Analytical Design, Control System Lag Lead and PID Compensators

Topics: **Lag Lead, and PID** with comparison to autotuned compensation in matlab

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Introduction

In this report, analytical design discussed in Control of Mechatronic Systems by Levent Guvenc is used to design a lead lag and PID controller for a **sevo motor -** accessible within the automated driving lab at Ohio State. The following sections are a mix of MATLAB code, generated plots, and comments. At the end, there is a comparison of the generated controllers from the analytical methods to a PID automatically tuned within Matlab.

Start of Script

Clearing the Variables

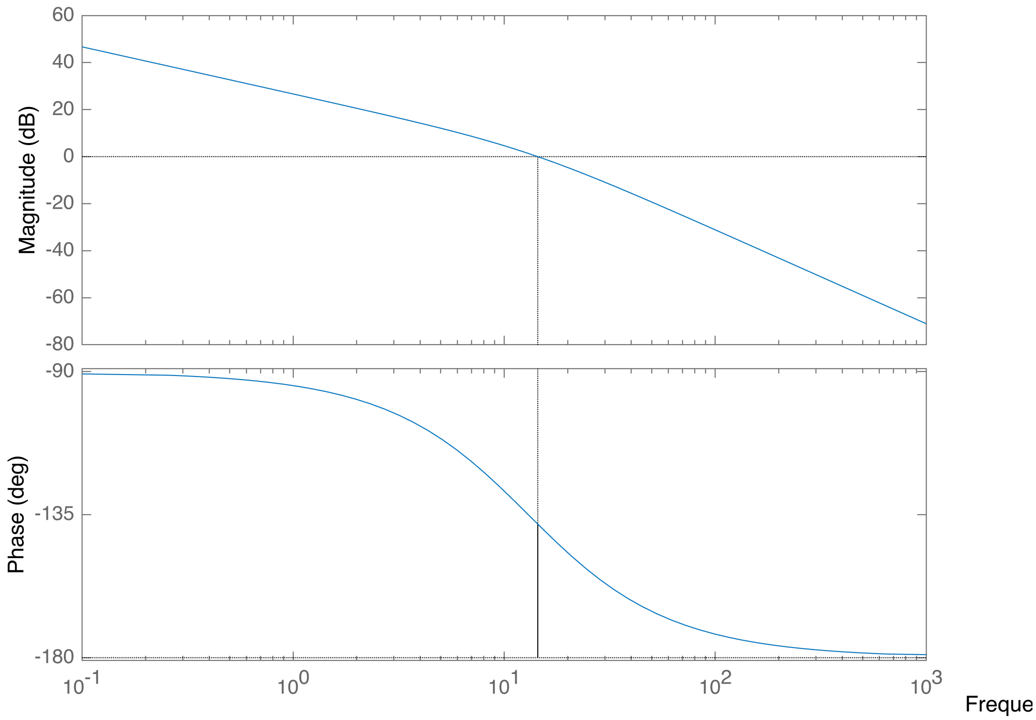
```
clc; clear; close all
```

Plant: Qube Servo Angular Position Plant, already in the Laplace Domain

```
%SISO System
num = 28; den = [0.1 1 0];
G = tf(num,den)
```

Plant Characteristics

```
margin(G)
```



this is a second order system

G has one pole at the origin, making it type 1

Symbolic Plant

```
syms s; num = 28; den = 0.1*s^2+s;
sG = num/den;
```

Inputs: static error constant, desired bandwidth (approximated by gain crossover frequency) and phase margin.

```
%Input a different plant here to test changes in time constant
%G = Gtau(1)
```

Analytical Design of a Phase Lead Compensator

Choose a gain K from error constant specification

Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0 BAD
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞ BAD	$K_a = 0$	∞	$K_a =$ Constant GOOD	$\frac{1}{K_a}$

The Velocity Error Constant gives constant error for a type 1 system

Picking a Steady State Error and deriving the new gain

```
Kold = 28; %the current plant has a gain of 28
%RULE OF THUMB, do not increase the gain too much (as increasing the gain is asking fo
%steady state error, which is less likely to be stable)
sse = 0.03; %picking a desired steady state error
Knew = 1/sse; %from the table above
K=Knew/Kold; %what the current plant must be multiplied by
alpha_guess = 0.1;
KcAlpha = K;
```

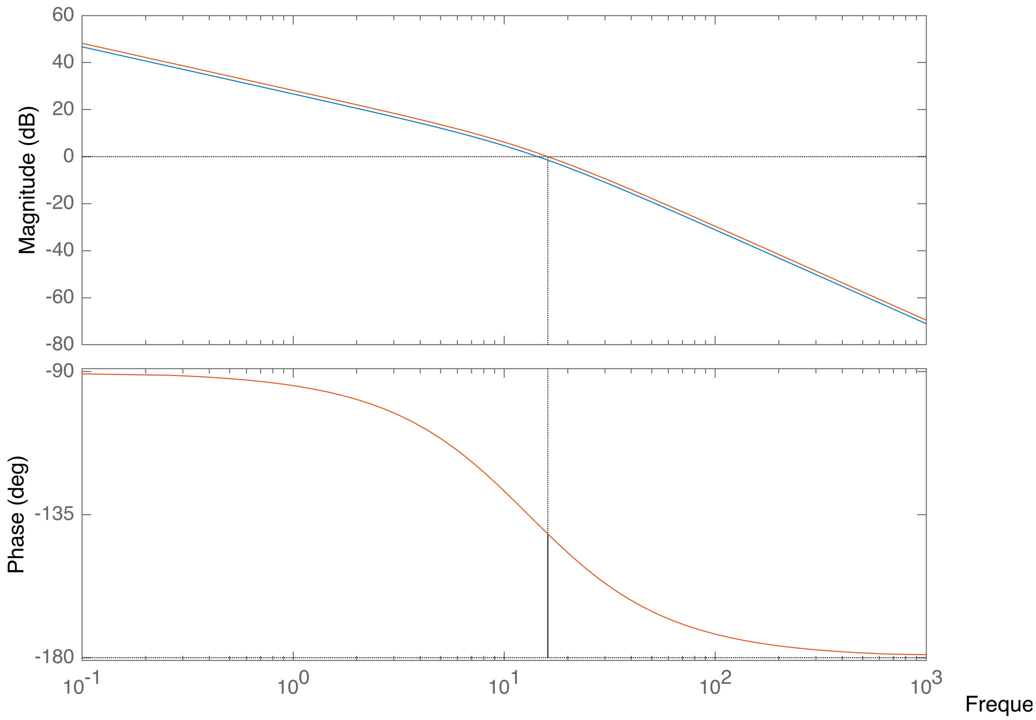
The New Plant

```
Gnew = K*G
```

```
Gnew =

       33.33
  ---------------
  0.1 s^2 + 1.3 s

Continuous-time transfer function.
```

