\ \vec{\Delta y} &= -8 \text{ m} \\ \vec{a} &= -9,8 \text{ ms}^{-2} \\ \vec{\Delta y} &= \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2 \\ -8 &= 0 + \frac{1}{2}(-9,8)(\Delta t^2)\end{aligned}$$

$$\Delta t = 1,28 \text{ s}$$

but $t_{\text{up}} = t_{\text{down}}$

\therefore the ball takes 1,28 s to reach a height of 8 m.



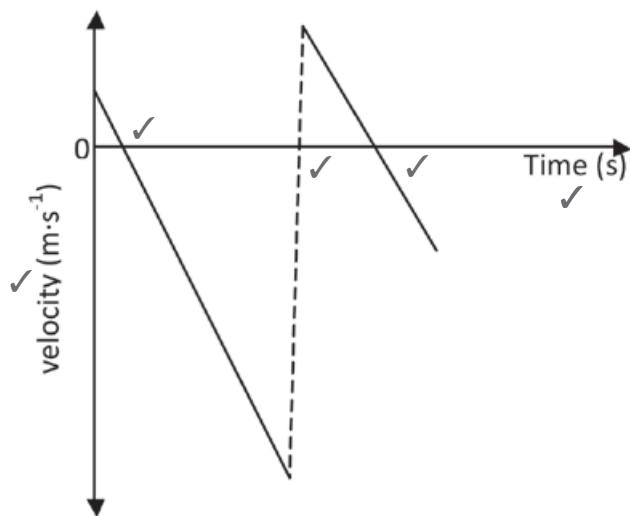
(10)

9. $\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$ ✓
 $\vec{v}_f = 0 + (-9,8)(1,28)$ ✓
 $\vec{v}_f = -12,54 \text{ ms}^{-1}$

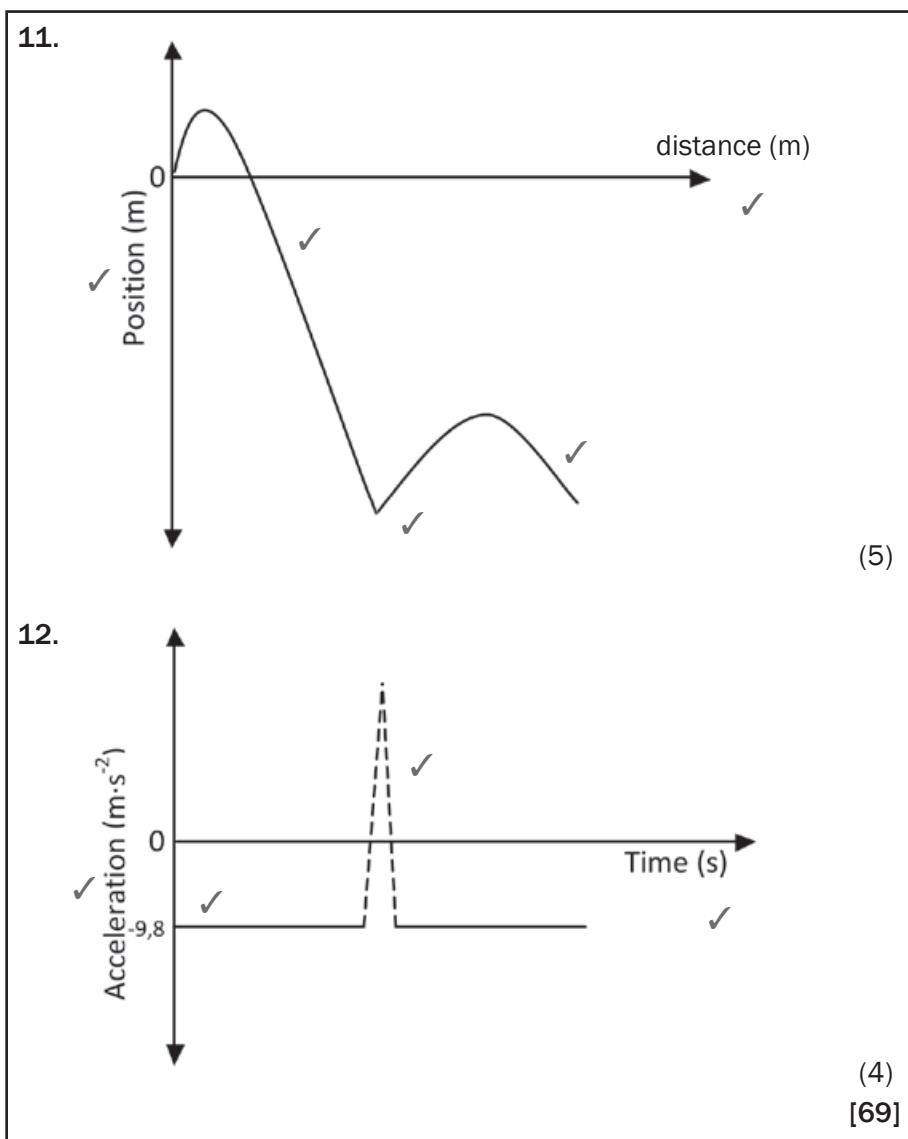
ball hits the ground at $12,54 \text{ ms}^{-1}$ downwards after the first bounce

(4)

10.



(5)





Activity 5

1. A 30 kg iron sphere and a 10 kg aluminium sphere with the same diameter fall freely from the roof of a tall building. Ignore the effects of friction. When the spheres are 5 m above the ground, they have the same ...
 - A momentum.
 - B Acceleration
 - C kinetic energy
 - D potential energy(2)

2. An object is thrown vertically into the air at $12 \text{ m}\cdot\text{s}^{-1}$ in the absence of air friction. When the object is at the highest point, the velocity of the object in $\text{m}\cdot\text{s}^{-1}$ is?
 - A 0
 - B 9,8 downwards
 - C 9,8 upwards
 - D 12(2)

3. An object is projected vertically upwards and then falls back to the ground level. The acceleration of the object is
 - A Directed upwards throughout its movement.
 - B Zero at the greatest height.
 - C Directed downwards throughout its movement.
 - D Directed upwards and then downwards.(2)

[6]

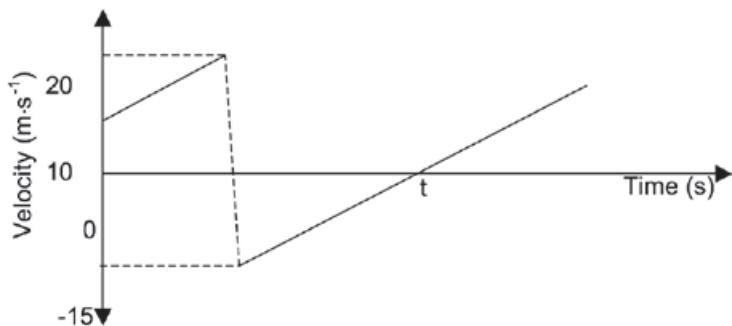
Solutions

- | | |
|---------|-----|
| 1. B ✓✓ | (2) |
| 2. A ✓✓ | (2) |
| 3. C ✓✓ | (2) |
- [6]



Activity 6

A ball of mass 0,15 kg is thrown vertically downwards from the top of a building to a concrete floor below. The ball bounces off the floor. The velocity versus time graph below shows the motion of the ball. Ignore the effects of air friction. TAKE DOWNWARD MOTION AS POSITIVE.



1. From the graph, write down the magnitude of the velocity at which the ball bounces off the floor. (1)
2. Is the collision of the ball with the floor ELASTIC or INELASTIC? Refer to the data on the graph to explain the answer. (2)
3. Calculate the:
 - a. Height from which the ball is thrown (3)
 - b. Size of the displacement of the ball from the moment it is thrown until time t (5)

(in an exam, you might see the word “magnitude” — this means “size”).
4. Sketch a position versus time graph for the motion of the ball from the moment it is thrown until it reaches its maximum height after the bounce. USE THE FLOOR AS THE ZERO POSITION.

Indicate the following on the graph:

- The height from which the ball is thrown
 - Time t (4)
- [15]

Solutions

1. $15 \text{ m}\cdot\text{s}^{-1}$ ✓ (1)

2. Inelastic ✓

The speed/velocity at which the ball leaves the floor is less / different than that at which it strikes the floor OR The speed/velocity of the ball changes during the collision. ✓

Therefore the kinetic energy changes/is not conserved. (2)

3a. $v_f^2 = v_i^2 + 2a\Delta y$ ✓

$(20)^2 = (10)^2 + 2(9,8)\Delta y$ ✓

$\therefore \Delta y = 15,31 \text{ m}$ ✓

(3)

3b Displacement from floor to maximum height

$v_f^2 = v_i^2 + 2a\Delta y$ ✓

$(0)^2 = (-15)^2 + 2(9,8)\Delta y$ ✓

$\Delta y = -11,48 \text{ m}$ ✓

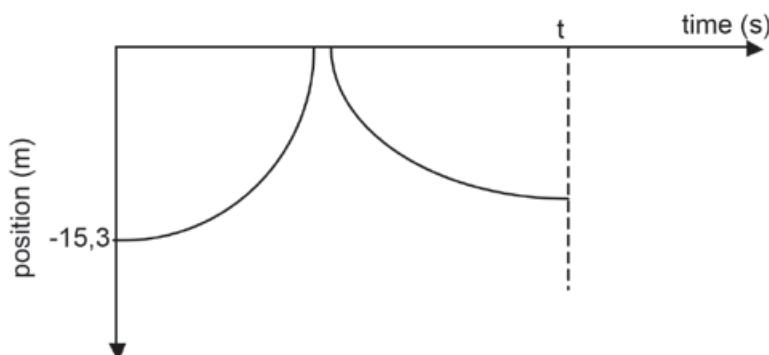
Total displacement

$= -11,48 + 15,3$ ✓

$= 3,82 \text{ m}$ ✓ or $3,83 \text{ m}$

(5)

4.

**Marking criteria for graph:**

Correct shape as shown for first part.	✓
Correct shape as shown for the second part up to $t / 2,55 \text{ s}$.	✓
Graph starts at $-15,3 \text{ m}$ at $t = 0 \text{ s}$.	✓
Maximum height after bounce at time $t / 2,55 \text{ s}$.	✓
Maximum height after bounce less than $15,3 \text{ m}$.	✓

(4)

[15]

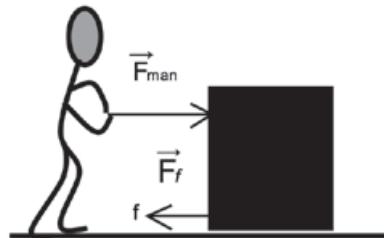


Work, energy and power

4.1 Work

Summary

- Work is a **scalar** quantity and therefore does not have a direction.
- The measuring unit of work is Joule. The symbol of Joule is J.
- **Net Work** is the sum of all work done on an object.
- **Net Work is done by a Net Force.**
- **Positive work** is the work done on an object to move it in the direction of the force (or component of the force). Positive work increases the kinetic energy of an object.
- **Negative work** is the work done by an opposing force. Negative work decreases the kinetic energy of an object.
- Work done by the man is positive.
- Work done by the friction is negative.



You must remember:

- **Work** is defined as the product of the force parallel to the movement of an object and the displacement of the object.
- **Work** can be defined Mathematically as: $W = F \Delta x \cos \theta$
 - W is the magnitude of work
 - F is the magnitude of the applied force
 - Δx is the magnitude of the displacement
 - θ is the angle between the applied force and the displacement of the object.
- **Work-energy theorem:** The net work done on an object is equal to the change in the object's kinetic energy OR work done on an object by a net force is equal to the change in the object's kinetic energy.
- **Work-energy theorem formula:** $W_{\text{net}} = \Delta K = K_f - K_i$
This is just saying that the total work is the the difference between the initial and the final kinetic energy state.



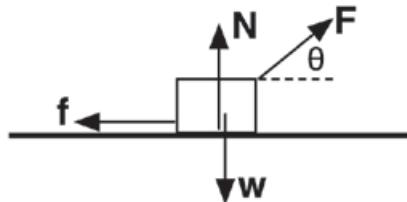
Activity 1

Multiple Choice Questions

Four options are provided as possible answers to the following questions. Each question has only **ONE** correct answer. Write only the letter (A–D) next to the question number (1.1 – 1.2).

1. An object moves in a straight line on a ROUGH horizontal surface.
If the net work done on the object is zero, then
 - A the object has zero kinetic energy.
 - B the object moves at constant speed.
 - C the object moves at constant acceleration.
 - D there is no frictional force acting on the object.(2)

2. An object is pulled along a straight horizontal road to the right without being lifted. The force diagram below shows all the forces acting on the object.



Which ONE of the above forces does POSITIVE WORK on the object?

- A W
 - B N
 - C f
 - D component F
- (2)
-
- [4]

Solutions

- | | |
|---------|-----|
| 1. B ✓✓ | (2) |
| 2. D ✓✓ | (2) |
| | [4] |



Activity 2: Work

A 220 N force is applied horizontally to a box of mass 50 kg which rests on a rough horizontal surface and the box moves 10 m. The kinetic friction between the surface and the box is 40 N. Calculate:

1. The work done on the box by the applied force. (3)
2. The work done on the box by the normal force. (3)
3. The work done on the box by the friction. (3)
4. The net work done on the box. (3)
5. The net force acting on the box. (5)
6. The work done on the box by the net force. (3)

[20]

Solutions

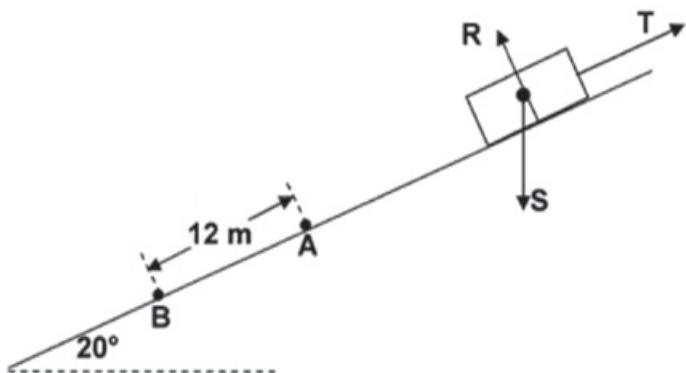
1. $W_{\text{applied}} = F_{\text{applied}} \cdot \Delta x \cdot \cos \theta = (220)(10)(\cos 0^\circ) = (220)(10)(1) = 2200 \text{ J}$ ✓ (3)
2. $W_{\text{normal}} = F_{\text{normal}} \cdot \Delta x \cdot \cos \theta = mg \cdot \Delta x \cdot \cos 90^\circ = (50)(9,8)(10)(0) = 0 \text{ J}$ ✓ (3)
3. $W_{\text{friction}} = F_{\text{friction}} \cdot \Delta x \cdot \cos \theta = (40)(10)(\cos 180^\circ) = (40)(10)(-1) = -400 \text{ J}$ ✓ (3)
4. $W_{\text{net}} = \sum W = W_{\text{applied}} + W_{\text{friction}} = (2200) + (-400) = 1800 \text{ J}$ ✓ (3)
5. Let F_{applied} act in the positive direction ($\therefore F_{\text{friction}}$ acts in the negative direction)
 $F_{\text{net}} = \sum F = (F_{\text{applied}}) + (-F_{\text{friction}}) = (220) + (-40) = +180 \text{ N}$
 $\therefore 180 \text{ N in the direction of the applied force}$ ✓ (5)
6. $W_{\text{net}} = F_{\text{net}} \cdot \Delta x \cdot \cos \theta = (180)(10)(\cos 0^\circ) = (180)(10)(1) = 1800 \text{ J}$ ✓ (3)

[20]



Activity 3: Work

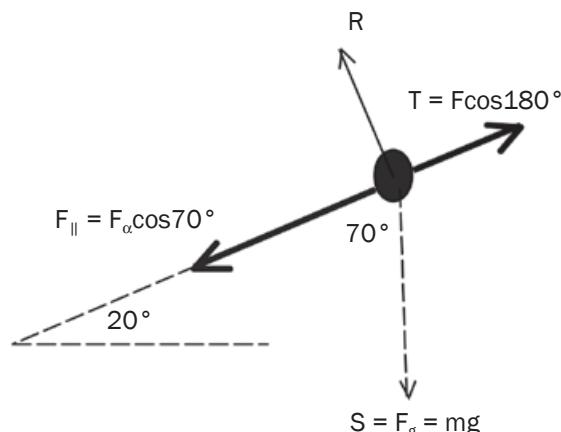
A crate of mass **70 kg** slides down a rough incline that makes an angle of **20°** with the horizontal, as shown in the diagram below. The crate experiences a constant frictional force of magnitude **190 N** during its motion down the incline. The forces acting on the crate are represented by **R**, **S** and **T**.



- Label the forces **R**, **S** and **T**. (3)
- The crate passes point A at a speed of $2 \text{ m}\cdot\text{s}^{-1}$ and moves a distance of 12 m before reaching point B lower down on the incline. Calculate the net work done on the crate during its motion from point A to point B. (5) [8]

Solutions

- R** is the Normal Force✓, **S** is the force of gravity✓ and **T** is the Frictional force✓ (3)
-



$$\begin{aligned} W_{\text{net}} &= F_g \Delta x \cdot \cos \theta + F_t \cdot \Delta x \cdot \cos \theta \checkmark \\ W_{\text{net}} &= (686)(12)(\cos 70^\circ) \checkmark + (190)(12)(\cos 180^\circ) \checkmark \\ W_{\text{net}} &= 2815,51 - 2280 \checkmark \\ W_{\text{net}} &= 535,51 \text{ J} \checkmark \end{aligned}$$



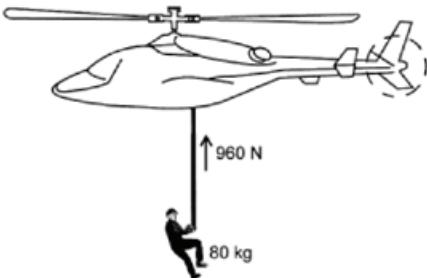
- Draw a free body diagram and label all the forces.
- Resolve the Force of gravity into its components to determine the applied force acting down the incline.
- To determine the net work apply the formula ($W = F\Delta x \cdot \cos(180^\circ)$) to both the applied force and the frictional force and add to find the net work done.

(5)[8]



Activity 4: Work-energy theorem

A rescue helicopter is stationary (hovers) above a soldier. The soldier of mass 80 kg is lifted vertically through a height of 20 m by a cable at a **CONSTANT SPEED** of $4 \text{ m}\cdot\text{s}^{-1}$. The tension in the cable is 960 N. Assume that there is no sideways motion during the lift. Air friction is not to be ignored.



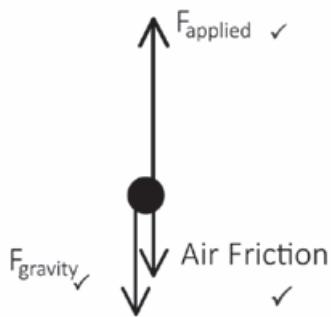
1. State the work-energy theorem in words. (2)
2. Draw a labelled free body diagram showing ALL the forces acting on the soldier while being lifted upwards. (3)
3. Write down the name of a non-constant force that acts on the soldier during the upward lift. (1)
4. Use the WORK-ENERGY THEOREM to calculate the work done on the soldier by friction after moving through the height of 20 m. (5)
5. Identify TWO forces which do negative work. (2)

[13]

Solutions

1. The net (total) work done on an object✓ is equal to the change in kinetic energy of the object. ✓OR The work done on an object by a net (resultant) force✓ is equal to the change in kinetic energy of the object. ✓ (2)

2.



(3)

3. Gravitational force or weight of the soldier.✓ (1)

4. Solution as follows:

$$W_{\text{net}} = \Delta K \checkmark$$

$$W_{\text{Gravity}} + W_{\text{Tension}} + W_{\text{friction}} = \Delta K$$

$$F_g \Delta y \cdot \cos 0^\circ + F_T \Delta y \cdot \cos 0^\circ + F_f \Delta y \cdot \cos 0^\circ = \Delta K$$

$$(960)(20) \cdot \cos 0^\circ \checkmark + (80)(9,8) \cdot \cos 180^\circ \checkmark + W_f = 0 \checkmark$$

$$19200 - 15680 + W_f = 0$$

$$\therefore W_f = 3520 \text{ J} \checkmark$$

(5)

5. Air friction✓ and Force of gravity✓ (2)

[13]

4.2 Energy

Summary

- Energy is a **scalar quantity** and therefore it does not have a direction
- The measuring unit of energy is called **Joule**. The symbol of Joule is J.
- The **principle of conservation of mechanical energy** states that the **total mechanical energy** (sum of gravitational potential energy and kinetic energy) **in an isolated system remains constant**. (A system is isolated when the resultant/net external force acting on the system is zero.)
- Solve conservation of energy problems using the equation:

$$W_{nc} = \Delta E_k + \Delta E_p$$
- The formula $E_k = \frac{1}{2} mv^2$ is used to calculate the **kinetic energy**.
- The formula $E_p = mgH$ is used to calculate the **potential energy**.
- **The Law of Conservation of Energy** states that energy cannot be created or destroyed. Energy can only be transferred from one object to another or transformed from one type of energy to another type.

You must remember:

- Energy is the ability to do work
- A **conservative force** is defined as a force for which the work done in moving an object between two points is independent of the path taken. Examples are gravitational force, the elastic force in a spring and electrostatic forces (coulomb forces).
- A **non-conservative force** is defined as a force for which the work done in moving an object between two points depends on the path taken. Examples are frictional force, air resistance, tension in a cable, etc.





