What you should be familiar with before taking this lesson

$$\begin{array}{c} \text{3 columns} \\ \downarrow \quad \downarrow \quad \downarrow \\ A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \text{2 rows} \end{array}$$

A **matrix** is a rectangular arrangement of numbers into rows and columns. Each number in a matrix is referred to as a **matrix element** or **entry**.

If this is new to you, you might want to check out our intro to matrices. You should also make sure you know how to add and subtract matrices.

What you will learn in this lesson

We can multiply matrices by real numbers. This article explores how this works.

Scalars and scalar multiplication

When we work with matrices, we refer to real numbers as **scalars**.

The term **scalar multiplication** refers to the product of a real number and a matrix. In scalar multiplication, each entry in the matrix is multiplied by the given scalar.

For example, given that $A = \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix}$, let's find $2A$.

To find $2A$, simply multiply each matrix entry by 2:

$$2A = 2 \cdot \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 10 & 2 \cdot 6 \\ 2 \cdot 4 & 2 \cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 12 \\ 8 & 6 \end{bmatrix}$$

Check your understanding

1) Given $B = \begin{bmatrix} -4 & -2 \\ 7 & 1 \end{bmatrix}$**, find** $-3B$**.**

$-3B = \begin{bmatrix} \ \end{bmatrix}$

Check

[I need help!]

2) Given $C = \begin{bmatrix} -42 \\ 27 \\ -3 \end{bmatrix}$**, find** $\frac{1}{3}C$**.**

$\frac{1}{3}C = \begin{bmatrix} & \end{bmatrix}$

Check

[I need help!]

Scalar multiplication as repeated addition

Recall that to add (or subtract) two matrices, we can simply add (or subtract) the corresponding entries.

For example,

$$\begin{bmatrix} 10 & 12 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 22 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+1 & 12+4 \\ 6+22 & 3+7 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 16 \\ 28 & 10 \end{bmatrix}$$

Now, suppose we wanted to consider the *repeated addition* of a matrix.

If $A = \begin{bmatrix} 4 & 8 \\ 2 & 1 \end{bmatrix}$, let's find $A + A + A$.

$$A + A + A$$

$$= \begin{bmatrix} 4 & 8 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4+4 & 8+8+8 \\ 2+2+2 & 1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 4 & 3 \cdot 8 \\ 3 \cdot 2 & 3 \cdot 1 \end{bmatrix}$$

$$= 3 \cdot \begin{bmatrix} 4 & 8 \\ 2 & 1 \end{bmatrix}$$

$$= 3A$$

Here we see that $A + A + A = 3A$.

Therefore, we can interpret scalar multiplication in the same way as we interpret multiplication with real numbers – as repeated matrix addition!

Solving matrix equations

A **matrix equation** is simply an equation in which the variable stands for a matrix.

For example, the equation below is a matrix equation.

$$3A = \begin{bmatrix} 3 & 24 \\ 18 & 0 \end{bmatrix}$$

To solve for A, we can multiply both sides of the matrix equation by the scalar $\frac{1}{3}$.

$$3A = \begin{bmatrix} 3 & 24 \\ 18 & 0 \end{bmatrix}$$

$$\frac{1}{3} \cdot 3A = \frac{1}{3} \cdot \begin{bmatrix} 3 & 24 \\ 18 & 0 \end{bmatrix}$$

$$1 \cdot A = \begin{bmatrix} \frac{1}{3} \cdot 3 & \frac{1}{3} \cdot 24 \\ \frac{1}{3} \cdot 18 & \frac{1}{3} \cdot 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 8 \\ 6 & 0 \end{bmatrix}$$

In general, we solve a matrix equation just like we would solve any linear equation, except that the operations we perform are with matrices!

Check your understanding

3) Solve for X.

$$5X = \begin{bmatrix} -25 & -5 \\ 10 & 20 \end{bmatrix}$$

$$X = \begin{bmatrix} \quad \end{bmatrix}$$

Check

[I need help!]

4) Solve for Y.

$$\frac{1}{3}Y = \begin{bmatrix} 4 & -2 & 2 \\ 7 & 1 & -3 \end{bmatrix}$$

$Y =$ [ ]

Check

[I need help!]

Challenge problem

5*) Solve for Z.

$$2Z + \begin{bmatrix} 3 & 6 \\ 4 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 10 & 23 \end{bmatrix}$$

$Z =$