The New Plants Margins and crossover frequencies

```
close all, figure(4), hold on, bode(G), margin(Gnew), hold off
```



The gain crossover approximates the current bandwidth

```
[Gm,Pm,W,WgainCrossover] = margin(Gnew)
```

```
Gm = Inf
Pm = 38.9102
W = Inf
WgainCrossover = 16.1052
```

Choose a new gain crossover smaller than the current crossover refrenced above

```
wgc = WgainCrossover*2; %RULE OF THUMB: for Lead we want to increase gain crossover to
% Lead Compensator Analytical Formula Design
%Revision Brian Lesko
PM=60; % desired phase margin in degrees
Gn = subs(sG,'s',[wgc*j]); % enter plant
theta=-180+PM-angle(Gn)*180/pi; % calculate required phase
    theta=theta*pi/180; % convert to radians
theta = double(theta);
% use formulas for control parameters
p=sin(theta)*wgc/(cos(theta)-KcAlpha*abs(Gn)); p=double(p);
z=sin(theta)*wgc/((1/KcAlpha/abs(Gn))-cos(theta)); z= double(z);
% substitute and check
s=tf('s');
alpha=z/p; % alpha of lead compensator
Kc=KcAlpha/alpha; % gain Kc of lead compensator
```

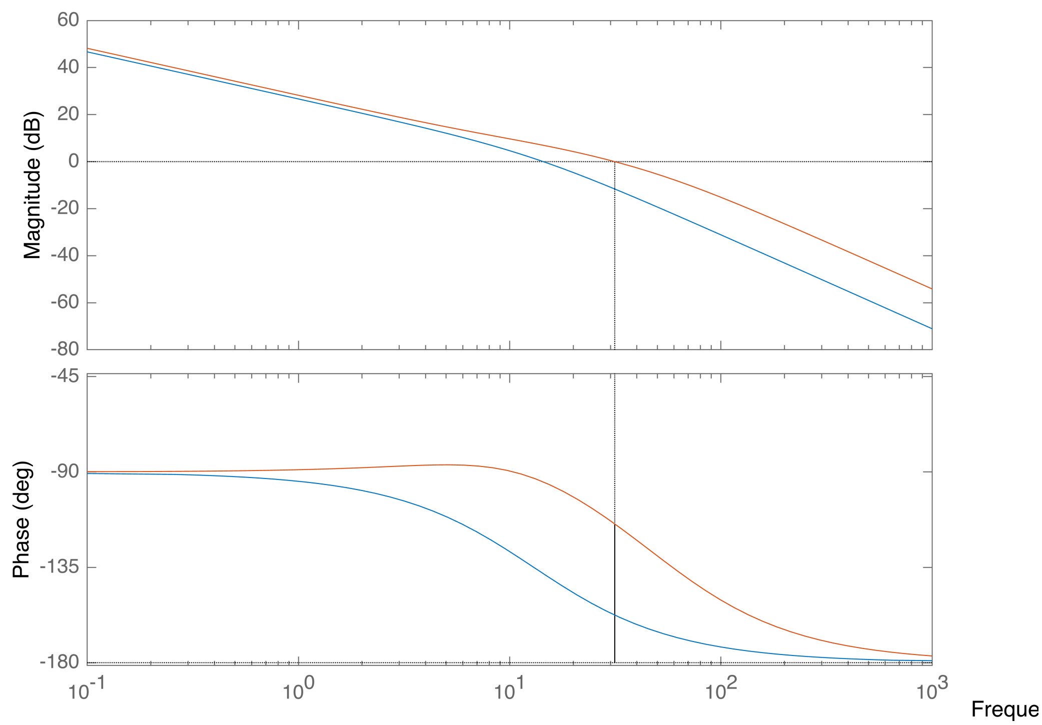
```
Cp=((1/z)*s+1)/((1/p)*s+1); % normalized lead compensator with unity dc gain
Clead=Kc*alpha*Cp; % lead compensator
max(pole(Clead*G))
```

```
ans = 0
```

```
pzplot(Clead*G), ax = gca; ax.XAxisLocation = 'origin'; ax.YAxisLocation = 'origin';
```

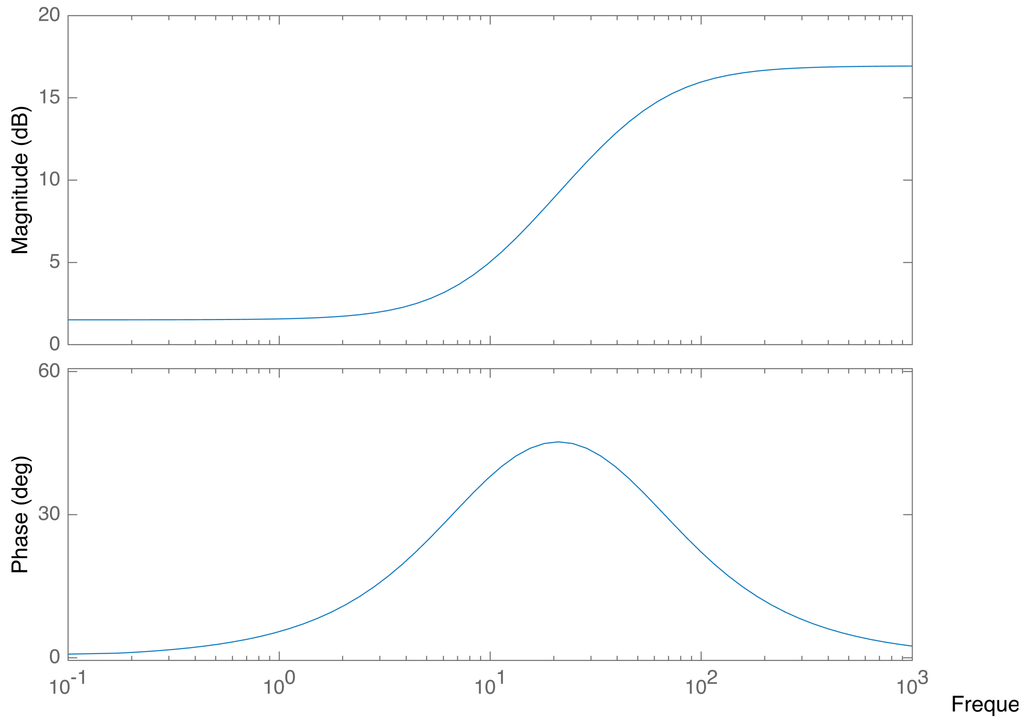
Bode Plot Comparison G and G*Clead

```
close all, figure(6), hold on, bode(G), margin(Clead*G), hold off
```



