Let A be a rectangular matrix given by

$$A = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}.$$

Compute $A^{T}A$ and show that it is a symmetric matrix and that the sum of its diagonal elements is the sum of the squares of all the elements of A.

$$A^{T}A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

$$\underbrace{2 \times 3}_{ok} \times \underbrace{3 \times 2}_{} \Rightarrow 2 \times 2$$

$$= \begin{bmatrix} a \cdot a + b \cdot b + c \cdot c & a \cdot d + b \cdot e + c \cdot f \\ d \cdot a + e \cdot b + f \cdot c & d \cdot d + e \cdot e + f \cdot f \end{bmatrix}$$

$$= \begin{bmatrix} a^{2} + b^{2} + c^{2} & a \cdot d + b \cdot e + c \cdot f \\ a \cdot d + b \cdot e + c \cdot f & d^{2} + e^{2} + f^{2} \end{bmatrix}$$