# Matrix Inverse



The inverse of a square matrix $\mathsf{A}$, sometimes called a reciprocal matrix, is a matrix $\mathsf{A}^{-1}$ such that

$$\mathsf{A}\,\mathsf{A}^{-1} = \mathsf{I},$$

where $\mathsf{I}$ is the identity matrix. Courant and Hilbert (1989, p. 10) use the notation $\check{\mathsf{A}}$ to denote the inverse matrix.

A square matrix $\mathsf{A}$ has an inverse iff the determinant $|\mathsf{A}| \neq 0$ (Lipschutz 1991, p. 45). The so-called invertible matrix theorem is major result in linear algebra which associates the existence of a matrix inverse with a number of other equivalent properties. A matrix possessing an inverse is called nonsingular, or invertible.

The matrix inverse of a square matrix $m$ may be taken in the Wolfram Language using the function `Inverse`[*m*].

For a $2 \times 2$ matrix

$$\mathsf{A} \equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

the matrix inverse is

$$\begin{aligned} \mathsf{A}^{-1} &= \frac{1}{|\mathsf{A}|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{a\,d - b\,c} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \end{aligned}$$

negative

flipped

For a $3 \times 3$ matrix

$$\mathsf{A} \equiv \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

the matrix inverse is

$$\mathsf{A}^{-1} = \frac{1}{|\mathsf{A}|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}.$$

A general $n \times n$ matrix can be inverted using methods such as the Gauss-Jordan elimination, Gaussian elimination, LU decomposition.

### In relation to its adjugate

The adjugate of a matrix $A$ can be used to find the inverse of $A$ as follows:

If $A$ is an $n \times n$ invertible matrix, then

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A).$$

## 2 × 2 generic matrix

The adjugate of the 2×2 matrix

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is

$$\operatorname{adj}(\mathbf{A}) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$