What you should be familiar with before taking this lesson

A **matrix** is a rectangular arrangement of numbers into rows and columns.

$$\begin{array}{ccc} & \text{3 columns} & \\ & \downarrow \quad \downarrow \quad \downarrow & \\ A = & \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} & \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \text{2 rows} \end{array}$$

The **dimensions** of a matrix tell the number of rows and columns of the matrix *in that order*. Since matrix A has 2 rows and 3 columns, it is called a 2×3matrix.

If this is new to you, we recommend that you check out our intro to matrices.

In **matrix multiplication**, each entry in the product matrix is the dot product of a row in the first matrix and and a column in the second matrix.

$$\begin{array}{cccccc} & & & b_1 \quad b_2 & & \\ & & & \downarrow \quad \downarrow & & \\ \begin{array}{c} a_1 \rightarrow \\ a_2 \rightarrow \end{array} & \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} & \cdot & \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} & = & \begin{bmatrix} \vec{a}_1 \cdot \vec{b}_1 & \vec{a}_1 \cdot \vec{b}_2 \\ \vec{a}_2 \cdot \vec{b}_1 & \vec{a}_2 \cdot \vec{b}_2 \end{bmatrix} \\ & A & & B & & C \end{array}$$

If this is new to you, we recommend that you check out our matrix multiplication article.

Definition of identity matrix

The $n \times n$ **identity matrix**, denoted I_n, is a matrix with n rows and n columns. The entries on the diagonal from the upper left to the bottom right are all 1's, and all other entries are 0.

For example:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The identity matrix plays a similar role in operations with matrices as the number 1 plays in operations with real numbers. Let's take a look.

Investigation: Multiplying by the identity matrix

Try a few multiplication problems involving the appropriate identity matrix.

1) $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$.

$I_2 \cdot A = \begin{bmatrix} \end{bmatrix}$

Check

[I need help!]

2) $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 5 & 4 \\ 3 & 2 & 2 \\ 4 & 1 & 3 \end{bmatrix}$.

$A \cdot I_3 = \begin{bmatrix} \; \end{bmatrix}$

Check

[I need help!]

The conclusion

The product of any square matrix and the appropriate identity matrix is always the original matrix, regardless of the order in which the multiplication was performed! In other words, $A \cdot I = I \cdot A = A$.

Connections to the real numbers

Multiplicative Identities

The identity matrix I plays a similar role to what the number 1 plays in the real number system.

The number 1	**The identity matrix I**
The product of 1 and any number a is a. $(a \cdot 1 = 1 \cdot a = a)$	The product of a square matrix A and the appropriate identity matrix I is A. $(A \cdot I = I \cdot A = A)$

Multiplicative Inverses

Two real numbers whose product is the multiplicative identity are called multiplicative inverses. For example, the numbers $\frac{1}{3}$ and 3 are multiplicative inverses because $\frac{1}{3} \cdot 3 = 1$ and $3 \cdot \frac{1}{3} = 1$.

In fact, all nonzero real numbers have multiplicative inverses. But does this connection hold with matrix operations?

Consider matrices A and B.

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

We can multiply to see that $AB = I_2$ and $BA = I_2$. [I'd like to see this, please!]

This means that A and B are multiplicative inverses.

However, as we will see, not all matrices have multiplicative inverses. This is one place where the properties of real numbers differ from the properties of matrices!