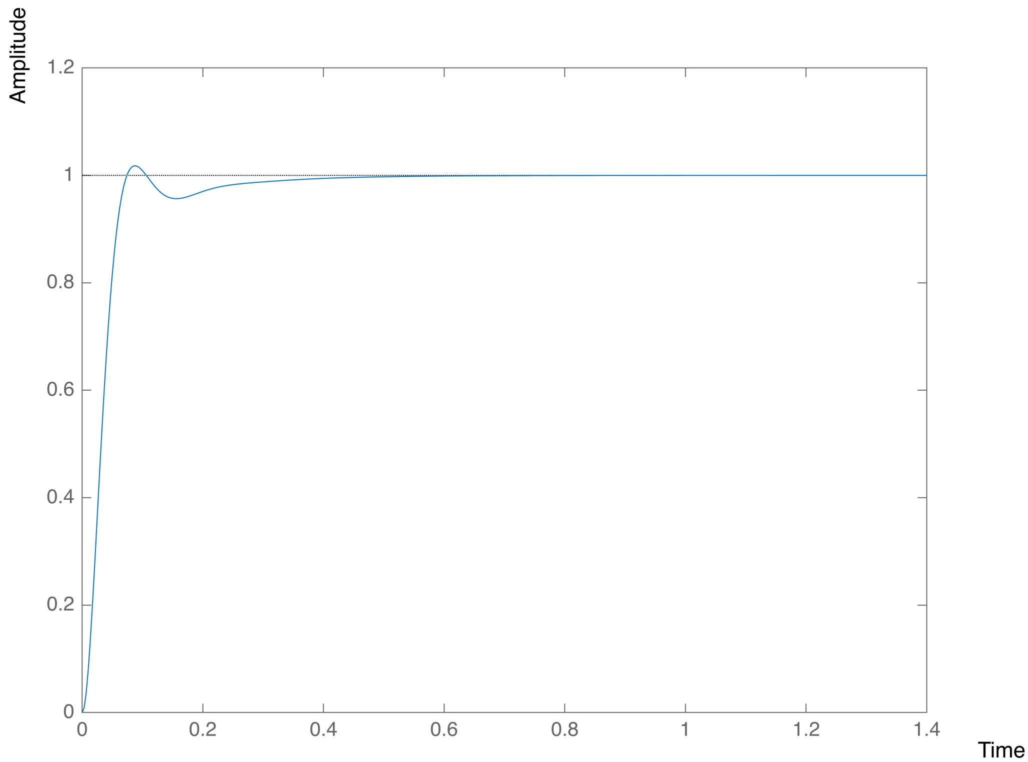
Bode of the Lead Controller

```
bode(Clead), title('Bode of the lead controller alone')
```



Step Response with The Lead Compensator

```
sys = feedback(G*Clead,1);
step(sys), title('Lead Compensation Step Response')
```



Everything looks as expected

Analytical Design of a Phase Lag Compensator for the Lead System

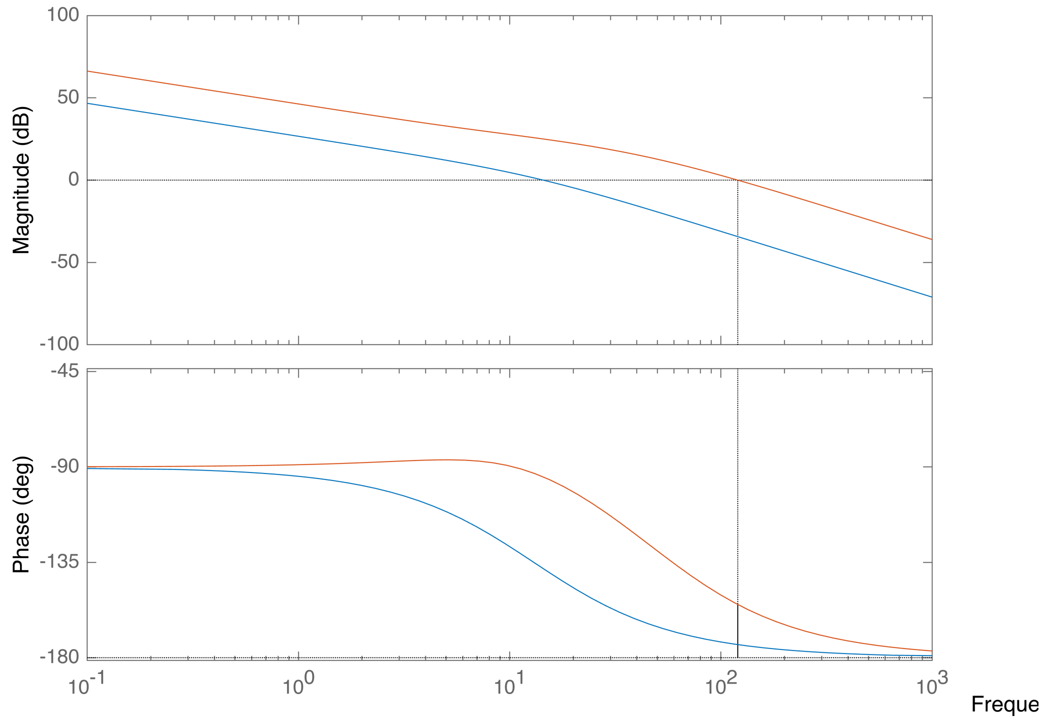
Static Error Constant Kp (Kp is for step) steady state error = 1/(1+Kp)

RULE OF THUMB. for lead lag, in the lag part, we only want to increase Kv, try 8*Kvold, OVERALL GAIN increased around 10.

```
close all
KcBeta = 8;
```