Activity 5

Multiple Choice Questions:

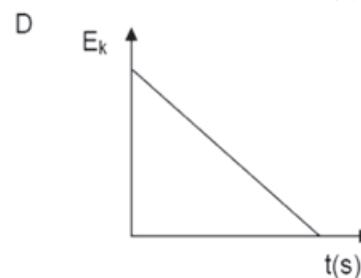
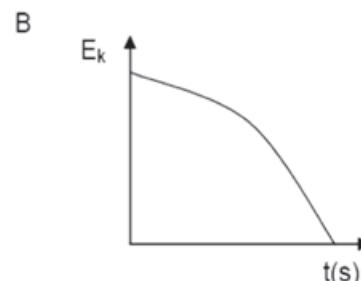
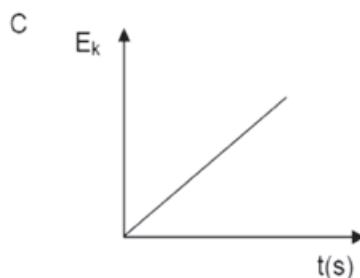
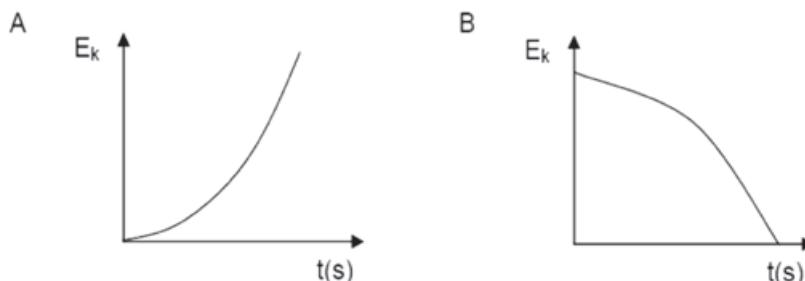
Four options are provided as possible answers to the following questions. Each question has only **ONE** correct answer. Write only the letter (A–D) next to the question number (5.1 – 5.2).

1. The kinetic energy of a car moving at a constant velocity v is K . The velocity of the car changes to $2v$. What is the new kinetic energy of the car?

- A 0,25 K
- B 0,5 K
- C 2 K
- D 4 K

(2)

2. A stone is dropped from the edge of a cliff. Which ONE of the following graphs best represents the change in kinetic energy of the stone during its fall?

(2)
[4]

Solutions

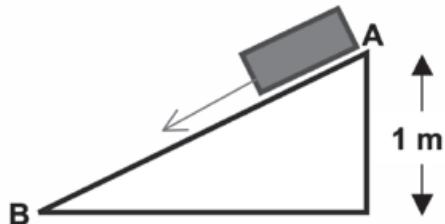
- 1. D ✓✓
- 2. A ✓✓

(2)
(2)
[4]



Activity 6: Work done by Non-Conservative Forces

A box of mass 100 kg slides down a slope. Its velocity increases from $0 \text{ m}\cdot\text{s}^{-1}$ at point A to $4 \text{ m}\cdot\text{s}^{-1}$ at point B as in the diagram. Calculate the work done by the non-conservative force while it slides from A to B.



[4]

Solution

$$\begin{aligned}
 W_{nc} &= \Delta E_p + \Delta E_k \checkmark \\
 &= [mgh_B - mgh_A] + [\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2] \\
 &= [(100)(9,8)(0) - (100)(9,8)(1)] \checkmark + [(\frac{1}{2})(100)(4)^2 - (\frac{1}{2})(100)(0)^2] \checkmark \\
 &= -980 + 800 = 180 \text{ J} \\
 \therefore W_{\text{friction}} &\text{ is } 180 \text{ J} \checkmark
 \end{aligned}$$

[4]

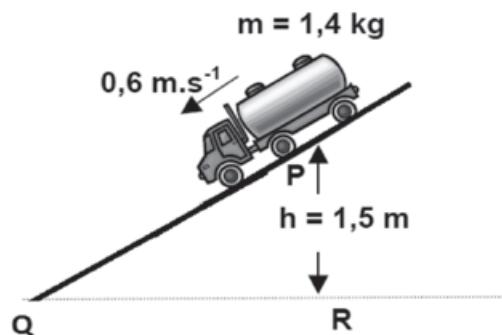


Remember that a frictional force is an example of a non-conservative force, therefore
 $W_{nc} = W_f$



Activity 7

A toy truck, mass 1,4 kg, moving down an inclined track, has a speed of $0,6 \text{ m}\cdot\text{s}^{-1}$ at point P, which is at a height of 1,5 m above the ground level QR. The curved section of the track, PQ, is 1,8 m long. When the truck reaches point Q it has a speed of $3 \text{ m}\cdot\text{s}^{-1}$. There is friction between the track and the truck.



1. State the principle of conservation of mechanical energy (2)
2. Is mechanical energy conserved? Explain. (2)
3. Assume that the average frictional force between the track and the truck is constant along PQ and calculate the average frictional force experienced by the truck as it moves along PQ. (6)
[10]

hint

- Use the formula $W_{\text{net}} = \Delta E_k + \Delta E_p$ to calculate the work done by the non-conservative force (friction).
- Use the work done by the non-conservative force to calculate friction.
- Remember, Work is a scalar, it does not have a direction and therefore do not leave your answer in a negative form.

Solutions

1. The total mechanical energy in an isolated system ✓ remains constant or is conserved. ✓ (2)
2. The mechanical energy is not conserved ✓ due to the presence of non-conservative force (frictional force). ✓ (2)
3. $W_{\text{nc}} = \Delta E_k + \Delta E_p$ ✓

$$W_{\text{nc}} = \left(\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2\right) + (mgh_f - mgh_i) \checkmark$$

$$W_{\text{nc}} = (0,5)(1,4)(32) - (0,5)(1,4)(0,62) + 0 - (1,4)(9,8)(1,5) \checkmark$$

$$W_{\text{nc}} = 6,3 - 0,252 + 0 - 20,58$$

$$W_{\text{nc}} = -14,532 = 15,532 \text{ J uphill} \checkmark$$

$$W_f = f\Delta x \cdot \cos \theta \checkmark$$

$$14,532 = f(1,8) \cdot \cos 0^\circ$$

$$f = 8,07 \text{ N} \checkmark$$

(6)

[10]

4.3 Power

Summary

- Power is the rate at which work is done or energy is transferred (or converted).
- Power** is an indication of the **rate** at which (how fast) work is done or energy is transferred or transformed and is a **scalar** quantity.

You must remember:

- The unit of **Power** is the watt
- The symbol of watt is **W**.
- The formula $P = \frac{W}{\Delta t}$ is used to calculate power, where **P** is power, **W** is work and Δt is the change in time.
- Average power can be calculated by applying the formula $P_{av} = FV_{av}$, where P_{av} is the average power, **F** is the force or net force and V_{av} is the average velocity or average speed.



Activity 8

Multiple Choice Questions:

Four options are provided as possible answers to the following questions. Each question has only **ONE** correct answer. Write only the letter (A–D) next to the question number (8.1 – 8.2).

- Power is defined as the rate.....
 - A of change in velocity.
 - B at which work is done.
 - C of change of momentum.
 - D of change of displacement. (2)
 - Which ONE of the following physical quantities is equal to the product of force and average velocity?
 - A Work
 - B Average power
 - C Energy
 - D Average acceleration (2)
- [4]

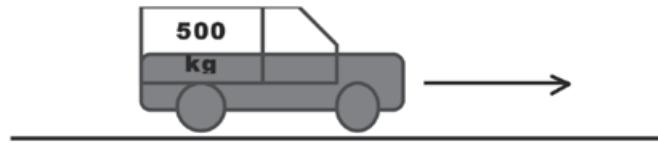
Solutions

- | | |
|---------|-----|
| 1. B ✓✓ | (2) |
| 2. B ✓✓ | (2) |
| | [4] |



Activity 9: Power

A car of mass 500 kg accelerates from $10 \text{ m}\cdot\text{s}^{-1}$ to $30 \text{ m}\cdot\text{s}^{-1}$ in 20 s.
Calculate the power of the car. [5]



Solution

$$\begin{aligned} W_{\text{net}} &= \Delta E_k = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) \checkmark \\ W_{\text{net}} &= (0,5)(500)(302) - (0,5)(500)(102) \\ W_{\text{net}} &= 200\,000 \text{ J} \\ P &= \frac{W}{\Delta t} = 200\,000 \div 20 \checkmark \\ P &= 10\,000 \text{ W} \checkmark \end{aligned}$$

[5]



1. First calculate the work done by applying the work-energy theorem.
2. Calculate the power (P).



Activity 10: Power

A pump is needed to lift water through a distance of 25 m from a borehole at a steady rate of 180 kg/min. What is the minimum power motor that could operate the pump if the velocity at the intake is $4 \text{ m}\cdot\text{s}^{-1}$ but at the outlet the water is moving with a speed of $9 \text{ m}\cdot\text{s}^{-1}$.



[8]

Solution

$$\begin{aligned} W_{\text{nc}} &= \Delta E_k + \Delta E_p \checkmark \\ W_{\text{nc}} &= \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (mgh_f - mgh_i) \checkmark \\ W_{\text{nc}} &= (0,5)(3)(9^2) - (0,5)(3)(4^2) + (3)(9,8)(25) - 0 \\ W_{\text{nc}} &= 121,5 - 24 + 735 - 0 \\ W_{\text{nc}} &= 832,5 \text{ J} \checkmark \\ \text{But} \\ P &= \frac{W}{\Delta t} \checkmark = 832,5/1 = 832,5 \text{ W} \checkmark \end{aligned}$$

[8]



In 1 second, the mass of water lifted up is $180 \text{ kg}/60 \text{ s} = 3 \text{ kg}$



Activity 11: Power

A 0,5 horsepower electric pump is used to bring water out of a borehole that is 80 m deep. 1 horsepower = 745,7 W. Calculate the mass of water that is let out of the borehole in one minute. [5]

Solution

$$P = 0,5 \times 745,7 = 372,85 \text{ W}$$

$$P = \frac{W}{\Delta t} \quad \checkmark$$

$$372,85 = W / 60$$

$$W = 22\ 371 \text{ J}$$

$$W = F\Delta y \cdot \cos\theta \quad \checkmark$$

$$F = mg$$

$$W = mg \Delta y \cdot \cos 0^\circ \quad \checkmark$$

$$22\ 371 = m \times (9,8)(80)\cos 0^\circ$$

$$22\ 371 = 784 \times m \quad \checkmark$$

$$\therefore m = 28,53 \text{ kg} \quad \checkmark$$

[5]



Activity 12: Power

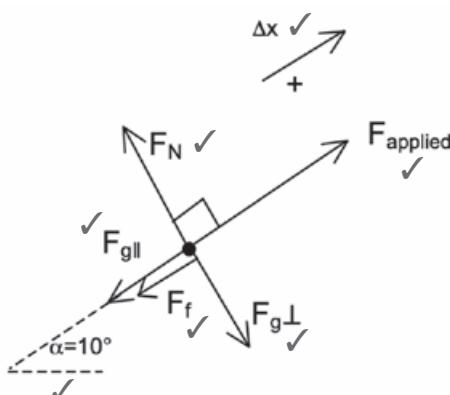
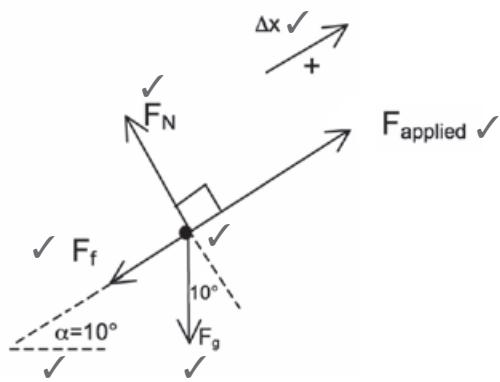
A motor car of mass 1 400 kg moves with a constant speed up a slope that makes an angle of 10° with the horizontal. The motor car experiences a frictional force of 700 N as it moves up the slope.

1. Draw a free body diagram to indicate the forces acting on the car. (7)
2. Calculate the applied force necessary to move the motor car up the slope at a constant speed. (5)
3. If the motor car moves at $80 \text{ km}\cdot\text{h}^{-1}$, calculate the power delivered by the motor car's engine. (3)

[15]

Solutions

1.



F_{applied} : pulling force exerted by the engine ✓

F_N : normal force upwards on the car by the surface ✓

F_g : gravitational attraction of the earth on the car vertically downwards ✓

$F_{g\parallel}$: component parallel to incline ✓

$F_{g\perp}$: component perpendicular to incline ✓

F_f : frictional force between the car and the surface, in the direction opposite to the motion ✓ (7)

2. If v is constant, $a = 0 \text{ m}\cdot\text{s}^{-2}$ and $F_{\text{net}} = 0 \text{ N}$

Let direction up the incline be positive.

$$F_{\text{net}} = ma = \sum F \checkmark$$

$$0 = F_{\text{applied}} - F_f - F_{g\parallel}$$

$$0 = F_{\text{applied}} - 700 - (1400)(9,8)(\sin 10^\circ) \checkmark$$

$$F_{\text{applied}} = 700 + (1400)(9,8)(\sin 10^\circ)$$

the applied force is 3 082,45 N up the incline. ✓ (5)

REMEMBER

First convert km/h to $\text{m}\cdot\text{s}^{-1}$. An easy way to do this is to divide by 3,6

3. $v = 80 \text{ km/h} = (80 000 \text{ m}) / (3600 \text{ s}) = 22,22 \text{ ms}^{-1} \checkmark$

$$P = Fv = (3 082,45)(22,22) = 68 498,89 \text{ W} \checkmark$$

(3)

[15]

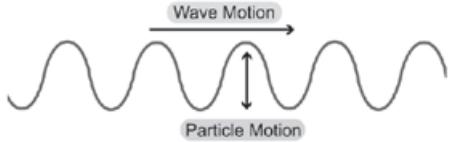
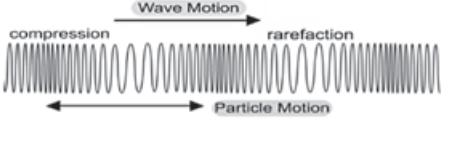


Doppler Effect

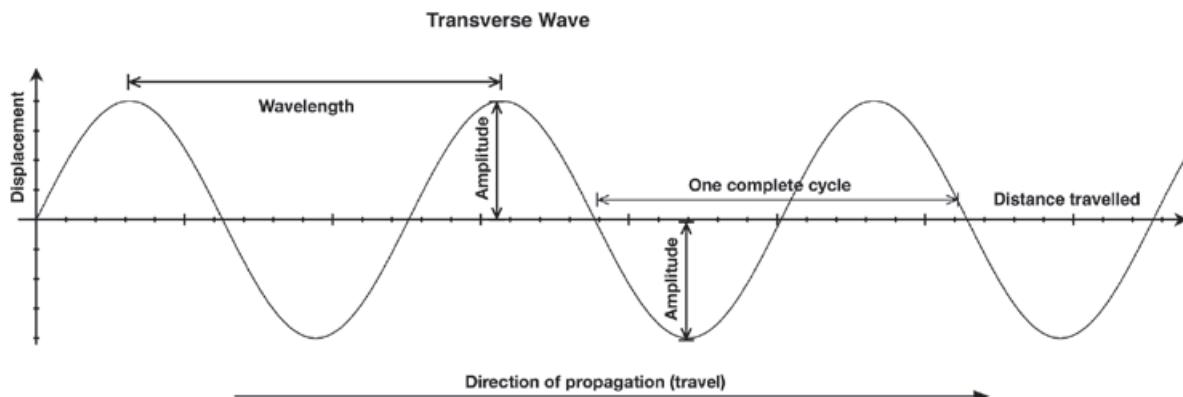
5.1 Waves: Revision

- Vibrations cause waves and waves cause vibrations.

There are two kinds of waves: transverse waves and longitudinal waves.

Transverse waves	Longitudinal waves
 <ul style="list-style-type: none">The disturbance of the medium is perpendicular to the direction in which the wave is propagated (transmitted).Examples: water waves, electromagnetic waves (light, radio waves, X-rays etc.)	 <ul style="list-style-type: none">The disturbance of the medium is parallel to the direction of propagation of the pulse.Example: sound waves, slinky spring

5.1.1 Wave properties



- The **amplitude** (height) of a wave motion is the maximum displacement of the particles from their equilibrium (rest) position. The amplitude determines the volume of a sound wave.
- The **wavelength (λ)** of a wave is the distance between two consecutive points in the wave which are in phase and is measured in metres (m). It is therefore also the **distance** between two successive crests or the distance between two successive troughs.
- The **frequency (f)** of a wave motion is the number of complete waves passing a specific point per second and is measured in **hertz (Hz)**. The frequency of a sound wave determines its pitch. The frequency of a light wave determines its **colour**.
- The frequency of a wave determines the energy of the wave.
- The higher the frequency, the higher the energy. So $E \propto f$



The **period (T)** of a wave motion is the time taken for one complete wave to pass a fixed point.

$$f = \frac{\text{number of waves passing a point}}{\text{time}}$$

and

$$T = \frac{\text{time}}{\text{number of waves passing a point}}$$

$$T = \frac{1}{f} \quad \text{and}$$

$$f = \frac{1}{T}$$

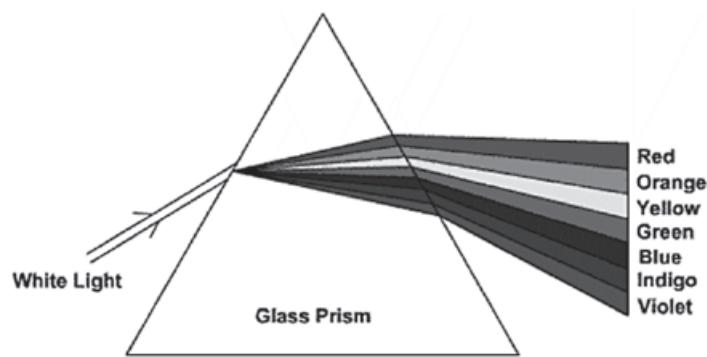
T is the period of the graph
(s) f is the frequency (Hz)

The **speed (v)** of a wave is the rate at which the energy is propagated by the wave and is measured in **$\text{m}\cdot\text{s}^{-1}$** .

$$v = f\lambda$$

where	v is speed	$(\text{m}\cdot\text{s}^{-1})$
	f is frequency	(Hz)
	λ is wavelength	(m)

5.1.2 Light



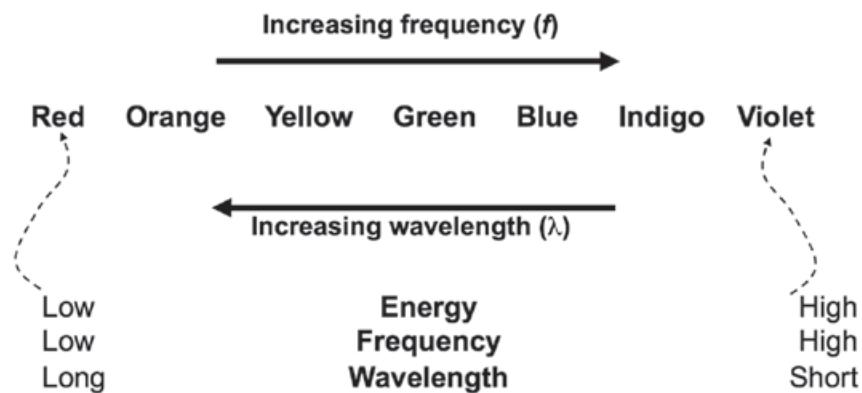
- The visible spectrum of light is just a small section of a much greater series of wavelengths called the **electromagnetic spectrum**.
- The speed of light (and all other electromagnetic radiation) is constant ($3 \times 10^8 \text{ m}\cdot\text{s}^{-1}$).
- The colour of light depends on its frequency.
- In the colour spectrum, red has the longest wavelength and lowest frequency and violet has the shortest wavelength and the highest frequency.



NB

For light: $c = f \lambda$
 c is the speed of an electromagnetic wave ($\text{m}\cdot\text{s}^{-1}$)
 $c = 3 \times 10^8 \text{ m}\cdot\text{s}^{-1}$
 f is frequency (Hz)
 λ is wavelength (m)

The Visible Light Spectrum



5.2 The Doppler Effect



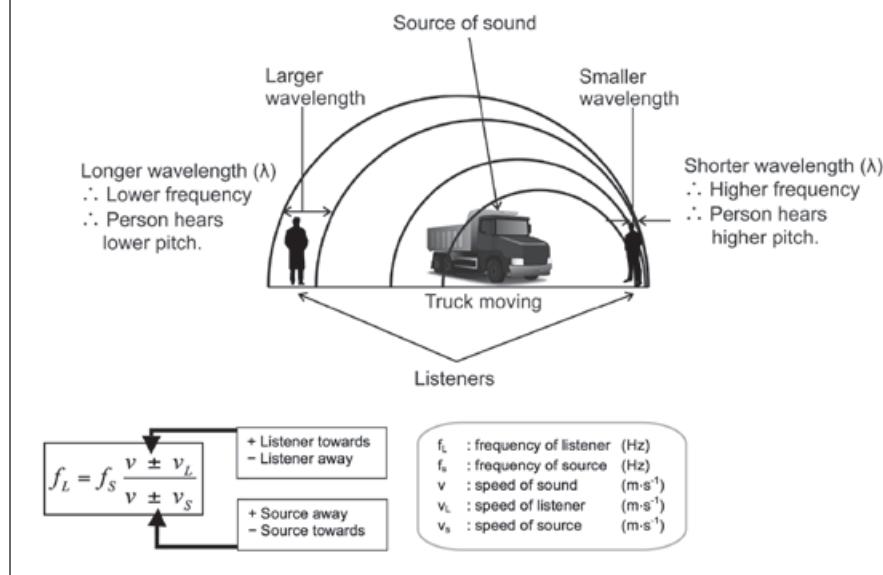
The Doppler Effect is the change in **frequency** or **pitch** of the sound or the colour of light that is detected when the wave source and the observer **move relative to each other**.

Vocabulary: Frequency means “how often”. “Observer” means person who sees, hears, or otherwise comes to know through the senses.

Example

When a car approaches a listener:

- the sound waves emitted by the car’s hooter are compressed in front of the car;
- more sound waves reach the listener per second and
- the pitch appears to be higher than the sound emitted by the source (the car’s hooter). The opposite is true when the car moves away from the listener.





Activity 1

1. A sound source approaches a stationary (not moving) observer at constant velocity. Which ONE of the following describes how the observed frequency and wavelength differ from that of the sound source?

Observed Wavelength	Observed Frequency
A. Greater than	Greater than
B. Less than	Less than
C. Greater than	Less than
D. Less than	Greater than

(2)

2. Which one of the following is the main principle applied when using the rate of blood flow or the heartbeat of a foetus in the womb?

