Practice: Gaussian Elimination

Using Gaussian elimination with back substitution, solve the following two systems of equations:

(a)

$$3x_1 - 7x_2 - 2x_3 = -7,$$
$$-3x_1 + 5x_2 + x_3 = 5,$$
$$6x_1 - 4x_2 = 2;$$

(b)

$$x_1 - 2x_2 + 3x_3 = 1,$$
$$-x_1 + 3x_2 - x_3 = -1,$$
$$2x_1 - 5x_2 + 5x_3 = 1.$$

Next Item

(a)

$$3x_1 - 7x_2 - 2x_3 = -7,$$
$$-3x_1 + 5x_2 + x_3 = 5,$$
$$6x_1 - 4x_2 = 2;$$

pivots

$$\begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

need to zero out

pivot

$$\begin{bmatrix} 3 & -7 & -2 & -7 \\ 0 & -2 & -1 & -2 \\ 0 & 10 & 4 & 16 \end{bmatrix}$$

need to zero out

$$\begin{bmatrix} 3 & -7 & -2 & -7 \\ 0 & -2 & -1 & -2 \\ 0 & 0 & -1 & 6 \end{bmatrix}$$

$$3x_1 - 7x_2 - 2x_3 = -7$$
$$-2x_2 - 1x_3 = -2$$
$$-1x_3 = 6$$

(a)

$$x_3 = -6$$
$$x_2 = 4$$
$$x_1 = 3$$

(b)

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ -1 & 3 & -1 & -1 \\ 2 & -5 & 5 & 1 \end{bmatrix}$$

pivot

need to zero out

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

pivot 2

zero out

$$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$x_1 - 2x_2 + 3x_3 = 1$$
$$x_2 + 2x_3 = 0$$
$$x_3 = -1$$

(b)

$$x_3 = -1$$
$$x_2 = 2$$
$$x_1 = 8$$