

Representing linear systems with matrices

Learn how systems of linear equations can be represented by augmented matrices.

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A **matrix** is a rectangular arrangement of numbers into rows and columns.

3 columns

↓ ↓ ↓

$$A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix}$$

← ← 2 rows

Matrices can be used to solve systems of equations. But first, we must learn how to represent systems with matrices.

Representing a linear system with matrices

A system of equations can be represented by an **augmented matrix**.

System of equations:

$$\begin{aligned} 2x + 5y &= 10 \\ 3x + 4y &= 24 \end{aligned}$$

Augmented matrix:

$$\begin{bmatrix} 2 & 5 & 10 \\ 3 & 4 & 24 \end{bmatrix}$$

↑ ↑ ↑
 x y constants

← Eq. 1
← Eq. 2

In an augmented matrix, each row represents one equation in the system and each column represents a variable or the constant terms.

In this way, we can see that augmented matrices are a shorthand way of writing systems of equations. The organization of the numbers into the matrix makes it unnecessary to write various symbols like x , y , and $=$, yet all of the information is still there!

Check your understanding

1) Which matrix represents the system?

$$2x + 3y = 8$$

$$5x + 2y = 2$$

Choose 1 answer:

☐ A

$$\begin{bmatrix} 2 & 3 & 8 \\ 2 & 5 & 2 \end{bmatrix}$$

☐ B

$$\begin{bmatrix} 2 & 3 & 8 \\ 5 & 2 & 2 \end{bmatrix}$$

Check

[I need help!]

2) Write the following system of equations as an augmented matrix.

$$7x + 4y = 3$$

$$6x + 3y = 5$$

[]

Check

[I need help!]

Let's look at another example

Now that we have the basics, let's take a look at a slightly more complicated example.

Example

Write the following system of equations as an augmented matrix.

$$3x - 2y = 4$$

$$x + 5z = -3$$

$$-4x - y + 3z = 0$$

Solution

To make things easier, let's rewrite the system to show each of the coefficients clearly. If a variable term is not written in an equation, it means that the coefficient is 0.

$$3x + (-2)y + 0z = 4$$

$$1x + 0y + 5z = -3$$

$$-4x + (-1)y + 3z = 0$$

This corresponds to the following augmented matrix.

$$\left[\begin{array}{cccc|c} 3 & -2 & 0 & 4 & \\ 1 & 0 & 5 & -3 & \\ -4 & -1 & 3 & 0 & \end{array} \right] \begin{array}{l} \leftarrow \text{Eq. 1} \\ \leftarrow \text{Eq. 2} \\ \leftarrow \text{Eq. 3} \end{array}$$

$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ x & y & z & \text{constants} \end{array}$

Again, notice how each column corresponds to a variable (x , y , z) or the **constants**. Also notice that the numbers in each row correspond to the coefficients in the same equation.

In general, before converting a system into an augmented matrix, be sure that the variables appear in the same order in each equation, and that the constant terms are isolated on one side.

Check your understanding

3) Which matrix represents the system?

$$3w - 2x + y + 5z = 10$$

$$w + 2y - 4z = 5$$

Choose 1 answer:

A

$$\left[\begin{array}{ccccc} 3 & -2 & 1 & 5 & 10 \\ 1 & 0 & 2 & -4 & 5 \end{array} \right]$$

B

$$\left[\begin{array}{ccccc} 3 & -2 & 1 & 5 & 10 \\ 0 & 1 & 2 & -4 & 5 \end{array} \right]$$

Check

[I need help!]

4) Write the following system of equations as an augmented matrix.

$$-a + b - 2c = 12$$

$$3a + b = -8$$

[]

Check

[I need help!]

Challenge problems

5*) Which system is represented by the augmented matrix?

$$\left[\begin{array}{cccc|c} 0 & -3 & -5 & 8 & \\ 4 & 1 & 1 & 2 & \\ 5 & 2 & -1 & 0 & \end{array} \right]$$

Choose 1 answer:

A

$$-3x - 5y = 8$$

$$4x + y + z = 2$$

$$5x + 2y - z = 0$$

B

$$-3y - 5z = 8$$

$$4x + y + z = 2$$

$$5x + 2y - z = 0$$

Check

[I need help!]

6*) Which matrix represents the system?

$$3x + 2 = 12y$$

$$-8y = 2x + 15$$

Choose 1 answer:

A

$$A = \begin{bmatrix} 3 & 2 & 12 \\ -8 & 2 & 15 \end{bmatrix}$$

B

$$B = \begin{bmatrix} 3 & 12 & 2 \\ 2 & -8 & 15 \end{bmatrix}$$

C

$$C = \begin{bmatrix} 3 & -12 & -2 \\ -2 & -8 & 15 \end{bmatrix}$$

D

$$D = \begin{bmatrix} 3 & -12 & -2 \\ -8 & -2 & 15 \end{bmatrix}$$