- A. Doppler Effect.
- B. Photoelectric effect
- C. Huygens principle
- D. Diffraction

(2)

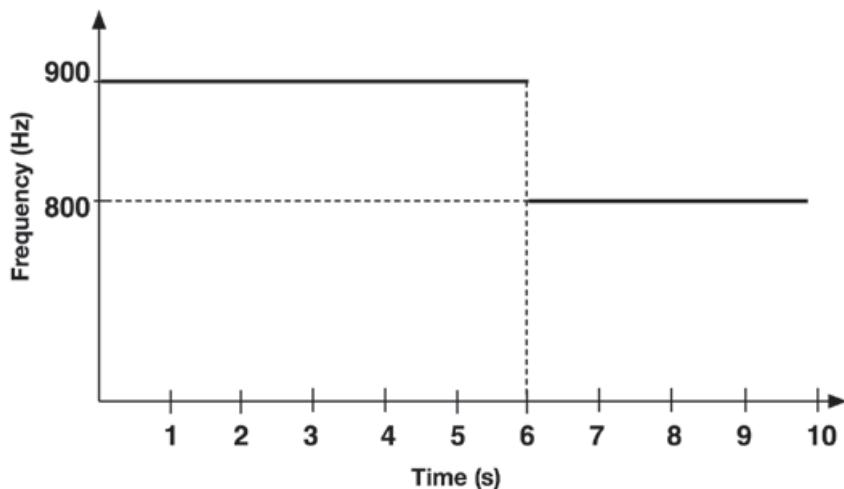
3. An ambulance approaches an accident scene at constant velocity.

The siren of the ambulance emits sound waves at a frequency of 980 Hz. A detector at the scene measures the frequency of the emitted sound waves as 1 050 Hz.

- a. Calculate the speed at which the ambulance approaches the accident scene. Use the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$ (6)
- b. Explain why the measured frequency is higher than the frequency of the source. (2)
- c. The principle of the Doppler Effect is applied in the Doppler flow meter. State ONE positive impact of the use of the Doppler flow meter on humans. (2)

4. The siren of a stationary (not moving) ambulance emits sound waves at a frequency of 850 Hz.

An observer (person witnessing this) who is travelling in a car at a constant speed in a straight line, begins measuring the frequency of the sound waves emitted by the siren when he is at a distance x from the ambulance. The observer continues measuring the frequency as he approaches, passes, and moves away from the ambulance. The results obtained are shown in the graph below.



- a. The observed frequency suddenly changes at $t = 6\text{ s}$. Give a reason for this sudden change in frequency. (1)
- b. Calculate
- b(1) The speed of the car (Take the speed of sound in air as $340 \text{ m}\cdot\text{s}^{-1}$) (5)
- b(2) Distance x between the car and the ambulance when the observer BEGINS measuring the frequency. (3)
- [23]

Solutions

1. D ✓ ✓ (2)

2. A ✓ ✓ (2)

3.

$$f_L = \frac{V \pm V_s(f_s)}{V \pm V_s} \quad \text{OR} \quad f_L = \frac{V(f_s)}{(V - V_s)} \quad \checkmark$$

$$\therefore 1050 \checkmark - (340 - 0) / (340 - V_s) \times 980 \checkmark$$

$$\therefore V_s = 22,67 \text{ m/s} \quad \checkmark \quad (6)$$

- a. Waves in front of the moving source are compressed. The observed wavelength decreases (✓). For the same speed of sound (✓), a higher frequency will be observed. (2)
- b. Determine whether arteries are clogged or narrowed (✓), so that precautions can be taken to prevent heart attack or stroke (✓), OR Determine the heartbeat (✓) of a foetus to assure that the child is alive or does not have a heart defect (✓). (2)

- 4.a The approaching observer (higher f) passes the source at $t = 6\text{ s}$ and moves away (lower f) from the source. (✓) (1)

4.b(1)

Option 1. Approaching observer:

$$f_L = \frac{V \pm V_s(f_s)}{V \pm V_s} \quad \text{OR} \quad f_L = \frac{V + V_s(f_s)}{V} \quad \checkmark$$

$$\therefore 900 \checkmark = (340 + V_s) (850) \checkmark / (340)$$

$$\therefore V_s = 20 \text{ m/s} \quad \checkmark \quad (5)$$

Option 2. Observer moving away:

$$f_L = \frac{V \pm V_L (f_s)}{V \pm V_s} \quad \text{OR} \quad f_L = \frac{V - V_L (f_s)}{V}$$

$$\therefore 800 \checkmark = (340 - V_L) (850) \checkmark / (340)$$

$$\therefore V_L = 20 \text{ m/s} \checkmark$$

(5)

Notes: any other correct Doppler Effect formula gets maximum $\frac{3}{4}$ marks.

4b(2)**Option 1.**

$$\Delta x = v_1 \Delta t + \frac{1}{2} a \Delta t^2 \checkmark$$

$$= (20)(6) + \frac{1}{2}(0)\Delta t^2 \checkmark$$

$$= 120 \text{ m} \checkmark$$

Option 2.

$$\Delta x = v \Delta t \checkmark$$

$$= (20)(6) \checkmark$$

$$= 120 \text{ m} \checkmark$$

(3)

Note: accept $s = ut$ or $s = vt$, as well as $s = ut + \frac{1}{2} at^2$, as well as $\Delta y = v_1 \Delta t + \frac{1}{2} a \Delta t^2$

[23]



Activity 2

An ambulance moving at $40 \text{ m}\cdot\text{s}^{-1}$ approaches a traffic light where a blind man and his dog wait to cross a road. The siren of the ambulance (**source**) emits sound waves at a frequency of 350 Hz (f_s). The pitch of the sound that the man hears increases as the ambulance moves towards him and decreases as the ambulance passes him and moves away.

1. If the speed of sound in air is $340 \text{ m}\cdot\text{s}^{-1}$, determine the frequency (f_L) of the sound waves that the man hears while the ambulance approaches him. (3)
2. Explain how this effect can help a blind person waiting to cross the road. (2)

[5]

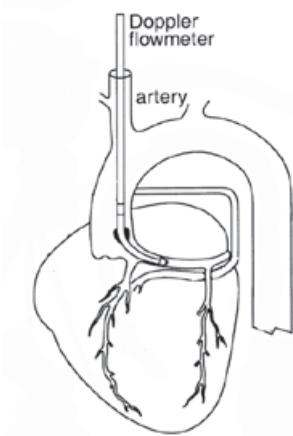
Solutions

1. $f_L = \frac{(V \pm V_L)}{(V \pm V_s)f_s} \checkmark$
 $f_L = \frac{(340+40)}{(340-40) \times 350 \text{ Hz}} \checkmark$
 $f_L = 396,67 \text{ Hz} \checkmark$ (3)

2. When crossing a street, a blind person can determine whether a car is moving towards or away from him. If the pitch of a vehicle decreases, ✓ the person knows that the vehicle is moving away from him, and vice versa. ✓ (2)

[5]

5.3 Applications of the Doppler Effect with ultrasound waves



The Doppler flow meter is used to measure the rate of blood flow in a patient's blood vessels.

- Ultrasound is a longitudinal wave with very high frequency of above 20 kHz that we cannot hear.
- A catheter connected to a Doppler flow meter is inserted in a blood vessel. It gives out a sound wave at ultrasound frequency. The blood velocity through the heart causes a 'Doppler shift' in the frequency of the returning waves. The meter measures this and compares the frequencies.
- The receiver detects the reflected sound and an electronic counter measures the reflected frequency.
- From the change in frequency, the speed of the blood flow can be determined and narrowing of blood vessels identified.

5.4 Applications of the Doppler Effect with light

The electromagnetic Spectrum

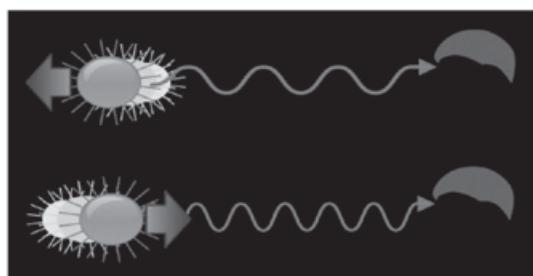
The electromagnetic spectrum is the full range of types of electromagnetic radiation.

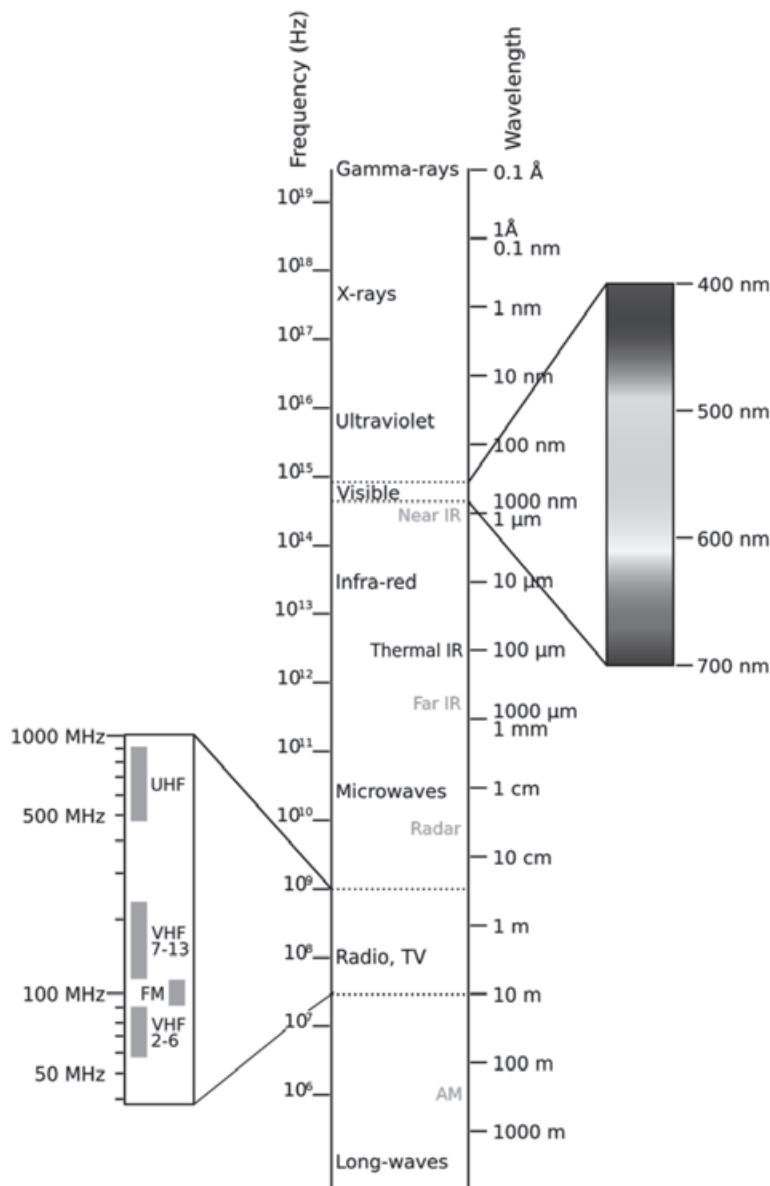
Electromagnetic radiation consists of waves which have both an electric and magnetic component. They are transverse waves. They are emitted by many objects eg. the sun, lights, fires, stoves, persons.

All forms of light, radio, and heat at a distance, are electromagnetic radiation. Note that electromagnetic radiation is NOT the same as radioactivity except for gamma waves, which come from nuclear reactions. The electromagnetic spectrum is shown on the next page.

Visible light is part of the electromagnetic spectrum.

- NB**
- Remember:**
- The wavelength of light is usually measured in nm (nanometers) and must be converted to m (metres) before doing any calculations.
 $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$
- Stars, like the sun, emit light.
 - When a star moves away from the Earth, its spectrum shifts to longer wavelengths (lower frequencies) – in other words, the red side of the spectrum. The star appears red.
 - When a star moves towards the Earth, its spectrum shifts to shorter wavelengths (higher frequencies) – in other words, the blue side of the spectrum. The star appears blue.





Ultrasound machines use the Doppler Effect to get readings of unborn babies in the womb so that doctors can see if the baby is healthy.



Electrostatics

Summary

- Electrostatics is the study of static (rest or stationary) positive or negative charges. Think for example of how you can get little shocks from scuffing your feet on a carpet when wearing rubber-soled shoes. This kind of electricity is not flowing, unlike, for example, the electricity in a plug point.
- Define electrostatics, electric field and electric field strength.
- Give evidence for the existence of two kinds of electric charge (like charges repel, unlike charges attract).
- Describe and demonstrate a method for determining whether an unknown charge is positive or negative.
- Name the unit of charge, and discuss its size with respect to common electrostatic situations and in terms of the number of unit charges it represents.
- Describe what it means to say that charge is conserved.
- Coulomb's Law
- Drawing of electric field lines.
- Application of Coulomb's Law and electric field strength (by calculations).

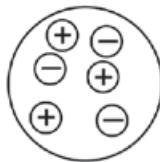
6.1 Definitions: Electrical charge and electric force

Electrical Charge

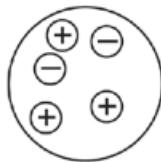
- At the atomic level, charge is associated with protons and electrons. They have the same magnitude of charge, **but their charge is opposite in sign**. Protons have Positive charge and Electrons have Negative charge. The symbol for a proton is p^+ and the symbol for an electron is e^- .
- Charge is measured in Coulombs, abbreviated C. It takes $6,25 \times 10^{18}$ charges to make 1C of charge, i.e. $6\ 250\ 000\ 000\ 000\ 000\ 000$ charged particles. $6,25 \times 10^{18}$ electrons will make -1C of charge. A coulomb is defined technically as one ampere-second (1 As), in other words, the amount of charge carried by one ampere in one second.
- Coulomb's Law is a measure of how strong the force is between two charged objects. Its formula is:
$$F = \frac{kQ_1Q_2}{r^2}$$
- Electrical charges will exert forces on each other. Two positive charges will repel each other and two negative charges will repel each other. (Like charges, or charges of the same sign, repel each other.) A positive and a negative charge will attract each other. (Unlike charges attract each other.)
- Electrostatic charge is a strong force.



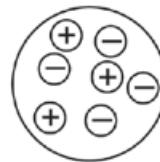
- As a result most objects usually have about the same amount of positive and negative charge. If they have exactly the same amount of positive and negative charge the net charge is zero and we say they are neutral.
- For convenience, we can abbreviate “positive” and “negative” as +ve and -ve respectively.



no. of +ve = no. of - ve charges
 \therefore neutral



no. of +ve charge > no. of -ve charges
 \therefore net positive charge



no. of +ve charges < no. of -ve charges
 \therefore net negative charge

Electric Force

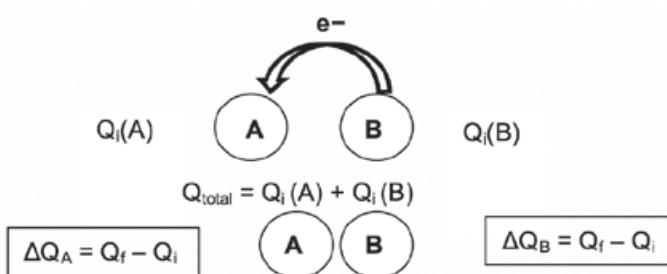
- The force is proportional to the product of the charges and inversely proportional to the square of the distance between them. (If we double one charge the force doubles. If both charges are doubled then the force increases by four. If the distance between the charges is increased, the force decreases, and vice versa.) This is similar to Newton's Law of Gravitation, which has the same formula structure.
- Electrical forces can do work and there is a potential energy associated with this force.

6.2 The Law of Conservation of Charge

- When two charged spheres are brought into contact with each other, electrons flow from the sphere with more electrons to the sphere with fewer electrons.
- The symbol for charge is Q . Do not confuse this with current, I . Q is measured in coulombs and I is measured in amperes.
- If sphere B has more electrons than sphere A:



Charge cannot be destroyed or created, but can only be transferred from one object to another.



REMEMBER
 when charges are brought into contact they must be added and then divided by TWO to get an average.

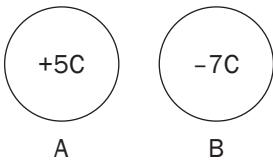
$$Q_{\text{new on each}} = \frac{Q_{\text{total}}}{2}$$

No. of e^- transferred = $\frac{\Delta Q(A)}{Qe^-} = \frac{\Delta Q(B)}{Qe^-}$





Worked example 1



Two spheres A and B carry charges of +5 C and -7 C respectively. They are brought into contact and are then separated.

1. What is the nature of the force between the charges before they are allowed to touch? Explain.
2. In which direction are electrons transferred during the contact? Explain.
3. Calculate the total charge in the system.
4. Calculate the charge on each sphere when they are separated.
5. Calculate the change in the charge on A and on B.
6. Calculate the number of electrons transferred from one sphere to the other.

Solutions

1. Attraction. Opposite charges (+ and -) attract.
2. From sphere B to sphere A. Electrons are transferred from the sphere with the most electrons (B in this case) to the sphere with the least electrons (A).
3. $Q_{\text{total}} = Q_i(A) + Q_i(B) = +5 + (-7) = -2 \text{ C}$
4. $Q_{\text{new on each}} = Q_{\text{total}}^{\frac{1}{2}} = \frac{-2}{2} = -1 \text{ C} = Q_f$
5. $\Delta Q_A = Q_f - Q_i = -1 - (+5) = -6 \text{ C} \therefore 6 \text{ C charge was transferred from B to A}$
 $\Delta Q_B = Q_f - Q_i = -1 - (-7) = +6 \text{ C} \therefore 6 \text{ C charge was transferred from B to A}$
6. No. of e^- transferred $\frac{\Delta Q(A)}{Qe^-} = \text{No. of } e^- \text{ transferred} = \frac{\Delta Q(B)}{Qe^-}$
 $= \frac{6}{1.6 \times 10^{-19}} \text{ or } = \frac{6}{1.6 \times 10^{-19}}$
 $= 3.75 \times 10^{19} \text{ electrons}$

6.3 Coulomb's Law

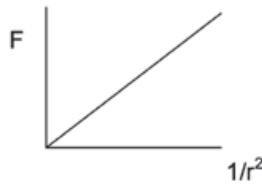
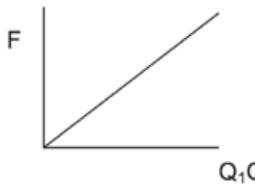
Coulomb's Law is the electrostatic force of attraction or repulsion between two charged objects is directly proportional to the product of the charges and inversely proportional to the square of the distance between their centres.



For any two charges, where F is electrostatic force (N), Q is charge (C), r is the distance between the centres of the objects (m), and k is Coulomb's constant ($9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$)

$$F \propto Q_1 \cdot Q_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

$$\therefore F = \frac{kQ_1Q_2}{r^2}$$



6.3.1 Using Coulomb's Law

We apply Coulomb's Law to determine how the electrostatic force between two charged objects (or point charges) changes when the charge on one or both of the objects changes and when the distance between their centres changes.

We can also use Coulomb's Law to calculate the electrostatic force between two charges, the distance between them, or the magnitudes (sizes) of the charges.

e.g.

Worked example 2

The original force between two charges is F . If both charges are doubled and the distance is a third of the original distance, what is the magnitude of the new force relative to the original force?

Solution

$$F_{\text{original}} = \frac{kQ_1Q_2}{r^2}$$

$$F_{\text{new}} = \frac{k(2Q_1)(2Q_2)}{\left(\frac{1}{3}r\right)^2}$$

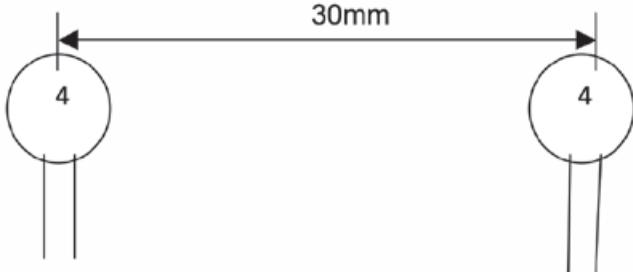
$$F_{\text{new}} = \frac{4kQ_1Q_2}{\frac{1}{9}r^2} = \frac{36kQ_1Q_2}{r^2} = 36 F_{\text{original}}$$



Remember to square the distance.



e.g.

Worked example 3

Two small identical metal spheres carry equal but opposite charges. If their centres are 30mm apart, and the electrostatic force between them is 2.56×10^{-3} N. Calculate:

1. The magnitude (size) of the charge on each sphere
2. The number of electrons that would flow from the negatively charged sphere to the positively charged sphere if they were brought into contact.

Solutions

1. $F = \frac{kQ_1Q_2}{r^2}$

$$2.56 \times 10^{-3} \times 0.0009 (9 \times 10^{-4}) = 9 \times 10^9 \times Q \times Q$$

$$2.304 \times 10^{-6} / (9 \times 10^9) = Q^2$$

$$Q^2 = \sqrt{2.304 \times 10^{-16}} ; Q = 1.6 \times 10^{-8}\text{C}$$

2. number of electrons = $\frac{\text{total charge}}{\text{charge on one electron}}$

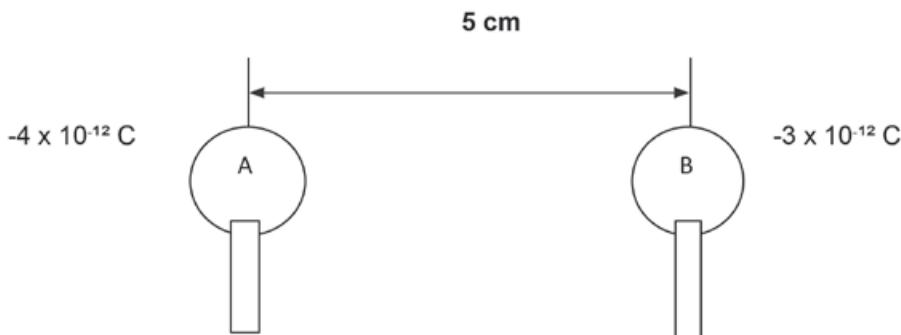
$$= (1.6 \times 10^{-8}) / (1.6 \times 10^{-19})$$

$$= 1 \times 10^{11} \text{ electrons}$$



Activity 1

Two small identical metal spheres, A and B carrying charge of $-4 \times 10^{-12}\text{C}$ and $-3 \times 10^{-12}\text{C}$ respectively, are mounted on insulated stands as shown. The distance between the centres of the spheres is 5 cm.



