n-tuples and the dot product

We are familiar with ordered pairs, for example $(2, 5)$, and perhaps even ordered triples, for example $(3, 1, 8)$.

An n-tuple is a generalization of this. It is an ordered list of n numbers.

We can find the **dot product** of two n-tuples of equal length by summing the products of corresponding entries.

For example, to find the dot product of two ordered pairs, we multiply the first coordinates and the second coordinates and add the results.

$$\begin{aligned}(2, 5) \cdot (3, 1) &= 2 \cdot 3 + 5 \cdot 1 \\ &= 6 + 5 \\ &= 11\end{aligned}$$

A **matrix** is a rectangular arrangement of numbers into rows and columns. Each number in a matrix is referred to as a **matrix element** or **entry**.

$$\begin{array}{cc} & \overbrace{\downarrow \quad \downarrow \quad \downarrow}^{\text{3 columns}} \\ A = & \begin{bmatrix} -2 & 8 & 6 \\ 5 & 2 & 7 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \text{2 rows}$$

The **dimensions** of a matrix give the number of rows and columns of the matrix *in that order*. Since matrix A has 2 rows and 3 columns, it is called a 2×3 matrix.

If this is new to you, we recommend that you check out our intro to matrices.

In **matrix multiplication**, each entry in the product matrix is the dot product of a row in the first matrix and a column in the second matrix.

$$\begin{array}{l} \vec{a_1} \rightarrow \\ \vec{a_2} \rightarrow \end{array} \underset{A}{\begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix}} \cdot \underset{B}{\begin{array}{c} \begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} \end{array}} = \underset{C}{\begin{bmatrix} \vec{a_1} \cdot \vec{b_1} & \vec{a_1} \cdot \vec{b_2} \\ \vec{a_2} \cdot \vec{b_1} & \vec{a_2} \cdot \vec{b_2} \end{bmatrix}}$$

If this is new to you, we recommend that you check out our matrix multiplication article.

Multiplying matrices

Given $A = \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix}$, let's find matrix $C = AB$.

To help our understanding, let's label the rows in matrix A and the columns in matrix B. We can define the product matrix, matrix C, as shown below.

$$\begin{array}{c} \vec{a_1} \rightarrow \\ \vec{a_2} \rightarrow \end{array} \underset{A}{\begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix}} \cdot \underset{B}{\overset{\begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow \end{array}}{\begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix}}} = \underset{C}{\begin{bmatrix} \vec{a_1} \cdot \vec{b_1} & \vec{a_1} \cdot \vec{b_2} \\ \vec{a_2} \cdot \vec{b_1} & \vec{a_2} \cdot \vec{b_2} \end{bmatrix}}$$

Notice that each entry in matrix C is the ***dot product*** of a row in matrix A and a column in matrix B. Specifically, the entry $c_{i,j}$ is the dot product of $\vec{a_i}$ and $\vec{b_j}$.

For example, $c_{1,2}$ is the dot product of $\vec{a_1}$ and $\vec{b_2}$

$$\begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} \vec{a_1} \cdot \vec{b_1} & 17 \\ \vec{a_2} \cdot \vec{b_1} & \vec{a_2} \cdot \vec{b_2} \end{bmatrix}$$

Got it, thanks!

$$\begin{aligned} c_{1,2} &= (1, 7) \cdot (3, 2) \\ &= 1 \cdot 3 + 7 \cdot 2 \\ &= 3 + 14 \\ &= 17 \end{aligned}$$

When is matrix multiplication defined?

In order for matrix multiplication to be defined, the number of columns in the first matrix must be equal to the number of rows in the second matrix.

> $$(m \times n) \cdot (n \times k)$$
>
> product is defined