What you should be familiar with before taking this lesson

A **matrix** is a rectangular arrangement of numbers into rows and columns.

$$\begin{array}{c} \text{3 columns} \\ \downarrow \quad \downarrow \quad \downarrow \\ A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \text{2 rows} \end{array}$$

The **dimensions** of a matrix give the number of rows and columns of the matrix *in that order*. Since matrix A has 2 rows and 3 columns, it is called a 2×3 matrix.

If this is new to you, you might want to check out our intro to matrices. You should also make sure you know how to add and subtract matrices and how to multiply a matrix by a scalar.

Definition of zero matrix

A **zero matrix** is a matrix in which all of the entries are 0. Some examples are given below.

3×3 zero matrix: $$O_{3\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2×4 zero matrix: $$O_{2\times4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A zero matrix is indicated by O, and a subscript can be added to indicate the dimensions of the matrix if necessary.

Zero matrices play a similar role in operations with matrices as the number zero plays in operations with real numbers. Let's take a look.

Investigation: What happens when we add a zero matrix?

Recall that to add two matrices, we simply add the corresponding entries.

[I want to see an example before I practice.]

Now try the following matrix addition problems. Notice that each problem involves the sum of a matrix and a zero matrix.

1)

$$\begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Check

[I need help!]

2)

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 4 & 8 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Check

[I need help!]

The conclusion

When we add the $m \times n$ zero matrix to any $m \times n$ matrix A, we get matrix Aback. In other words, $A + O = A$ and $O + A = A$.

Here the dimensions of the zero matrix are not explicitly given. It is understood that the dimensions of the zero matrix match the dimensions of matrix A.

Reflection question

What are the dimensions of the zero matrix in the equation $B + O = B$ given that $B = \begin{bmatrix} -2 & 5 & 6 \\ 8 & 1 & 8 \end{bmatrix}$?

$\square \times \square$

Check

[I need help!]

Investigation: What happens when we add opposite matrices?

The **opposite** of a matrix A is the matrix $-A$, where each element in this matrix is the *opposite* of the corresponding element in matrix A.

For example, if $A = \begin{bmatrix} 4 & 1 \\ -6 & 2 \end{bmatrix}$, then $-A = \begin{bmatrix} -4 & -1 \\ 6 & -2 \end{bmatrix}$.

Now try the following matrix addition problems. Notice that each problem involves the sum of a matrix and its opposite.

3)

$$\begin{bmatrix} 4 & -3 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ -8 & -7 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Check

[I need help!]

4)

$$\begin{bmatrix} -4 & 2 & 5 \\ 1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -2 & -5 \\ -1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Check

[I need help!]

The conclusion

When we add any $m \times n$ matrix to its opposite, we get the $m \times n$ zero matrix. So if A is any matrix, then $A + (-A) = O$ and $-A + A = O$.

It is also true that $A - A = O$. This is because subtracting a matrix is like adding its opposite. [Can you show me why?]

Investigation: What happens when we multiply a matrix by the scalar 0?

When we multiply a matrix by a scalar, each entry in the matrix is multiplied by the given scalar.

[I want to see an example before I practice.]

Now try the following matrix scalar multiplication problems. Notice that each problem involves multiplying a matrix by the scalar 0.

5)

$0 \cdot \begin{bmatrix} 5 & 4 \\ 9 & 1 \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$

Check

[I need help!]

6)

$0 \cdot \begin{bmatrix} -2 & 4 & 10 \\ 7 & -1 & 5 \\ -3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$

Check

[I need help!]

The conclusion

When we multiply any $m \times n$ matrix by the scalar 0, we get the $m \times n$ zero matrix.

Mathematically, this means that $0A = O$.

Summary: Comparing the zero matrix to the real number zero

In the investigations above, we saw that a zero matrix behaves much like the real number zero.

In particular, we can make the following connections:

The number zero	The zero matrix
Adding zero to any number $a$gives back that number a. (eg. $a + 0 = a$)	Adding a zero matrix to any matrix Agives back the matrix A. (eg. $A + O = O + A = A$)
Adding any number to its opposite will give zero. (eg. $a + (-a) = 0$)	Adding any matrix to its opposite will give a zero matrix. (e.g. $A + (-A) = O$)
Any number times zero is zero. (e.g $a \cdot 0 = 0$).	Scalar multiplication of a matrix by 0 will give a zero matrix. (eg. $0A = O$)

Understanding these connections can help make matrix calculations involving a zero matrix much easier!