## Matrix Inverse





The inverse of a square matrix A, sometimes called a reciprocal matrix, is a matrix  $A^{-1}$  such that

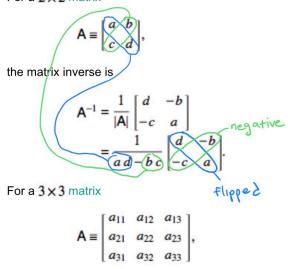
$$A A^{-1} = I$$
,

where I is the identity matrix. Courant and Hilbert (1989, p. 10) use the notation  $\check{\mathbf{A}}$  to denote the inverse matrix.

A square matrix  $\mathbf{A}$  has an inverse iff the determinant  $|\mathbf{A}| \neq 0$  (Lipschutz 1991, p. 45). The so-called invertible matrix theorem is major result in linear algebra which associates the existence of a matrix inverse with a number of other equivalent properties. A matrix possessing an inverse is called nonsingular, or invertible.

The matrix inverse of a square matrix m may be taken in the Wolfram Language using the function Inverse[m].

## For a 2 x 2 matrix



the matrix inverse is

$$\mathsf{A}^{-1} = \frac{1}{|\mathsf{A}|} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{bmatrix} \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$$

$$\mathsf{A}^{-1} = \frac{1}{|\mathsf{A}|} \begin{bmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{bmatrix} \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} \begin{bmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{bmatrix}.$$

$$\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}.$$

A general  $n \times n$  matrix can be inverted using methods such as the Gauss-Jordan elimination, Gaussian elimination, LU decomposition.

## In relation to its adjugate

The adjugate of a matrix  $\boldsymbol{A}$  can be used to find the inverse of  $\boldsymbol{A}$  as follows:

If A is an n imes n invertible matrix, then

$$A^{-1}=rac{1}{\det(A)}\operatorname{adj}(A).$$

## 2 × 2 generic matrix

The adjugate of the 2×2 matrix

$$\mathbf{A} = egin{pmatrix} a & b \ c & d \end{pmatrix}$$

is

$$\operatorname{adj}(\mathbf{A}) = \left( egin{array}{cc} d & -b \ -c & a \end{array} 
ight).$$