1. Calculate the magnitude and direction of the force that A exerts on B. (6)
Sphere A is moved and makes contact with sphere B. It is then moved back to its original position.
 2. Calculate the new charge on each of the spheres. (3)
 3. How does the magnitude of the force that the sphere A exerts on sphere B change? Answer by writing ONLY increases, decreases or remains the same. (2)
- [11]

Solutions

1.
$$F = \frac{kQ_1Q_2}{r^2} \checkmark$$

$$= \frac{(9 \times 10^9) \times (-4 \times 10^{-12}) \times (3 \times 10^{-12})}{(0,05)^2} \checkmark$$

$$= \frac{-1,08 \times 10^{-13}}{0,0025}$$

$$= -4,32 \times 10^{-11} \text{ N} \checkmark$$

$$= 4,32 \times 10^{-11} \text{ N} \checkmark \text{ Force of attraction} \checkmark \quad (6)$$



Start by converting the distance 5cm to m. Look at the conversion table.

2.
$$Q = \frac{(Q_1 + Q_2)}{2} \checkmark$$

$$= \frac{(-4 \times 10^{-12}) + (3 \times 10^{-12})}{2} \checkmark$$

$$= -5 \times 10^{-13} \text{ C} \checkmark$$

This is a new charge (3)

3. Increases $\checkmark \checkmark$ (2)

[11]

**DEFINITION**

An **electric field** is a region or space in which an electric charge experiences an electric force.

6.4 Electric fields around charged objects

- **Electric fields** are represented by field lines as illustrated in the diagrams below.
- **Electric field** is a **vector quantity**.
- An **electric field line** indicates the direction in which a positive test charge would move if placed at a point in the electric field.

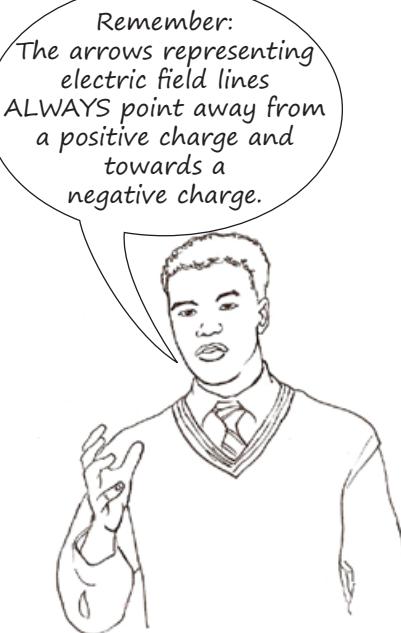
6.4.1 Properties of electric field lines

Electric field lines:

- start and end perpendicular to the surface of a charged object
- never cross each other
- are closer where the electric field is stronger
- are directed from positive to negative

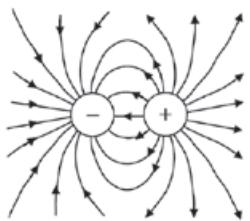
6.4.2 Representing electric fields

You must be able to draw simple diagrams to show the electric fields around charged objects.

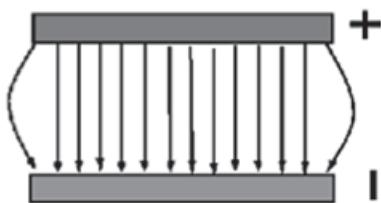


a) Around a positive point charge	b) Around a negative point charge
c) Between two like point charges of equal magnitude (both positive or both negative)	d) Between two like point charges that are not equal in magnitude
 If the point charges are negative, the arrows point inwards, towards the point charges.	 The field lines are closer together when the electric field is stronger (around the greater charge).

- e) Between two unlike (opposite) point charges (one positive and the other negative)



- f) Between two oppositely charged (one positive and one negative) parallel plates



This electric field is uniform – it is equally strong everywhere between the two plates, so the electric field lines are equally spaced and parallel.



Step by step

- Step 1.** ALWAYS calculate the electric field strength at the given point (P in this case) due to each of the point charges first. The negative sign for a negative charge is NOT used in this equation.
- Step 2.** Then choose an electric field direction as positive and state this clearly.
- Step 3.** Then find the resultant (or net) electric field strength by adding the two field strength values. Lastly, remember the signs for the directions!

6.5 Electric field strength



Formula

The electric field strength at a point is the electric force per unit positive charge experienced at a point in an electric field.

For any charge:

$$E = \frac{F}{Q} \text{ and } E = \frac{kQ}{r^2}$$

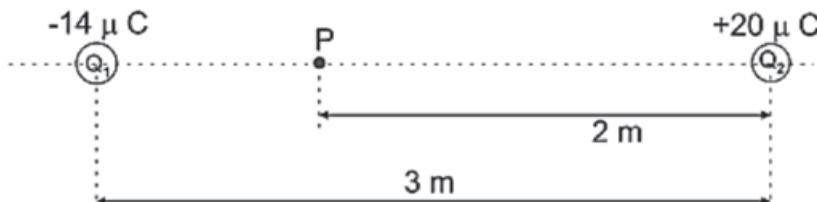
e.g.

Worked example 4

Two point charges, Q_1 and Q_2 , at a distance of 3 m apart, are shown below. The charge on Q_1 is $-14 \mu\text{C}$ and the charge on Q_2 is $+20 \mu\text{C}$.

REMEMBER:

First convert μC to C : $-14 \mu\text{C} = -14 \times 10^{-6} \text{ C}$ and $20 \mu\text{C} = 20 \times 10^{-6} \text{ C}$



- Define the electric field strength at a point.
- Calculate the net (resultant) electric field at point P situated 2 m from Q_2 .

Solutions

a) Electric field strength at a point is the electric force per unit positive charge experienced at the point.

b) Electric field at P due to Q_1 :

$$E = \frac{kQ}{r^2} = \frac{(9 \times 10^9)(14 \times 10^{-6})}{1^2} = 1,26 \times 10^5 \text{ N}\cdot\text{C}^{-1} \text{ to the left}$$

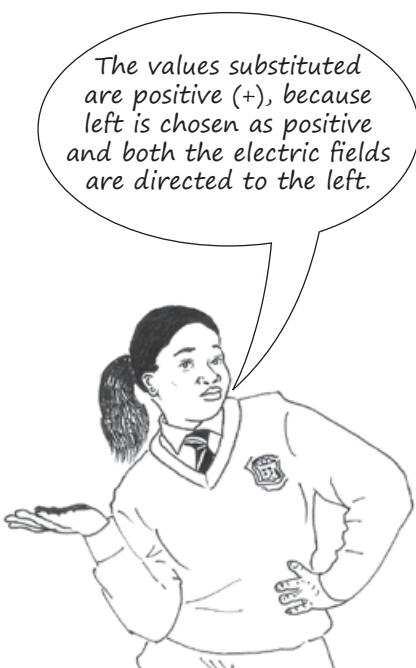
Electric field at P due to Q_2 :

$$E = \frac{kQ}{r^2} = \frac{(9 \times 10^9)(20 \times 10^{-6})}{1^2} = 4,5 \times 10^4 \text{ N}\cdot\text{C}^{-1} \text{ to the left}$$

Let $\leftarrow E^+$

$$\begin{aligned} E_{\text{net}} &= E_{Q_1} + E_{Q_2} \\ &= (+1,26 \times 10^5) + (+4,5 \times 10^4 \text{ N}\cdot\text{C}^{-1}) \\ &= +1,71 \times 10^5 \text{ N}\cdot\text{C}^{-1} \\ \therefore & 1,71 \times 10^5 \text{ N}\cdot\text{C}^{-1} \text{ to the left} \end{aligned}$$

The values substituted are positive (+), because left is chosen as positive and both the electric fields are directed to the left.

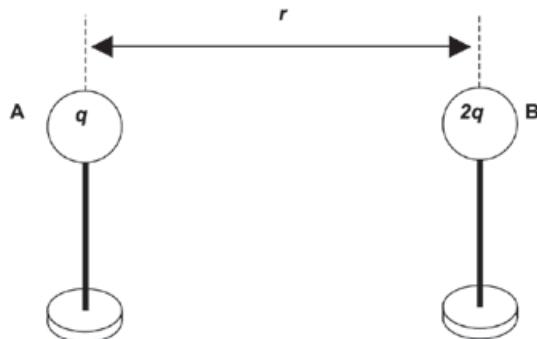




Activity 2

Multiple Choice Questions:

1. The sketch below shows two small metal spheres, A and B, on insulated stands carrying charges of magnitude q and $2q$ respectively. The distance between the centres of the two spheres is r .



Sphere A exerts a force of magnitude F on sphere B. What is the magnitude of the force that sphere B exerts on sphere A?

- A $\frac{1}{2}F$
 - B F
 - C $2F$
 - D $4F$
- (2)

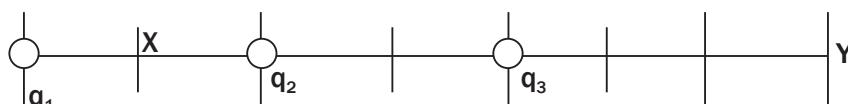
2. Two identical small metal spheres on insulated stands carry equal charges and are a distance d apart. Each sphere experiences an electrostatic force of magnitude F .

The spheres are now placed a distance $\frac{1}{2}d$ apart.

The magnitude of the electrostatic force each spheres now experience is..

- A $\frac{1}{2}F$
 - B $F\frac{1}{2}$
 - C $2\frac{1}{2}F$
 - D $4F$
- (2)

3. Three identical point charges, q_1 , q_2 and q_3 , are placed in a straight line, as shown below. Point charge q_2 is placed midway between point charges q_1 and q_3 . X and Y are two points on the straight line as shown.



Which ONE of the following best describes how the electric field E at a point X compares to that at point Y?

	DIRECTION OF E	MAGNITUDE OF E
A	Same	$E_x > E_y$
B	Same	$E_x < E_y$
C	Opposite	$E_x > E_y$
D	Opposite	$E_x < E_y$

(2)
[6]

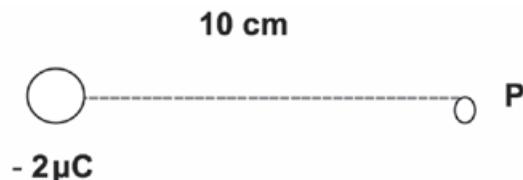
Solutions

1. B ✓✓ (2)
 2. D ✓✓ (2)
 3. D ✓✓ (2)
- [6]

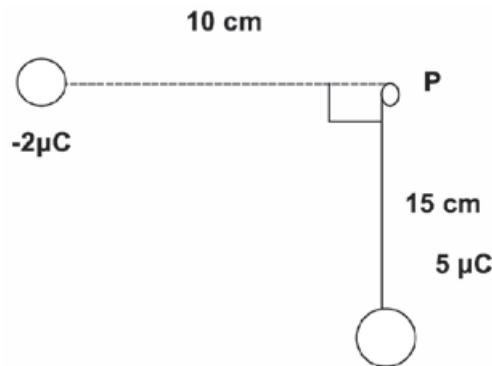


Activity 3

A negative charge of $2 \mu\text{C}$ is positioned 10 cm from point P, as shown below.



1. Define the electric field at point P in words. (2)
2. Draw the electric field lines associated with this charge. (2)
3. A positive charge of $5 \mu\text{C}$ is now positioned 15 cm from point P, as showed in the diagram below. (12)



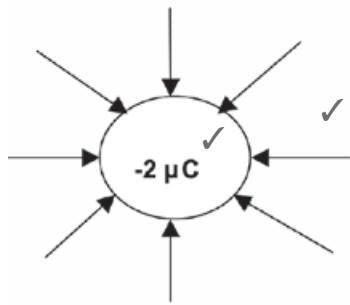
Calculate the magnitude of the electric field at point P due to both charges.

(12)
[16]

Solutions

1. The force per unit charge. (2)

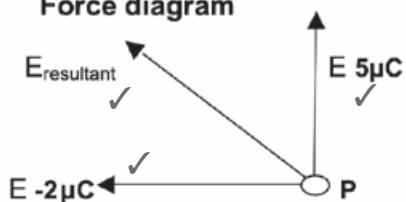
2.

**Marking criteria**

Shape of field lines	✓
Direction of field lines (towards charge)	✓

(2)

3. **Force diagram**



$$\begin{aligned} E_{2 \mu C} &= \frac{kQ}{r^2} \quad \checkmark \\ &= \frac{(9 \times 10^9)(2 \times 10^{-6})}{(0,1)^2} \quad \checkmark \\ &= 1,8 \times 10^6 \text{ N}\cdot\text{C}^{-1} \quad \checkmark \text{ towards the } 2 \mu\text{C} \quad \checkmark \end{aligned}$$

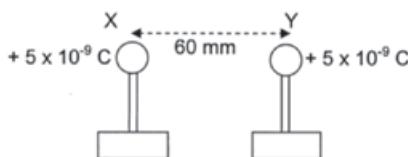
$$\begin{aligned} E_{5 \mu C} &= \frac{kQ}{r^2} \\ &= \frac{(9 \times 10^9)(5 \times 10^{-6})}{(0,15)^2} \quad \checkmark \\ &= 2 \times 10^6 \text{ N}\cdot\text{C}^{-1} \quad \checkmark \text{ away from the } 5 \mu\text{C} \text{ charge} \end{aligned}$$

$$\begin{aligned} E_{\text{resultant}} &= \sqrt{(1,8 \times 10^6)^2 + (2 \times 10^6)^2} \quad \checkmark \text{ pythagoras} \quad (12) \\ &= 2,69 \times 10^6 \text{ N}\cdot\text{C}^{-1} \quad \checkmark \end{aligned}$$

[16]



Activity 4



The centres of two small, charged conducting spheres, X and Y, on insulated stands, are separated by a distance of 60 mm. Sphere X initially carries a charge of $+12 \times 10^{-9}$ C.

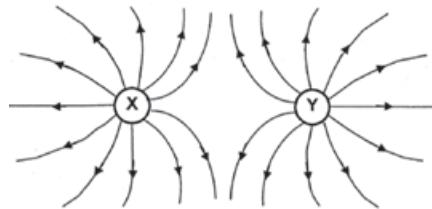
X and Y are brought into contact with each other and are separated again. After separation, each sphere carries EQUAL charges of $+5 \times 10^{-9}$ C.

1. Draw a neat diagram of the resultant electric field pattern that surrounds X and Y. (4)
2. Calculate the number of electrons that must be added to Y to make it neutral. (3)
3. Calculate the magnitude of the force which X exerts on Y once they are back in their original positions (after separation). (4)
4. Calculate the original charge on sphere Y. (4)

[15]

Solutions

1. The field is curved; lines on the outside are important.



Marks for: field lines between charges (✓); field lines outside the charges (✓); direction: away from X and Y (✓); field lines not going into spheres but touching surface and not touching each other. (✓) (4)

2. Number of electrons on Y = $\frac{Q}{q_e}$
 $= \frac{-5 \times 10^{-9}}{-1,6 \times 10^{-19}}$ (✓)
 $= 3,125 \times 10^{10}$ electrons (✓) (3)

3. Force $F = \frac{kq_1q_2}{r^2}$ (✓)
 $= \frac{(9 \times 10^9)(5 \times 10^{-9})(5 \times 10^{-9})}{(0,06)^2}$ (✓)
 $= 6,25 \times 10^{-5}$ N (✓)

If you forget to convert mm to metres, or forget to square the distance between the two (r^2) you will lose a mark. (4)

4. Total charge $Q = \frac{(Q_1 + Q_2)}{2}$ ✓
 $Q = \frac{(12 \times 10^{-9}) Q_Y}{2}$ (where Q_Y is the charge on sphere Y)
 $2(5 \times 10^{-9} \text{ C}) = (12 \times 10^{-9}) Q_Y$
 $\frac{2(5 \times 10^{-9} \text{ C})}{12 \times 10^{-9}}$ ✓ $= Q_Y = 8,3 \times 10^{-17} \text{ C}$ ✓ (4)

[15]



Electric circuits

Summary

You must remember:

- For a current to flow, we need a source of electrical energy (cell or battery) and a closed circuit (or at least a magnetic field moving near a conductor).
- The direction of the **conventional current** is from the positive pole or terminal of the cell through the circuit to the negative pole or terminal of the cell. This flow in one direction is called **Direct Current**.
- The **potential difference** between two points in a conductor is the work done per unit charge to move a positive charge from one point to another. Potential difference is measured in volts (V) with a voltmeter which is connected in **parallel** in a circuit.



$$\text{Potential difference} = \frac{\text{work done}}{\text{charge}}$$

$$V = \frac{W}{Q}$$

$$E = \frac{W}{Q}$$

Electric current is the amount of charge per second which flows past a point. It is measured in **amperes (A)** with an ammeter which is connected in **series** in a circuit.

$$I = \frac{Q}{t}$$

Resistance is a measure of how much a conductor opposes the flow of charge through it. It is measured in **Ohms (Ω)**. A **resistor** is a component in a circuit that resists (opposes) the flow of current.



7.1 Factors influencing the resistance of a wire conductor

- The **length** of the resistor (the longer the wire, the greater the resistance)
- The **diameter** (thickness) of the resistor (the thinner the wire, the greater the resistance)
- The **temperature** of the resistor (the higher the temperature, the greater the resistance)
- The **type** of material that the resistor is made from. Different substances have different resistances, e.g. tungsten (W) has a very high resistance but copper (Cu) has a very low resistance.

7.2 Ohm's Law

- Potential difference across a conductor is directly proportional to the current in the conductor at constant temperature.
- The mathematical formula of Ohm's Law

$$R = \frac{V}{I}$$



DEFINITION OF THE OHM:

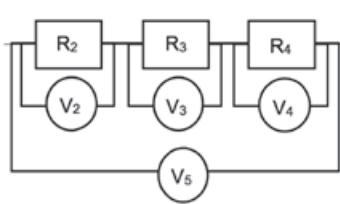
A conductor has a resistance of **1 ohm (1 Ω)** if the potential difference of **1 volt (1 V)** applied across its ends, causes a current of **1 ampere (1 A)** to flow through it.

7.2.1 Circuit connection

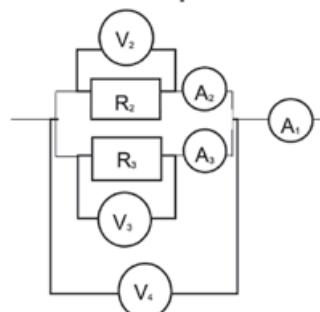
Circuits can be connected into two ways: Series and Parallel. A series circuit requires the electricity to travel one path that does not split. A parallel circuit allows the electricity to go down different paths (there's a split in the circuit). See the diagrams below showing resistors in series and parallel for an illustration.

Resistors in a circuit

Resistors in series



Resistors in parallel



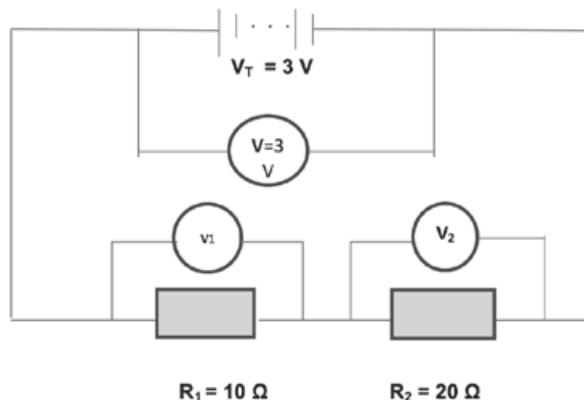
- Resistors are connected in series or in parallel.
- Resistors, rheostats and light bulbs resist the flow of current.

<ul style="list-style-type: none"> • Resistors are connected in series or in parallel. • Resistors, rheostats and light bulbs resist the flow of current. 	<ul style="list-style-type: none"> • Work is done in a resistor when the electric energy is transformed to heat energy or to light energy.
<ul style="list-style-type: none"> • Resistors in series are potential (voltage) dividers. • The total resistance in a circuit increases when <i>more resistors</i> are added in series. • The current through all resistors in series is the same. • The total current in a circuit decreases when <i>more resistors</i> are added in series. • If one resistor burns out, the circuit is broken and no current flows (in a series circuit). 	<ul style="list-style-type: none"> • The total current in a circuit increases when <i>more resistors</i> are added in parallel. • If one resistor burns out, current still flows through the other resistor (in a parallel circuit). • Resistors in parallel are current (amperage) dividers. • The total resistance in a circuit decreases when <i>more resistors</i> are added in parallel.

7.2.2 Comparison between series and parallel circuits

Series connection	Parallel connection
$I_1 = I_2 = I_3\dots$ The total current across the series is the same.	$V_1 = V_2 = V_3\dots$ The total voltage across the parallel component is the same.
$V_T = V_1 + V_2 + V_3\dots$ This is a potential divider.	$I_T = I_1 + I_2 + I_3\dots$ This is a current divider.
$R_T = r_1 + r_2 + r_3\dots$ Addition of resistors.	$\frac{1}{R_T} = \frac{1}{r_1} + \frac{1}{r_2} + \dots$ Addition of the ratio of resistors.

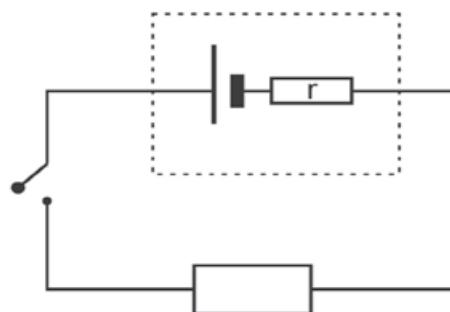
7.3 Voltage (Potential Difference) and Electromotive Force (emf)



- Voltage in which the charge is **losing energy** is a **potential difference**, V .
- Voltage in which the charge is **gaining** energy is an **electromotive force (emf)**, ϵ
- Therefore the voltage across the battery is an electromotive force (emf), while the voltage across each resistor is potential difference (p.d.).