Bode plot with KcAlpha Gain vs Original

```
close all, figure(1), hold on, bode(G), margin(KcBeta*Clead*G)
```



bandwidth increase to account for phase lag's tendency to reduce bandwidth

```
[Gm,Pm,W,wgcold] = margin(G*KcBeta*Clead)
```

```
Gm = Inf
Pm = 25.1060
W = Inf
wgcold = 120.1920
```

```
wgc = wgcold*1;
% Lag Compensator Analytical Formula Design
%Revision: Brian Lesko
PM=80; % desired phase margin in degrees
%KcBeta=10; % static gain from error constant specification
%sn=wgc*j;
Gn = subs(sG,{'s'},[wgc*j]);
theta=-180+PM-angle(Gn)*180/pi; % calculate required phase
    theta=double(theta);
theta=theta*pi/180;% convert to radians
% use formulas for control parameters
p=sin(theta)*wgc/(cos(theta)-KcBeta*abs(Gn));
z=sin(theta)*wgc/((1/KcBeta/abs(Gn))-cos(theta));
    z = double(z); p = double(p);
% substitute and check
s=tf('s');
beta=z/p; % beta of lag compensator
```

```
Kc=KcBeta/beta; % gain Kc of lag compensator
Cp=((1/z)*s+1)/((1/p)*s+1); % normalized lag compensator with unity dc gain
Clag=Kc*beta*Cp % lag compensator
```

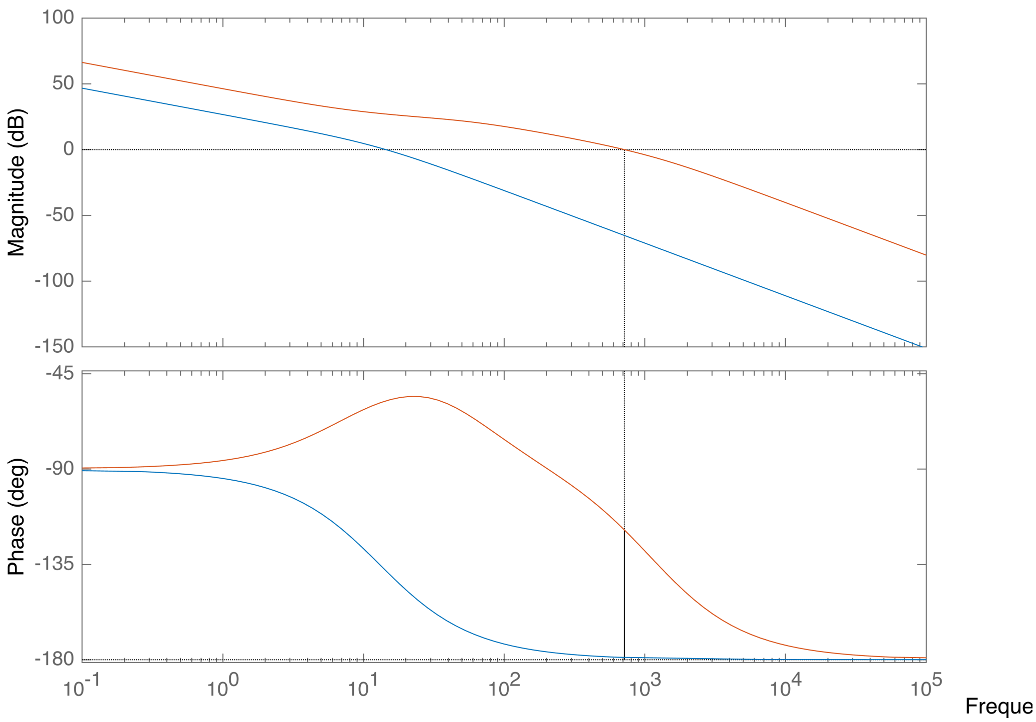
```
Clag =

   0.4279 s + 8
  ---------------
  0.0008619 s + 1

Continuous-time transfer function.
```

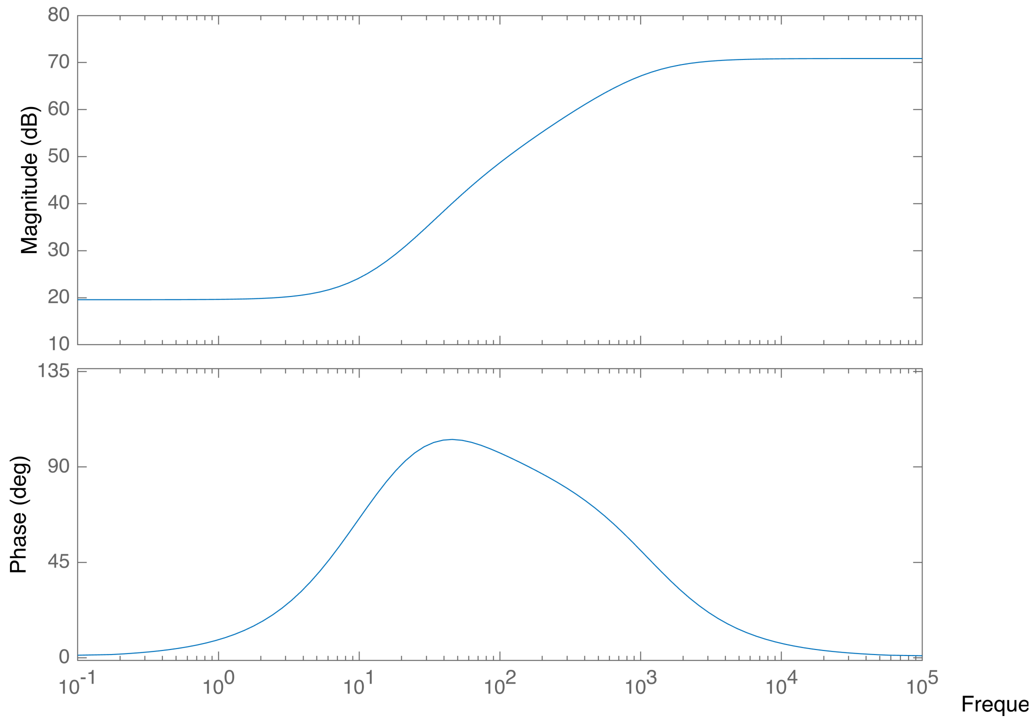
phase Lag vs original plant Bode plot

```
close all, hold on, figure(2), bode(G), margin(Clag*G*Clead);shg % check if desired PM
```

