Practice: Solving (LU)x = b

Using

$$A = \begin{pmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{pmatrix}}_{U} = LU,$$

compute the solution to $Ax = b$ with

(a) $b = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$;

(b) $b = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(a)

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{pmatrix}}_{U} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 2 \end{bmatrix}$$

$x_1 = -3$ $\quad \boxed{x_1 = -3}$

$-x_1 + x_2 = 3$ $\quad \boxed{x_2} = 3 + (-3) = \boxed{0}$

$2x_1 - 5x_2 + x_3 = 2$ $\quad \boxed{x_3} = 2 - 2(-3) + 5(0) = \boxed{8}$

(b)

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{pmatrix}}_{U} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$x = b$ with

$x_1 = 1$ $\quad \boxed{x_1 = 1}$

$-x_1 + x_2 = -1$ $\quad \boxed{x_2} = -1 + 1 = \boxed{0}$

$2x_1 - 5x_2 + x_3 = 1$ $\quad \boxed{x_3} = 1 - 2(1) + 5(0) = \boxed{-1}$

Steps to get L and U

① Calculate upper using gaussian elimination and save m_1, m_2 and m_3 fundamental matrices

② Create L pivots and zeroes $\begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ & & 1 \end{bmatrix}$

③ Combine m_1, m_2, m_3 and make all non identity numbers negative

①

$$A = \begin{pmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{pmatrix} \times \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{m_1} =$$

$$\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 6 & -4 & 0 \end{bmatrix} \times \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}}^{m_2} =$$

$$\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 10 & 4 \end{bmatrix} \times \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}}^{m_3} =$$

$$\boxed{\begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix} = U}$$

③

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} = L}$$