7.4 Internal Resistance

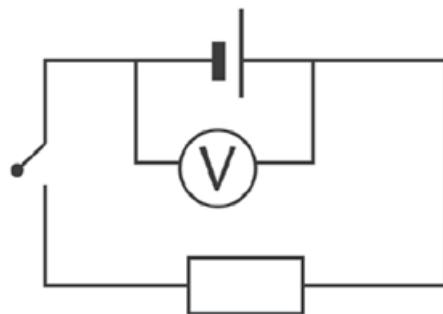
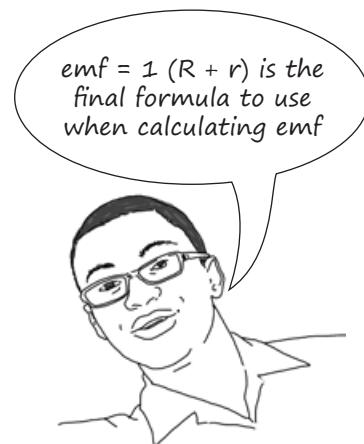
- When the **switch is open**, the voltmeter reads the emf of the battery which is 12,5 V. This means the battery can transfer 12,5 J of energy for every 1 C of charge.
- When the **switch is closed**, the voltmeter reads the p.d of the external circuit.
- **Internal resistance** is found inside the cell or battery, which is the small amount of energy that is used up inside the cell or battery.
- An ideal cell would have zero internal resistance, $r = 0 \Omega$.
- The volts used inside the cell are referred to as the **lost volts**, V' .



7.4.1 emf

The emf (ϵ) of a cell is:

- the electrical potential difference across the terminals (poles) of a cell while no current flows
- the total amount of electric energy supplied by the cell per coulomb of charge
- measured with a voltmeter connected in parallel over a cell or battery when no current flows (the switch is open)(see diagram below)
- measured in volts (V).

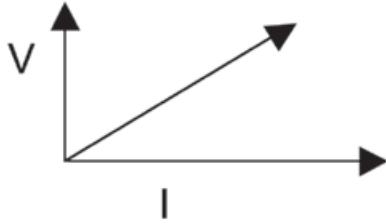
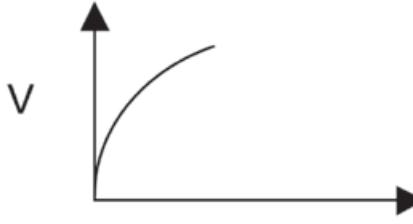


NB Formula to calculate the emf

Derivation: since $R = V/I$, $V = IR$, so if $\text{emf} = V + V'$ (the sum of the two voltage measurements), then $\text{emf} = IR + IR'$. For convenience, we write IR' as Ir . Now, since I is common to both factors IR and Ir , we can take it out as a multiplier, like so: $I(R+r)$. Hence, $\text{emf} = I(R+r)$.

$$\begin{aligned}\text{given: } \text{emf} &= V + V' \\ \therefore \text{emf} &= IR + Ir \\ \therefore \text{emf} &= I(R + r)\end{aligned}$$

7.4.2 Ohmic and non-ohmic conductors: differences and examples

	Ohmic conductor	Non-ohmic conductor
Obeys Ohm's Law or not?	Obeys Ohm's Law when voltage or current is varied	Does not obey Ohm's Law when voltage or current is varied
Graph of the voltage vs. the current across conductors	 Shape: Straight line from the origin $V / I = R$	 Shape: Curve $V / I \neq R$
Examples	circuit resistors nickel wire.	light bulb diodes transistors

7.5 Electric energy

When current flows or charges move through a resistor:

- electric energy is transferred from the moving charges to the particles in the resistor
- the particles in the resistor gain kinetic energy
- the temperature of the resistor increases as the kinetic energy of the particles increases.

Formulas

The work done (W) is equal to the energy (E) transferred.

$$W = E \text{ (in joule J)}$$

$$W = VQ = Vit = I2Rt = \frac{V^2t}{R}$$

W:	work done	(J) joule
V:	potential difference	(V) volt
I:	current	(A) ampere
R:	resistance	(Ω) ohm
t:	time	(s) seconds

If you are asked to calculate the amount of energy transferred, use the correct formula to calculate the work done

7.6 Power



DEFINITION

Power is the rate at which work is done or energy is transferred.

$$P = \frac{W}{\Delta t} = VI = I^2R = \frac{V^2}{R}$$

P:	power	(W) watt
W:	work done	(J) joule
V:	potential difference	(V) volt
I:	current	(A) ampere
R:	resistance	(Ω) ohm
t:	time	(s) seconds

7.6.1 The brightness of light bulbs

The brightness of a light bulb is determined by the rate at which energy is transformed in the bulb, that is, by the power (P).

- As the power increases, the brightness increases.
- For bulbs in series:

$P \propto R$ (power is directly proportional to the resistance of the bulb)

∴ if the resistance increases, the power increases and

∴ the brightness increases.

- For bulbs in parallel:

$P \propto \frac{1}{R}$ (power is inversely proportional to the resistance of the bulb)
 \therefore if the resistance decreases, the power increases and
 \therefore the brightness increases.

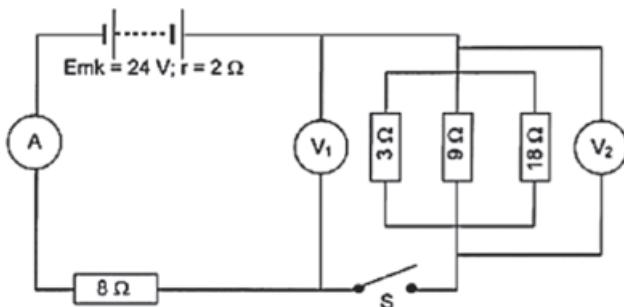
Note: the above applies to incandescent bulbs only (the type with a filament of wire).

Compact Fluorescent Lights (CFL) come in a range of power/wattages but they will simply blow if overpowered by a significant increase in voltage/amperage.



Worked example 1

In this circuit the battery has an emf of 24 V and an internal resistance of 2Ω . Voltmeter V_1 is connected as shown and voltmeter V_2 is connected over the three resistors in parallel. The resistance of the connectors and of the ammeter may be ignored.



- Switch S is open.
 - What is the reading on V_1 ?
 - What is the reading on V_2 ?
- Switch S is now closed. Calculate:
 - the total resistance in the circuit.
 - the current that flows through the 8Ω resistor.
 - the charge that flows past a cross section of the 8Ω resistor in one minute.

Solutions

1a) $V_1 = 24\text{ V}$ (when S is open, no current flows and V_1 is connected across the battery)

1b) $V_2 = 0\text{ V}$ (S is open, no current flows, V is connected on one side of the switch, \therefore only to one pole of the battery)

$$\frac{1}{R_{II}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{9} + \frac{1}{18} = \frac{6+2+1}{18} = \frac{9}{18}$$

$$\therefore R_{II} = 18/9 = 2\Omega$$

Solutions

2a) $R_{\text{external}} = R_{\text{series}} + R_{\parallel} = 8 + 2 = 10 \Omega$ and

$$R_{\text{external}} = R_{\text{ext}} + r_{\text{int}} = 10 + 2 = 12 \Omega$$

2b) The 8Ω resistor is connected in series \therefore the total current flows through it

\therefore calculate the total current (I) that flows through the circuit.

$$R_{\text{tot}} = \frac{V_{\text{tot}}}{I_{\text{tot}}}$$

$$\therefore 12 = \frac{24}{I_{\text{tot}}}$$

$$\therefore I_{\text{tot}} = \frac{24}{12} = 2 \text{ A}$$

2c) $I = \frac{Q}{\Delta t}$

$$\therefore 2 = \frac{Q}{(1)(60)}$$

$$\therefore Q = (2)(1)(60) = 120 \text{ C}$$

This is a current that flows through all the circuits.



Remember: Convert minutes to seconds: $\times 60$ or hours to seconds: $\times 3600$
 Remember: Current is the rate at which electric charge passes a fixed point in a conductor.

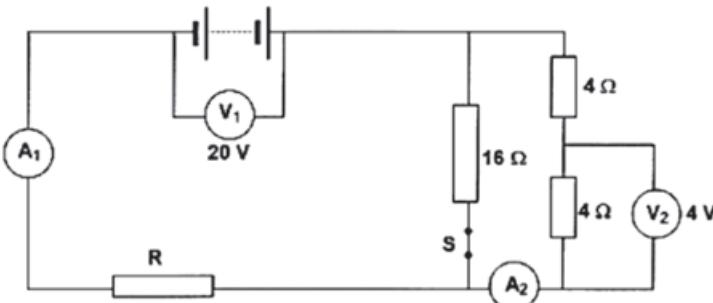
$$I \propto Q$$

$$I = \frac{Q}{\Delta t}$$



Activity 1

In the circuit below the battery has an emf of 24 V and an unknown internal resistance. Voltmeter V_1 is connected across the battery. The resistance of the connectors and of the ammeter may be ignored. When switch S is closed, voltmeter V_2 reads 4 V and voltmeter V_1 reads 20 V.



1. Calculate:
 - a) The reading on ammeter A_2 . (2)
 - b) The reading on ammeter A_1 . (5)
 - c) The resistance of resistor R . (4)
 - d) The internal resistance of the battery. (3)
 - e) The energy converted in resistor R in 10 minutes. (3)
2. Switch S is now opened. Will the reading on voltmeter V_1 increase, decrease or remain constant? Explain. (6)

[23]

Solutions

1a. Ammeter A_2 reads the current that flows through the 4Ω resistor.

$$R_{4\Omega} = \frac{V_{4\Omega}}{I_{4\Omega}} \checkmark \therefore I_{4\Omega} = \frac{4}{4} = 1 \text{ A } \checkmark \quad (2)$$

1b. The resistance of the 16Ω resistor is **DOUBLE** the resistance of the $(4 + 4) = 8 \Omega$ resistors
 \therefore the current that flows through the 16Ω resistor is **HALF** the current that flows through the $(4 + 4) = 8 \Omega$ resistor.
 1 A flows through the $(4 + 4) = 8 \Omega$ resistor
 $\therefore 0,5 \text{ A}$ flows through the 16Ω resistor \checkmark

$$\therefore I_{\text{total}} = 1 + 0,5 = 1,5 \text{ A } \checkmark \quad (5)$$

1c. $\frac{1}{R_{\parallel}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{16} + \frac{1}{(4+4)} = 1 + \frac{2}{16} = \frac{3}{16} \checkmark$

$$R_{\parallel} = \frac{16}{3} = 5,33 \Omega \checkmark$$

$$\text{and } R_{\text{ext}} = \frac{V_{\text{ext}}}{I_{\text{ext}}} \checkmark \therefore R_{\text{ext}} = \frac{20}{1,5} = 13,33 \Omega \checkmark \quad (4)$$

1d. $\text{emf} = V_{\text{tot}} = V_{\text{external}} + V_{\text{internal}} \checkmark \therefore 24 = 20 + V_{\text{internal}}$
 $\therefore V_{\text{internal}} = 24 - 20 = 4 \text{ V } \checkmark \quad (3)$

1e. Energy transferred (or transformed) = E = work done = W

$$W = I^2 R \Delta t \checkmark = (1,5)^2 (8)(10)(60) \checkmark = 10 800 \text{ J } \checkmark \quad (3)$$

2. When current flows, V_1 reads the external potential difference: when S is opened there are fewer resistors in parallel \checkmark

$\therefore R_{\text{external}} \text{ increases } \checkmark \therefore I_{\text{total}} \text{ decreases } \checkmark \therefore V_{\text{int}} \text{ decreases } \checkmark$
 $\text{and emf is constant } \therefore V_{\text{external}} \text{ increases } \checkmark \quad (6)$

[23]

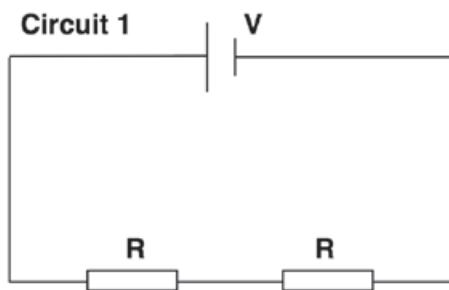


Activity 2

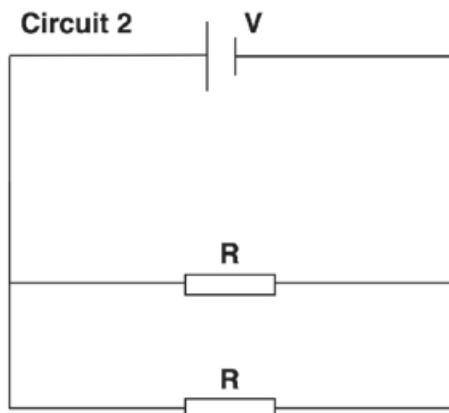
Multiple Choice questions:

1. Which ONE of the following is the unit of measurement for the rate of flow of charge?
 A watt
 B coulomb
 C volt
 D ampere (2)

2. The two resistors in circuit 1 below are identical. They are connected in series to a cell of emf V and negligible internal resistance. The power dissipated by each resistor is P .



The two resistors are now connected in parallel, as shown in circuit 2 below.



The power dissipated by each resistor in the circuit 2 is...

- A $2P$
 - B $4P$
 - C $8P$
 - D $16P$ (2)
- [4]**

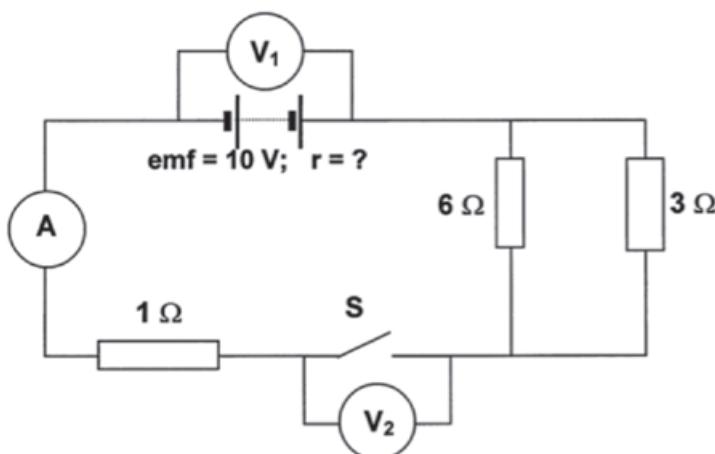
Solutions

- | | |
|----------|------------|
| 1. D ✓ ✓ | (2) |
| 2. B ✓ ✓ | (2) |
| | [4] |



Activity 3

In the circuit represented below, the battery has an emf of 10 V and an unknown internal resistance. Voltmeter V_1 is connected across the battery and voltmeter V_2 is connected across the open switch S. The resistance of the connecting wires and ammeter can be ignored.



Switch S is open

1. What is the reading on V_1 ? (2)
2. What is the reading on V_2 ? (2)

When Switch S is closed, the reading on V_1 drops to 7,5 V.

3. What is the reading on V_2 ? (2)
4. Calculate the reading on the ammeter. (8)
5. Calculate the internal resistance of the battery. (5)

[19]

Solutions

1. 10 V ✓✓ (2)

2. 10 V ✓✓ (2)

3. Zero or 0 V ✓✓ (2)

4. $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$ ✓
 $\frac{1}{R_p} = \frac{1}{6} (\checkmark) + \frac{1}{3} (\checkmark) = \frac{1}{2}$

$R_p = 2 \Omega (\checkmark)$

Now use 2 Ω in the next calculation...

OR $R_p = \frac{\text{product}}{\text{sum}} (\checkmark)$
 $R_p = \frac{(6 \times 3)}{(6 + 3)}$
 $R_p = 2 \Omega (\checkmark)$

$R_{\text{ext}} = 2 \Omega + 1 \Omega = 3 \Omega$

$I = \frac{V}{R}$ ✓

= 7,5 (\checkmark) / 3 (\checkmark)

= 2,5 A (\checkmark)

(8)

5. $\text{emf} = V_{\text{circ}} + V_{\text{lost}}$ ✓

$10 (\checkmark) = (7,5 + 2,5)r$ ✓✓

$r = 1 \Omega (\checkmark)$

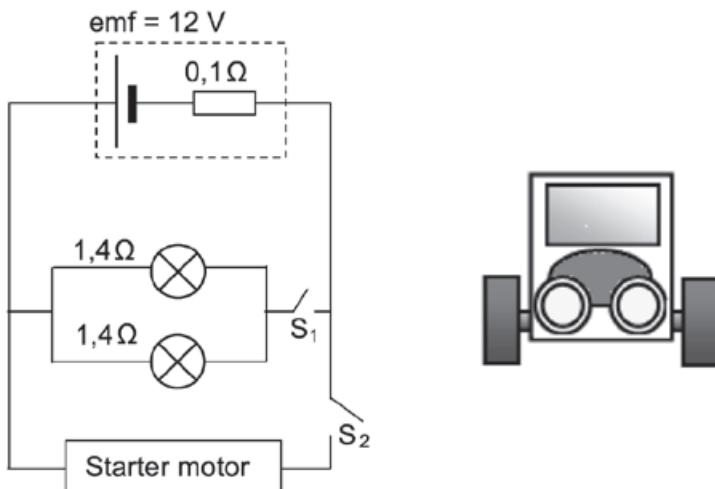
(5)

[19]



Activity 4

The headlights of a car are connected in parallel to a 12 V battery, as shown in the simplified circuit diagram below. The internal resistance of the battery is $0,1\Omega$ and each headlight has a resistance of $1,4\Omega$. The starter motor is connected in parallel with the headlights and controlled by the ignition switch, S_2 . The resistance of the connecting wires may be ignored.



1. State Ohm's Law in words. (2)
2. With only switch S_1 closed, calculate the following:
 - a) Effective resistance of the two headlights (3)
 - b) Potential difference across the two headlights (4)
 - c) Power dissipated by one of the headlights (3)
3. Ignition switch S_2 is now closed (whilst S_1 is also closed) for a short time and the starter motor, with VERY LOW RESISTANCE, rotates.
How will the brightness of the headlights be affected while switch S_2 is closed? Write down only INCREASES, DECREASES or REMAINS THE SAME.
Fully explain how you arrived at the answer. (9)

[21]

Solutions

1. The current in a conductor is directly proportional to the potential difference across its ends at constant temperature. ✓✓

OR

The ratio of potential difference to current is constant at constant temperature (2)

$$\begin{aligned} 2.a) \quad \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_{21}} \quad \checkmark \\ &= \frac{1}{1,4} + \frac{1}{1,4} \quad \checkmark \\ &= 0,7 \Omega \quad \checkmark \end{aligned} \quad (3)$$

$$\begin{aligned} 2.b) \quad emf &= I(R + r) \quad \checkmark \\ 12 &= I(0,7 + 0,1) \quad \checkmark \\ I &= 15 \text{ A} \\ R &= \frac{V}{I} \\ V &= 0,7 \times 15 \quad \checkmark = 10,5 \text{ V} \quad \checkmark \end{aligned} \quad (4)$$

$$\begin{aligned} 2.c) \quad I(\text{light}) &= 7,5 \text{ A} \text{ (from 2b: 2 headlights = 15 A)} \quad \checkmark \\ P &= VI \\ &= (10,5)(7,5) \quad \checkmark \\ &= 78,75 \text{ W} \quad \checkmark \end{aligned} \quad (3)$$

3. Decreases ✓
 (Effective/total) resistance decreases. ✓
 (Total) current increases. ✓
 “Lost volts” / V_{internal} / Ir increases, thus potential difference / V (across headlights) decreases. ✓
 $P = V / R$ decreases. ✓

$$\begin{aligned} P &= W / \Delta t \quad \checkmark \\ 60 &= W / 2(60) \\ W &= 7200 \text{ J} \quad \checkmark \\ \text{Starter motor } W &= Vq \quad \checkmark \\ q &= 7200 / 12 \\ q &= 600 \text{ C} \quad \checkmark \end{aligned} \quad (9)$$

[21]



Electrodynamics: electrical machines (generators and motors)

Summary

- Definitions
- Faraday's law of electromagnetic induction
- Differences between motors and generators.
- Operations of motors and generators.
- Differences between direct current (DC) and alternating current (AC) in the cases of both motors and generators.
- The graphs of AC and DC.
- Right hand rule to determine the direction of the force on the conductor.
- The use of motors in everyday life.
- Calculations of the Root Mean Square.



DEFINITIONS AND LAWS YOU MUST REMEMBER

Magnetic flux (Φ) is the product of the strength of a magnetic field and the surface area the field cuts perpendicularly. It is measured in Wb (weber) units.

Electromagnetic induction is when a magnet moves relative to a conductor, and the magnet's magnetic field is at right angles to the conductor, the maximum **electric current** is induced in the conductor.

• Faraday's Law of Electromagnetic Induction

- The induced emf in a conductor is directly proportional to the rate of change of the magnetic flux in the conductor.
- So $\varepsilon \propto \frac{\Delta\Phi}{\Delta t}$
- $\varepsilon = - N \frac{\Delta\Phi}{\Delta t}$ where

N is the number of turns in the coil

ε is emf in (V) volts

$\Delta\Phi$ is change in magnetic flux in (Wb) weber

Δt is change in time in (s) seconds

- The negative sign shows that the emf creates a current and a magnetic field B that opposes the change in the magnetic flux Φ .