Bode of Lag Controller * Lead Controller

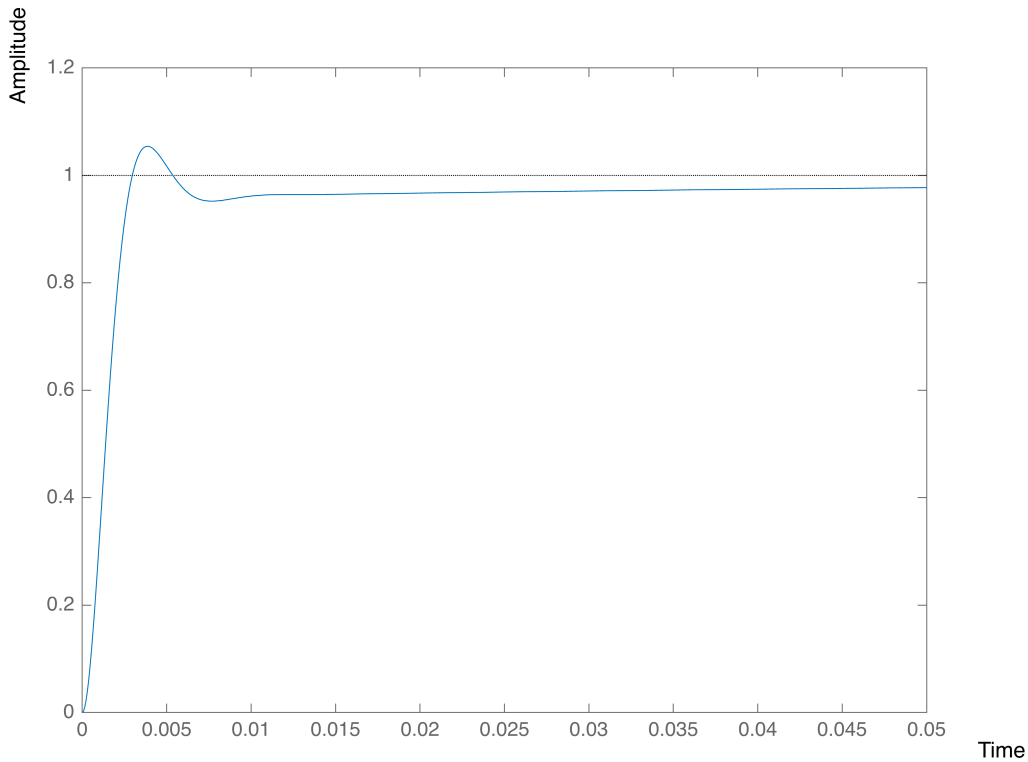
```
hold off, figure(3), bode(Clag*Clead)
```





Step of Lag Lead Compensated Plant

```
step(feedback(Clag*Clead*G,1))
```

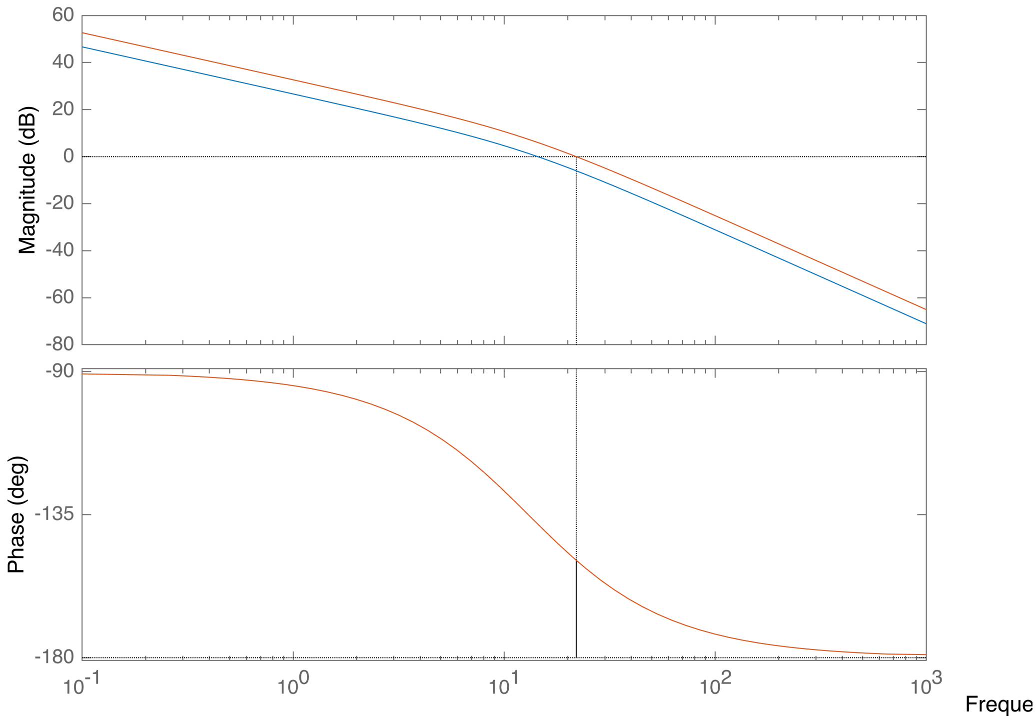


This is the Lag Lead Controller analytical result

3. Analytical Design of a PID Controller

Error Constant Defines the Gain used

```
%The plant is Type 1, use some error constant
K = 2;
close all, figure(7), hold on, bode(G), margin(G*K)
```



