Matrix row operations

The following table summarizes the three elementary **matrix row operations.**

Matrix row operation	Example
Switch any two rows	$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 4 & 6 \\ 2 & 5 & 3 \end{bmatrix}$ (Interchange row 1 and row 2.)
Multiply a row by a nonzero constant	$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3\cdot 2 & 3\cdot 5 & 3\cdot 3 \\ 3 & 4 & 6 \end{bmatrix}$ (Row 1 becomes 3 times itself.)
Add one row to another	$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 5 & 3 \\ 3+2 & 4+5 & 6+3 \end{bmatrix}$ (Row 2 becomes the sum of rows 2 and 1.)

Matrix row operations can be used to solve systems of equations, but before we look at why, let's practice these skills.

Systems of equations and matrix row operations

Recall that in an augmented matrix, each row represents one equation in the system and each column represents a variable or the constant terms.

For example, the system on the left corresponds to the augmented matrix on the right.

System	Matrix
$\begin{matrix} 1x+3y=5 \\ 2x+5y=6 \end{matrix}$	$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 6 \end{bmatrix}$

When working with augmented matrices, we can perform any of the **matrix row operations** to create a new augmented matrix that produces an equivalent system of equations. Let's take a look at why.

Switching any two rows

Equivalent Systems	Augmented matrix
$\begin{aligned} 1x + 3y &= 5 \\ 2x + 5y &= 6 \end{aligned}$	$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 6 \end{bmatrix}$
	$\downarrow$
$\begin{aligned} 2x + 5y &= 6 \\ 1x + 3y &= 5 \end{aligned}$	$\begin{bmatrix} 2 & 5 & 6 \\ 1 & 3 & 5 \end{bmatrix}$

Multiply a row by a nonzero constant

We can multiply both sides of an equation by the same nonzero constant to obtain an equivalent equation.

In solving systems of equations, we often do this to eliminate a variable. Because the two equations are equivalent, we see that the two systems are also equivalent.

Equivalent Systems	Augmented matrix
$\begin{aligned} 1x + 3y &= 5 \\ 2x + 5y &= 6 \end{aligned}$	$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 6 \end{bmatrix}$
	$\downarrow$
$\begin{aligned} -2x + (-6)y &= -10 \\ 2x + \quad 5y &= 6 \end{aligned}$	$\begin{bmatrix} -2 & -6 & -10 \\ 2 & 5 & 6 \end{bmatrix}$

This means that when using an augmented matrix to solve a system, we can **multiply any row by a nonzero constant**.

We do this often when solving systems of equations. For example, in this system $\begin{aligned} -2x - 6y &= -10 \\ 2x + 5y &= 6 \end{aligned}$, we can add the equations to obtain $-y = -4$.

Pairing this new equation with either original equation creates an equivalent system of equations.
Why do we keep one of the original equations?

Equivalent Systems	Augmented matrix
$\begin{aligned} -2x - 6y &= -10 \\ 2x + 5y &= 6 \end{aligned}$	$\begin{bmatrix} -2 & -6 & -10 \\ 2 & 5 & 6 \end{bmatrix}$
	$\downarrow$
$\begin{aligned} -2x + (-6)y &= -10 \\ 0x + (-1)y &= -4 \end{aligned}$	$\begin{bmatrix} -2 & -6 & -10 \\ 0 & -1 & -4 \end{bmatrix}$

So when using an augmented matrix to solve a system, we can **add one row to another**.