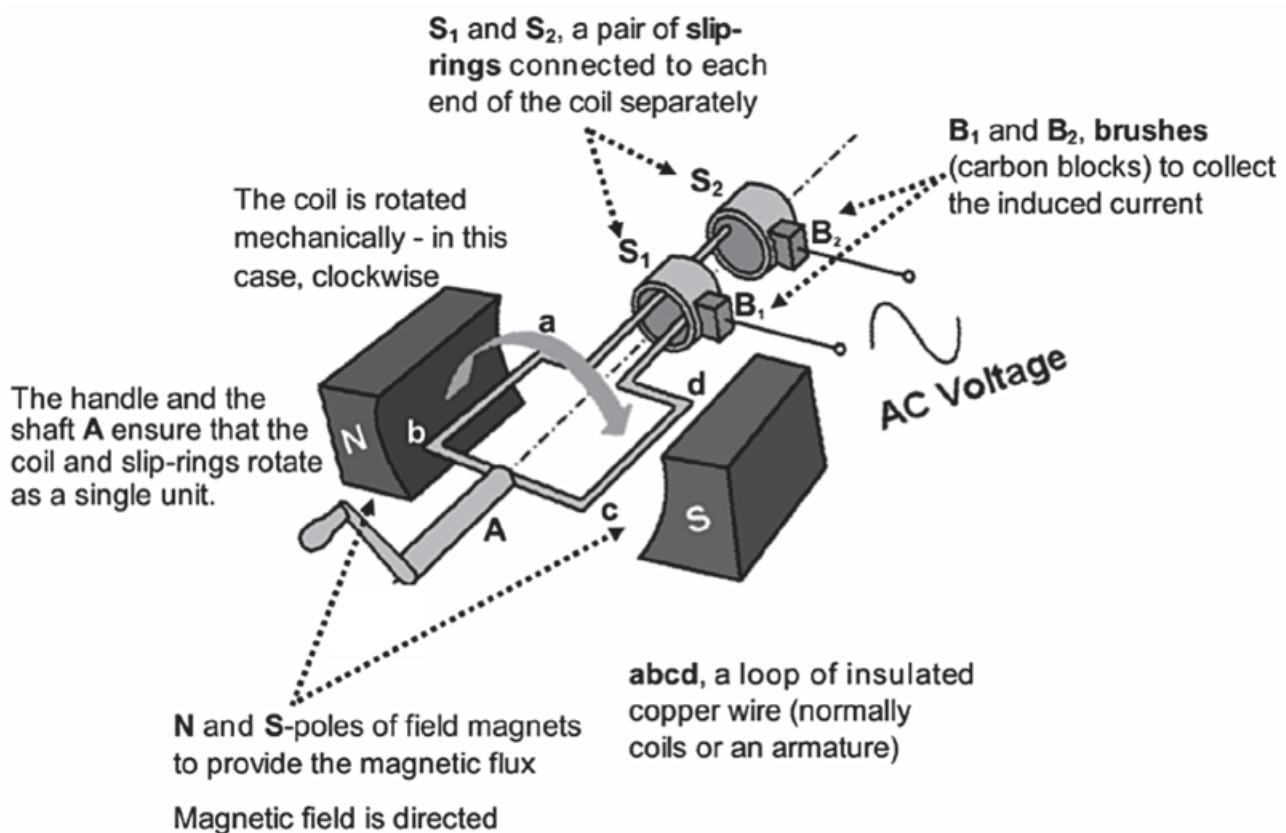
8.1 Motors and generators

8.1.1. Alternating current generators

The principle of the AC generator

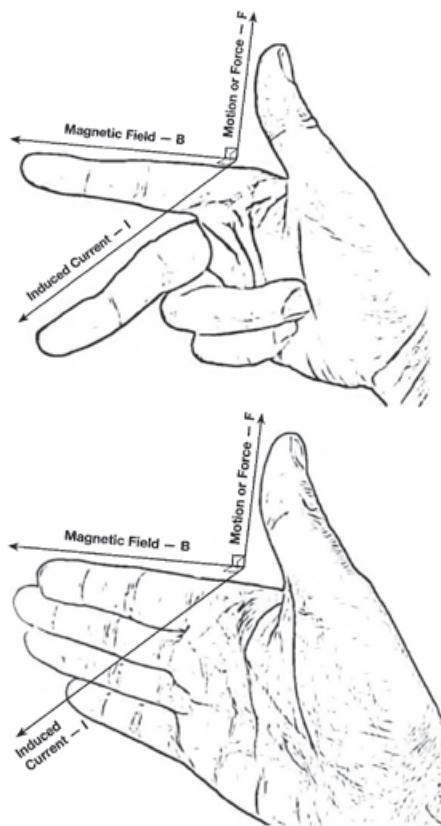
We know that, according to the phenomenon of electromagnetic induction:

- when an electric conductor moves in a magnetic field, there is a change in the magnetic flux which induces an emf that causes a current flow in the conductor;
- the magnetic field strength (B) that passes perpendicularly through a surface area A (in m^2) is called the magnetic flux (Φ) and is measured in weber (Wb).



NB: Use the Right Hand Rule to determine the direction of the force on the charges (F) in the conductor of the generator – the conventional current direction (I) of the induced current. The magnetic field (B) is in the North to South direction. Remember it like so: First Finger is Field; SeCond finger is Current; ThuMb is Movement or Thrust.

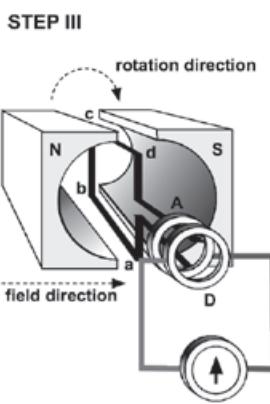
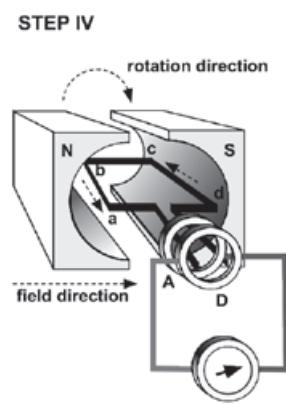
"Fleming's left-hand rule is used for electric motors, while Fleming's right-hand rule is used for electric generators. Different hands need to be used for motors and generators because of the differences between cause and effect." (Wikipedia). So, if you're trying to work out the direction of current in a generator, you need to use the Right Hand Rule, and, vice versa, if you're trying to work out which way an electric motor will turn, you need to use the Left Hand Rule. The fingers are the same; just the hand changes. Note also that an alternative hand positioning is to place all four fingers forwards (Field), and then the thumb indicates the thrust or motion, and a line perpendicular to the palm indicates the current.

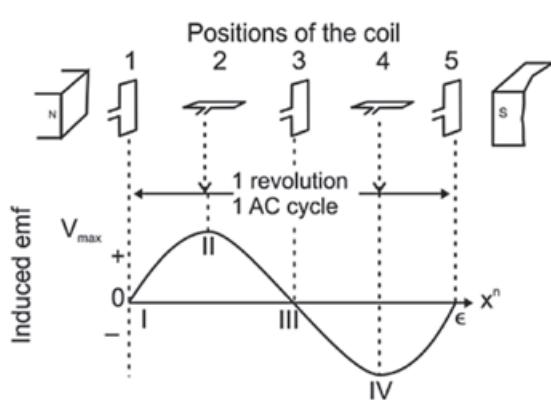


Step-by-step: The AC Generator

Suggestion: watch the video <http://www.youtube.com/watch?v=wpCYiSFBQOU> (where 0 is zero)

Step I: Coil vertical:	Step II: Coil horizontal:
<ul style="list-style-type: none"> • ab and cd are parallel to the normal to the magnetic field • which is directed from N to S • and is parallel to the motion of sides ab and cd • there is no change in the magnetic flux • ∴ no emf is induced in the coil and • ∴ no current flows in the coil <p>$V = 0 \text{ V}$ and $I = 0 \text{ A}$</p> <p style="text-align: center;">STEP I</p>	<ul style="list-style-type: none"> • Coil is rotated in a clockwise direction • ab and cd cut through the magnetic field which is directed from N to S. • ∴ there is a change in the magnetic flux and ab moves down and an emf is induced across the ends of the coil which induces current in the coil. • ∴ conventional current direction is from b to a, so a and A are - (negative) AND • cd moves up and an emf is induced across the ends of the coil which induces current in the coil. So conventional current direction is from d to c, so d and D are + (positive) • V and I increase to V_{\max} and I_{\max} when the coil is horizontal and it cuts the magnetic field perpendicularly • V and I decrease from V_{\max} and I_{\max} as the coil turns further. <p>STEP II</p>

Step III: Coil vertical:	Step IV: Coil horizontal:
<ul style="list-style-type: none"> • ab and cd are again parallel to the normal to the magnetic field • which is directed from N to S • and is parallel to the motion of sides ab and cd • ∴ there is no change in the magnetic flux • ∴ no emf is induced in the coil and • ∴ no current flows in the coil <p>$V = 0 \text{ V}$ and $I = 0 \text{ A}$</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> hint Because each side of the coil or armature is always connected to the same slip ring and brush, the current changes direction. </div>	<ul style="list-style-type: none"> • ab and cd again start to cut through the magnetic field which is directed from N to S. • ∴ there is again a change in the magnetic flux as ab moves up (thumb points up) and an emf is induced across the ends of the coil which induces current in the coil. • ∴ conventional current direction is from a to b, so a and A are + (positive) <p>AND</p> <ul style="list-style-type: none"> • cd moves down (thumb points down) and an emf is induced across the ends of the coil which induces current in the coil. Palm faces forwards from c to d ∴ conventional current direction is from c to d, so d and D are - (negative) • V and I increase to V_{\max} and I_{\max} when the coil is horizontal and it cuts the magnetic field perpendicularly. V and I decrease from V_{\max} and I_{\max} as the coil turns further.  



The alternating current (AC) cycle

In AC (alternating current), the current changes voltage (and direction) every cycle; that is, every time the generator or dynamo turns over through one revolution (full cycle).

When the coil is vertical (in coil positions 1, 3 and 5)	When the coil is horizontal (in coil positions 2 and 4)
<ul style="list-style-type: none"> • ab and cd are parallel to the normal to the magnetic field, and do not cut through the magnetic field • There is no changing magnetic flux • ∴ no emf or current is induced in the coil • ∴ $V = 0 \text{ V}$ and $I = 0 \text{ A}$ 	<ul style="list-style-type: none"> • ab and cd are perpendicular to the normal to the magnetic field, and therefore cut through the magnetic field • There is a changing magnetic flux • ∴ emf and current are induced in the coil • ∴ $V = V_{\max}$ and $I = I_{\max}$ but the emf and current are reversed.



Increasing the induced emf and current

The induced emf (and therefore the amount of induced current) increases if:

- The conductor (wire) is rotated faster so that the rate at which the magnetic flux changes, increases;
- the magnetic field is stronger (use stronger magnets);
- there are more turns (loops) on the coil, so that the length of the conductor (wire) moving through the field is increased.

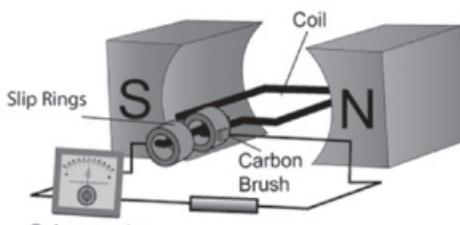
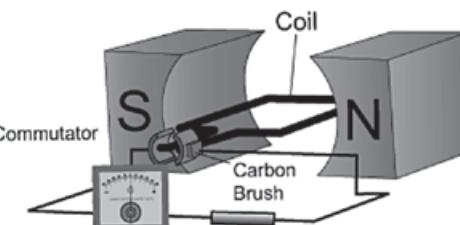
The direct current (DC) cycle

In DC (direct current), the current keeps the same voltage (and direction) in every cycle; that is, every time the generator or dynamo turns over through one revolution (full cycle).

The direct current (DC) cycle	
	<p>Induced emf</p> <p>V_{\max}</p> <p>+</p> <p>0</p> <p>1 Revolution 1 AC cycle</p>
<p>When the coil is vertical</p> <ul style="list-style-type: none"> • ab and cd are parallel to the normal to the magnetic field, and do not cut through the magnetic field • there is no changing magnetic flux • ∴ no emf or current is induced in the coil • ∴ $V = 0 \text{ V}$ and $I = 0 \text{ A}$ 	<p>When the coil is horizontal</p> <ul style="list-style-type: none"> • ab and cd are perpendicular to the normal to the magnetic field, and therefore cut through the magnetic field • there is a changing magnetic flux • ∴ emf and current are induced in the coil • ∴ $V = V_{\max}$ and $I = I_{\max}$ • The emf and current are always positive.

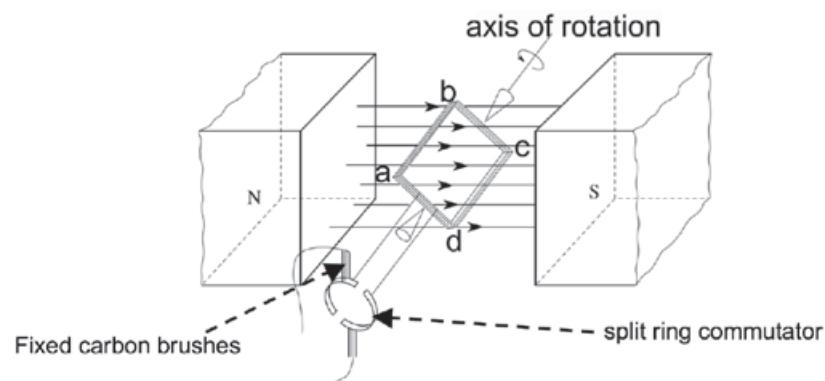
8.1.2 The difference between AC and DC generators

A direct current generator (dynamo) generates direct current instead of alternating current.

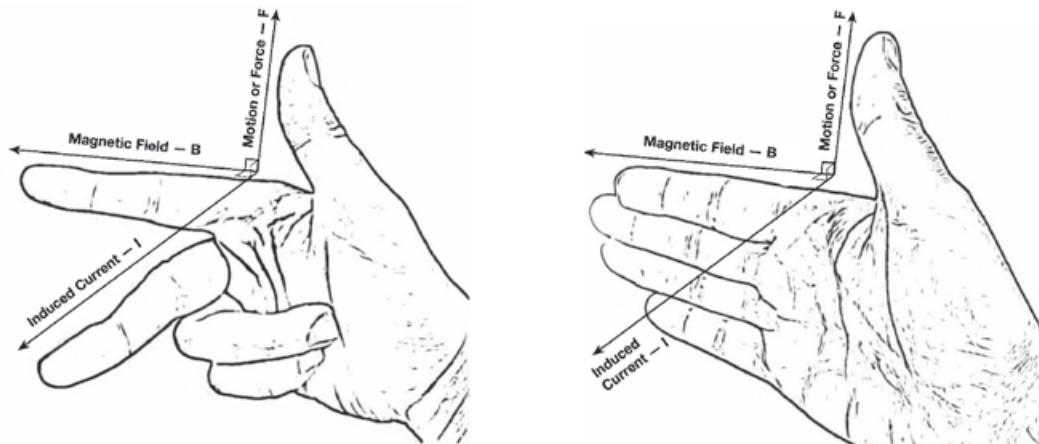
Alternating current (AC) generator	Direct current (DC) generator
	
Similarities between AC and DC generators	
<ul style="list-style-type: none"> Both convert mechanical energy to electrical energy. The coils are turned mechanically (e.g. by steam, flowing water or wind). The induced emf increases and decreases during each cycle. When the coil cuts through the magnetic field, the changing magnetic flux induces an emf and electric current in the coil. The induced V and I have maximum values twice during every cycle. Carbon brushes collect the current. 	
Differences between AC and DC generators	
AC generator	DC generator
<ul style="list-style-type: none"> The coil is connected to slip rings. The same part of the coil is always connected to the same slip ring. The current in the slip rings changes direction when the current in the coil reverses. The brushes collect the alternating current (AC) from the slip rings. 	<ul style="list-style-type: none"> The coil is connected to a split ring commutator. A brush makes contact with a different half of the split ring commutator during each half of the rotation (cycle). One brush always makes contact with the positive half of the split ring commutator and the other brush always makes contact with the negative half of the split ring commutator. The brushes collect DC from the split ring commutator.

8.1.3 Electric motors

Parts of the direct current (DC) motor



Use Fleming's Right Hand Rule to determine the direction of the force on the conductor –the direction in which the coil turns. Remember: the current is in the direction of the middle finger or palm, whereas the magnetic field is in the direction of the fingers (or index finger), and the thrust (motion) is in the direction of the thumb.



8.1.4 The working of a simple DC-motor

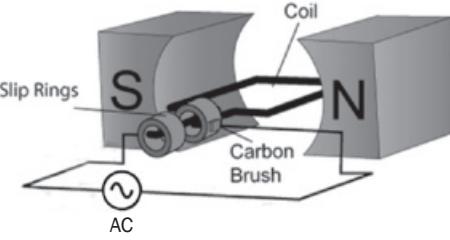
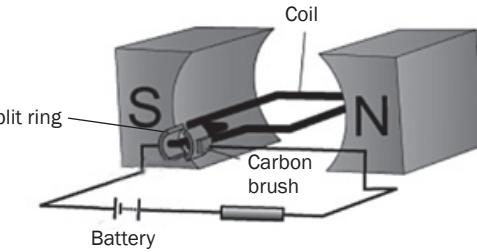
Step 1: Coil horizontal at 0°	Step 2: Coil vertical at 90°
<ul style="list-style-type: none"> the split ring commutator makes contact with the brushes ∴ current flows through the coil ab and cd are at 90° to the magnetic field the magnetic field is from N to S (fingers left to right) ab is connected to the + terminal ∴ conventional current from b to a (thumb from b to a) ∴ palm faces up ∴ DOWNWARD force on ab cd is connected to the – terminal ∴ conventional current from d to c (thumb from d to c) ∴ palm faces down ∴ UPWARD force on cd the 2 forces cause a resultant torque (turning force) on the coil ∴ it rotates anticlockwise <p style="text-align: center;">STEP I</p>	<ul style="list-style-type: none"> the openings in the split ring commutator are opposite the brushes ∴ no current flows through the coil ∴ no resultant force on the coil BUT coil's momentum carries it past the vertical position <p style="text-align: center;">STEP II</p>
<p style="text-align: center;">Step 3: Coil horizontal at 180°</p> <ul style="list-style-type: none"> The split ring commutator makes contact with the brushes again ∴ current flows through the coil ab and cd are at 90° to the magnetic field the magnetic field is from N to S (fingers left to right) ab is now connected to the – terminal ∴ conventional current from a to b (thumb from a to b) ∴ palm faces down ∴ UPWARD force on ab cd is connected to the + terminal ∴ conventional current from c to d (thumb from c to d) ∴ palm faces up ∴ DOWNWARD force on cd the 2 forces cause a resultant torque (turning force) on the coil ∴ it continues to rotate anticlockwise <p style="text-align: center;">STEP III</p>	<p style="text-align: center;">Step 4: Coil vertical at 270°</p> <ul style="list-style-type: none"> The openings in the split ring commutator are opposite the brushes again ∴ no current flows through the coil ∴ no resultant force on the coil BUT coil's momentum carries it past the vertical position back to the original position in Step 1 the first cycle is complete and the process is repeated. <p style="text-align: center;">STEP IV</p>

Increasing the speed at which the DC motor rotates (turns)

The coil will turn faster if:

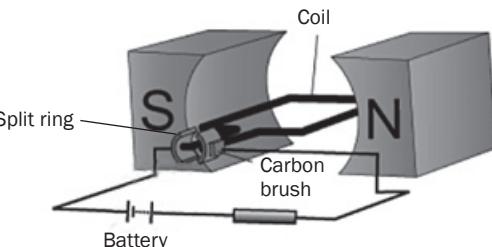
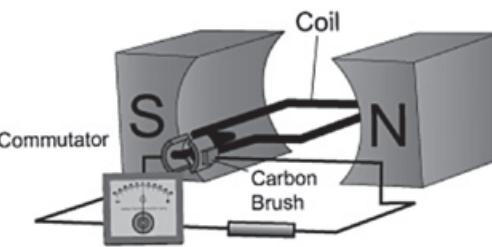
- the current in the coil increases;
- the number of turns on the coil increases;
- the strength of the magnetic field increases.

8.1.5 The differences between AC and DC motors

Alternating current (AC) motor	Direct current (DC) motor
 <p>Slip Rings S N Carbon Brush AC</p>	 <p>Split ring S N Carbon brush Battery</p>
<ul style="list-style-type: none"> • AC power supply • Fixed magnets supply a fixed magnetic field from N to S and the brushes make contact with slip rings to supply the AC to the coil OR • AC electromagnets supply a magnetic field that changes direction during each AC cycle and the brushes make contact with a split ring commutator to supply the AC to the coil • Used for heavy loads e.g. washing machines, electric drills 	<ul style="list-style-type: none"> • DC power supply (battery) • Brushes contact with split ring commutator to supply the DC to the coil • Used for small loads e.g. hair dryers, toy cars

8.1.6 Differences between a motor and a generator

An electric motor and an electric generator are basically the same device. The primary difference is in the case of a motor, electricity is used to turn it, whereas in the case of a generator, turning it mechanically generates electricity.

Direct Current (DC) Motors	Direct Current (DC) Generators
	
<ul style="list-style-type: none"> Connected to a battery. The battery is a source of electric energy. Converts electrical energy to mechanical energy. The magnetic fields of the magnets and those around the current-carrying conductor, interact. The resultant magnetic field exerts a magnetic force on the coil. The coil turns due to the resultant magnetic force acting on it. 	<ul style="list-style-type: none"> NOT connected to a battery. Converts mechanical energy to electrical energy. The coil is turned mechanically (e.g. by steam, flowing water or wind). When the coil cuts through the magnetic field of the magnets, there is a change in the magnetic flux. According to Faraday's Law of Electromagnetic Induction, a change in magnetic flux induces an emf across the ends of the coil and current is induced in the coil.

8.1.7 Electrical motors in everyday life

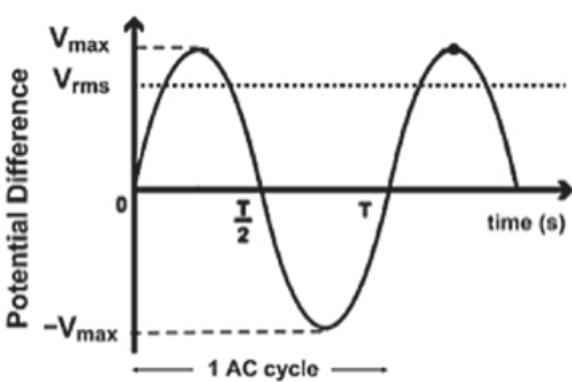
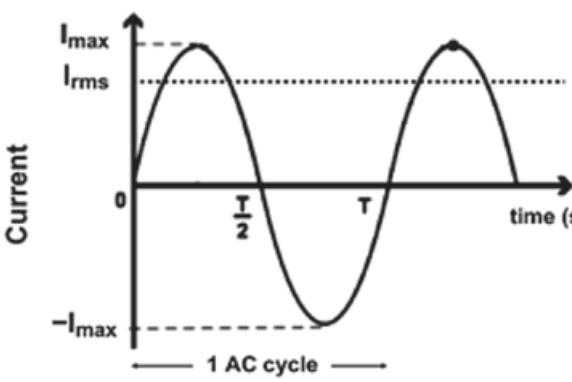
In practice, motors turn evenly at a high speed. The coil in a motor consists of a soft iron core, surrounded by coils. This coil forms the armature. Most armatures have many coils, which are placed at different angles. Each coil in the armature has its own commutator. This results in a bigger turning effect which makes the motor turn evenly. A very important example of an electric motor is the starter motor of a car, which turns the car engine over in order to start it. The purpose of the car battery is to power the starter motor (and other things like lights). When the car is running, the petrol motor turns a generator over which then recharges the battery.