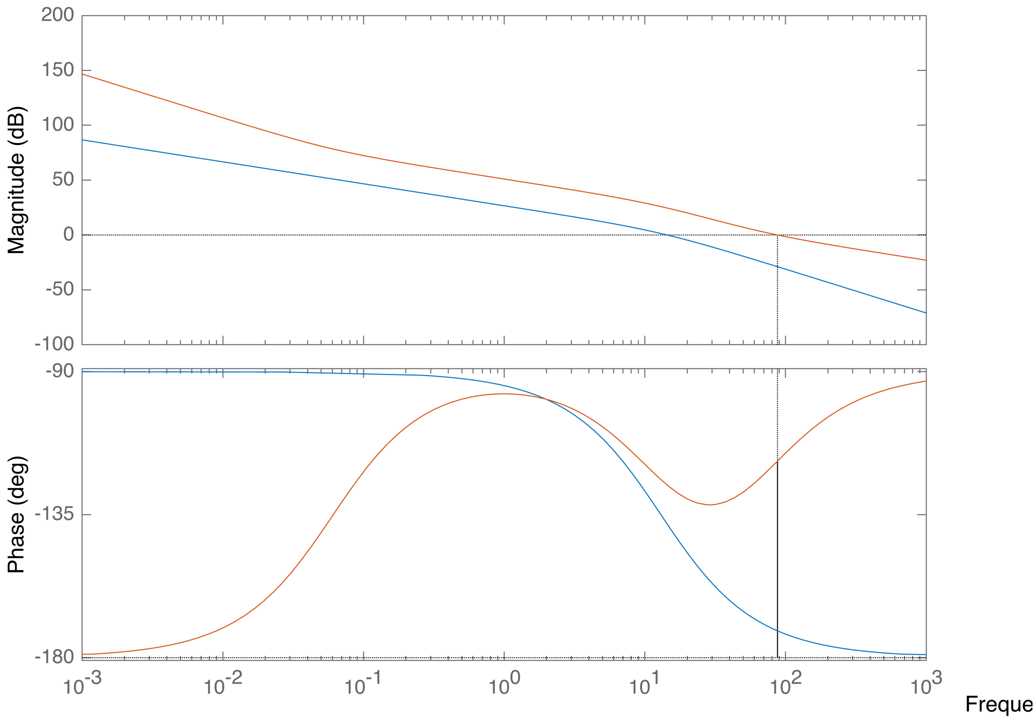
The current Bandwidth can be increased

```
%increase
wgc=22*4; % Enter desired gain crossover frequency in rad/s
```

```
PM=60; % Enter desired phase margin in degrees
Ki=1; % Integral gain based on error constant for PID
%%%
sn=wgc*j;
Gn = subs(sG,{'s'},[sn]); % Enter plant here
theta=-180+PM-angle(Gn)*180/pi;% This is the angle in degrees required from controller
    theta = double(theta);
theta=theta*pi/180; %Angle converted to radians for Matlab commands
Kp=cos(theta)/ abs(Gn);
    Kp=double(Kp); % Kp for PID - Change with other formula for different controller
Kd=Ki/wgc/wgc+sin(theta)/wgc/abs(Gn);
    Kd=double(Kd); % Kd for PID - Change with other formula for different controller
s=tf('s'); % Define s as Laplace transform variable for transfer functions
Cpid=(Kd*s*s+Kp*s+Ki)/s
```

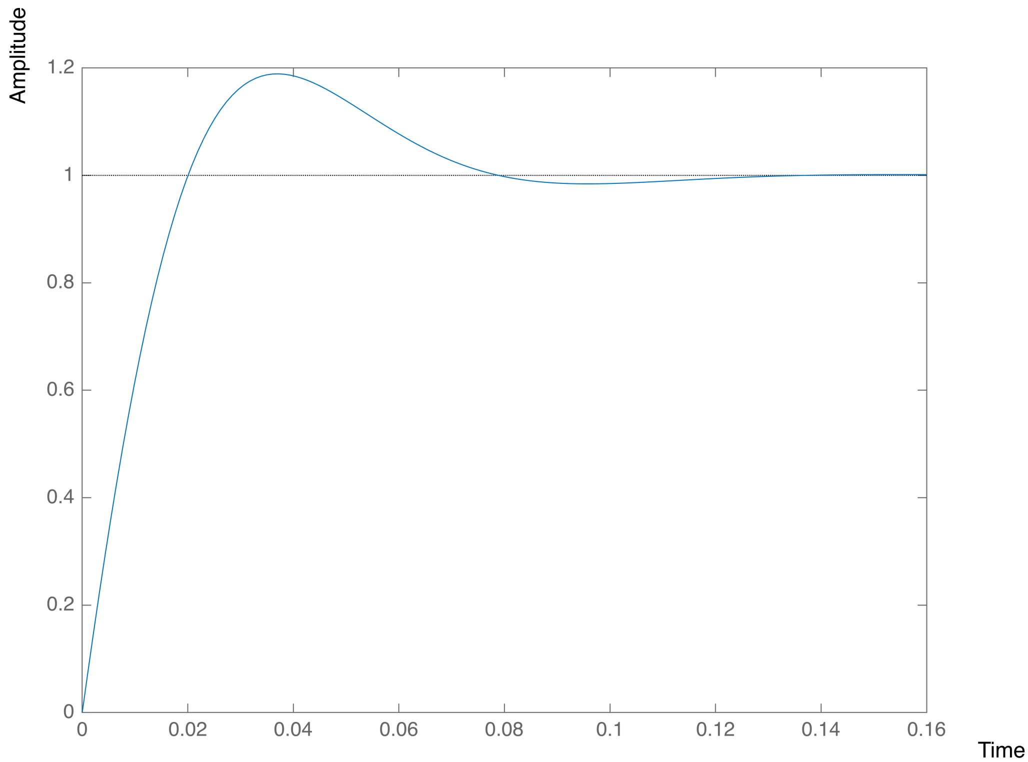
```
Cpid =

  0.2545 s^2 + 16.55 s + 1
  ------------------------
             s

Continuous-time transfer function.
```

```
close all, figure(8), hold on, bode(G), margin(G*Cpid)
```



PID Step Response

```
close all, figure(9), hold on, step(feedback(Cpid*G,1))
```



