In the table below, A, B, and C are matrices of equal dimensions.

Property	Example
Commutative property of addition	$A+B=B+A$
Associative property of addition	$A+(B+C)=(A+B)+C$
Additive identity property	For any matrix A, there is a unique matrix Osuch that $A+O=A$.
Additive inverse property	For each A, there is a unique matrix $-A$ such that $A+(-A)=O$.
Closure property of addition	$A+B$ is a matrix of the same dimensions as $A$and B.

This article explores these matrix addition properties.

Matrices and matrix addition

$$\begin{array}{ccc} & \text{3 columns} & \\ A = & \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} & \text{2 rows} \end{array}$$

A **matrix** is a rectangular arrangement of numbers into rows and columns. The **dimensions** of a matrix give the number of rows and columns of the matrix *in that order*. Since matrix A has 2 rows and 3 columns, it is called a 2×3 matrix.

To add two matrices of the same dimensions, simply add the entries in the corresponding positions.

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 3+5 & 7+2 \\ 2+8 & 4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 9 \\ 10 & 5 \end{bmatrix}$$

If any of this is new to you, you should check out the following articles before you proceed:

- Intro to matrices
- Adding & subtracting matrices
- Intro to zero matrices

Dimensions considerations

Notice that the sum of two 2×2 matrices is another 2×2 matrix. In general, the sum of two $m \times n$ matrices is another $m \times n$ matrix. This describes the **closure property** of matrix addition.

If the dimensions of two matrices are not the same, the addition is not defined. This is because if A is a 2×3 matrix and B is a 2×2 matrix, then some entries in matrix A will not have corresponding entries in matrix B!

$$\begin{bmatrix} 2 & 7 & 8 \\ 2 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 8 & 1 \end{bmatrix} = \text{undefined}$$

Matrix addition & real number addition

Because matrix addition relies heavily on the addition of real numbers, many of the addition properties that we know to be true with real numbers are also true with matrices.

Let's take a look at each property individually.

Commutative property of addition: $A + B = B + A$

This property states that you can add two matrices in any order and get the same result.

This parallels the commutative property of addition for real numbers. For example, $3 + 5 = 5 + 3$.

The following example illustrates this matrix property.

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 3+5 & 7+2 \\ 2+8 & 4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5+3 & 2+7 \\ 8+2 & 1+4 \end{bmatrix} \qquad \text{(Real \# addition is comm}$$

$$= \begin{bmatrix} 5 & 2 \\ 8 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$$

Notice how the commutative property of addition for matrices holds thanks to the commutative property of addition for real numbers!

Associative property of addition: $(A + B) + C = A + (B + C)$

This property states that you can change the grouping in matrix addition and get the same result. For example, you can add matrix A to B first, and then add matrix C, **or**, you can add matrix B to C, and then add this result to A.

This property parallels the associative property of addition for real numbers. For example, $(2+3)+5=2+(3+5)$.

Let's justify this matrix property by looking at an example.

$(A+B)+C$	$A+(B+C)$
$\left(\begin{bmatrix}5 & 2\\ 2 & 8\end{bmatrix}+\begin{bmatrix}3 & 4\\ 9 & 6\end{bmatrix}\right)+\begin{bmatrix}1 & 0\\ 3 & 7\end{bmatrix}$	$\begin{bmatrix}5 & 2\\ 2 & 8\end{bmatrix}+\left(\begin{bmatrix}3 & 4\\ 9 & 6\end{bmatrix}+\begin{bmatrix}1 & 0\\ 3 & 7\end{bmatrix}\right)$
$\downarrow$	$\downarrow$
$\begin{bmatrix}5+3 & 2+4\\ 2+9 & 8+6\end{bmatrix}+\begin{bmatrix}1 & 0\\ 3 & 7\end{bmatrix}$	$\begin{bmatrix}5 & 2\\ 2 & 8\end{bmatrix}+\begin{bmatrix}3+1 & 4+0\\ 9+3 & 6+7\end{bmatrix}$
$\downarrow$	$\downarrow$
$\begin{bmatrix}(5+3)+1 & (2+4)+0\\ (2+9)+3 & (8+6)+7\end{bmatrix}$ $=$	$\begin{bmatrix}5+(3+1) & 2+(4+0)\\ 2+(9+3) & 8+(6+7)\end{bmatrix}$

$\uparrow$

Real # addition is associative

In each column we simplified one side of the identity into a single matrix. The two resulting matrices are equivalent thanks to the real number associative property of addition. For example, $(5+3)+1=5+(3+1)$.

Because of this property, we can write down an expression like $A+B+C$and have this be completely defined. We do not need parentheses indicating which addition to perform first, as it doesn't matter!

Additive identity property: $A+O=A$

A **zero matrix**, denoted O, is a matrix in which all of the entries are 0.

Notice that when a zero matrix is added to any matrix A, the result is always A.

- $\begin{bmatrix} 3 & -1 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 7 & 9 \end{bmatrix}$

- $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 8 & 3 \\ -1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 8 & 3 \\ -1 & 5 & 7 \end{bmatrix}$

These examples illustrate what is meant by the additive identity property; that the sum of any matrix A and the the appropriate zero matrix is the matrix A.

A zero matrix can be compared to the number zero in the real number system. For all real numbers a, we know that $a + 0 = a$. The number 0 is the additive identity in the real number system just like O is the additive identity for matrices.

Additive inverse property: $A + (-A) = O$

The **opposite** of a matrix A is the matrix $-A$, where each element in this matrix is the *opposite* of the corresponding element in matrix A.

For example, if $A = \begin{bmatrix} -2 & 8 \\ -3 & 1 \end{bmatrix}$, then $-A = \begin{bmatrix} 2 & -8 \\ 3 & -1 \end{bmatrix}$.

If we add A to $-A$ we get a zero matrix, which illustrates the additive inverse property.

$$A + (-A) = \begin{bmatrix} -2 & 8 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -8 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+2 & 8+(-8) \\ -3+3 & 1+(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The sum of a real number and its opposite is always 0, and so the sum of any matrix and its opposite gives a zero matrix. Because of this, we refer to opposite matrices as **additive inverses**.

Check your understanding

For the problems below, let A, B, and C be 2×2 matrices.

1) Which of the following matrix expressions are equivalent to $(A + B) + C$?

Choose all answers that apply:

(A)
$A + (B + C)$

(B)
$(C + A) + B$

(C)
$A + B + C$

(D)
$(A + C) + (B + C)$

Check

[I need help!]

2) Which of the following matrix expressions are equivalent to $(A+(-A))+B$?

Remember A and B are 2×2 matrices.

Choose all answers that apply:

(A)
B

(B)
$A+(-A+B)$

(C)
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}+B$

(D)
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}+B$