Some motors, e.g. an electric drill, can also use alternating current because they contain electromagnets and not permanent magnets. As the alternating current flows in the coil, the magnetic field changes direction. Thus the motor continues to turn in the same direction.

8.2 Alternating current circuits

- Frequency:** The frequency (f) of an alternating current supply is the number of complete cycles per second and is measured in hertz (Hz). In South Africa electricity is supplied by Eskom power stations and has a frequency of 50 Hz.
- Period:** The period (T) of an alternating supply is the time taken to complete one cycle. If the frequency of the AC current is 50 Hz, $T = \frac{1}{f} = \frac{1}{50} = 0,02$ s

8.2.1 Voltage and current in an AC circuit

emf induced by an AC generator	Current induced by an AC generator
 <p>Potential Difference</p> <p>V_{\max}</p> <p>V_{rms}</p> <p>$-V_{\max}$</p> <p>$\frac{T}{2}$</p> <p>T</p> <p>time (s)</p> <p>— 1 AC cycle —</p>	 <p>Current</p> <p>I_{\max}</p> <p>I_{rms}</p> <p>$-I_{\max}$</p> <p>$\frac{T}{2}$</p> <p>T</p> <p>time (s)</p> <p>— 1 AC cycle —</p>
<p>V varies in cycles</p> <ul style="list-style-type: none"> – between zero and V_{\max} – between + and – values <p>as a function of time (i.e. over time)</p> <p>Voltage changes polarity twice in one AC cycle</p> <p>V_{\max} is read at the crest (top) of the voltage curve and is the amplitude of the voltage curve</p> <p>V_{\max} is reached twice in 1 AC cycle</p> <p>V_{rms} is root mean squared voltage, measured in volts (V)</p> $V_{\text{rms}} = \frac{V_{\max}}{\sqrt{2}}$	<p>I varies in cycles</p> <ul style="list-style-type: none"> – between zero and I_{\max} – between + and – values <p>as a function of time (i.e. over time)</p> <p>Current changes direction twice in one AC cycle</p> <p>I_{\max} is read at the crest (top) of the current curve and is the amplitude of the current curve</p> <p>I_{\max} is reached twice in 1 AC cycle</p> <p>I_{rms} is root mean squared current measured in amperes (A)</p> $I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}}$



DEFINITION

Root mean squared voltage

The root mean squared voltage (V_{rms}) is the equivalent DC voltage value that produces the same heating effect or power as the changing AC.

$$V_{\text{rms}} = \frac{V_{\max}}{\sqrt{2}}$$

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}}$$

The root mean squared current (I_{rms}) is the effective current value of alternating current.

- Root mean square (rms) values are the AC equivalent of DC emf.
- If a DC circuit has an emf of 100 V and an AC circuit has a V_{rms} of 100 V, the circuits would use the same amount of power.

8.2.2 Electric power in an AC circuit

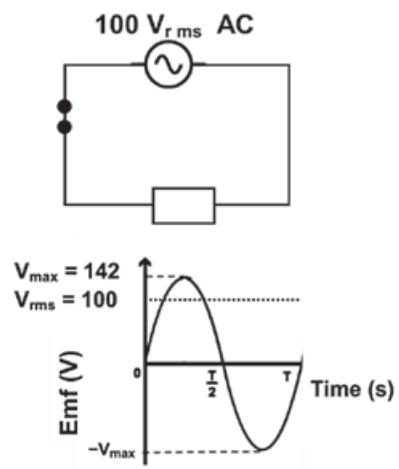


DEFINITION

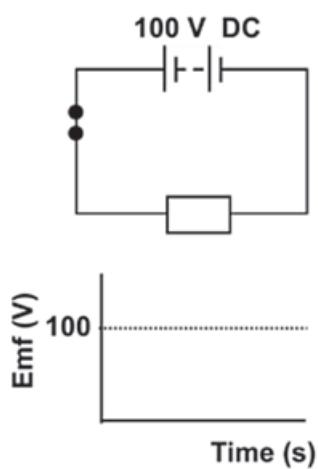
Electrical power (P) is the rate at which energy is transferred or transformed from one type to another.

Summary

AC circuit



DC circuit



$$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} = 100 \text{ V}$$

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} \therefore V_{\text{rms}} = I_{\text{rms}}R$$

$$P_{\text{avg}} = V_{\text{rms}}I_{\text{rms}} = I_{\text{rms}}^2R = \frac{R_{\text{rms}}^2}{R}$$

$$V = 100 \text{ V}$$

$$R = \frac{V}{I} \therefore V = IR$$

$$P = VI - I^2R = \frac{V^2}{R}$$

P_{avg}	:	average power
V_{rms}	:	rms potential difference
I_{rms}	:	rms current
R	:	resistance

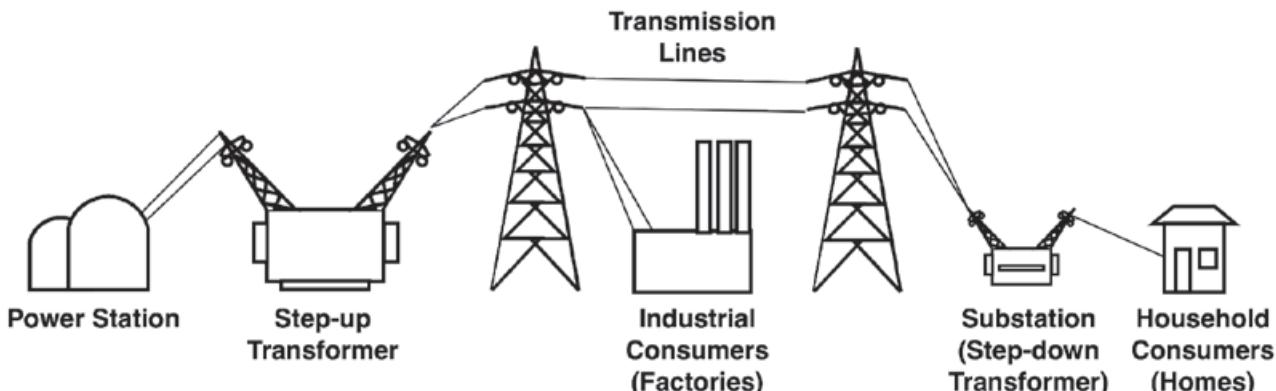
watt	(W)
volt	(V)
ampere	(A)
ohm	(Ω)

Remember:
1 megavolt = 1×10^6 volt
1 MV = 1×10^6 V



8.2.3 Advantages of alternating current

- The most important advantage of AC is the fact that the potential difference can be changed by using transformers.
- Transformers can only function with alternating current.



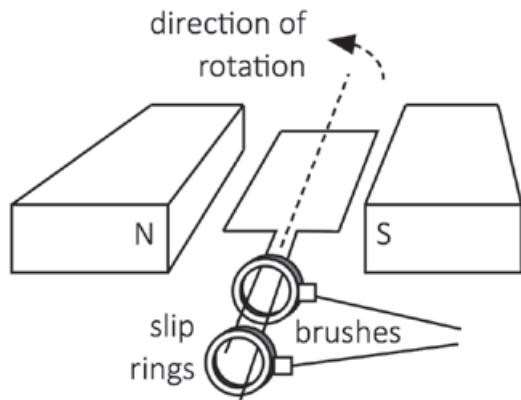
- The power in a transformer remains constant and $P = VI \therefore V \propto \frac{1}{I}$
- At power stations step-up transformers are used to increase (step-up) the voltage which decreases the current.
- The voltage is increased to between 130 and 750 MV
- This allows electrical energy to be transmitted in electric cables over long distances while the current is low.
- The loss in energy due to the heating effect of the cables is low when the current (I) is small:

$$W = I^2Rt \therefore W_{\text{transformed to heat}} \propto I^2$$
- Conducting cables are thick to help decrease the energy lost as heat during transmission.
- Factories need high voltage (± 10 kV).
- In towns **step-down transformers** are used to **decrease** (step-down) the **voltage** to ± 220 V. You can see these at the side of the road in most suburbs; they are painted dark green.



Activity 1

A simplified sketch of a generator is shown below.



1. Is the output voltage AC or DC? Give a reason for your answer. (2)
 2. What type of energy conversion takes place in the above generator? (2)
 3. State TWO effects on the output voltage if the coil is made to turn faster. (2)
 4. What is the position of the coil relative to the magnetic field when the output voltage is a maximum? (1)
- [7]

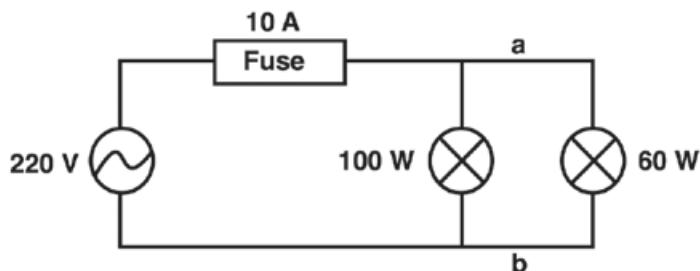
Solutions

1. AC – The generator has slip rings. ✓✓ (2)
 2. Mechanical ✓ energy is converted to electrical energy. ✓ (2)
 3. Output voltage increases and the number of cycles per second increases. ✓ (2)
 4. The coil position is parallel to the magnetic field. (1)
- [7]



Activity 2

Lights in most households are connected in parallel, as shown in the simplified circuit below. Two light bulbs rated at 100 W; 220 V and 60 W; 220 V respectively are connected to an AC source of rms value 220 V. The fuse in the circuit can allow a maximum current of 10 A.



1. Calculate the peak voltage of the source. (3)
2. Calculate the resistance of the 100 W light bulb when operating at optimal conditions. (3)
3. An electric iron, with a power rating of 2 200 W, is now connected across points **a** and **b**. Explain, with the aid of a calculation, why this is not advisable. (5)

[11]

Solutions

1. $V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$ ✓

$$220 = \frac{V_{\text{max}}}{\sqrt{2}} \therefore V_{\text{max}} = 311,1 \text{ V} \quad \checkmark \quad (3)$$

2. $P = \frac{V^2}{R}$ ✓

$$100 = \frac{220^2}{R} \therefore R = 484 \Omega \quad \checkmark \quad (3)$$

3. $P_{\text{ave}} = V_{\text{rms}} I_{\text{rms}}$ ✓ $2 200 = (220) I_{\text{rms}}$ ✓

$$I_{\text{rms}} = \frac{2200}{220}$$

$$I_{\text{rms}} = 10 \text{ A} \quad \checkmark$$

The iron draws 10 A of current. Together with the lights the total current will exceed 10 A causing the fuse to blow. ✓✓

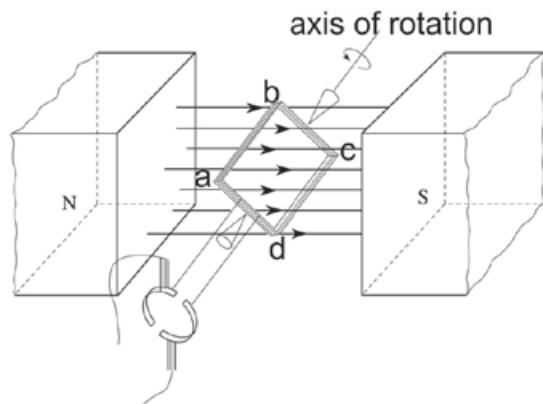
(5)

[11]



Activity 3

The essential components of a simplified DC motor are shown in the diagram below.



When the motor is functioning, the coil rotates in a clockwise direction as shown.

1. Write down the function of each of the following components:
 - a) Split-ring commutator (1)
 - b) Brushes (1)
 2. What is the direction of the conventional current in the part of the coil labeled AB? Write down only FROM A TO B or FROM B TO A. (1)
 3. Will the coil experience a maximum or minimum turning effect (torque) if the coil is in the position as shown in the diagram above? (1)
 4. State ONE way in which this turning effect (torque) can be increased. (1)
 5. Alternating current (AC) is used for the long-distance transmission of electricity.
Give a reason why AC is preferred over DC for long-distance transmission of electricity. (2)
 6. An electrical appliance with a power rating of 2 000 W is connected to a 230 V rms household mains supply.
Calculate the:
 - a) Peak (maximum) voltage (3)
 - b) rms current passing through the appliance (3)
- [13]

Solutions

1. a) Reverses current direction in the coil every half cycle. (1)
- b) Connects external circuit to split ring commutator. (1)
2. B to A ✓ (1)
3. Maximum ✓ (1)
4. Increase current strength ✓ / Increase number of coils ✓ / Use stronger magnets. ✓ (any one) (1)
5. AC can be stepped up to high voltages and low current. ✓ Less energy loss with low current ($W = I^2R\Delta t$). ✓ (2)
6. a) $V_{rms} = \frac{V_{max}}{\sqrt{2}}$ ✓ ∴ 230 ✓ = $\frac{V_{max}}{\sqrt{2}}$ ∴ $V_{max} = 325,27$ V ✓ (3)
- b) $P_{ave} = V_{rms}I_{rms}$ ✓
 $2\ 000 = (230)I_{rms}$ ✓
 $I_{rms} = 2000 / 230$
 $I_{rms} = 8,695$ A ✓ (3)

[13]

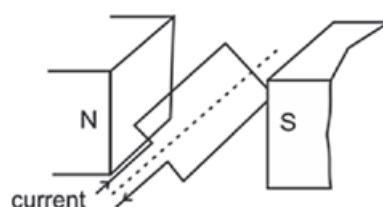


Activity 4

Electric motors are important components of many modern electrical appliances. AC motors are used in washing machines and vacuum cleaners, and DC motors are used in toys and some tools.

1. What energy conversion takes place in electric motors? (2)
2. What is the essential difference in the design between DC motors and AC motors? (4)
3. List THREE ways in which the efficiency of the motor can be improved. (3)
4. Consider the diagram. The conventional current direction is indicated by the arrows.
 - a) In which direction (clockwise or anti-clockwise) will the coiled armature rotate if the switch is closed? (1)
 - b) Why does the armature continue moving in the same direction once it has reached the vertical position? (2)

[12]



Solutions

1. Electric energy converted to mechanical energy. (2)
2. A DC motor reverses current direction with the aid of the commutator whenever the coil is in the vertical position to ensure continuous rotation.

An AC motor, with alternating current as input, works without commutators since the current alternates/slip rings can be used. (4)

3. Increase the number of turns on each coil; increased current; stronger magnets. ✓ (3)
4. a) Anticlockwise (1)
- b) The armature's own momentum ✓/ the split ring commutator changes direction of current, every time the coil reaches the vertical position. ✓ (2)

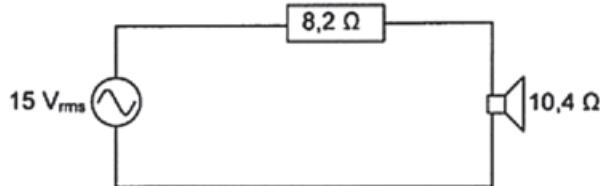
[12]

**Activity 5**

In the circuit the AC source delivers alternating voltages at audio frequency to the speaker.

1. What is the peak voltage that the source can deliver? (3)
2. Calculate the average power delivered to the speaker. (7)

[10]

**Solutions**

1. $V_{rms} = V_{max}/\sqrt{2}$ ✓
 $\therefore V_{max} = \frac{15}{\sqrt{2}}$ ✓ = 21,21 V ✓ (3)
2. $R_{total} = 8,2 + 10,4$ ✓ = 18,6 Ω ✓
 $I = \frac{V}{R}$ ✓ = 15 / 18,6 = 0,81 A ✓
 $P = I^2R$ ✓ = $(0,81)^2(10,4)$ ✓ = 6,76 W ✓ (7)

[10]



Optical phenomena and properties of materials

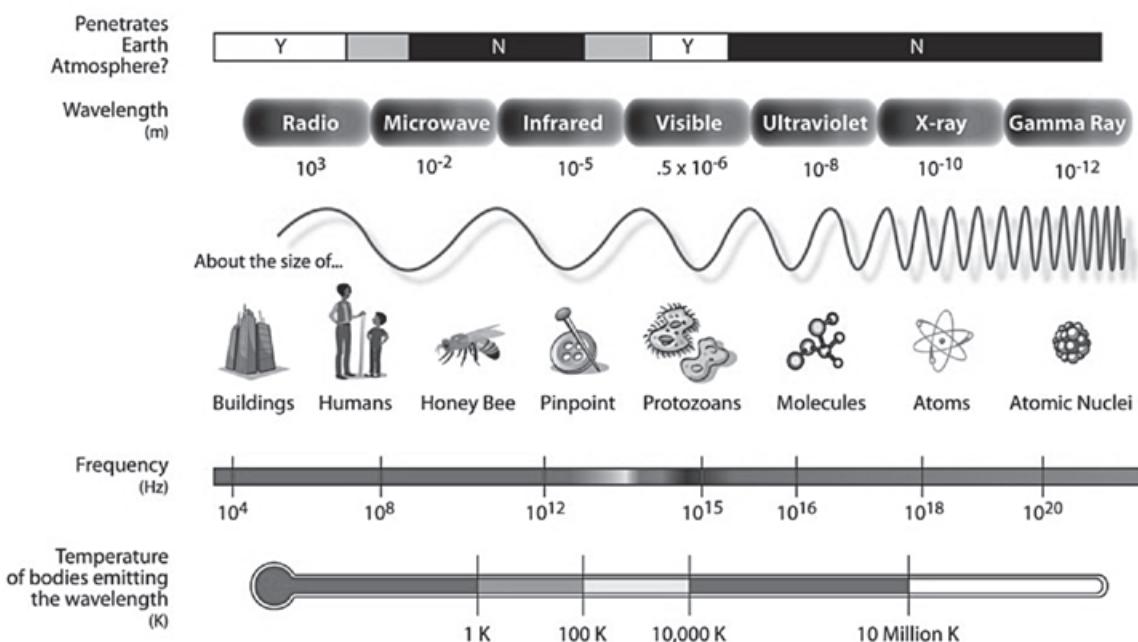
9.1 Electromagnetic waves and visible light: Revision

9.1.1 Electromagnetic waves

Electromagnetic waves consist of electric and magnetic fields. These fields are perpendicular to each other and to the direction in which the wave is propagated (the direction in which it travels).

You must remember:

- Electromagnetic waves travel through a vacuum at $3 \times 10^8 \text{ m}\cdot\text{s}^{-1}$
- They increase in frequency and energy from radio waves (lower frequency and less energy) to gamma rays (higher frequency and more energy)
- They increase in wavelength from gamma rays (shorter wavelength) to radio waves (longer wavelength)
- The visible light spectrum is part of the electromagnetic spectrum (shown below).



The electromagnetic spectrum (Source:http://en.wikipedia.org/wiki/Electromagnetic_spectrum)

9.1.2 Visible light

Visible light is part of the electromagnetic spectrum. You must remember:

- Visible light increases in frequency and energy from red (lower frequency, less energy) to violet (higher frequency, more energy)
- Visible light increases in wavelength from violet (shorter wavelength) to red (longer wavelength).
- Visible light has a dual nature because it has **wave properties** while it is propagated (transmitted) and it has **particle properties** when it strikes and interacts with other matter (the photoelectric effect).



Dual nature of light:

- **Wave nature** during propagation proved by **diffraction and interference**
- **Particle nature** during interaction with matter proved by the **photoelectric effect**

9.2 The photoelectric effect

The photoelectric effect is used in solar panels to generate electricity. The photoelectric effect refers to the ability of light to cause metals to release electrons. You must remember:

- Light energy is transmitted in 'packages' which are called **photons**.
- Each photon consists of a certain amount of energy which is called a **quantum**.
- The amount of energy (E) in a quantum is directly proportional to the **frequency (f)** of the light.
- An electron needs a minimum amount of energy to be released from an atom. So the photon providing this energy must have a minimum frequency before it will allow an electron to be released from the metal surface.
- When light shines on a surface (like a metal), the photons collide with the atoms in the surface.
- All the energy of the photon ($E = hf$) is transferred to the atom with which the photon collides.
- If an electron in an atom on the surface of the metal gains sufficient energy during the collision, it is ejected from the metal surface and is called a **photoelectron**.



DEFINITIONS

- The **photoelectric effect** is the process whereby electrons are ejected from a metal surface when light of suitable frequency is incident on (falls on) that surface.
- The **work function (W_0)** of a metal is the minimum energy that is required to emit a photoelectron from the surface of the metal.
- The **threshold frequency or cut-off frequency (f_0)** is the minimum frequency of the incident photons (light) that is required to emit a photoelectron from the surface of the metal.

$$\begin{aligned} \text{energy} &\propto \text{frequency} & \text{and} & \quad \text{work function} \propto \text{threshold (cut-off) frequency} \\ \therefore E &\propto f & \text{and} & \quad W_0 \propto f_0 \\ \therefore E &= hf & \text{and} & \quad W_0 = hf_0 \end{aligned}$$



- The formula $c = f\lambda$ is used to calculate the speed of light, where **c** ($3 \times 10^8 \text{ m}\cdot\text{s}^{-1}$) is the speed of light in metres per second, λ is the wavelength in meters (m) and f is the frequency in hertz (Hz).
- The formula $E = hf$ is used to calculate the energy of radiation, where E is the energy of radiation in Joule (J), **h** is Planck's constant ($6.63 \times 10^{-34} \text{ J}\cdot\text{s}^{-1}$) and f is the frequency in hertz (Hz).

9.2.1 Changing the frequency and intensity of the incident light

- The intensity of a light wave is measured by the power (wattage) of the light source. An 8 W (watt) CFL lamp is dim but a 16 W CFL lamp is bright.
- It also depends on the type of light. So, for example, Light Emitting Diode (LED) lights use very little power but are very bright, Compact Fluorescent Lights (CFLs) use a mid-range amount of power and are bright for the amount of power they use, and Tungsten Filament or Incandescent Lamps use a lot of power for the amount of light they provide E.G. 60 W, 100 W.
- This means that the same type of light with a *higher* wattage will be *brighter* than the same type of light with a *lower* wattage.