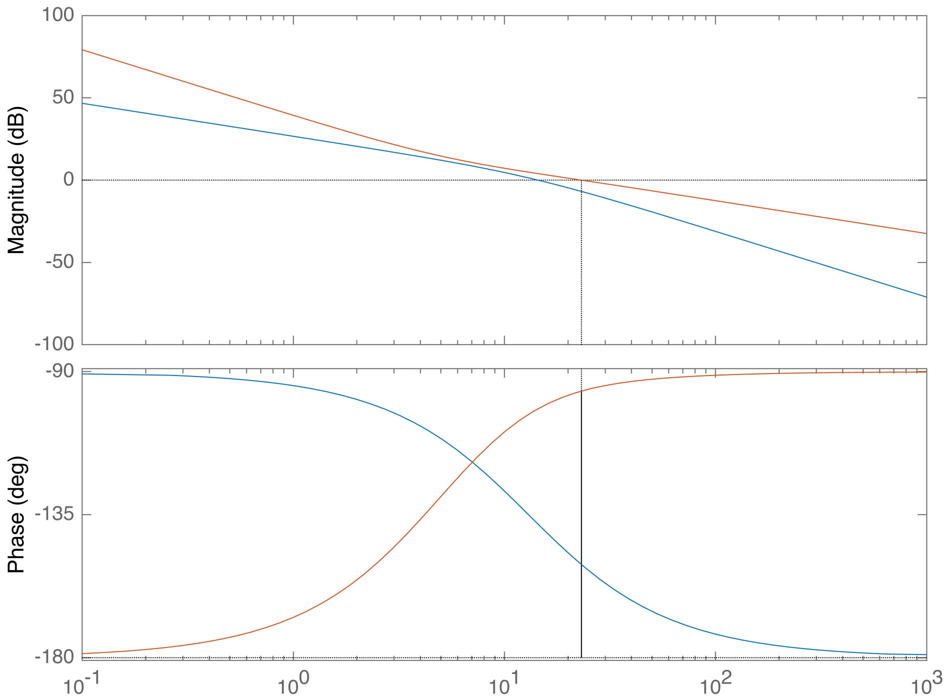
Automatic Tuned PID from Matlab

The Generated Controller

```
Cpidtune = pidtune(G,'PID');
```

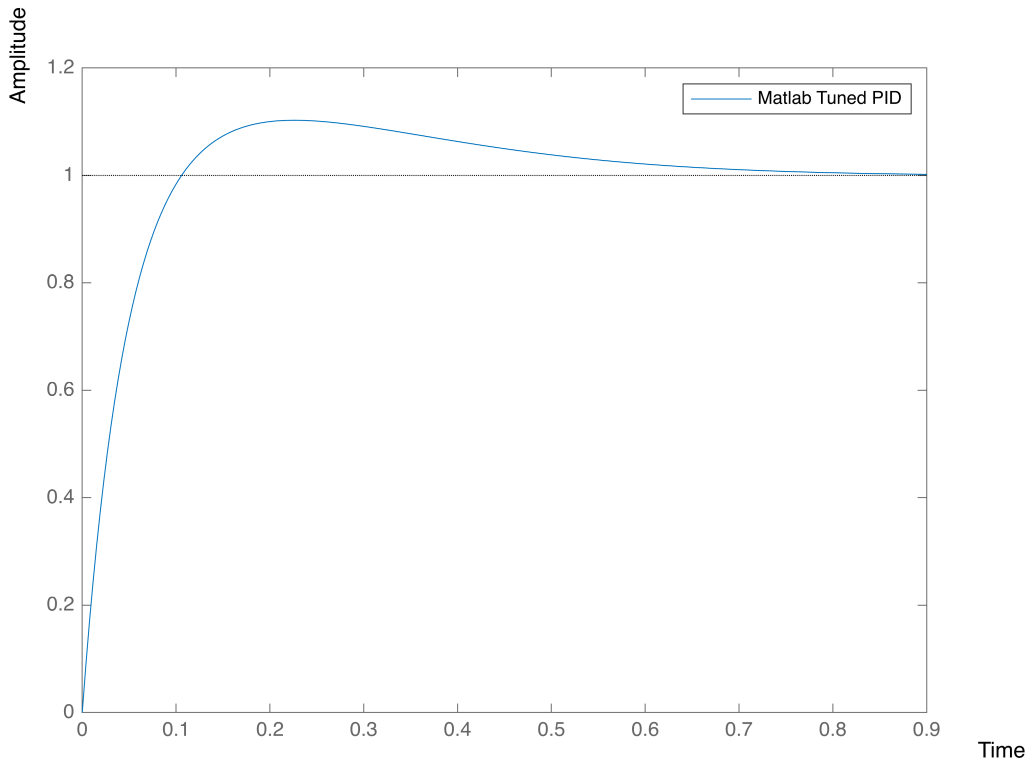
Matlab Tuned PID Bode plot

```
close all, figure(10), hold on, bode(G), margin(G*Cpidtune)
```



Matlab Tuned PID Step Response

```
close all, figure(11), hold on, step(feedback(Cpidtune*G,1)), legend('Matlab Tuned PID
```



Change the Time Constants: at 10% 20% 30% increases and evaluate the change in performance of each design

The Current Plant, G, is a second order plant. Second order plants have the general form

```
syms a Wn zeta s
G_2nd_order = a*Wn^2 / ( s^2 + 2*zeta*Wn*s + Wn^2)
```

```
G_2nd_order =
```

$$\frac{\mathrm{Wn}^2\, a}{\mathrm{Wn}^2 + 2\,\zeta\,\mathrm{Wn}\, s + s^2}$$

Where Tau, the time constant, is equal to 1/(zeta*wn)

```
current_zeta_x_Wn = 1/2; current_tau = 1/current_zeta_x_Wn;
```

The second order plant in terms of tau is

```
s=tf('s');
G_2nd_order_tau = 28 / (0.1*s^2 + 2*(1/current_tau)*s)
```

```
G_2nd_order_tau =

      28
  -----------
  0.1 s^2 + s
```

```
Continuous-time transfer function.
```

Creating new Plants with 10% 20% and 30% increases in the time constant, tau

```
G_tau_10_percent_increase = 28 / (0.1*s^2 + 2*(1/current_tau*1.1)*s);
G_tau_20_percent_increase = 28 / (0.1*s^2 + 2*(1/current_tau*1.2)*s);
G_tau_30_percent_increase = 28 / (0.1*s^2 + 2*(1/current_tau*1.3)*s);
%Putting the plants into an array variable
Gtau(1) = G;
Gtau(2) = G_tau_10_percent_increase;
Gtau(3) = G_tau_20_percent_increase;
Gtau(4) = G_tau_30_percent_increase;
```

The new plants to test are:

```
Gtau
```

```
Gtau =

  From input 1 to output:
         28
    ---------------
    0.1 s^2 + 1.3 s

  From input 2 to output:
         28
    ---------------
    0.1 s^2 + 1.1 s

  From input 3 to output:
         28
    ---------------
    0.1 s^2 + 1.2 s

  From input 4 to output:
         28
    ---------------
    0.1 s^2 + 1.3 s

Continuous-time transfer function.
```

The following figures were created by running the above script 4 times with the respective tau value, saving the values using the following system, and commenting out the beginning variable clearing.