When the frequency of the **incident** light (the light falling on the metal) is greater than the threshold frequency, changes to the intensity (brightness) and the frequency cause these changes:

Increasing the frequency of incident light

The **frequency** of the incident light is **increased** while its **intensity** (brightness) remains **constant**:

- The same number of photoelectrons are emitted from the surface of the metal
- the kinetic energy of the photoelectrons increases
- the speed at which the photoelectrons move away from the metal increases

BUT

- the same number of photons strike the metal surface per second and
- the same number of photoelectrons are emitted from the metal surface per second; i.e. the rate at which photoelectrons are emitted, remains constant.

Increasing the intensity of incident light

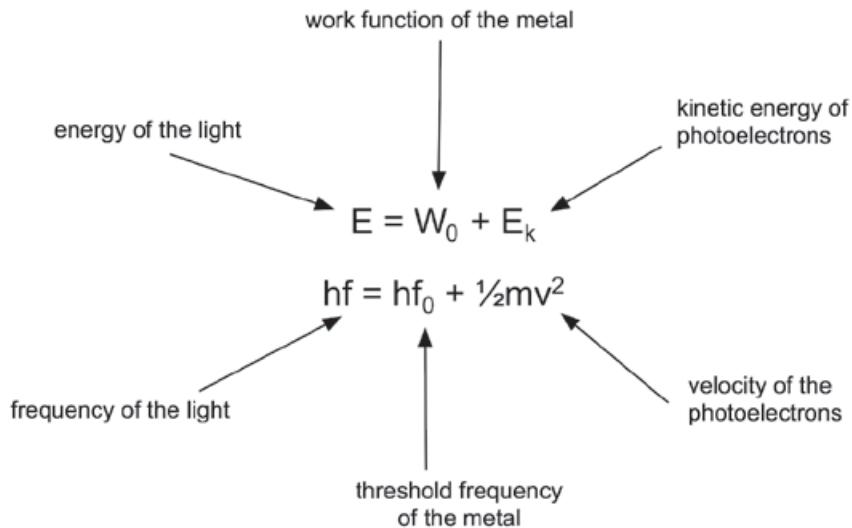
The **intensity** (brightness) of the incident light is **increased** while its **frequency** remains **constant**:

- More photoelectrons are emitted from the surface of the metal
- the energy of each photon remains constant as the frequency is constant and
- the kinetic energy and speed of the emitted photoelectrons remains constant so that
- the speed at which the photoelectrons move away from the metal remains constant

BUT

- more photons strike the metal surface per second and
- more photoelectrons are emitted from the metal surface per second; i.e. the rate at which photoelectrons are emitted, increases.

9.2.2 Calculating energy of photoelectrons



9.2.3 Conditions for emission of photoelectrons

<p>e.g. 1 $f_{\text{light}} < f_0$</p> $E_{\text{light}} < W_0$ $hf_{\text{light}} < hf_0$ <p><i>Photoelectrons:</i></p> <ul style="list-style-type: none"> are not emitted. 	<p>e.g. 2 $f_{\text{light}} = f_0$</p> $E_{\text{light}} = W_0$ $hf_{\text{light}} = hf_0$ <p><i>Photoelectrons:</i></p> <ul style="list-style-type: none"> are emitted onto the surface of the metal don't have kinetic energy don't move away. 	<p>e.g. 3 $f_{\text{light}} > f_0$</p> $E_{\text{light}} > W_0$ $hf_{\text{light}} > hf_0$ <p><i>Photoelectrons:</i></p> <ul style="list-style-type: none"> are emitted from the surface of the metal have kinetic energy move away.
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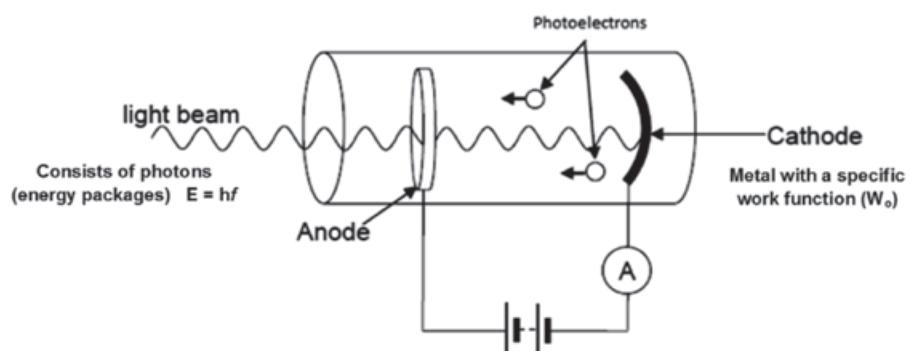
E:	energy of light	(J)
h:	Planck's constant	$(6,63 \times 10^{-34} \text{ J}\cdot\text{s}^{-1})$
f:	frequency	(Hz)
W_0 :	work function	(J)
E_k :	kinetic energy	(J)
v:	velocity of electrons	$(\text{m}\cdot\text{s}^{-1})$
m:	mass of electron	$(9,11 \times 10^{-31} \text{ kg})$

9.2.4 Another example of the photoelectric effect

A photoelectric diode in an electric circuit is another example of the application of the photoelectric effect. When photons (light) with a frequency higher than the cut-off frequency of the metal cathode shines on the cathode, photoelectrons are emitted.

Photoelectric diodes are used in:

- smoke detectors
- light meters in cameras
- remote controls and
- CD players.



A photoelectric diode in an electric circuit

e.g.

Worked example 1

Calculate the energy of a light wave with a wavelength of 660 nm.

Solutions

$$c = \lambda f$$

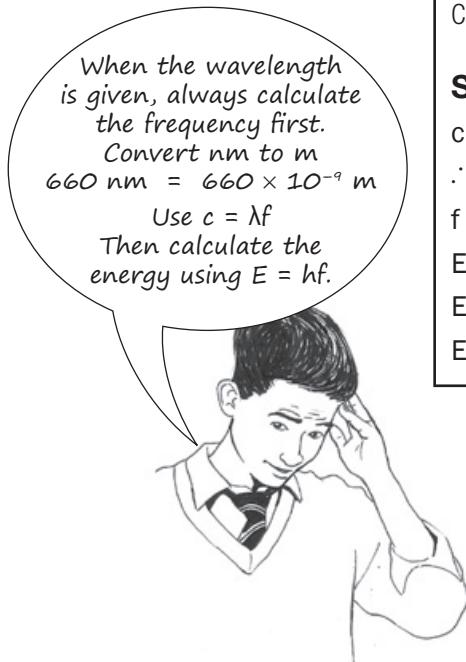
$$\therefore 3 \times 10^8 = 660 \times 10^{-9} f$$

$$f = 3 \times 10^8 / 660 \times 10^{-9} \text{ Hz}$$

$$E = hf$$

$$E = (6,63 \times 10^{-34})(4,55 \times 10^{14})$$

$$E = 3,02 \times 10^{-19} \text{ J}$$





Worked example 2

A learner wants to demonstrate the photoelectric effect.

He uses a disk of zinc placed on an electroscope.

The work function (W_0) of zinc is 6.9×10^{-19} J.

1. Define the concept **work function**.
 2. Calculate the **maximum wavelength** of light that will eject electrons from the zinc.
 3. The electroroscope is negatively charged and then exposed to ultraviolet light from a mercury discharge lamp. One of the wavelengths of the light is 260 nm. Calculate the kinetic energy of an electron emitted from the zinc disk by a photon of this light.

Solutions

$$\begin{aligned}
 3. \quad E &= W_0 + E_k \\
 E &= hf \\
 hf &= W_0 + E_k \\
 (6,63 \times 10^{-19})(1,15 \times 10^{15}) &= 6,9 \times 10^{-19} + E_k \\
 E_k &= (7,63 \times 10^{-19}) - (6,9 \times 10^{-19}) \\
 E_k &= 7,3 \times 10^{-20} \text{ J}
 \end{aligned}$$

And

$$\begin{aligned}c &= \lambda f \\3 \times 10^8 &= (260 \times 10^{-9}) f \\f &= 3 \times 10^8 / (260 \times 10^{-9}) \\f &= 1,15 \times 10^{15} \text{ Hz}\end{aligned}$$

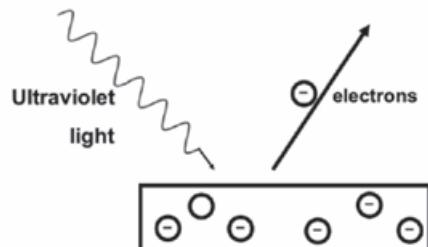
Remember:
Convert nm to m
 260 nm
 $= 260 \times 10^{-9} \text{ m}$





Activity 1

A metal surface is illuminated with ultraviolet light of wavelength 330 nm. Electrons are emitted from the metal surface. The minimum amount of energy required to emit an electron from the surface of this metal is $3,5 \times 10^{-19}$ J.



1. Name the phenomenon illustrated. (1)
 2. Give ONE word or term for the underlined sentence in the above paragraph. (1)
 3. Calculate the frequency of the ultraviolet light. (3)
 4. Calculate the kinetic energy of a photoelectron emitted from the surface of the metal when the ultraviolet light shines on it. (3)
 5. The intensity of the ultraviolet light illuminating the metal is now increased.
What effect will this change have on the following?
 - a) Kinetic energy of the emitted photoelectrons. (Write down only INCREASES, DECREASES or REMAINS THE SAME.) (1)
 - b) Number of photoelectrons emitted per second. (Write down only INCREASES, DECREASES or REMAINS THE SAME.) (1)
- [10]

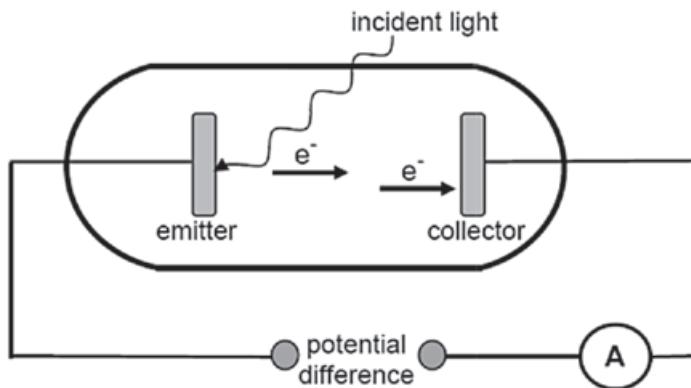
Solutions

1. Photoelectric effect. ✓ (1)
 2. Work function. ✓ (1)
 3. $c = \lambda f$ ✓
 $3 \times 10^8 = (330 \times 10^{-9}) f$ ✓
 $f = 9,09 \times 10^{14} \text{ Hz}$ ✓ (3)
 4. $E_k = hf - W_0$ ✓ $330 \text{ nm} = 330 \times 10^{-9} \text{ m}$
 $= (6,6 \times 10^{-34})(9,09 \times 10^{14}) - 3,5 \times 10^{-19}$
 $= 6,0 \times 10^{-19} - 3,5 \times 10^{-19}$ ✓
 $= 2,5 \times 10^{-19} \text{ J}$ ✓ (3)
 5. a) Remains the same. ✓ (1)
b) Increases. ✓ (1)
- [10]



Activity 2

In the simplified diagram below, light is incident on the emitter of a photocell. The emitted photoelectrons move towards the collector and the ammeter registers a reading.



1. Name the phenomenon illustrated above. (1)
 2. The work function of the metal used as emitter is 8.0×10^{-19} J. The incident light has a wavelength of 200 nm. Calculate the maximum speed at which an electron can be emitted. (6)
 3. Incident light of a higher frequency is now used.
How will this change affect the maximum kinetic energy of the electron emitted in the question above?
Write down only INCREASES, DECREASES or REMAINS
THE SAME. (1)
 4. The intensity of the incident light is now increased.
How will this change affect the speed of the electron calculated in QUESTION 11.1.2? Write down INCREASES, DECREASES or REMAINS
THE SAME. Give a reason for the answer. (2)
 5. A metal worker places two iron rods, A and B, in a furnace.
After a while he observes that A glows deep red while B glows orange. Which rod A or B has higher energy of radiation?
Give a reason for your answer. (2)
 6. Neon signs illuminate many buildings.
What type of spectrum is produced by neon signs? (1)
- [13]

Solutions

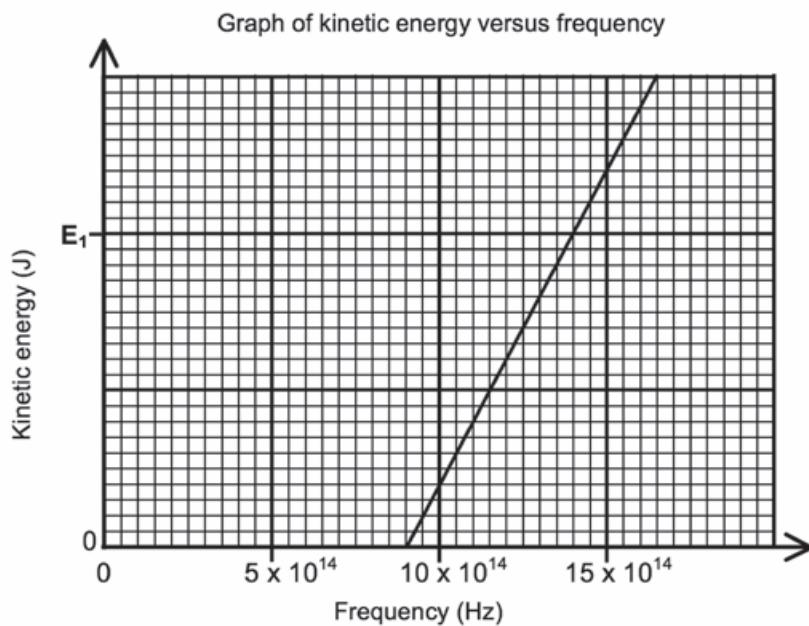
1. Photo-electric effect ✓ (1)
2. $E = W_0 + E_k \checkmark$
 $hf = hf_0 + E_k \checkmark$
 $\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2 \checkmark$

$$\frac{(6,63 \times 10^{34})(3 \times 10^8)}{200 \times 10^{-9}} \checkmark = 8 \times 10^{-19} \checkmark + \frac{1}{2}(9,11 \times 10^{-31})v^2 \checkmark \quad (6)$$
3. Increases ✓ (1)
4. Remains the same ✓
Intensity only affects number of photoelectrons emitted per second. ✓ (2)
5. B ✓
Orange light has a higher frequency than red light. ✓ (2)
6. Line emission (spectra) ✓ (1)

[13]

**Activity 3**

During an investigation, light of different frequencies is shone onto the metal cathode of a photocell. The kinetic energy of the emitted photoelectrons is measured. The graph below shows the results obtained.



1. For this investigation, write down the following:
 - a) Dependent variable (1)
 - b) Independent variable (1)
 - c) Controlled variable (1)
2. Define the term threshold frequency. (2)

3. Use the graph to obtain the threshold frequency of the metal used as cathode in the photocell. (1)
 4. Calculate the kinetic energy at E_1 shown on the graph. (4)
 5. How would the kinetic energy calculated in QUESTION 11.4 be affected if light of higher intensity is used? Write down only INCREASES, DECREASES or REMAINS THE SAME. (1)
- [11]

Solutions

- 1.a) Kinetic energy ✓ (1)
 - b) Frequency ✓ (1)
 - c) (Type of) metal ✓ (1)
 2. The minimum frequency needed to emit electrons ✓ from the surface of a metal. ✓ (2)
 3. 9×10^{14} Hz ✓ (1)
 4. $E = W_0 + E_k$ ✓
 $hf = hf_0 + E_k$
 $(6,63 \times 10^{-34})(14 \times 10^{14})$ ✓ =
 $(6,63 \times 10^{-34})(9 \times 10^{14}) + E_k$ ✓
therefore $E_k = 3,32 \times 10^{-19}$ J ✓ (4)
 5. Remains the same ✓ (1)
- [11]



Emission and absorption spectra

All images on pages 144, 145 and 146 are in colour on the inside front cover.

Summary

- When a light ray passes from one optical medium to another, the ray is refracted and its speed and direction change.
- The pattern which forms when a ray of light is broken up into its component frequencies is called a spectrum. The light is broken up by the individual rays of different frequencies being refracted or having their path “bent” as they go through optical (transparent) media of different optical density (different degrees of transparency/light conductivity).
- Spectra can be observed with a diffraction grating, a spectroscope or a prism, or in a rainbow after a storm.



(Source: <http://en.wikipedia.org/wiki/Prism>)

10.1 Continuous emission spectra

- The spectrum produced when white light passes through a prism is called a continuous spectrum.
- The spectrum emitted by the sun is a continuous emission spectrum.
- The colours in the spectrum follow on each other without any gaps between them. A familiar example of a spectrum is a rainbow that one sees after a thunderstorm.

10.2 Atomic emission spectra

- Atomic emission spectra are produced when a gas is heated or by passing an electric current through it in a gas discharge tube.
- The electrons in the atoms of the gas absorb the energy and become excited and move to a higher (excited) energy level. This high energy state is unstable.
- When excited electrons return to the ground state or a lower energy level, the energy is released in specific energy packets called “photons”, or light particles.
- The gas becomes incandescent (glowing).
- The energy of the emitted photon equals the energy difference between the two energy levels. The energy of light is directly proportional to its frequency and the frequency of light determines its colour.
- Only the frequencies (colours) of light that are in the visible range, that are emitted by the atoms, are seen by the eye. Colours out of the visible range, such as ultraviolet and infrared are not seen. The range of frequencies emitted by a particular substance are called a line emission spectrum as most substances do not emit the full spectrum; instead, they emit a particular pattern of frequencies.
- The atoms of each element have a unique set of energy levels, so the line emission spectrum is a set of discrete coloured lines with dark spaces in between where those frequencies are not being emitted.
- The line emission spectrum for each element is unique to that element, and can be used to identify that element. For example, amber street lamps have a sodium lamp in them, and thus produce an amber light, as sodium emits primarily in the yellow band. Likewise, fireworks’ colours are determined by the chemicals used in them. So, for example, bright red is produced by strontium (Sr), blue-green by copper (Cu), and so on.
- Scientists are able to tell what elements are present on distant planets and stars by projecting their light through a prism and capturing the line emission spectrum.

Emission spectrum of Iron (Fe)



Emission spectrum of Hydrogen (H)



**DEFINITIONS**

discrete: clear and individual, separate

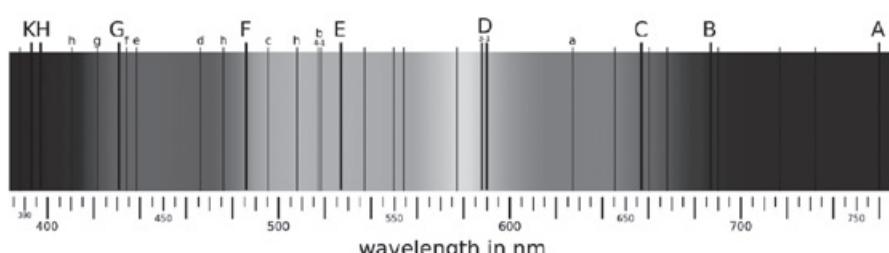
incandescent: glowing

ground state: lowest stable energy state

excited state: high and unstable energy state

10.3 Atomic absorption spectra

An atomic absorption spectrum is a continuous spectrum where certain colours or frequencies are missing. These frequencies appear as dark lines in the spectrum. The region A-B in the diagram below is Infrared. The region B-C and part way to D is red. The region C-D is orange. The region D-E is green. The region E-F is cyan/light blue. The region around F is blue. The region F-G is indigo, and G-H violet. The region KH is ultraviolet.



(Source: Wikimedia Commons)

You must remember:

- Atomic absorption spectra are produced when light passes through a cold gas.
- The electrons in the atoms of the gas absorb energy from the light and become excited and move to a higher (excited) energy level.
- The energy of the absorbed light energy equals the energy difference between the two energy levels.
- The energy of light is directly proportional to its frequency and the frequency of light determines its colour.
- The light that has not been absorbed by the gas, reaches the eye and therefore shows the range of frequencies in the atomic absorption spectrum.
- The atoms of each element have a unique set of energy levels, so the atomic absorption spectrum is a continuous spectrum with a few black lines. These lines represent the colours (and hence frequencies) of the light that were absorbed by the gas atoms' electrons.
- The atomic absorption spectrum for each element is unique to that element, and can be used to identify that element.
- The dark lines represent the same frequencies of light that are emitted in the same element's atomic emission spectrum. If an atomic emission spectrum and an atomic absorption spectrum are combined for a specific element, we see a continuous spectrum.



Activity 1

1. What is the approximate wavelength range of visible light? (2)
2. Name five wavelength ranges and their uses. (5)
3. How can a scientist tell what elements are present on a star? (3)
4. What is the approximate wavelength of red light? And violet? (2)
5. What does the wavelength of UV tell you about its energy levels? (2)
6. Does microwave radiation or gamma radiation have more energy per photon? (1)
7. Give one example of a colour in fireworks achieved through emission spectra. (1)

[16]

Solutions

1. 400 nm (✓) to 700 nm (✓) (One mark per correct value) (2)
2. Visible light: to see (✓); Xrays: to inspect bones without surgery (✓); Gamma rays: to kill bacteria (✓); UV: suntanning (✓), helps bees navigate (✓), powers photosynthesis (✓); Infrared: night vision (✓), heat radiation (✓), some lasers (✓); Microwaves: telecommunications (✓), radar (✓), ovens (✓); Radio waves: telecommunications (✓). (any 5) (5)
3. She can project the light from the star through a spectroscope (✓) which splits it into its components (✓). She can then compare the spectrum to known emission spectra of known elements (✓). (3)
4. Any value 700-600 nm (✓) (it's continuous); Any value near 400-450 nm (✓). (2)
5. UV has a short wavelength (✓) which means that it has high (✓) energy levels. (2)
6. Gamma. (✓) (1)
7. Cu / Copper: blue / green / cyan / blue-green / turquoise (✓); Strontium / Lithium: Red (✓); Iron / Sodium / Calcium / Na / Fe / Ca: orange / yellow (✓); Magnesium / Mg / Aluminium / Al: White (✓); Potassium/K: lilac / violet (✓); Green: Barium / Ba (✓) (light green), possibly Copper / Cu (darker green) (✓). (any one) (1)

[16]





The Mind the Gap study guide series assists you to make the leap by studying hard to achieve success in the Grade 12 exam.

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