C_lag_tau(i) = Clag

C_lead_tau(i) = Clead

C_pid_tau(i) = Cpid

Gtau(i)

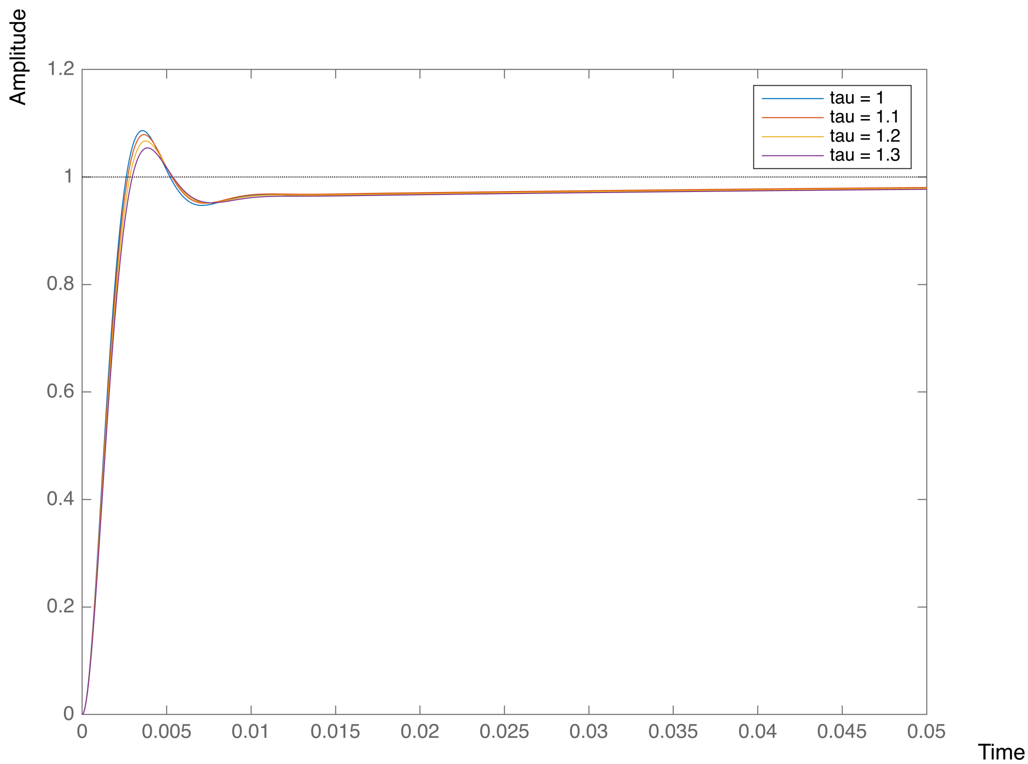
This section can only be run with the downloaded plants and controllers saved from the previous step

Loading the plants and controllers

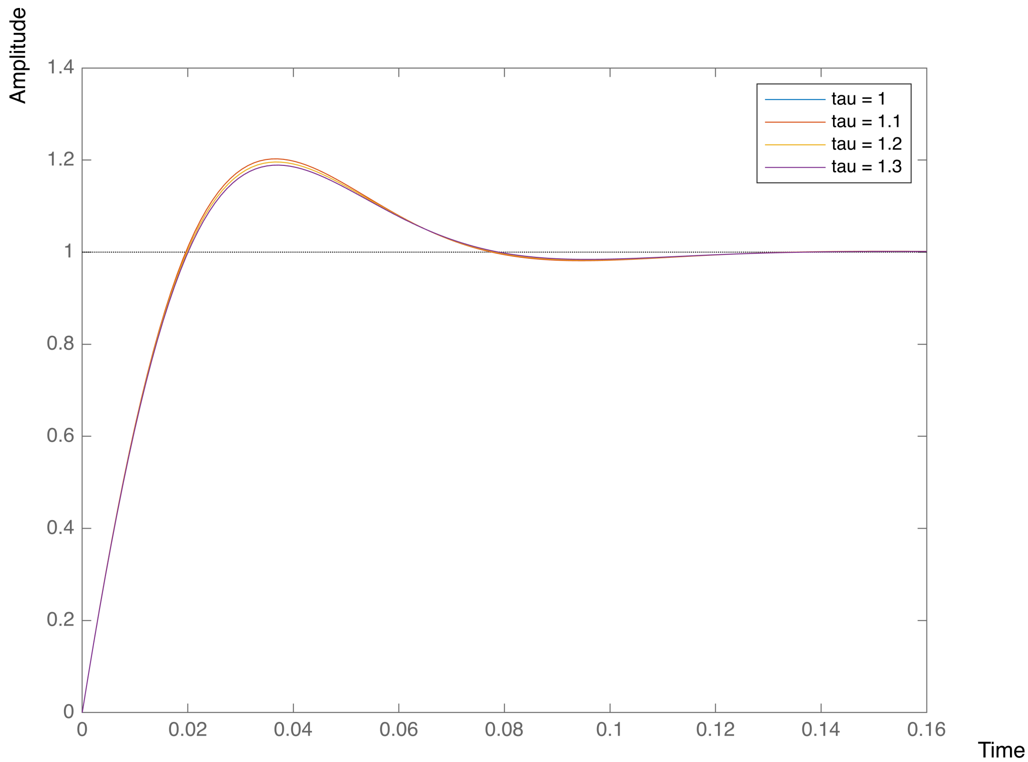
```
load('C_pid_tau.mat'); load('C_lead_tau.mat'); load('C_lag_tau.mat'); load('Gtau.mat')
```

Creating the feedback systems

```
for i = 1:4
    LagLeadsys(i) = feedback( C_lag_tau(i)*C_lead_tau(i)*Gtau(i) ,1 );
    PIDsys(i) = feedback( C_pid_tau(i)*Gtau(i) , 1 );
end
figure(), step(LagLeadsys(1),LagLeadsys(2),LagLeadsys(3),LagLeadsys(4))
title('Lag Lead Step Responses'), legend('tau = 1','tau = 1.1','tau = 1.2','tau = 1.3'
```




```
figure(), step(PIDsys(1),PIDsys(2),PIDsys(3),PIDsys(4))
title('PID Step Responses'), legend('tau = 1','tau = 1.1','tau = 1.2','tau = 1.3')
```



It seems that the choice for the time constant has a somewhat small impact on the step response for this plant, especially for the PID controller